

A Comparison of Wiener and Kalman Filters for the Artifact Suppression from EEG Signal

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Abstract: *Electroencephalogram (EEG) is a noninvasive method to record electrical activity of brain and it has been used extensively in research of brain function due to its high time resolution. However raw EEG is a mixture of signals, which contains noises such as Ocular Artifact (OA) that is irrelevant to the cognitive function of brain. To remove OAs from EEG, many methods have been proposed, such as Independent Components Analysis (ICA), Empirical Mode Decomposition (EMD), Discrete Wavelet Transform (DWT), Adaptive filtering and Adaptive Noise Cancellation (ANC). In this paper, a comprehensive overview of techniques that can be used for the removal of artifacts from an EEG. For this purpose, the Wiener and Kalman filters are used to compare to remove OAs in EEG. Firstly, the artifact removing method using two filters are applied on synthetic data. Two factors are used to compare the result of filter on EEG signal; that is Signal to Artifact Ratio (SAR) and Mean Squared Error (MSE). The SAR value is 4.34 dB for Kalman filter while for Wiener filter it is 5.30 dB. The Mean Squared Error (MSE) of Wiener filter is 7.195×10^{-05} , significantly lower compared to results using Kalman method is 0.067. Then these approaches are applied to the contaminated EEG data. The experimental result shows that comparatively the Wiener filter is more effective in removing the artifact without losing the original information.*

Keywords: artifact reduction, EEG signal, MSE, Kalman filter, SAR, Wiener filter

1. Introduction

Electroencephalography (EEG) records carry information about abnormalities or responses to certain stimuli in the human brain. However, during recordings, many physiological or technical artifacts can be observed. Such artifacts might hide the brain information and should be removed. Conventional filtering cannot be applied to eliminate those types of artifacts because EEG signal and artifacts have overlapping spectra. In previous research, many simple and complex methods have been proposed for detecting and removing artifacts. The simple signal processing filter, known as Butterworth bandpass filter is used to remove the artifact. The clean EEG signal is easily achieved from raw EEG by applying 4th order of bandpass Butterworth filter and compare it with DWT. However, this type of filter does not suitable for EEG signal processing because some of the original information are loss during the filtration process.[1].

Adaptive filter is also used to remove OAs from EEG by LMS algorithm [2] and VSSLMS algorithm [3]. These methods usually take the output of adaptive filter as OA interference, then subtract it from EEG [4]. Adaptive filter can automatically adjust parameters under the condition that priori knowledge of noise is unknown, and de-noise adaptively. The precondition of using adaptive filtering is an extra EOG reference channel. Several documents [5-8] use independent components analysis (ICA) to attenuate OAs in EEG. ICA is a kind of blind signal separation method; it does not need EOG reference channel. However, the independent components separated by ICA are of uncertainty in terms of sorting, which leads to visual classification of components [9, 10]. In addition, how to determine the number of independent components effectively and reducing computation complexity is problems which need to be considered [11].

The artifacts are suppressed by modeling EEG activity by an autoregressive model and eye blink by an output-error model, and then use Kalman filter to estimate the true EEG based on

integrating two models [12]. Recently, the ocular artifacts are suppressed by combining Independent Component Analysis (ICA) and an adaptive Wiener filter ("ICAWF"). The idea is to obtain pure eye blinking components and suppress them from the original independent components in order to remove the artifacts and preserve the physiological information. These methods are applied to real EEG data from a healthy subject [13]. To remove the EOG artifact in real-time applications, two filtering methods Wiener filter and Kalman filter is illustrated in this paper. To demonstrate the performance of two methods, we compare them and obtained a conclusion that Wiener filter is better than Kalman filter for ocular artifact removing from EEG signal.

This paper is arranged as follows: research background of EEG and some methods of OAs removing are stated in the first part. In the second part, two models used for comparison and described in detail. In the third part, some experiments and performance evaluation are made. Lastly, the paper ends up with a brief conclusion.

