Number of cars X	0	1	2	3	4	5	6
Probability	0.09	0.29	0.38	0.16	0.05	0.02	0.01

A housing company builds houses with two-car garages. What percent of households have more cars than the garage can hold?

(a) 16% (b) 24% (c) 62%

10.30 Choose a common fruit fly *Drosophila melanogaster* at random. Call the length of the thorax (where the wings and legs attach) Y. The random variable Y has the Normal distribution with mean $\mu = 0.800$ millimeter (mm) and standard deviation $\sigma = 0.078$ mm. The probability $P(Y > 1)$ that the fly you choose has a thorax more than 1 mm long is about

(a) 0.995. (b) 0.5. (c) 0.005.

CHAPTER 10 EXERCISES

10.31 Sample space. In each of the following situations, describe a sample space S for the random phenomenon.

(a) A basketball player shoots four free throws. You record the sequence of hits and misses.

(b) A basketball player shoots four free throws. You record the number of baskets she makes.

Darrell Walker/HWMS/Icon SMI/Newscom

10.32 Probability models? In each of the following situations, state whether or not the given assignment of probabilities to individual outcomes is legitimate, that is, satisfies the rules of probability. Remember, a legitimate model need not be a practically reasonable model. If the assignment of probabilities is not legitimate, give specific reasons for your answer.

(a) Roll a six-sided die and record the count of spots on the up-face:

$$P(1) = 0 \quad P(2) = 1/6 \quad P(3) = 1/3$$
$$P(4) = 1/3 \quad P(5) = 1/6 \quad P(6) = 0$$

(b) Deal a card from a shuffled deck:

$$P(\text{clubs}) = 12/52 \quad P(\text{diamonds}) = 12/52$$
$$P(\text{hearts}) = 12/52 \quad P(\text{spades}) = 16/52$$

(c) Choose a college student at random and record sex and enrollment status:

$$P(\text{female full-time}) = 0.56 \quad P(\text{male full-time}) = 0.44$$
$$P(\text{female part-time}) = 0.24 \quad P(\text{male part-time}) = 0.17$$

10.33 Education among young adults. Choose a young adult (aged 25 to 29) at random. The probability is 0.13 that the person chosen did not complete high school, 0.31 that the person has a high school diploma but no further education, and 0.29 that the person has at least a bachelor's degree.

(a) What must be the probability that a randomly chosen young adult has some education beyond high school but does not have a bachelor's degree?

(b) What is the probability that a randomly chosen young adult has at least a high school education?

10.34 Land in Canada. Canada's national statistics agency, Statistics Canada, says that the land area of Canada is 9,094,000 square kilometers. Of this land, 4,176,000 square kilometers are forested. Choose a square kilometer of land in Canada at random.

(a) What is the probability that the area you choose is forested?

(b) What is the probability that it is not forested?

10.35 Foreign-language study. Choose a student in a U.S. public high school at random and ask if he or she is studying a language other than English. Here is the distribution of results:

Language	Spanish	French	German	All others	None
Probability	0.30	0.08	0.02	0.03	0.57

(a) Explain why this is a legitimate probability model.

(b) What is the probability that a randomly chosen student is studying a language other than English?

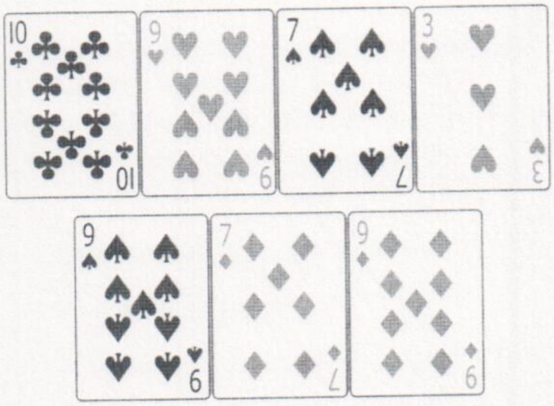
(c) What is the probability that a randomly chosen student is studying French, German, or Spanish?

