## Chapter 3 Solutions

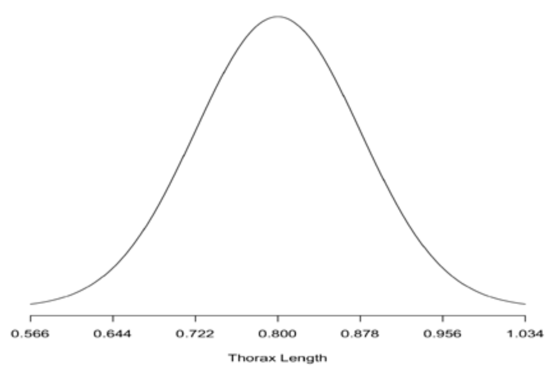
3.1. Sketches will vary. Use them to confirm that students understand the meaning of (a) symmetric and (b) skewed to the left.

3.2. (a) It is on or above the horizontal axis everywhere, and because it forms a 1/5 × 5 rectangle, the area beneath the curve is 1. (b) One-fifth of accidents occur in the first mile: This is a 1/5 × 1 rectangle, so the proportion is 1/5, or 0.20. (c) The length of path along the stream is (1.3 – 0.8) = 1/2 mile. Hence, this is a (1/2)(1/5) rectangle, so the proportion is 1/10, or 0.10. (d) The part of the bike path more than a mile from either road is the 3-mile stretch from the 1-mile marker to the 4-mile marker. This is a (3)(1/5) rectangle, so the proportion is 3/5, or 0.6.

3.3: $\mu = 2.5$, which is the obvious balance point of the rectangle. The median is also 2.5 because the distribution is symmetric (so that median = mean), and half the area under the curve lies to the left and half to the right of 2.5.

3.4: (a) Mean is C, median is B (the right skew pulls the mean to the right). (b) Mean is B, median is B (this distribution is symmetric). (c) Mean is A, median is B (the left skew pulls the mean to the left).

3.5: Here is a sketch of the distribution of the Normal curve describing thorax lengths of fruit flies. The tick marks are placed at the mean, and at one, two and three standard deviations above and below the mean for scale.




3.6: Use the sketch from Exercise 3.5 and shade in the appropriate areas to answer these questions. (a) 99.7% of all thorax lengths are within three standard deviations of the mean, or between 0.566 mm and 1.034 mm. (b) This is the area one or more standard deviations above the mean. Hence, 16% of thorax lengths exceed 0.878 mm.

3.7: (a) In 95% of all years, monsoon rain levels are between 688 and 1016 mm—two standard deviations above and below the mean: 852 ± 2(82) = 688 to 1016 mm. (b) The driest 2.5% of monsoon rainfalls are less than 688 mm; this is more than two standard deviations below the mean.

3.8: Alysha's standardized score is $z = \frac{670-516}{116} = 1.33$. John's standardized score is $z = \frac{26-21}{5.3} = 0.94$. Alysha's score is relatively higher than John's.

3.9: We need to use the same scale, so recall that 6 feet = 72 inches. A woman 6 feet tall has standardized score $z = \frac{72-64.3}{2.7} = 2.85$ (quite tall, relatively). A man 6 feet tall has standardized score $z = \frac{72-69.9}{3.1} = 0.68$. Hence, a woman 6 feet tall is 2.85 standard deviations taller than average for women. A man 6 feet tall is only 0.68 standard deviations above average for men.

3.10: (a) 0.0778. (b) 0.9222. (c) 0.9906. (d) 0.9906 – 0.0778 = 0.9128.

3.11: Let $x$ be the monsoon rainfall in a given year. (a) $x \le 697$ mm corresponds to $z \le \frac{697-852}{82} = -1.89$, for which Table A gives 0.0294 = 2.94%. (b) $683 < x < 1022$ corresponds to $\frac{683-852}{82} < z < \frac{1022-852}{82}$, or $-2.06 < z < 2.07$. This proportion is

0.9808 − 0.0197 = 0.9611 = 96.11%.

3.12: (a) Let $x$ be the MCAT score of a randomly selected student. Then $x > 30$ corresponds to $z > \frac{30-25.0}{6.4} = 0.78$, for which Table A gives 0.7823 as an area to the left. Hence, the answer is 1 – 0.7823 = 0.2177, or 21.77%. (b) $20 \le x \le 25$ corresponds to $\frac{20-25.0}{6.4} \le z \le \frac{20-25.0}{6.4}$, or $-0.78 \le z \le 0$. Hence, using Table A, the area is 0.5000 – 0.2177 = 0.2833, or 28.33%.

