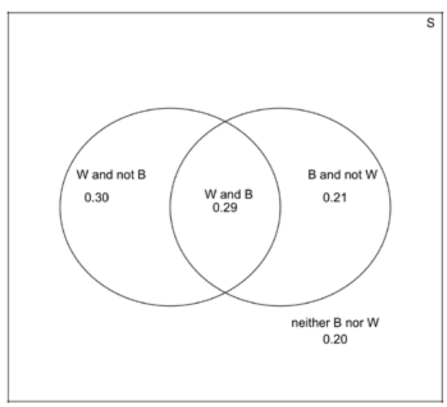
Chapter 12 Solutions

12.1: It is unlikely that these events are independent. In particular, it is reasonable to expect that younger adults are more likely than older adults to be college students. **Note**: *Using the notation of conditional probability introduced later in this chapter,* P(*college student* | *over 55*) < *0.08.*

12.2: This would not be surprising: assuming that all the authors are independent (for example, none were written by siblings or married couples), we can view the nine names as being a random sample, and the probability that none of these are among the 10 most common names is $P(N = 0) = (1 - 0.096)^9 = 0.4032$.

12.3: If we assume that each site is independent of the others (and that they can be considered as a random sample from the collection of sites referenced in scientific journals), then *P*(all seven are still good) = $(0.87)^7 = 0.3773$.

12.4: A Venn diagram is provided. *B* is the event "the degree is a bachelor's degree," and *W* is the event "the degree was earned by a woman." The probability of the overlap is given. Subtracting this from the given probabilities for *B* and *W* gives the probabilities of the rest of those events. Those probabilities add to 0.80, so *P*(neither *B* nor *W*) = 0.20. (a) Because $P(W) = 0.59$, *P*(degree was earned by a man) = *P*(not *W*) = 1 – 0.59 = 0.41, or 41%. (b) *P*(*B* and not *W*) = 0.50 – 0.29 = 0.21, or 21%. (c) Since *P*(*B* and *W*) = 0.29, but $P(B)\times P(W) = (0.50)(0.59) = 0.295$, then $P(B \text{ and } W) \neq P(B)\times P(W)$. Hence, *B* and *W* are not independent.



12.5: (a) A Venn diagram is provided. (b) The events are

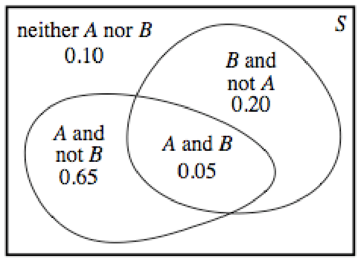
{*A* and *B*} = {student is at least 25 and local}

{*A* and not *B*} = {student is at least 25 and not local}

{*B* and not *A*} = {student is less than 25 and local}

{neither *A* nor *B*} = {student is less than 25 and not local}

(c) *P*(*A* and *B*) is given. Subtracting this from the given probabilities for *A* and *B* gives *P*(*A* and not *B*) and *P*(*B* and not *A*). Those probabilities add to 0.90, so *P*(neither *B* nor *W*) = 0.10.



12.6: Refer to the Venn diagram in the solution of Exercise 12.4. Using the notation given in that solution, $P(W \mid B) = \frac{P(B \text{ and } W)}{P(B)} = 0.29/0.50 = 0.58$.

12.7: $P(B \mid \text{not } A) = \frac{P(B \text{ and not } A)}{P(\text{not } A)} = \frac{P(B) - P(B \text{ and } A)}{P(\text{not } A)} = \frac{0.2}{0.3} = 0.667$.

12.8: Let *R* be the event "game is a role playing game," while *S* is the event "game is a strategy game." Then $P(\text{not } S) = 1 - 0.354 = 0.646$, and $P(R \mid \text{not } S) = \frac{P(R \text{ and not } S)}{P(\text{not } S)} = \frac{0.139}{0.646} = 0.2152$. (Note that "*R* and not *S*" is equivalent to *R*.)