10.36 Car colors. Choose a new car or light truck at random and note its color. Here are the probabilities of the most popular colors for vehicles sold globally in 2010:[10]

Color	Silver	Black	White	Gray	Red	Blue	Beige, brown
Probability	0.26	0.24	0.16	0.16	0.06	0.05	0.03

(a) What is the probability that the vehicle you choose has any color other than those listed?
(b) What is the probability that a randomly chosen vehicle is neither silver nor white?

10.37 Drawing cards. You are about to draw a card at random (that is, all choices have the same probability) from a set of 7 cards. Although you can't see the cards, here they are:



(a) What is the probability that you draw a 9?
(b) What is the probability that you draw a red 9?
(c) What is the probability that you do not draw a 7?

10.38 Loaded dice. There are many ways to produce crooked dice. To *load* a die so that 6 comes up too often and 1 (which is opposite 6) comes up too seldom, add a bit of lead to the filling of the spot on the 1 face. If a die is loaded so that 6 comes up with probability 0.2 and the probabilities of the 2, 3, 4, and 5 faces are not affected, what is the assignment of probabilities to the six faces?

10.39 A door prize. A party host gives a door prize to one guest chosen at random. There are 48 men and 42 women at the party. What is the probability that the prize goes to a woman? Explain how you arrived at your answer.

10.40 Race and ethnicity. The U.S. Census Bureau allows each person to choose from a long list of races. That is, in the eyes of the U.S. Census Bureau, you belong to whatever race you say you belong to. "Hispanic/Latino" is a separate category; Hispanics may be of any race. If we choose a resident of the United States at random, the U.S. Census Bureau gives these probabilities:[11]

	Hispanic	Not Hispanic
Asian	0.001	0.044
Black	0.006	0.124
White	0.144	0.667
Other	0.005	0.009

(a) Verify that this is a legitimate assignment of probabilities.
(b) What is the probability that a randomly chosen American is Hispanic?
(c) Non-Hispanic whites are the historical majority in the United States. What is the probability that a randomly chosen American is not a member of this group?

Choose at random a person aged 15 to 44 years. Ask their age and who they live with (alone, with spouse, with other persons). Here is the probability model for 12 possible answers:[12]

	Age in Years			
	15–19	20–24	25–34	35–44
Alone	0.001	0.011	0.031	0.030
With spouse	0.001	0.023	0.155	0.216
With others	0.169	0.132	0.142	0.089

Exercises 10.41 to 10.43 use this probability model.

10.41 Living arrangements.

(a) Why is this a legitimate finite probability model?
(b) What is the probability that the person chosen is a 15- to 19-year-old who lives with others?
(c) What is the probability that the person is 15 to 19 years old?
(d) What is the probability that the person chosen lives with others?

10.42 Living arrangements, continued.

(a) List the outcomes that make up the event

A = {The person chosen is *either* 15 to 19 years old *or* lives with others, or both}

(b) What is $P(A)$? Explain carefully why $P(A)$ is not the sum of the probabilities you found in parts (b) and (c) of the previous exercise.

10.43 Living arrangements, continued.

(a) What is the probability that the person chosen is 20 years old or older?
(b) What is the probability that the person chosen does not live alone?

10.44 Spelling errors. Spell-checking software catches "nonword errors" that result in a string of letters that is not a word, as when "the" is typed as "teh." When undergraduates are asked to type a 250-word essay (without spell-checking), the number X of nonword errors has the following distribution:

Value of X	0	1	2	3	4
Probability	0.1	0.2	0.3	0.3	0.1

(a) Is the random variable X discrete or continuous? Why?
(b) Write the event "at least one nonword error" in terms of X. What is the probability of this event?
(c) Describe the event $X \leq 2$ in words. What is its probability? What is the probability that $X < 2$?

10.45 First digits again. A crook who never heard of Benford's law might choose the first digits of his faked invoices so that all of 1, 2, 3, 4, 5, 6, 7, 8, and 9 are equally likely. Call the first digit of a randomly chosen fake invoice W for short.