3.13: (a) We want the value such that the proportion below is 0.15. Using Table A, looking for an area as close as possible to 0.1500, we find this value has $z = –1.04$ (software would give the more precise $z = –1.0364$). (b) Now we want the value such that the proportion above is 0.70. This means that we want a proportion of 0.30 below. Using Table A, looking for an area as close to 0.3000 as possible, we find this value has $z = –0.52$ (software gives $z = –0.5244$).

3.14: Since the Normal distribution is symmetric, its median and mean are the same. Hence, the median MCAT score is 25.0. Now, following Example 3.11, the first quartile has $z = –0.67$, since the area under the curve to the left of the first quartile is 0.2500 (software gives $z = –0.6745$). Similarly, the third quartile has $z = 0.67$ since the area under the curve to the left of the third quartile is 0.7500. Hence, the first quartile is 25.0 – (0.67)(6.4) = 20.71, and the third quartile is 25.0 + (0.67)(6.4) = 29.29.

3.15. (b) Income distributions are typically skewed to the right. Also, in a forest, there are likely to be many more relatively short trees than there are relatively tall trees. Although the distribution of home prices in a very large metropolitan area tends to be right-skewed, perhaps in a suburb, where the houses tend to be similar, the distribution is more symmetric.

3.16. (a) Mean and standard deviation tell you center and spread, which is all you need for a Normal distribution.

3.17. (b) The curve is centered at 2.

3.18. (b) Estimating a standard deviation is more difficult than estimating the mean, but among the three options, 2 is clearly too small and 5 is clearly too large, so 3 seems to be the most reasonable for the standard deviation.

3.19. (b) 266 ± 2(16) = 234 to 298 days.

3.20: (c) 130 is two standard deviations above the mean, so 2.5% of adults have IQs of 130 or more.

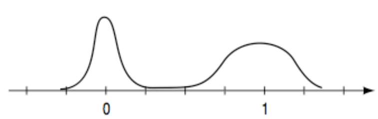
3.21: (b) $z = \dfrac{127-100}{15} = 1.80$.

3.22: (c) 1 – 0.9664 = 0.0336.

3.23: (a) 0.2266.

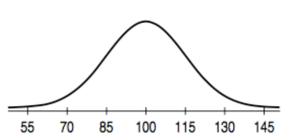
3.24: (c) About 96%. As in Exercise 3.21, $z$ = 1.80, and by Table A, the proportion below is 0.9641.

3.25: Sketches will vary, but should be some variation on the one shown here: the peak at 0 should be "tall and skinny," while near 1, the curve should be "short and fat."




3.26. For each distribution, take the mean plus or minus two standard deviations. For mildly obese people, this is 373 ± 2(67) = 239 to 507 minutes. For lean people, this is 526 ± 2(107) = 312 to 740 minutes.

3.27. 70 is two standard deviations below the mean (that is, it has standard score $z = -2$), so about 2.5% (half of the outer 5%) of adults would have WAIS scores below 70.



3.28: (a) 0.1056. (b) 1 – 0.1056 = 0.8944. (c) 1 – 0.9850 = 0.0150. (d) 0.9850 – 0.1056 = 0.8794.

3.29: (a) We want the proportion less than $z$ to be 0.60, so looking up a left-tail area of 0.6000 in the table, we find $z$ = 0.25. (Software gives $z$ = 0.2533.) (b) If 15% are more than $z$, then 85% are less than or equal to $z$. Hence, $z$ = 1.04. (Software gives $z$ = 1.0364.)

3.30: (a) Let $x$ be the length of a thorax for a randomly selected fruit fly. (a) $x < 0.7$ mm corresponds to $z < \frac{0.7-0.800}{0.078} = -1.28$. Hence, the area is 0.1003, or 10.03%. (b) $x > 1$ mm corresponds to $z > \frac{1-0.800}{0.078} = 2.56$. Hence, the area is 1 – 0.9948 = 0.0052, or 0.52%. (c) 0.7 mm < $x$ < 1 mm corresponds to $-1.28 < z < 2.56$. Hence, the area is 0.9948 – 0.1003 = 0.8945, or 89.45%.

3.31: About 0.2119: The proportion of rainy days with rainfall pH below 5.0 is about 0.2119: $x <$ 5.0 corresponds to $z < \frac{5.0-5.43}{0.54} = -0.80$, for which Table A gives 0.2119.

