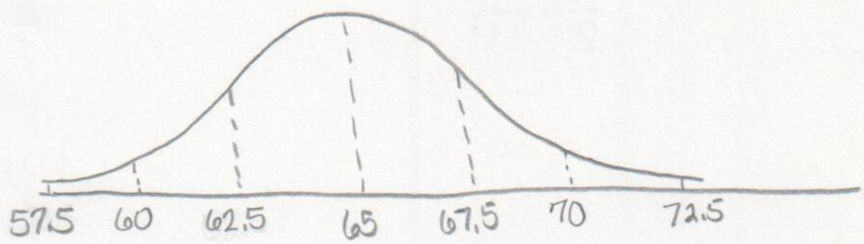
Lisa

Normal Distribution Practice

Part 1: The 68-95-99.7 Rule and Introduction to Standard Scores

1. Heights of women follow a normal distribution with mean 65 inches and standard deviation 2.5 inches – N(65, 2.5). Draw a normal curve with a mean of 65 and a standard deviation of 2.5.
   a) Draw a horizontal line for the x-axis and a bell-shaped normal curve above it.
   b) Mark the center of the curve with a vertical line from the peak to the x-axis and label the point on the axis as 65.
   c) Draw out and mark plus or minus one, two, and three standard deviations from the mean. Label these on the horizontal axis with the appropriate numbers.



2. Find the standard scores of the numbers you have labeled along the horizontal axis above.

$$z = \frac{x - \mu}{\sigma}$$

$\frac{57.5-65}{2.5} = {}^{-}3$

$\frac{60-65}{2.5} = -2$

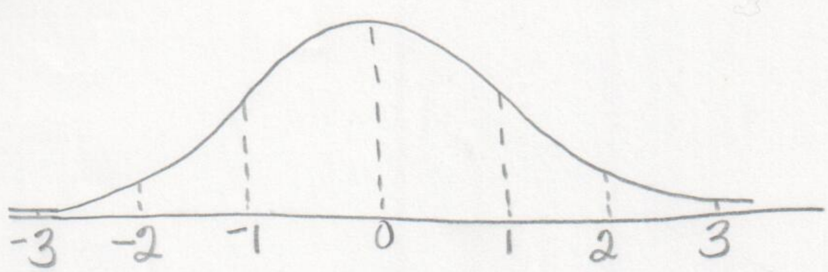
$\frac{62.5-65}{2.5} = -1$

$\frac{67.5-65}{2.5} = 1$

$\frac{70-65}{2.5} = 2$

$\frac{72.5-65}{2.5} = 3$

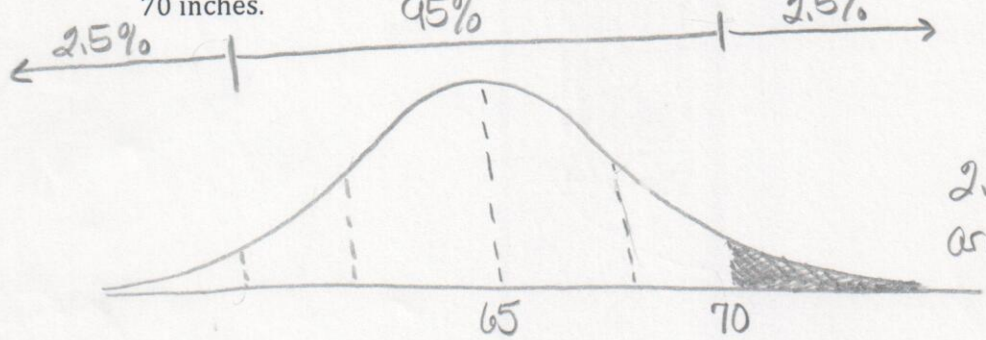
3. Now draw the standard normal curve with a mean of 0 at the center and with the standard scores along the horizontal axis.



- The standard scores represent how many standard deviations an observation is above or below the mean.
- Notice how the observation 70 is two standard deviations above the mean of 65, and it corresponds to the 2 on the standard normal curve.
- Notice also, the observations that fall below the mean have negative standard scores.

