

prob1

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Prob 1. 1) $E(r_t) = M + E(B_t \times (M_J + \sigma_J S_t))$

$$= M + P \cdot E(M_J + \sigma_J S_t)$$

$$= M + P \cdot M_J$$

$$E(J_t) = P \cdot (M_J + \sigma_J S_t) = P \cdot M_J$$

$$E(r_t^2) = E(M^2 + 2M \sigma_J S_t + 2M J_t + \sigma^2 S_t^2 + 2\sigma S_t J_t + J_t^2)$$

$$= M^2 + 2M \cdot P M_J + \sigma^2 + P E[(M_J + \sigma_J S_t)^2]$$

$$= M^2 + 2M P M_J + \sigma^2 + P M_J^2 + P \sigma_J^2$$

$$\text{Var}(r_t) = M^2 + 2M P M_J + \sigma^2 + P M_J^2 + P \sigma_J^2 - M^2 - 2M P M_J - P^2 M_J^2$$

$$= \sigma^2 + P M_J^2 + P \sigma_J^2 - P^2 M_J^2$$

$$S(r_t) = E\left(\frac{(r_t - E(r_t))^3}{\text{Var}(r_t)^{3/2}}\right)$$

$$E[(\sigma S_t + J_t - P M_J)^3]$$

$$= E[\sigma^3 S_t^3 + J_t^3 - 3P M_J^3 + 3\sigma^2 S_t^2 J_t - 3\sigma^2 S_t P M_J + 3\sigma S_t J_t^2 - 3J_t^2 P M_J + 3\sigma S_t P M_J^2 + 3J_t P M_J^2 - 6\sigma S_t J_t P M_J]$$

$$= E[P \cdot (M_J^3 + \sigma_J^3 S_t^3 + 3M_J^2 \sigma_J S_t + 3M_J \sigma_J^2 S_t^2) - P^3 M_J^3 + 3\sigma^2 P M_J^2]$$

$$= P M_J^3 + 3P M_J \sigma_J^2 + 2P^3 M_J^3 - 3P^2 M_J^3 - 3P^2 M_J \sigma_J^2$$

$$S(r_t) = \frac{P M_J^3 + 3P M_J \sigma_J^2 + 2P^3 M_J^3 - 3P^2 M_J^3 - 3P^2 M_J \sigma_J^2}{(\sigma^2 + P M_J^2 + P \sigma_J^2 - P^2 M_J^2)^{3/2}}$$

$$E((r_t - E(r_t))^4) = E[(\sigma S_t + J_t - P M_J)^4] = E[3\sigma^4 + 6\sigma^2 J_t^2 + 6\sigma^2 P^2 M_J^2 + J_t^4 + 6J_t^2 P^2 M_J^2 - 4J_t P^3 M_J^3 - 12\sigma^2 J_t P M_J + P^4 M_J^4 - 4P M_J J_t^3]$$

$$= 3\sigma^4 + 6\sigma^2 P M_J^2 + 6\sigma^2 P^2 \sigma_J^2 + 6\sigma^2 P^2 M_J^2 + P(M_J^4 + 6M_J^2 \sigma_J^2 + 3\sigma_J^4) + 6(P M_J^2 + P \sigma_J^2) P^2 M_J^2 - 3P^4 M_J^4 - 12\sigma^2 P^2 M_J^2 - 4P M_J (P M_J^3 + 3P M_J \sigma_J^2)$$

$$= 3\sigma^4 + 6\sigma^2 P M_J^2 + 6\sigma^2 P^2 \sigma_J^2 - 6\sigma^2 P^2 M_J^2 + P M_J^4 + 6P M_J^2 \sigma_J^2 + 3P \sigma_J^4 + 6P^3 M_J^4 + 6P^3 M_J^2 \sigma_J^2 - 3P^4 M_J^4 - 4P^2 M_J^4 - 12P^2 M_J^2 \sigma_J^2$$

