A recent study claims that girls and boys do not do equally well on math tests taken from the 2nd to 11th grades (*Chicago Tribune*, July 25, 2008). Suppose in a representative sample, 344 of 430 girls and 369 of 450 boys score at proficient or advanced levels on a standardized math test. Use <u>Table 1</u>.

Let p_1 represent the population proportion of girls and p_2 the population proportion of boys.

a. Construct the 95% confidence interval for the difference between the population proportions of girls and boys who score at proficient or advanced levels. (Negative values should be indicated by a minus sign. Round intermediate calculations to at least 4 decimal places. Round your answers to 2 decimal places.)

Confidence interval is
$$-7.19 \pm 0.2$$
 % to 3.19 ± 0.2 %.

- **b.** Select the appropriate null and alternative hypotheses to test whether the proportion of girls who score at proficient or advanced levels differs from the proportion of boys.
 - $Oldsymbol{Olds$
 - \bigcirc H_0 : $p_1 p_2 \le 0$; H_A : $p_1 p_2 > 0$
 - H_0 : $p_1 p_2 \ge 0$; H_A : $p_1 p_2 < 0$
- c. At the 5% significance level, what is the conclusion to the test? Do the results support the study's claim?
 - Reject H_0 ; the study's claim is supported by the sample data.
 - Reject H₀; the study's claim is not supported by the sample data.
 - Do not reject H_0 ; the study's claim is supported by the sample data.
 - \bigcirc Do not reject H_0 ; the study's claim is not supported by the sample data.

Explanation:

a.
$$p_1 = \frac{x_1}{n_1} = \frac{344}{430} = 0.80; \ \ p_2 = \frac{x_2}{n_2} = \frac{369}{450} = 0.82$$

With $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$.

$$\left(p_1 - p_2 \right) \pm z_{\alpha/2} \sqrt{\frac{p_1^{(1-p_1)}}{n_1} + \frac{p_2^{(1-p_2)}}{n_2}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.82)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.82)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.82)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.80)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.80)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.80)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.80)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.80)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.80)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.80)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.80)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.80)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480} + \frac{0.82(1-0.80)}{450}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{480}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{10.80}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{10.80}} = (0.80 - 0.82) \pm 1.96 \times \sqrt{\frac{0.80(1-0.80)}{10.80}$$

= 0.02 ± 0.0519 or [-0.0719, 0.0319]. The difference in the population proportions is between -7.19% and 3.19%, at the 95% confidence level.

Since the hypothesized difference of 0 is included in the above 95% confidence interval, we do not reject H_0 . The study's claim is not supported by the sample data, at the 5% significance level.

References

Worksheet

Difficulty: 3 Hard

Learning Objective: 10-03 Make inferences about the difference between two population proportions based on independent sampling.

According to the Pew report, 14.6% of newly married couples in 2008 reported that their spouse was of another race or ethnicity (*CNNLiving*, June 7, 2010). In a similar survey in 1980, only 6.8% of newlywed couples reported marrying outside their race or ethnicity. Suppose both of these surveys were conducted on 120 newly married couples. Use <u>Table 1</u>.

Let p_1 represent the population proportion in 2008 and p_2 the population proportion in 1980.

- **a.** Specify the competing hypotheses to test the claim that there is an increase in the proportion of people who marry outside their race or ethnicity.
 - H_0 : $p_1 p_2 = 0$; H_A : $p_1 p_2 \neq 0$
 - $OPD H_0$: $p_1 p_2 \le 0$; H_A : $p_1 p_2 > 0$
 - H_0 : $p_1 p_2 \ge 0$; H_A : $p_1 p_2 < 0$
- b. What is the value of the test statistic and the associated p-value? (Round intermediate calculations to at least 4 decimal places, "Test statistic" value to 2 decimal places and "p-value" to 4 decimal places.)

Test statistic	1.95 ± 2%
p-value	0.0256 ± 0.01

- c. At the 5% level of significance, what is the conclusion?
 - Reject H₀ since the p-value is less than α.
 - Reject H_0 since the p-value is more than α .
 - On not reject H_0 since the p-value is less than α .
 - Do not reject H_0 since the p-value is more than α .