3.32: (a) Less than 2% of runners have heart rates above 130 bpm: For the $N$ (104, 12.5) distribution, $x > 130$ corresponds to $z > \frac{130-104}{12.5} = 2.08$. Table A gives 1 – 0.9812 = 0.0188 = 1.88%. (b) About 50% of nonrunners have heart rates above 130 bpm: For the $N$ (130, 17) distribution, $x > 130$ corresponds to $z > 0$.

3.33: About 0.9876: For the $N$ (0.8750, 0.0012) distribution, $0.8720 < x < 0.8780$ corresponds to $\frac{0.8720-0.8750}{0.0012} < z < \frac{0.8780-0.8750}{0.0012}$, or $-2.50 < z < 2.50$, for which Table A gives 0.9938 − 0.0062 = 0.9876.

3.34: Let $x$ be the BMI for a randomly selected young woman aged 20 to 29. (a) Being underweight corresponds to $x < 18.5$. This gives $z < \frac{18.5-26.5}{6.4} = -1.25$. Hence, 0.1056, or 10.56% are underweight. (b) Being obese corresponds to $x > 30$. This gives $z > \frac{30-26.5}{6.4} = 0.55$. Hence, 1 – 0.7088 = 0.2912, or 29.12% are obese.

For problems 3.35 – 3.38, let $x$ denote the gas mileage of a randomly selected vehicle type from the population of 2010 model vehicles (excluding the high mileage outliers, as mentioned).

3.35: Cars with better mileage than the Camaro correspond to $x > 19$, which corresponds to $z > \frac{19-20.3}{4.3} = -0.30$. Hence, this proportion is 1 – 0.3821 = 0.6179, or 61.79%.

3.36: We need the proportion below our vehicle's mileage to be 0.10. Looking for 0.1000 as a left-tail area in the table gives $z = -1.28$, so our vehicle would need mileage to be 20.3 – (1.28)(4.3) = 14.80 mpg. A car would need to have gas mileage of 14.80 mpg or lower to be in the bottom 10% for all 2010 models.

3.37: As seen in Example 3.11, the first and third quartiles have $z = -0.67$ and $z = 0.67$, respectively. Hence, the first quartile is 20.3 – (0.67)(4.3) = 17.42 mpg, and the third quartile is 20.3 + (0.67)(4.3) = 23.18 mpg.

3.38: The first quintile is the mileage so that 20% of models have a lower mileage. This has $z = -0.84$ (find the number closest to 0.2000 in Table A as a left-tail area). Similarly, the second, third and fourth quintiles have $z = -0.25$, $z = 0.25$ and $z = 0.84$, respectively. The first quintile is then 20.3 – (0.84)(4.3) = 16.69 mpg. Similarly, the second, third, and fourth quintiles are, respectively, 19.23 mpg, 21.38 mpg, and 23.91 mpg.

3.39: If William scored 32, his percentile is simply the proportion of all scores lower than 32. Let $x$ be the MCAT score for a randomly selected student that took it. The event $x < 32$ corresponds to $z < \frac{32-25.0}{6.4} = 1.09$. Hence, 0.8621 is the corresponding proportion, or 86.21%. William's MCAT score is the 86.21 percentile.

3.40: About 0.0031: a score of 1600 standardizes to $z = \frac{1600-1021}{211} = 2.74$, for which Table A gives a proportion of 0.9969 below. Therefore, the proportion above 1600 (which are reported as 1600) is about 0.0031.

3.41: If $x$ is the height of a randomly selected woman in this age group, we want the proportion corresponding to $x > 69.9$ inches. This corresponds to $z > \frac{69.9-64.3}{2.7} = 2.07$, which has proportion 1 – 0.9808 = 0.0192, or 1.92%.

3.42: The distribution of weights of women is right-skewed. First, the mean weight is larger than the median weight. Another clue comes from the greater distance between the median and third quartile (173.7 – 144.0 = 29.7) than between the median and first quartile (144 – 124.1 = 19.9).

3.43: (a) Let $x$ be a randomly selected man's SAT math score. $x > 750$ corresponds to $z > \frac{750-534}{118} = 1.83$. Hence, the proportion is 1 – 0.9664 = 0.0336. (b) Let $x$ be a randomly selected woman's SAT math score. $x > 750$ corresponds to $z > \frac{750-500}{112} = 2.23$. Hence, the proportion is 1 – 0.9871 = 0.0129.

