General Rules of Probability
Guided Notes

Example 5

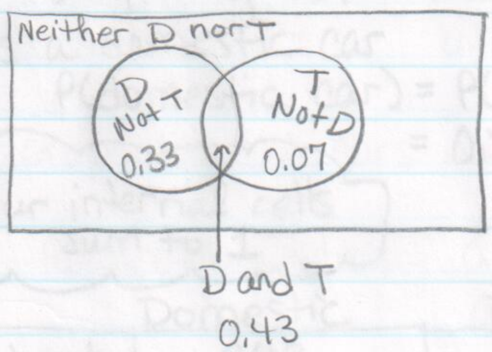
In 2010, 76% of new vehicles sold were domestic, 50% were light trucks, and 43% were domestic light trucks.

Define the following events and choose a vehicle sale at random. Are the events disjoint?

$\boxed{5.1}$ A = sale of a light truck
B = Sale of a domestic light truck

Not Disjoint

$\boxed{5.2}$ a. What is the probability the sale chosen involved a domestic vehicle or a light truck?



0.76 D = domestic
0.50 T = light truck

If 50% of sales were of light trucks,
0.5 − 0.43 = 0.07 (T Not D)

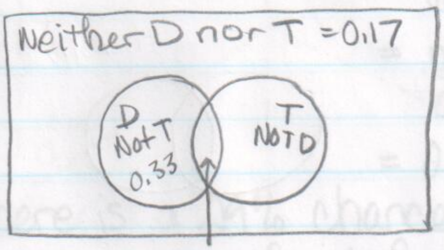
D = 0.76
D and T = 0.43
(D NOT T) = 0.76 − 0.43 = 0.33

P(domestic or light truck) = P(domestic) + P(light truck) − P(domestic and light truck)
= 0.76 + 0.5 − 0.43
= 0.83

b. What is the probability the vehicle chosen is an imported car?

Anything that is not domestic or a light truck is an imported car:

P(imported car) = 1 - 0.83 = 0.17



D and T

c. What is the probability that the vehicle chosen is a domestic car?

Anything not a truck and not imported is a domestic car

P(domestic car) = P(domestic) - P(domestic truck)
= 0.76 - 0.43 = 0.33

5.3

Four internal cells sum to 1

Total column sums to 1

Total row sums to 1

	Domestic	Imported	Total
light truck	0.43	0.07	0.5
car	0.33	0.17	0.5
Total	0.76	0.24	1

P(dom. light truck <u>or</u> imported light truck) = 0.43 + 0.07 = 0.5

P(dom. car <u>or</u> imported car) = 0.33 + 0.17 = 0.5

P(domestic car <u>or</u> domestic truck) = 0.43 + 0.33 = 0.76

P(imported car <u>or</u> imported truck) = 0.07 + 0.17 = 0.24

	Domestic	Imported	total
Light truck	0.43	0.07	0.5
car	0.33	0.17	0.5
total	0.76	0.24	1

P(truck | imported) = proportion of imported vehicles that are cars

$= \dfrac{0.17}{0.24}$ ← imported cars / ← imported vehicles

$= 0.29$

There is a 29% chance that you sampled a truck given the information that you sampled an imported vehicle.

what proportion of domestic vehicles are cars?

