Lisa Over HW 1

19.5/20

Random Numbers
Seed 1 = 12345
Seed 2 = 54321

1. $X_0 = 5$ $\quad X_n = 3X_{n-1} \bmod 150$

$X_1 = 3 \cdot 5 \bmod 150 = 15$
$X_2 = 3 \cdot 15 \bmod 150 = 45$
$X_3 = 3 \cdot 45 \bmod 150 = 135$
$X_4 = 3 \cdot 135 \bmod 150 = 105$
$X_5 = 3 \cdot 105 \bmod 150 = 15$
$X_6 = 3 \cdot 15 \bmod 150 = 45$ ✓
$X_7 = 3 \cdot 45 \bmod 150 = 135$
$X_8 = 3 \cdot 135 \bmod 150 = 105$
$X_9 = 3 \cdot 105 \bmod 150 = 15$
$X_{10} = 3 \cdot 15 \bmod 150 = 45$

2. $X_0 = 3$ $\quad X_n = (5X_{n-1} + 7) \bmod 200$

$X_1 = (5 \cdot 3 + 7) \bmod 200 = 22$
$X_2 = (5 \cdot 22 + 7) \bmod 200 = 117$
$X_3 = (5 \cdot 117 + 7) \bmod 200 = 192$
$X_4 = (5 \cdot 192 + 7) \bmod 200 = 167$
$X_5 = (5 \cdot 167 + 7) \bmod 200 = 42$
$X_6 = (5 \cdot 42 + 7) \bmod 200 = 17$
$X_7 = (5 \cdot 17 + 7) \bmod 200 = 92$ ✓
$X_8 = (5 \cdot 92 + 7) \bmod 200 = 67$
$X_9 = (5 \cdot 67 + 7) \bmod 200 = 142$
$X_{10} = (5 \cdot 142 + 7) \bmod 200 = 117$

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3. $\int_0^1 \exp[e^x]\,dx \approx 6.316576$ (calc)

mean(exp(exp(x.s))) ≈ 6.327965

4. $\int_0^1 (1-x^2)^{3/2}\,dx \approx 0.58905$ (calc)

mean((1-x.s^2)^(3/2)) ≈ 0.5892747

5. $\int_{-2}^{2} e^{x+x^2}\,dx \approx 93.1628$ (calc)

b-a = 4

mean(exp((x.s×4 -2)+(x.s×4 -2)^2)×4) ≈ 95.31651

6. $\int_0^\infty x(1+x^2)^{-2}\,dx \approx 0.5$ (calc)

mean((1/x.s-1)×(1+(1/x.s-1)^2)^-2×(1/x.s^2)) ≈ 0.494

7. $\int_{-\infty}^{\infty} e^{-x^2}\,dx \approx 1.77245$ (calc)

mean(2×(exp(-(1/x.s-1)^2)×(1/x.s^2)) ≈ 1.755853

$2\int_0^\infty e^{-x^2}\,dx$

8. $\int_0^1\int_0^1 e^{(x+y)^2}\,dy\,dx \approx 4.89916$ (calc)

mean(exp((x.s+y.s)^2)) ≈ 4.85899

9. $\int_0^\infty\int_0^x e^{-(x+y)}\,dy\,dx \approx 0.5$ (calc)

mean ≈ 0.492 code attached

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10. $Cov(U, e^U)$ U is uniform on $(0,1)$.

$\int_0^1 x\,dx \approx 0.5$ $\int_0^1 e^x\,dx \approx 1.7183$

$\int_0^1 (x-0.5)(e^x-1.7183)(1)(1)\,dx \approx 0.140859$ (calc) ✓

mean((x.S − 0.5)x(exp(x.5)−1.7177)) ≈ 0.1435019

11. a. −0.0605 code attached
b. −0.0667

12. a. E[N] ≈ 2.66 code attached
b. E[N] ≈ 2.7
c. E[N] ≈ 2.7448 ✓
d. E[N] ≈ 2.75

13. a. E[N] ≈ 3.97 → Should be 3? code attached
b. P(N=1) ≈ 0.2074
P(N=2) ≈ 0.2177
P(N=3) ≈ 0.2338
P(N=4) ≈ 0.1602
P(N=5) ≈ 0.0982
P(N=6) ≈ 0.0471

−0.5