(a) Write the probability distribution for the random variable W.
(b) Find $P(W \geq 6)$ and compare your result with the Benford's law probability from Example 10.7.

10.46 Who gets interviewed? Abby, Deborah, Mei-Ling, Sam, and Roberto are students in a small seminar course. Their professor decides to choose two of them to interview about the course. To avoid unfairness, the choice will be made by drawing two names from a hat. (This is an SRS of size 2.)

(a) Write down all possible choices of two of the five names. This is the sample space.
(b) The random drawing makes all choices equally likely. What is the probability of each choice?
(c) What is the probability that Mei-Ling is chosen?
(d) Abby, Deborah, and Mei-Ling liked the course. Sam and Roberto did not like the course. What is the probability that both people selected liked the course?

10.47 Birth order. A couple plans to have three children. There are 8 possible arrangements of girls and boys. For example, GGB means the first two children are girls and the third child is a boy. All 8 arrangements are (approximately) equally likely.

(a) Write down all 8 arrangements of the sexes of three children. What is the probability of any one of these arrangements?
(b) Let X be the number of girls the couple has. What is the probability that $X = 2$?
(c) Starting from your work in (a), find the distribution of X. That is, what values can X take, and what are the probabilities for each value?

Picture Press/Alamy

10.48 Unusual dice. Nonstandard dice can produce interesting distributions of outcomes. You have two balanced, six-sided dice. One is a standard die, with faces having 1, 2, 3, 4, 5, and 6 spots. The other die has three faces with 0 spots and three faces with 6 spots. Find the probability distribution for the total number of spots Y on the up-faces when you roll these two dice. (*Hint:* Start with a picture like Figure 10.2 for the possible up-faces. Label the three 0 faces on the second die 0a, 0b, 0c in your picture, and similarly distinguish the three 6 faces.)

10.49 Random numbers. Many random number generators allow users to specify the range of the random numbers to be produced. Suppose that you specify that the random number T can take any value between 0 and 2. Then the density curve of the outcomes has constant height between 0 and 2, and height 0 elsewhere.

(a) Is the random variable Y discrete or continuous? Why?
(b) What is the height of the density curve between 0 and 2? Draw a graph of the density curve.
(c) Use your graph from (b) and the fact that probability is area under the curve to find $P(Y \leq 1)$.

10.50 More random numbers. Find these probabilities as areas under the density curve you sketched in Exercise 10.49.

(a) $P(0.5 < Y < 1.3)$
(b) $P(Y \geq 0.8)$

10.51 Survey accuracy. A sample survey contacted an SRS of 3050 registered voters shortly before the 2008 presidential election and asked respondents whom they planned to vote for. Election results show that 53% of registered voters voted for Barack Obama. We will see later that in this situation the proportion of the sample who planned to vote for Barack Obama (call this proportion V) has approximately the Normal distribution with mean $\mu = 0.53$ and standard deviation $\sigma = 0.009$.

(a) If the respondents answer truthfully, what is $P(0.51 \leq V \leq 0.55)$? This is the probability that the sample proportion V estimates the population proportion 0.53 within plus or minus 0.02.
(b) In fact, 55% of the respondents said they planned to vote for Barack Obama ($V = 0.55$). If respondents answer truthfully, what is $P(V \geq 0.55)$?

10.52 Friends. How many close friends do you have? Suppose that the number of close friends adults claim to have varies from person to person with mean $\mu = 9$ and standard deviation $\sigma = 2.5$. An opinion poll asks this question of an SRS of 1100 adults. We will see later that in this situation the sample mean response $\bar{x}$ has approximately the Normal distribution with mean 9 and standard deviation 0.075. What is $P(8.9 \leq \bar{x} \leq 9.1)$, the probability that the sample result $\bar{x}$ estimates the population truth $\mu = 9$ to within ± 0.1?

