Lisa

Test 2 PRACTICE TEST
Probability and Distributions

Section 1: Probability

1. A basketball player makes 70% of her free throws. When fouled, she gets to take 2 free throws. Let *X* represent the number of free throws made in two tries. The probability distribution for the number of free throws she makes in two attempts is summarized in the following table:

Number of free throws made	0	1	2
Probability	0.09	0.42	0.49

Let X denote the number of free throws made.

a. Express in terms of X the probability that she makes at least one free throw.

$P(X \geq 1)$

b. Calculate the probability that she makes at least one free throw:

$P(X \geq 1) = P(X=1) + P(X=2) = 0.42 + 0.49 = 0.91$

OR $P(X \geq 1) = 1 - P(X=0) = 1 - 0.09 = \boxed{0.91}$

2. You roll a pair of fair dice and compute the number of spots on the two sides facing up. Denote this total by *X*. The probability distribution of *X* is

x	2	3	4	5	6	7	8	9	10	11	12
P(x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

a. The probability that *X* is a 2, 11, or 12 is $P(X=2 \text{ or } X=11 \text{ or } X=12) =$

$P(X=2) + P(X=11) + P(X=12) = 1/36 + 2/36 + 1/36 = 4/36 = 1/9$

$= \boxed{0.111}$

b. The probability that *X* is at least 7 is

$P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$+ P(X=11) + P(X=12)$

$= \dfrac{6+5+4+3+2+1}{36} = \dfrac{21}{36} = \dfrac{7}{12} = \boxed{0.583}$

Test 2
Probability and Distributions

3. The following table gives the sex and age group of college students at a Midwestern university.

	Female	Male	Total
15 to 17 years	89	61	150
18 to 24 years	5,668	4,697	10,365
25 to 34 years	1,904	1,589	3,493
35 years or older	1,660	970	2,630
Total	9,321	7,317	16,638

One student is to be selected at random.

The probability that the selected student is 25 to 34 years old is

$\frac{3493}{16,638} = \boxed{0.2099} \approx 21\%$

4. A survey of college students finds that 20% like country music, 15% like gospel music, and 10% like both country music and gospel music. Make a table and answer the following questions.

	Like Gospel	Not Like Gospel	Total
Like Country	0.1	0.1	0.2
Not Like Country	0.05	0.75	0.8
Total	0.15	0.85	1

a. The proportion of students that like gospel music but not country music is
0.05

b. The proportion of students that like neither country music nor gospel music is
0.75

c. The proportion of students that like either country music or gospel music is include those who like both but don't count twice

① $0.15+0.2-0.1=\boxed{0.25}$ OR ② $0.05+0.1+0.1=0.25$

d. The conditional probability that a student likes country music given that he or she likes gospel music is

The new whole is that the student likes gospel music. What proportion of those students also like country?

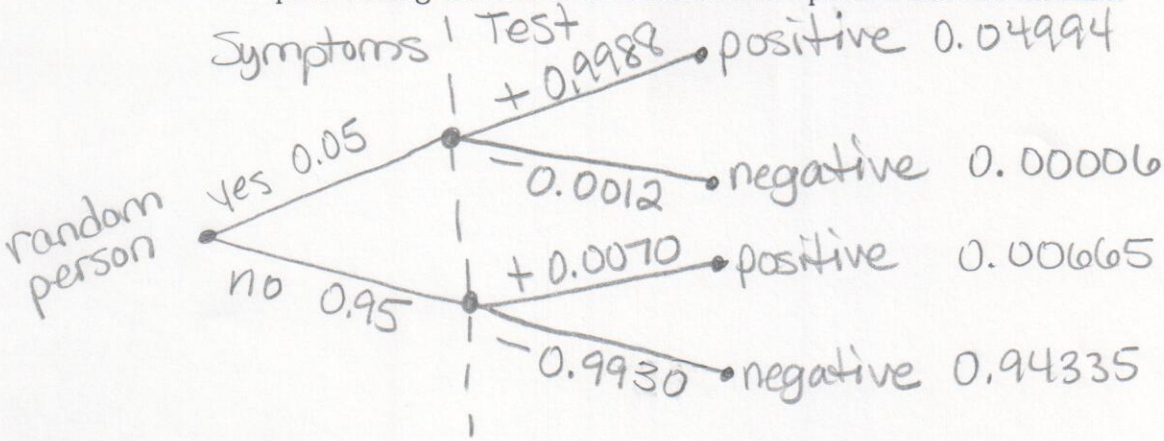
$\frac{0.1}{0.15} = \boxed{0.67}$

5. The following table lists the probabilities of positive and negative test results for diagnosing a particular disease when patients experience symptoms and when patients do not experience symptoms.

