Ch07 Study Guide
Sampling Distributions Practice Problems

Sampling Distribution of Sample Means

Example 1

Over the entire six years that students attend an Ohio elementary school, they are absent, on average, 27 days due to influenza. Assume that the standard deviation over this time period is σ = 8 days. Upon graduation from elementary school, a random sample of 34 students is taken and asked how many days of school they missed due to influenza.

a. What is the expected value for the sampling distribution of the number of school days missed due to influenza? $E(x) = 27$ days

b. What is the standard deviation for the sampling distribution of the number of school days missed due to influenza? $se(\bar{x}) = \frac{8}{\sqrt{34}} = 1.37$

c. The probability that the sample mean is less than 30 school days is _______. $Z = \frac{30-27}{1.37}$

$(\bar{X} < 30)$ =norm.dist(30, 27, 1.37, 1)

d. The probability that the sample mean is between 24 and 30 school days is _______.

$P(24 \le \bar{x} < 30)$ $\quad Z_1 = \frac{24-27}{1.37}$ $\quad Z_2 = \frac{30-27}{1.37}$

$\bar{X} = 24$ $\quad \bar{x} = 30$

Example 2

Suppose that, on average, electricians earn approximately μ = $58,000 per year in the United States. Assume that the distribution for electricians' yearly earnings is normally distributed and that the standard deviation is σ = $14,000.

a. Given a sample of four electricians, what is the standard deviation for the sampling distribution of the sample mean? $se(\bar{X}) = \frac{14000}{\sqrt{4}} = 7000$

b. What is the probability that the average salary of four randomly selected electricians exceeds $60,000? $P(\bar{X} > 60{,}000)$ $\quad Z = \frac{60{,}000 - 58{,}000}{7000}$

c. What is the probability that the average salary of four randomly selected electricians is less than $50,000? $P(\bar{x} < 50{,}000)$ $\quad Z = \frac{50{,}000 - 58{,}000}{7000}$

d. What is the probability that the average salary of four randomly selected electricians is more than $50,000 but less than $60,000?

$P(50{,}000 < \bar{X} < 60{,}000)$ $\quad Z_1 = \frac{50{,}000 - 58{,}000}{7000}$

$Z_2 = \frac{60{,}000 - 58{,}000}{7000}$

Example 3

Susan has been on a bowling team for 14 years. After examining all of her scores over that period of time, she finds that they follow a normal distribution. Her average score is 220, with a standard deviation of 10.

a. What is the probability that in a one-game playoff, her score is more than 225?
$P(X>225)$ single value so use σ $Z=\frac{225-220}{10}$

b. If during a typical week Susan bowls 16 games, what is the probability that her average score is more than 225?
$se(\bar{x})=\frac{10}{\sqrt{16}}=2.5$ $Z=\frac{225-220}{2.5}$ $P(\bar{x}>225)$

c. If during a typical week Susan bowls 16 games, what is the probability that her average score for the week is between 216 and 224?
$se(\bar{x})=2.5$ $Z_1=\frac{216-220}{2.5}$ $Z_2=\frac{224-220}{2.5}$ $P(216<\bar{x}<224)$

d. If during a typical month Susan bowls 64 games, what is the probability that her average score in this month is above 224?
$se(\bar{x})=\frac{10}{\sqrt{64}}=1.25$ $Z=\frac{224-220}{1.25}$ $P(\bar{x}>224)$

Example 4

Professor Elderman has given the same multiple-choice final exam in his Principles of Microeconomics class for many years. After examining his records from the past 10 years, he finds that the scores have a mean of 75 and a standard deviation of 8.

a. What is the probability that a class of 15 students will have a class average greater than 70 on Professor Elderman's final exam?
Cannot determine $n<30$

b. What is the probability that a class of 36 students will have an average greater than 70 on Professor Elderman's final exam?
$se(\bar{x})=\frac{8}{\sqrt{36}}=1.33$ $P(\bar{x}>70)$ $Z=\frac{70-75}{1.33}$

c. Professor Elderman offers his class of 36 a pizza party if the class average is above 80. What is the probability that he will have to deliver on his promise? (i.e., What is the probability the class will earn a pizza party?) $Z=\frac{80-75}{1.33}$
$se(\bar{x})=1.33$

d. What is the probability Professor Elderman's class of 36 has a class average below 77?
$se(\bar{x})=1.33$ $P(\bar{x}<77)$ $Z=\frac{77-75}{1.33}$

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Sampling Distribution of Sample Proportions

Example 1

A university administrator expects that 20% of students in a core course will receive an A. He looks at the grades assigned to 66 students. $P = 0.2$ $n = 66$

a. What are the expected value and the standard error for the proportion of students that receive an A? $E(\bar{p}) = 0.2$

b. The probability that the proportion of students that receive an A is 0.15 or less is _______.
$se(\bar{p}) = \sqrt{\frac{(0.2)(0.8)}{66}} = 0.0492$ $Z = \frac{0.15 - 0.2}{0.0492}$ $P(\bar{p} < 0.15)$

c. The probability that the proportion of students who receive an A is between 0.15 and 0.30 is _______.
$se(\bar{p}) = 0.0492$ $Z_1 = \frac{0.30 - 0.2}{0.0492}$ $Z_2 = \frac{0.15 - 0.2}{0.0492}$ $P(0.15 < \bar{p} < 0.3)$

d. The probability that the proportion of students who receive an A is *not* between 0.15 and 0.25 is _______.
$1 - [P(\bar{p} < 0.25) - P(\bar{p} < 0.15)]$
$se(\bar{p}) = 0.0492$

