Arcs and Chords
Properties for Understanding and Use[i]

Grade Level and Content: Geometry, 9th or 10th Grade Mathematics

Big Idea: Students will be assigned one of four circle and chord constructions based on a theorem. Each student will construct the circle and chords following given specifications. Students will measure angles or line segments and interpret and discuss the results with other students assigned the same construction/theorem. In a jigsaw activity, each student will report and demonstrate their assigned theorem to students who were assigned a different theorem. As a class, students will prove the theorems and then use the theorems to analyze and solve a real-life problem.

Objectives:

Students will be able to …

1. Correctly construct circles, chords, and perpendicular line segments (either bisecting and non-bisecting).
2. Correctly use mathematical terminology and notation to discuss a given construction and to report on and demonstrate the results of their constructions.
3. Correctly use properties and relations of geometric figures (knowledge of properties of central angles and methods for proving triangles congruent) to make conjectures about their constructions and to prove given theorems.
4. Correctly analyze a given problem: list known information, the information to be found, and missing information.
5. Correctly develop a plan to solve a given problem.

Standards: Standard 2.4.G.A (Geometry Reasoning) which states, "Write formal proofs (direct proofs, indirect proofs/proofs by contradiction, use of counter-examples, truth tables, etc.) to validate conjectures or arguments."

Standard 2.5.G.A (Geometry Problem Solving) which states, "Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts."

Standard 2.5.G.B (Geometry Communication) which states, "Use symbols, mathematical terminology, standard notation, mathematical rules, graphing, and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas, and results."

Standard 2.9.G.A (Geometry Definitions, Properties, and Relations) which states, "Identify and use properties and relations of geometric figures; create justifications for arguments related to geometric relations."

Materials:

- 1 SMART™ Board with computer and projector
- 25 copies: Arcs and Chords Student Assignment Sheet

Technology: The SMART Board will be used to present the anticipatory set, the real-life problem, and the theorem proofs. The SMART Board enables the teacher and the students to work out solutions and proofs together.

Anticipatory Set:

<table>
<tr>
<td>3 minutes,
3 minutes</td>
<td>

Display Prior Knowledge slide.

Display Making Waves slide.

Can you think of an action from your own experience that caused waves to radiate out from a central point?

- ✓ Jumping into a pool
- ✓ Rain in a puddle
- ✓ Throwing rocks into a stream
- ✓ Beating with an electric mixer

Where were the effects the strongest?

- ✓ Center

You know where the center is in all of these cases, because you created or witnessed the cause. We have to use what we can witness about an earthquake to figure out where the epicenter is, because we cannot see underneath the earth.

Display Japan Quake 2011 slide.[ii]

The **epicenter** of an earthquake is the point on the earth's surface that is directly above the point where the earthquake originated.

The epicenter of the 2011 earthquake in Japan is located in the ocean east of the country.

How and why do scientists locate the epicenter of an earthquake?

Note: Locating the epicenter of an earthquake helps scientists predict and prepare for future quakes.

</td>
</tr>
</table>

Procedure:

<table>
<tr>
<td>Objectives 1 and 2

10 minutes,
13 minutes</td>
<td>

Construct and Discuss

Display Construct and Discuss slide and explain/direct.

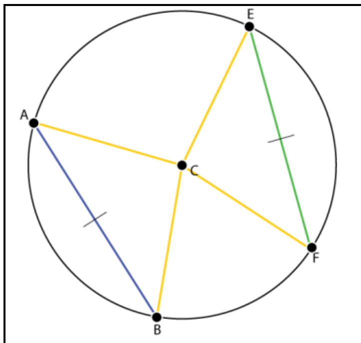
- Use your compass and ruler to create your assigned construction.
- Get into a group with students who have the same construction.
- Interpret and discuss
  - What can you propose about your construction?
  - What does the theorem on your colored card tell you about your construction?

</td>
</tr>
</table>

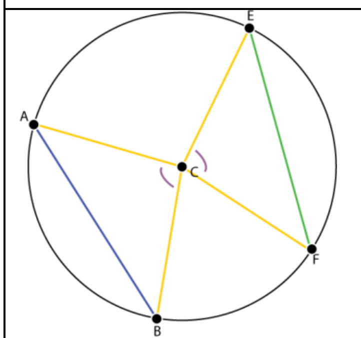
	Display next slide and monitor group discussions. Construction 1 1. What do you know about the measure of two minor arcs when their central angles are congruent? ✓ The two minor arcs are congruent. 2. What can you propose about the measure of two minor arcs when their corresponding chords are congruent? ✓ When chords are congruent, their corresponding arcs are congruent. Construction 2 1. What is the chord **DM**? ✓ Diameter 2. What can you propose about the relationship between two chords that are perpendicular to one another when one passes through the center? ✓ The diameter bisects the chord. Construction 3 1. What point does **DM** pass through? ✓ Center 2. What can you propose about the relationship between two chords when one is the perpendicular bisector of the other? ✓ The 'bisecting' chord (the one that bisects the other) is a diameter. Construction 4 1. What can you propose about the relationship between two congruent chords and the center of a circle? ✓ Two congruent chords are equidistant from the center. *Pass out theorem cards when students start discussing their constructions.* Construction 1 get Theorem 10.5 Construction 2 get Theorem 10.6 Construction 3 get Theorem 10.7 Construction 4 get Theorem 10.8
Objective 1 and 2 7 minutes, 20 minutes	**Report and Demonstrate** *Display Report and Demonstrate slide and explain/direct.* • Form groups based on your colored card. There should be four students per group. • Report and demonstrate your construction to the other three students. • Explain the theorem and how your construction is an example of a specific case.
Objective 3 10 minutes, 30 minutes	**Prove and Support** *Display Prove and Support slide and explain.* The diameter of the wave we analyzed is hundreds of miles long. You demonstrated a specific case for each of your theorems. How do scientists know that these theorems work for very large circles?