2. Artifact Reduction Methods

There are a number of general techniques used for artifact separation and removal. This separation can often be accomplished using simple classical filtering techniques, such as low-pass filtering. However, this can only be employed when the frequency bands of the artifact and the desired signal do not overlap. When there is spectral overlap, alternative techniques must be adopted. Filtering approaches such as adaptive, Wiener or Bayes filtering have the advantage that they can be automated, however they need a measured or reliably estimated reference in order to operate. Some of these methods can operate on single channels, a characteristic that makes them attractive for the artifact removing.

2.1 Wiener filter

The Wiener filter is a filter used to produce an estimate of a desired or target random process by linear time-invariant (LTI) filtering of an observed noisy process, assuming known stationary signal and noise spectra, and additive noise. The Wiener filter can be used to filter out the noise from the corrupted signal to provide an estimate of the underlying signal of interest. The Wiener filter is based on a statistical approach, and a more statistical account of the theory is given in the minimum mean square error (MMSE) estimator article. The Wiener filter minimizes the mean square error between the estimated random process and the desired process [14]. Wiener filters are characterized by the following:

- 1) Assumption: signal and (additive) noise are stationary linear stochastic processes with known spectral characteristics or known autocorrelation and cross-correlation.
- 2) Requirement: the filter must be physically realizable / causal (this requirement can be dropped, resulting in a non-causal solution).
- 3) Performance criterion: minimum mean-square error (MMSE)

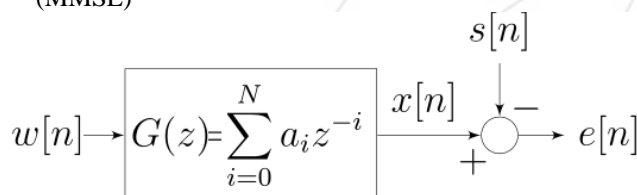


Figure 1: Block diagram of the FIR Wiener filter for discrete series. An input signal $w[n]$ is convolved with the Wiener filter $g[n]$ and the result is compared to a reference signal $s[n]$ to obtain the filtering error $e[n]$.

Figure 1 shows the block diagram of a finite impulse response (FIR) Wiener filter. In order to derive the coefficients of the Wiener filter, consider the signal $w[n]$ being fed to Wiener filter of order N and with coefficients $\{a_0, \dots, a_N\}$. The output of the filter is denoted $x[n]$ which is given by the expression

$$x[n] = \sum_{i=0}^N a_i w[n-i].$$

The residual error is denoted $e[n]$ and is defined as $e[n] = x[n] - s[n]$. The Wiener filter is designed so as to minimize the mean square error.

So, Wiener filtering is parametric technique used to remove unwanted artifacts from the measured signals. The Wiener algorithm is based on a statistical approach and thus does not require the use of an external reference signal. The signal and the (additive) artifact are assumed to be stationary linear stochastic processes with known spectral characteristics or known autocorrelation and cross-correlation. The desired signal and artifact are also assumed to be uncorrelated with each other. The purpose of the Wiener filter is to produce a linear time invariant filter so that the mean square error between the true desired signal $s(n)$ and the estimated one

$\hat{s}(n)$ is minimized [15]. Figure 2 shows the block diagrams for artifact removing using Wiener filter for EEG signal.

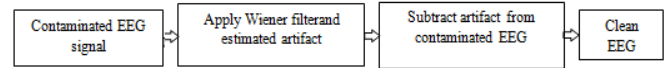


Figure 2: Block diagram of artifact removing using Wiener filter

2.2 Kalman filter

Kalman filtering, also known as linear quadratic estimation (LQE), is an algorithm that uses a series of measurements observed over time, containing statistical noise and other inaccuracies, and produces estimates of unknown variables that tend to be more precise than those based on a single measurement alone, by using Bayesian inference and estimating a joint probability distribution over the variables for each timeframe. Kalman filtering was first described by Kalman in 1960 [16]. The Kalman filter essentially implements a mathematical predictor-corrector type estimator. The filter uses feedback control to estimate a process: the filter first estimates the process state at a given time and then obtains feedback in the form of (noisy) measurements [17]. This creates two layers of calculations: time update equations and measurement update equations as detailed by Welch and Bishop in [17].

