$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ $\mu_{\bar{x}} = \mu$

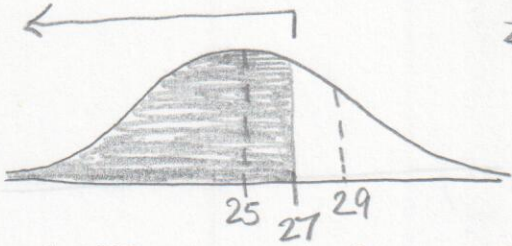
Sampling Distribution Practice

Sampling distribution of the mean of normally distributed data

1. I eat a breakfast cereal every day. The amount of saturated fat in a serving is normally distributed with mean 25 g and standard deviation 4 g.

"n" may be small if population is normal.

   a. Find the probability that my saturated fat intake on any day is below 27 g.



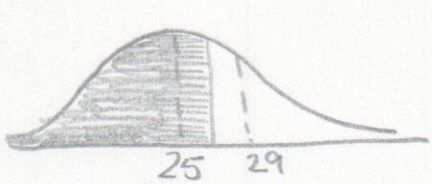
$$z = \frac{x-\mu}{\sigma} = \frac{27-25}{4} \cdot \frac{2}{4} = \frac{1}{2}$$

$$P(x < 27) = 0.6915$$

   b. If I eat it every day for a week, find the sampling distribution of the daily average fat intake over the week. Use it to find the probability that the average daily saturated fat intake for the week was less than 27 g.

$n = 7$

$$z = \frac{\mu_{\bar{x}} - \mu}{\sigma/\sqrt{n}} = \frac{27-25}{4/\sqrt{7}} = \frac{2}{1.5119} = 1.32$$



$$P(\mu_{\bar{x}} < 27) = 0.9032$$

   c. If I eat it for a 30-day month, what is the probability that the average daily saturated fat intake for the month was less than 27 g.

$n = 30$

$$z = \frac{\mu_{\bar{x}} - \mu}{\sigma/\sqrt{n}} = \frac{27-25}{4/\sqrt{30}} = \frac{2}{0.7303} = 2.7386$$

$$P(\mu_{\bar{x}} < 27) = 0.9968$$

These three examples show how increasing sample size leads to a sample mean that tends to be closer and closer to the true mean μ. Although on any given day there is quite a high probability of fat intake over 27 g, once you aggregate over several days this probability goes down dramatically.

   d. What is the probability that my total saturated fat intake for the month was less than 810g?

$$z = \frac{27-25}{4/\sqrt{30}}$$

$\mu_{\bar{x}} = \frac{810}{30} = 27$ g on average $n = 30$

$P(\mu_{\bar{x}} < 27) = 0.9968$ (same as part c)

Sampling Distribution Practice

e. What is the probability that it was less than780 g? for the month $n=30$

$z = \frac{26-25}{4/\sqrt{30}} = \frac{1}{0.7303}$ $\mu_{\bar{x}} = \frac{780}{30} = 26$

$z = 1.37$

$P(\mu_{\bar{x}} < 26) = 0.9147$

Central limit theorem

2. The weight of adult males has a mean of around 65 kg and a standard deviation of 20 kg. The distribution is not normal. n must be 30 or more.

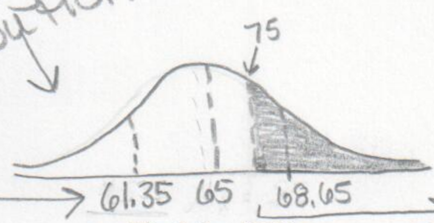
   a. Can you calculate the probability that a randomly selected individual weighs more than 75 kg?

   No, because $n=1$ is not large enough. We cannot infer anything about an individual in a population where we don't know the distribution.

   b. What is the probability that the average weight of 30 randomly selected males will exceed 75 kg? $\sqrt{30} = 5.48$

   $n=30$ $z = \frac{75-65}{20/\sqrt{30}} = \frac{10}{3.65} = 2.74$

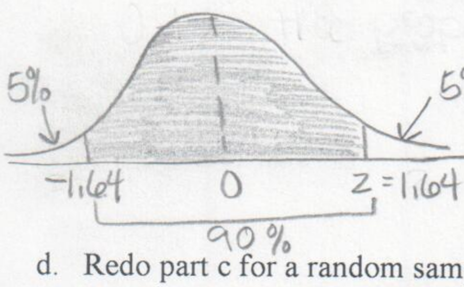
   sampling distribution



   $(65-\sigma/\sqrt{n})$

   $P(\mu_{\bar{x}} < 75) = 0.9969$ $P(\mu_{\bar{x}} > 75) = 1-0.9969$
   $= 0.0031$

   c. Find the 90% range ~~within which~~ [that will contain] the average weight of a random sample of 30 adult males ~~will lie~~? $P(Z<z) = 0.95$ $z=1.64$



   $1.64 = \frac{\mu_{\bar{x}} - 65}{20/\sqrt{30}} = \frac{\mu_{\bar{x}} - 65}{3.65}$

   $\mu_{\bar{x}} = 65 \pm (1.64)(3.65)$
   $(65+5.99, 65-5.99)$
   $(59, 71)$

   d. Redo part c for a random sample of 100 adult males.

