Identifying Factors, Primes, and Composites[i];
Shikaku Puzzles and Deductive Reasoning[ii]

Grade Level and Content: Pre-algebra, 7th or 8th Grade Mathematics

Big Idea: Students will review what it means for a number to be a factor of another, review the difference between prime and composite numbers, review divisibility rules and create a quick reference, and apply this knowledge to logic puzzles to develop deductive reasoning skills.

Objectives:

Students will be able to…

1. Correctly define key terms: prime number, composite number, divisible, multiple, and factor.
2. Correctly classify numbers as either prime or composite given the numbers.
3. Correctly identify factor pairs of given numbers.
4. Correctly construct different rectangles from the same number of tiles with each based on different set of factor pairs of that number given graph paper and the number of tiles to be used.
5. Articulate, orally and in writing, solutions and strategies to given Shikaku puzzles using mathematical terms and language.
6. Adequately demonstrate deductive reasoning skills given Shikaku puzzles.

Standards: Foundational concept for standard 2.1.8.E (Grade 8 Number Theory) which states, "Apply concepts of prime and composite numbers to calculate GCFs (Greatest Common Factor) and LCNs (Least Common Multiple) of numbers."

Meets standard 2.4.8.A (Grade 8 Reasoning) which states, "Draw inductive and deductive conclusions within mathematical contexts."

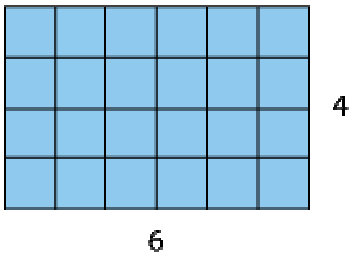
Meets standard 2.4.8.B (Grade 8 Connections) which states, "Use *if…then* statements to construct simple valid arguments."

Materials:

- 1 SMART™ Board with computer and projector
- 25 sheets of graph paper
- 25 copies: Guided Notes - Key Terms
- 25 copies: 2-page Shikaku puzzles[iii]
- 5 copies: 2-page 24-Tile Connect-the-Dots
- 25 copies: Deductive Puzzling Project
- 25 copies: Deductive Puzzling Rubric

Technology: The SMART Board will be used to present the anticipatory set, the key terms review, and the Shikaku puzzles and rules so the teacher and the students can explore concepts, solutions, and strategies together.

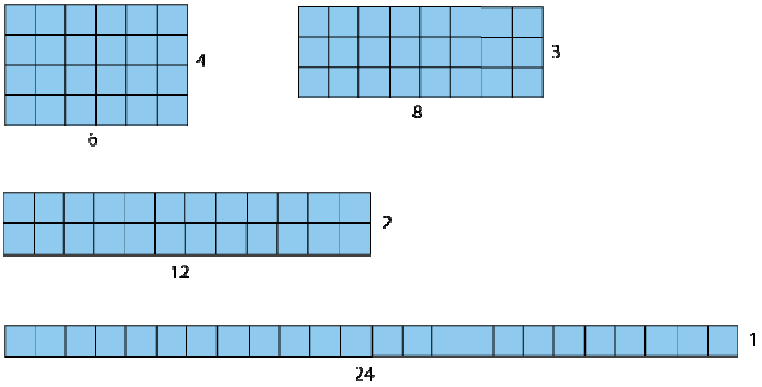
Anticipatory Set:

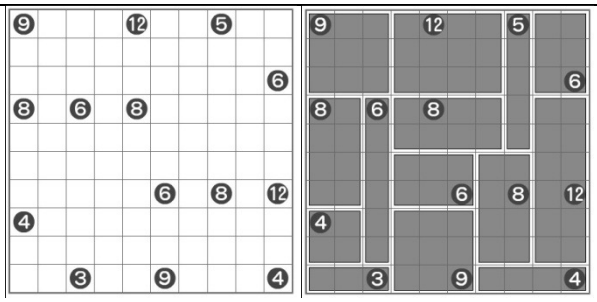
Objectives 3 and 4 3 minutes, 3 minutes	**Hand out** graph paper as students enter the room. **Hand out** 24-tile connect the dots to students with special needs. **Display** the following rectangle on the SMART Board along with the question for students to answer about the rectangle. Consider this 4 X 6 rectangular arrangement of 24 tiles. On your graph paper, draw all the other rectangular arrangements that can be made using exactly 24 tiles. Label the dimensions of each arrangement. 

