Lisa Over
Final Exam Problem 2
May 5, 2015

CODE

```
#Read file and attach
data = read.csv(file.choose(), header=TRUE)
attach(data)

#Set gibbs parameters

y = data$IgG
x = data$Max.O2.Uptake

output = summary(glm(y ~ x + I(x^2)))

N = 25000
lag = 500
burnin = 0
b0 = output$coef[1,1]
b1 = output$coef[2,1]
b2 = output$coef[3,1]
```

#Gibbs function receives eight parameters: "x" and "y" data values; "b0", "b1", and "b2" initial regression coefficients, "N" for the number of realizations, "lag" for determining how many realizations to skip between saves, and "burnin" for determining how many realizations to skip before starting to save. Gibbs generates N independent random normal values for each regression coefficient and N independent inverse gamma values for the regression variance.

```
gibbs <- function(x,y,b0,b1,b2,N,lag,burnin) {

    #obtain length of data
    n = length(x)
    #Set N to be N*lag+burnin
    N <- N*lag + burnin

    #Initialize vectors to hold the beta coefficients and sigsq realizations
    b0s = NULL
    b1s = NULL
    b2s = NULL
    s2s = NULL

    for(i in 1:N) {

        #Generate a sigsq, s2, based on current regression coefficients b0, b1, b2
        s2 = 1/rgamma(1, n/2, sum((y-b0-b1*x-b2*x^2)^2)/2)
        #Generate coefficients based on current values of coefficients and sigsq, s2
        b0 = rnorm(1, sum(y-b1*x-b2*x^2)/n, sqrt(s2/n))
        b1 = rnorm(1, sum(x*(y-b0-b2*x^2))/sum(x^2), sqrt(s2/sum(x^2)))
        b2 = rnorm(1, sum(x^2*(y-b0-b1*x))/sum(x^4), sqrt(s2/sum(x^4)))
```

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```
        #if i is greater than burnin and if i is a multiple of the lag, store alpha, beta, and siqsq
         if(i > burnin) {
          if(i %% lag == 0) {
                b0s <- c(b0s,b0)
                b1s <- c(b1s,b1)
                b2s <- c(b2s,b2)
                s2s <- c(s2s,s2)
    }
   }
        }

        vectors <- list("beta0s" = b0s, "beta1s" = b1s, "beta2s" = b2s, "sigsqs" = s2s)
        return(vectors)

}

v = gibbs(x,y,b0,b1,b2,N,lag,burnin)

beta0s = v$beta0s
beta1s = v$beta1s
beta2s = v$beta2s
sigsqs = v$sigsqs

mean(beta0s)
mean(beta1s)
mean(beta2s)
mean(sigsqs)

#Plot each parameter with ts.plot, hist, and acf

par(mfrow=c(3,1)) #split plotting window into 3 rows and 1 column
ts.plot(beta0s,xlab="Iterations")
hist(beta0s,probability=T, cex.lab=1.5, cex.axis=1.5)
acf(beta0s,lag.max=500)

par(mfrow=c(3,1)) #split plotting window into 3 rows and 1 column
ts.plot(beta1s,xlab="Iterations")
hist(beta1s,probability=T, cex.lab=1.5, cex.axis=1.5)
acf(beta1s,lag.max=500)

par(mfrow=c(3,1)) #split plotting window into 3 rows and 1 column
ts.plot(beta2s,xlab="Iterations")
hist(beta2s,probability=T, cex.lab=1.5, cex.axis=1.5)
acf(beta2s,lag.max=500)

par(mfrow=c(3,1)) #split plotting window into 3 rows and 1 column
ts.plot(sigsqs,xlab="Iterations")
hist(sigsqs,probability=T, cex.lab=1.5, cex.axis=1.5)
```

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```
acf(sigsqs,lag.max=500)

#Create a matrix of regreesion coefficients - N rows and 3 columns
k=1
mcoef <- matrix(, nrow = N, ncol = 3)
for(i in 1:N) {
			coord = c(beta0s[i], beta1s[i], beta2s[i])
			mcoef[k ,] = coord
			k = k + 1
 }

#Create a matrix of yhat values - N rows and 30 columns (number of x values)
yhats <- matrix(, nrow = N, ncol = length(x))
for(i in 1:N) {
		yhat <- mcoef[i,1] + mcoef[i,2]*x + mcoef[i,3]*x^2
		yhats[i ,] = yhat
}
#Calculate the means of the columns (2) of the yhats matrix
means = apply(yhats, 2, mean)


#Select 3 random numbers between 0 and 25000 to represent the indices of the 3 randomly selected coefficient coordinates from the mcoef matrix
rLines = sample(1:25000, 3, replace=F)

#Create a 3-row matrix of yhat values for each x using the randomly selected coordinates
yhats3 <- matrix(, nrow = 3, ncol = length(x))
for(i in 1:3) {
		yhat <- mcoef[rLines[i],1] + mcoef[rLines[i],2]*x + mcoef[rLines[i],3]*x^2
		yhats3[i ,] = yhat
}

#Plot x and y with possible fitted curves after spline smoothing
smooth1 = smooth.spline(x,yhats3[1,], spar=0.35)
smooth2 = smooth.spline(x,yhats3[2,], spar=0.35)
smooth3 = smooth.spline(x,yhats3[3,], spar=0.35)
smooth4 = smooth.spline(x, means, spar=0.35)

plot(x,y)
lines(smooth1, lty=2)
lines(smooth2, lty=2)
lines(smooth3, lty=2)
lines(smooth4, lwd=2.5)
```

