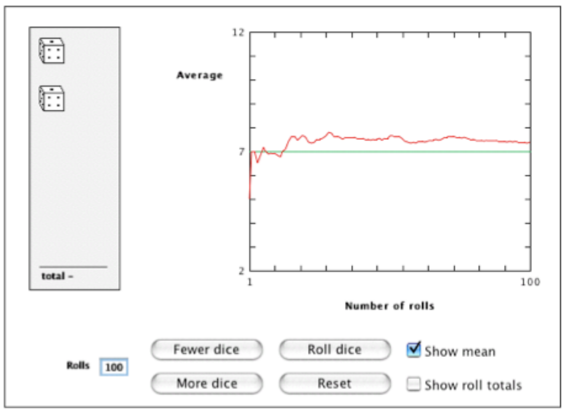
Chapter 11 Solutions

11.1: Both 3.8 and 160.2 active cells per 100,000 cells are statistics (related to one sample — the subjects before infusion and the same subjects after infusion).

11.2: Both 41% and 36% are parameters (related to the population of all registered voters in Florida); 34% is a statistic (related to the sample of registered voters among those called).

11.3: Both 12% and 23 are statistics, as they describe the sample of 230 American male weight lifters.

11.4: Sketches will vary; one result is shown below.



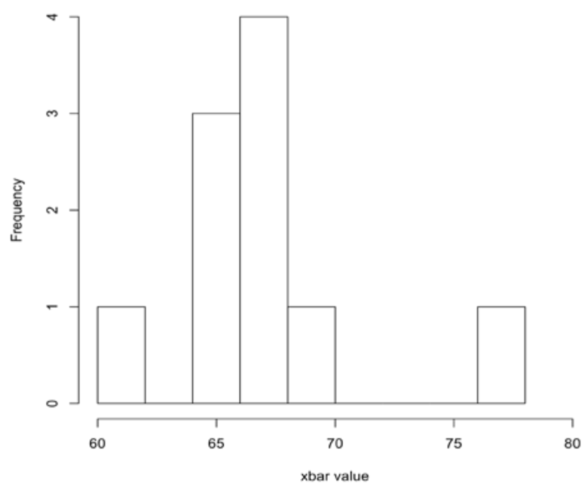
11.5: Although the probability of having to pay for a total loss for 1 or more of the 12 policies is very small, if this were to happen, it would be financially disastrous. On the other hand, for thousands of policies, the law of large numbers says that the average claim on many policies will be close to the mean, so the insurance company can be assured that the premiums they collect will (almost certainly) cover the claims.

11.6: (a) The population is the 12,000 students; the population distribution (Normal with mean 7.11 minutes and standard deviation 0.74 minute) describes the time it takes a randomly selected individual to run a mile. (b) The sampling distribution (Normal with mean 7.11 minutes and standard deviation 0.074 minute) describes the average mile-time for 100 randomly selected students.

11.7: (a) $\mu = 694/10 = 69.4$. (b) The table below shows the results for line 116. Note that we need to choose 5 digits because the digit 4 appears twice. (When choosing an SRS, no student should be chosen more than once.) (c) The results for the other lines are in the table; the histogram is shown next to the table. (Students might choose different intervals than those shown here.) The center of the histogram is a bit lower than 69.4 (it is 66.9), but for a small group of x-values, we should not expect the center to be in exactly the right place.

Note: *You might consider having students choose different samples from those prescribed in this exercise, and then pooling the results for the whole class. With more values of* x, *a better picture of the sampling distribution begins to develop.*

Line	Digits	Scores	x-bar
116	14459	63 + 72 + 72 + 59 = 266	66.5
117	3816	55 + 75 + 63 + 65 = 258	64.5
118	7319	66 + 55 + 63 + 59 = 243	60.75
119	95857	59 + 72 + 75 + 66 = 272	68
120	3547	55 + 72 + 72 + 66 = 265	66.25
121	7148	66 + 63 + 72 + 75 = 276	69
122	1387	63 + 55 + 75 + 66 = 259	64.75
123	54580	72 + 72 + 75 + 86 = 305	76.25
124	7103	66 + 63 + 86 + 55 = 270	67.5
125	9674	59 + 65 + 66 + 72 = 262	65.5



11.8: (a) $\bar{x}$ is not systematically higher than or lower than μ; that is, it has no particular tendency to underestimate or overestimate μ. (b) With large samples, $\bar{x}$ is more likely to be close to μ, because with a larger sample comes more information (and therefore less uncertainty).