$$K(r_t) - 3 = \frac{E((r_t - E(r_t))^4)}{\text{Var}(r_t)^2} - 3 = \frac{E((r_t - E(r_t))^4) - 3\sigma^4 + P^2 M_J^4 + P^2 \sigma_J^4 + P^4 M_J^4 + 2\sigma^2 P M_J^2 + 2\sigma^2 P \sigma_J^2 - 2\sigma^2 P^2 M_J^2 + 2P^2 M_J^2 \sigma_J^2 - 2P^3 M_J^4 - 2P^3 \sigma_J^2 M_J^2}{\text{Var}(r_t)^2}$$

$$+ 12P^3 M_J^2 \sigma_J^2 - 6P^4 M_J^4 - 7P^2 M_J^4 - 18P^2 M_J^2 \sigma_J^2 - 3P^2 \sigma_J^4$$

$$\text{Var}(r_t)^2$$

$$\text{so, } K(t) - 3 = \frac{p\mu_J^4 + 6p\mu_J^2\sigma_J^2 + 3p\sigma_J^4 + 12p^3\mu_J^4 + 12p^3\mu_J^2\sigma_J^2 - 6p^4\mu_J^4 - 7p^2\mu_J^4 - 18p^2\mu_J^2\sigma_J^2 - 3p^2\sigma_J^4}{\sigma^4 + p^2\mu_J^4 + p^2\sigma_J^4 + p^4\mu_J^4 + 2\sigma^3p\mu_J^2 + 2\sigma^3p\sigma_J^2 - 2\sigma^3p\mu_J^2 + 2p^2\mu_J^2\sigma_J^2 - 2p^3\mu_J^4 - 2p^3\sigma_J^2\mu_J^2}$$

Prob 3. 1. max. $p(N_t | \lambda)$

$$\lambda_{MLE} = \arg \max_{\lambda} \frac{(\lambda t)^{N_t} e^{-\lambda t}}{N_t!}$$

$$\text{FOC: } N_t (\lambda t)^{N_t-1} e^{-\lambda t} - (\lambda t)^{N_t} e^{-\lambda t} (-t) = 0$$

$$\lambda t = N_t$$

$$\lambda^* = \frac{N_t}{t}$$

2. max $p(N_t=0 | \lambda) = e^{-\lambda t}$

$$\text{FOC: } e^{-\lambda t} (-t) = 0 \quad t \geq 0$$

if $t=0$, λ can be anything in \mathbb{R}^+ , which makes sense bc $N_0=0$.

if $t>0$, $e^{-\lambda t}=0$, which is impossible, so $\lambda_{MLE}=0$.

and Poisson distr. requires $\lambda t > 0$ so contradiction.

Therefore, prior to the 1st event, $t=0 \Rightarrow \lambda \in \mathbb{R}^+$ and $t>0$, λ doesn't exist

3. $p(\lambda | N_t) \propto p(N_t | \lambda) \cdot p(\lambda)$

$$= \frac{(\lambda t)^{N_t} e^{-\lambda t}}{N_t!} \cdot \frac{A^a}{\Gamma(a)} \lambda^{a-1} e^{-A\lambda}$$

$$= \frac{A^a t^{N_t} \lambda^{N_t+a-1} e^{-\lambda(t+A)}}{N_t! \cdot \Gamma(a)}$$

