Binomial Practice Problems

1. The Normal distribution can be used to approximate binomial probabilities when $n \cdot p \geq 10$ and $n(1-p) \geq 10$

2. Three choices of dessert – ice cream, apple pie, and chocolate cake. Each dessert is equally likely to be chosen.

$P(\text{choose ice cream}) = 1/3$

$P(\text{do not choose ice cream}) = 2/3$

(a.) $n=4 \quad p=0.33$ Let X be number of people who choose ice cream.

$X = 0, 1, 2, 3, \text{ or } 4$

$P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$

$_4C_2 (0.33)^2 (0.67)^2 = 0.2933$

$_4C_3 (0.33)^3 (0.67)^1 = 0.0963$

$_4C_4 (0.33)^4 (0.67)^0 = 0.0119$

$P(X \geq 2) = \boxed{0.4015}$

$n=21$ (b) Cannot approximate b/c $np = 6.93 < 10$ However,

$P(X \geq 2) = 1 - P(X<2) = 1 - [P(X=0) + P(X=1)]$

$= 1 - [{}_{21}C_0 (0.33)^0 (0.67)^{21} + {}_{21}C_1 (0.33)^1 (0.67)^{20}] = \boxed{0.9975}$

3. 40% pay w/ credit card

If $n=3$ customers are sampled and we are interested in whether or not each pays with a credit card, define X to be the number who pay with CC. $\quad n=3 \quad p=0.4 \quad (1-p)=0.6$

(a) $P(X=0) = {}_3C_0 (0.4)^0 (0.6)^3 = \boxed{0.216}$

(b) $P(X=2) = {}_3C_2 (0.4)^2 (0.6)^1 = \boxed{0.288}$

(c) $P(X \geq 2) = P(X=2) + P(X=3)$

$= \underbrace{0.288}_{\text{from (b)}} + {}_3C_3 (0.4)^3 (0.6)^0 = \boxed{0.352}$

3 continued

d. $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$P(X=0) = 0.216$ from b

$P(X=3) = 0.064$ from c

$P(X=1) = {}_3C_1 (0.4)^1 (0.6)^2 = 0.432$

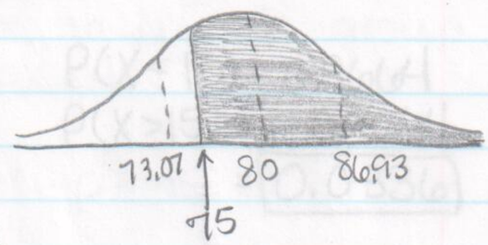
$P(X \le 2) = 0.216 + 0.064 + 0.432 = \boxed{0.712}$

If $n = 200$ customers are chosen, let X be defined as the number who pay w/ cc.
$X = 0, 1, 2, \ldots, 200$. $p = 0.4$

$np = 80$ $n(1-p) = 120$ Use Normal approx.

(a) $P(X > 75) = 0.7642$

$\mu = 80$
$\sigma = 6.93$



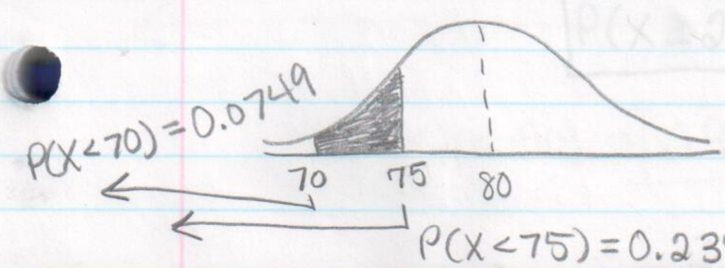
$z = \frac{75-80}{6.93} = {}^{-}0.72$

$P(X > 75) = 1 - P(X < 75)$
$= 1 - 0.2358$
$= \boxed{0.7642}$

(b) $P(X < 70) = \boxed{0.0749}$

$z = \frac{70-80}{6.93} = -1.44$

(c) $P(70 < X < 75) = 0.2358 - 0.0749$
$= \boxed{0.1609}$



On a 40 question test...

