# Assignment 2 STAT 315-463: Multivariable Statistical Methods and Applications

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**Due date: Friday 24 March 2023**

- Your assignment needs to show the R code you used, and your well discussed answers to the questions.
- Submit your assignments on Learn.

## Background

In the dataset, `USJudgeRatings.csv`, you are presented with ratings of State Judges on the Superior Court on 12 variables provided by 43 Lawyers in 1977.

| | | | | | |
|---|---|---|---|---|---|
| CONT | Number of contacts of lawyer with judge | INTG | Judicial integrity | DMNR | Demeanour |
| DILG | Diligence | CFMG | Case flow managing | DECI | Prompt decisions |
| PREP | Preparation for trial | FAMI | Familiarity with law | ORAL | Sound oral rulings |
| WRIT | Sound written rulings | PHYS | Physical ability | RTEN | Worthy of retention |

```
# Read in the data
dataset <- read.csv("USJudgeRatings.csv")
```

## Principal Component Analysis of the Rating Data.

Perform a PCA on the standardised ratings. Note, you will need to standardise the ratings yourself. Then answer the following questions.

```
# Standardise the ratings
library(dplyr , quietly = T)
```

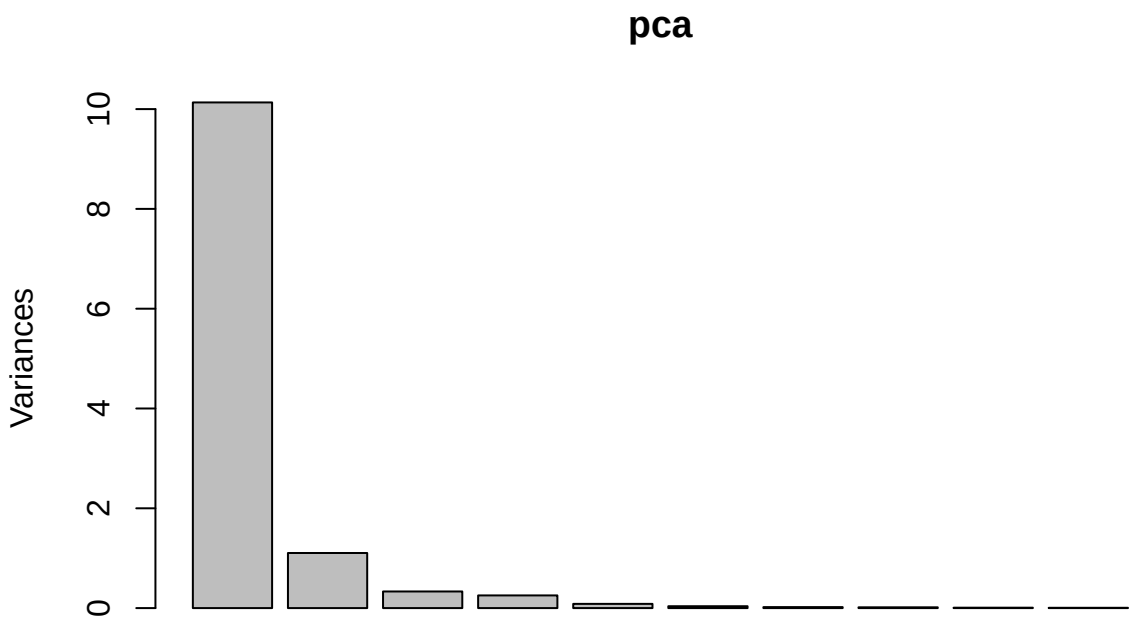
```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
dataset_std <- dataset %>% mutate(across(where(is.numeric),scale))
summary(dataset_std)
```

```
##     Lawyer                CONT.V1             INTG.V1
##  Length:43          Min.   :-1.846373   Min.   :-2.7539372
##  Class :character   1st Qu.:-0.624109   1st Qu.:-0.6114828
##  Mode  :character   Median :-0.145831   Median : 0.1026687
##                     Mean   : 0.000000   Mean   : 0.0000000
##                     3rd Qu.: 0.491872   3rd Qu.: 0.6869745
##                     Max.   : 3.361535   Max.   : 1.5309717
##        DMNR.V1              DILG.V1              CFMG.V1
##  Min.   :-2.8121570   Min.   :-2.8782657   Min.   :-2.4172132
##  1st Qu.:-0.5388442   1st Qu.:-0.6027579   1st Qu.:-0.5569865
##  Median : 0.1606366   Median : 0.1187446   Median : 0.1405985
##  Mean   : 0.0000000   Mean   : 0.0000000   Mean   : 0.0000000
##  3rd Qu.: 0.7289648   3rd Qu.: 0.8402471   3rd Qu.: 0.6637873
##  Max.   : 1.2972929   Max.   : 1.4507492   Max.   : 1.4195044
##        DECI.V1              PREP.V1              FAMI.V1
##  Min.   :-2.3228698   Min.   :-2.7979078   Min.   :-2.5167602
##  1st Qu.:-0.5792693   1st Qu.:-0.5951957   1st Qu.:-0.5673125
##  Median : 0.1679881   Median : 0.2439327   Median : 0.1176285
##  Mean   : 0.0000000   Mean   : 0.0000000   Mean   : 0.0000000
##  3rd Qu.: 0.7284311   3rd Qu.: 0.7683879   3rd Qu.: 0.8025696
##  Max.   : 1.5379600   Max.   : 1.7124074   Max.   : 1.6982618
##        ORAL.V1              WRIT.V1              PHYS.V1
##  Min.   :-2.5672387   Min.   :-2.5841599   Min.   :-3.442921
##  1st Qu.:-0.4386179   1st Qu.:-0.5032821   1st Qu.:-0.249989
##  Median : 0.2049186   Median : 0.2250252   Median : 0.175735
##  Mean   : 0.0000000   Mean   : 0.0000000   Mean   : 0.000000
##  3rd Qu.: 0.6999467   3rd Qu.: 0.6932227   3rd Qu.: 0.601459
##  Max.   : 1.5909973   Max.   : 1.6816397   Max.   : 1.240046
##        RTEN.V1
##  Min.   :-2.5453217
##  1st Qu.:-0.4108424
##  Median : 0.1795455
##  Mean   : 0.0000000
##  3rd Qu.: 0.5882756
##  Max.   : 1.4511502
```