12.9: Let *H* be the event that an adult belongs to a club, and *T* be the event that he/she goes at least twice a week. We have been given $P(H) = 0.15$ and $P(T \mid H) = 0.50$. Note also that *P*(*T* and *H*) = P(T), since one has to be a member of the club in order to attend. So $P(T) = P(H)P(T \mid H) = (0.15)(0.50) = 0.075$. About 7.5% of all adults go to health clubs at least twice a week.

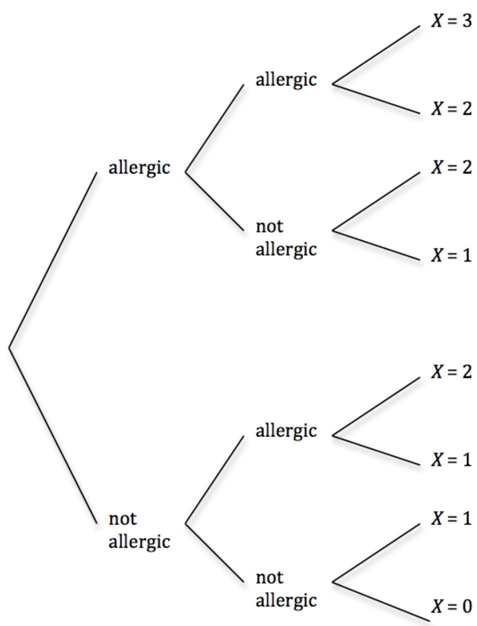
12.10: PLAN: Express the information we are given in terms of events and their probabilities: let *A* = {the teen is online}, *B* = {the teen has a profile}, and *C* = {the teen has commented on a friend's blog}. Then $P(A) = 0.93$, $P(B \mid A) = 0.55$, and $P(C \mid A \text{ and } B) = 0.76$. We want to find *P*(*A* and *B* and *C*). SOLVE: Use the multiplication rule: $P(A \text{ and } B \text{ and } C) = P(A)P(B \mid A)P(C \mid A \text{ and } B) = (0.93)(0.55)(0.76) = 0.3887$. CONCLUDE: About 39% of all teens are online, have a profile, and have placed comments on a friend's blog.

12.11: (a) and (b) These probabilities are provided in the table. (c) The product of these conditional probabilities gives the probability of a flush in spades by the general multiplication rule: we must draw a spade, and then another, and then a third, a fourth, and a fifth. The product of these probabilities is about 0.0004952. (d) Because there are four possible suits in which to have a flush, the probability of a flush is four times that found in (c), or about 0.001981.

$$
\begin{aligned}
P(\text{1st card } \spadesuit) &= \tfrac{13}{52} = \tfrac{1}{4} &&= 0.25 \\
P(\text{2nd card } \spadesuit \mid 1 \spadesuit \text{ picked}) &= \tfrac{12}{51} = \tfrac{4}{17} &&\doteq 0.2353 \\
P(\text{3rd card } \spadesuit \mid 2 \spadesuit\text{s picked}) &= \tfrac{11}{50} &&= 0.22 \\
P(\text{4th card } \spadesuit \mid 3 \spadesuit\text{s picked}) &= \tfrac{10}{49} &&\doteq 0.2041 \\
P(\text{5th card } \spadesuit \mid 4 \spadesuit\text{s picked}) &= \tfrac{9}{48} = \tfrac{3}{16} &&= 0.1875
\end{aligned}
$$

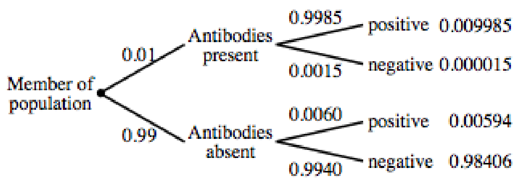
12.12: (a) There are a total of 976 professors, of which 272 are women, so P(woman) = 272/976 = 0.2787. (b) P (woman | full professor) = 73/375 = 0.1947. (c) Rank and sex are not independent; if they were, the probabilities in (a) and (b) would be equal.

