

# ECE368: Probabilistic Reasoning

## Lab 2: Bayesian Linear Regression

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**You should hand in:** 1) A scanned .pdf version of this sheet with your answers (file size should be under 2 MB); 2) four figures for Question 2 and three figures for Question 4 in the .pdf format; and 3) one Python file `regression.py` that contains your code. All these files should be uploaded to Quercus.

1. Express the posterior distribution  $p(\mathbf{a}|x_1, z_1, \dots, x_N, z_N)$  using  $\sigma^2, \beta, x_1, z_1, x_2, z_2, \dots, x_N, z_N$ . (1 pt)

$$\Sigma = \sigma^2 \left( \frac{\sigma^2}{\beta} \mathbf{I} + \mathbf{X}^T \mathbf{X} \right)^{-1} \quad \text{where} \quad \mathbf{X} = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_N \end{bmatrix} \in \mathbb{R}^{N \times 2} \quad \mathbf{z} = \begin{bmatrix} z_1 \\ \vdots \\ z_N \end{bmatrix} \in \mathbb{R}^N$$

$$\mu = \frac{1}{\sigma^2} \Sigma \mathbf{X}^T \mathbf{z}$$

$$p(\mathbf{a}|x_1, z_1, \dots, x_N, z_N) \propto \mathcal{N}(\mu, \Sigma)$$

2. Let  $\sigma^2 = 0.1$  and  $\beta = 1$ . Draw four contour plots corresponding to the distributions  $p(\mathbf{a})$ ,  $p(\mathbf{a}|x_1, z_1)$ ,  $p(\mathbf{a}|x_1, z_1, \dots, x_5, z_5)$ , and  $p(\mathbf{a}|x_1, z_1, \dots, x_{100}, z_{100})$ . In all contour plots, the x-axis represents  $a_0$ , and the y-axis represents  $a_1$ . Please save the figures with names **prior.pdf**, **posterior1.pdf**, **posterior5.pdf**, **posterior100.pdf**, respectively. (1.5 pt)
3. Suppose that there is a new input  $x$ , for which we want to predict the corresponding target value  $z$ . Write down the distribution of the prediction  $z$ , i.e.  $p(z|x, x_1, z_1, \dots, x_N, z_N)$ . (1 pt)

$$\text{Let } \tilde{\mathbf{x}} = \begin{bmatrix} 1 \\ x \end{bmatrix}$$

$$p(z|x, x_1, z_1, \dots, x_N, z_N) \propto \mathcal{N}(\mu^T \tilde{\mathbf{x}}, \tilde{\mathbf{x}}^T \Sigma \tilde{\mathbf{x}} + \sigma^2 \mathbf{I})$$

4. Let  $\sigma^2 = 0.1$  and  $\beta = 1$ . Given a set of new inputs  $\{-4, -3.8, \dots, 3.8, 4\}$ , plot three figures, whose x-axis is the input and y-axis is the prediction, corresponding to three cases:
- (a) The predictions are based on one training sample, i.e., based on  $p(z|x, x_1, z_1)$ .
  - (b) The predictions are based on 5 training samples, i.e., based on  $p(z|x, x_1, z_1, \dots, x_5, z_5)$ .
  - (c) The predictions are based on 100 training samples, i.e., based on  $p(z|x, x_1, z_1, \dots, x_{100}, z_{100})$ .

The range of each figure is set as  $[-4, 4] \times [-4, 4]$ . Each figure should contain the following three components: 1) the new inputs and the corresponding predicted targets; 2) a vertical interval at each predicted target, indicating the range within one standard deviation; 3) the training sample(s) that are used for the prediction. Use `plt.errorbar` for 1) and 2); use `plt.scatter` for 3). Please save the figures with names **predict1.pdf**, **predict5.pdf**, **predict100.pdf**, respectively. (1.5 pt)