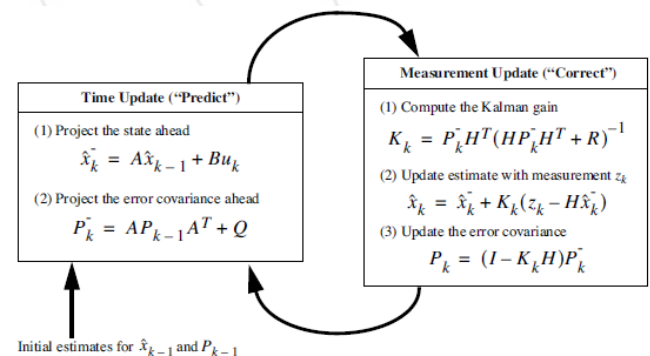


Figure 3: A complete picture of the operation of the Kalman filter

The filter is a widely applied concept in time series analysis used in fields such as signal processing. The algorithm works in a two-step process. In the prediction step, the Kalman filter produces estimates of the current state variables, along with their uncertainties. Once the outcome of the next measurement (necessarily corrupted with some amount of error, including random noise) is observed, these estimates are updated using a weighted average, with more weight being given to estimates with higher certainty. The algorithm is recursive. It can run in real time, using only the present input measurements and the previously calculated state and its uncertainty matrix; no additional past information is required. The Kalman filter does not make any assumption that the errors are Gaussian [16]. However, the filter yields the exact conditional probability estimate in the special case that all errors are Gaussian-distributed. Another way of thinking about the weighting by K is that as the measurement error covariance R approaches zero, the actual measurement z_k is "trusted" more and more, while the predicted measurement $H \hat{x}_k^-$ is trusted less and less. On the other hand,

as the a priori estimate error covariance P_k^- approaches zero the actual measurement z_k is trusted less and less, while the predicted measurement $H\hat{x}_k^-$ is trusted more and more. The equations for the Kalman filter fall into two groups: time update equations and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback, i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate. Figure 3. shows the ongoing discrete Kalman filter cycle. The time update projects the current state estimate ahead in

time. The measurement update adjusts the projected estimate by an actual measurement at that time.

After each time and measurement update pair, the process is repeated with the previous a posteriori estimates used to project or predict the new a priori estimates. This recursive nature is one of the very appealing features of the Kalman filter--it makes practical implementations much more feasible than an implementation of a Wiener filter which is designed to operate on all of the data directly for each estimate. The Kalman filter instead recursively conditions the current estimate on all of the past measurements.

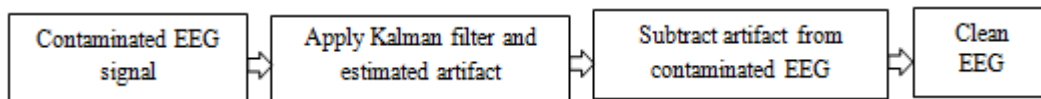


Figure 4: Block diagram of artifact removing using Kalman filter

Estimation of the noise covariances Q_k and R_k

Practical implementation of the Kalman filter is often difficult due to the difficulty of getting a good estimate of the noise covariance matrices Q_k and R_k . Extensive research has been done in this field to estimate these covariances from data. One of the more promising and practical approaches to do this is the auto covariance least-squares (ALS) technique that uses the time-lagged autocovariances of routine operating data to estimate the covariances [18,19]. The meaning of each variable from Figure 3 is as follows. A is state variable gain matrix, k presents observation matrix, Q_k means covariance of $u(k)$, R_k is covariance $v(k)$, P_k indicates posteriori estimation error covariance, P_k^- stands for priori estimation error covariance, H_k is Kalman gain, x_k is estimation of state variable at point k , and z_k means observed data (approximated OAs) where $z(k) = x(k) + u(k)$.

There are a number of approaches for achieving artifact removal using a Kalman filter. First, a model of both the desired signal and the contaminating artifact can be produced. The recorded signal can be described as the summation of these two model signals and, thus, the process and measurement models can be determined. The Kalman filter can then be implemented to estimate the unknown system parameters by reducing the variance of the error between the recorded output and the modeled output. With the known system parameters, artifact removal can be accomplished by setting the parameters affiliated with the artifact in the measurement model to zero. Figure 4. Shows the block diagram for artifact removing using Kalman filter for EEG signal. Finally, we just subtract the output of Kalman filtering from raw EEG and clean EEG is obtained.