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Explanation:

b.
$$\overline{p}_1 = 0.146, n_1 = 120; \overline{p}_2 = 0.068, n_2 = 120$$

$$\overline{p} = \frac{n_1 \overline{p}_1 + n_2 \overline{p}_2}{n_1 + n_2} = \frac{120(0.146) + 120(0.068)}{120 + 120} = 0.107$$

$$z = \frac{p_1 - p_2}{\sqrt{p(1-p)\left(\frac{1}{p_1} + \frac{1}{p_2}\right)}} = \frac{0.146 - 0.068}{\sqrt{0.107(1 - 0.107)\left(\frac{1}{120} + \frac{1}{120}\right)}} = 1.95$$

the p-value = $P(Z \ge 1.95) = 1 - 0.9744 = 0.0256$.

c.

Since the *p*-value = $0.0256 < 0.05 = \alpha$, we reject H_0 . There has been an increase in the proportion of individuals marrying outside their race or ethnicity.

References

Worksheet

Difficulty: 3 Hard

Learning Objective: 10-03 Make inferences about the difference between two population proportions based on

independent sampling.

More people are using social media to network, rather than phone calls or e-mails (US News & World Report, October 20, 2010). From an employment perspective, jobseekers are no longer calling up friends for help with job placement, as they can now get help online. In a recent survey of 150 jobseekers, 67 said they used LinkedIn to search for jobs. A similar survey of 140 jobseekers, conducted three years ago, had found that 58 jobseekers had used LinkedIn for their job search. Use Table 1.

Let p_1 represent the population proportion of recent jobseekers and p_2 the population proportion of job seekers three years ago. Let recent survey and earlier survey represent population 1 and population 2, respectively.

- a. Set up the hypotheses to test whether there is sufficient evidence to suggest that more people are now using LinkedIn to search for jobs as compared to three years ago.
 - \bigcirc H_0 : $p_1 p_2 = 0$; H_A : $p_1 p_2 \neq 0$
 - \bigcirc H_0 : $p_1 p_2 \le 0$; H_A : $p_1 p_2 > 0$
 - H_0 : $p_1 p_2 \ge 0$; H_A : $p_1 p_2 < 0$
- b. Calculate the value of the test statistic. (Round intermediate calculations to at least 4 decimal places and final answer to 2 decimal places.)

0.56 ± 2% Test statistic

c. Calculate the critical value at the 5% level of significance. (Round your answer to 3 decimal places.)

1.645 ± 0.01 Critical value

- d. Interpret the results.
 - Reject H_0 ; there is an increase in the proportion of people using LinkedIn
 - Reject H_0 ; there is no increase in the proportion of people using LinkedIn
 - \bigcirc Do not reject H_0 ; there is an increase in the proportion of people using LinkedIn
 - ullet Do not reject H_0 ; there is no increase in the proportion of people using LinkedIn

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Explanation:

$$\overline{p}_1 = \frac{x_1}{n_1} = \frac{67}{150} = 0.4467$$

$$\overline{P}_2 = \frac{x_2}{n_2} = \frac{58}{140} = 0.4143$$

$$\overline{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{67 + 58}{290} = 0.4310$$

$$z = \frac{p_1 - p_2}{\sqrt{p(1 - p)(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.4467 - 0.4143}{\sqrt{0.4310(1 - 0.4310)(\frac{1}{150} + \frac{1}{140})}} = 0.56$$

c. & d

With α = 0.05, the critical value $z_{0.05}$ = 1.645. Since z = 0.56 < 1.645, we do not reject H_0 . We cannot conclude that the proportion of recent workers finding jobs on LinkedIn is more than the proportion three years ago, at the 5% significance level.

References

Worksheet

Difficulty: 3 Hard

Learning Objective: 10-03 Make inferences about the difference between two population proportions based on independent sampling.

Only 26% of psychology majors are "satisfied" or "very satisfied" with their career paths as compared to 50% of accounting majors (*The Wall Street Journal*, October 11, 2010). Suppose these results were obtained from a survey of 300 psychology majors and 350 accounting majors.

Let p_1 represent the population proportion of satisfied accounting majors and p_2 the population proportion of satisfied psychology majors.