   $z = 1.64$ $20/\sqrt{100} = 2$

   $\mu_{\bar{x}} = 65 \pm (1.64)(2)$
   $(65+3.28, 65-3.28)$
   $(61.72, 68.28)$

Sampling Distribution Practice

3. The amount of baggage a plane passenger checks is random, with a mean of 20 lbs and a standard deviation of 30 pounds. Nothing is stated about the distribution.

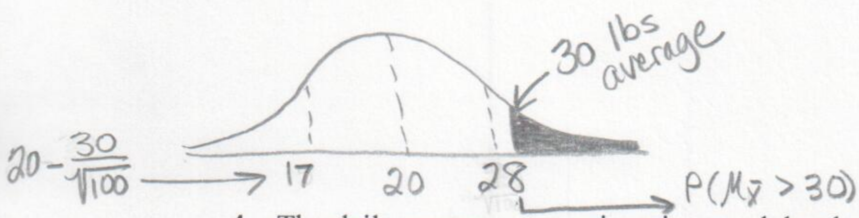
   a. What is the probability that a passenger checks more than 30 pounds of baggage?

   We cannot say anything about an individual because the distribution is unknown.

   b. A plane carries 100 passengers. It can handle 3000 pounds of checked baggage, that is, a maximum of 30 pounds per passenger. What is the probability that a random load of 100 passengers will check too much baggage for the plane to handle?

   $n=100$

   $$z = \frac{30-20}{30/\sqrt{100}} = \frac{10}{3} = 3.33$$



   $$P(\mu_{\bar{x}} > 30) = 1 - 0.9996 = 0.0004$$

4. The daily revenue at a university snack bar has been recorded for the past five years. Records indicate that the mean daily revenue is \$3500 and the standard deviation is \$550. The distribution is skewed to the right due to several high volume days (football game days). Suppose that 100 days are randomly selected and the average daily revenue computed. Which of the following describes the sampling distribution of the sample mean?

   $\mu_{\bar{x}} = \mu = 3500 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{550}{10} = 55$

   A) normally distributed with a mean of \$3500 and a standard deviation of \$550
   B) normally distributed with a mean of \$3500 and a standard deviation of \$55
   C) normally distributed with a mean of \$350 and a standard deviation of \$55
   D) skewed to the right with a mean of \$3500 and a standard deviation of \$550

5. The number of cars running a red light in a day, at a given intersection, possesses a distribution with a mean of 2.7 cars and a standard deviation of 4. The number of cars running the red light was observed on 100 randomly chosen days and the mean number of cars calculated. Describe the sampling distribution of the sample mean.

   $\sigma_{\bar{x}} = \frac{4}{\sqrt{100}} = 0.4$

   A) approximately normal with mean = 2.7 and standard deviation = 0.4
   B) shape unknown with mean = 2.7 and standard deviation = 4
   C) shape unknown with mean = 2.7 and standard deviation = 0.4
   D) approximately normal with mean = 2.7 and standard deviation = 4

   Sampling Distribution will be Normal since $n > 30$.

Sampling Distribution Practice

6. Suppose a random sample of n = 36 measurements is selected from a population with mean μ = 256 and variance $\sigma^2 = 144$. Find the mean and standard deviation of the sampling distribution of the sample mean $\bar{x}$.

$\sigma^2 = 144$ so $\sigma = 12$

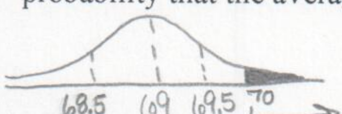
$\mu_{\bar{x}} = 256 \qquad \sigma_{\bar{x}} = \frac{12}{\sqrt{36}} = 2$

7. Suppose a random sample of n = 64 measurements is selected from a population with mean μ = 65 and standard deviation σ = 12. Find the z- score corresponding to a value of $\bar{x} = 68$.

$\mu_{\bar{x}} = 68 \qquad Z = \frac{68-65}{12/\sqrt{64}} = \frac{3}{1.5} = 2$

8. The average score of all golfers for a particular course has a mean of 69 and a standard deviation of 5. Suppose 100 golfers played the course today. Find the probability that the average score of the 100 golfers exceeded 70.

$Z = \frac{70-69}{0.5} = \frac{1}{1/2} = 2$



$\frac{\sigma}{\sqrt{n}} = \frac{5}{10} = \frac{1}{2}$

$Z = 2$

$P(\mu_{\bar{x}} > 70) = 1 - P(\mu_{\bar{x}} < 70)$

$= 1 - 0.9772 = 0.0228$

9. The scores of students on the ACT college entrance examination have a Normal distribution with mean 18.6 and standard deviation of 5.9.

   a. What is the probability that a single student randomly chosen has an ACT score less 21?