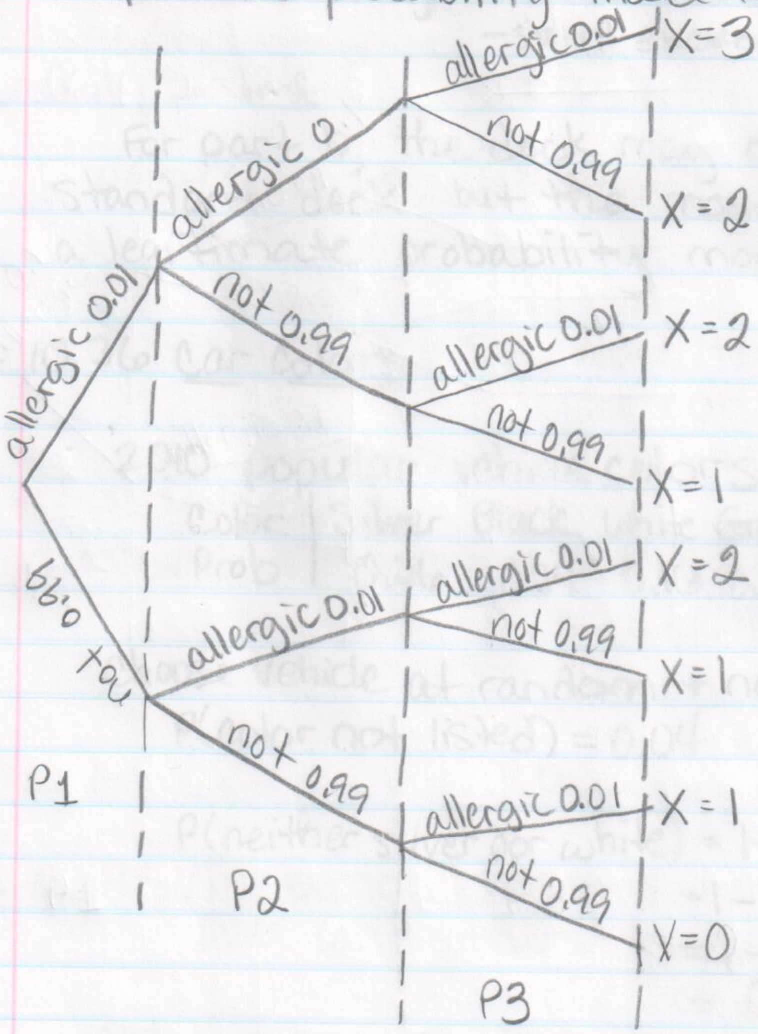
$$P(\text{car} \mid \text{domestic}) = \frac{0.33}{0.76} = 0.4342$$

what proportion of cars are imported?

$$P(\text{imported} \mid \text{car}) = \frac{0.17}{0.5} = 0.34$$

Example 6

- 1% of Americans are allergic to peanuts or tree nuts
- Choose 3 people at random and let the random variable X be the number in the sample who <u>are</u> <u>allergic</u> to peanuts or tree nuts
- Complete the probability model for X.



$(0.1)^3 = 0.000001$

$(0.1)^2 \cdot (0.99) = 0.000099$

$(0.1)^2 \cdot (0.99) = 0.000099$

$(0.1) \cdot (0.99)^2 = 0.009801$

$(0.1)^2 \cdot (0.99) = 0.000099$

$(0.1) \cdot (0.99)^2 = 0.009801$

$(0.1)(0.99)^2 = 0.009801$

$(0.99)^3 = 0.970299$

X	0	1	2	3
Prob(x)	0.970299	0.029403	0.000297	0.000001

3 ways to get X=1

3 ways to get X=2

$0.009801 + 0.009801 + 0.009801 = 0.029403$

$0.000099 + 0.000099 + 0.000099 = 0.000297$

Text Problems

legitimate Probability models?

10.32 a) > probabilities sum to 1 AND

b) all probabilities are between 0 and 1

c) Not legitimate model

- sum exceeds 1

For part b, the deck may not be a standard deck but the model is still a legitimate probability model

10.36 Car Colors

2010 popular vehicle colors

Color	Silver	Black	White	Grey	Red	Blue	Brown
Prob	0.26	0.24	0.16	0.16	0.06	0.05	0.03

Choose vehicle at random + note color...

P(color not listed) = 0.04

P(neither silver nor white) = 1 − [P(white) or P(silver)]

$= 1 - 0.26 + 0.16$

$= 1 - 0.42$

$= 0.58$

Age in Years

	15-19	20-24	25-34	35-44
Alone	0.001	0.011	0.031	0.03
w/ spouse	0.001	0.023	0.155	0.216
w/ others	0.169	0.132	0.142	0.089

10.41 a. why legitimate? · sum to 1 · all between 0 and 1

b. P(15-19 who lives with others) = 0.169 (pick out of table)

c. P(15-19 yrs) = [(15-19 / Alone) + (15-19 / spouse) + (15-19 / others)]

