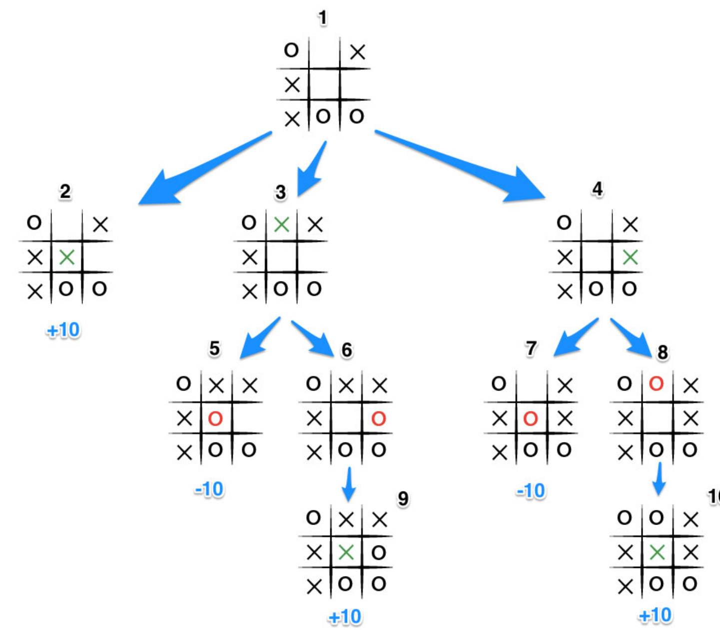
Terminology

- s: State
- a: Action to take
- r: Reward (or penalty)
- Episode: 1 round (e.g. 1 game)
- Q(s, a): Q-value, expected value of taking action *a* at state *s*
- π(s): Policy, **what** action to take at state *s* to maximise expected value



Markov Decision Process

$$\text{MDP} = (\mathcal{S}, \mathcal{A}, P, R, s_0, \gamma).$$

$\mathcal{S}$: observable state space

$\mathcal{A}$: action space

P: state transition probabilities

R: reward function

s_0: starting state

γ: reward discont rate.()

Markov assumption: $P(s_{t+1}|s_0, a_0, \ldots, s_t, a_t) = P(s_{t+1}|s_t, a_t)$

Reward assumption: $R(s_0, a_0, \ldots, s_t, a_t, s_{t+1}) = R(s_t, a_t, s_{t+1}) = r_{t+1} \in \mathbb{R}$

Policy: $\pi(s, a) = P(a_t|s_t) \in [0, 1]$, that is $a_t \sim \pi(s_t, \cdot)$

Goal : $\max_{\pi} \mathbb{E}[r_0 + r_1 + \ldots]$ (Maximise expected cumulative rewards) 7