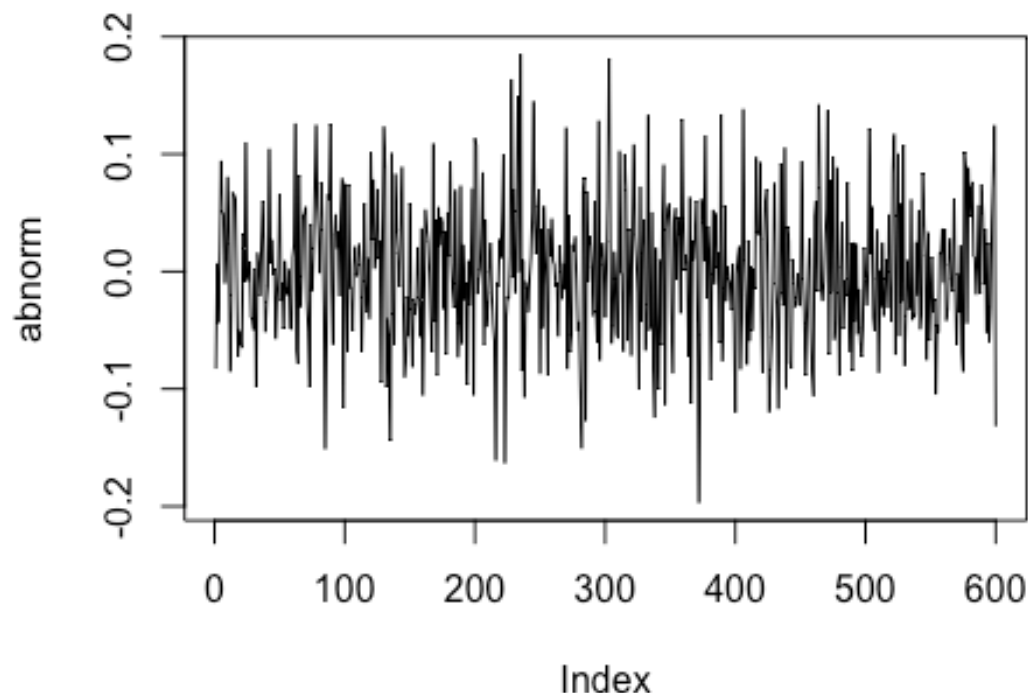
$$\propto \lambda^{N_t+a-1} e^{-\lambda(t+A)}$$

so the posterior distr is a gamma distr w/ $a' = N_t + a$ and $A' = t + A$.

4. $\mathbb{E}(\lambda | N_t) = \frac{N_t + a}{t + A}$ bc it's a gamma distr.

2. Because we have more flexibility with bernoulli normal mixture model and that we can see more negative jumps than positive jumps which is consistent with the data. the variance is not consistent over time which is also consistent with market data. and the stock return can have fat tails under the bernoulli normal mixture model

```
N <- 600
set.seed(124)
norm <- rnorm(N)
abnorm <- norm*0.063 + 0.0045
plot(abnorm, type = "l")
```



It does not look like the data, because the data is non-stationary. there are times where there's a lot of variance in the returns and times where there isn't, however, the simulation from normal distribution's variance is stationary across time.

```
mu <- 0.012
sig <- 0.05
p<- 0.15
muj<- -0.05
sigj <- 0.17
#mean 0.0045
mu + p*muj

## [1] 0.0045

#variance =0.00715375
sig**2+p*muj**2+p*sigj**2-p**2*muj**2

## [1] 0.00715375

#skewness = -0.9319173
(p*muj**3+3*p*muj*sigj**2+2*p**3*muj**3-3*p*p*muj**3-3*p*p*muj*sigj**2)/(sig*
*2+p*muj**2+p*sigj**2-p**2*muj**2)**(3/2)

## [1] -0.9319173
```