1. How many principal components do you believe should be retained. Justify your answer by looking at the variation in the data explained by each component.

```
# Perform Pricipal Component Analysis on the standardised dataset
pca <- prcomp(dataset_std[,-1])
# Use the scree plot
screeplot(pca)
```




From the scree plot, we can clearly see that the first two principal components can capture most of the variation from the data. Third and fourth components still capture a few variation but not as much. From the fifth onward, the values are very close to 0. Therefore, I think no more than the first four principal components should be retained.

```
# Get the percentage of variance explained by each component
summary(pca)
```

```
## Importance of components:
##                           PC1     PC2     PC3     PC4     PC5     PC6     PC7
## Standard deviation     3.1833 1.05078 0.57698 0.50383 0.29061 0.19310 0.14030
## Proportion of Variance 0.8445 0.09201 0.02774 0.02115 0.00704 0.00311 0.00164
## Cumulative Proportion  0.8445 0.93647 0.96421 0.98537 0.99240 0.99551 0.99715
##                           PC8     PC9    PC10    PC11    PC12
## Standard deviation     0.12416 0.08851 0.07491 0.05708 0.04539
## Proportion of Variance 0.00128 0.00065 0.00047 0.00027 0.00017
## Cumulative Proportion  0.99844 0.99909 0.99956 0.99983 1.00000
```

It is shown that the first principal component (PC1) explains 84.4% of variation, with PC2 accounts for 9.2%, PC3 for 2.8%, and PC4 for 2.1%. The first 3 PCs explain 96.4% of the variation and the first 4 PCs explain 98.5% of the variation. So from this result, I think even just take the first 3 PCs should be enough to cover a good percentage of variation.