https://www.cs.cmu.edu/~mgormley/courses/10601-s17/slides/lecture26-ri.pdf

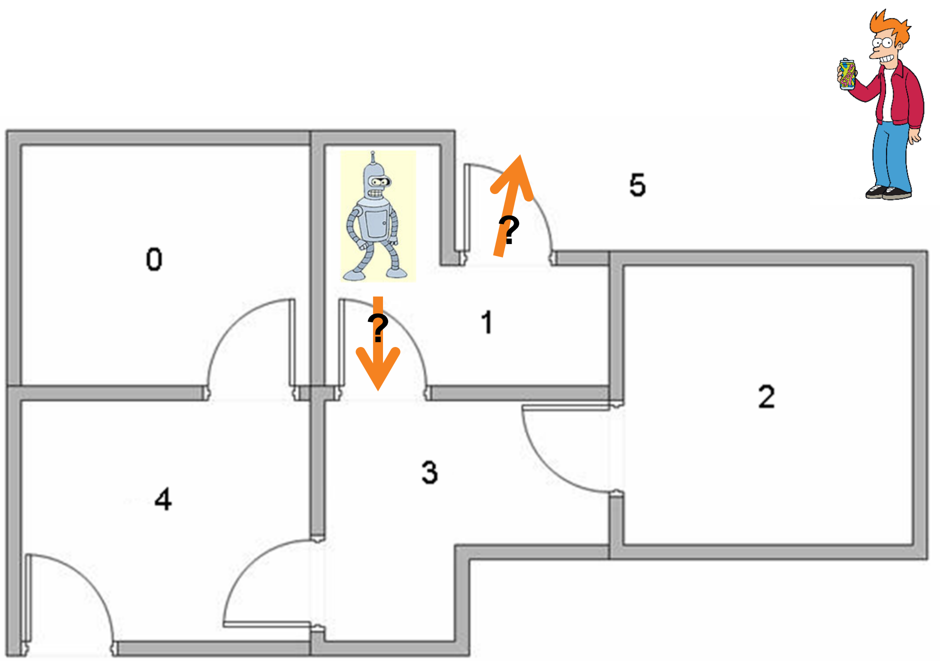
Q-Learning

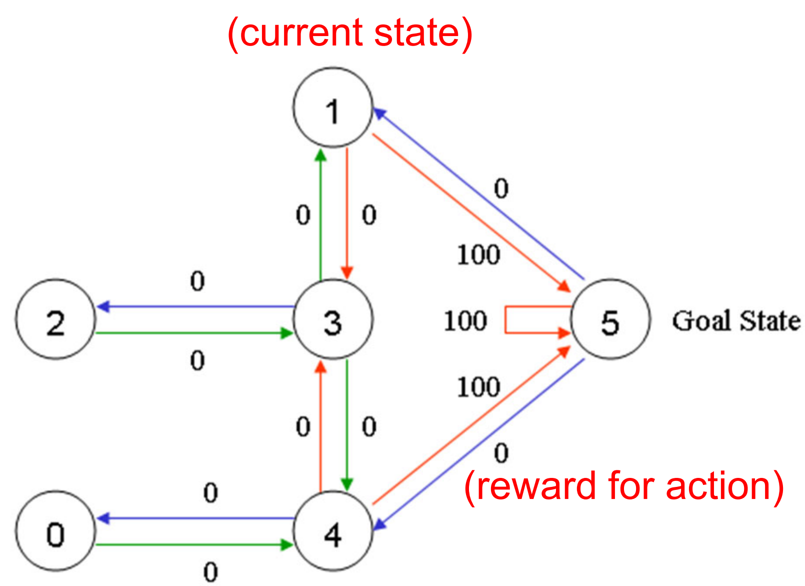
- Simple approach to solve Markov Decision Process

  Q(s, a) = R(s, a) + Gamma * Max[Q(next state, all actions)]

- Gamma: discount factor (between 0 and 1)
  - How much of future rewards to consider compared to present rewards
- "Learning from experience"

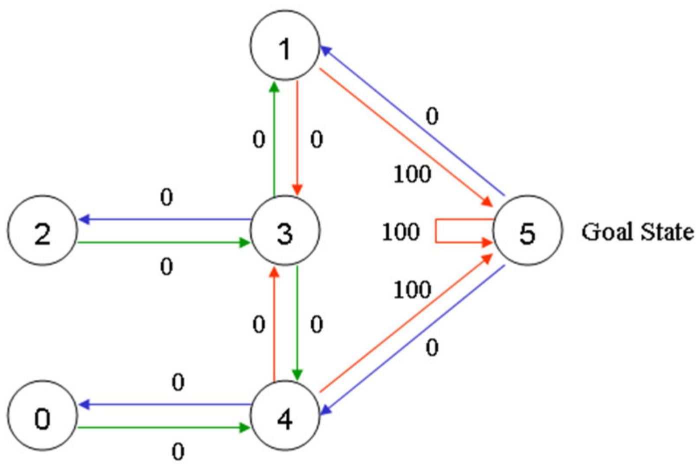
Q-Learning: Learn the Path through Rewards





Full example (with code): http://www.mnemstudio.org/path-finding-q-learning-tutorial.htm

Q-Learning: Path Finding



$$R = \begin{array}{cc} & \text{Action} \\ \text{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix} \end{array}$$

Q-Learning Task:

