Problem 2

Theoretical Expected Value

$$f(x) = \begin{cases} \frac{x-2}{2} & 2 \le x \le 3 \\ \frac{2-x/3}{2} & 3 \le x \le 6 \end{cases}$$

$$E[X] = \int_2^3 x \cdot \frac{x-2}{2}\,dx + \int_3^6 x \cdot \frac{2-x/3}{2}\,dx$$

$$= \frac{1}{2}\left[\int_2^3 x^2 - 2x\,dx + \int_3^6 2x - \frac{x^2}{3}\,dx\right]$$

$$= \frac{1}{2}\left[\frac{x^3}{3} - x^2\Big|_2^3 + x^2 - \frac{x^3}{9}\Big|_3^6\right]$$

$$= \frac{1}{2}\left(\frac{22}{3}\right) = \boxed{3.66}$$