3. Results and Discussion

3.1 Synthetic data

In this section, the different filtering method is tested with a synthetic signal. A noisy synthetic signal is created as the addition of the following components:

$$x(t) = s_1(t) + v$$

where, $s_1(t) = 4 \sin(0.05\pi t)$; v = additive white Gaussian noise

The noisy synthetic signal $x(t)$ (Figure 5) has two components which is shown in top panel, the white Gaussian noise is second top panel and the original sine wave is shown in bottom panel. The two digital filters are used here to remove the noise from the noisy synthetic signal. Figure 5. shows the Gaussian noise removing using Wiener filter and Figure 6. is illustrated the additive noise removing using Kalman filter. From Figure 5, it is observed that Wiener filter perfectly remove the noise and the filtering synthetic signal is look like same as the original sine wave. On the other hand, the sine wave is contained some white noise using Kalman filter is illustrated in Figure 6. It is visualized, from time domain plot of Figure 7 and Figure 8 that the Wiener filter is completely remove the noise than the Kalman filter.

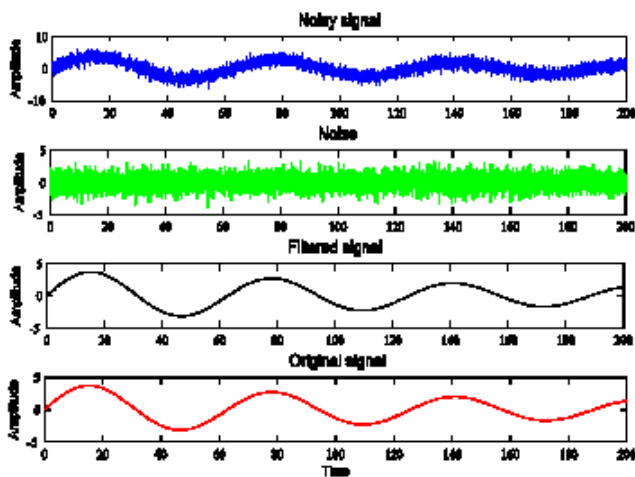


Figure 5: The separation of clean EEG from the synthetic data using Wiener filter.

3.2 Application to contaminated EEG data

Dataset description: The recorded EEG is modeled as linear combination of pure EEG and EOG artifact defined as:

$$S_{EEG}(t) = P_{EEG}(t) + A_{EOG}(t)$$

where, $S_{EEG}(t)$, $P_{EEG}(t)$, and $A_{EOG}(t)$ are measured EEG, pure EEG and EOG artifact respectively. The measured EEG signal is the superposition of original EEG signal due to brain activity and the fraction of EOG signal due to eye blink activity. The aim of the proposed scheme is to extract required EEG signal $P_{EEG}(t)$ from the measured signal $S_{EEG}(t)$ which consists of the required signal $P_{EEG}(t)$ plus the ocular artifact signal $A_{EOG}(t)$. In this paper, multichannel $S_{EEG}(t)$ is used as the primary input to suppress its EOG artifact using wiener and Kalman filter based techniques. No reference channel is used to estimate the pure EEG signals.

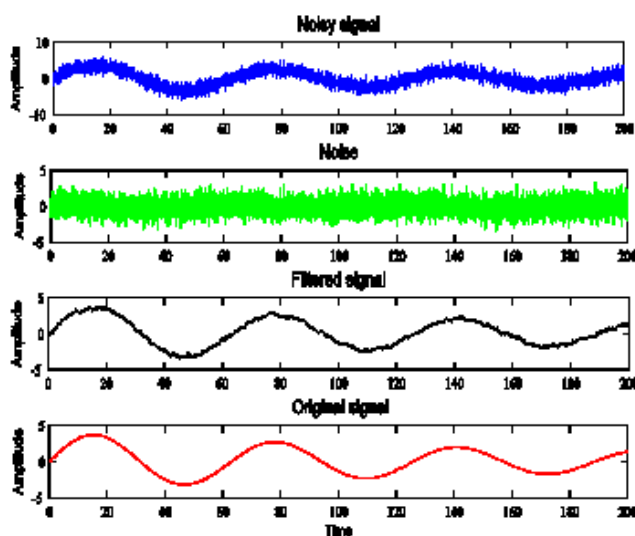


Figure 6: The separation of clean EEG from the synthetic data using Kalman filter