= 0.001 + 0.001 + 0.169 = 0.171

d. P(lives w/ others) = [15-19/others) + (20-24/others) + (25-34/others) + (35-44/others)

= 0.169 + 0.132 + 0.142 + 0.089

= 0.532

10.42 a. A = {The person chosen is either 15-19 OR lives with others OR Both}

Not the sum of b and c because there is a double count of (15-19/others)

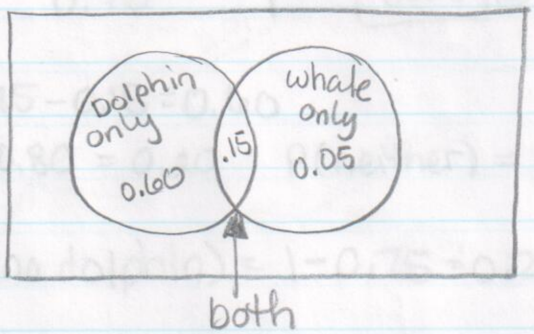
P(A) = P(15-19 yrs) + P(others) − P(15-19 and others)

= 0.171 + 0.532 − 0.169 = 0.534

12.32 80% chance of seeing a dolphin <u>or</u> a whale

75% chance of seeing a dolphin

15% chance of seeing both a dolphin and a whale



Dolphin

$15\% + \underline{60\%} = 75\%$

whale

$80\% - 15\% - 60\% = \underline{5\%}$

$P(\text{whale}) = 0.15 + 0.05 = 0.2$

$P(\text{whale}/\text{not dolphin}) = 0.05$

$P(\text{whale AND dolphin}) = 0.15$

Are these events independent?

check with numerical computation...

- we know $P(\text{whale AND dolphin}) = 0.15$
- if the events are independent, then

$P(W) \cdot P(D) = 0.15$

– check $P(W) = 0.2$

$P(D) = 0.75$

$0.2 \times 0.75 = 0.15$ ✓

The events are independent.

	Dolphin	No Dolphin	Total
whale	0.15	$\boxed{0.05}$ ④	$\boxed{0.20}$ ⑤
no whale	$\boxed{0.60}$ ①	$\boxed{0.20}$ ②	$\boxed{0.80}$ ⑥
Total	0.75	$\boxed{0.25}$ ③	1

① $0.75 - 0.15 = 0.60$

② $1 - 0.80 = 0.20$ P(neither) = 1 − P(dolphin or whale or both)