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```
#Create a vector of yhat values with O2 uptake = 60 and plot a histogram of the values
yhats60 = NULL
for(i in 1:N) {
	yhat60 <- mcoef[i,1] + mcoef[i,2]*60 + mcoef[i,3]*60^2
	yhats60 = c(yhats60, yhat60)
}
hist(yhats60)

mean(yhats60)
quantile(yhats60, 0.025)
quantile(yhats60, 0.975)
```

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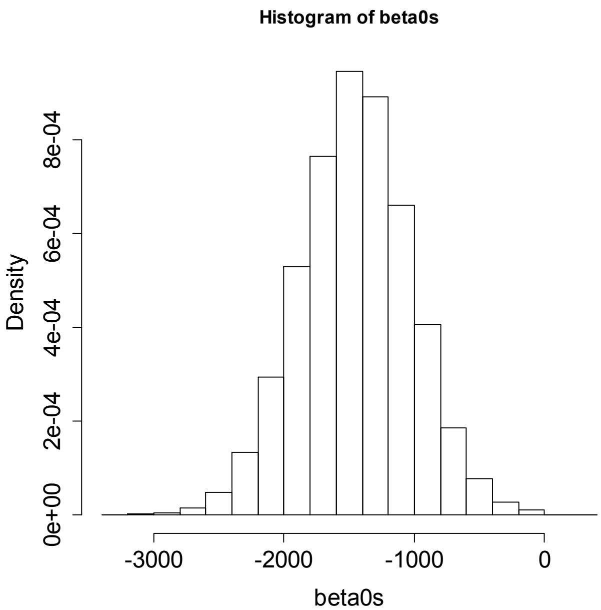