10.53 Playing Pick 4. The Pick 4 games in many state lotteries announce a four-digit winning number each day. Each of the 10,000 possible numbers 0000 to 9999 has the same chance of winning. You win if your choice matches the winning digits. Suppose your chosen number is 5974.

(a) What is the probability that the winning number matches your number exactly?

(b) What is the probability that the winning number has the same digits as your number *in any order?*

10.54 Nickels falling over. You may feel that it is obvious that the probability of a head in tossing a coin is about 1/2 because the coin has two faces. Such opinions are not always correct. Stand a nickel on edge on a hard, flat surface. Pound the surface with your hand so that the nickel falls over. What is the probability that it falls with heads upward? Make at least 50 trials to estimate the probability of a head.

10.55 What probability doesn't say. The idea of probability is that the *proportion* of heads in many tosses of a balanced coin eventually gets close to 0.5. But does the actual *count* of heads get close to one-half the number of tosses? Let's find out. Set the "Probability of heads" in the *Probability* applet to 0.5 and the number of tosses to 40. You can extend the number of tosses by clicking "Toss" again to get 40 more. Don't click "Reset" during this exercise.

(a) After 40 tosses, what is the proportion of heads? What is the count of heads? What is the difference between the count of heads and 20 (one-half the number of tosses)?

(b) Keep going to 120 tosses. Again record the proportion and count of heads and the difference between the count and 60 (half the number of tosses).

(c) Keep going. Stop at 240 tosses and again at 480 tosses to record the same facts. Although it may take a long time, the laws of probability say that the proportion of heads will always get close to 0.5 and also that the difference between the count of heads and half the number of tosses will always grow without limit.

10.56 LeBron's free throws. The basketball player LeBron James makes about three-quarters of his free throws over an entire season. Use the *Probability* applet or statistical software to simulate 100 free throws shot by a player who has probability 0.75 of making each shot. (In most software, the key phrase to look for is "Bernoulli trials." This is the technical term for independent trials with Yes/No outcomes. Our outcomes here are "Hit" and "Miss.")

(a) What percent of the 100 shots did he hit?

(b) Examine the sequence of hits and misses. How long was the longest run of shots made? Of shots missed? (Sequences of random outcomes often show runs longer than our intuition thinks likely.)

10.57 Simulating an opinion poll. A 2009 opinion poll showed that about 40% of the American public have very little or no confidence in big business. Suppose that this is exactly true. Choosing a person at random then has probability 0.40 of getting one who has very little or no confidence in big business. Use the *Probability* applet or statistical software to simulate choosing many people at random. (In most software, the key phrase to look for is "Bernoulli trials." This is the technical term for independent trials with Yes/No outcomes. Our outcomes here are "Favorable" or not.)

(a) Simulate drawing 20 people, then 80 people, then 320 people. What proportion have very little or no confidence in big business in each case? We expect (but because of chance variation we can't be sure) that the proportion will be closer to 0.40 in longer runs of trials.

(b) Simulate drawing 20 people 10 times and record the percents in each sample who have very little or no confidence in big business. Then simulate drawing 320 people 10 times and again record the 10 percents. Which set of 10 results is less variable? We expect the results of samples of size 320 to be more predictable (less variable) than the results of samples of size 20. That is "long-run regularity" showing itself.

EXPLORING THE WEB

10.58 Super Bowl odds. Oddsmakers often list the odds for certain sporting events on the Web. For example, one can find the current odds of winning the next Super Bowl for each NFL team. We found a list of such odds at www.vegas.com/gaming/futures/superbowl.html. When an oddsmaker says the odds are A to B of winning, he or she means that the probability of winning is $B/(A + B)$. For example, when we checked the Web site listed above, the odds that the Indianapolis Colts would win Super Bowl XLIV were 13 to 2. This corresponds to a probability of winning of $2/(13 + 2) = 2/15$.

On the Web, find the current odds, according to an oddsmaker, of winning the Super Bowl for each NFL team. Convert these odds to probabilities. Do these probabilities satisfy Rules 1 and 2 given in this chapter? If they don't, can you think of a reason why?

