Chapter 13 Solutions

13.1: Binomial. (1) We have a fixed number of observations (n = 15). (2) It is reasonable to believe that each call is independent of the others. (3) “Success” means reaching a live person, “failure” is any other outcome. (4) Each randomly dialed number has chance p = 0.2 of reaching a live person.

13.2: Not binomial. We do not have a fixed number of observations.

13.3: Not binomial. The trials aren’t independent. If one tile in a box is cracked, there are likely more tiles cracked.

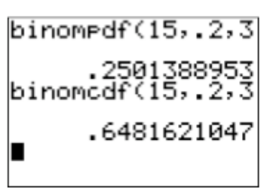
13.4: We have a fixed number of independent trials, each leading to success (used the Internet for personal reasons) or failure, with the probability of success constant from trial to trial. We’re counting the number of successes in our sample. Hence, the number in the sample who used the Internet is binomial in distribution with n = 1500 and p = 0.80.

13.5: (a) C, the number caught, is binomial with n = 10 and p = 0.7. M, the number missed, is binomial with n = 10 and p = 0.3. (b) We find $P(M = 3) = \binom{10}{3}(0.3)^3(0.7)^7 =$ $120(0.027)(0.08235) = 0.2668$. With software, we find $P(M \geq 3) = 0.6172$.

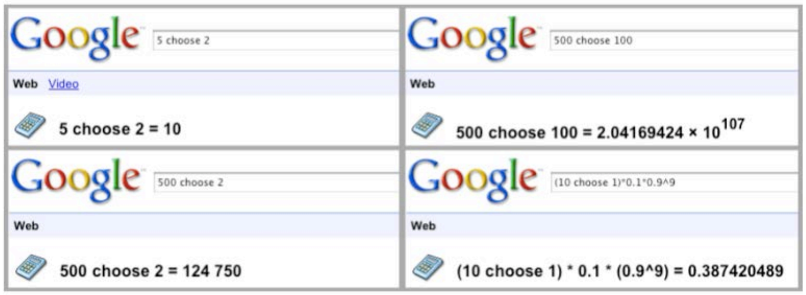
13.6: Let N be the number of live persons contacted among the 15 calls observed. Then N has the binomial distribution with n = 15 and p = 0.2.

(a) $P(N = 3) = \binom{15}{3}(0.2)^3(0.8)^{12} = 0.2501$.
(b) $P(N \leq 3) = P(N = 0) + \cdots + P(N = 3) = 0.6482$.
(c) $P(N \geq 3) = P(N = 3) + \cdots + P(N = 15) = 0.6020$.
(d) $P(N < 3) = P(N = 0) + \cdots + P(N = 2) = 0.3980$.
(e) $P(N > 3) = P(N = 4) + \cdots + P(N = 15) = 0.3518$.

The TI-83/84 screen shown illustrates the use of that calculator’s binompdf and binomcdf functions (found in the DISTR menu) to compute the first two probabilities. The first of these finds individual binomial probabilities, and the second finds cumulative probabilities (that is, it sums the probability from 0 up to a given number). Excel offers similar features with its BINOMDIST function. Calculators that do not have binomial probabilities may have a built-in function to compute the factorial, for example, $\binom{15}{3}$, which can then be multiplied by the appropriate probabilities.



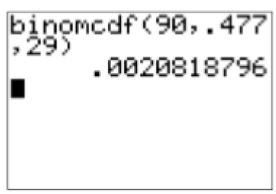
13.7: The screenshots below show Google's answers at the time these solutions were prepared. (a) 5 choose 2 returns 10. (b) 500 choose 2 returns 124,750, and 500 choose 100 returns $2.04169424 \times 10^{107}$. (c) (10 choose 1)*0.1*0.9^9 returns 0.387420489.



13.8: (a) With n = 15 and p = 0.2, we have $\mu = np$ = 3 calls. (b) $\sigma = \sqrt{np(1-p)} = \sqrt{2.4}$ = 1.5492 calls. (c) With p = 0.08, $\sigma = \sqrt{1.104}$ = 1.0507 calls; with p = 0.01, $\sigma = \sqrt{0.1485}$ = 0.3854 calls. As p approaches 0, the standard deviation decreases (that is, it also approaches 0).

