hw_06

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Loans

1.

```
y <- c(16, 5, 10, 15, 13, 22)
y
```

```
## [1] 16  5 10 15 13 22
```

2.

```
X <- matrix(c(1, 4,
              1, 1,
              1, 2,
              1, 3,
              1, 3,
              1, 4), ncol = 2, nrow = 6, byrow = TRUE)
X
```

```
##      [,1] [,2]
## [1,]    1    4
## [2,]    1    1
## [3,]    1    2
## [4,]    1    3
## [5,]    1    3
## [6,]    1    4
```

3.

Beta-hat = $(XTX)^{-1}$ * (XTY)

```
betahat = solve(t(X) %*% X) %*% t(X) %*% y
betahat
```

```
##           [,1]
## [1,] 0.4390244
## [2,] 4.6097561
```

4.

s^2 (beta hat) = MSE $(XTX)^{-1}$

with help from ChatGPT: https://chat.openai.com/c/6e20bc4c-a72c-4ab5-a278-0c5ff4fe543e

```
y_pred <- X %*% betahat
residuals <- y - y_pred
df <- length(y) - ncol(X)
MSE <- sum(residuals^2)/df
MSE
```

```
## [1] 5.073171
```

5.

s^2 (beta hat) = MSE $(XTX)^{-1}$

with help from ChatGPT: https://chat.openai.com/c/6e20bc4c-a72c-4ab5-a278-0c5ff4fe543e

```
cov_matrix <- MSE * solve(t(X) %*% X)
variances <- diag(cov_matrix)
SE_beta <- sqrt(variances)
SE_beta
```

```
## [1] 2.6087301 0.8616352
```

6.

```
summary(lm(y ~ X[, 2]))
```

```
##
## Call:
## lm(formula = y ~ X[, 2])
##
## Residuals:
##        1        2        3        4        5        6
## -2.87805 -0.04878  0.34146  0.73171 -1.26829  3.12195
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.4390     2.6087   0.168  0.87452
## X[, 2]        4.6098     0.8616   5.350  0.00589 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.252 on 4 degrees of freedom
## Multiple R-squared:  0.8774, Adjusted R-squared:  0.8467
## F-statistic: 28.62 on 1 and 4 DF,  p-value: 0.005886
```

Other questions

1.

system of equations in matrix notation is: W = AY

where:

W = [W1 W2 W3], Y = [Y1 Y2 Y3}

(imagine these are vertical, I just can't figure out how to make it that way in R, sorry!)

coefficient matrix A: (again, imagine there are brackets around this)

1 -1 2

A = 0 0 -1

1 0 0

so the system of linear equations can be represented as (imagine brackets, sorry)

W1 0 -1 2 Y1

W2 = 0 1 1 Y2

W3 1 0 0 Y3

2.

AB11 = 1(2) + 3(1) + 5(3) = 2 + 3 + 15 = 20

AB12 = 1(-2) + 3(-1) + 5(-3) = -2 - 3 - 15 = -20

AB21 = 2(2) + 4(1) + 6(3) = 4 + 4 + 18 = 26

AB22 = 2(-2) + 4(-1) + 6(-3) = -4 -4 - 18 = -26

so the resulting matrix AB is:

20 -20

26 -26

Verifying with R:

```
A <- matrix(c(1, 3, 5, 2, 4, 6), nrow = 2, byrow = TRUE)
B <- matrix(c(2, -2, 1, -1, 3, -3), nrow = 3, byrow = TRUE)

AB <- A %*% B
AB
```

```
##      [,1] [,2]
## [1,]   20  -20
## [2,]   26  -26
```