2. In your own words, describe what you believe the first principal component is measuring.

```
pca$rotation[, 1:2]
```

```
##               PC1          PC2
## CONT  0.003075143 -0.932890644
## INTG -0.288550775  0.182040993
## DMNR -0.286884206  0.197565743
## DILG -0.304354091 -0.036304667
## CFMG -0.302572733 -0.168393523
## DECI -0.301891969 -0.127877299
## PREP -0.309406446 -0.032230248
## FAMI -0.306679527  0.001315183
## ORAL -0.312708348  0.003625720
## WRIT -0.311061231  0.031378756
## PHYS -0.280723624 -0.089037698
## RTEN -0.309790218  0.039381306
```

Rotations, or loadings, specify the weight that each variable contribute to the component. In the result above, we can see that all the variables, except CONT, have negative values. Among those, the component has large negative associations with ORAL, WRIT

3. What do you think the second principal component represents?
4. You are told *Judicial Integrity* and *Demeanour* are particularly important traits, and should be given 5 times the weight of the other variables. Re-run the Principal Component Analysis such that Integrity and Demeanour is given 5 times the weight of all other variables. What impact does this have?

```
# Apply weights to INTG and DMNR
dataset_weighed <- dataset_std
dataset_weighed$INTG <- dataset_std$INTG * 5
dataset_weighed$DMNR <- dataset_std$DMNR * 5

# Run the principal component analysis on the weighed dataset
pca1 <- prcomp(dataset_weighed[,-1])
summary(pca1)
```

```
## Importance of components:
##                           PC1     PC2     PC3     PC4     PC5     PC6     PC7
## Standard deviation     7.4897 1.36069 0.98113 0.88014 0.41322 0.28404 0.16642
## Proportion of Variance 0.9349 0.03086 0.01604 0.01291 0.00285 0.00134 0.00046
## Cumulative Proportion  0.9349 0.96579 0.98183 0.99474 0.99759 0.99893 0.99940
##                           PC8     PC9    PC10    PC11    PC12
## Standard deviation     0.12790 0.09368 0.07540 0.05781 0.04546
## Proportion of Variance 0.00027 0.00015 0.00009 0.00006 0.00003
## Cumulative Proportion  0.99967 0.99982 0.99991 0.99997 1.00000
```

```
pca1$rotation[, 1:3]
```

```
##              PC1        PC2         PC3
## CONT  0.01683514  0.4550909  0.08123501
## INTG -0.66064656 -0.1535575  0.69375788
## DMNR -0.66007098 -0.2704810 -0.63435275
```

```
## DILG -0.11860621  0.2787158  0.08527462
## CFMG -0.11383471  0.3580957 -0.06369157
## DECI -0.11263597  0.3564752 -0.07551669
## PREP -0.12040199  0.2809923  0.03303414
## FAMI -0.11888533  0.2746566  0.04884273
## ORAL -0.12545426  0.2256858 -0.03326969
## WRIT -0.12442206  0.2228469  0.01365766
## PHYS -0.10681562  0.2886185 -0.28780330
## RTEN -0.12882861  0.1577163 -0.07117101
```

## Factor Analysis for the Rating Data

Perform Factor Analysis on the standardised Ratings Data.