CHAPTER 12 EXERCISES

12.27 Playing the lottery. New York State's "Quick Draw" lottery moves right along. Players choose between 1 and 10 numbers from the range 1 to 80; 20 winning numbers are displayed on a screen every four minutes. If you choose just 1 number, your probability of winning is 20/80, or 0.25. Lester plays 1 number 8 times as he sits in a bar. What is the probability that all 8 bets lose?

12.28 Universal blood donors. People with type O-negative blood are universal donors. That is, any patient can receive a transfusion of O-negative blood. Only 7.2% of the American population have O-negative blood. If 10 people appear at random to give blood, what is the probability that at least 1 of them is a universal donor?

12.29 Playing the slots. Slot machines are now video games, with outcomes determined by random number generators. In the old days, slot machines were like this: you pull the lever to spin three wheels; each wheel has 20 symbols, all equally likely to show when the wheel stops spinning; the three wheels are independent of each other. Suppose that the middle wheel has 9 cherries among its 20 symbols, and the left and right wheels have 1 cherry each.

Peter Dazeley/Getty

(a) You win the jackpot if all three wheels show cherries. What is the probability of winning the jackpot?
(b) There are three ways that the three wheels can show 2 cherries and 1 symbol other than a cherry. Find the probability of each of these ways.
(c) What is the probability that the wheels stop with exactly 2 cherries showing among them?

12.30 A whale of a time. Hacksaw's Boats of St. Lucia takes tourists on a daily dolphin/whale watch cruise. Their brochure claims an 80% chance of sighting a dolphin or a whale, and you can assume that sightings from day to day are independent.

Mark Conlin/Alamy

(a) If you take the dolphin/whale watch cruise on two consecutive days, what is the probability that you see a dolphin or a whale on both days?
(b) If you take the dolphin/whale watch cruise on two consecutive days, what is the probability that you see a dolphin or a whale on at least one day? (*Hint:* First compute the probability that you don't see a dolphin or a whale on either day.)
(c) If you want to have a 99% probability of seeing a dolphin or a whale at least once, what is the minimum number of days that you will need to take the cruise?

12.31 Tendon surgery. You have torn a tendon and are facing surgery to repair it. The surgeon explains the risks to you: infection occurs in 3% of such operations, the repair fails in 14%, and both infection and failure occur together in 1%. What percent of these operations succeed and are free from infection? Follow the four-step process in your answer.

12.32 A whale of a time, continued. Hacksaw's Boats of St. Lucia takes tourists on a daily dolphin/whale watch cruise. Their brochure claims an 80% chance of sighting a dolphin or a whale. Suppose that there is a 75% chance of seeing a dolphin and a 15% chance of seeing both a dolphin and a whale. Make a Venn diagram. Then answer these questions.
(a) What is the probability of seeing a whale on the cruise?
(b) What is the probability of seeing a whale but not a dolphin?
(c) Are seeing a whale and seeing a dolphin independent events?

12.33 Tendon surgery, continued. You have torn a tendon and are facing surgery to repair it. The surgeon explains the risks to you: infection occurs in 3% of such operations, the repair fails in 14%, and both infection and failure occur together in 1%. What is the probability of infection given that the repair is successful? Follow the four-step process in your answer.

12.34 Screening job applicants. A company retains a psychologist to assess whether job applicants are suited for assembly-line work. The psychologist classifies applicants as one of A (well suited), B (marginal), or C (not suited). The company is concerned about the event D that an employee leaves the company within a year of being hired. Data on all people hired in the past five years give these probabilities:

$P(A) = 0.4$	$P(B) = 0.3$	$P(C) = 0.3$
$P(A \text{ and } D) = 0.1$	$P(B \text{ and } D) = 0.1$	$P(C \text{ and } D) = 0.2$

Sketch a Venn diagram of the events A, B, C, and D and mark on your diagram the probabilities of all combinations of psychological assessment and leaving (or not) within a year. What is $P(D)$, the probability that an employee leaves within a year?