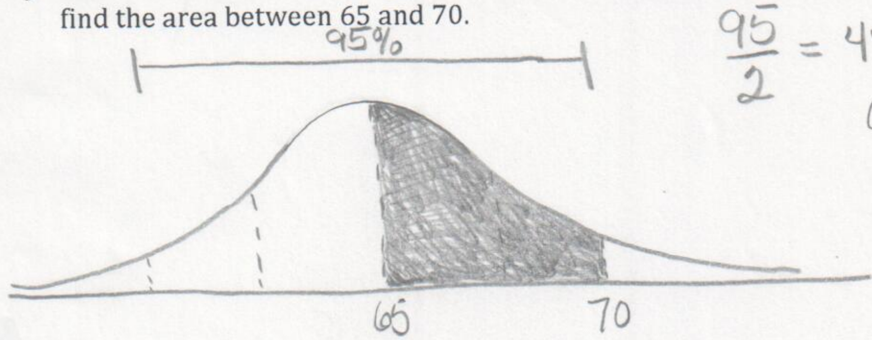
Normal Distribution Practice

4. Find the percentage of females who are taller than 70 inches.
   a) Draw a normal curve with the mean of 65 labeled on the horizontal axis
   b) Place 70 on the horizontal axis two standard deviations to the right of the mean of 65.
   c) Shade the area under the curve that we are interested in – everything to the right of 70.
   d) Use the **68-95-99.7** rule and the fact that the total area under the curve equals 1 to find the area to the right of 70.
   e) The area you found above represents the proportion of females who are taller than 70 inches.



2.5% of females are taller than 70"

5. Find the percentage of females that are between 65 and 70 inches tall.
   a) Draw a normal curve with the mean of 65 and the value 70 labeled on the horizontal axis.
   b) Shade the area under the curve that we are interested in.
   c) Use the **68-95-99.7** rule and the fact that the total area under the curve equals 1 to find the area between 65 and 70.

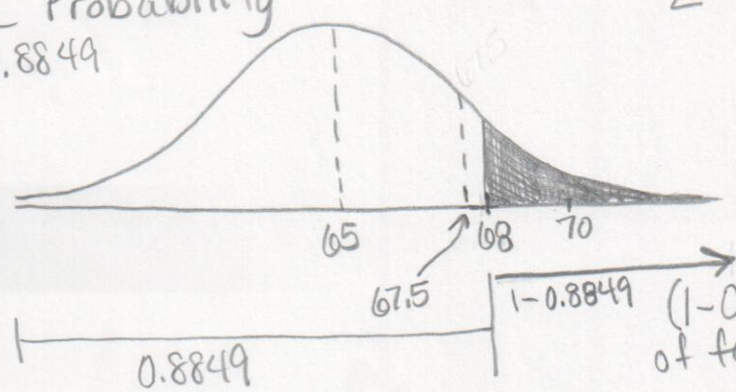


$\frac{95}{2} = 47.5\%$ of females are between 65 and 70 inches in height

6. How would you find the percentage of females taller than 68 inches?
   a) Draw the curve, label, and shade the desired region.
   b) Calculate the standard score for 68.
   c) Look up the standard value in Table A: Standard Normal cumulative proportions. The proportion you find will be the proportion of females shorter than 68.
   d) Use the fact that the total area under the curve equals 1 to calculate the percentage of females taller than 68 inches.

$Z = \frac{68-65}{2.5} = \frac{3}{2.5} = 1.2$

Cumulative Probability at z=1.2 is 0.8849



68 is 1.2 standard deviations above the mean.

$(1-0.8849)=0.1151$ About 11.5% of females are taller than 68"

Normal Distribution Practice

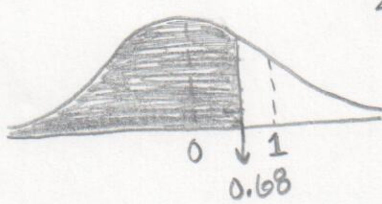
Part 2: Finding Proportions from z Scores and z Scores from Proportions

Use the standard Normal table to find proportions. In each case sketch the Normal curve and shade the area of interest.

- It is always a good idea to draw such a sketch, partly to remind yourself that probabilities correspond to areas, partly because the sketch can help you to figure out how to use the table correctly, and partly because it allows you to check visually whether your answer seems reasonable.