3.44: If the distribution is Normal, it must be symmetric about its mean—and in particular, the 10th and 90th percentiles must be equal distances below and above the mean—so the mean is 250 points. If 225 points below (above) the mean is the 10th (90th) percentile, this is 1.28 standard deviations below (above) the mean, so the distribution's standard deviation is 225/1.28 = 175.8 points.

3.45. (a) About 0.6% of healthy young adults have osteoporosis (the cumulative probability below a standard score of −2.5 is 0.0062). (b) About 31% of this population of older women has osteoporosis: The BMD level that is 2.5 standard deviations below the young adult mean would standardize to −0.5 for these older women, and the cumulative probability for this standard score is 0.3085.

3.46. (a) There are two somewhat low IQs—72 qualifies as an outlier by the 1.5 × IQR rule, while 74 is on the boundary. However, for a small sample, this stemplot looks reasonably Normal. (b) We compute $x$ = 105.84 and $s$ = 14.27 and find:

> 23/31 = 74.2% of the scores in the range $x \pm 1s$, or 91.6 to 120.1, and
>
> 29/31 = 93.5% of the scores in the range $x \pm 2s$, or 77.3 to 134.4.

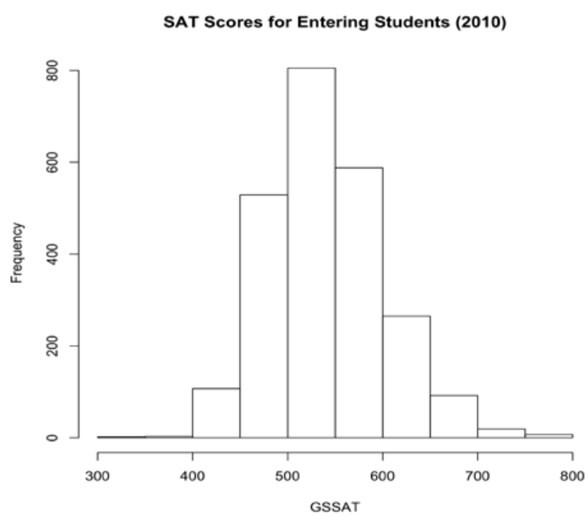
For an exactly Normal distribution, we would expect these proportions to be 68% and 95%. Given the small sample, this is reasonably close agreement.

| | |
|---|---|
| 7 | 24 |
| 7 | |
| 8 | |
| 8 | 69 |
| 9 | 1 3 |
| 9 | 68 |
| 10 | 023334 |
| 10 | 578 |
| 11 | 11222444 |
| 11 | 89 |
| 12 | 0 |
| 12 | 8 |
| 13 | 02 |

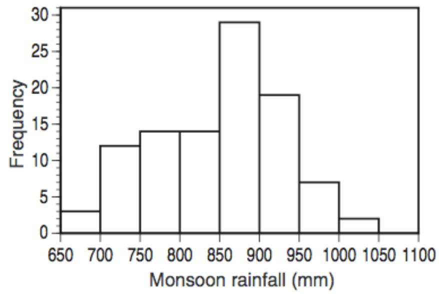
3.47: (a) 145,000/1,568,835 = 0.0924, or 9.24%. (b) There are 50,860 + 145,000 = 195,860 students with ACT score 28 or higher. This is 195,860/1,568,835 = 0.1248, or 12.48%. (c) If $x$ is the ACT score, then $x > 28$ corresponds to $z > \frac{28-21.0}{5.2} = 1.35$, so the corresponding proportion is 1 – 0.9115 = 0.0885, or 8.85%.

3.48: (a) The mean (5.43) is almost identical to the median (5.44), and the quartiles are similar distances from the median: $M - Q1 = 0.39$ while $Q3 - M = 0.35$. This suggests that the distribution is reasonably symmetric. (b) $x < 5.05$ corresponds to $z < \frac{5.05 - 5.43}{0.54} = -0.70$, and $x < 5.79$ corresponds to $z < \frac{5.79 - 5.43}{0.54} = 0.67$. Table A gives these proportions as 0.2420 and 0.7486. These are quite close to 0.25 and 0.75, which is what we would expect for the quartiles, so they are consistent with the idea that the distribution is close to Normal.