- Complete the Q-values table so that robot can find the **best path** to take at a **given state**
- -1 values indicate invalid directions

$$Q = \begin{array}{cc} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Q-Learning: Algorithm

Select parameter Gamma, set rewards in matrix R

Initialize matrix Q to zero

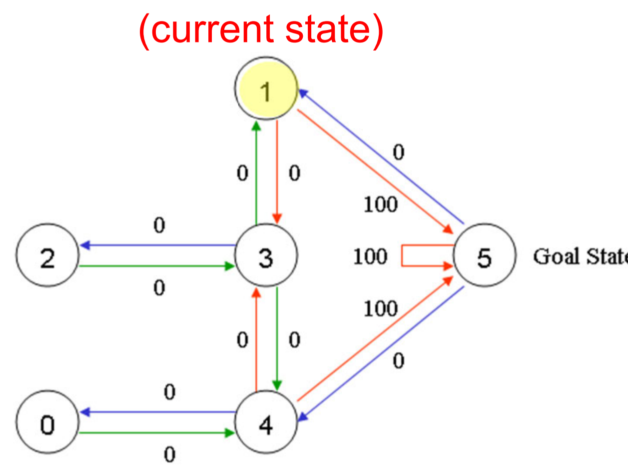
```
For each episode {
        Select a random initial state
        While (goal state not reached) {
                Select 1 possible action for current state
                Using the selected action, consider going to the next state

                Get maximum Q value for next state
                Set the next state as the current state
        }
}
```

Q-Learning: Episode 1

Gamma = 0.8

State = 1 (random)

$$
\begin{array}{cc}
 & \textbf{Action} \\
\textbf{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
R = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
-1 & -1 & -1 & -1 & 0 & -1 \\
-1 & -1 & -1 & 0 & -1 & 100 \\
-1 & -1 & -1 & 0 & -1 & -1 \\
-1 & 0 & 0 & -1 & 0 & -1 \\
0 & -1 & -1 & 0 & -1 & 100 \\
-1 & 0 & -1 & -1 & 0 & 100
\end{bmatrix}
\end{array}
$$



Possible action: 5 (selected randomly from 3 and 5)

Q(state, action) = R(state, action) + Gamma * Max[Q(next state, all actions)]

Q(1, 5) = R(1, 5) + 0.8 * Max[Q(5, 1), Q(5, 4), Q(5, 5)]

= 100 + 0.8 * 0 = **100**

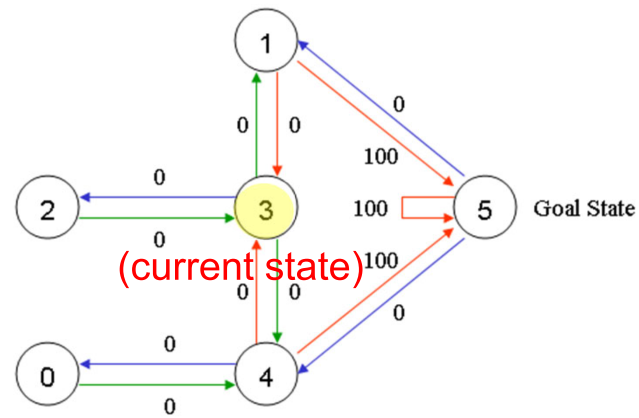
$$
\begin{array}{cc}
 & \textbf{Action} \\
\textbf{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
Q = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 100 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

Q-Learning: Episode 2

Gamma = 0.8

State = 3 (random)

$$
R = \begin{array}{c} \text{State} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\text{Action} \\
\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
-1 & -1 & -1 & -1 & 0 & -1 \\
-1 & -1 & -1 & 0 & -1 & 100 \\
-1 & -1 & -1 & 0 & -1 & -1 \\
-1 & 0 & 0 & -1 & 0 & -1 \\
0 & -1 & -1 & 0 & -1 & 100 \\
-1 & 0 & -1 & -1 & 0 & 100
\end{bmatrix}
\end{array}
$$



Possible action: 1 (selected randomly from 1, 2, and 4)

