Lisa

Compare Exact Probability to Normal Approximation

If you toss 20 coins, what is the probability that five coins or less will be heads. If X is the number of heads, then we want to find the value:

use $p = 0.51$ $(1-p) = 0.49$

Exact Probability Using the Binomial Formula $P(X = x) = {}_nC_x \cdot (p)^x \cdot (1-p)^{n-x}$

$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$.

$P(X=0) = {}_{20}C_0 \cdot (0.51)^0 (0.49)^{20} = 0.00000064$ ${}_{20}C_0 = 1$

$P(X=1) = {}_{20}C_1 \cdot (0.51)^1 (0.49)^{19} = 20(0.51)(0.49)^{19} = 0.000013$

$P(X=2) = {}_{20}C_2 \cdot (0.51)^2 (0.49)^{18} = 190\,(0.51)^2 (0.49)^{18} = 0.00013$

$P(X=3) = {}_{20}C_3 \cdot (0.51)^3 (0.49)^{17} = 1140 \cdot (0.51)^3 (0.49)^{17} = 0.00082$

$P(X=4) = {}_{20}C_4 \cdot (0.51)^4 (0.49)^{16} = 4845\,(0.51)^4 (0.49)^{16} = 0.0036$

$P(X=5) = {}_{20}C_5 (0.51)^5 (0.49)^{15} = 15504\,(0.51)^5 (0.49)^{15} = 0.0121$

$P(X \le 5) = 0.00000064 + 0.000013 + 0.00013 + 0.00082 + 0.0036 + 0.0121$

$= 0.0166 \approx 1.7\%$

Approximate Probability Using the Normal Distribution

$\mu = np = 10.2$

$\sigma = \sqrt{np(1-p)} = 2.2356$

$z = \dfrac{5 - 10.2}{2.2356} = -2.33$

$P(X \le 5) = 0.0099$

0.0166 (exact from above)

$- 0.0099$

0.0067

The difference between the exact probability and the approximate probability is 0.0067.