Lisa Over
Homework #5
February 16, 2015

Code

```
#Get data from file
data = read.table(file.choose(), header=F)
attach(data)
data = data$V1

#Set burnin to 10 and leave lag=1 and run Gibbs with N=2500
N = 2500
lag = 1
burnin = 10
m = 80
```

Method 1: Gibbs Sampler

#GIBBS

The following Gibbs sampler function assumes μ *and* σ^2 are a priori independent with prior densities $\pi(\mu) \propto 1$ *and* $\pi(\sigma^2) \propto \sigma^{-2}$. The function receives five parameters: "data" for the data values of interest, "m" for the population mean, "N" for number of realizations, "lag" for determining how many realizations to skip between saves, and "burnin" for determining how many realizations to skip before starting to save. Gibbs generates N independent random normal values of μ *and* σ^2 from marginal posteriors $\pi(\mu|\bar{y})$*and* $\pi(\sigma^2|\bar{y})$.

```
gibbs <- function(data,m,N,lag,burnin) {

    #obtain mean (y.bar), variance (s.sqr), and length (n) of data
    y.bar = mean(data)
    s.sqr = var(data)
    n = length(data)

    #Set N to be N*lag+burnin
    N <- N*lag + burnin

    #Initialize vectors to hold mu and sigma sqr values
    m.s <- NULL
    s.s <- NULL

  for(i in 1:N) {

    #generate a sigma sqr s using mean (y.bar), variance (s.sqr), and length (n) of data
    s <- 1/rgamma(1,n/2,0.5*((n-1)*s.sqr+n*(y.bar-m)^2))
    #generate a mu m using mean (y.bar) and length (n) of data and sigma sqr (s) from last step
    m <- rnorm(1, y.bar, sqrt(s/n))

  #if i is greater than burnin and if i is a multiple of the lag, store m and s
```

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```
  if(i > burnin) {
      if(i %% lag == 0) {
                 m.s <- c(m.s,m)
                 s.s <- c(s.s,s)
    }
   }
  }
  vectors <- list("mu" = m.s, "sigsqr" = s.s)
  return(vectors)
 }

vectors = gibbs(data,m,N,lag,burnin)

#vectors$mu represents Gibbs sampler realizations for mu
#vectors$sigsqr represents Gibbs sampler realizations for sigma sqr
```

#PLOTS

```
par(mfrow=c(2,2)) #split plotting window into 2 rows and 2 columns
ts.plot(vectors$mu,xlab="Iterations")
ts.plot(vectors$sigsqr,xlab="Iterations")
hist(vectors$mu,probability=T, cex.lab=1.5, cex.axis=1.5)
hist(vectors$sigsqr,probability=T, cex.lab=1.5, cex.axis=1.5)
yy = seq(10,70,by=.001)
alpha = (n/2)-1
lambda = ((n-1)*s.sqr)/2
lines(yy, ((lambda^alpha)/gamma(alpha))*((1/yy)^(alpha+1))*exp(-lambda/yy))

par(mfrow=c(2,1))
acf(vectors$mu)
acf(vectors$sigsqr)
```

#QUANTILES/CREDIBLE INTERVALS

```
quantile(vectors$mu, 0.025)
quantile(vectors$mu, 0.975)
quantile(vectors$sigsqr, 0.025)
quantile(vectors$sigsqr, 0.975)

quantile(vectors$mu, 0.005)
quantile(vectors$mu, 0.995)
quantile(vectors$sigsqr, 0.005)
quantile(vectors$sigsqr, 0.995)
```

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Method 2: Theoretical Draws

```
mt.s = NULL
vt.s = NULL
n = length(data)
s.sqr = var(data)
y.bar = mean(data)

for(i in 1:N) {
    v = 1/rgamma(1,(n/2-1),(n-1)*s.sqr/2)
  m = rnorm(1,y.bar,sqrt(v/n))
    mt.s = c(mt.s,m)
    vt.s = c(vt.s,v)
}

#PLOTS
par(mfrow=c(2,2)) #split plotting window into 2 rows and 2 columns
ts.plot(mt.s,xlab="Iterations")
ts.plot(vt.s,xlab="Iterations")
hist(mt.s,probability=T, cex.lab=1.5, cex.axis=1.5)
hist(vt.s,probability=T, cex.lab=1.5, cex.axis=1.5)
yy = seq(10,70,by=.001)
alpha = (n/2)-1
lambda = ((n-1)*s.sqr)/2
lines(yy, ((lambda^alpha)/gamma(alpha))*((1/yy)^(alpha+1))*exp(-lambda/yy))

#QUANTILES/CREDIBLE INTERVALS
quantile(mt.s, 0.025)
quantile(mt.s, 0.975)
quantile(vt.s, 0.025)
quantile(vt.s, 0.975)