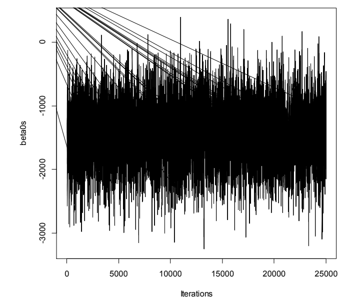
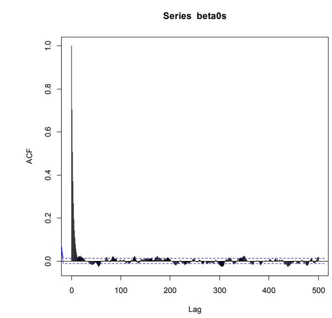
RESULTS

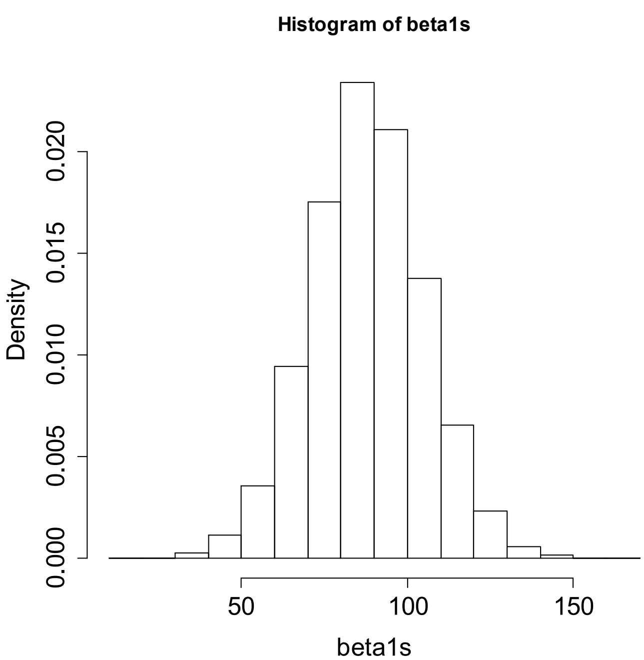
Beta0 mean: **-1452.651**

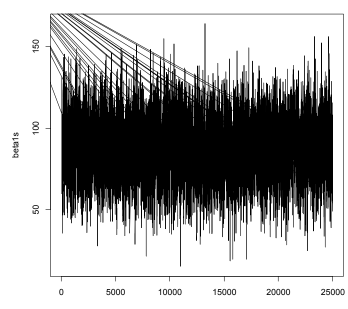


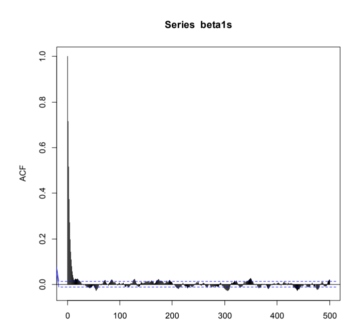




Beta1 mean: **87.85156**







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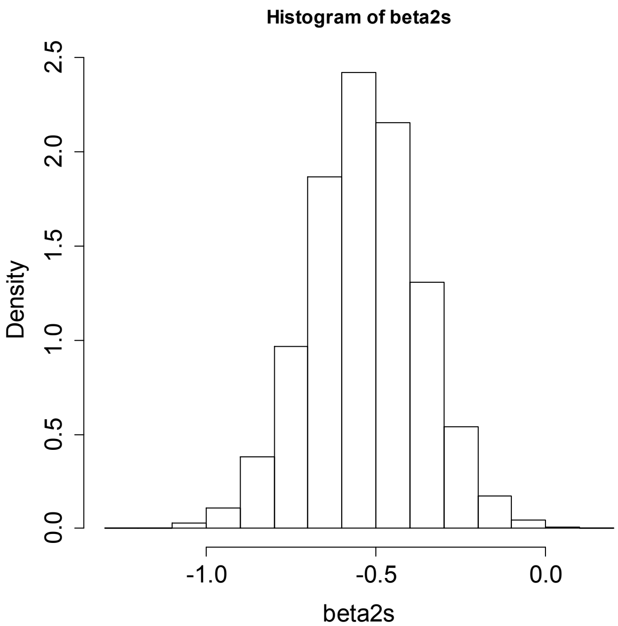
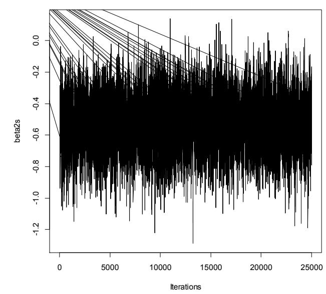
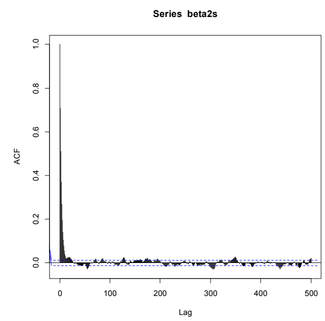
