Key

Test 2
Probability and Distributions

Section 1: Probability

1. The following table gives the sex and age group of college students at a Midwestern university.

	Female	Male	Total
15 to 17 years	89	61	150
18 to 24 years	5,668	4,697	10,365
25 to 34 years	1,904	1,589	3,493
35 years or older	1,660	970	2,630
Total	9,321	7,317	16,638

One student is to be selected at random.

a. The probability that the selected student is female is...(2pts)

$$P(\text{female}) = \frac{9321}{16638} = \boxed{0.5602}$$

b. The probability that the selected student is 25 to 34 years old given that the student is male is...(3pts)

$$P(25\text{-}34 \mid \text{male}) = \frac{1589}{7317} = \boxed{0.2172}$$

2. A survey of university recreation users finds that 52% are female, 68% are students, and 35% are female and a student. Make a table and answer the following questions.

	Female	Male	Total
Student	0.35	0.33	0.68
Employee	0.17	0.15	0.32
Total	0.52	0.48	1

a. Complete the table. (5pts)

b. The conditional probability that a user is male given that he is an employee is...(3pts)

$$P(\text{male} \mid \text{employee}) = \frac{0.15}{0.32} = \boxed{0.4688}$$

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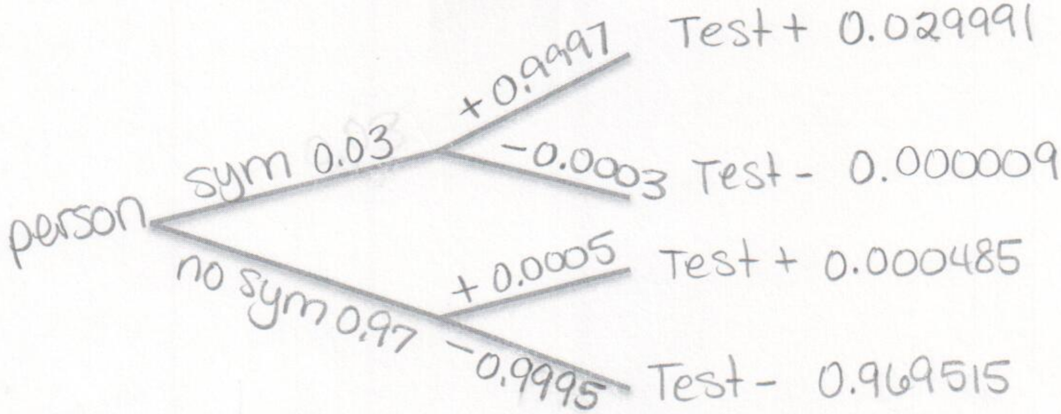
3. The following table lists the probabilities of positive and negative test results for diagnosing a particular disease when patients experience symptoms and when patients do not experience symptoms.

	Test positive	Test negative
Symptoms present	0.9997	0.0003
No symptoms present	0.0005	0.9995

Suppose 3% of a large population experience symptoms.

a. Make a tree diagram for selecting a person from this population and performing the test to determine if the person has the disease. (10pts)



b. What is the probability that the test is positive for a randomly chosen person from this population? (3pts)

$P(\text{positive}) = \boxed{0.030476}$

4. Based upon past experience, 40% of all customers at the Campus Pizza Place order the meat lover's pizza. If a random sample of five customers is selected, what is the probability that (9pts) $n=5$ $p=0.4$

Count those who order meat lover's so X is number of people who order meat lover's.

a. None order the meat lover's pizza?

$P(X=0) = {}_5C_0 (0.4)^0 (0.6)^5$
$= (1)(1)(0.07776) = \boxed{0.07776}$

b. At least four order the meat lover's pizza?

$P(X \geq 4) = P(X=4) + P(X=5)$
$= {}_5C_4 (0.4)^4 (0.6)^1 + {}_5C_5 (0.4)^5 (0.6)^0 = (5)(0.0256)(0.6) + (1)(0.01024)(1)$
$= \boxed{0.08704}$

c. Not more than three order the meat lover's pizza?