11.9: (a) The sampling distribution of $\bar{x}$ is $N(186, 41/\sqrt{100}) = N(186$ mg/dl, 4.1 mg/dl). Therefore, $P(183 < x < 189) = P(-0.73 < Z < 0.73) = 0.5346$. (b) With $n = 1000$, the sample mean has the $N(186$ mg/dl, 1.2965 mg/dl) distribution, so $P(183 < x < 189) = P(-2.31 < Z < 2.31) = 0.9792$.

11.10: (a) $\sigma/\sqrt{n} = 10/\sqrt{4} = 5$ mg. (b) Solve $\sigma/\sqrt{n} = 2$, or $10/\sqrt{n} = 2$, so $\sqrt{n} = 5$, or $n = 25$. The average of several measurements is more likely than a single measurement to be close to the mean.

11.11: No: the histogram of the sample values will look like the population distribution, whatever it might happen to be. (For example, if we roll a fair die many times, the histogram of sample values should look relatively flat—probability close to 1/6 for each value 1, 2, 3, 4, 5, and 6.) The central limit theorem says that the histogram of *sample means* (from many large samples) will look more and more Normal.

11.12: (a) $\mu_{\bar{x}}$ = 0.5 and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ = 0.7/$\sqrt{50}$ = 0.09899. (b) Because this distribution is only approximately Normal, it would be quite reasonable to use the 68–95–99.7 rule to give a rough estimate: 0.6 is about one standard deviation above the mean, so the probability should be about 0.16 (half of the 32% that falls outside ±1 standard deviation). Alternatively, $P(\bar{x} > 0.6) = P(Z > \frac{0.6-.5}{0.09899}) = P(Z > 1.01) = 0.1562$.

11.13: STATE: We ask what is the probability that the average loss for 10,000 such policies will be greater than \$85, when the long-run average loss is \$75? PLAN: Use the central limit theorem to approximate this probability. SOLVE: The central limit theorem says that, in spite of the skewness of the population distribution, the average loss among 10,000 policies will be approximately N (\$75, \$300/$\sqrt{10{,}000}$) = N (\$75, \$3) Now $P(\bar{x} >\$85) = \text{P}(Z > \frac{85-75}{3}) = P(Z > 3.33) = 1 - 0.9996 = 0.0004$. CONCLUDE: We can be about 99.96% certain that average losses will not exceed \$85 per policy.

11.14: (b) statistic. This is a proportion of the people interviewed in the sample of 60,000 households.

11.15: (c) parameter. 58.8% is a proportion of all registered voters (the population).

11.16: (b) The law of large numbers says that the mean from a large sample is close to the population mean. Statement (c) is also true, but is based on the central limit theorem, not on the law of large numbers.

11.17: (a) The mean of the sample means ($\bar{x}$’s) is the same as the population mean (μ).

11.18: (c) The standard deviation of the distribution of $\bar{x}$ is $\sigma/\sqrt{n}$.

11.19: (a) “Unbiased” means that the estimator is right “on the average.”

11.20: (c) The central limit theorem says that the mean from a large sample has (approximately) a Normal distribution. Statement (a) is also true, but is based on the law of large numbers, not on the central limit theorem.

11.21: (b) For n = 6 women, $\bar{x}$ has a N (266, 16/$\sqrt{6}$) = N (266, 6.5320) distribution, so $P(\bar{x} > 270) = P(Z > 0.61) = 0.2709$.

11.22: 1 is a parameter (the mean of the population of all conductivity measurements); 1.07 is a statistic (the mean of the 10 measurements in the sample).

11.23: Both 25.40 and 20.41 are statistics (related, respectively, to the two samples).