12.35 Type of high school attended. Choose a college freshman at random and ask what type of high school they attended. Here is the distribution of results:[9]

Type	Regular public	Public charter	Public magnet	Private religious	Private independent	Home school
Probability	0.781	0.018	0.031	0.105	0.059	0.006

What is the conditional probability that a college freshman was home schooled given that he or she did not attend a regular public high school?

12.36 Income tax returns. Here is the distribution of the adjusted gross income (in thousands of dollars) reported on individual federal income tax returns in 2008:[10]

Income	<15	15–29	30–74	75–199	≥200
Probability	0.265	0.209	0.315	0.180	0.031

(a) What is the probability that a randomly chosen return shows an adjusted gross income of $30,000 or more?
(b) Given that a return shows an income of at least $30,000, what is the conditional probability that the income is at least $75,000?

12.37 Thomas's pizza. You work at Thomas's pizza shop. You have the following information about the 7 pizzas in the oven: 3 of the 7 have thick crust, and of these 1 has only sausage and 2 have only mushrooms; the remaining 4 pizzas have regular crust, and of these 2 have only sausage and 2 have only mushrooms. Choose a pizza at random from the oven.

(a) Are the events {getting a thick-crust pizza} and {getting a pizza with mushrooms} independent? Explain.
(b) You add an eighth pizza to the oven. This pizza has thick crust with only cheese. Now are the events {getting a thick-crust pizza} and {getting a pizza with mushrooms} independent? Explain.

12.38 A probability teaser. Suppose (as is roughly correct) that each child born is equally likely to be a boy or a girl and that the sexes of successive children are independent. If we let BG mean that the older child is a boy and the younger child is a girl, then each of the combinations BB, BG, GB, GG has probability 0.25. Ashley and Brianna each have two children.

(a) You know that at least one of Ashley's children is a boy. What is the conditional probability that she has two boys?
(b) You know that Brianna's older child is a boy. What is the conditional probability that she has two boys?

12.39 College degrees. A striking trend in higher education is that more women than men reach each level of attainment. The National Center for Education Statistics provides projections for the number of degrees earned, classified by level and by the sex of the degree recipient. Here are the projected number of earned degrees (in thousands) in the United States for the 2015–2016 academic year:[11]

	Associate's	Bachelor's	Master's	Professional	Doctorate	Total
Female	556	1034	450	54	45	2139
Male	311	737	282	53	38	1421
Total	867	1771	732	107	83	3560

(a) If you choose a degree recipient at random, what is the probability that the person you choose is a man?
(b) What is the conditional probability that you choose a man given that the person chosen received a master's?
(c) Are the events "choose a man" and "choose a master's degree recipient" independent? How do you know?

12.40 College degrees. Exercise 12.39 gives the projected counts (in thousands) of earned degrees in the United States in the 2015–2016 academic year. Use these data to answer the following questions.

(a) What is the probability that a randomly chosen degree recipient is a woman?
(b) What is the conditional probability that the person chosen received an associate's degree given that she is a woman?
(c) Use the multiplication rule to find the probability of choosing a female associate's degree recipient. Check your result by finding this probability directly from the table of counts.

12.41 Deer and pine seedlings. As suburban gardeners know, deer will eat almost anything green. In a study of pine seedlings at an environmental center in Ohio, researchers noted how deer damage varied with how much of the seedling was covered by thorny undergrowth:[12]

	Deer Damage	
Thorny cover	Yes	No
None	60	151
<1/3	76	158
1/3 to 2/3	44	177
>2/3	29	176

Peter Skinner/Photo Researchers

(a) What is the probability that a randomly selected seedling was damaged by deer?

(b) What are the conditional probabilities that a randomly selected seedling was damaged given each level of cover?

(c) Does knowing about the amount of thorny cover on a seedling change the probability of deer damage? If so, cover and damage are not independent.

12.42 Deer and pine seedlings. In the setting of Exercise 12.41, what percent of the trees that were not damaged by deer were more than two-thirds covered by thorny plants?

