

Seminar Report

Applying Semi-Supervised Locally Linear Embedding

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Abstract

Storyline

- Goal: present SS-LLE as a local, graph-based manifold learning method incorporating prior knowledge
- Step 0: define basic mathematical concepts required to understand argumentation (plus notation)
- Step 1: introduce idea of **isometry** (most basic: MDS)
- Step 2: introduce idea of **graph-based** models
 - Achieve non-linearity
 - Common structure: build graph \rightarrow derive matrix as quadratic form over graph function \rightarrow derive embedding from eigenvalue problem
 - Most basic: ISOMAP (global, dense, convex)
- Step 3: introduce idea of **locality**
 - Relax global to local isometry
 - Find sparse rather than dense matrices
 - **Laplacian eigenmaps** as concept in which the others can be generalized
 - Define weighting scheme for neighborhood
 - Use Laplacian to derive matrix
 - Solve sparse eigenvalue problem
- Step 4: introduce **local linearity**
 - **LLE**
 - Obtain weights via linear reconstructions
 - Can be shown to approximate graph Laplacian (Belkin & Niyogi (2006))
 - **Hessian LLE**
 - Replace Laplacian by Hessian
- Step 5: introduce **prior knowledge**
 - **SS-LLE**
 - Improve results by pre-specifying some manifold coordinates

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1 Introduction

- Why is dimensionality reduction desirable? Not only because it's easier to handle and visualize lower-dimensional data but because data-generating process is often truly of much lower dimension
- Our goal is to find the mapping from latent feature space embedded in the m -dimensional Euclidean space we observe to the d -dimensional space the embedding is locally homeomorphic to (unrolling the Swiss roll)
- This mapping can be constructed linearly or non-linearly (slapping the roll flat vs unrolling it), thereby defining the complexity of the manifolds we are able to learn
- Brief intuition to manifold learning with simple example (e.g., rotated letters A)
- Different methods out there (linear, non-linear, ...)

2 Mathematical Framework

2.1 “Mathematical Objects” (find proper title)

- Topologies
- Geodesics
- Manifolds
- Tangent spaces

2.2 Spectral Decomposition

- Eigenvalues/eigenvectors
- Spectral decomposition

3 Local Graph-Based Manifold Learning

3.1 Concept of Isometry

- Notion of distance
- Preserving distances in manifold learning
- MDS (very brief)

3.2 Graph-Based Models

3.2.1 Neighborhoods

- k -/ ϵ -neighborhoods and neighborhood graphs
- Linear reconstruction and reconstruction error

3.2.2 Basics of Spectral Graph Theory

- Degree and adjacency matrices
- Laplacian operators

3.2.3 General Structure of Graph-Based Models

- Neighborhood graph
- Weight matrix
- Eigenwert problem

3.2.4 ISOMAP

- (One of the) earliest, simplest variant(s)
- MDS with geodesics

3.3 Laplacian Eigenmaps

- Notion of locality
- Laplacian eigenmaps

3.4 Locally Linear Embedding (LLE)

- Notion of local linearity
- Approximation of graph Laplacian

3.5 Hessian Locally Linear Embedding (HLLE)

- Hessian instead of Laplacian (eigenmaps)
- Hessian instead of LS fit (LLE)

4 Semi-Supervised Locally Linear Embedding (SS-LLE)

4.1 Employment of Prior Information

- Why use labels in the first place?
- How will that help?
- How do we even find prior points?
- Exact vs inexact knowledge

4.2 SS-LLE Algorithm

- What is different wrt standard LLE?

4.3 Strengths and Drawbacks of SS-LLE

5 Experiment Results

5.1 Data

5.2 Experimental Design

- Implementation details
- Hyperparameters
- Evaluation criteria

5.3 Results and Discussion

6 Conclusion

Lorem ipsum

A Appendix

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B Electronic Appendix

Data, code and figures are provided in electronic form.

Declaration of Authorship

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