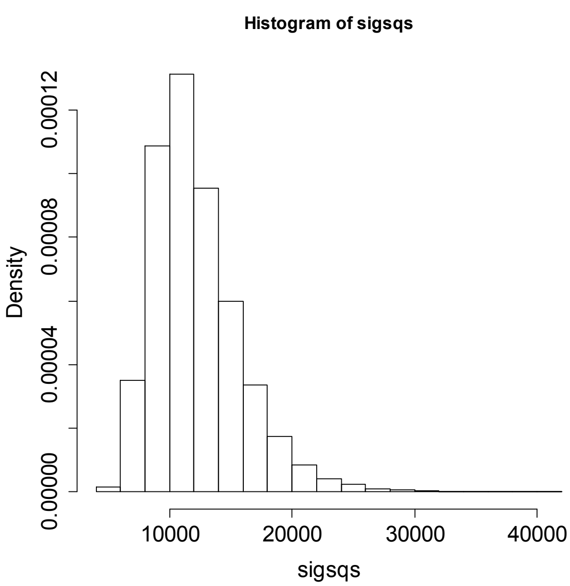
Beta2 mean: **-0.5320706**

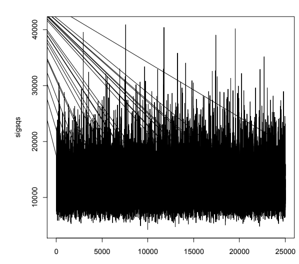


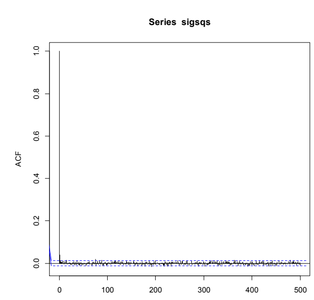




Sigma Sq mean: **12219.16**







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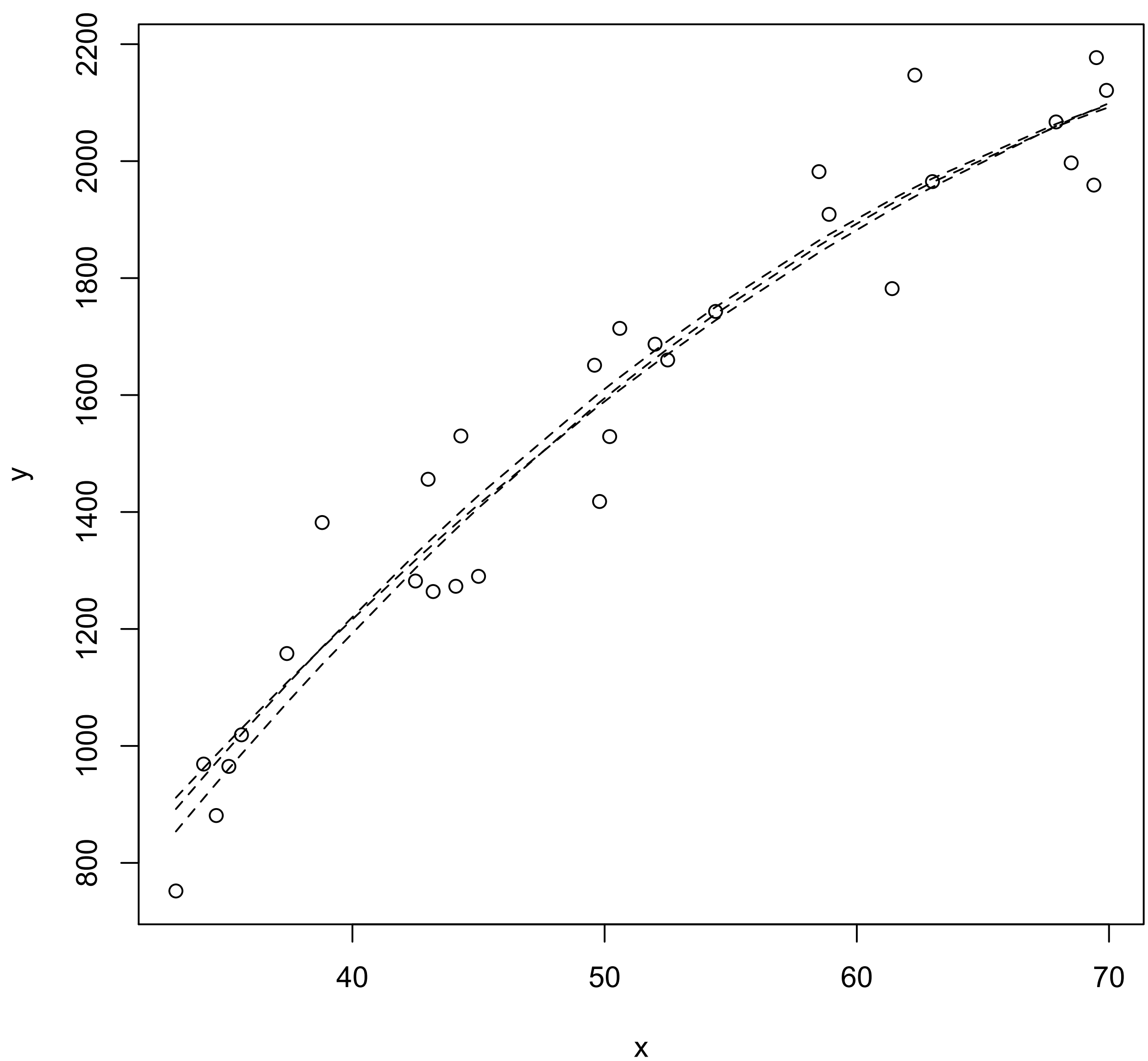
The following scatterplot shows the data with three possible regression curves that model the data. The regression coefficients for these curves were selected randomly from a matrix