UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

School of Computer Science & Applied Mathematics

Artificial Intelligence (COMS4033A/7044A)
Weather Prediction

1 Introduction

In this short lab, we'll be implementing some standard algorithms on Hidden Markov Models for the purpose of weather prediction!

Submission 1: Today's Weather (60 marks)

One part of being a meteorologist is the need to predict the weather. However, as we well know, it often doesn't turn out as expected! We have decided to write our own weather predicting program. Instead of using atmospheric conditions to predict the weather, we will instead use our neighbour, who has a knack for knowing when it's going to rain.

Specifically, we are trying to model two weather conditions: `rainy` and `not rainy`. Through data capture over several days, you have calculated the following statistics:

- If it was raining yesterday, then there is a 70% chance that it will also rain today.
- If it was not raining yesterday, then there is a 30% chance that it will rain today.
- On days in which it rained, your neighbour left home carrying an umbrella 90% of the time.
- On days in which it did not rain, your neighbour left home carrying an umbrella 20% of the time.

We can model the system as an HMM with a single random Boolean state variable that specifies whether it will rain today, and a single observation variable that specifies whether our neighbour leaves home with an umbrella. Assuming a uniform prior and given a sequence of observations $o_1, o_2, \ldots, o_n$, write a Python program to compute $P(s_n = Rain)$ where s_n is the hidden variable at timestep n. Be sure to normalise your probabilities at each step by dividing by their sum to ensure they sum to 1.

Input

Input is a single line containing 0s and 1s separated by spaces. The number of integers varies between different test cases, but each integer i represents whether an umbrella was observed at timestep i. For example, 10 means that at timestep 1, the umbrella was observed, and at timestep 2, no umbrella was observed.

Output

For each timestep (including timestep 0), output the probability that it is raining. Round the probability that it is raining to 3 decimal places.

Example Input-Output Pairs

Sample Input #1

```
1 1
```

Sample Output #1

```
Timestep 0: 0.5
Timestep 1: 0.818
Timestep 2: 0.883
```

Sample Input #2

```
0
```

Sample Output #2

```
Timestep 0: 0.5
Timestep 1: 0.111
```

Sample Input #3

```
1 1 0 1 0
```

Sample Output #3

```
Timestep 0: 0.5
Timestep 1: 0.818
Timestep 2: 0.883
Timestep 3: 0.191
Timestep 4: 0.731
Timestep 5: 0.154
```

Submission 2: Tomorrow's Weather (30 marks)

Now that we can estimate the weather for today, let's use our model to estimate the weather into the future! We will apply the same kind of forward algorithm to predict $s_{k+1}, s_{k+2}, \ldots$ given only evidence up to timestep k. In other words, to predict the future, we use the same algorithm but ignore the sensor model (since we do not have observations in the future just yet). The algorithm's update then becomes:

$$F(k,t) = \sum_i P(s_k|s_i)F(i,t-1)$$

Input

Input is a single line containing 0s and 1s separated by spaces. The number of integers varies between different test cases, but each integer i represents whether an umbrella was observed at timestep i. For example, 10 means that at timestep 1, the umbrella was observed, and at timestep 2, no umbrella was observed.

Output

Your output should be exactly the same as in the previous submission, but should also include an additional two timesteps into the future.

Example Input-Output Pairs

Sample Input #1

```
1 1
```

Sample Output #1

```
Timestep 0: 0.5
Timestep 1: 0.818
Timestep 2: 0.883
Timestep 3: 0.653
Timestep 4: 0.561
```

Sample Input #2

```
0
```

Sample Output #2

```
Timestep 0: 0.5
Timestep 1: 0.111
Timestep 2: 0.344
Timestep 3: 0.438
```

Sample Input #3

```
1 1 0 1 0
```

Sample Output #3

```
Timestep 0: 0.5
Timestep 1: 0.818
Timestep 2: 0.883
Timestep 3: 0.191
Timestep 4: 0.731
Timestep 5: 0.154
Timestep 6: 0.361
Timestep 7: 0.445
```

Submission 3: Some Questions (10 marks)

Use your previous submission and modify it in whatever way you see fit to answer the following questions. Write down your answers by completing the relevant Moodle quiz submission.

1. What happens when we try make predictions many timesteps into the future without evidence?
2. How are these far-off predictions affected by the prior distribution? What if you assign a prior probability of 1 to rain?
3. What is the effect on these future predictions if the transition probabilities are changed?