1. The proportion of Z-values less than 0.68—that is, $P(Z < 0.68)$. What about $P(Z \leq 0.68)$?

$Z = 0.68$

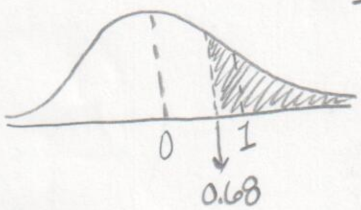


$P(Z<0.68) = 0.7517$

$P(Z \leq 0.68 = 0.7517$

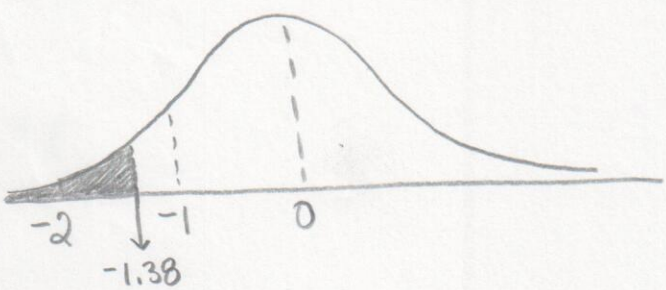
2. $P(Z > 0.68)$

$P(Z>0.68)$
$= 1-0.7517 = 0.2483$

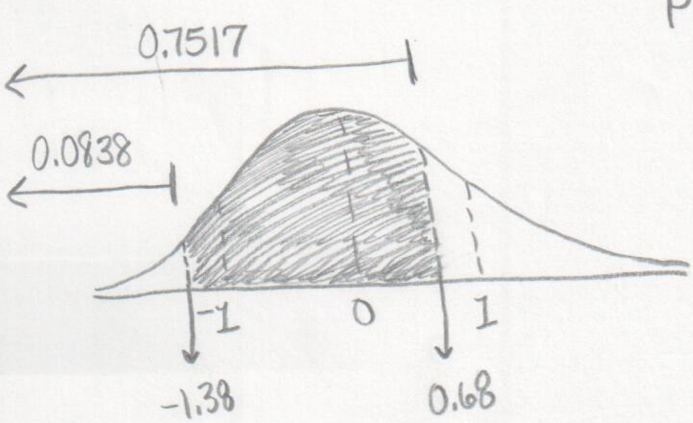


3. $P(Z < -1.38)$

$P(Z<-1.38) = 0.0838$



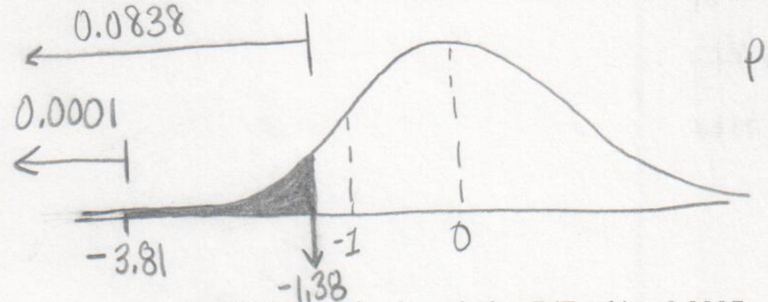
4. $P(-1.38 < Z < 0.68)$



$P(-1.38<Z<0.68) =$
$0.7517-0.0838 = 0.6679$

Normal Distribution Practice

5. P(-3.81 < Z < -1.38) [The limitations of your table force you to make an estimate here.]



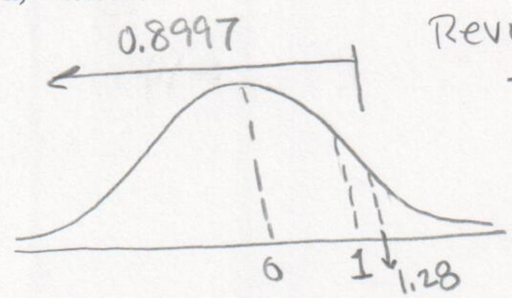
$P(-3.81 < Z < -1.38)$

$= 0.0838 - 0.0001$

$= 0.0837$

6. Find the value k such that P(Z < k) = 0.8997.

$k = 1.28$



Reverse lookup - find the probability 0.8997 in the body of the table then see what the z-score is.

7. Find Q1 and Q3 (the first and 3rd quartiles) for the standard normal distribution.

25% of data is below Q1

Find 0.25 in the body of the table

→ $^-0.675$

$Q1 = {^-0.675}$ $Q3 = 0.675$

75% of the data is below Q3 - Find 0.75 in the body of the table.

