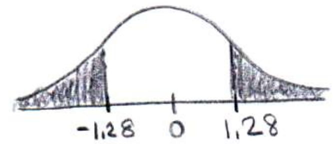
Ch06 Study Guide
Normal Distribution Practice Problems

Z-Table Lookup



Example 1

a. The probability $P(Z < -1.28)$ is closest to 0.1003

b. The probability $P(Z > 1.28)$ is closest to $1 - 0.8997 = 0.1003$

Example 2

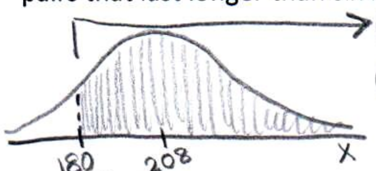
a. Find the probability $P(-1.96 \le Z \le 0)$. $P(Z \le 0) - P(Z \le -1.96) = 0.5 - 0.0250 = 0.4750$

b. Find the probability $P(-1.96 \le Z \le 1.96)$. $P(Z \le 1.96) - P(Z \le -1.96) = 0.975 - 0.25 = 0.95$

c. Find the probability $P(-2 \le Z \le 2)$. $P(Z \le 2) - P(Z \le -2) = 0.9772 - 0.0228 = 0.9544$

Example 3

You work in marketing for a company that produces work boots. Quality control has sent you a memo detailing the length of time before the boots wear out under heavy use. They find that the boots wear out in an average of 208 days, but the exact amount of time varies, following a normal distribution with a standard deviation of 14 days. For an upcoming ad campaign, you need to know the percent of the pairs that last longer than six months—that is, 180 days.



① want to know this probability and convert to %

② $\mu = 208$, $\sigma = 14$, $X = 180$

③ $z = \frac{180-208}{14} = -2$

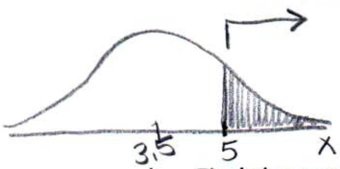
④ $P(X \ge 180) = P(Z \ge -2) = 1 - P(Z \le -2) = 1 - 0.0228 = 0.9772$

⑤ 97.72% last longer

Example 4

The time to complete the construction of a soapbox derby car is normally distributed with a mean of 3.5 hours and a standard deviation of 1.2 hours.

a. Find the probability that it would take more than 5 hours to construct a soapbox derby car.

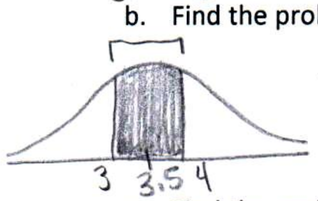


$\mu = 3.5$ $\quad Z = \frac{5-3.5}{1.2} = 1.25$

$\sigma = 1.2$

$X = 5$ $\quad P(X \ge 5) = P(Z \ge 1.25) = 1 - P(Z \le 1.25) = 1 - 0.8944 = 0.1056$

b. Find the probability that it would take between 3 and 4 hours to construct a soapbox derby car.



$\mu = 3.5$ $\quad Z_1 = \frac{3-3.5}{1.2} = -0.42$ $\quad P(3 \le X \le 4) = P(-0.42 \le Z \le 0.42)$

$\sigma = 1.2$ $\quad = P(Z \le 0.42) - P(Z \le -0.42)$

$X_1 = 3$ $\quad Z_2 = \frac{4-3.5}{1.2} = 0.42$ $\quad = 0.6628 - 0.3372 = 0.3256$

$X_2 = 4$

c. Find the probability that it would take exactly 4.2 hours to construct a soapbox derby car.

$P(X = 4.2) = 0$

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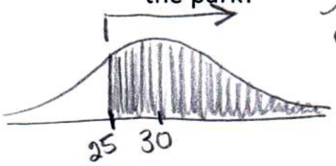
Example 5
You are planning a May camping trip to Denali National Park in Alaska and want to make sure your sleeping bag is warm enough. The average low temperature in the park for May follows a normal distribution with a mean of 30°F and a standard deviation of 12°F. One sleeping bag you are considering advertises that it is good for temperatures down to 25°F.

a. What is the probability that this bag will be warm enough on a randomly selected May night at the park?

$\mu = 30$
$\sigma = 12$
$X = 25$

$Z = \frac{25-30}{12} = -0.42$

$P(X \geq 25) = P(Z \geq -0.42) = 1 - P(Z \leq -0.42) =$
$1 - 0.3372 = 0.6628$

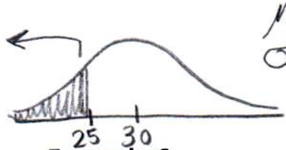


b. What is the probability that the bag will *not* be warm enough?

