Continuous Finance HW1

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1 Problem 4

$\Delta B_i \sim N(0, t_{i+1} - ti)$
So, $(\Delta B_i^2) = var(\Delta B_i) = \Delta t_i$
$\mathbb{E}(\Delta B_i^4) = 3var(\Delta B_i)^2 = 3(\Delta t_i)^2$
Therefore,
$LHS = \mathbb{E}[(\Delta B_i)^4 - 2(\Delta B_i)^2 \Delta t_i + (\Delta t_i)^2 \parallel F_{t_i}]$
$= 3(\Delta t_i)^2 - 2(\Delta t_i)^2 + (\Delta t_i)^2$
$= 2(\Delta t_i)^2$
$= RHS$

$p_t(x, y) = f_t(y - x) = \frac{1}{\sigma\sqrt{2\pi t}} \exp\left[-\frac{1}{2\sigma^2 t}(y - x - \mu t)^2\right], \quad t \in (0, \infty);\ x,\ y \in$