11.24: In the long run, the gambler earns an average of 94.7 cents per bet. In other words, the gambler loses (and the house gains) an average of 5.3 cents for each \$1 bet.

11.25: $\bar{x}$ has mean μ = 852 mm, and standard deviation $\sigma/\sqrt{n}$ = $82/\sqrt{10}$ = 25.93 mm.

11.26: (a) $P(20 < X < 30) = P(\frac{20-25}{6.4} < Z < \frac{30-25}{6.4})$ = P(–0.78 < Z < 0.78) = 0.7823 – 0.2177 = 0.5646. (b) If n = 25 students, the sampling distribution of $\bar{x}$ is $N(25, 6.4/\sqrt{25}) = N(25, 1.28)$. (c) $P(20 < \bar{x} < 30) = P(\frac{20-25}{1.28} < Z < \frac{30-25}{1.28}) = P(-3.91 < Z < 3.91) \approx 1$.

11.27: Let X be Shelia's measured glucose level. (a) $P(X > 140) = P(Z > 1.5) = 0.0668$. (b) If $\bar{x}$ is the mean of four measurements (assumed to be independent), then $\bar{x}$ has a $N(122, 12/\sqrt{4})$ = N(122 mg/dl, 6 mg/dl) distribution, and $P(\bar{x} > 140) = P(Z > 3) = 0.0013$.

11.28: (a) Let $\bar{x}$ be the mean number of minutes per day that the 5 randomly selected mildly obese people spend walking. Then $\bar{x}$ has the $N(373, 67/\sqrt{5})$ = N(373 min., 29.96 min.) distribution. Now $P(\bar{x} > 420) = P(Z > \frac{420-373}{29.96}) = P(Z > 1.57) \approx 0.0582$. (b) Let $\bar{x}$ be the sample mean number of minutes per day for the 5 randomly selected lean people. $\bar{x}$ has the $N(526, 107/\sqrt{5})$ = N(526 min., 47.85 min.). $P(\bar{x} > 420) = P(Z > -2.22) = 0.9868$.

11.29: As shown in Exercise 11.27(b), the mean of four measurements has a N(122 mg/dl, 6 mg/dl) distribution, and $P(Z > 1.645) = 0.05$ if Z is $N(0,1)$, so $L = 122 + 1.645 \times 6 = 131.87$ mg/dl.

11.30: (a) For the emissions E of a single car, $P(E > 0.07) = P(Z > \frac{0.07-0.05}{0.01}) = P(Z > 2) = 0.0228$. (b) The average $\bar{x}$ is Normal with mean 0.05 g/mi and standard deviation $0.01/\sqrt{25}$ = 0.002 g/mi. Therefore, $P(\bar{x} > 0.07) = P(Z > \frac{0.07-0.05}{0.002}) = P(Z > 10) \approx 0$.

11.31: (a) The central limit theorem gives that $\bar{x}$ will have a Normal distribution with mean 8.8 beats per five seconds, and standard deviation $1/\sqrt{12}$ = 0.288675 beats per five seconds. (b) $P(\bar{x} < 8) = P(Z < -2.77) = 0.0028$. (c) If the total number of beats in one minute is less than 100, then the average over 12 5-seceond intervals needs to be less than 100/12 = 8.333 beats per five seconds. $P(\bar{x} < 8.333) = P(Z < -1.62) = 0.0526$.

11.32: The mean NOX level for 25 cars has a N(0.05 g/mi, 0.002 g/mi) distribution, and $P(Z > 2.326) = 0.01$ if Z is $N(0,1)$, so $L = 0.05 + (2.326)(0.002) = 0.054652$ g/mi.