Display [Theorem 10.5](#) slide.

Students work on the SMART board to prove the theorem.



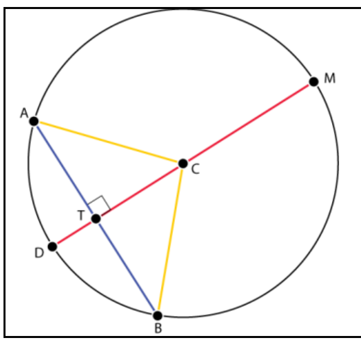
Statement	Reason
AB ≅ EF	Given
CA ≅ CB≅ CE ≅ CF	Radii (Constructed)
ΔACB ≅ ΔECF	SSS
∠ACB ≅ ∠ECF	CPCTC



Statement	Reason
CA ≅ CB≅ CE ≅ CF; ∠ACB ≅ ∠ECF	Given (Constructed)
Arc AB ≅ Arc EF	Congruent central angles ⇔congruent arcs
ΔACB ≅ ΔECF	SAS
AB ≅ EF	CPCTC

Display [Theorem 10.6](#) slide and explain.

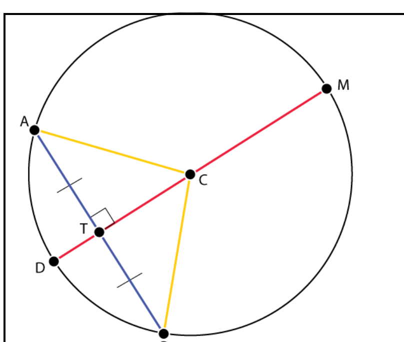
Students work on the SMART board to prove the theorem.



Statement	Reason
Diameter DM ⊥ Chord AB	Given (Constructed)
CA ≅ CB	Radii (Constructed)
CT ≅ CT	Reflexive
ΔACB ≅ ΔECF	Hypotenuse Leg
AT ≅ BT	CPCTC

Display [Theorem 10.7](#) slide and explain.

Students work on the SMART board to prove the theorem.



Statement	Reason
DM ⊥ Chord AB; AT ≅ BT	Given (Constructed)
CA ≅ CB	Radii (Constructed)
∠ACB ≅ Minor Arc AB	Congruent central angles ⇔congruent arcs
DM is a diameter	DM bisects chord AB ⇔ DM bisects Minor Arc AB ⇔ DM bisects ∠ACB and thus passes through C

Objectives 4 and 5 10 minutes, 40 minutes	**Analyze, Plan, and Solve: Earthquake** *Display Analyze, Plan, and Solve: Earthquake slide.* Seismometers measure the quake's intensity and the distance from the device to the epicenter. But this distance does not tell the direction. The epicenter could be **x** miles away in any number of directions. At the close of this lesson, I will explain how the seismometer works. *Display next slide.* For now, look at these three black dots. They represent seismometer locations where the intensity reading is the same. **Note:** The method we will use in this lesson is only one method and not the most common method. Typically, scientists have different readings from three seismometers. I will explain how they find the epicenter with these readings if we have time at the end of class. What do you think these same readings mean? ✓ The identical readings indicate that the seismometers are the same distance from the epicenter. If they are the same distance from the epicenter, what can we say about their location with respect to each other? ✓ They are all on the same circle. *Click to show the intensity wave.* This is the intensity wave where our three seismometers reside. What do we know about chords that will help us find the epicenter? ✓ Diameters, which pass through the center, are perpendicular bisectors of other chords. What can we add to this drawing to help us solve it? ✓ Draw lines to connect the dots and to create chords of the outer wave ring. ✓ Construct two diameters by drawing the perpendicular bisector of each chord. *Students draw chords and perpendicular bisectors on the SMART board to find the epicenter.* Where is the epicenter? ✓ The epicenter is where the two diameters intersect. Let's see if we were accurate… *Click twice to show the chords and then the perpendicular bisectors.* *Click to show the epicenter.*

Closing:

5 minute, 45 minutes	*Display Measuring Seismic Waves slide.* Just for fun, let's look at how seismometers determine the distance to the epicenter. Earthquakes produce two different kinds of waves. Primary waves (P-waves) and secondary waves (S-waves). The P-waves travel almost twice as fast as the S-waves. In my animation, the red arc represents a P-wave and the blue arc represents an S-wave. *Click to demonstrate the waves.* Seismometers measure the time difference between the arrival of the P-waves and the S-waves.

	Watch again and notice that the P-wave arrives much sooner than the S-wave. *Click to demonstrate the waves again.* Scientists use this time difference to determine the distance to the epicenter. *Display Intersecting Circles slide.* The most common method of finding the epicenter uses three seismometer readings like we did, but the readings do not have to be identical. Scientists draw circles around each seismometer location making each location the center of a circle. Where the circles intersect is the location of the epicenter. This more common method uses the timing of the P-waves and S-waves to determine the distance to the epicenter. Those distances then become the radii of the respective circles. **Homework:** Finish Earthquake problem if we didn't in class. Prove Theorem 10.8. Solve assigned application, problem solving, and reasoning problems in the book.

---

[i] Content idea from Larson, Roland E, Boswell, Laurie, and Stiff, Lee (1998), Geometry: An Integrated Approach, McDougal Littell, Chapter 10, p. 499-504

[ii] Japan Map from http://en.wikipedia.org/wiki/2011_T%C5%8Dhoku_earthquake_and_tsunami