③ P(no dolphin) $= 1 - 0.75 = 0.25$

④ $0.25 - 0.20 = 0.05$

P(D|W) = proportion of dolphin sightings when see whale

$= \frac{0.15}{0.20} = 0.75$

P(W|D) = proportion of whale sightings when also see dolphin

$= \frac{0.15}{0.75} = 0.20$

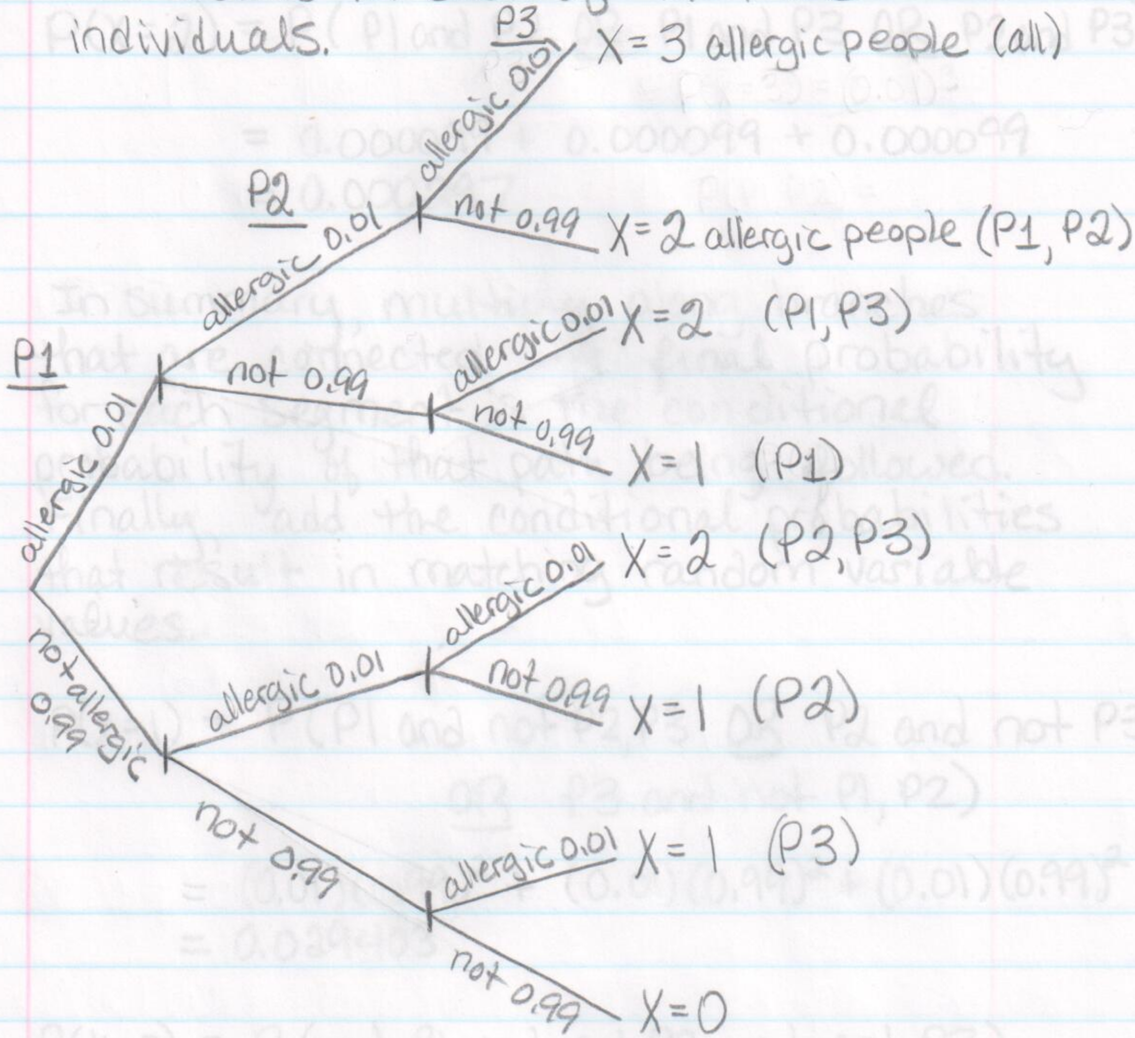
Independent?

we know P(D and W) = 0.15

If they are independent, P(W)P(D) = 0.15

$P(W)P(D) = 0.2 \cdot 0.75 = 0.15$

12.13 Construct a tree diagram showing the results for each of the three individuals.



X	0	1	2	3
Prob	0.970299	0.029403	0.000297	0.000001

P(P1 and P2 and P3)

= P(P1 is allergic) × P(P2 is allergic) × P(P3 is allergic) $= (0.01)^3$

$= 0.000001$

P(P1 and P2) = (0.01)(0.01)(0.99) = 0.000099

P(P1 and P3) = (0.01)(0.99)(0.01) = 0.000099

$P(P2 \text{ and } P3) = (0.99)(0.01)(0.01) = 0.000099$

$P(X=2) = P(P1 \text{ and } P2 \ \underline{OR} \ P1 \text{ and } P3 \ \underline{OR} \ P2 \text{ and } P3)$

$$= 0.000099 + 0.000099 + 0.000099$$
$$= 0.000297$$

In summary, multiply along branches that are connected. The final probability for each segment is the conditional probability of that path being followed. Finally, add the conditional probabilities that result in matching random variable values.

$P(X=1) = P(P1 \text{ and not } P2, P3 \ \underline{OR} \ P2 \text{ and not } P3, P1 \ \underline{OR} \ P3 \text{ and not } P1, P2)$

$$= (0.01)(0.99)^2 + (0.01)(0.99)^2 + (0.01)(0.99)^2$$
$$= 0.029403$$

$P(X=0) = P(\text{not } P1 \text{ and not } P2 \text{ and not } P3)$
$$= (0.99)^3 = 0.970299$$