Lisa Over
Final Exam Problem 1
May 5, 2015

**METHOD**

The 25,000 independent values from the following probability density were obtained using rejection sampling with a uniform proposal density.

$$f(x) = \begin{cases} \dfrac{x-2}{2} & if\ 2 \le x \le 3 \\ \dfrac{2 - {}^{x}/_{3}}{2} & if\ 3 \le x \le 6 \end{cases}$$

To generate the 25,000 independent values from $f(x)$, a random uniform value, u, was drawn, transformed to be within the domain of $f(x)$, i.e., $x = u * 4 + 2$, and then “proposed” as being a value from the target density $f(x)$. This value was accepted as being from $f(x)$ if the probability that it was from $f(x)$ was greater than the probability represented by another independent uniform random value.

The probability that the value x was from $f(x)$ was calculated as one of the following two ratios depending on the value of x:

$p = \frac{\frac{x-2}{2}}{A*0.25}$ when $2 \le x \le 3$ and where A = 2

$p = \frac{\frac{2-{}^{x}/_{3}}{2}}{A*0.25}$ when $3 < x \le 6$ and where A = 2

The height of the uniform proposal density over the 4-unit interval [2, 6] is 0.25. To guarantee that the calculated ratio is a value between (0, 1), the denominator must always be greater than or equal to any value in the numerator, i.e., any value in the range of $f(x)$. The factor required to raise the height of the proposal density to 0.5, which is the maximum height of $f(x)$, is A = 2.

A summary of the acceptance procedure and criteria for x follow:

Draw a second random uniform value, u2, to act as a comparison probability. If the probability that x is a value from the target density, $p$, is greater than u2, accept x as being a value from $f(x)$. Otherwise, reject x.

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**CODE**

```
#The proposal density is uniform over a 4-unit interval so the height of the proposal density is 0.25. The maximum height of the target density is 0.5 so multiply 0.25 by 2 (A=2) so the height of the proposal density is greater than the height of the target density for all f(x).
A = 2
real.s = NULL
N = 25000

for(i in 1:N) {
	#Draw a uniform independent random variable and transform it to be within the domain of the given density, 2 <= x <= 6.
	x = runif(1) * 4 + 2

	#Determine what part of the piecewise function to use based on the value of x and set the numerator equal to the appropriate function and the denominator equal to A times the height of the uniform density over the four unit interval.
	if (x <= 3) {
		num = (x - 2)/2
		denom = A*0.25
		}
	else {
		num = (2 - (x/3))/2
		denom = A*0.25
		}
	#Calculate the probability that the previously generated uniform is from the target density
	prob <- num/denom

	#Generate a uniform independent random variable to use as a comparison probability: if the probability, prob, that the transformed x uniform could be from the given distribution is greater than the probability represented by this newly generated uniform random variable, select the x transformed uniform and store in real.s
	if(runif(1) < prob) {
		real.s <- c(real.s,x)
	}
}
hist(real.s)
mean(real.s)
#[1] 3.664185
```

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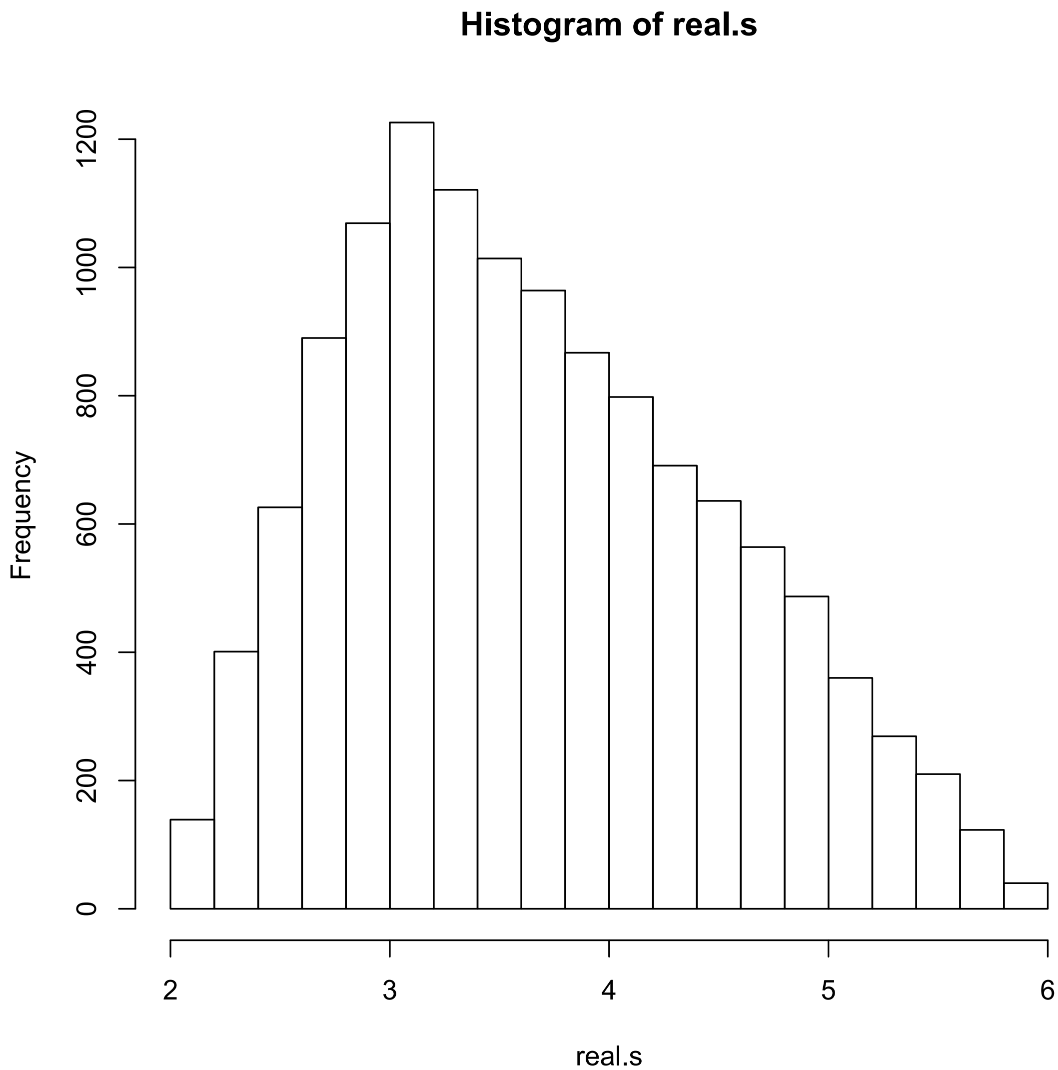
**RESULTS**




The mean of the 25,000 independent realizations from $f(x)$ is 3.664185.
The theoretical mean of $f(x)$ is 3.66 (see notes).