$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$= 1 - P(X \geq 4)$
$= 1 - 0.08704 = \boxed{0.91296}$

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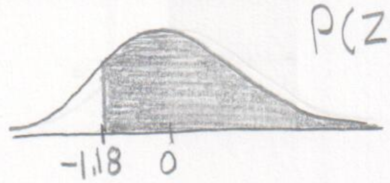
Section 2: Distributions

Draw a picture, label and shade it, and find the specified value.

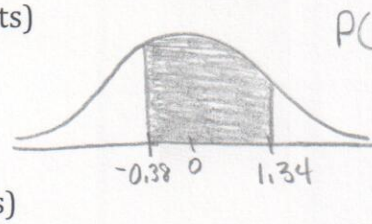
5. P(Z > -1.18) (3pts)



$P(Z<-1.18)=0.1190$

$P(Z>-1.18)=\boxed{0.881}$

6. P(-0.38 < Z < 1.34) (3pts)



$P(Z<-0.38)=0.3520$

$P(Z<1.34)=0.9099$

$P(-0.38<Z<1.34)$

$=0.9099-0.3520$

$=\boxed{0.5579}$

7. P(Z < k) = 0.9750 (5pts)

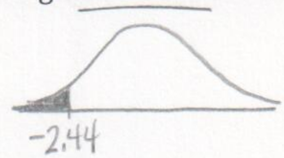
Reverse lookup

$\boxed{k=1.96}$

8. Suppose that the package of cookies lists the weight as 16 ounces. The actual weights of bags of cookies vary according to a Normal distribution with mean μ = 16.5 ounces and standard deviation σ = 0.205 ounces

   a) What proportion of bags of cookies weigh less than the advertised 16 ounces? (5pts)

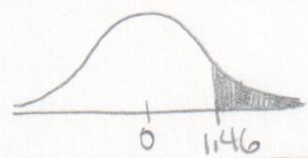
$Z=\dfrac{16-16.5}{0.205}=-2.44$



$P(X<16)=P(Z<-2.44)=\boxed{0.0073}$

   b) What proportion of bags of cookies weigh more than 16.8 ounces? (5pts)

$Z=\dfrac{16.8-16.5}{0.205}=\dfrac{0.3}{0.205}=1.46$



$P(X>16.8)=P(Z>1.46)=1-P(Z<1.46)=1-0.9279=\boxed{0.0721}$

   c) What is the weight such that only 1 bag of cookies in 500 weighs less than that amount? (5pts)

$\text{prob}=\dfrac{1}{500}=0.002$

Reverse lookup Z

$Z=-2.88$

$-2.88=\dfrac{X-16.5}{0.205}$

$X=(-2.88)(0.205)+16.5$

$\boxed{X=15.9\text{ oz}}$

CHOOSE EITHER **part d** OR **part e**. That is, only answer **one** of the following two questions. (6pts)

d) If the manufacturer wants to adjust the production process so that only 1 bag of cookies in 500 weighs less than the advertised weight, what should the mean of the actual weights be (assuming that the standard deviation of the weights remains 0.205 ounces)?

prob = 0.002 $\quad Z = -2.88$

$$-2.88 = \frac{16 - \mu}{0.205}$$

$$(-2.88)(0.205) = 16 - \mu$$
$$-0.5904 = 16 - \mu$$
$$\mu - 0.5904 = 16$$
$$\mu = 16 + 0.5904 = \boxed{16.59}$$

e) If the manufacturer wants to adjust the production process so that the mean remains at 16.5 ounces but only 1 bag of cookies in 500 weighs less than the advertised weight, how small does the standard deviation of the weights need to be?

