

Quiz 7. Two Topics: (1) Splines, (2) Shrinkage Methods

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Attempt (circle one): BEFORE **AFTER**Splines

1. A B-spline is fit with the following basis functions

$$\begin{cases} b_1(X) = 0 & \text{for } -10 \leq X \leq r \\ b_2(X) = X^2 & \text{for } r < X \leq s \\ b_3(X) = X - 1 & \text{for } s < X \leq 10 \end{cases}$$

The corresponding coefficients are: $\beta_0 = 0$, $\beta_1 = 1$, $\beta_2 = 0.5$, $\beta_3 = 2$.(a) Find the knots r and s that will be suitable for the b-spline. (Hint: the neighboring splines are continuous at the knots.)

$$\hat{Y} = 0 + 1 \cdot b_1(x) + 0.5 \cdot b_2(x) + 2 \cdot b_3(x)$$

$$\text{If } -10 \leq X \leq r, \hat{Y} = 0 + 1 \cdot 0 + 0.5 \cdot 0 + 2 \cdot 0 = 0$$

$$\text{If } r < x \leq s, \hat{Y} = 0 + 1 \cdot 0 + 0.5 \cdot x^2 + 2 \cdot 0 = 0.5 \cdot x^2$$

$$\text{If } s < x \leq 10, \hat{Y} = 0 + 1 \cdot 0 + 0.5 \cdot 0 + 2(X-1) = 2(x-1)$$

$$\text{At } x = r, \text{ by A, } \hat{Y} = 0$$

$$\text{By B, } \hat{Y} = r^2$$

$$\text{Continuous} \rightarrow 0 = 0.5r^2, \text{ solve for } r \rightarrow r = 0$$

$$\text{At } x = s, \text{ by B, } \hat{Y} = 0.5s^2$$

$$\text{By C, } \hat{Y} = 2(s-1)$$

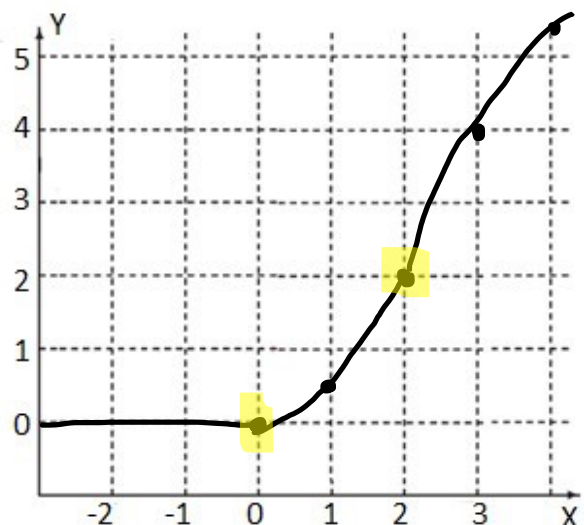
$$\text{Continuous} \rightarrow \text{the above two expressions are equal} \rightarrow \text{solve for } s$$

$$0.5s^2 = 2(s-1) = 2s-2 \rightarrow 0.5s^2 = 2s-2 \rightarrow 0.5s^2 - 2s + 2 = 0 \rightarrow s^2 - 4s + 4 = 0 \rightarrow (s-2)^2 = 0 \rightarrow s = 2$$

The suitable knots are $r = 0$ and $s = 2$.

(b) (Stat-627 Only). Sketch the B-spline.

Visually verify if the connections at the knots are smooth.



Continue to next page for more questions.

1

Stat 427/627 Statistical Machine Learning

Quiz 7

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Shrinkage Methods:

Circle **all** correct answers. There may be more than one correct choice.

2. Which of the following are shrinkage methods?

- ☒ (a) Ridge regression ☒ (b) LASSO (c) Neither Ridge regression nor Lasso

3. Which of the following(s) can be used as a variable selection method?

- (a) Ridge regression ☒ (b) LASSO (c) Neither Ridge regression nor Lasso

• Fill the blanks with “***will not***”, or “***may***”, or “***always***”.

4. Compared to the least squares regression, the ridge regression and lasso:

- (a) Will not reduce the bias of the slope's estimate .
(b) may reduce the standard error of the slopes' estimate.
(c) May reduce the mean-squared error of prediction.

There will be another question on Shrinkage Methods in the next quiz.