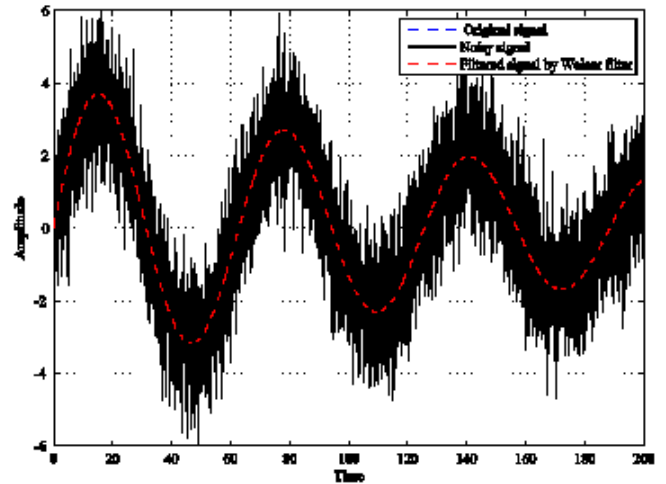


Figure 7: The separation of clean EEG from the synthetic data using Wiener filter.

4. Experimental Results

In this section we give a comprehensive overview of techniques that can be used for the removal of artifacts from EEG signal. For each method, we explain the reasoning behind its use for the removal of artifacts from an EEG and highlight some of its advantages and deficiencies. The Wiener filter based artifact reduction method is tested with synthetic signal and the artificially corrupted EEG signal and after that the signal is comprised with Kalman filter based artifact reduction method.

Figure 2. shows the artifact removing using Wiener filter for EEG signal. Calibration is needed prior to usage in Wiener filter. On the other hand, when properly calibrated, it can achieve a better SAR for corrected data as compared to the adaptive filter. It eliminates the requirement for additional hardware on the recording device necessary with the adaptive filter. The output of this filter is electrooculogram (EOG). To get the desired clean EEG data, the artifactual component is subtracted from contaminated EEG. By subtracting electrooculogram from raw EEG, we get the purified electroencephalography that reflects the actual neural activities. The electro-oculogram suppression results for a single channel of recorded electroencephalography are illustrated in Figure 9. in which the separated electrooculogram and purified electroencephalography signals are shown in the second and third rows respectively. From Figure 9.it is observed that the purified EEG signal contains more original information although the artifact has cancel out. When Kalman filter is applied to our dataset, the noise variances play an important role in stability and efficacy of the Kalman filter. We chose the following values based on the experiments performed with different values of variances on different signal like synthetic and EEG signal,

$$\sigma_E^2 = 1 \times 10^{-2} \text{ for synthetic signal}$$

$$\sigma_E^2 = 1 \times 10^{-4} \text{ for real EEG}$$

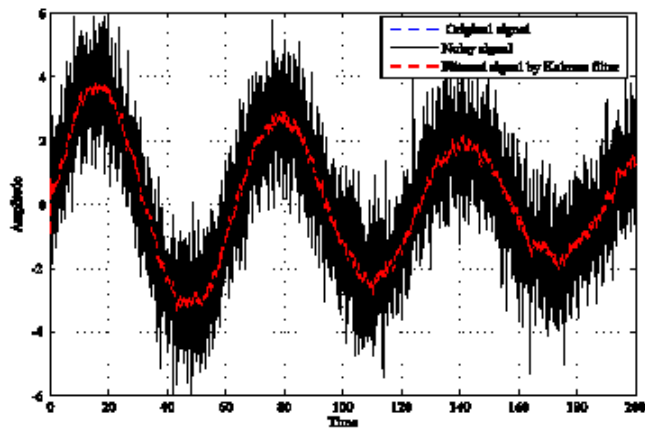


Figure 8: The separation of clean EEG from the synthetic data using Kalman filter

In other words, there is a tradeoff between how much an algorithm removes the artifact and how much it distorts the EEG signal. By tuning the Kalman filter parameters, we use measurement noise variance $R=0.05$. It is apparent from Figure 10 that using Kalman filter for artifact correction, underlying EEG or low frequency cerebral data may be lost and EOG signal contains EEG information.