12.13: PLAN: We construct a tree diagram showing the results (allergic or not) for each of the three individuals. SOLVE: In the tree diagram, each "up-step" represents an allergic individual (and has probability 0.01), and each "down-step" is a non-allergic individual (and has probability 0.99). At the end of each of the 8 complete branches are the value of X. Any branch with 2 up-steps and 1 down-step has probability $0.01^2 \times 0.99^1 = 0.000099$, and yields $X = 2$. Any branch with 1 up-step and 2 down-steps has probability $0.01^1 \times 0.99^2 = 0.009801$, and yields $X = 1$.



There are three branches each corresponding to $X = 2$ and $X = 1$, and only one branch each for $X = 3$ and $X = 0$. Because $X = 0$ and $X = 3$ appear on one branch each, $P(X = 0) = 0.99^3 = 0.970299$ and $P(X = 3) = 0.01^3 = 0.000001$. Meanwhile, $P(X = 1) = 3(0.01)^1(0.99)^2 = 0.029403$, and $P(X = 2) = 3(0.01)^2(0.99)^1 = 0.000297$. CONCLUDE: $P(X = 0) = 0.970299$, $P(X = 1) = 0.029403$, $P(X = 2) = 0.000297$, and $P(X = 3) = 0.000001$.

12.14: (a) The tree diagram follows. (b) P(positive) = 0.009985 + 0.00594 = 0.015925.



12.15: $P(X = 2 \mid X \geq 1) = \dfrac{P(X = 2 \text{ and } X \geq 1)}{P(X \geq 1)} = \dfrac{P(X = 2)}{P(X \geq 1)} = \dfrac{0.000297}{1 - 0.970299} = 0.010$.

12.16: P (has antibody | positive) = $\dfrac{0.009985}{0.015925}$ = 0.627.

12.17: (b) This probability is $(0.98)^3 = 0.9412$.

12.18: (b) This probability is $1 - (0.98)^3 = 0.0588$.

12.19: (a) P(at least one positive) = 1 – P(both negative) = 1 – P(first negative)P(second negative) = 1 – (0.1)(0.2) = 0.98.

12.20: (b) There were 23,190 female recipients out of a total of 26,338 + 23,190 = 49,528 doctorates. Hence, P(female) = 23,190/49,528 = 0.4682, or 0.47.

12.21: (c) Of 6,006 + 1,623 = 7,629 Engineering doctorates, 1,623 were awarded to females. Then P(female | engineering) = 1,623/7,629 = 02127, or 0.21.

12.22: (b) Of 23,190 female doctorates, 1,623 were awarded in Engineering. Hence, P(engineering | female) = 1,623/23,190 = 0.0699, or 0.07.

12.23: (c) We want the fraction of engineering doctorates conferred to women. Hence, A (engineering degree) is what has been given. Hence, $P(B \mid A)$.

12.24: (b) $P(W \text{ or } S) = P(W) + P(S) - P(W \text{ and } S) = 0.52 + 0.25 - 0.11 = 0.66$.

12.25: (c) $P(W \text{ and } D) = P(W)P(D \mid W) = (0.86)(0.028) = 0.024$.

12.26: (b) $P(D) = P(W \text{ and } D) + P(B \text{ and } D) + P(A \text{ and } D) = (0.86)(0.028) + (0.12)(0.044) + (0.02)(0.035) = 0.030$.

12.27: P(8 losses) = $(0.75)^8 = 0.1001$.

12.28: P(none are O-negative) = $(1 - 0.072)^{10}$ = 0.4737, so P(at least one is O-negative) = 1 − 0.4737 = 0.5263.

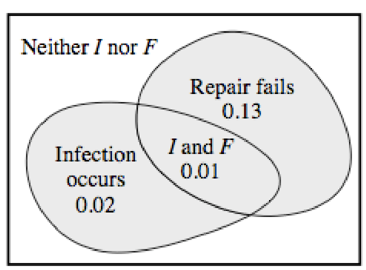
12.29: (a) P(win the jackpot) = $\left(\frac{1}{20}\right)\left(\frac{9}{20}\right)\left(\frac{1}{20}\right)$ = 0.001125. (b) The other (non-cherry) symbol can show up on the middle wheel, with probability $\left(\frac{1}{20}\right)\left(\frac{11}{20}\right)\left(\frac{1}{20}\right)$ = 0.001375, or on either of the outside wheels, with probability = $\left(\frac{19}{20}\right)\left(\frac{9}{20}\right)\left(\frac{1}{20}\right)$ (each). (c) Combining all three cases from part (b), we have P(exactly two cherries) = 0.001375 + 2 · 0.021375 = 0.044125.