	Test positive	Test negative
Symptoms present	0.9988	0.0012
No symptoms present	0.0070	0.9930

Suppose 5% of a large population have ~~the disease~~ symptoms.

a. Make a tree diagram for selecting a person from this population and performing the test to determine if the person has the disease.



b. What is the probability that the test is positive for a randomly chosen person from this population?

$P(+) = 0.04994 + 0.00665 = \boxed{0.05659}$

6. Based upon past experience, ~~30%~~ 33% of all customers at Danny's Hamburger Shop order the special. If a random sample of three customers is selected, what is the probability that

$p = 0.33$

a. None order the special?

$P(X=0) = {}_3C_0\,(0.33)^0(0.67)^3 = (1)(1)(0.3008) = \boxed{0.3008}$

b. Two order the special?

$P(X=2) = {}_3C_2\,(0.33)^2(0.67)^1 = (3)(0.1089)(0.67) = \boxed{0.2189}$

c. At least two order the special?

$P(X\geq 2) = P(X=2) + P(X=3) = {}_3C_2(0.33)^2(0.67)^1 + {}_3C_3(0.33)^3(0.67)^0$

$= 0.2189 + (1)(0.0359)(1)$

$= \boxed{0.2548}$

d. Not more than two order the special?

$P(X\leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.3008 + {}_3C_1(0.33)^1(0.67)^2 + 0.2189$

$= 0.3008 + 0.4444 + 0.2189$

$\boxed{= 0.9641}$

Test 2
Probability and Distributions

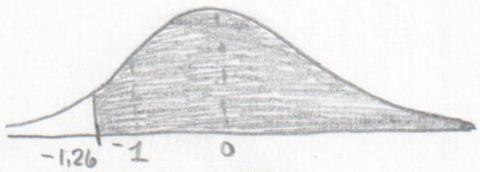
Section 2: Distributions

Draw a picture, label and shade it, and find the specified value.

7. $P(Z \geq -1.26)$

$P(Z < -1.26) = 0.1038$

$P(Z > -1.26) = \boxed{0.8962}$



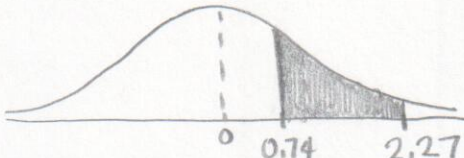
8. $P(0.74 < Z < 2.27)$

$P(Z < 2.27) = 0.9884$

$P(Z < 0.74) = 0.7704$

$P(0.74 < Z < 2.27) = 0.9884 - 0.7704$

$= \boxed{0.218}$



9. $P(Z < k) = \underbrace{0.9838}$

This is the probability so reverse lookup the z-score.

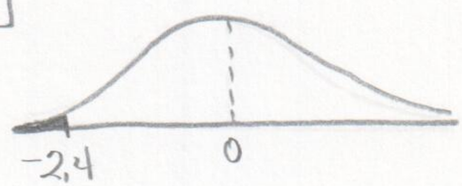
$z = 2.14$ $P(Z < 2.14) = 0.9838$

10. Suppose that the package of a bag of potato chips lists the weight as 16 ounces. The actual weights of bags of chips vary according to a Normal distribution with mean μ = 16.3 ounces and standard deviation σ = 0.125 ounces

a) What proportion of potato chip bags weigh less than the advertised 16 ounces?

$P(X < 16) = \boxed{0.0082}$

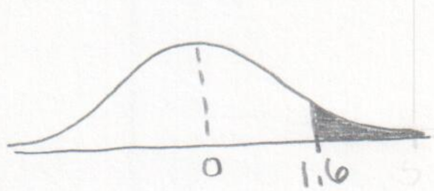
$z = \frac{16 - 16.3}{0.125} = -2.4$



b) What proportion of potato chip bags weigh more than 16.5 [redacted] ounces?

$P(X > 16.5) = \boxed{0.0548}$

$z = \frac{16.5 - 16.3}{0.125} = 1.6$



0.9452 $\quad$ $1 - 0.9452 = 0.0548$

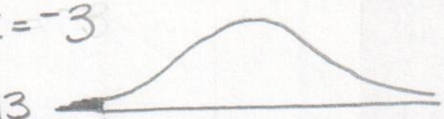
c) What is the weight such that only 1 bag of chips in 800 weighs less than that amount?

