Worksheet: Normal Approximation to Binomial Distribution

Multiple Choice
Identify the choice that best completes the statement or answers the question.

_____ 1. Under what conditions is a normal probability distribution a good approximation for a discrete binomial distribution?
a. np and nq greater than 5
c. $\sigma = \sqrt{npq}$

b. np and nq less than 5
d. continuity correction applied

_____ 2. For which of the binomial distributions listed below is the normal distribution *not* a reasonable approximation?
a. $n = 50, p = 0.4$ b. $n = 40, p = 0.12$ c. $n = 75, p = 0.11$ d. $n = 40, p = 0.8$

_____ 3. Assuming that a normal distribution is a reasonable approximation for a binomial distribution, what value is used to approximate the mean?
a. nq
b. np
c. $\dfrac{q}{p}$
d. $\dfrac{p}{q}$

_____ 4. Assuming that a normal distribution is a reasonable approximation for a binomial distribution, what value is used to approximate the standard deviation?
a. $\sqrt{npq}$
b. np
c. npq
d. $\sqrt{np}$

Short Answer

5. Is a normal distribution a reasonable approximation for a binomial distribution with $n = 50$ and $p = 0.85$? Explain your reasoning.

6. Use the normal approximation to find μ and σ in a binomial distribution with $n = 1000$ and $p = 0.5$.

7. A really tough trivia quiz has 50 multiple-choice questions, each with four possible answers. Use a normal approximation to calculate the probability of randomly guessing the correct answers for at least half of the questions.

8. A really tough trivia quiz has 50 multiple-choice questions, each with four possible answers. Use a normal approximation to calculate the probability of randomly guessing the correct answers for 10 to 15 (inclusive) of the questions.

9. QuenCola, a soft-drink company, knows that it has a 42% market share in one region of the province. QuenCola's marketing department conducts a blind taste test with 100 people at a mall in the region. Use a normal approximation to calculate the probability that fewer than 40 of these people will choose QuenCola.

10. QuenCola, a soft-drink company, knows that it has a 42% market share in one region of the province. QuenCola's marketing department conducts a blind taste test with 100 people at a mall in the region. Use a normal approximation to calculate the probability that exactly 40 of these people will choose QuenCola.

Problem

11.   The probability of an airline flight arriving on time is 90%. Use the normal approximation to find the probability that at least 300 of a random sample of 350 flights will arrive on time. Explain each step in the calculation.

12.   A bank found that 25% of its loans to new small businesses become delinquent. If 500 small businesses are selected randomly from the bank's files, what is the probability that at least 130 of them are delinquent? Compare the result from the normal approximation with that from a calculation using the binomial distribution. Explain whether the difference between the two results is significant.

13.   In his best year, Mickey Mantle's batting average was 0.365. In that year, what was the probability that he would get exactly 5 hits in 15 times at bat? Compare the results from the normal approximation with the results from a calculation using a binomial distribution.

14.   It is estimated that 10% of the vehicles entering Canada from the United States carry undeclared goods. Use the normal approximation to calculate the probability that a search of 500 randomly selected vehicles will find fewer than 50 with undeclared goods.

15.   It is estimated that 10% of the vehicles entering Canada from the United States carry undeclared goods. Use the normal approximation to calculate the probability that a search of 500 randomly selected vehicles will find more than 60 with undeclared goods.

16.   Surveys indicate that 60% of the vehicles in some metropolitan areas operate below peak performance, causing excessive emissions of pollutants. Transport Canada inspected 100 randomly selected vehicles in one of these areas.
a) Predict how closely a normal approximation will agree with the binomial calculation of the probability that exactly half of these vehicles cause unnecessary pollution.
b) Use the normal approximation to calculate the probability that exactly half of the vehicles inspected cause unnecessary pollution.
c) Use the formula for binomial probabilities to calculate the probability that exactly half of the vehicles inspected cause unnecessary pollution.
d) Do these calculations support your prediction in part a)? Explain why or why not.