Q(state, action) = R(state, action) + Gamma * Max[Q(next state, all actions)]

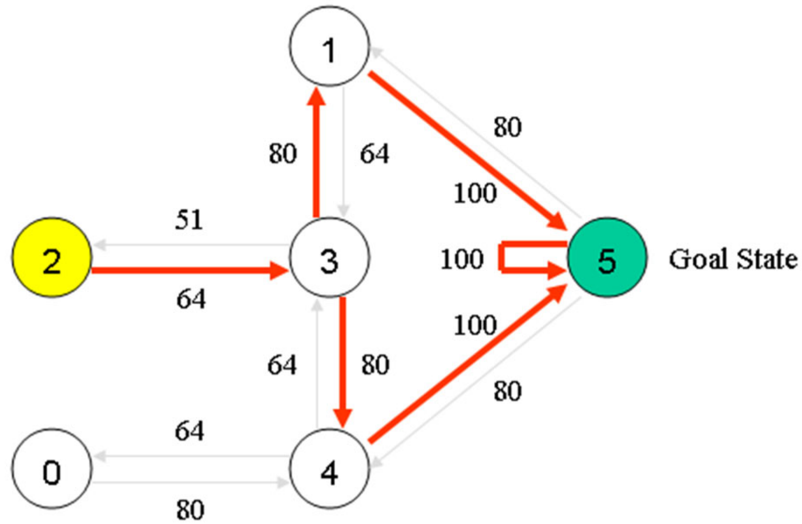
Q(3, 1) = R(3, 1) + 0.8 * Max[Q(1, 2), Q(1, 5)]

= 0 + 0.8 * Max(0, 100) = **80**

$$
Q = \begin{array}{c} \text{State} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\text{Action} \\
\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 100 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 80 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

Q-Learning: Convergence

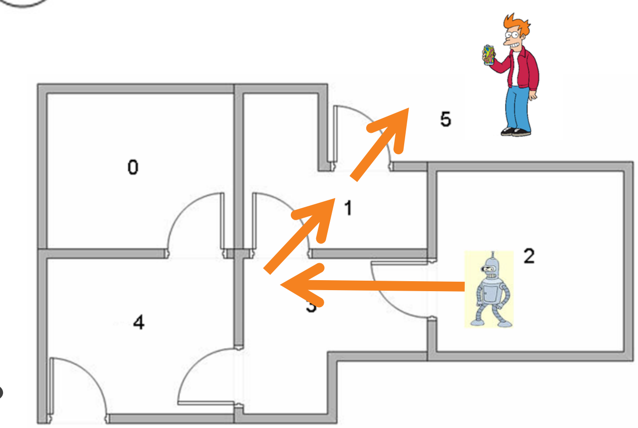
$$
Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\left[\begin{array}{cccccc}
0 & 0 & 0 & 0 & 80 & 0 \\
0 & 0 & 0 & 64 & 0 & 100 \\
0 & 0 & 0 & 64 & 0 & 0 \\
0 & 80 & 51 & 0 & 80 & 0 \\
64 & 0 & 0 & 64 & 0 & 100 \\
0 & 80 & 0 & 0 & 80 & 100
\end{array}\right]
\end{array}
$$



What path should the Robot use if starting from 2?

State 2: Maximum Q-value is for state 3
State 3: Maximum Q-value is same for state 1 or 4 => random choice

State 1: Maximum Q-value is state 5. Path: 2-3-1-5
State 4: Maximum Q-value is state 5. Path: 2-3-4-5



Q: 2-3-4-5 is a longer path, how do you take distance into account?

Q-Learning Enhancements

- Q-Learning when rewards for all state, action combinations *not fully known*
  - In real-world or complex environments, not all possible actions and states can be enumerated (e.g. autonomous driving, Starcraft)
- Strategies:
  - **Monte Carlo:** take average of the Q-values observed so far
  - **Temporal Differencing**: use the difference with previous step's Q-value to estimate the next Q-value
  - **Deep Q-Learning**: use a deep neural network to estimate the Q-values