12.43 Deer and pine seedlings. In the setting of Exercise 12.41, what percent of the trees that were damaged by deer were less than one-third covered by thorny plants?

Julie graduates from college. *Julie has studied biology, chemistry, and computing and hopes to use her science background in crime investigation. Late one night she thinks about some jobs for which she has applied. Let A, B, and C be the events that Julie is offered a job by*

A = the Connecticut Office of the Chief Medical Examiner

B = the New Jersey Division of Criminal Justice

C = the federal Disaster Mortuary Operations Response Team

Julie writes down her personal probabilities for being offered these jobs:

$P(A) = 0.5 \qquad P(B) = 0.4 \qquad P(C) = 0.2$

$P(A \text{ and } B) = 0.1 \qquad P(A \text{ and } C) = 0.5 \qquad P(B \text{ and } C) = 0.05$

$P(A \text{ and } B \text{ and } C) = 0$

Make a Venn diagram of the events A, B, and C. As in Figure 12.4 (see page 313), mark the probabilities of every intersection involving these events. Use this diagram for Exercises 12.44 to 12.46.

12.44 Will Julie get a job offer? What is the probability that Julie is not offered any of the three jobs?

12.45 Will Julie get just this offer? What is the probability that Julie is offered the Connecticut job but not the New Jersey or federal job?

12.46 Julie's conditional probabilities. If Julie is offered the federal job, what is the conditional probability that she is also offered the New Jersey job? If Julie is offered the New Jersey job, what is the conditional probability that she is also offered the federal job?

12.47 The geometric distributions. You are rolling a pair of balanced dice in a board game. Rolls are independent. You land in a danger zone that requires you to roll doubles (both faces show the same number of spots) before you are allowed to play again. How long will you wait to play again?

(a) What is the probability of rolling doubles on a single toss of the dice? (If you need review, the possible outcomes appear in Figure 10.2 (page 265). All 36 outcomes are equally likely.)

(b) What is the probability that you do not roll doubles on the first toss, but you do on the second toss?

(c) What is the probability that the first two tosses are not doubles and the third toss is doubles? This is the probability that the first doubles occurs on the third toss.

(d) Now you see the pattern. What is the probability that the first doubles occurs on the fourth toss? On the fifth toss? Give the general result: what is the probability that the first doubles occurs on the kth toss?

(e) What is the probability that you get to go again within 3 turns?

(*Comment:* The distribution of the number of trials to the first success is called a *geometric distribution*. In this problem you have found geometric distribution probabilities when the probability of a success on each trial is 1/6. The same idea works for any probability of success.)

12.48 Winning at tennis. A player serving in tennis has two chances to get a serve into play. If the first serve is out, the player serves again. If the second serve is also out, the player loses the point. Here are probabilities based on four years of the Wimbledon Championship:[13]

$$P(\text{1st serve in}) = 0.59$$
$$P(\text{win point} \mid \text{1st serve in}) = 0.73$$
$$P(\text{2nd serve in} \mid \text{1st serve out}) = 0.86$$
$$P(\text{win point} \mid \text{1st serve out and 2nd serve in}) = 0.59$$

Make a tree diagram for the results of the two serves and the outcome (win or lose) of the point. (The branches in your tree have different numbers of stages depending on the outcome of the first serve.) What is the probability that the serving player wins the point?

12.49 Urban voters. The voters in a large city are 40% white, 40% black, and 20% Hispanic. (Hispanics may be of any race in official statistics, but here we are speaking of political blocks.) A black mayoral candidate anticipates attracting 30% of the white vote, 90% of the black vote, and 50% of the Hispanic vote. Draw a tree diagram with probabilities for the race (white, black, or Hispanic) and vote (for or against the candidate) of a randomly chosen voter. What percent of the overall vote does the candidate expect to get? Use the four-step process to guide your work.

12.50 Winning at tennis, continued. Based on your work in Exercise 12.48, in what percent of points won by the server was the first serve in? (Write this as a conditional probability and use the definition of conditional probability.)