Figure 11. depicts the original EEG and clean EEG using two methods. From this Figure, it is observed that the Wiener filtering based method is best for reduce the EOG from contaminated EEG without cutting the information and assist to get clean EEG. The pure EEG do not show any large EOG. As a result, the purified EEG signal is found as completely artifact free in Wiener filtering method.

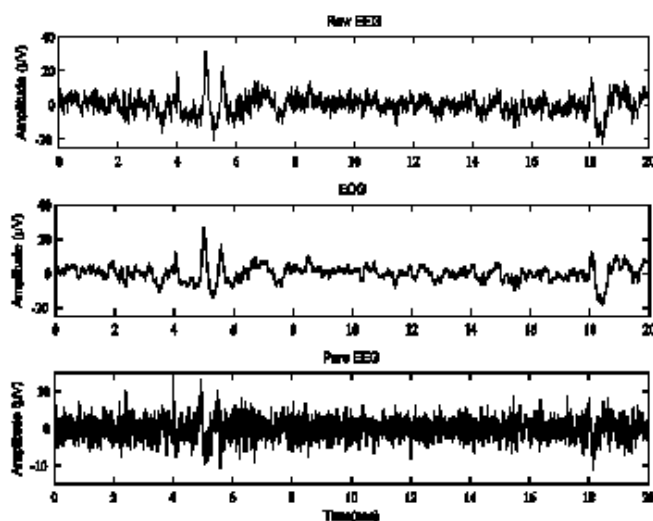


Figure 9: Mixture of EOG like contaminated EEG signal and pure EEG. (Top): contaminated EEG signal. (Middle): EOG signal. (Bottom): the pure EEG signal using Wiener filter.

Performance metrics:

In order to determine whether the method is successful at removing ocular artifact (OA) from EEG, the performance is assessed using two statistical parameters i.e. signal to artifact ratio (SAR) and mean square error (MSE).

Signal to artifact ratio (SAR):

The metrics commonly employed to represent the energy of the signal compared to the energy of the artifact is the signal to artifact ratio (SAR)

$$SAR_{dB_{contaminated_EEG}} = 20 \log_{10} \frac{rms[x(t)]}{rms[s(t) - \hat{s}(t)]}$$

Here, $x(t)$ is the clean EEG signal. $s(t)$ stands for contaminated EEG signal, $\hat{s}(t)$ is artifact free EEG signal and N for the signal length or the number of samples.

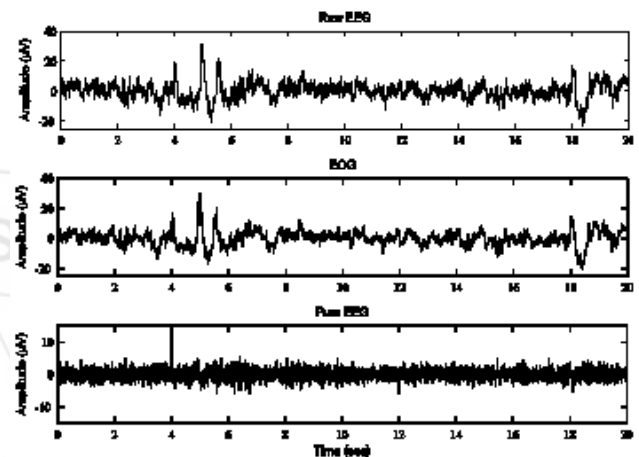


Figure 10: Mixture of EOG like contaminated EEG signal and pure EEG. (Top): contaminated EEG signal. (Middle): EOG signal. (Bottom): the pure EEG signal using Kalman filter.