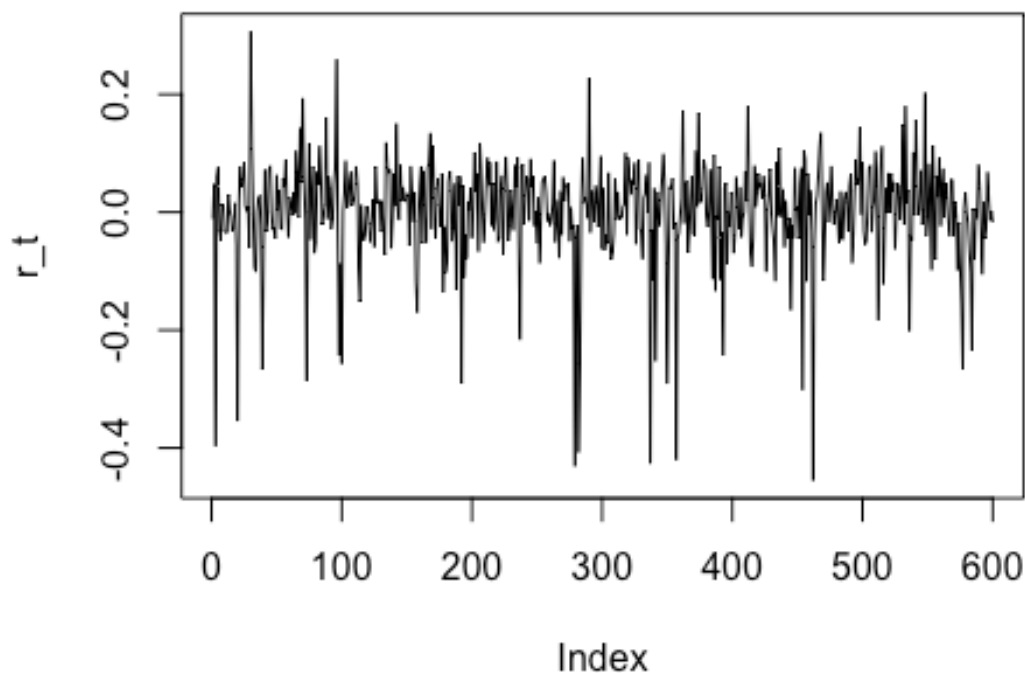
```

#kurtosis =7.002188
(3*sig**4+6*sig*sig*p*muj**2+6*sig*sig*p*sigj*sigj-6*sig*sig*p*p*muj*muj+p*muj**4+6*p*muj*muj*sigj*sigj+3*p*sigj**4+6*p**3*muj**4+6*p**3*muj**2*sigj**2-3*p**4*muj**4-4*p*p*muj**4-12*p*p*muj**2*sigj**2)/(sig**2+p*muj**2+p*sigj**2-p*2*muj**2)**2-3

## [1] 7.002188

delta_t <- rnorm(N)
epsilon_t <- rnorm(N)
B_t <- rbinom(N, size = 1, prob = p)
J_t <- B_t*(muj+sigj*delta_t)
r_t <- mu + sig*epsilon_t+J_t
plot(r_t,type = "l")

```



It doesn't look like the data, but there is a lot more negative jumps than positive jumps. what's missing is that in the real data, there's periods of times with a lot of negative jumps and positive jumps and some periods with neither. whereas the simulation doesn't achieve that

```

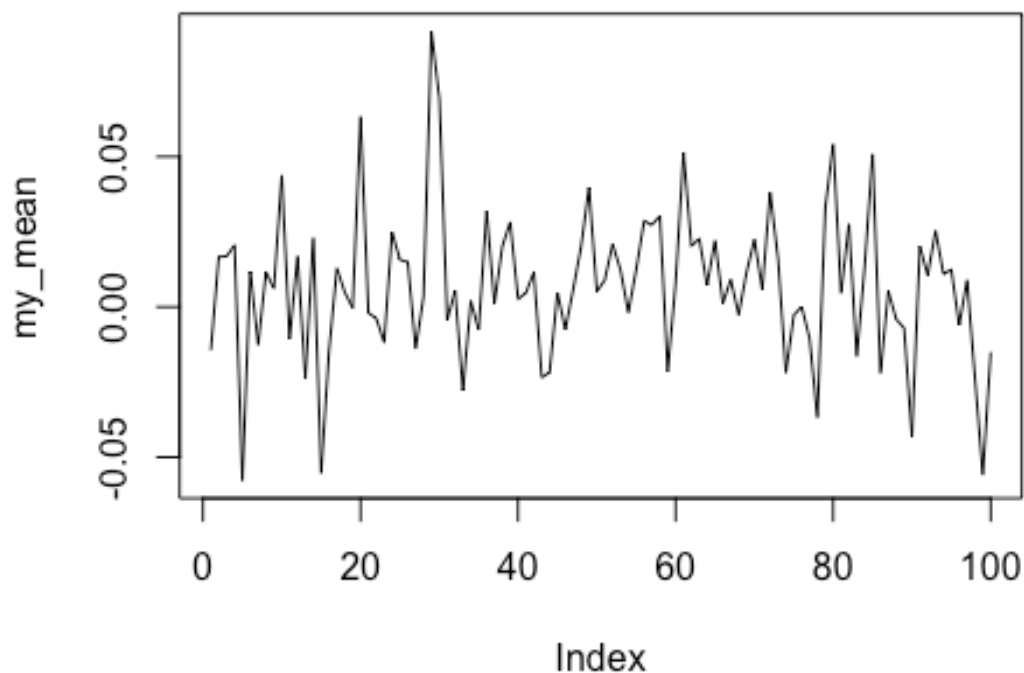
numN <- 100
N<- 12
my_mean <- c()
my_error <-c()