13.9: (a) X is binomial with n = 10 and p = 0.3; Y is binomial with n = 10 and p = 0.7 (b) The mean of Y is (10)(0.7) = 7 errors caught, and for X the mean is (10)(0.3) = 3 errors missed. (c) The standard deviation of Y (or X) is $\sigma = \sqrt{10(0.7)(0.3)}$ = 1.4491 errors.

13.10: Let X be the number of 1's and 2's; then X has a binomial distribution with n = 90 and p = 0.477 (in the absence of fraud). This should have a mean of 42.93 and standard deviation $\sigma = \sqrt{22.4524}$ = 4.7384. Therefore, $P(X \le 29) = P\left(Z \le \frac{29-42.93}{4.7384}\right) = P(Z \le -2.94) = 0.0016$. (Using software, we find that the exact value is 0.0021.) This probability is quite small, so we have reason to be suspicious.



13.11: (a) μ = (1520)(0.31) = 471.2 and $\sigma = \sqrt{1520(0.31)(1-0.31)} = \sqrt{325.128}$ = 18.0313 students. (b) Note that $np = (1520)(0.31) = 471.2 \ge 10$ and $n(1-p) = (1520)(0.69) = 1048.8 \ge 10$, so n is large enough for the Normal approximation to be reasonable. The college wants 475 students, so $P(X \ge 476) = P\left(Z \ge \frac{476-471.2}{18.0313}\right) = P(Z \ge 0.27) = 0.3936$. (c) The exact probability is 0.4045 (obtained from software), so the Normal approximation is 0.0109 too low. For a better approximation, consider using the continuity correction, described in Exercise 13.43.

13.12: (a) μ = (1000)(0.24) = 240 and σ = $\sqrt{1000(0.24)(1-0.24)}$ = 13.5056 first generation Canadians. (b) To check whether the Normal approximation can be applied, note that *np* = 240 and *n*(1 − *p*) = 760 are both more than 10. We compute $P(210 \le X \le 270)$ = $P\left(\frac{210-240}{13.5056} \le Z \le \frac{270-240}{13.5056}\right) = P(-2.22 \le Z \le 2.22) = 0.9736$.

13.13: (b) He has 3 independent eggs, each with probability 1/4 of containing salmonella.

13.14: (b) $P(S \ge 1) = P(S > 0) = 1 - P(S = 0) = 1 - 0.4219 = 0.5781$.

13.15: (c) The selections are not independent; once we choose one student, it changes the probability that the next student is a business major.

13.16: (c) We must choose 3 of the 5 shots to be "made"; $\binom{5}{3}$ = 10. Note that answer (b) is only wrong for its computation… in fact, a correct answer to this problem would also be $\binom{5}{2}$ = 10 (not 20), since the act of deciding which 3 shots are made is equivalent to choosing which 2 shots are missed.

13.17: (a) This probability is $(0.60)^2(0.40)^3 = 0.02304$.

13.18: (a) Missing 3 shots means making 2 shots, so this probability is $\binom{5}{2}(0.4)^2(0.6)^3 = 0.2304$.

13.19: (b) This is the event that a single digit is 8 or 9, so the probability is 0.20.

13.20: (a) Two lines of the table means that we have 2(40) = 80 digits. This is the number of "successes" (8 or 9) in *n* = 80 independent trials with *p* = 0.20.

13.21: (a) The mean is *np* = (80)(0.20) = 16.

13.22: (a) No: There is no fixed number of observations. (b) A binomial distribution is reasonable here; a "large city" will have a population much larger than 100 (the sample size), and each randomly selected juror has the same (unknown) probability *p* of opposing the death penalty. (c) In a Pick 3 game, Joe's chance of winning the lottery is the same every week, so assuming that a year consists of 52 weeks (observations), this would be binomial.