Example 2

The labor force participation rate is the number of people in the labor force divided by the number of people in the country who are of working age and not institutionalized. The BLS reported in February 2012 that the labor force participation rate in the United States was 63.7% (Calculatedrisk.com). A marketing company asks 110 working-age people if they either have a job or are looking for a job, or, in other words, whether they are in the labor force.

a. What are the expected value and the standard error for a labor participation rate in the company's sample? $E(\bar{p}) = 0.637$ $se(\bar{p}) = \sqrt{\frac{(0.637)(0.363)}{110}} = 0.0458$

b. For the company's sample, the probability that the proportion of people who are in the labor force is greater than 0.67 is _______. $Z = \frac{0.67 - 0.637}{0.0458}$
$P(\bar{p} > 0.67)$

c. What is the probability that fewer than 60% of those surveyed are members of the labor force?
$P(\bar{p} < 0.6)$ $Z = \frac{0.6 - 0.637}{0.0458}$

d. What is the probability that between 58% and 62.5% of those surveyed are members of the labor force?
$P(0.58 < \bar{p} < 0.625)$ $Z_1 = \frac{0.58 - 0.637}{0.0458}$

$Z_2 = \frac{0.625 - 0.637}{0.0458}$

Example 3

Super Bowl XLVI was played between the New York Giants and the New England Patriots in Indianapolis. Due to a decade-long rivalry between the Patriots and the city's own team, the Colts, most Indianapolis residents were rooting heartily for the Giants. Suppose that 95% of Indianapolis residents wanted the Giants to beat the Patriots. $E(\bar{p}) = 0.95$

a. What is the probability that, of a sample of 100 Indianapolis residents, at least ~~20%~~ 12% were rooting for the Patriots in Super Bowl XLVI? $P(\bar{p} > 0.12)$ $Z = \frac{0.12 - 0.95}{0.0218}$ $\boxed{z = (0.12 - 0.05)/0.0218}$

$se(\bar{p}) = \sqrt{\frac{(0.95)(0.05)}{100}} = 0.0218$

b. What is the probability that from a sample of 100 Indianapolis residents, fewer than 90% were rooting for the Giants in Super Bowl XLVI?

$se(\bar{p}) = 0.0218$ $P(\bar{p} < 0.9)$ $Z = \frac{0.9 - 0.95}{0.0218}$

c. What is the probability that from a sample of 40 Indianapolis residents, fewer than 90% were rooting for the Giants in Super Bowl XLIV?

Cannot Determine $np = 38$ $n(1-p) = 2$

d. What is the probability that from a sample of 200 Indianapolis residents, fewer than ~~175~~ 185 were rooting for the Giants in Super Bowl XLIV?

$n = 200$ $se(\bar{p}) = 0.0154$ $\bar{p} = \frac{185}{200} = 0.925$ $P(\bar{p} < 0.925)$ $Z = \frac{0.925 - 0.95}{0.0154}$

Example 4

According to the 2011 Gallup daily tracking polls (www.gallup.com, February 3, 2012), Mississippi is the most conservative U.S. state, with 53.4 percent of its residents identifying themselves as conservative.

$E(\bar{x}) = 0.534$

a. What is the probability that at least 58% of a random sample of 200 Mississippi residents identify themselves as conservative? $P(\bar{p} > 0.58)$ $Z = \frac{0.58 - 0.534}{0.0353}$

$se(\bar{p}) = \sqrt{\frac{(0.534)(0.466)}{200}} = 0.0353$

b. What is the probability that at least 90 but fewer than 110 respondents of a random sample of 200 Mississippi residents identify as conservative? $Z_1 = \frac{0.45 - 0.534}{0.0353}$ $Z_2 = \frac{0.55 - 0.534}{0.0353}$

$se(\bar{p}) = 0.0353$ $P\left(\frac{90}{200} < \bar{p} < \frac{110}{200}\right) = P(0.45 < \bar{p} < 0.55)$

c. What is the probability that at least ~~50~~ 60 respondents of a random sample of 100 Mississippi residents do *not* identify themselves as conservative? $Z = \frac{0.6 - 0.466}{0.0499}$

$se(\bar{p}) = \sqrt{\frac{(0.534)(0.466)}{100}} = 0.0499$ $[1 - P(\bar{p} > 0.6)]$

d. What is the probability that fewer than 50 respondents of a random sample of 100 Mississippi residents do *not* identify themselves as conservative?

$se(\bar{p}) = 0.0499$ $[1 - P(\bar{p} < 0.5)]$ $Z = \frac{0.5 - 0.466}{0.0499}$