```

```

for (i in 1:numN) {
  delta_t <- rnorm(N)
  epsilon_t <- rnorm(N)
  B_t <- rbinom(N, size = 1, prob = p)
  J_t <- B_t*(mu_j+sig_j*delta_t)
  r_t <- mu + sig*epsilon_t+J_t
  my_mean[i] <- mean(r_t)
  my_error[i] <- std.error(r_t)
}
t_mean <- mean(my_mean)
t_sd <- sqrt(var(my_mean))
#var is var(my_mean) = 0.0006082747
t_stat <- t_mean/(t_sd/sqrt(numN))
plot(my_mean,type= "l")

```



```

# it looks like a distribution from the plot
skewness(my_mean) #0.1260242

## [1] 0.1707318

kurtosis(my_mean)#1.373097

## [1] 1.406444

```

```

jarque.bera.test(my_mean)

##
##  Jarque Bera Test
##
## data:  my_mean
## X-squared = 9.8247, df = 2, p-value = 0.007355

#p-value = 0.01017 so it couldn't be a normal distribution
mean(my_error)

## [1] 0.02123394

#the mean of the standard errors are 0.02217715 which is slightly higher than
the standard deviation

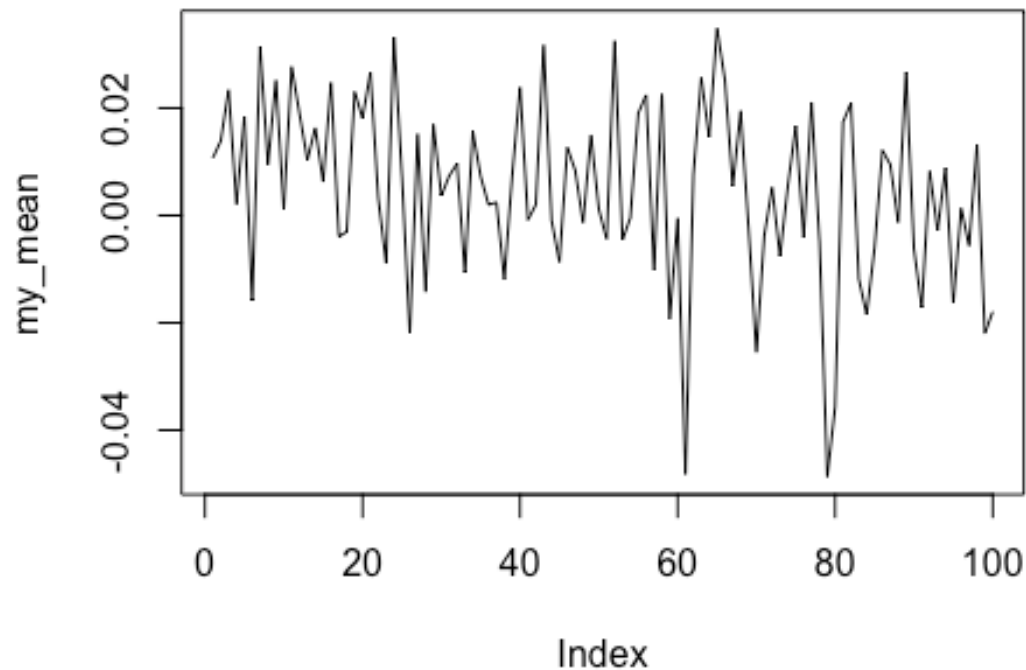
```

t_stat is 3.044169 which is greater than 1.96 so we reject the null hypothesis

```

numN <- 100
N<- 24
my_mean <- c()
my_error <-c()
for (i in 1:numN) {
  delta_t <- rnorm(N)
  epsilon_t <- rnorm(N)
  B_t <- rbinom(N, size = 1, prob = p)
  J_t <- B_t*(mu_j+sig_j*delta_t)
  r_t <- mu + sig*epsilon_t+J_t
  my_mean[i] <- mean(r_t)
  my_error[i] <- std.error(r_t)
}
t_mean <- mean(my_mean)
t_sd <- sqrt(var(my_mean))
#var is var(my_mean) = 0.0003389792
t_stat <- t_mean/(t_sd/sqrt(numN))
plot(my_mean,type= "l")