of possible combinations of coefficients, β_0, β_1, and β_2, with each combination of coefficients representing a possible regression curve. The coefficients were drawn from their respective posterior predictive distributions.



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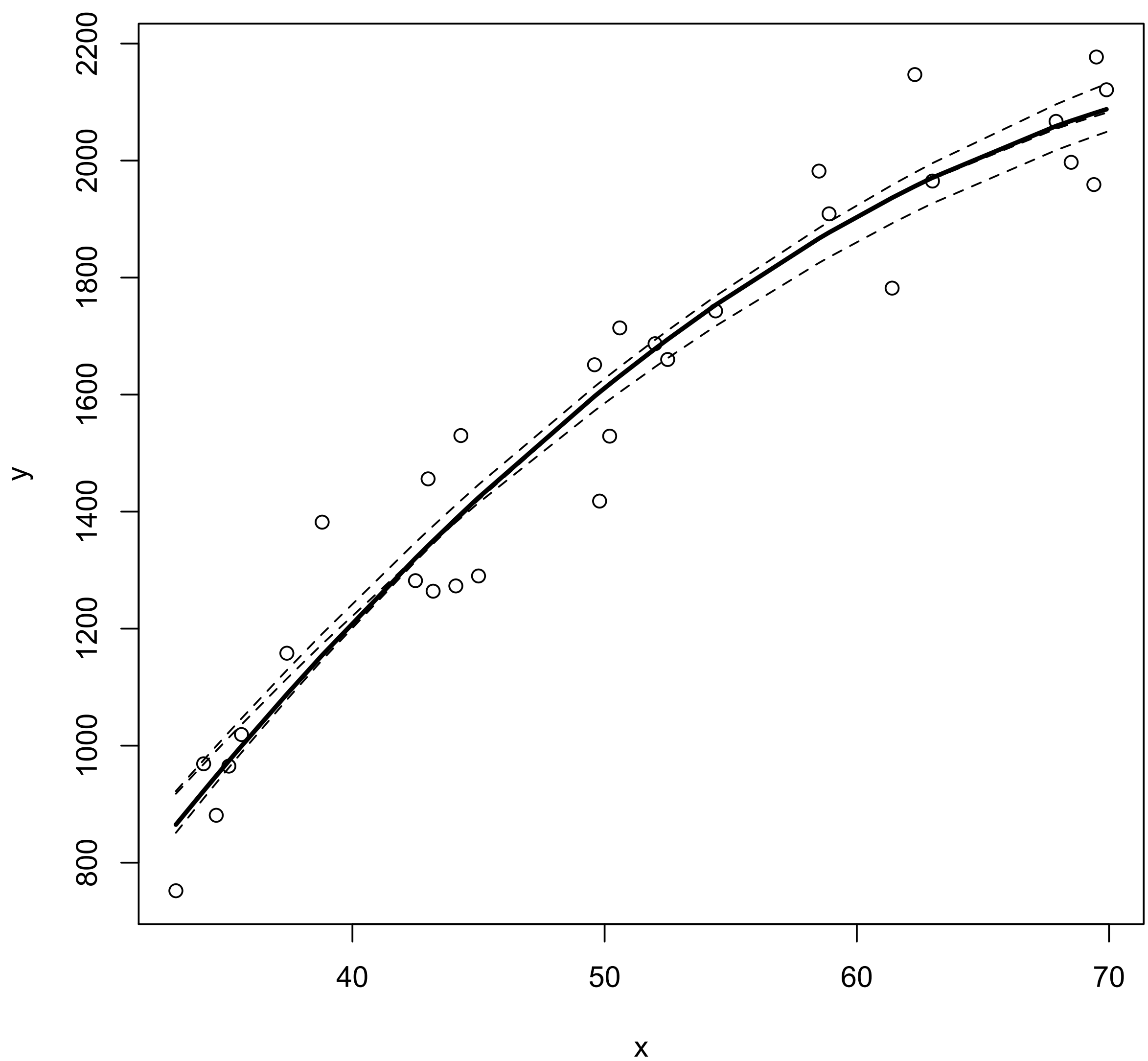
The following scatterplot shows data with the three randomly selected regression curves superimposed as dotted lines and the mean