Neural Networks
Lecture Notes
Self-organizing maps

Marcel van Gerven

1 Learning Goals

After studying the lecture material you should:

1. know what is meant by a topographic map
2. be able to give an example of a topographic map in the brain
3. be able to explain in words and recognize in mathematical form the four stages of the self-organising map algorithm

2 Notes

2.1 Self-organizing maps

Neurobiological studies indicate that sensory inputs (motor, visual, auditory, etc.) up to higher-order semantic categories are mapped onto corresponding areas of the cerebral cortex in an orderly fashion. Examples are orientation columns, ocular dominance columns and color blobs, maps of the visual field and auditory input, somatosensory maps, brain regions that code for semantic categories and distributed yet clustered representations of categorical information.

This can be formalized in terms of topographic maps, that have two important properties:

1. At each stage of representation, or processing, each piece of incoming information is kept in its proper context/neighbourhood.
2. Neurons dealing with closely related pieces of information are kept close together

Self-organizing maps (SOMs) are computational models of topographic map formation. The goal is to transform an incoming signal pattern of arbitrary dimension into a low-dimensional discrete map. This transformation should be done in a topologically ordered fashion. The neurons become selectively tuned to various input patterns (stimuli) or classes of input patterns during the course of competitive learning. The locations of the neurons so tuned (i.e. the winning neurons) become ordered. A meaningful coordinate system for the input features is created on the map. The SOM thus forms the required topographic map of the input patterns. This map can be used to cluster the inputs.

Let $\mathbf{x}$ denote the input and let a_{ij} denote the activation of the output unit at location (i, j). Weights w_{ijk} are used to link an input x_k to the output at location (i, j). Training of SOMs consists of four stages:

- *Initialization*: All connection weights are initialized with small random values.

- *Competition*: For each input pattern only one neuron is the winner (winner-take-all). Winner is the node (i^*, j^*) which minimizes Euclidean distance:

$$(i^*, j^*) = \arg\min_{i,j} ||\mathbf{x} - \mathbf{w}_{ij}|| = \arg\min_{i,j} \left[\sum_k (x_k - w_{ijk})^2 \right]$$

  where $\mathbf{w}_{ij}$ is the prototype vector associated with output unit (i, j).

- *Cooperation*: The winning neuron determines how neighbouring neurons cooperate. Activity of each output node is given by the distance to the winning node. The Gaussian neighbourhood function is commonly used:

$$a_{ij} = e^{\frac{-(i-i^*)^2-(j-j^*)^2}{2\sigma(t)^2}}.$$

  The neighbourhood function has several important properties:

  – it is maximal at the winning neuron
  – it is symmetrical about that neuron
  – it decreases monotonically to zero as the distance goes to infinity
  – it is translation invariant (i.e. independent of the location of the winning neuron)

  A special feature of the SOM is that the size of the neighbourhood needs to decrease with time. A popular time dependence is an exponential decay:

$$\sigma(t) = \sigma_0 \exp(-t/\tau)$$

- *Adaptation*: Neurons adapt their connection weights according to their excitation. Weights are updated by:

$$\Delta w_{ijk} = \epsilon \cdot a_{ij} \cdot (x_k - w_{ijk})$$

  This makes connections around the winning node more similar to the input. It can be justified by assuming the traditional Hebbian law for synaptic modification, and an additional nonlinear, "active" forgetting process for the synaptic strengths [KKSO84] since $\Delta w_{ijk} \propto a_{ij}x_k - a_{ij}w_{ijk}$.

3 Reading material

For self-organizing maps consult [Koh90, Koh13]. For applications in cognitive science and neuroscience, consult [RK89, OS01, Afl06].

References

[Afl06] T N Aflalo. Possible Origins of the Complex Topographic Organization of Motor Cortex: Reduction of a Multidimensional Space onto a Two-Dimensional Array. *J. Neurosci.*, 26(23):6288–6297, 2006.

[KKSO84] Teuvo Kohonen, M Kai, Tapio Saram, and Others. Phonotopic maps-insightful representation of phonological features for speech recognition. 1984.

[Koh90] T Kohonen. The self-organizing map. *Proc. IEEE*, 78(9):1464–1480, 1990.

[Koh13] Teuvo Kohonen. Essentials of the self-organizing map. *Neural Networks*, 37:52–65, 2013.

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