Module 5 Project: Using Linear Programming Models to maximize profits

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Introduction

1. Mathematical formulation of the problem

First of all, I calculated **the total warehouse space** is **2460ft** because:

- The warehouse has 82 shelves.

- Each shelf is 30 ft long and 5 ft wide.

-> 82 x 30 = 2460ft

I define decision variables:

- x1: Number of Pressure Washers purchased

- x2: Number of Go Karts purchased

- x3: Number of Generators purchased

- x4: Number of Water Pump cases purchased

Objective Function (Maximize Net Profit):

maxZ = (499.99x1 + 729.99x2 + 700.99x3 + 269.99x4)−(330x1 + 370x2 + 410x3 + 635x4)

-> maxZ = 169.99x1 + 359.99x2 + 290.99x3 − 365.01x4

Constraints:

- Budget constraint (Cannot exceed $170,000): 330x1 + 370x2 + 410x3 + 635x4 =<170000
- Warehouse space constraint (Total available: 2460 ft): 5x1 + 8x2 + 5x3 + 1.25x4 =< 2460
- Inventory allocation constraint (At least 30% must be pressure washers and go-karts):

  x1 + x2 >= 0.3(x1 + x2 + x3 + x4)
- Sales ratio constraint (Generators must be at least twice the water pumps): x3 >= 2x4
- Non-negativity constraints: x1, x2, x3, x4 >= 0

2. Set up the linear programming formulation in R, 3. Use R to solve the problem, and generate a sensitivity report.

Then, I use lpSolve package in R to set up and solve the model.

```r
# Load the required package
library(lpSolve)
```

```r
# Define cost and revenue
costs <- c(330, 370, 410, 635)
revenues <- c(499.99, 729.99, 700.99, 269.99)

# Profit per unit
profit <- revenues - costs

# Constraints
## Budget constraint
budget <- 170000

## Warehouse space constraint (Total available: 2460 ft)
space_per_item <- c(5, 8, 5, 5/4)  # Water pumps case occupies 1.25 ft
warehouse_capacity <- 2460

## Inventory allocation constraint (30% allocated to pressure washers and go-karts)
## x1 + x2 >= 0.3 * (x1 + x2 + x3 + x4)

## Sales ratio constraint (Generators must be at least 2 times water pumps)
## x3 >= 2 * x4

# Coefficients matrix
constraints <- matrix(
  c(330, 370, 410, 635,       # Budget constraint
    5, 8, 5, 5/4,            # Warehouse space constraint
    0.7, 0.7, -0.3, -0.3,    # Inventory allocation
    0, 0, 1, -2),            # Sales ratio constraint
  nrow = 4, byrow = TRUE
)

# Right-hand side values for constraints
rhs <- c(budget, warehouse_capacity, 0, 0)

# Constraint directions
constraints_dir <- c("<=", "<=", ">=", ">=")

# Solve the LP model
solution <- lp(
  direction = "max",  # Maximization problem
  objective.in = profit,
  const.mat = constraints,
  const.dir = constraints_dir,
  const.rhs = rhs,
  all.int = TRUE  # Ensure integer values
)

# Display results
cat("Optimal Solution:")
```

```
## Optimal Solution:
```

```r
print(solution$solution)  # Number of items to purchase
```

```
## [1]   1 125 291   0
```

```r
cat("\nMaximum Profit: ", solution$objval)
```

```
##
## Maximum Profit:  129846.8
```

4. Describe the optimal solutions

With the output above, we can see that:

- Optimal Solution:

- Pressure Washers (x1): 1 unit
- Go-Karts (x2): 125 units
- Generators (x3): 291 units
- Water Pump Cases (x4): 0 units

- Maximum Profit: 129846.8

Why is x4 zero? Answer: A zero value in a linear programming solution means it is not profitable or not feasible to purchase water pumps. This could be due to:

1. Low profitability

- Water pumps have a negative profit per unit: 269.99 - 635 = -365.01
- Since selling water pumps causes a loss, the model does not include them.

2. Budget Constraints:

- Higher-profit items could make greater use of the budget or warehousing.
- Due to their larger revenues, the model gives priority to go-karts and generators.

3. Impact of Sales Ratio Constraint:

- Selling water pumps compels us to have additional generators on hand since generators must be at least twice as large as water pumps.
- Since this might waste money and space, it would be best to fully avoid using water pumps.

5. Use the sensitivity report to determine the smallest selling price

```r
# Sensitivity Analysis (Shadow Prices and Reduced Costs)
print(solution$reducedcost)
```

```
## NULL
```

Because the output I got is NULL, I will calculate the smallest selling price manually. Then, I used the table below to compare x4 with the most profitable item (generators, x3) to estimate the required price.

Item	Cost	Selling price	Profit
x3	410	700.99	290.99
x4	635	269.99	-365.01

Estimate Minimum Selling Price for x4:

New Selling Price of x = Cost + Profit of Most Profitable Item

= 635 + 290.99 = 925.99

```r
revenues[4] <- 925.99  # Increase water pump price
profit <- revenues - costs  # Recalculate profit per unit
```

Re-run the LP model:

```r
solution <- lp(
  direction = "max",
  objective.in = profit,
  const.mat = constraints,
  const.dir = constraints_dir,
  const.rhs = rhs,
  all.int = TRUE
)

print(solution$solution)  # Check if x  is now included
```

```
## [1]   0 219 127  58
```

```r
cat("\nNew Maximum Profit: ", solution$objval)
```

```
##
## New Maximum Profit:  132671
```

6.

```r
print(solution$duals)
```

```
## [1] 0
```

Since the shadow price is 0, this tells us:

- The company already has enough budget to buy the most profitable combination of products.

- More money won't change the optimal solution because the company cannot store or sell more products under the current constraints.

7.

Since the shadow price is 0, this tells us:

- The company's potential to optimize profit is not limited by warehouse area.

- Changing the warehouse's size will not change the optimal solution or profit.

- It is not necessary for the company to lease a bigger warehouse.