12.30: (a) $(0.80)^2 = 0.64$. (b) P(sighting on at least one day) = 1 – P(sighting on neither day) = 1 – $(0.20)^2$ = 0.96. (c) We want the number of trips, k, such that $1 – (0.20)^k \geq 0.99$. You can solve this algebraically for k, yielding $k \geq 2.86$, which must be rounded up to 3. Details follow:

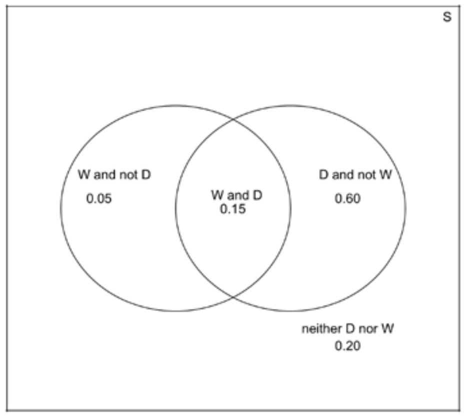
$$1-(0.20)^k \geq 0.99$$
$$(0.20)^k \leq 1-0.99 = 0.01$$
$$k\ln(0.20) \leq \ln(0.01)$$
$$k(-1.609) \leq -4.605$$
$$k \geq \frac{-4.605}{-1.609} = 2.862$$

Alternatively, we can solve this by trial and error: If we try k =2, for example, $1 – (0.20)^2 = 0.96$, which fails. However, with k = 3, $1 – (0.20)^3 = 0.992$, which satisfies the requirement. Hence, at least three trips are needed to assure at least 0.99 probability of at least one sighting.

12.31: PLAN: Let I be the event “infection occurs” and let F be “the repair fails.” We have been given $P(I) = 0.03$, $P(F) = 0.14$, and $P(I$ and $F) = 0.01$. We want to find P(not I and not F). SOLVE: First use the general addition rule: $P(I$ or $F) = P(I) + P(F) − P(I$ and $F)$ = 0.03 + 0.14 − 0.01 = 0.16. This is the shaded region in the Venn diagram provided. Now observe that the desired probability is the complement of “I or F” (the *unshaded* region): P(not I and not F) = 1 − $P(I$ or $F)$ = 0.84. CONCLUDE: 84% of operations succeed and are free from infection.

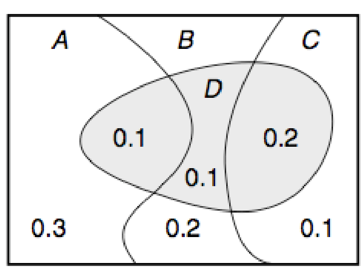


12.32: Let W be the event that a whale is seen. Let D be the event that a dolphin is seen. The Venn diagram is provided below. (a) $P(W) = 0.05 + 0.15 = 0.20$. (b) $P(W$ and not $D) = 0.05$. (c) Yes, since $P(W$ and $D) = 0.15 = P(W)P(D) = (0.20)(0.75) = 0.15$.



12.33: PLAN: Let I be the event "infection occurs" and let F be "the repair fails." Refer to the Venn diagram in Exercise 12.31 (ignoring the shading). We want to find $P(I \mid \text{not } F)$. SOLVE: We have $P(I \mid \text{not } F) = \dfrac{P(I \text{ and not } F)}{P(\text{not } F)} = \dfrac{0.02}{0.86} = 0.0233$. CONCLUDE: The probability of infection given that the repair is successful is 0.0233. That is, in 2.33% of all successful operation cases, the patient develops infection.

12.34: Note that in this diagram, events A, B, and C should not overlap and should account for all possibilities (that is, those three events fill the entire diagram). Meanwhile, D intersects all three of the others. The probabilities $P(A$ and $D)$, $P(B$ and $D)$, and $P(C$ and $D)$ give the probability of each overlapping region, and the portion of each event A, B, and C outside of D must account for the rest of that event's probability. As can be seen from the diagram, $P(D) = 0.4 = 0.1 + 0.1 + 0.2$.