Worksheet: Normal Approximation to Binomial Distribution
Answer Section

MULTIPLE CHOICE

1. ANS: A PTS: 1 REF: Knowledge & Understanding
OBJ: Section 8.4 TOP: Normal Approximation to binomial distribution
2. ANS: B PTS: 1 REF: Knowledge & Understanding
OBJ: Section 8.4 TOP: Normal Approximation to binomial distribution
3. ANS: B PTS: 1 REF: Knowledge & Understanding
OBJ: Section 8.4 TOP: Normal Approximation to binomial distribution
4. ANS: A PTS: 1 REF: Knowledge & Understanding
OBJ: Section 8.4 TOP: Normal Approximation to binomial distribution

SHORT ANSWER

5. ANS:
$np = 45$
$nq = 7.5$
Since both np and nq are greater than 5, a normal distribution is a good approximation for this binomial distribution.

PTS: 1 REF: Applications | Communication OBJ: Section 8.4
TOP: Normal Approximation to binomial distribution

6. ANS:

$$\mu = np \qquad \sigma = \sqrt{npq}$$
$$= 1000 \times 0.5 \qquad = \sqrt{1000 \times 0.5 \times 0.5}$$
$$= 500 \qquad = 15.81$$

PTS: 1 REF: Applications OBJ: Section 8.4
TOP: Normal Approximation to binomial distribution

7. ANS:

$$P(X > 24.5) \doteq 4.44 \times 10^{-5}$$

PTS: 1 REF: Applications OBJ: Section 8.4
TOP: Normal Approximation to binomial distribution

8. ANS:

$$P(9.5 < X < 15.5) \doteq 0.673$$

PTS: 1 REF: Applications OBJ: Section 8.4
TOP: Normal Approximation to binomial distribution

9. ANS:

$$P(X < 39.5) \doteq 0.306$$

PTS: 1 REF: Applications OBJ: Section 8.4
TOP: Normal Approximation to binomial distribution

10.   ANS:
$P(39.5 < X < 40.5) \doteq 0.074$

PTS: 1   REF: Applications   OBJ: Section 8.4
TOP: Normal Approximation to binomial distribution

PROBLEM

11.   ANS:
$np = 350 \times 0.90$
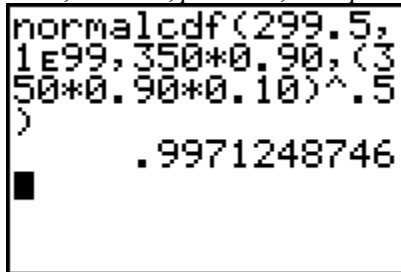$\quad = 315$
$nq = 350 \times 0.10$
$\quad = 35$
Since both *np* and *nq* are greater than 5, the normal distribution is a good approximation.

Since the data are discrete, a continuity correction must be applied. To find the probability of at least 300 flights being on time, use a continuity correction with a range of 299.5 to ∞.
A graphing-calculator solution is shown below. The probability can also be calculated by using the **NORMDIST** function in a spreadsheet, or by converting to *z*-scores and using the table of Areas Under the Normal Distribution Curve on page 606 of the student textbook.

Here, *n* = 350, *p* = 0.90, and *q* = 0.10.

```
normalcdf(299.5,
1ᴇ99,350*0.90,(3
50*0.90*0.10)^.5
)
        .9971248746
■
```

The probability of at least 300 of the 350 flights being on time is about 99.7%.

PTS: 1   REF: Communication | Thinking/Inquiry/Problem Solving
OBJ: Section 8.4   TOP: Normal Approximation to binomial distribution
12.   ANS:
The probabilities can be calculated with a graphing calculator using the method shown below or with a spreadsheet using the **NORMDIST** and **BINOMDIST** functions.

Both *np* and *nq* are well over 5, so the normal distribution is a reasonable approximation. To include the probability that exactly 130 loans are delinquent, use a continuity correction with 129.5 as the lower limit.

$\mu = np$ $\sigma = \sqrt{npq}$
$\quad = 500 \times 0.25$ $\quad = \sqrt{500 \times 0.25 \times 0.75}$
$\quad = 125$ $\quad = 9.68$

For the binomial calculation, $P(X \geq 130) = 1 - P(X \leq 129)$.

```
normalcdf(129.5,
1E99,125,(500*.2
5*.75)^.5)
         .3210523961
1-binomcdf(500,.
25,129)
         .3186634539
■
```

Both methods give about a 32% probability that at least 130 of the 500 loans are delinquent. The normal approximation differs from the binomial probability by less than 0.3%. This discrepancy is unimportant for this particular calculation because p is given to only two significant digits.