The 68-95-99.7 Rule. Continue to use Z to denote a variable with a standard normal distribution. Use the table of standard normal probabilities to find:

8. P(-1 < Z < 1) $0.8413 - 0.1587 = 0.6826$ ~68%

9. P(-2 < Z < 2) $0.9772 - 0.0228 = 0.9544$ ~95%

10. P(-3 < Z < 3) $0.9987 - 0.0013 = 0.9974$ ~99.7%

Critical Values. Use the table of standard normal probabilities (in "reverse") to find as accurately as possible the values of z satisfying:

11. P(Z > z) = 0.10 $z = {^-1.285}$

12. P(Z > z) = 0.05 $z = {^-1.795}$

13. P(Z > z) = 0.01 $z = {^-2.3}$

14. P(Z > z) = 0.001 $z = {^-3.1}$

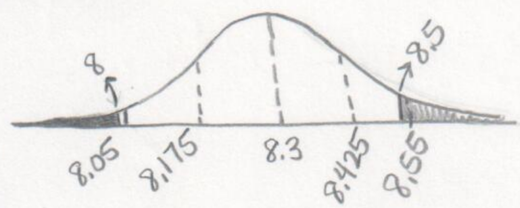
Normal Distribution Practice

Package Weights. Suppose that the wrapper of a candy bar lists its weight as 8 ounces. The actual weights of individual candy bars naturally vary to some extent, however. Suppose that these actual weights vary according to a normal distribution with mean $\mu = \cancel{8.3}$ ounces and standard deviation $\sigma = 0.125$ ounces.
8.3

15. What proportion of the candy bars weigh less than the advertised 8 ounces?

$z = \frac{8-8.3}{0.125} = -2.4$ $P(z<8) = 0.0082$

$\sim 0.8\%$



16. What proportion of the candy bars weigh more than 8.5 ounces?

$z = \frac{8.5-8.3}{0.125} = 1.6$ $P(z<1.6) = 0.9452$

$P(z>1.6) = 1 - 0.9452 = 0.0548$ $\sim 5.5\%$

17. What is the weight such that only 1 candy bar in 1000 weighs less than that amount?

Proportions are probabilities — $1 \div 1000 = 0.001$

$P(z<k) = 0.001$ The standard weight of such a bar is -3.08

$k = -3.08$ $-3.08 = \frac{x-8.3}{0.125}$ $x = (-3.08)(0.125) + 8.3 = 7.915$

18. If the manufacturer wants to adjust the production process so that only 1 candy bar in 1000 weighs less than the advertised weight, what should the mean of the actual weights be (assuming that the standard deviation of the weights remains 0.125 ounces)?

$-3.08 = \frac{8-\mu}{0.125}$

We know:
- z at 0.001 is -3.8
- Advertised weight is 8 oz
- Std. Dev. remains the same

Solve for μ $(-3.08)(0.125) = 8 - \mu$

$\mu = (3.08)(0.125) + 8$

$\mu = 8.385$

19. If the manufacturer wants to adjust the production process so that the mean remains at 8.3 ounces but only 1 candy bar in 1000 weighs less than the advertised weight, how small does the standard deviation of the weights need to be?

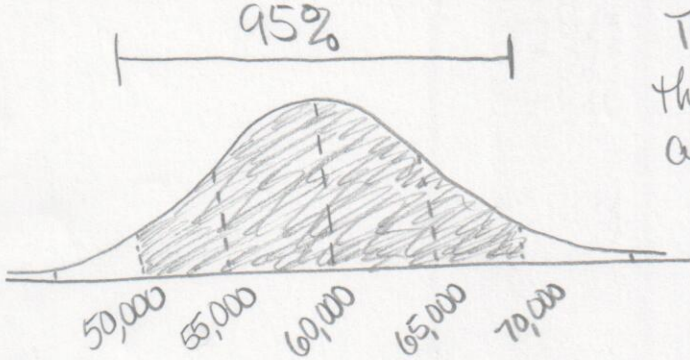
$-3.08 = \frac{8-8.3}{\sigma}$ Solve for σ