quantile(mt.s, 0.005)
quantile(mt.s, 0.995)
quantile(vt.s, 0.005)
quantile(vt.s, 0.995)
```

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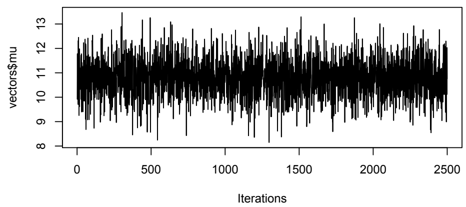
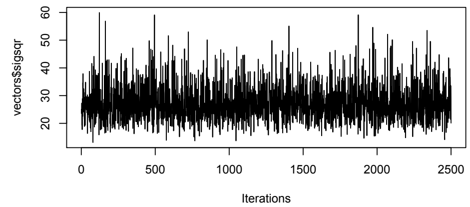
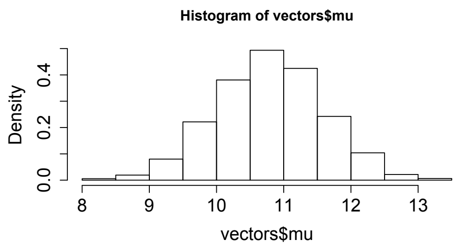
Results

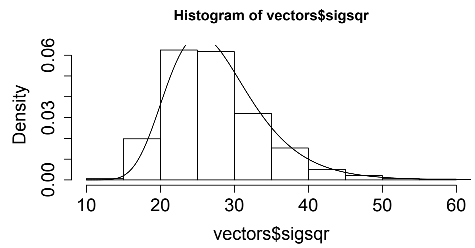
Method 1: Gibbs Sampler

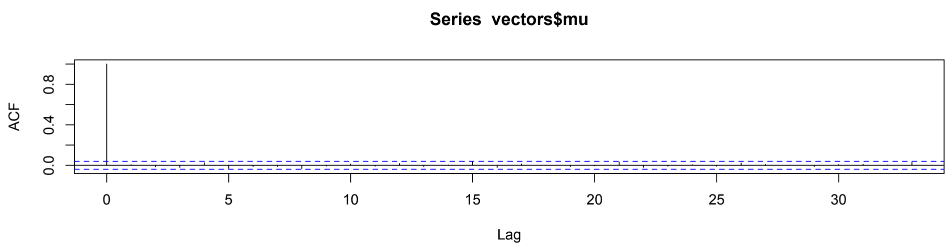
(a) PLOTS

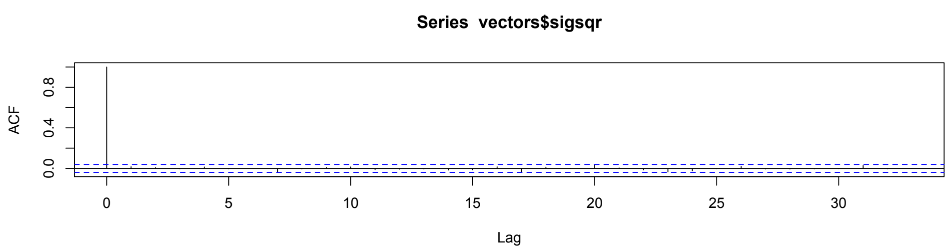












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(b) QUANTILES

	$P(b_1 < \mu < b_2) = 0.95$		$P(b_1 < \mu < b_2) = 0.99$	
μ	$b_1 = 9.2466$	$b_2 = 12.4249$	$b_1 = 8.5788$	$b_2 = 13.0917$
	$P(c_1 < \sigma^2 < c_2) = 0.95$		$P(c_1 < \sigma^2 < c_2) = 0.99$	
σ^2	$c_1 = 17.7869$	$C_2 = 41.4715$	$c_1 = 15.9805$	$c_2 = 50.6254$

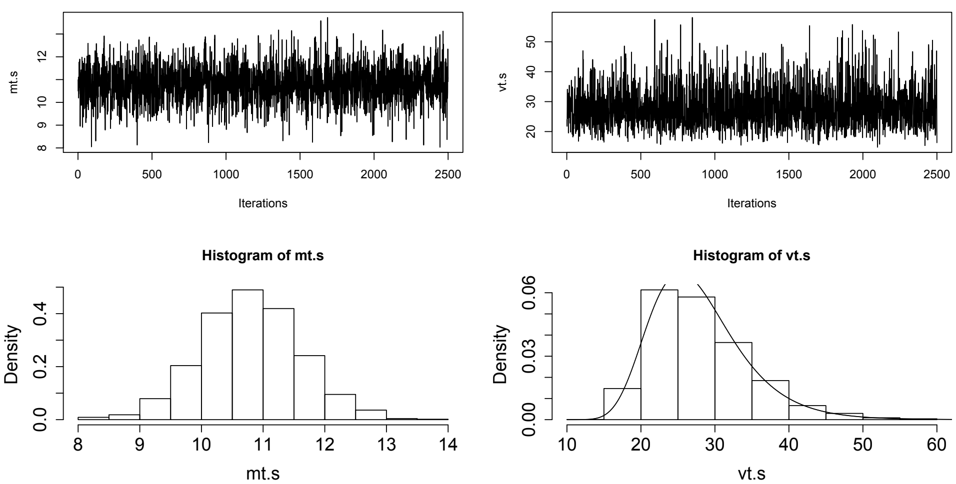
(c) CREDIBLE INTERVALS

Lisa Over 6/5/2015 11:18 AM
Comment [1]: For credible intervals, also include estimates for mu and sigsqr.

	Lower 95%	Upper 95%	Lower 99%	Upper 99%
μ	9.2466	12.4249	8.5788	13.0917
σ^2	17.7869	41.4715	15.9805	50.6254

Method 2: Theoretical Draws

(a) PLOTS



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(b) QUANTILES

	$P(b_1 < \mu < b_2) = 0.95$		$P(b_1 < \mu < b_2) = 0.99$	
μ	$b_1 = 9.18$	$b_2 = 12.3514$	$b_1 = 8.6776$	$b_2 = 12.9029$
	$P(c_1 < \sigma^2 < c_2) = 0.95$		$P(c_1 < \sigma^2 < c_2) = 0.99$	
σ^2	$c_1 = 17.9174$	$c_2 = 43.9736$	$c_1 = 15.9959$	$c_2 = 50.4055$

(c) CREDIBLE INTERVALS

Lisa Over 6/5/2015 11:18 AM
Comment [2]: For credible intervals, also include estimates for mu and sigsqr.

	Lower 95%	Upper 95%	Lower 99%	Upper 99%
μ	9.18	12.3514	8.6776	12.9029
σ^2	17.9174	43.9736	15.9959	50.4055