$\mu = 30$
$\sigma = 12$
$X = 25$

$Z = \frac{25-30}{12} = -0.42$

$P(X \leq 25) = P(Z \leq -0.42) = 0.3372$



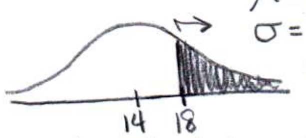
Example 6
Gold miners in Alaska have found, on average, 14 ounces of gold per 1,000 tons of dirt excavated with a standard deviation of 4 ounces. Assume the amount of gold found per 1,000 tons of dirt is normally distributed.

a. What is the probability the miners find more than 18 ounces of gold in the next 1,000 tons of dirt excavated?

$\mu = 14$
$\sigma = 4$
$X = 18$

$Z = \frac{18-14}{4} = 1$

$P(X \geq 18) = P(Z \geq 1) = 1 - P(Z \leq 1) = 1 - 0.8413$
$= 0.1587$



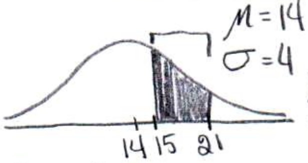
b. What is the probability the miners find between 15 and 21 ounces of gold in the next 1,000 tons of dirt excavated?

$\mu = 14$
$\sigma = 4$
$X_1 = 15$
$X_2 = 21$

$Z_1 = \frac{15-14}{4} = 0.25$

$Z_2 = \frac{21-14}{4} = 1.75$

$P(15 \leq X \leq 21) = P(0.25 \leq Z \leq 1.75) =$
$P(Z \leq 1.75) - P(Z \leq 0.25) =$
$0.9599 - 0.5987 = 0.3612$

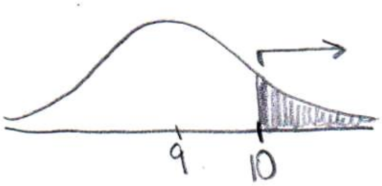


Example 7
Suppose the life of a particular brand of laptop battery is normally distributed with a mean of 9 hours and a standard deviation of 0.7 hours. What is the probability that the battery will last more than 10 hours before running out of power?

$\mu = 9$
$\sigma = 0.7$
$X = 10$

$Z = \frac{10-9}{0.7} = 1.43$

$P(X \geq 10) = P(Z \geq 1.43) = 1 - P(Z \leq 1.43) =$
$1 - 0.9236 = 0.0764$



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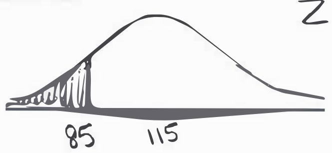
Example 8

A superstar major league baseball player just signed a new deal that pays him a record amount of money. The star has driven in an average of 115 runs over the course of his career, with a standard deviation of 28 runs. An average player at his position drives in 85 runs. What is the probability the superstar bats in fewer runs than an average player next year? Assume the number of runs batted in is normally distributed.

$\mu = 115$
$\sigma = 28$
$X = 85$



$Z = \frac{85-115}{28} = -1.07$

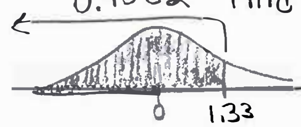
$P(X \le 85) = P(Z \le -1.07) = 0.1423$

Reverse Z-Table Lookup

Example 1

a. Find the z value such that $P(Z \le z) = 0.9082$.

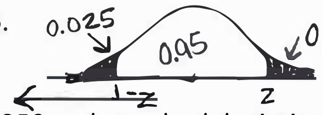
$Z = 1.33$



Find 0.9082 in body of the table

b. Find the z value such that $P(-z \le Z \le z) = 0.95$.

$Z = 1.96$

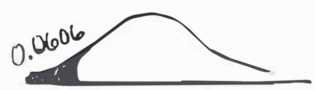


Lookup 0.025 or 0.975 in body of the table

Example 2

a. Let X be normally distributed with mean $\mu = 250$ and standard deviation $\sigma = 80$. Find the value x such that $P(X \le x) = 0.0606$. $= P(Z \le z)$ $\quad Z = -1.495$

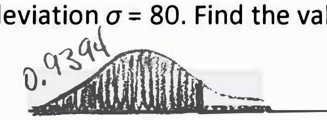
$X = (-1.495)(80) + 250 = 130.4$



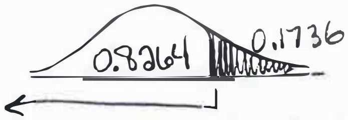
z = -1.5
x = 130

b. Let X be normally distributed with mean $\mu = 250$ and standard deviation $\sigma = 80$. Find the value x such that $P(X \le x) = 0.9394$. $= P(Z \le z)$ $\quad Z = 1.55$