12.51 Where do the votes come from? In the election described in Exercise 12.49, what percent of the candidate's votes come from black voters? (Write this as a conditional probability and use the definition of conditional probability.)

12.52 Teens and texting. The Pew Internet and American Life Project finds that 75% of teenagers (ages 12 to 17) now own cell phones, and of the teens who own cell phones, 87% use text messaging.[14]

(a) What percent of teens own cell phones *and* are "texters?"

(b) Among teens who own cell phones and are texters, 15% send more than 200 texts a day, or more than 6,000 texts a month. What percent of all teens own a cell phone, are texters and send more than 6,000 texts a month?

12.53 Lactose intolerance. Lactose intolerance causes difficulty digesting dairy products that contain lactose (milk sugar). It is particularly common among people of African and Asian ancestry. In the United States (ignoring other groups and people who consider themselves to belong to more than one race), 82% of the population is white, 14% is black, and 4% is Asian. Moreover, 15% of whites, 70% of blacks, and 90% of Asians are lactose intolerant.[15]

(a) What percent of the entire population is lactose intolerant?

(b) What percent of people who are lactose intolerant are Asian?

12.54 Fundraising by telephone. Tree diagrams can organize problems having more than two stages. Figure 12.6 shows probabilities for a charity calling potential donors by telephone.[16] Each person called is either a recent donor, a past donor, or a new prospect. At the next stage, the person called either does or does not pledge to contribute, with conditional probabilities that depend on the donor class the person belongs to. Finally, those who make a pledge either do or don't actually make a contribution.

(a) What percent of calls result in a contribution?

(b) What percent of those who contribute are recent donors?

DNA forensics. *When a suspect's DNA is compared with a sample of DNA collected at a crime scene, the comparison is made between certain sections of the DNA called loci. Each locus has two alleles (gene forms), one inherited from the mother and the other from the father. Suppose that there are two alleles, called A and B, for a particular locus. These alleles can be present at the locus in three combinations. A person's alleles at the locus could both be A, one allele could be A and the other B, or both alleles could be B, giving the three combinations (A and A), (A and B), and (B and B). Here's how the math works. If the proportion of the population with allele A as one of their alleles at the locus is a, and the proportion of the population with allele*

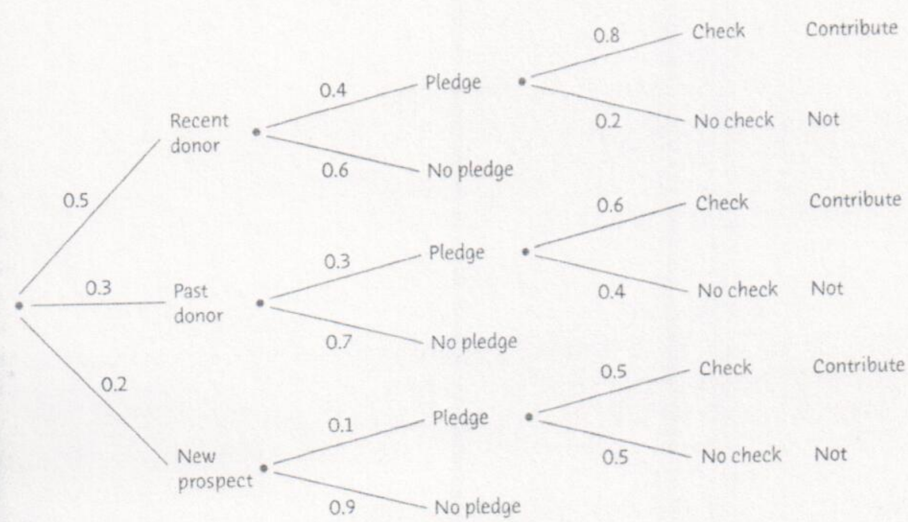


FIGURE 12.6

Tree diagram for fundraising by telephone, for Exercise 12.54. The three stages are the type of prospect called, whether or not the person makes a pledge, and whether or not a person who pledges actually makes a contribution.