12.35: Let H be the event student was home schooled. Let R be the event student attended a regular public school. We want $P(H \mid \text{not } R)$. Note that the event "H and not R" = "H" since the events are disjoint. Then $P(H \mid \text{not } R) = \dfrac{P(H)}{P(\text{not } R)} = \dfrac{0.006}{1-0.781} = 0.0274$.

12.36: (a) P (income $\geq$ \$30,000) = 0.315 + 0.180 + 0.031 = 0.526. (b) P (income $\geq$ \$75,000 |

income ≥ \$30,000) = $\dfrac{P(\text{income} \ \geq \ \$75{,}000)}{P(\text{income} \ \geq \ \$30{,}000)} = \dfrac{0.211}{0.526} = 0.4011.$

12.37: (a) These events are not independent, because *P*(pizza with mushrooms) = 4/7, but *P* (mushrooms | thick crust) = 2/3 (if the events were independent, these probabilities would be equal). Alternatively, note that *P*(thick crust with mushrooms) = 2/7, which is not equal to the product of *P*(mushrooms) = 4/7 and *P*(thick crust pizza) = 3/7. (b) With the eighth pizza, *P* (mushrooms) = 4/8 = 1/2, and *P* (mushrooms | thick crust) = 2/4 = 1/2 , so these events are independent.

12.38: (a) *P*(two boys | at least one boy) = $\dfrac{P(\text{two boys})}{\mathrm{P}(\text{at least one boy})} = \dfrac{0.25}{0.75} = \dfrac{1}{3}$. (b) *P*(two boys |older child is a boy) = $\dfrac{P(\text{two boys})}{\mathrm{P}(\text{older child is boy})} = \dfrac{0.25}{0.50} = \dfrac{1}{2}$. Note that we can also find this by reasoning that *P*(two boys | older child is a boy) = *P* (younger child is a boy | older child is a boy). Because the two children's genders are independent, this probability is the same as the unconditional probability *P*(younger child is a boy) = 0.5.

12.39: Let *W* be the event "the person is a woman" and *M* be "the person earned a Master's degree." (a) *P*(not *W*) = 1421/3560 = 0.3992. (b) *P*(not *W* | *M*) = 282/732 = 0.3852. (c) The events "choose a man" and "choose a Master's degree recipient" are not independent. If they were, the two probabilities in (a) and (b) would be equal.

12.40. Let *W* be the event "the person is a woman" and *A* be "the person earned a Associate's degree." (a) *P*(*W*) = 2139/3560 = 0.6008. (b) *P*(*A* | *W*) = 556/2139 = 0.2599. (c) *P*(*W* and *A*) = *P*(*W*)*P*(*A* | *W*) = (0.6008)(0.2599) = 0.1561. Except for rounding, this agrees with the directly computed probability: *P*(*W* and *A*) = 556/3560 = 0.1562.

12.41: Let *D* be the event "a seedling was damaged by a deer." (a) *P*(*D*) = 209/871 = 0.2400. (b) The conditional probabilities are:

P(*D*| no cover) = 60/211 = 0.2844

P(*D*| cover < 1/3) = 76/234 = 0.3248

P(*D* | 1/3 to 2/3 cover) = 44/221 = 0.1991

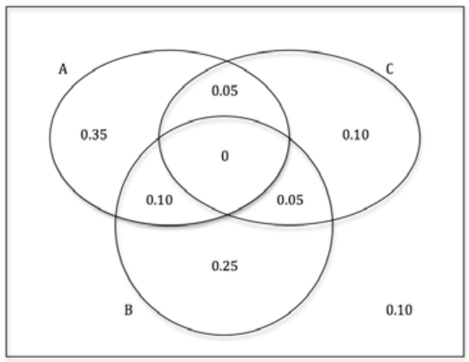
P(*D* | cover > 2/3) = 29/205 = 0.1415

(c) Cover and damage are not independent; *P*(*D*) decreases noticeably when thorny cover is 1/3 or more.