11.33: STATE: What are the probabilities of an average return over 10%, or less than 5%? PLAN: Use the central limit theorem to approximate this probability. SOLVE: The central limit theorem says that over 40 years, $\bar{x}$ (the mean return) is approximately Normal with mean μ = 10.8% and standard deviation 17.1%/$\sqrt{40}$ = 2.704%. Therefore, $P(\bar{x} > 10\%) = P(Z > -0.30)$ = 0.6179, and $P(\bar{x} < 5\%) = P(Z < -2.14) = 0.0162$. CONCLUDE: There is about a 62% chance of getting average returns over 10%, and a 1.6% chance of getting average returns less than 5%. Note: than 5%. **Note**: *We have to assume that returns in separate years are independent.*

11.34: STATE: What is the probability that the total weight of the 22 passengers exceeds 4500 lb? PLAN: Use the central limit theorem to approximate this probability. SOLVE: If W is total weight, then the sample mean weight is $\bar{x} = W/22$. The event that the total weight exceeds 4500 pounds is equivalent to the event that $\bar{x}$ exceeds 4500/22 = 204.55 lb. The central limit theorem says that $\bar{x}$ is approximately Normal with mean 190 lb and standard deviation $35/\sqrt{22} = 7.462$ lb. Therefore, $P(W > 4500) = P(\bar{x} > 204.55) = P(Z > \frac{204.55-190}{7.462}) = P(Z > 1.95) = 0.0256$. CONCLUDE: There is a small chance—about 2.56%—that the total weight exceeds 4500 lb.

11.35: We need to choose n so that $6.4/\sqrt{n} = 1$. That means $\sqrt{n} = 6.4$, so $n = 40.96$.

Because n must be a whole number, take $n = 41$.

11.36: (a) 99.7% of all observations fall within 3 standard deviations, so we want

$3\sigma/\sqrt{n} = 1$. The standard deviation of x must therefore be 1/3 = 0.33 point. (b) We need to choose n so that $6.4/\sqrt{n} = 0.33$. This means $\sqrt{n} = 19.2$, so $n = 368.64$. Because n must be a whole number, take $n = 369$.

11.37: On the average, Joe loses 40 cents each time he plays (that is, he spends \$1 and gets back 60 cents).

11.38: (a) With n = 14,000, $\mu_{\bar{x}} = \$0.60$ and $\sigma_{\bar{x}} = \$18.96/\sqrt{14{,}000} = \$0.1602$.

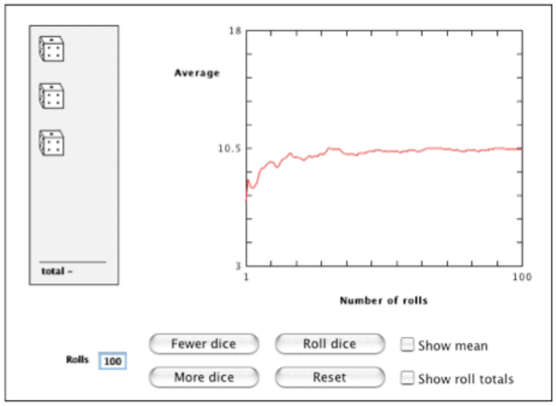
(b) $P(\$0.50 < \bar{x} < \$0.70) = P(-0.62 < Z < 0.62) = 0.4648$.

11.39: (a) With n = 150,000, $\mu_{\bar{x}} = \$0.40$ and $\sigma_{\bar{x}} = \$18.96/\sqrt{150{,}000} = \$0.0490$.

(b) $P(\$0.30 < \bar{x} < \$0.50) = P(-2.04 < Z < 2.04) = 0.9586$.

11.40: (a) The estimate in Exercise 11.38 was 0.4648 (Table A) or 0.4674, so the Normal approximation slightly underestimates the exact answer. (b) With n = 3500, the Normal approximation gives $P(\$0.50 < \bar{x} < \$0.70) = P(-0.31 < Z < 0.31)$, which is 0.2434 (Table A). This is just a bit smaller than the exact answer. (c) The probability that their average winnings fall between \$0.50 and \$0.70 is the same as the probability found in part (b) of the previous exercise, for which the Normal approximation gives 0.9586 (Table A) or 0.9589 (software), so the approximation differs from the exact value by only about 0.0003.

11.41: The mean is 10.5 (= (3)(3.5) because a single die has a mean of 3.5). Sketches will vary, as will the number of rolls; one result is shown.



11.42 and 11.43 are Web-based exercises.