13.23: (a) A binomial distribution is *not* an appropriate choice for field goals made, because given the different situations the kicker faces, his probability of success is likely to change from one attempt to another. (b) It would be reasonable to use a binomial distribution for free throws made because we have *n* = 150 attempts, presumably independent (or at least approximately so), with chance of success *p* = 0.8 each time.

13.24: (a) n = 20 and p = 0.25. (b) $\mu = np$ = 5 correct guesses. (c) $P(X = 5) =$

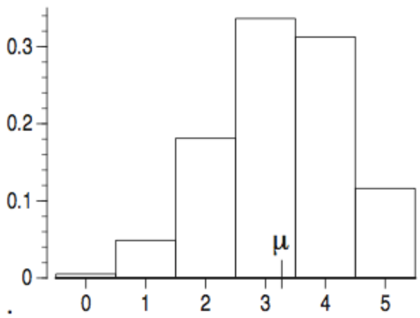
$\binom{20}{5}(0.25)^5(0.75)^{15} = 0.2023.$

13.25: (a) n = 5 and p = 0.65. (b) The possible values of X are the integers 0, 1, 2, 3, 4, 5. (c) All cases are computed:

$P(X = 0) = \binom{5}{0}(0.65)^0(0.35)^5 = 0.00525$ $P(X = 1) = \binom{5}{1}(0.65)^1(0.35)^4 = 0.04877$

$P(X = 2) = \binom{5}{2}(0.65)^2(0.35)^3 = 0.18115$ $P(X = 3) = \binom{5}{3}(0.65)^3(0.35)^2 = 0.33642$

$P(X = 4) = \binom{5}{4}(0.65)^4(0.35)^1 = 0.31239$ $P(X = 5) = \binom{5}{5}(0.65)^5(0.35)^0 = 0.11603.$



(d) $\mu = np$ = (5)(0.65) = 3.25 and $\sigma = \sqrt{5(0.65)(1-0.65)}$ = 1.0665 years. The mean μ is indicated on the probability histogram.

13.26: (a) This probability is 18/38 = 0.47368. (b) X has the binomial (n = 4, p = 0.47368) distribution. (c) P(break even) = $P(X = 2) = \binom{4}{2}(0.47368)^2(1-0.47368)^2 = 0.37292$. (d) P(lose money) = $P(X < 2) = P(X = 0) + P(X = 1) = \binom{4}{0}(0.47368)^0(1-0.47368)^4 +$

$\binom{4}{1}(0.47368)^1(1-0.47368)^3 = 0.07674 + 0.27625 = 0.35299.$

13.27: (a) All women are independent, and each has the same probability of getting pregnant. (b) Under ideal conditions, the number who get pregnant is binomial with n = 20 and p = 0.01; $P(N \geq 1) = 1 - P(N = 0) = 1 - 0.8179 = 0.1821$. In typical use, p = 0.03, and $P(N \geq 1)$= 1−0.5438 = 0.4562.

13.28: X, the number of wins betting on "red" 200 times, is binomial with $n = 200$ and $p = 0.47368$, using the information from Exercise 13.26. The Normal approximation is quite safe: $np = 94.736 > 10$ and $n(1-p) = 105.264 > 10$. The mean is $\mu = np = 94.736$ and the standard deviation is $\sigma = \sqrt{200(0.47368)(1-0.47368)} = 7.06126$, so $P(X < 100) = P(X \le 99)$ $= P\left(Z \le \frac{99-94.736}{7.06126}\right) = P(Z \le 0.60) = 0.7257$. The exact binomial probability is 0.7502. As the number of plays (n) increases, the probability of losing money will increase. For example, if $n = 400$, $P(X < 200)$ = standardize = $P(Z \le 0.95) = 0.8289$. The exact binomial probability is 0.8424.

13.29: (a) X, the number of women who get pregnant in typical use, is binomial with $n = 600$ and $p = 0.03$. The Normal approximation is safe: $np = 18$ and $n(1-p) = 582$ are both larger than 10. The mean is 18 and the standard deviation is 4.1785, so $P(X \ge 20) = P\left(Z \ge \frac{20-18}{4.1785}\right) = P(Z \ge 0.48)$ = 0.3156. The exact binomial probability is 0.3477. (b) Under ideal conditions, $p = 0.01$, so $np = 6$ is too small.