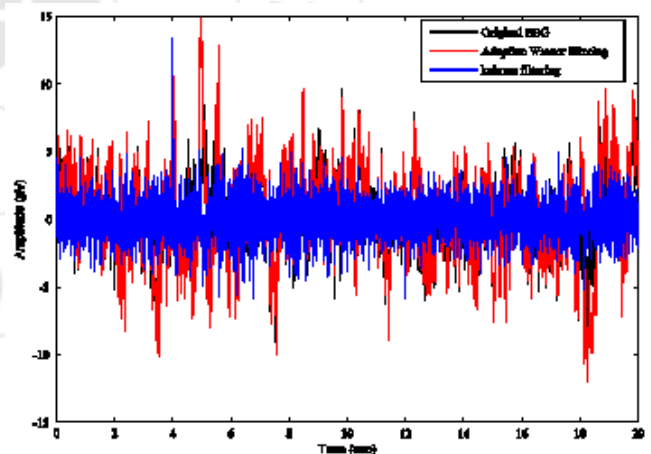


Figure 11: Visual comparison of the original EEG signal and the corrected EEG signals after applying different ocular artifact removal methods

Mean square error (MSE):

It is used to describe similarity between the original signal and de-noised signal.

$$MSE = \frac{1}{N} \sum_{i=1}^N [s_i(t) - \hat{s}_i(t)]^2$$

$s_i(t)$ stands for original signal, $\hat{s}_i(t)$ estimates signal de-noised by mentioned filter, N for the signal length or the number of samples.

Table 1: A comparative summary of the Wiener and Kalman filter for synthetic data

| No. | Methods | SAR in dB | MSE |
|-----|---------------|-----------|-------------------------|
| 1. | Wiener filter | 5.30 | 7.195×10^{-05} |
| 2. | Kalman filter | 4.34 | 0.061 |

TABLE I and TABLE II shows the comparison results of the SAR and MSE values from Wiener filter and Kalman filter, respectively. TABLE I shows the comparison between Wiener filter and Kalman filter for synthetic signal. In TABLE II, the table shows that the Wiener filter based technique yields the best SAR (lower the SAR value able to clean more artifact) result than Kalman filter. The SAR value is -0.0405 dB for Wiener filter while for Kalman filter it is -5.6158 dB. The Wiener filter shows lower MSE value compared to the Kalman filter. Based on the MSE and SAR values, it is observed that the Wiener filter is able to filter out more noise compare to Kalman filter. Because higher the SAR value better the artifact removing. And lower the MSE value, better the method for the artifact removing.

Table 2: A comparative summary of the Wiener and Kalman filter for contaminated EEG data

| No. | Methods | SAR in dB | MSE |
|-----|---------------|-----------|--------|
| 1. | Wiener filter | -0.0405 | 3.5516 |
| 2. | Kalman filter | -5.6158 | 7.9914 |

5. Conclusions

In this paper, we aim at suppressing the ocular artifacts by Wiener filter and compare it with Kalman filter. A comparison between two methods to suppress eye blinking artifacts in EEG signal. These methods are applied to real EEG data. The Wiener filter algorithm causes a little less EEG signal distortion. The eye blinking and also the eye movement artifacts have been suppressed, while the physiological information is preserved. Kalman filter shows similar graphical results but the artifactual components have been completely removed, physiological information is lost. Wiener filter removes the segments where the ocular artifacts are present and preserves the physiological information. The performance of this method is evaluated based on two different metrics. The MSE and SAR is a popular parameter to determine the quality of signal after filtering. From TABLE I, it is understandable that the SAR value of Wiener filter is 5.30 dB which is higher than 4.34 dB in Kalman filter. Similarly, the MSE value of Wiener filter is 7.195×10^{-05} which is lower than 0.061 in Kalman filter. According to a visual inspection of the signals, the Wiener filter yields the best results. In all cases, artifacts are adequately attenuated, without removing significant useful information. We conclude that Wiener filter based artifact reduction method is an efficient processing technique for improving the quality of EEG signals in biomedical signal analysis.

6. Acknowledgement

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7. Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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