$Y_i \sim N(\beta_0+\beta_1 x+\beta_2 x^2, \sigma^2) \rightarrow$ Likelihood

Y_i is normal with mean $\beta_0+\beta_1 x+\beta_2 x^2$ and constant variance σ^2.

Joint Posterior with uniformed priors

i. $$\left[\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{1}{2\sigma^2}(y_i-(\beta_0+\beta_1 x_i+\beta_2 x_i^2))^2}\right]\frac{1}{\sigma^2}\cdot\frac{1}{10000}\cdot\frac{1}{10000}\cdot\frac{1}{10000}$$

$$\propto \left[\frac{1}{\sigma^2}\right]^{\frac{n}{2}+1} e^{-\frac{1}{2\sigma^2}(y_i-(\beta_0+\beta_1 x_i+\beta_2 x_i^2))^2}$$

ii. Full conditional for σ^2

$$\alpha = n/2 \quad \lambda = \frac{\sum(y_i-(\beta_0+\beta_1 x_i+\beta_2 x_i^2))^2}{2}$$

$$(\sigma^2 \mid \beta_0, \beta_1, \beta_2, \vec{y}, \vec{x}, \vec{x^2}) \sim IG\left(\frac{n}{2}, \frac{\sum(y_i-(\beta_0+\beta_1 x_i+\beta_2 x_i^2))^2}{2}\right)$$

Full conditional for β_0

$$\prod_{i=1}^{n} e^{-\frac{1}{2\sigma^2}(y_i-\beta_1 x_i-\beta_2 x_i^2-\beta_0)^2}$$

$$= \prod_{i=1}^{n} e^{-\frac{1}{2\sigma^2}\left[(y_i-\beta_1 x_i-\beta_2 x_i^2)^2-2\beta_0(y_i-\beta_1 x_i-\beta_2 x_i^2)+\beta_0^2)\right]}$$

$$= e^{-\frac{1}{2\sigma^2}\left[\sum_{i=1}^{n}(y_i-\beta_1 x_i-\beta_2 x_i^2)^2-2\beta_0\sum_{i=1}^{n}(y_i-\beta_1 x_i-\beta_2 x_i^2)-n\beta_0^2\right]}$$

divide by n $\rightarrow$

$$= e^{-\frac{n}{2\sigma^2}\left[\frac{\sum_{i=1}^{n}(y_i-\beta_1 x_i-\beta_2 x_i^2)^2}{n} - 2\beta_0 \frac{\sum_{i=1}^{n}(y_i-\beta_1 x_i-\beta_2 x_i^2)}{n} + \beta_0^2\right]}$$

$$\propto e^{-\frac{n}{2\sigma^2}\left(\beta_0 - \frac{\sum_{i=1}^{n}(y_i-\beta_1 x_i-\beta_2 x_i^2)}{n}\right)^2}$$

$$(\beta_0 \mid \sigma^2, \beta_1, \beta_2, \vec{y}, \vec{x}, \vec{x^2}) \sim N\left(\frac{\sum_{i=1}^{n}(y_i-\beta_1 x_i-\beta_2 x_i^2)}{n}, \frac{\sigma^2}{n}\right)$$

Full Conditional for β_1

$$\prod_{i=1}^{n} e^{-\frac{1}{2\sigma^2}(y_i-\beta_0-\beta_2 x_i^2-\beta_1 x_i)^2}$$

$$= \prod_{i=1}^{n} e^{-\frac{1}{2\sigma^2}\left((y_i-\beta_0-\beta_2 x_i^2)^2 - 2\beta_1 x_i(y_i-\beta_0-\beta_2 x_i^2) + (\beta_1 x_i)^2\right)}$$

$$= e^{-\frac{1}{2\sigma^2}\left(\sum_{i=1}^{n}(y_i-\beta_0-\beta_2 x_i^2) - 2\beta_1 \sum_{i=1}^{n} x_i(y_i-\beta_0-\beta_2 x_i^2) + \beta_1^2 \sum_{i=1}^{n} x_i^2\right)}$$

$$= e^{-\frac{\sum_{i=1}^{n} x_i^2}{2\sigma^2}\left(\frac{\Sigma(y_i-\beta_0-\beta_2 x_i^2)}{\Sigma x_i^2} - \frac{2\beta_1 \Sigma x_i(y_i-\beta_0-\beta_2 x_i^2)}{\Sigma x_i^2} + \beta_1^2\right)}$$

$$\propto e^{-\frac{\sum_{i=1}^{n} x_i^2}{2\sigma^2}\left(\beta_1 - \frac{\Sigma x_i(y_i-\beta_0-\beta_2 x_i^2)}{\Sigma x_i^2}\right)^2}$$

$$(\beta_1 \mid \sigma^2, \beta_0, \beta_2, \vec{y}, \vec{x}, \vec{x}^2) \sim N\left(\frac{\Sigma x_i(y_i-\beta_0-\beta_2 x_i^2)}{\Sigma x_i^2}, \frac{\sigma^2}{\Sigma x_i^2}\right)$$

Full Conditional for β_2

$$\prod_{i=1}^{n} e^{-\frac{1}{2\sigma^2}(y_i-\beta_0-\beta_1 x_i-\beta_2 x_i^2)^2}$$

$$= \prod_{i=1}^{n} e^{-\frac{1}{2\sigma^2}\left[(y_i-\beta_0-\beta_1 x_i)^2-2\beta_2 x_i^2(y_i-\beta_0-\beta_1 x_i)+(\beta_2 x_i^2)^2\right]}$$

$$= e^{-\frac{1}{2\sigma^2}\left[\sum_{i=1}^{n}(y_i-\beta_0-\beta_1 x_i)^2-2\beta_2\sum_{i=1}^{n}x_i^2(y_i-\beta_0-\beta_1 x_i)+\beta_2^2\sum_{i=1}^{n}x_i^4\right]}$$

$$= e^{\frac{-\sum_{i=1}^{n}x_i^4}{2\sigma^2}\left[\frac{\sum_{i=1}^{n}(y_i-\beta_0-\beta_1 x_i)^2}{\Sigma x_i^4}-2\beta_2\frac{\sum_{i=1}^{n}x_i^2(y_i-\beta_0-\beta_1 x_i)}{\Sigma x_i^4}+\beta_2^2\right]}$$

$$\propto e^{\frac{-\sum_{i=1}^{n}x_i^4}{2\sigma^2}\left[\beta_2-\frac{\Sigma x_i^2(y_i-\beta_0-\beta_1 x_i)}{\Sigma x_i^4}\right]^2}$$

$$(\beta_2 \mid \sigma^2, \beta_0, \beta_1, \vec{y}, \vec{x}, \vec{x}^2) \sim N\left(\frac{\sum_{i=1}^{n}x_i^2(y_i-\beta_0-\beta_1 x_i)}{\sum_{i=1}^{n}x_i^4}, \frac{\sigma^2}{\sum_{i=1}^{n}x_i^4}\right)$$