regression curve superimposed as a bold line. The mean regression curve was obtained by averaging each column of coefficients, β_0, β_1, and β_2, in the coefficient matrix.



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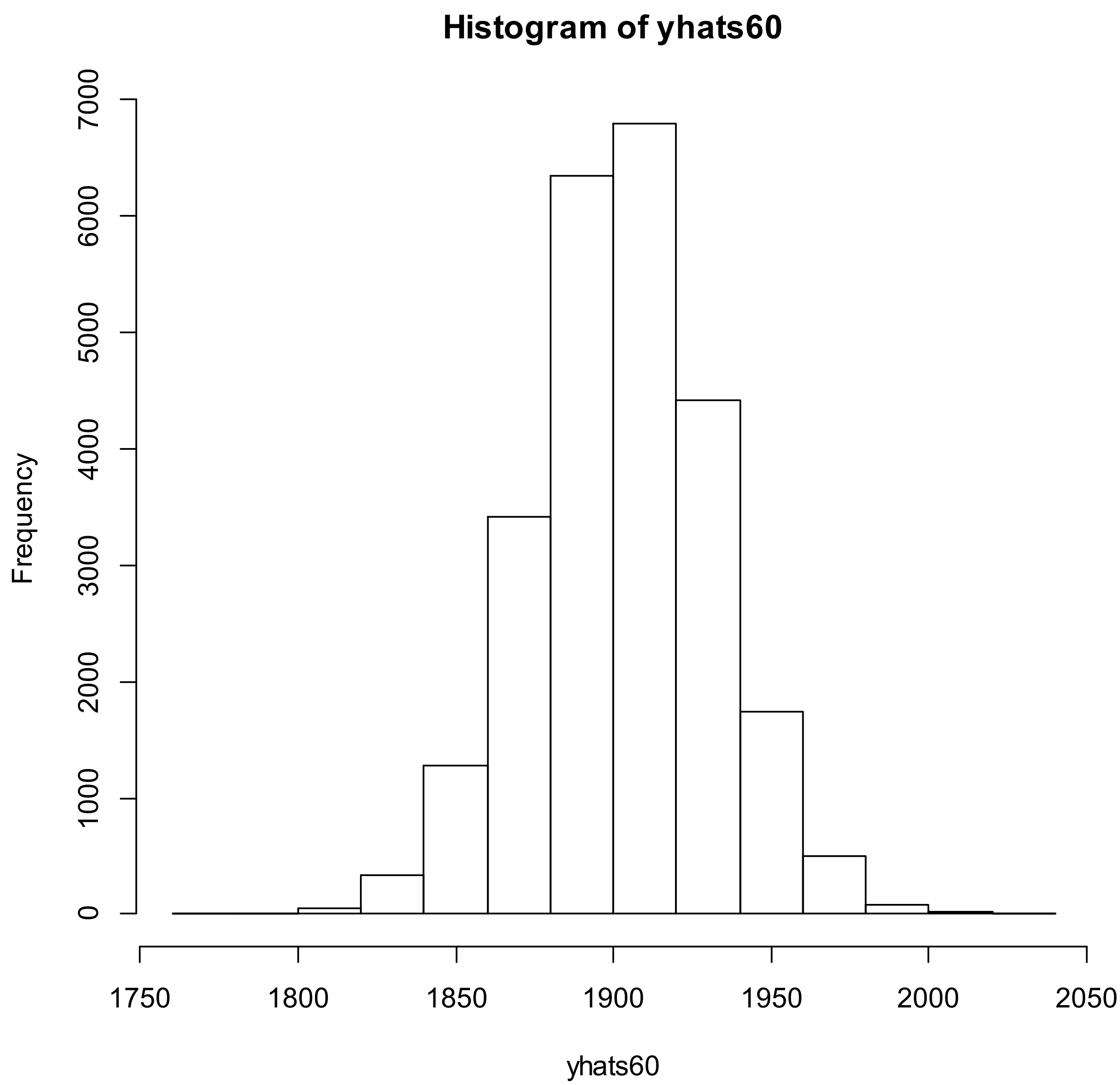
The following histogram shows the distribution of possible IgG measurements for a person with maximal oxygen uptake equal to 60. The possible values were computed using the regression