

Sampling Distributions of Sample Means and Sample Proportions

Applied Statistics • Dr. Rachel Hall

Tips. The techniques for finding the distribution of a sample mean and finding the distribution of a sample proportion are quite similar, although the formulas are different. For “the rules,” see Sections 8.2 and 8.3.

1. The sample must be selected randomly for any of these techniques to work.
Check that selection was random.
2. Before you do anything, **check that the sampling distribution is approximately normal**. Generally this involves checking that the sample size (n) is large enough to guarantee normality.
 - (a) For sample means, the sampling distribution is normal if the population distribution is normal. If the population distribution is not normal, n needs to be at least 30 to guarantee that the sampling distribution is approximately normal.
 - (b) For sample proportions, check that **both** $n\pi \geq 10$ **and** $n(1 - \pi) \geq 10$.
3. Once you know the sampling distribution is normal, you must always **calculate the standard deviation** of the sample means ($\sigma_{\bar{x}}$) or sample proportions (σ_p). It will not be the same as the population standard deviation! Also, write down the mean of the sample means ($\mu_{\bar{x}}$) or sample proportions (μ_p).

Sample Means. This example is Problem 8.20. An elevator has a maximum occupancy of 16 people and a weight limit of 2500 lbs. The average weight of the population is 150 lbs, with a standard deviation of 27 lbs, and the distribution of weight in the population is approximately normal. Suppose that a random sample of size 16 people is chosen.

Before we do anything, check that (1) the sample is selected randomly (YES) and (2) the sampling distribution is normal. (YES) Why? _____

1. The expected value of the sample mean = the average value of the sample mean = the population mean (μ) = _____.
2. The standard deviation of the sample means is calculated by the formula _____. Its value in this problem is _____.
3. Since (average weight) = (sum of weights)/(sample size), the sum of the weights is (average weight) \times 16. So the total weight exceeds 2500 lbs if the average weight is more than _____.

4. The chance that the total weight of a random sample of 16 people will exceed 2500 lbs is $P(\bar{x} > 156.25)$. The z -score for $\bar{x} = 156.25$ is _____. So the probability they will exceed the weight limit is _____.
5. By the way, what is the probability that any *individual* weighs more than 156.25 lbs? _____

Sample Proportions. Many categorical variables you can think of have only two possible values (think of them as “yes” and “no,” or “success” and “failure”). An example is the outcome of a free throw. Reggie Miller of the Indiana Pacers had a 90% free throw percentage in the 2002–2003 season. That is, on average, Miller made 9 of every 10 free throws. Suppose we were to randomly select 100 free throws he made throughout the season and record the number of successes in these 100 free throws. What is the probability that he makes less than 85 of the 100 selected? (Think: is the probability more than 50%?)

1. What is the sample? The sample size? Is selection of the sample random? What is π in this problem? What does p represent in this problem?
2. Check that the probability distribution is normal. Since this is a sample proportion, we must check that _____ and _____ (write the formulas).
3. Write down the average sample proportion $= \mu_p =$ _____. Calculate the standard deviation of the sample proportion $= \sigma_p =$ _____.
4. We’re trying to find the probability that p is less than _____. Find the appropriate z -score: _____. Use the chart to find $P(z < \text{_____})$. Answer: _____.
5. By the way, if Miller shoots only once, what is the probability that he makes the shot? _____