Procedure:

Objectives 1, and 2 8 minutes, 11 minutes	Today we are going to review factors, prime numbers, and composite numbers. After this lesson, you will be able to identify factor pairs of a number and the rectangular arrangement of the number based on its factor pairs. You will also have a chance to apply your new knowledge to solve some fun puzzles. Let's get started. **Hand out** Guided Notes – Key Terms to special needs students. **Display** on the SMART Board and reveal gradually to follow discussion below: 1. A *factor* is a smaller part of a larger number. Both the factor and the larger number are whole numbers. 2. When a whole number greater than 1 is *divisible* by a second whole number without a remainder, the second number is a *factor* of the first and the first is a multiple of the second. Does anyone remember what a *factor* is? Can you tell me? What does it mean for a number to be *divisible* by another? What does it mean for a number to be a *multiple* of another? Let's complete these sentences. **Display** sentences on the SMART Board and write in missing terms as students say them: 1. We say that 20 is _____ by 5 because 20 divided by 5 has no remainder. ✓ divisible 2. We say that 20 is a _____ of 5 because 20 is divisible by 5. ✓ multiple 3. We say that 5 is a _____ of 20 because 20 is divisible by 5. ✓ Factor 4. We say that 44 is _____ by 11 because 44 divided by 11 has no remainder.

	✓ divisible 5. We say that 44 is a _____ of 11 because 44 is divisible by 11. ✓ multiple 6. We say that 11 is a _____ of 44 because 44 is divisible by 11. ✓ factor Now let's review some other terms. **Display** on the SMART Board and reveal gradually to follow discussion below: • A *prime number* has exactly two factors: 1 and itself. • A *composite number* has more than two factors. Does anyone remember what a *prime number* is? Are there any whole numbers that are a part of a prime number? Can you tell me what they are? What about a *composite number*? How many factors does a composite number have? Work with your partner to tell me whether the following numbers are prime or composite. For each number, list the number of factors and group them in factor pairs. You have 2 minutes. **Set Timer.** Go. **Display** numbers and instructions on the SMART Board. Tell whether the following numbers are prime or composite and group their factors in pairs: 1. 2 2. 10 3. 11 4. 17 5. 21 6. 28 7. 31 8. 51 **Display** on the SMART Board gradually to follow discussion: 1. 2 Prime with two factors (2 X 1) 2. 10 Composite with four factors (10 X 1) and (2 X 5) 3. 11 Prime with two factors (11 X 1) 4. 17 Prime with two factors (17 X 1) 5. 21 Composite with four factors (21 X 1) and (3 X 7)

	6. 28 Composite with six factors (28 X 1), (2 X 14), and (4 X 7) 7. 31 Prime with two factors (31 X 1) 8. 51 Composite with four factors (51 X 1) and (3 X 17)
Objectives 3 and 4 5 minutes, 16 minutes	**Display** grid on the SMART Board. Now take out your graph paper with the rectangle problem you did at the beginning of class. **Select** three students randomly to come up and draw another way to represent 24 tiles. Can you draw another way to represent 24 tiles in a rectangle? What are the factors of 24? We want to arrange the tiles in a rectangle so we need two numbers that, when multiplied together, make 24. Our example used the factors 6 and 4. What are some other factors of 24? ✓ 8 and 3 ✓ 12 and 2 ✓ 24 and 1 **Display** 24-tiles rectangle on the SMART Board if needed. 
Objectives 5 and 6 20 minutes, 36 minutes	Now we are going to apply our knowledge of factor pairs and rectangles to solve a fun puzzle called Shikaku. Look at the sample puzzle and its solution on the board while I pass out the puzzle sheets. **Hand out** 2-page Shikaku puzzles**.** **Display** puzzle and solution on the SMART Board.[iv]



What do you think are the rules and goal for solving this game?