   $Z = \frac{21-18.6}{5.9} = \frac{2.4}{5.9} = 0.41$

   $P(x < 21) = 0.6591$

   b. What is the probability that 10 randomly chosen students have a mean score less than 18?

   $n = 10 \qquad \sigma/\sqrt{n} = \frac{5.9}{\sqrt{10}} = 1.866$

   $Z = \frac{18-18.6}{1.866}$

   $Z = -0.32 \qquad P(\mu_{\bar{x}} < 18) = 0.3745$

   c. What is the probability that 20 randomly chosen students have a sum score greater than 380?

   $\frac{\sigma}{\sqrt{n}} = \frac{5.9}{\sqrt{20}} = 1.319$

   average score

   $\frac{380}{20} = 19$

   $Z = \frac{19-18.6}{1.319} = 0.3$

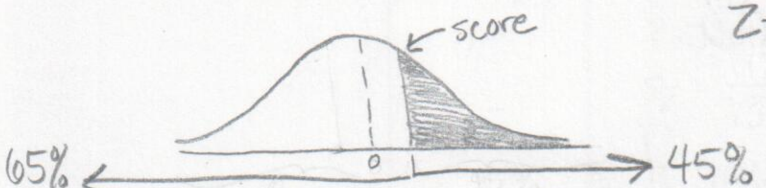
   $P(\mu_{\bar{x}} > 19) = 1 - P(\mu_{\bar{x}} < 19)$

   $= 1 - 0.6179$

   $= 0.3821$

d. The probability that a single student randomly chosen has an ACT score above a certain score is 0.45. What is the score?

$P(Z > z) = 0.45$ look up 0.65 in body of z-table to find z



$z = 0.38$

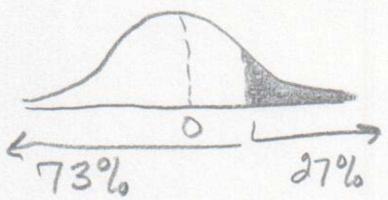
$0.38 = \frac{X - 18.6}{5.9}$ $X = (5.9)(0.38) + 18.6$

$X = 20.8$

e. The probability that 10 randomly chosen students have an average score greater than a certain average is 0.27. What is the average score?

look up 0.73 in body of z-table

$z = 0.61$



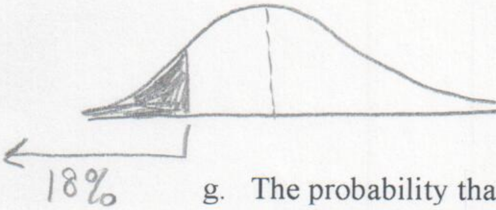
$0.61 = \frac{\mu_{\bar{x}} - 18.6}{5.9/\sqrt{10}}$ $\mu_{\bar{x}} = (0.61)(1.866) + 18.6$

$\mu_{\bar{x}} = 19.74$

f. The probability that 10 randomly chosen students have an average score less than a certain average is 0.18. What is the average score?

look up 0.18 in body of z-table

$z = -0.89$



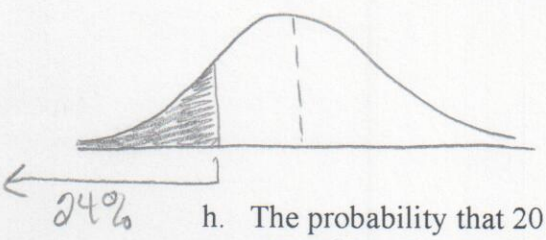
$-0.89 = \frac{\mu_{\bar{x}} - 18.6}{5.9/\sqrt{10}}$ $\mu_{\bar{x}} = (-0.89)(1.866) + 18.6$

$\mu_{\bar{x}} = 16.9$

g. The probability that 20 randomly chosen students have a total score less than a certain total 0.24. What is the total score?

look up 0.24 in body of z-table

$z = -0.69$



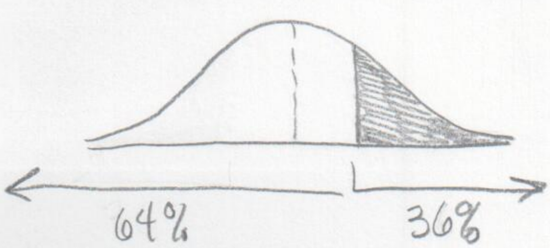
$-0.69 = \frac{\mu_{\bar{x}} - 18.6}{5.9/\sqrt{20}}$ $\mu_{\bar{x}} = (-0.69)(1.319) + 18.6$

$\mu_{\bar{x}} = 17.7$ Total is $17.7 \times 20 = 353.8$

h. The probability that 20 randomly chosen students have a total score more than a certain total 0.36. What is the total score?

look up 0.64 in body of z-table

$z = 0.36$



$0.36 = \frac{\mu_{\bar{x}} - 18.6}{5.9/\sqrt{20}}$ $\mu_{\bar{x}} = (0.36)(1.319) + 18.6$

$\mu_{\bar{x}} = 19.1$

Total is $19.1 \times 20 = 381.5$