Find a weight

$P(Z < \_\_) = \frac{1}{800} = 0.0013$

$z = -3$

0.0013



$-3 = \frac{X - 16.3}{0.125}$

$(-3)(0.125) + 16.3 = X$

$X = \boxed{15.93}$

d) If the manufacturer wants to adjust the production process so that only 1 bag of chips in 800 weighs less than the advertised weight, what should the mean of the actual weights be (assuming that the standard deviation of the weights remains 0.125 ounces)?

Find a new mean

$P(Z < \_\_) = \frac{1}{800} = 0.0013$

$z = -3$

$-3 = \frac{16 - \mu}{0.125}$

$(-3)(0.125) - 16 = -\mu$

$-16.38 = -\mu$

$\boxed{\mu = 16.38}$

e) If the manufacturer wants to adjust the production process so that the mean remains at 16.3 ounces but only 1 bag of chips in 800 weighs less than the advertised weight, how small does the standard deviation of the weights need to be?

$P(Z < \_\_) = 0.0013$

$z = -3$

$-3 = \frac{16 - 16.3}{\sigma}$

$(-3)\sigma = 16 - 16.3$

$\sigma = \frac{-0.3}{-3} = 0.1$

$\boxed{\sigma = 0.1}$

Test 2
Probability and Distributions

11. What two checks do you need to perform before you can use the facts $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ to evaluate a sampling distribution of the sample mean? Do both have to be true?

(1) Population is Normal
(2) $n \geq 30$
One or the other – both are not necessary

12. What two checks do you need to perform before you can use the Normal approximation to the binomial distribution to evaluate binomial probabilities? Do both have to be true?

$np \geq 10$
$n(1-p) \geq 10$

Both must be true.

13. The amount of baggage a plane passenger checks is random, with a mean of 25 lbs and a standard deviation of 40 pounds.

a. What is the probability that a passenger checks more than 35 pounds of baggage?

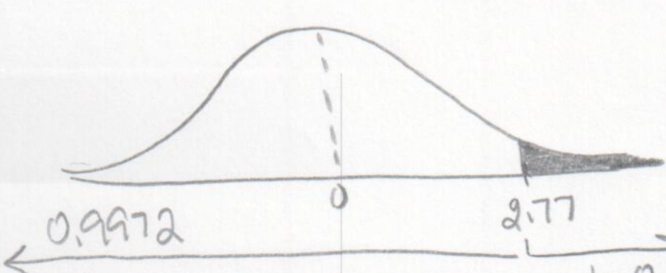
Cannot answer this one because nothing is stated about the population distribution and $n < 30$.

b. A plane carries 250 passengers. It can handle 8000 pounds of checked baggage, that is, a maximum of 32 pounds per passenger. What is the probability that a random load of 250 passengers will check too much baggage for the plane to handle?

$n = 250$

$$z = \frac{32-25}{40/\sqrt{250}} = \frac{7}{2.53} = 2.77$$

$$P(\bar{x} > 32) = \boxed{0.0028}$$



$1 - 0.9972 = 0.0028$

14. Twenty percent of American households own three or more cars. A random sample of 144 American households is selected. Let X be the number of households selected that own three or more cars.

a. The standard deviation of X is $n = 144 \quad p = 0.2 \quad (1-p) = 0.8$

$$\sigma = \sqrt{np(1-p)} = 4.8$$

b. The mean of X is

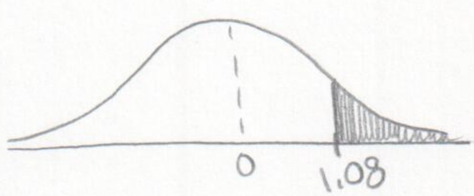
$$\mu = np = 28.8$$

c. Interpret the mean and standard deviation in terms of the situation.

In repeated samples, the average number of households that own three or more cars is 28.8. This number can vary by 4.8 households.

d. Using the Normal approximation, the probability that at least 34 of the households selected own at least 3 or more cars is

$$Z = \frac{34 - 28.8}{4.8} = 1.08$$



At least 34:

$$P(X \geq 34)$$
$$= P(Z \geq 1.08)$$
$$= 1 - P(Z \leq 1.08)$$
$$= 1 - 0.8599$$
$$= \boxed{0.1401}$$