$\sigma = \frac{(8-8.3)}{-3.08} = 0.097$

Normal Distribution Practice

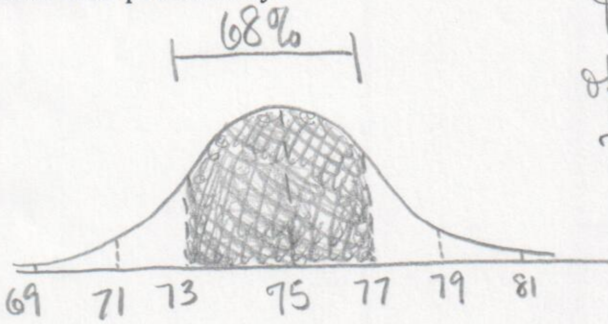
Part 3: Word Problems

1. The length of wear on Spinning Tires is normally distributed with a mean of 60,000 miles and a standard deviation of 5,000 miles. Shade the region under the curve that represents the fraction of tires that last between 50,000 miles and 70,000 miles. What fraction of tires does that represent?



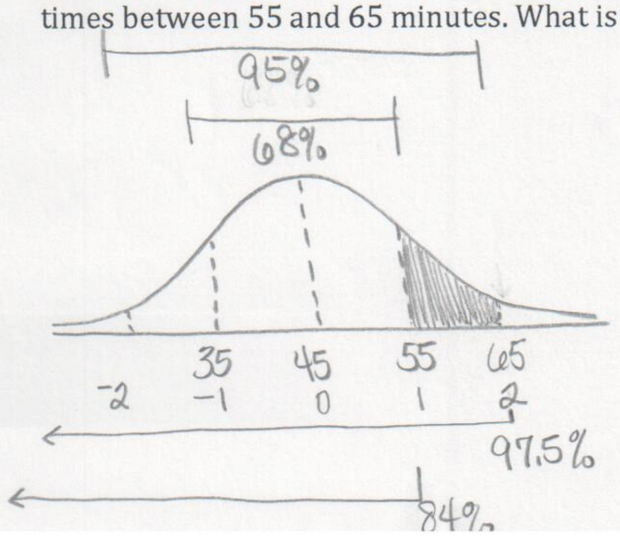
The fraction of tires that last between 50,000 and 70,000 miles is 95%.

2. The number of crackers in a box of Crackerbox Crackers is normally distributed with a mean of 75 and a standard deviation of 2. Shade the region under the curve that represents the probability that a box has between 73 and 77 crackers. What is that probability?



The probability that a box of crackers has between 73 and 77 crackers is about 0.68.

3. The length of time it takes to groom a dog at Shaggy's Pet Shoppe is normally distributed with a mean of 45 minutes and a standard deviation of 10 minutes. Shade the region under the curve that represents the percent of dog grooming times between 55 and 65 minutes. What is that percent?



97.5 - 84 = 13.5%

The percent of dog grooming times between 55 and 65 minutes is about 13.5%

Normal Distribution Practice

when you have a z-score that is a fraction - between -1 and 1 - it is easy to see it as a probability and think you are done. Be careful!

4. The College of Knowledge gives an admission qualifying exam. The results are normally distributed with a mean of 500 and a standard deviation of 100. The admissions department would like to accept only students who score in the 65th percentile or better. Complete the chart below, and then determine which students would qualify and what score is associated with the 65th percentile. Which students qualify for admission? $\mu = 500 \quad \sigma = 100$

Student Score	z-Score	Percentile
530	$\frac{530-500}{100} = 0.3$	$0.6179 \approx 61.8\%$
570	$\frac{570-600}{100} = 0.7$	$0.7580 \approx 75.8\%$
650	$\frac{650-500}{100} = 1.5$	$0.9332 \approx 93.3\%$
800	$\frac{800-500}{100} = 3$	$0.9987 \approx 99.9\%$
540	$\frac{540-500}{100} = 0.4$	$0.6554 \approx 65.5\%$

Student performed better than ____ % of other students.

Student with score = 530 will not get accepted.

5. The MP3 player, aPod, made by Mango Corp., has an average battery of 400 hours. Battery life for the aPod is normally distributed with a standard deviation of 25 hours. The MP3 player, PeaPod, made by Pineapple Inc., has an average battery life of 390 hours. The distribution for its battery life is also normally distributed with a standard deviation of 30 hours.

a) Find the z- scores for each battery with lives of 250, 350, 410, and 450 hours.

	aPod	PeaPod
250	$z = \frac{250-400}{25} = -6$	$z = \frac{250-390}{30} = -4.67$
350	-2	-1.33
410	0.4	0.667
450	2	2

b) Which battery lasting 410 hours performed better? PeaPod because it's standard score is greater than that of the aPod.

b.) PeaPods lasting 410 hours performed better than ~67% of others.

aPod performed better than only 40%.

c) What percent of aPod batteries last between 375 and 410 hours?