12.42: This conditional probability is *P*(cover > 2/3 | not *D*) = 176/(151 + 158 + 177 + 176) = 176/662 = 0.2659, or 26.59%.

12.43. This conditional probability is *P*(cover < 1/3 | *D*) = 76/(60 + 76 + 44 + 29) = 76/209 = 0.3636, or 36.36%.

12.44: We first construct the Venn diagram for this set of exercises. To find the probabilities in this Venn diagram, begin with *P*(*A* and *B* and *C*) = 0 in the center of the diagram. Then the two-way intersections *P*(*A* and *B*), *P*(*A* and *C*), and *P*(*B* and *C*) go in the remainder of the overlapping areas; if *P* (*A* and *B* and *C*) had been something other than 0, we would have subtracted this from each of the two-way intersection probabilities to find, for example, *P* (*A* and *B* and not *C*). Next, determine *P*(*A* only) so that the total probability of the regions that make up the event *A* is 0.50. Finally, *P*(none) = *P*(not *A* and not *B* and not *C*) = 0.10 because the total probability inside the three sets *A*, *B*, and *C* is 0.90. The completed Venn diagram is shown.



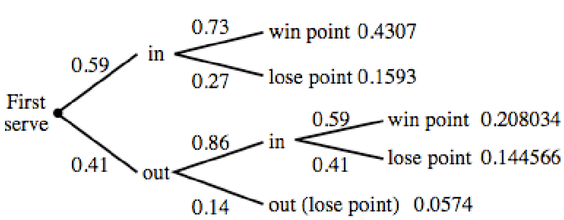
We seek *P*(no offer) = *P*(not *A* and not *B* and not *C*) = 0.10.

12.45: This is *P*(*A* and not *B* and not *C*) = 0.35.

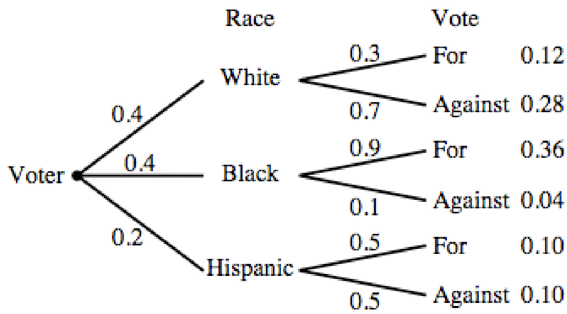
12.46: *P*(*B* |*C*) = 0.05/0.20 = 0.25. *P*(*C* | *B*) = 0.05/0.40 = 0.125.

12.47: (a) *P*(doubles on first toss) = 1/6, since 6 of the 36 equally likely outcomes enumerated in Figure 10.2 involve rolling doubles. (b) We need no doubles on the first roll (which happens with probability 5/6), then doubles on the second toss. *P*(first doubles appears on toss 2) = (5/6)(1/6) = 5/36. (c) Similarly, *P*(first doubles appears on toss 3) = $(5/6)^2(1/6)$ = 25/216. (d) *P*(first doubles appears on toss 4) = $(5/6)^3(1/6)$, etc. In general, *P*(first doubles appears on toss *k*) = $(5/6)^{k-1}(1/6)$. (e) *P*(go again within 3 turns) = *P*(roll doubles in 3 or fewer rolls) = *P*(roll doubles on 1st, 2nd, or 3rd try) = (1/6) + (5/6)(1/6) + $(5/6)^2(1/6)$ = 0.4213.

12.48: The tree diagram provided organizes this information; the probability of each outcome is the product of the individual branch probabilities leading to it. The total probability of the serving player winning a point is 0.4307 + 0.208034 = 0.6387.