PTS: 1   REF: Applications | Communication   OBJ: Section 8.4
TOP: Normal Approximation to binomial distribution
13.   ANS:
The probabilities can be calculated with a graphing calculator using the method shown below or with a spreadsheet using the **NORMDIST** and **BINOMDIST** functions. The normal approximation can also be determined by converting to z-scores and using the table of Areas Under the Normal Distribution Curve on page 606 of the student textbook. The binomial probability can be found using the formula $P(x) = {_n}C_x p^x q^{n-x}$, as described in section 7.2 of the student textbook.

Here, $n = 15$, $p = 0.365$, and $q = 0.635$. Both np and nq are over 5, so the normal distribution is a reasonable approximation. To find the probability of exactly 5 hits, use a continuity correction with a range of 4.5 to 5.5.

```
normalcdf(4.5,5.
5,15*0.365,(15*0
.365*0.635)^.5)
         .2048299016
binompdf(15,0.36
5,5)
         .2073752427
```

The binomial calculation is more accurate, so the probability of 5 hits is about 20.7%. The normal approximation gives a probability of 20.5%, which is reasonably close, but not accurate to the three significant digits given in the data.

PTS: 1   REF: Applications   OBJ: Section 8.4
TOP: Normal Approximation to binomial distribution
14.   ANS:
A graphing-calculator solution is shown below. The probability can also be calculated by using the **NORMDIST** function in a spreadsheet or by converting to z-scores and using the table of Areas Under the Normal Distribution Curve on page 606 of the student textbook.

Here, $n = 500$, $p = 0.10$, and $q = 0.90$. To find the probability of fewer than 50 vehicles with undeclared goods, use a continuity correction with 49.5 as the upper limit.

```
normalcdf(-1E99,
49.5,500*0.1,(50
0*0.1*0.9)^.5)
        .4702920381
```

The probability of less than 50 vehicles with undeclared goods is about 47%.

PTS: 1   REF: Applications   OBJ: Section 8.4
TOP: Normal Approximation to binomial distribution
15.   ANS:
A graphing-calculator solution is shown below. The probability can also be calculated by using the **NORMDIST** function in a spreadsheet or by converting to z-scores and using the table of Areas Under the Normal Distribution Curve on page 606 of the student textbook.

Here, $n = 500$, $p = 0.10$, and $q = 0.90$. To find the probability of more than 60 vehicles with undeclared goods, use a continuity correction with 60.5 as the lower limit.

```
normalcdf(60.5,1
E99,500*0.1,(500
*0.1*0.9)^.5)
        .0587624423
■
```

The probability of more than 60 vehicles with undeclared goods is about 59%.

PTS: 1   REF: Applications   OBJ: Section 8.4
TOP: Normal Approximation to binomial distribution
16.   ANS:
a) Answers may vary. Since np and nq are both larger than 5, the normal approximation should be accurate to at least the nearest percent.
b) A graphing-calculator solution is shown below. The normal approximation can also be calculated by using the **NORMDIST** function in a spreadsheet or by converting to z-scores and using the table of Areas Under the Normal Distribution Curve on page 606 of the student textbook.
    Here, $n = 100$, $p = 0.60$, and $q = 0.40$. To find the probability that exactly half the vehicles pollute more than necessary, use a continuity correction with a range of 49.5 to 50.5.

```
normalcdf(49.5,5
0.5,100*0.6,(100
*0.6*0.4)^.5)
        .0101953711
■
```

The probability that exactly half of the vehicles inspected cause unnecessary pollution is about 1.0%.

c) $P(x) = {}_nC_x p^x q^{n-x}$

$\quad = {}_{100}C_{50} \times (0.6)^{50} \times (0.4)^{100-50}$

$\quad = 0.010\ 34$

The probability that exactly half of the vehicles inspected cause unnecessary pollution is about 1.0%.

d) Answers may vary.

PTS: 1   REF: Communication | Thinking/Inquiry/Problem Solving
OBJ: Section 8.4   TOP: Normal Approximation to binomial distribution