13.30: (a) We must assume that each drive is independent, and that he has a 52% chance of hitting the fairway each time. Both of these assumptions are suspect, and student opinions about which is less realistic may vary. (Hitting the fairway is not like, say, shooting a free throw; some fairways are harder to hit than others. Having an *average* success rate of 52% does not necessarily mean that Phil has a 52% chance of hitting *any* fairway.) (b) For a binomial distribution with $n = 14$ and $p = 0.52$, the average number of fairways hit is $np = (14)(0.52) = 7.28$ fairways hit. In fact, 7 hit fairways is the most likely outcome, with P(7 hit fairways) = 0.2071.

13.31: (a) If R is the number of red-blossomed plants out of a sample of 4, then $P(R = 3) = \binom{4}{3}(0.75)^3(0.25)^1 = 0.4219$, using a binomial distribution with $n = 4$ and $p = 0.75$. (b) With $n = 60$, the mean number of red-blossomed plants is $np = 45$. (c) If R is the number of red-blossomed plants out of a sample of 60, then $P(R \ge 45) = P(Z \ge 0) = 0.5000$ (software gives 0.5688 using the binomial distribution).

13.32: (a) X, the number of positive tests, has a binomial distribution with $n = 1000$ and $p = 0.004$. (b) $\mu = np = (1000)(0.004) = 4$ positive tests. (c) To use the Normal approximation, we need np and $n(1-p)$ both bigger than 10, and as we saw in (b), $np = 4$.

13.33: (a) Of 1,498,000 total vehicles in these top 5 nameplates, Impalas accounted for proportion 184,000/1,498,000 = 0.12283. (b) If I is the number of Impala buyers in the 1000 surveyed buyers, then I has the binomial distribution with $n = 1000$, and $p = 0.12283$. Hence, $\mu = np = (1000)(0.12283) = 122.83$ and $\sigma = \sqrt{np(1-p)} = \sqrt{1000(0.12283)(1-0.12283)} = 10.38$ Impala buyers. (c) $P(I > 100) = P(I \ge 101) = P(Z \ge -2.10) = 0.9821$.

13.34: Let D be the number of members that drop out in the first 4 weeks. Then D has the binomial distribution with $n = 300$ and $p = 0.25$, assuming customer results are independent. Let $S = 300 - D$ be the number still enrolled after 4 weeks. Then S has the binomial distribution with $n = 300$ and $p = 0.75$. (a) For D, $\mu = np = (300)(0.25) = 75$ and $\sigma = \sqrt{np(1-p)}$ = $\sqrt{300(0.25)(1-0.25)}$ = 7.5 customers. (b) We use the Normal approximation to the distribution of S, since $np = 225$ and $n(1 - p) = 75$ are both larger than 10. Now, $P(S \geq 210) = P(Z \geq -2) =$ 0.9772.

13.35: (a) With $n = 100$, the mean and standard deviation are $\mu = 75$ and $\sigma = 4.3301$ questions, so $P(70 \leq X \leq 80) = P(-1.15 \leq Z \leq 1.15) = 0.7498$ (software gives 0.7967). (b) With $n = 250$, we have $\mu = 187.5$ and $\sigma = 6.8465$ questions, and a score between 70% and 80% means 175 to 200 correct answers, so $P(175 \leq X \leq 200) = P(-1.83 \leq Z \leq 1.83) = 0.9328$ (software gives 0.9428).

13.36: We have $\mu = 5000$ and $\sigma = 50$ heads, so using the Normal approximation, we compute $P(X \geq 5067 \text{ or } X \leq 4933) = 2P(Z \geq 1.34) = 0.1802$. If Kerrich's coin were "fair," we would see results at least as far from 5000 as what he observed in about 18% of all repetitions of the experiment of flipping the coin 10,000 times. This is not unreasonable behavior for a fair coin.