12.49: PLAN: Let *W*, *B*, and *H* be the events that a randomly selected voter is (respectively) white, black, and Hispanic. We have been given *P*(*W*) = 0.4, *P*(*B*) = 0.4, *P*(*H*) = 0.2. If *F* = “a voter votes for the candidate," then $P(F \mid W)=0.3$, $P(F \mid B) = 0.9$, $P(F \mid H) = 0.5$. We want to find $P(F)$. SOLVE: The tree diagram provided organizes the information. The numbers on the right side of the tree are found by the general multiplication rule; for example, P("white" and "for") = $P(W \text{ and } F) = P(W)P(F \mid W) = (0.4)(0.3) = 0.12$. We find $P(F)$ by adding all the numbers next to the branches ending in "for": $P(F) = 0.12 + 0.36 + 0.10 = 0.58$. CONCLUDE: The black candidate expects to get 58% of the vote.



12.50: $P(\text{first serve in} \mid \text{server won point}) = \dfrac{P(\text{first serve in and server won point})}{P(\text{server won point})} = \dfrac{0.4307}{0.6387} =$ 0.6743, or 67.43%.

12.51: $P(B \mid F) = \dfrac{P(B \text{ and } F)}{P(F)} = \dfrac{0.36}{0.58} = 0.6207$, or about 62%.

12.52 (a) Let C = {teen owns a cell phone} and T = {texts}. We are given: $P(C) = 0.75$ and $P(T|C) = 0.87$. So, $P(C \text{ and } T) = P(C)*P(T|C) = (0.75)(0.87) = 0.6525$. (b) Let M = {more than 6,000 texts a month}. We want $P(C \text{ and } T \text{ and } M) = P(C)*P(T|C)*P(M|C \text{ and } T) = (0.75)(0.87)(0.15) = 0.0979$.

12.53: For a randomly selected resident of the United States, let W, B, A, and L be (respectively) the events that this person is white, black, Asian, and lactose intolerant. We have been given

$P(W) = 0.82$ $P(B) = 0.14$ $P(A) = 0.04$

$P(L|W) = 0.15$ $P(L|B) = 0.70$ $P(L|A) = 0.90$

(a) $P(L) = (0.82)(0.15) + (0.14)(0.70) + (0.04)(0.90) = 0.257$, or 25.7%.

(b) $P(A \mid L) = P(A \text{ and } L)/P(L) = (0.04)(0.90)/0.257 = 0.1401$, or 14%.

12.54: Let R = {recent donor}, P = {pledged}, and C = {contributed}. (We could also give names to the "past donor" and "new prospect" events, but we do not need these for this explanation.) (a) The percent of calls resulting in a contribution can be found by considering all the branches of the tree that end in a contribution, meaning that we compute, for example, $P(C \text{ and } R) = P(R) \cdot P(P \mid R) \cdot P(C \mid R \text{ and } P)$. This gives

$P(C) = (0.5)(0.4)(0.8) + (0.3)(0.3)(0.6) + (0.2)(0.1)(0.5) = 0.224$, or 22.4%. (b) $P(R|C) = \frac{P(C \text{ and } R)}{P(C)} = \frac{(0.5)(0.4)(0.8)}{0.224} = 0.7143$, or 71.4%.

12.55: In this problem, allele 29 is playing the role of A, and 0.181 is the proportion with this allele ($a = 0.181$). Similarly, allele 31 is playing the role of B, and the proportion having this allele is $b = 0.071$. The labels aren't important – you can reverse assignments of A and B. The proportion of the population with combination (29,31) is therefore 2(0.181)(0.071) = 0.025702. The proportion with combination (29,29) is (0.181)(0.181) = 0.032761. Of course under these assignments, there are other alleles possible for this locus.

12.56: The proportion having combination (16,17) is 2(0.232)(0.212) = 0.098368.

12.57: In Exercise 12.55, we found that the proportion of the population with allele (29,31) at loci D21S11 is 0.025702. In Exercise 12.56, we found that the proportion with allele (16,17) at loci D3S1358 is 0.098368. Assuming independence between loci, the proportion with allele (29,31) at D21S11 and (16,17) at D3S1358 is (0.098368)(0.025702) = 0.002529.

12.58: If the DNA profile found on the hair is possessed by 1 in 1.6 million individuals, then we would expect about 3 individuals in the database of 4.5 million convicted felons to demonstrate a match. This comes from (4.5 million)/(1.6 million) = 2.8125, which was rounded up to 3.

12.59 and 12.60 are Web-based exercises.