$Z = -2.88$

$$-2.88 = \frac{16 - 16.5}{\sigma}$$

$$\sigma = \frac{-0.5}{-2.88} = \boxed{0.1736}$$

9. What two checks do you need to perform before you can use the facts $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ to evaluate a sampling distribution of the sample mean? **Do both have to be true?** (3pts)

1. $n \geq 30$
2. Population is Normal

Either must be true – not both

10. What two checks do you need to perform before you can use the Normal approximation to the binomial distribution to evaluate binomial probabilities? **Do both have to be true?** (3pts)

1. $np \geq 10$
2. $n(1-p) \geq 10$

Both must be true

11. Twenty percent of American households own three or more TVs. A random sample of 200 American households is selected. Let X be the number of households selected that own three or more TVs.

a. The mean of X is...(2pts)

$\mu = np = (200)(0.2) = \boxed{40}$

b. The standard deviation of X is...(2pts)

$\sigma = \sqrt{np(1-p)} = \sqrt{32} = \boxed{5.6569}$

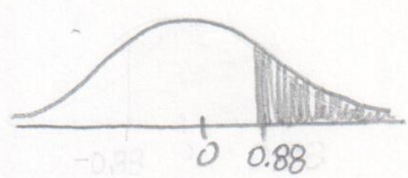
c. Interpret the mean and standard deviation in terms of the situation. (3pts)

In repeated sampling, the average number of households that own 3 or more TVs is 40. Typically this will vary by about 5.6569 or between 5 and 6 households.

d. Using the Normal approximation, the probability that at least 45 of the households selected own at least 3 or more TVs is...(5pts)

$np = 40$ ✓
$n(1-p) = 160$ ✓

$P(\text{household} \geq 45) = 1 - P(\text{household} < 45)$



$Z = \dfrac{45-40}{5.6569} = 0.88$

$P(\text{household} < 45) = P(Z < 0.88)$
$= 0.8106$

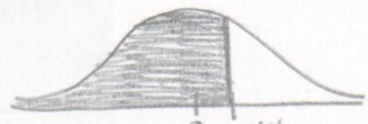
$P(Z > 0.88) = 1 - 0.8106 = \boxed{0.1894}$

Test 2
Probability and Distributions

12. The scores of students on the ACT college entrance examination have a Normal distribution with mean 19.3 and standard deviation of 6.2.

   a. What is the probability that a single student randomly chosen has an ACT score less 22? (5pts)

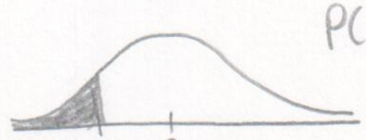
$$Z = \frac{22-19.3}{6.2} = 0.44 \qquad P(Z<0.44) = \boxed{0.67}$$



   b. What is the probability that 30 randomly chosen students have a mean score less than 18? (5pts)

$$n=30 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6.2}{\sqrt{30}} = 1.132 \qquad \mu_{\bar{x}} = \mu = 19.3$$

$$Z = \frac{18-19.3}{1.132} = -1.15 \qquad P(Z<-1.15) = \boxed{0.1251}$$

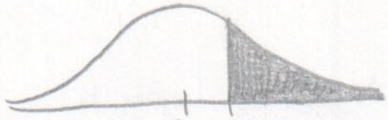


   c. **EXTRA CREDIT:** What is the probability that 30 randomly chosen students have a sum score greater than 600? (3pts)

$$\bar{X} = \frac{600}{30} = 20 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6.2}{\sqrt{30}} = 1.132 \qquad \mu_{\bar{x}} = \mu = 19.3$$

$$Z = \frac{20-19.3}{1.132} = 0.62 \qquad P(Z<0.62) = 0.7324$$

$$P(Z>0.62) = 1-0.7324 = \boxed{0.2676}$$



   d. **EXTRA CREDIT:** The probability that a single student randomly chosen has an ACT score above a certain score is 0.45. What is the score? (3pts)

$$P(Z>z) = 0.45 \qquad P(Z<z) = 0.55 \qquad z = 0.12$$

$$Z = 0.12$$

$$0.12 = \frac{X-19.3}{6.2} \qquad X = (0.12)(6.2) + 19.3$$

$$\boxed{X = 20.04}$$