- a. Develop the appropriate null and alternative hypotheses to test whether the proportion of accounting majors satisfied with their career paths differs from psychology majors by more than 20 percentage points.
 - OH_0 : $p_1 p_2 = 0.20$; H_A : $p_1 p_2 \neq 0.20$
 - \bigcirc H_0 : $p_1 p_2 \le 0.20$; H_A : $p_1 p_2 > 0.20$
 - H_0 : $p_1 p_2 \ge 0.20$; H_A : $p_1 p_2 < 0.20$
- b. Calculate the value of the test statistic and the p-value. (Round intermediate calculations to at least 4 decimal places. Round "Test statistic" value to 2 decimal places and "p-value" to 4 decimal places.)

Test statistic	1.09 ± 2%
<i>p</i> -value	0.1379 ± 0.02

- c. At the 5% significance level, what is the conclusion?
 - Reject H_0 ; the difference in the proportions is more than 20%
 - Reject H_0 ; the difference in the proportions is not more than 20%
 - Do not reject H₀; the difference in the proportions is more than 20%
 - On not reject H_0 ; the difference in the proportions is not more than 20%.

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Explanation:

b.
$$p_1 = 0.50, n_1 = 350; \overline{p}_2 = 0.26, n_2 = 300$$

$$z = \frac{\frac{(p_1 - p_2) - d_0}{\sqrt{\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}}} = \frac{\frac{(0.50 - 0.28) - 0.20}{\sqrt{\frac{0.50(1 - 0.50)}{350} + \frac{0.28(1 - 0.28)}{300}}} = 1.09$$

The *p*-value = $P(Z \ge 1.09) = 0.1379$.

C.

Since the p-value = 0.1379 > 0.05 = α , we do not reject H_0 . There is not enough evidence to say that the proportion of accounting majors satisfied with their career path differs from psychology majors by more than 20%.

References

Worksheet Difficulty: 3 Hard Learning Objective: 10-03 Make inferences

about the difference between two population proportions based on

independent sampling.

A recent report suggests that business majors spend the least amount of time on course work than all other college students (*New York Times*, November 17, 2011). A provost of a university decides to conduct a survey where students are asked if they study hard, defined as spending at least 20 hours per week on course work. Of 120 business majors included in the survey, 20 said that they studied hard, as compared to 48 out of 150 nonbusiness majors who said that they studied hard. Use Table 1.

Let p_1 represent the population proportion of business majors who study hard, and p_2 the population proportion of non-business majors who study hard. Let business majors who study hard and non-business majors who study hard represent population 1 and population 2, respectively.

- **a-1.** State the hypotheses to test if proportion of business majors who study hard is less than that of the non-business majors.
 - H_0 : $p_1 p_2 \le 0$; H_A : $p_1 p_2 > 0$
 - $OPD H_0$: $p_1 p_2 \ge 0$; H_A : $p_1 p_2 < 0$
 - H_0 : $p_1 p_2 = 0$; H_A : $p_1 p_2 \neq 0$
- **a-2.** State the conclusion about whether the business majors who study hard is less than that of the non-business majors at the 5% level.
 - Reject H₀; there is enough evidence to support the claim that the proportion of business majors who study hard is less than that of the non-business majors.
 - Reject H₀; there is not enough evidence to support the claim that the proportion of business majors who study hard is less than that of the non-business majors.
 - Do not reject H_0 ; there is enough evidence to support the claim that the proportion of business majors who study hard is less than that of the non-business majors.
 - Do not reject H_0 ; there is not enough evidence to support the claim that the proportion of business majors who study hard is less than that of the non-business majors.

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Explanation:

$$p_1 = \frac{x_1}{n_1} = \frac{20}{120} = 0.1667$$

$$p_2 = \frac{x_2}{n_2} = \frac{\text{(expression error)}}{\text{(expression error)}} = \text{(expression error)}$$

$$p = \frac{x_1 + x_2}{n_1 + n_2} = \frac{\text{(expression error)}}{\text{(expression error)}} = \text{(expression error)}$$

$$z = \frac{p_1 - p_2}{\sqrt{p(1 - p)(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.1667 - 0.32}{\sqrt{0.2519(1 - 0.2519)(\frac{1}{120} + \frac{1}{150})}} = -2.88.$$

Since the *p*-value = $P(Z \le -2.88)$ = 0.0020 < 0.05 = α , we reject H₀. At the 5% significance level, the proportion of business majors who study hard is significantly less than the proportion of non-business majors who study hard.

References

Worksheet Difficulty: 3 Hard Learning Objective: 10-03 Make inferences

about the difference between two population proportions based on

independent sampling.