$X = (1.55)(80) + 250 = 374$



c. Let X be normally distributed with mean $\mu = 25$ and standard deviation $\sigma = 5$. Find the value x such that $P(X \ge x) = 0.1736$. $= P(Z \ge z)$ $\quad P(Z \le z) = 0.8264$ $\quad Z = 0.94$

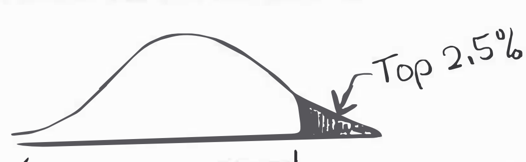


$X = (0.94)(5) + 25 = 29.7$

Example 3

The salary of teachers in a particular school district is normally distributed with a mean of \$60,000 and a standard deviation of \$3,500. Due to budget limitations, it has been decided that the teachers who are in the top 2.5% of the salaries would not get a raise. What is the salary level that divides the teachers into one group that gets a raise and one that doesn't?

$\mu = 60{,}000$
$\sigma = 3500$



Find $P(Z \le z) = 0.975$ $\quad Z = 1.96$

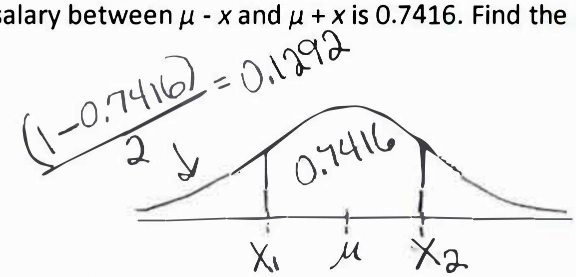
$X = (1.96)(3{,}500) + 60{,}000$
$X = \$66{,}860$

Teachers making \$66,860 or more will not get a raise.

Example 4

The starting salary of an administrative assistant is normally distributed with a mean of \$55,000 and a standard deviation of \$2,700. We know that the probability of a randomly selected administrative assistant making a salary between μ - x and μ + x is 0.7416. Find the salary range referred to in this statement.

$\mu = 55{,}000$
$\sigma = 2{,}700$



Find $P(Z \le z) = 0.1292$ $\quad Z = -1.13$

Salary range
[\$51,949, \$58,051]

$X_1 = (-1.13)(2700) + 55000$
$X_1 = \$51{,}949$
$X_2 = (1.13)(2700) + 55000$
$X_2 = \$58{,}051$

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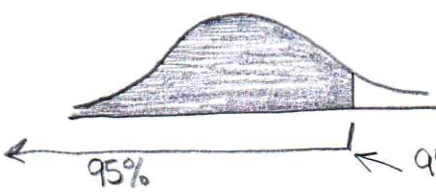
Example 5

The stock price of a particular asset has a mean and standard deviation of \$62.50 and \$7.25, respectively. Use the normal distribution to compute the 95th percentile of this stock price.

$\mu = 62.50$
$\sigma = 7.25$



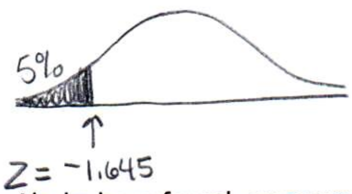
$X = (1.645)(7.25) + 62.50$
$X = \$74.43$ 95th percentile

95th percentile $Z = 1.645$

Example 6

You are planning a May camping trip to Denali National Park in Alaska and want to make sure your sleeping bag is warm enough. The average low temperature in the park for May follows a normal distribution with a mean of 30°F and a standard deviation of 12°F. Above what temperature must the sleeping bag be suited such that the temperature will be too cold only 5% of the time?

$\mu = 30$
$\sigma = 12$



$X = (-1.645)(12) + 30 = 10.26$

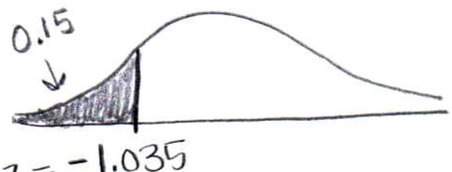
The sleeping bag must be suited for about 10° for the temperature to be too cold only 5% of the time.

Example 7

Gold miners in Alaska have found, on average, 14 ounces of gold per 1,000 tons of dirt excavated with a standard deviation of 4 ounces. Assume the amount of gold found per 1,000 tons of dirt is normally distributed. If the miners excavated 1,000 tons of dirt, how little gold must they have found such that they find that amount or less only 15% of the time?

$\mu = 14$
$\sigma = 4$

$X = (-1.035)(4) + 14 = 9.86$



They must have found 9.86 ounces of gold to have found that amount or less only 15% of the time.