13.37: (a) Answers will vary, but over 99.8% of samples should have 0 to 4 bad CDs. (b) Each time we choose a sample of size 10, the probability that we have exactly 1 bad CD is 0.3874; therefore, out of 20 samples, the number of times that we have exactly 1 bad CD has a binomial distribution with parameters $n = 20$ and $p = 0.3874$. This means that most students—99.8% of them—will find that between 2 and 14 of their 20 samples have exactly 1 bad CD, giving a proportion between 0.10 and 0.70. (If anyone has an answer outside of that range, which would be significant evidence that he or she did the exercise incorrectly.)

13.38: The number N of new infections is binomial with $n = 20$ and $p = 0.80$ (for unvaccinated children) or 0.05 (for vaccinated children). (a) For vaccinated children, the mean is (20)(0.05) = 1 new infection, and $P(N \leq 2) = 0.9245$. (b) For unvaccinated children, the mean is (20)(0.80) = 16 new infections, and $P(N \geq 18) = 0.2061$.

13.39: The number N of infections among untreated BJU students is binomial with $n = 1400$ and $p = 0.80$, so the mean is 1120 and the standard deviation is 14.9666 students. 75% of that group is 1050, and the Normal approximation is safe:

$$P(N \geq 1050) = P(Z \geq \frac{1050 - 1120}{14.9666}) = P(Z \geq -4.68)$$, which is very near to 1. (Exact

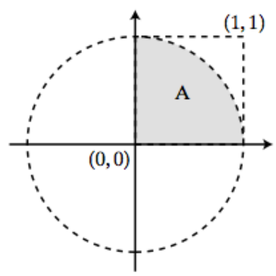
computation gives 0.999998.)

13.40: Let V and U be (respectively) the number of new infections among the vaccinated and unvaccinated children. (a) V is binomial with $n = 17$ and $p = 0.05$, with mean 0.85 infections. (b) U is binomial with $n = 3$ and $p = 0.80$, with mean 2.4 infections. (c) The overall mean is 3.25 infections (2.4 + 0.85).

13.41: Define V and U as in the previous exercise. (a) $P(V = 1) = 0.3741$ and

$P(U = 1) = 0.0960$. Because these events are independent, $P(V = 1 \text{ and } U = 1) = P(V = 1)P(U = 1) = 0.0359$. (b) Considering all the possible ways to have a total of 2 infections, we have $P(2 \text{ infections}) = P(V = 0 \text{ and } U = 2) + P(V = 1 \text{ and } U = 1) + P(V = 2 \text{ and } U = 0) = P(V = 0)P(U = 2) + P(V = 1)P(U = 1) + P(V = 2)P(U = 0) = (0.4181)(0.3840) + (0.3741)(0.0960) + (0.1575)(0.0080) = 0.1977$.

13.42: (a) and (b) The unit square and circle are shown; the intersection A is shaded. (c) The circle has area π, and A is a quarter of the circle, so the area of A is $\pi/4$. This is the probability that a randomly selected point (X, Y) falls in A, so T is binomial with $n = 2000$ and $p = \pi/4 = 0.7854$. (d) The mean of T is $np = 2000(\pi/4) = 500\pi = 1570.7963$ and the standard deviation is $\sqrt{np(1-p)} = \sqrt{2000\frac{\pi}{4}\left(1-\frac{\pi}{4}\right)} = 18.3602$. (e) Because the mean of T is 500π, $T/500$ is an estimate of π.



13.43: The number X of fairways Phil hits is binomial with $n = 24$ and $p = 0.52$. (a) $np = 12.48$ and $n(1 - p) = 11.52$, so the Normal approximation is (barely) safe. (b) The mean is $np = 12.48$ and the standard deviation is $\sqrt{24(0.52)(0.48)} = 2.447529$. Using the Normal approximation, $P(X \geq 17) = P(Z \geq 1.85) = 0.0322$. (c) With the continuity correction, $P(X \geq 17) = P(X \geq 16.5) = P(Z \geq 1.64) = 0.0505$ (using Table A). Indeed, the answer using the continuity correction is closer to the exact answer (0.0487).

13.44 and 13.45 are Web-based exercises.