$P(X < 375) = P(Z < -1) = 0.1587$

$P(Z < 0.4) = 0.6554$

$0.6554 - 0.1587 = 0.4967$

About 50% of aPod batteries last between 375 and 410 hours.

d) What percent of PeaPod batteries last more than 370 hours?

$z = \frac{370-390}{30} = -0.667$

$P(Z < -0.667) = 0.25 \qquad P(Z > -0.667) = 1 - 0.25 = 0.75$

About 75% of PeaPod batteries last longer than 370 hours.

Normal Distribution Practice

6. The braking distance for a Krazy-Car traveling at 50 mph is normally distributed with a mean of 50 ft. and a standard deviation of 5 ft. Answer the following. ~~without using a calculator or a table.~~ You could use 68-95-99.7 and no table or calculator!

a) What is the likelihood a Krazy-Car will take more than 65 ft. to stop?

$P(Z<3) = 0.9987$

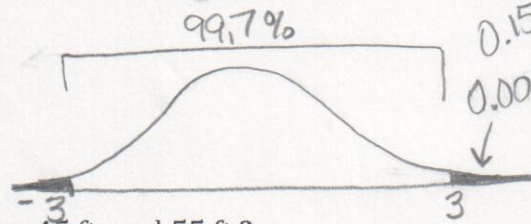
$P(Z>3) = 1-0.9987 = \boxed{0.0013}$

OR

$P(Z>3) = 1-(0.997+0.0015)$

$= 1-0.9985 = \boxed{0.0015}$

$Z = \frac{65-50}{5} = 3$



b) What is the probability a Krazy-Car will stop between 45 ft. and 55 ft.?

$Z = \frac{45-50}{5} = -1$ $\qquad$ $Z = \frac{55-50}{5} = 1$

$P(-1<Z<1) = 0.8413-0.1587 = \boxed{0.6826}$

OR $\boxed{0.68}$

Recall that 68% of scores are within 1 standard dev. from the mean.

c) What percent of the time will a Krazy-Car traveling at 50 mph stop between 35 and 55 ft.?

$Z = \frac{35-50}{5} = -3$ $\qquad$ $Z = \frac{55-50}{5} = 1$

$P(Z<-3) = 0.0013$

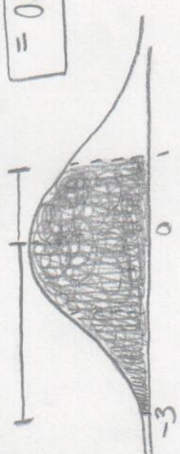
$P(Z<1) = 0.8413$

$P(-3<Z<1) = 0.8413-0.0013$

$\boxed{= 0.85}$

Krazy-car will stop between 35' and 55' 85% of the time.

c. OR

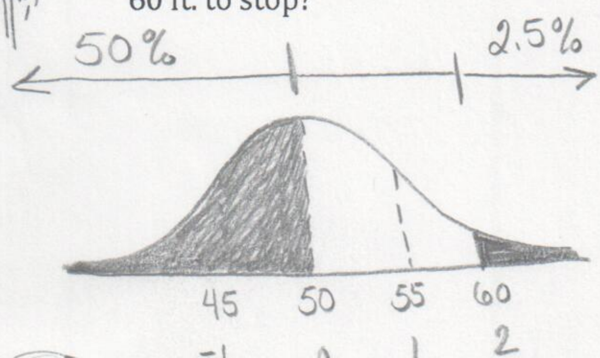


$99.7 \div 2 + 68 \div 2$

$0.4985 + 0.34$

$\boxed{= 0.8385}$

d) What is the probability a Krazy-Car will require less than 50 ft. or more than 60 ft. to stop?



The probability of Krazy-car will require less than 50' or more than 60' is $0.5 + 0.025 = \boxed{0.525.}$

OR

$P(Z<0) = 0.50$

$P(Z<2) = 0.9772$ $\quad$ $P(Z>2) = 1-0.9772 = 0.0228$

$P(Z<0)$ OR $P(Z>2) = 0.50 + 0.0228 = \boxed{0.523}$