1. What happens when you try to fit a 3 and a 4 factor solution with no rotation. Hint: For the three factor solution, you may need to add `control=list(nstart=100)` as an additional argument in the `factanal` function.

```
factor3 <- factanal(dataset_std[,-1], factors = 3, rotation = "none", control=list(nstart=100));factor3
```

```
##
## Call:
## factanal(x = dataset_std[, -1], factors = 3, rotation = "none",     control = list(nstart = 100))
##
## Uniquenesses:
##  CONT  INTG  DMNR  DILG  CFMG  DECI  PREP  FAMI  ORAL  WRIT  PHYS  RTEN
## 0.709 0.052 0.020 0.050 0.009 0.027 0.011 0.005 0.006 0.005 0.189 0.016
##
## Loadings:
##      Factor1 Factor2 Factor3
## CONT          0.418   0.342
## INTG  0.905  -0.359
## DMNR  0.894  -0.409   0.119
## DILG  0.969
## CFMG  0.961   0.171   0.195
## DECI  0.961   0.175   0.139
## PREP  0.991
## FAMI  0.988          -0.126
## ORAL  0.996
## WRIT  0.996
## PHYS  0.877           0.203
## RTEN  0.975  -0.155
##
##                Factor1 Factor2 Factor3
## SS loadings     10.067   0.574   0.262
## Proportion Var   0.839   0.048   0.022
## Cumulative Var   0.839   0.887   0.909
##
## Test of the hypothesis that 3 factors are sufficient.
## The chi square statistic is 112.37 on 33 degrees of freedom.
## The p-value is 1.38e-10
```

```
factor4 <- factanal(dataset_std[,-1], factors = 4, rotation = "none");factor4
```

```
##
## Call:
## factanal(x = dataset_std[, -1], factors = 4, rotation = "none")
##
## Uniquenesses:
##  CONT  INTG  DMNR  DILG  CFMG  DECI  PREP  FAMI  ORAL  WRIT  PHYS  RTEN
## 0.749 0.013 0.032 0.023 0.014 0.025 0.008 0.005 0.005 0.005 0.047 0.006
##
## Loadings:
##      Factor1 Factor2 Factor3 Factor4
## CONT          0.316   0.343   0.183
## INTG  0.916  -0.356  -0.101
## DMNR  0.904  -0.385
## DILG  0.968                  0.177
## CFMG  0.956   0.160   0.177   0.123
## DECI  0.956   0.179   0.148
## PREP  0.989   0.102
## FAMI  0.985   0.112  -0.101
## ORAL  0.997
## WRIT  0.995
## PHYS  0.879           0.361  -0.221
## RTEN  0.981  -0.144   0.102
##
##                Factor1 Factor2 Factor3 Factor4
## SS loadings     10.091   0.487   0.338   0.154
## Proportion Var   0.841   0.041   0.028   0.013
## Cumulative Var   0.841   0.881   0.910   0.922
##
## Test of the hypothesis that 4 factors are sufficient.
## The chi square statistic is 49.78 on 24 degrees of freedom.
## The p-value is 0.00151
```

2. Which variables are grouped by the first two factors? (e.g. threshold $|\text{loading}| \geq 0.25$)

3. Compare the factor loadings for the first two factors to the first two principal components of the standardised data found in the previous section. Comment on any similarities and/or differences.

4. Comment on the observed variable specific variances (the uniquenesses). Do you believe all observed variables are explained by the factors discovered.

5. Re-fit the 3 factor solution with a varimax rotation. How does this change the interpretation of the factors? In this case, do you find the rotated or non-rotated solution easier to interpret. Explain why or why not?

```
factanal(dataset_std[,-1], factors = 3, rotation = "varimax", control=list(nstart=100))
```

```
##
## Call:
## factanal(x = dataset_std[, -1], factors = 3, rotation = "varimax",     control = list(nstart = 100))
##
## Uniquenesses:
```

```
##  CONT  INTG  DMNR  DILG  CFMG  DECI  PREP  FAMI  ORAL  WRIT  PHYS  RTEN
## 0.709 0.052 0.020 0.050 0.009 0.027 0.011 0.005 0.006 0.005 0.189 0.016
##
## Loadings:
##      Factor1 Factor2 Factor3
## CONT                  0.537
## INTG  0.743   0.567  -0.273
## DMNR  0.695   0.664  -0.238
## DILG  0.920   0.316
## CFMG  0.899   0.364   0.226
## DECI  0.912   0.323   0.190
## PREP  0.953   0.284
## FAMI  0.969   0.227
## ORAL  0.917   0.386
## WRIT  0.937   0.330
## PHYS  0.774   0.444   0.120
## RTEN  0.845   0.513
##
##                Factor1 Factor2 Factor3
## SS loadings      8.402   1.952   0.550
## Proportion Var   0.700   0.163   0.046
## Cumulative Var   0.700   0.863   0.909
##
## Test of the hypothesis that 3 factors are sufficient.
## The chi square statistic is 112.37 on 33 degrees of freedom.
## The p-value is 1.38e-10
```