Quiz 5
Confidence Intervals

1. A nylon rope maker claims its repelling rope's average breaking strength is at least 800 pounds. A government inspector is interested in whether or not the actual breaking strength is living up to the company's claim. The inspector randomly selects 30 ropes from a large shipment and tests the breaking strength. She observed a mean breaking strength of 750 pounds with a standard deviation of 64 pounds.

   a. Compute a 90% confidence interval for the true mean breaking strength of all ropes.

   $t^*_{0.9,29} = 1.699$ $\quad \bar{x} \pm t^* \frac{s}{\sqrt{n}} = 750 \pm \frac{(1.699)(64)}{\sqrt{30}} = 750 \pm 19.85$

   $\bar{x} = 750$

   $s = 64$ $\quad (730.15, 769.85)$

   $n = 30$

   we are 90% confident the interval (730.15, 769.85) contains the true mean breaking strength.

   b. What is the point estimate for the population mean? 750 lbs

   c. What is the standard error of your point estimate? $\frac{64}{\sqrt{30}} = 11.68$

   d. What is the critical value? 1.699

   e. What is the margin of error? 19.85 lbs

   f. Does this confidence interval contain the value 800 pounds (the breaking strength that is claimed by the company)? What do you infer?

   The 90% confidence interval (730.15, 769.85) does not contain 800. This is evidence that the true mean breaking strength is not the 800 lbs claimed by the company.

   g. The inspector wants to reduce the bound on the error of estimation to 10 pounds by conducting another study using a larger sample of ropes. How large of a sample is needed to estimate the true mean breaking strength to within 10 pounds with 95% level of confidence?

   $M = 10$ lbs $\quad n = \left(\frac{z^* s}{M}\right)^2 = \left[\frac{(1.96)(64)}{10}\right]^2 = 157.35$

   $n = 158$

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Confidence Intervals

1. A physician wants to estimate the true proportion of American adults who exercise at least three times a week. In a random sample of 400 American adults, 120 reported exercising three times a week.

   a. Construct a 95% confidence interval for the true proportion of American adults that exercise three times a week.

$z^* = 1.96$

$\hat{p} = \frac{120}{400} = 0.3$

$n = 400$

$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.3 \pm (1.96)\sqrt{\frac{(0.3)(0.7)}{400}}$

$= 0.3 \pm (1.96)(0.0229)$

$= 0.3 \pm 0.0449$

$(0.26, 0.34)$

We are 95% confident the interval (0.26, 0.34) contains the true proportion of adults who exercise at least 3 times a week.

   b. What is the point estimate for the population proportion? 0.3

   c. What is the standard error of your point estimate? $\sqrt{\frac{(0.3)(0.7)}{400}} = 0.0229$

   d. What is the critical value? 1.96

   e. What is the margin of error? 0.0449

   f. It is desired to reduce the bound on the error of estimation to 0.03 with a 95% level of confidence. Find the minimum sample size required to accomplish this goal. Use the estimate of *p* from the existing sample.

$M = 0.03$

$n = \left(\frac{z^*}{M}\right)^2 \cdot \hat{p} \cdot (1-\hat{p})$

$n = \left(\frac{1.96}{0.03}\right)^2 (0.3)(0.7) = 896.37$

$n = 897$