3.49: (a) A histogram is provided below, and appears to be roughly symmetric with no outliers. (b) Mean = 544.42, Median = 540, Standard deviation = 61.24, Q1 = 500, Q3 = 580. The mean and median are close, and the distances of each quartile to the median are equal. These results are consistent with a Normal distribution. (c) If $x$ is the score of a randomly selected GSU entering student, then we are assuming $x$ has the $N(544.42, 61.24)$ distribution. The proportion of GSU students scoring higher than the national average of 501 corresponds to the proportion of $x > 501$, or $z > \frac{501 - 544.42}{61.24} = –0.71$, or 1 – 0.2389 = 0.7611, or 76.11%. (d) In fact, 1776 entering GSU students scored higher than 501, which represents 1776/2417 = 0.7348, or 73.48%. The nominal Normal probability in (c) fits the actual data well.

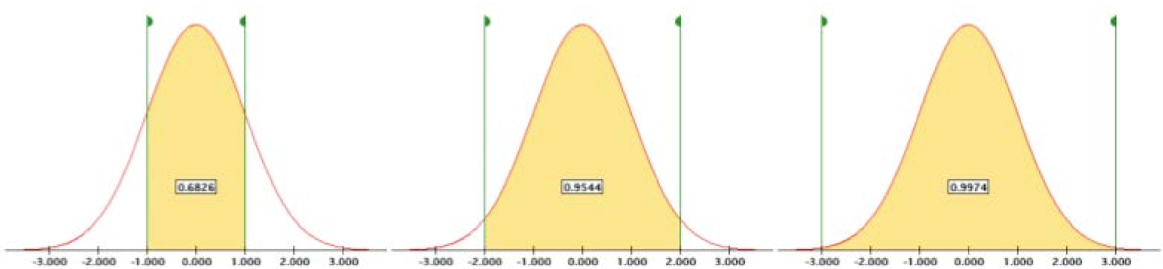


3.50: (a) One possible histogram is provided. The mean is 847.58 mm, and the median is 860.8 mm. (b) The histogram shows a left skew; this makes the mean lower than the median.

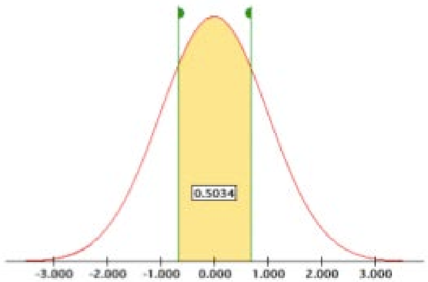



3.51: (a) The 65 Canadians with earnings greater than \$375 represent 65/200 = 0.325, or 32.5%. $x > 375$ corresponds to $z > \dfrac{375-350.30}{292.20} = 0.08$, which has proportion 1 – 0.5319 = 0.4681, or 46.81% above. (b) $x < 0$ corresponds to $z < \dfrac{0-350.30}{292.20} = -1.20$, or 0.1151, or 11.51%. (c) The Normal distribution model predicts 11.5% of Canadians to earn less than \$0, while (of course) none do. This is a substantial error since the Normal model predicts 11.5% of values more than 375, where we actually observed 32.5% more than 375. The standard deviation (\$292.20) is large relative to the average (\$350.30), which suggests a strong right-skew in the distribution, given that no values can be negative. In this application, the data seem to be far from Normal in distribution.

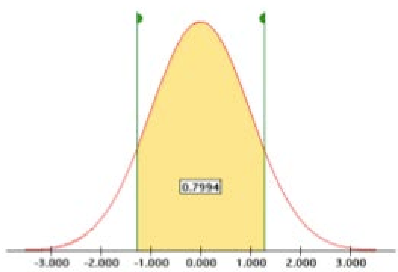
3.52: (a) The applet shows an area of 0.6826 between −1.000 and 1.000, while the 68–95–99.7 rule rounds this to 0.68. (b) Between −2.000 and 2.000, the applet reports 0.9544 (compared with the rounded 0.95 from the 68–95–99.7 rule). Between −3.000 and 3.000, the applet reports 0.9974 (compared with the rounded 0.997).



3.53: Because the quartiles of any distribution have 50% of observations between them, we seek to place the flags so that the reported area is 0.5. The closest the applet gets is an area of 0.5034, between −0.680 and 0.680. Thus the quartiles of any Normal distribution are about 0.68 standard deviations above and below the mean. **Note:** *Table A places the quartiles at about 0.67; other statistical software gives ±0.6745.*




3.54: Placing the flags so that the area between them is as close as possible to 0.80, we find that the A/B cutoff is about 1.28 standard deviations above the mean, and the B/C cutoff is about 1.28 standard deviations below the mean.




3.55 and 3.56 are Web-based exercises.