curves generated from the posterior predictive distributions of the regression coefficients, β_0, β_1, and β_2, and hence from the posterior predictive distribution of IgG. The mean IgG is 1902.988 with a 95% credible interval (1845.851, 1959.284).



Mean: **1902.988**

95% Credible Interval: **(1845.851, 1959.284)**

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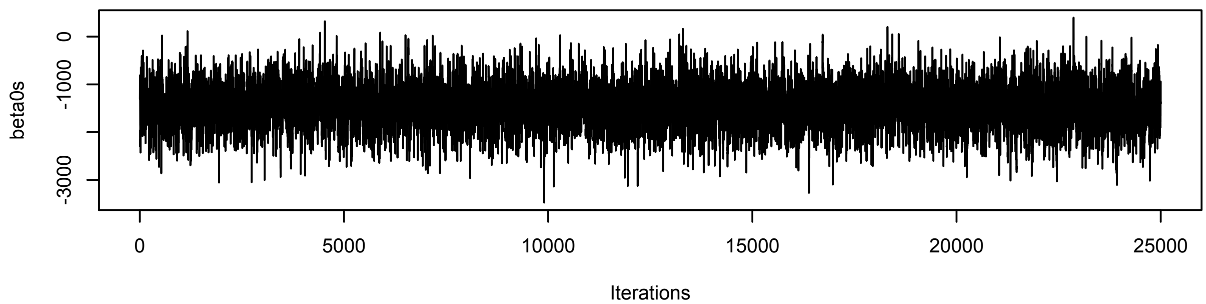

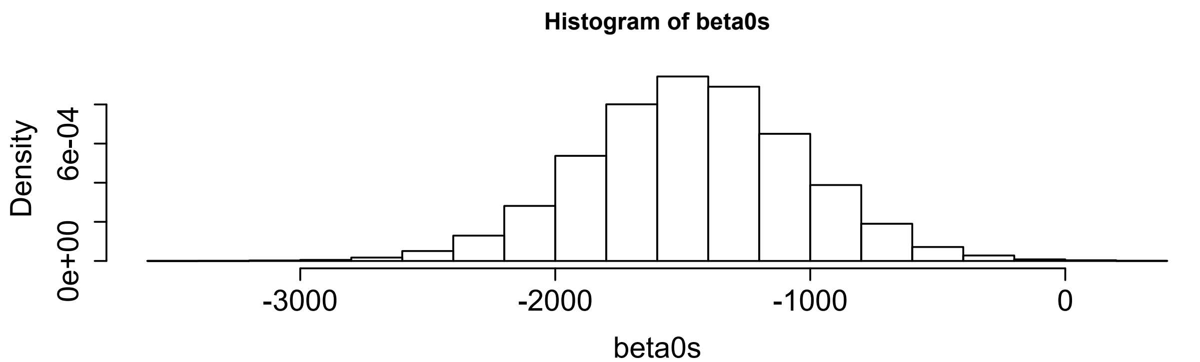