4. $n=40$ $p=0.25$ (4 selections for each problem)

Guess on every answer

Let X be the number of questions correct.

ⓐ $P(X=5) = {}_{40}C_5 (0.25)^5 (0.75)^{35}$
$= 658008 \cdot 0.00098 \cdot 0.000042$
$= \boxed{0.027} \approx 2.7\%$

ⓑ $P(X>15)$ Use Normal approx. b/c $40 \cdot 0.25 \geq 10$ and $40 \cdot 0.75 \geq 10$

$n=40$ $p=0.25$
$\mu = (40)(0.25) = 10$
$\sigma = \sqrt{(40)(0.25)(0.75)} = 2.7386$

$z = \frac{15-10}{2.7386} = 1.83$

$P(X<15) = 0.9664$
$P(X>15) = 1-0.9664$
$= \boxed{0.0336}$

ⓒ 70% is passing $(0.7)(40)=28$
Student must get 28 out of 40 correct to pass the test

$P(X \geq 28)$

$\mu = 10$
$\sigma = 2.7386$

$z = \frac{28-10}{2.7386} = 6.57$

Estimate $P(X \leq 28) \approx 0.9999$

$\boxed{P(X \geq 28) \approx 0.0001}$

5. Batting average is 0.275. Use Normal approx. to find, in next 100 times at bat, $P(X \geq 20)$.

$n = 100 \quad p = 0.275 \quad \mu = np = 27.5$

$\sigma = \sqrt{np(1-p)} = 4.465$

$$Z = \frac{20 - 27.5}{4.465} = -6.16$$

Estimate $P(X < 20) \approx 0.0001$

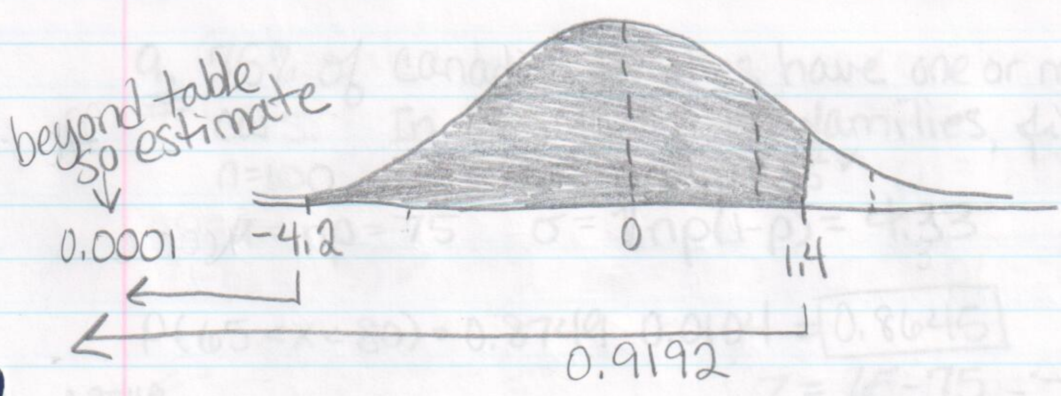
$\boxed{P(X \geq 20) \approx 0.9999}$

6. Probability a person is right-handed is 85%. Use Normal approx. to find, in group of 100 people, $P(70 < X < 90)$.

$n = 100 \quad np = 85 \quad n(1-p) = 15$

$\mu = np = 85 \quad \sigma = \sqrt{np(1-p)} = 3.57$

$$Z = \frac{70 - 85}{3.57} = -4.2 \qquad Z = \frac{90 - 85}{3.57} = 1.4$$



$P(70 < X < 90) = \boxed{0.9191}$

7. Probability that a tire will be defective is 0.03.

We are counting defective tires so probability of success, p, is 0.0

In 350 tires, find $P(X \le 5)$

$n = 350 \quad np = 10.5 \checkmark \quad n(1-p) = 339.5 \checkmark$

$\mu = 10.5 \quad \sigma = 3.19$

$Z = \frac{5 - 10.5}{3.19} = -1.72 \quad P(X \le 5) = \boxed{0.0427}$

8. Failure rate of calculus students is 30%.

