9.

(1) 母体∈常態分配　σ点估計值

$$S=\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}=\sqrt{\frac{\Sigma x_i^2-n\bar{x}^2}{n-1}}=\sqrt{\frac{1284-6\times 14.33^2}{5}}$$

$$=\sqrt{10.38}=3.22$$　σ点估計：3.22 //

(2) $1-\alpha=0.9$　$\frac{\alpha}{2}=0.05$　$n-1=5$

$$\chi^2_{1-\frac{\alpha}{2}}(n-1)=\chi^2_{0.95}(5)=1.15$$

∴ σ之90%信賴區間為：

$$\left[\sqrt{\frac{5\times 10.38}{\chi^2_{0.05}(5)}},\sqrt{\frac{5\times 10.38}{\chi^2_{0.95}(5)}}\right]=\left[\sqrt{\frac{51.9}{11.07}},\sqrt{\frac{51.9}{1.15}}\right]$$

$$=(2.17,6.72)$$ //

20.

(1) $n_1=9$　$\bar{x}=7.67$　$S_1=9.27$　$n_2=9$

$\bar{y}=6.78$　$S_2=21.15$　∴ $\sigma_1^2\neq\sigma_2^2$

$$V=\frac{\left(\frac{9.27^2}{9}+\frac{21.15^2}{9}\right)^2}{\frac{\left(\frac{9.27^2}{9}\right)^2}{8}+\frac{(21.15)^2}{8}}=10.96\fallingdotseq 11$$

∴ $\mu_1-\mu_2$之95%信賴區間：

$$(\bar{x}-\bar{y})\pm t_{\frac{\alpha}{2}}(V)\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}=$$

$$(7.67-6.78)\pm t_{0.025}(11)\sqrt{\frac{9.27^2}{9}+\frac{21.15^2}{9}}$$

$$=0.89\pm 2.201\times 7.70=0.89\pm 16.95$$

即 $(-16.06,17.84)$

(2) $1-\alpha=0.9$ $\chi^2_{\frac{\alpha}{2}}(n_1-1)=\chi^2_{0.05}(8)=15.51$

$\chi^2_{1-\frac{\alpha}{2}}(n_1-1)=\chi^2_{0.95}(8)=2.73$

∴ σ_1^2 の 90% 信頼区間：

$$\left(\sqrt{\frac{8\times 9.27^2}{\chi^2_{0.05}(8)}},\sqrt{\frac{8\times 9.25^2}{\chi^2_{0.95}(8)}}\right)=\left(\sqrt{\frac{687.46}{15.51}},\sqrt{\frac{687.46}{2.73}}\right)$$

$=(6.66, 15.87)$

(3) $1-\alpha=0.90$ $F_{\frac{\alpha}{2}}(n_1-1, n_2-1)=$

$F_{0.05}(8,8)=3.44$

$$F_{1-\frac{\alpha}{2}}(n_1-1, n_2-1)=F_{0.95}(8,8)=\frac{1}{F_{0.05}(8,8)}=0.29$$

∴ $\frac{\sigma_1^2}{\sigma_2^2}$ の 90% 信頼区間為：

$$\left[\frac{S_1^2}{S_2^2}\times\frac{1}{F_{\frac{\alpha}{2}}(n_1-1, n_2-1)}, \frac{S_1^2}{S_2^2}\times\frac{1}{F_{1-\frac{\alpha}{2}}(n_1-1, n_2-1)}\right]=$$

$$\left[\frac{9.27^2}{21.15^2}\times\frac{1}{3.44}, \frac{9.27^2}{21.15^2}\times\frac{1}{0.29}\right]=(0.06, 0.66)$$