Student's *T* Distribution

The *t* distributions were discovered by William S. Gosset in 1908. Gosset was a statistician employed by the Guinness brewing company which had stipulated that he not publish under his own name. He therefore wrote under the pen name ``Student.'' These distributions arise in the following situation.

Suppose we have a simple random sample of size *n* drawn from a Normal population with mean μ and standard deviation σ. Let $\overline{x}$ denote the sample mean and *s*, the sample standard deviation. Then the quantity

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}} \qquad (1)$$

has a *t* distribution with *n*-1 degrees of freedom.

Note that there is a different *t* distribution for each sample size, in other words, it is a class of distributions. When we speak of a specific *t* distribution, we have to specify the *degrees of freedom*. The degrees of freedom for this *t* statistics comes from the sample standard deviation *s* in the denominator of equation 1.

The *t* density curves are symmetric and bell-shaped like the normal distribution and have their peak at 0. However, the spread is more than that of the standard normal distribution. This is due to the fact that in formula 1, the denominator is *s* rather than σ. Since *s* is a random quantity varying with various samples, the variability in *t* is more, resulting in a larger spread.

The larger the degrees of freedom, the closer the *t*-density is to the normal density. This reflects the fact that the standard deviation *s* approaches σ for large sample size *n*. You can visualize this in the applet below by moving the sliders.

The stationary curve is the standard normal density.

About this document ...

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