- ✓ Divide large square into regions so that areas match the number given
- ✓ Regions must be rectangles (squares are a kind of rectangle)

The Japanese word "Skikaku" means "four corners" or "rectangle."

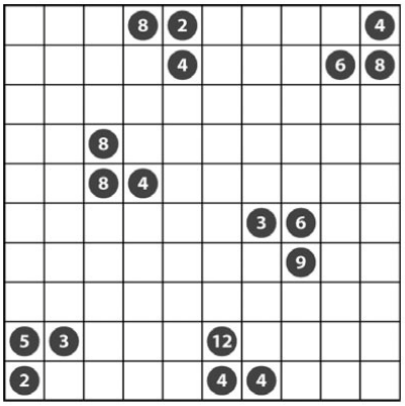
Write these instructions on your handout.

Display the rules on the SMART Board and discuss.

Rules:

1. Subdivide the large square along the grid lines into rectangular regions
2. Only one number can appear in each rectangle
3. Each square on the grid can be used in only one rectangle (no rectangles can overlap)

Display on the SMART Board the first Shikaku from the handout - without the solution. [v]



Puzzle 1

The handout I just gave you has a few Shikaku puzzles. Look at the first one and work with your partner to find a solution.

Consider and write a response to the question, "What can I see that could be a possible starting point in solving this Shikaku puzzle?"

You have 3 minutes. **Set Timer.** Go.

Time is up. Now let's talk about how you approached the puzzle. Any ideas on how to get started?

- ✓ Find places where there is only one way to place a rectangle.

Let's find a rectangle that can only go one way?

There are a couple of places you could start this particular puzzle, but let's look together at the 2 in the lower left corner? What factors does the number 2 have?

- ✓ 2 and 1

What do we know about a rectangle with only two tiles?

- ✓ We know that 2 can be represented by only 2 tiles in a line.

How can the 2-tile rectangle be oriented in our puzzle? Orientation means direction. There are two ways we can orient a 2-tile rectangle.

- ✓ We can orient a 2-tile rectangle vertically or horizontally.

When we write the dimensions of a rectangle, we write the horizontal side first and the vertical side second. So, a 2 X 1 rectangle has its longest side on the horizontal, while a 1 X 2 rectangle has its longest side on the vertical. Switching the factors in a pair does not increase the number of factors. The factors of the number 2 are still 1 and 2.

Refer to the Shikaku puzzle on the SMART Board for the following discussion.

If 2 can be represented by only a 2 X 1 or a 1 X 2 rectangle, what tiles can we block off to represent this number 2 in our puzzle? What do we know?

- ✓ We know there are no tiles below or to the left of our number 2 so we cannot orient our rectangle horizontally to the left of or vertically down from our number 2.
- ✓ We know we cannot include any other numbers in our rectangle so we cannot orient our rectangle vertically up from our number 2.
- ✓ If we cannot orient our rectangle up or down from or to the left of our number 2, then we can only orient our rectangle horizontally to the right of our number 2.

Draw the 2-tile rectangle on the SMART Board.

Now look at the number 3 in the lower left corner.

What tiles could we block out for this number 3? What do we know about its factor pairs?

- ✓ A rectangle of 3 tiles can only be a 3 X 1 or a 1 X 3 rectangle.

What orientation possibilities do we have with this number?

- ✓ Horizontally to the right or left of the number 3 or vertically up or down from the number 3.

What do we know about the rules and our rectangle that tells us where we can orient this rectangle?

- ✓ We know we cannot orient it down from its number because we do not have enough tiles.
- ✓ We know we cannot orient it to the left of its number because of the number 5.
- ✓ If we cannot orient our rectangle down from or to the left of our number, then we can only orient it up from or to the right of its number.

Do we have enough information to complete this 3-tile rectangle? Is there only

one possible way t orient it?

- ✓ No

Let's come back to this later when we have more information. Now, let's look at the number 5 in the lower left corner.

What tiles could we block out for this number 5? What do we know about its factor pairs?