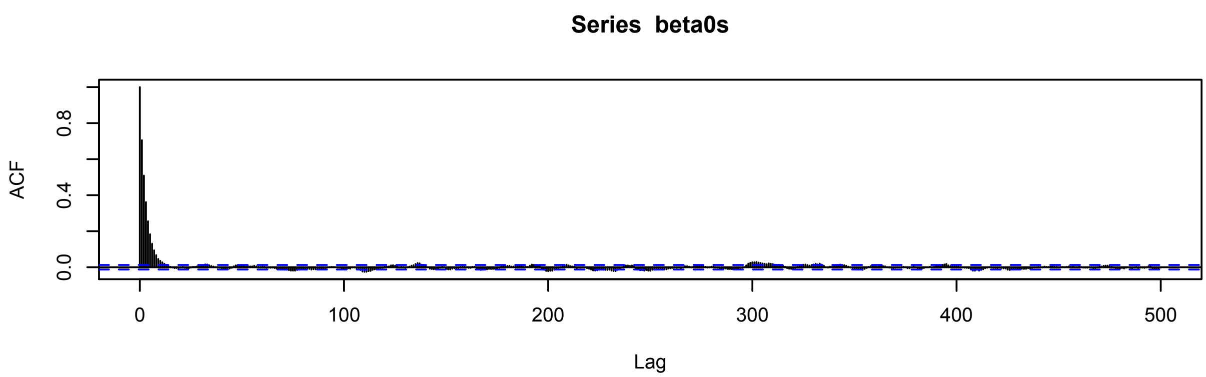

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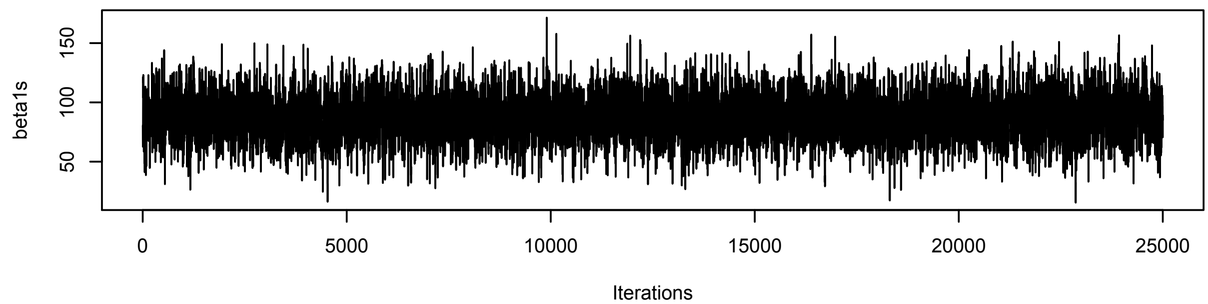

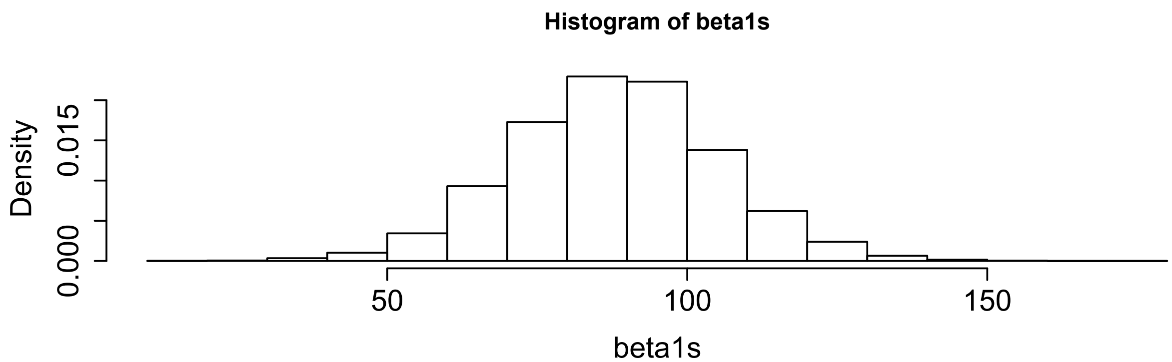

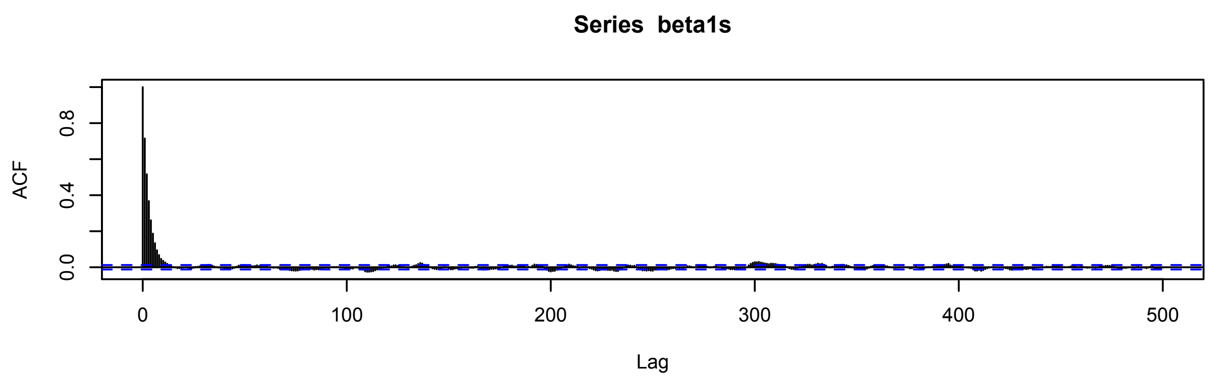

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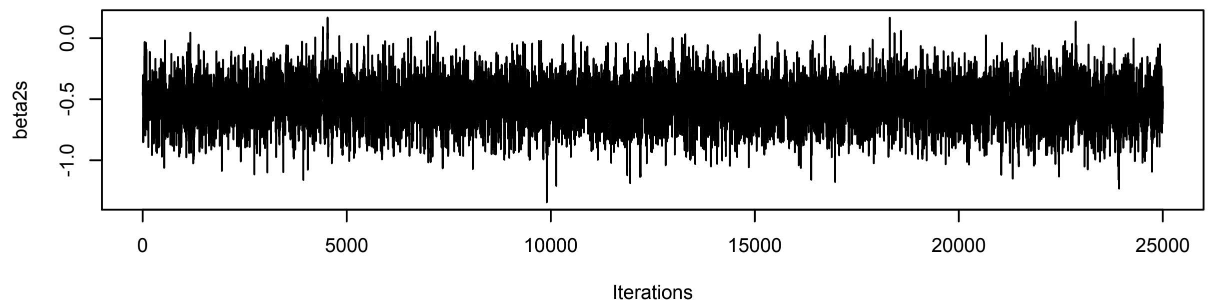

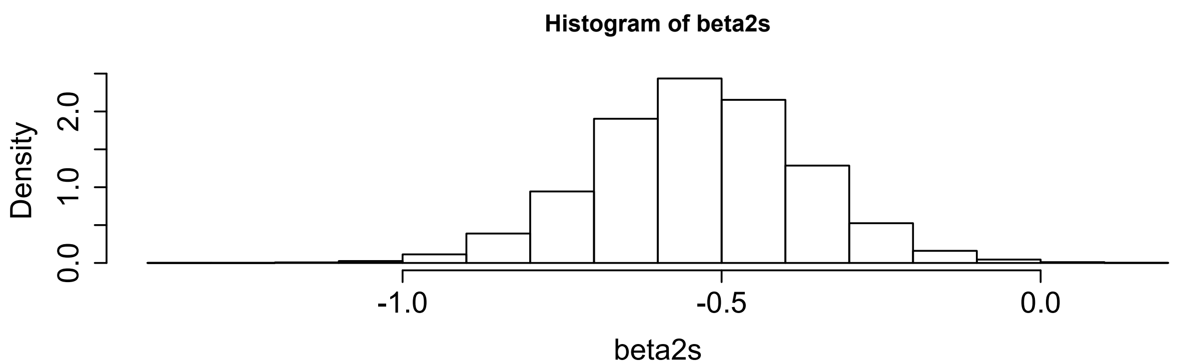

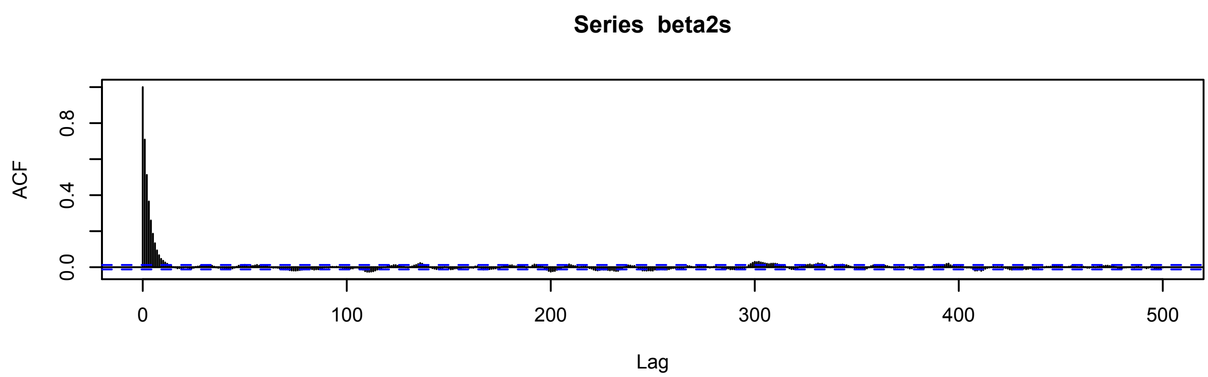

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