$n = 35$

$P(X \le 6) \quad np = 10.5 \quad n(1-p) = 24.5$

$\mu = np = 10.5 \quad \sigma = \sqrt{np(1-p)} = 2.71$

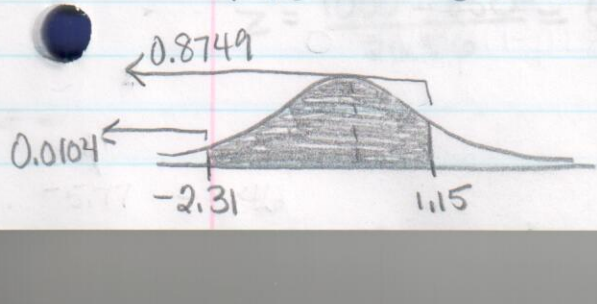
$Z = \frac{6 - 10.5}{2.71} = -1.66 \quad P(X \le 6) = \boxed{0.0485}$

9. 75% of canadian families have one or more cars. In a SRS of 100 families, find

$n = 100 \quad np = 75 \checkmark \quad n(1-p) = 25 \checkmark$

$\mu = np = 75 \quad \sigma = \sqrt{np(1-p)} = 4.33$

$P(65 < X < 80) = 0.8749 - 0.0104 = \boxed{0.8645}$



$Z = \frac{65 - 75}{4.33} = -2.31$

$Z = \frac{80 - 75}{4.33} = 1.15$

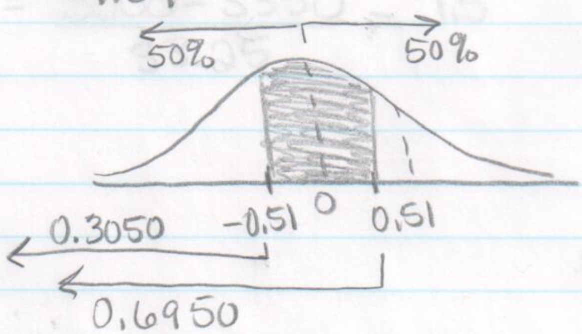
10. 11% of glass jars are defective. Out of 1000 jars, what is $P(105 < X < 115)$

$n = 1000 \quad p = 0.11$ $\qquad$ $np = 110$ ✓ $\quad n(1-p) = 890$ ✓

$\mu = np = 110$

$\sigma = \sqrt{np(1-p)} = 9.89$

$z = \frac{105-110}{9.89} = -0.51$ $\qquad$ $z = \frac{115-110}{9.89} = 0.51$



$P(110 < X < 115)$

$= 0.6950 - 0.3050$

$\boxed{= 0.39}$

OR...

$P(105 < X < 115) = 2 \times P(110 < X < 115)$

$= 2 \times (0.6950 - 0.5)$

$= 2 \times 0.195$

$\boxed{= 0.39}$

11. roll die 5000 times

(a) Probability of rolling more than 1000 ones

Define X to be the number of ones

$n = 5000 \quad p = 1/6 = 0.17 \quad (1-p) = 0.83$

$np = 850$ ✓

$n(1-p) = 4150$ ✓ $\quad \mu = np = 850 \quad \sigma = \sqrt{np(1-p)} = 26.56$

$z = \frac{1000-850}{26.56} = 5.65$

$P(X > 1000) = 1 - P(X < 1000)$

Estimate $P(X < 1000) \approx 0.9999$

$P(X > 1000) \approx 1 - 0.9999$

≈ 0.0001

11 (b) $n = 5000$ Define x to be the number of primes

Primes are 1, 2, 3, 5

$P(X > 3300)$

$p = 4/6 = 0.67$

$(1-p) = 0.33$

$np = 3350$ ✓ $n(1-p) = 1650$

$\mu = np = 3350$

$\sigma = \sqrt{np(1-p)} = 33.25$

$$Z = \frac{3300 - 3350}{33.25} = -1.5$$

$P(X > 3300)$

$= 1 - P(X < 3300)$

$= 1 - 0.0668$

$\boxed{= 0.9332}$