- ✓ We know a rectangle of 5 tiles can only be a 5 X 1 or a 1 X 5 rectangle.
- ✓ We know we cannot orient it down from or to the left of its number because we do not have enough tiles.
- ✓ If we can only have 5 tiles in a line and we cannot go down or left, we can only go up or to the right.

Do we have enough information to decide where to place this rectangle? What else do we know?

- ✓ We know we cannot include any other numbers in our rectangle.
- ✓ If we cannot include the number 3, we can only block 5 tiles up from its number.

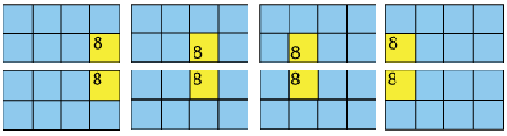
Draw the 5-tile rectangle on the SMART Board.

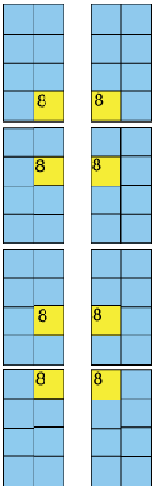
Now look at the number 8 in the upper right corner.

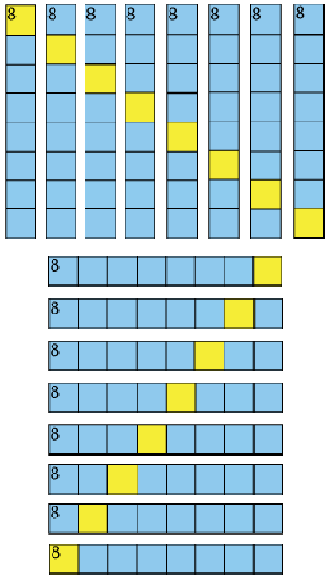
What are the possible arrangements of 8 tiles? What are the factors of 8?

- ✓ 8X 1
- ✓ 2 X 4

Display all of the possible arrangements of 8-tiles on the SMART Board and reveal gradually for the following discussion.







	What tiles could we block out for this number 8? What do we know about its possible orientation? ✓ A rectangle of 8 tiles could be a 4 X 2 rectangle oriented in one of four directions: Up and right, up and left, down and right, or down and left from its number. ✓ A rectangle of 8 tiles could be a 2 X 4 rectangle oriented in one of four directions: Up and right, up and left, down and right, or down and left from its number. ✓ A rectangle of 8 tiles could be an 8 X 1 rectangle oriented to the right or left of its number. ✓ A rectangle of 8 tiles could be a 1 X 8 rectangle oriented up from or down from its number. Of all those choices, can you see that there is only one way our 8-tile rectangle can be drawn? ✓ We see that there is not enough room on the puzzle to go up from or right of the number 8. ✓ We know we cannot go left because of the number 6. ✓ If we cannot go up, right, or left, we must go down. What 8-tile rectangle will fit without also infringing on the number 6? Can we pick with certainty? ✓ We know a 2 X 4 rectangle or a 4 X 2 rectangle would include the number 6 even if we orient them down from the number. ✓ If we cannot make a 2 X 4 or a 4 X 6 rectangle, we must make a 1 X 8 rectangle. **Draw** a 1 X 8 rectangle on the SMART Board.

Closing:

3 minute, 39 minutes	**Display** homework on the SMART Board. **Homework:** Tonight, work on the other three Shikaku puzzles on that page. For each rectangle that you find, write some notes to justify the rectangle's dimensions and orientation. Consider the following when you write: • "I know" and "If…then…" language • The facts of factor pairs, prime numbers, and composite numbers • The Shikaku puzzle rules **Hand out** Deductive Puzzling Project and Rubric. This is a Deductive Puzzling Project that is due in one week. We will talk more about this tomorrow when we go over the puzzles you are looking at tonight.

[i] Content idea from Foresman and Wesley, 1999

[ii] Content idea from Wanko, 2010
[iii] Page 2 of handout from Wanko, 2010
[iv] Nikoli.com, 2010
[v] Wanko, 2010