```



```
# it doesnt look like a distribution from the plot
skewness(my_mean) #-0.5748082
```

```
## [1] -0.647919
```

```
kurtosis(my_mean)#0.5307763
```

```
## [1] 0.6570971
```

```
jarque.bera.test(my_mean)
```

```
##
```

```
## Jarque Bera Test
```

```
##
```

```
## data: my_mean
```

```
## X-squared = 9.4395, df = 2, p-value = 0.008918
```

```
#p-value = 0.02749 so it couldn't be a normal distribution
```

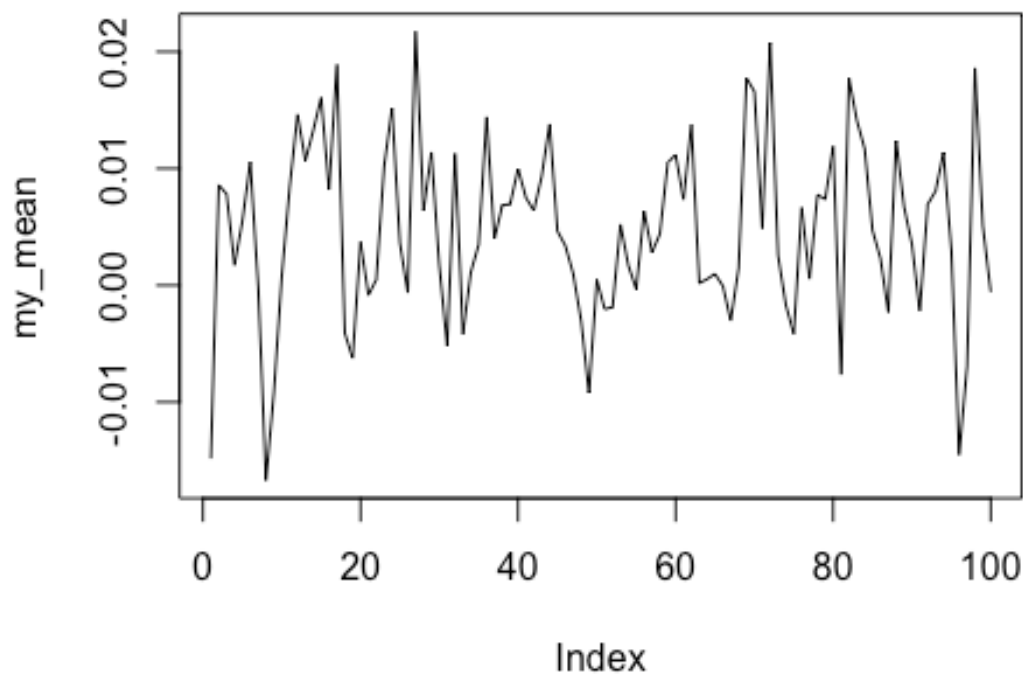
```
mean(my_error)
```

```
## [1] 0.01653043
```

```
#the mean of the standard errors are 0.01663182 which is slightly lower than the standard deviation
```


t_{stat} is 0.15277 which is less than 1.96 so we keep the null hypothesis

```
numN <- 100
N<- 120
my_mean <- c()
my_error <-c()
for (i in 1:numN) {
  delta_t <- rnorm(N)
  epsilon_t <- rnorm(N)
  B_t <- rbinom(N, size = 1, prob = p)
  J_t <- B_t*(mu_j+sig_j*delta_t)
  r_t <- mu + sig*epsilon_t+J_t
  my_mean[i] <- mean(r_t)
  my_error[i] <- std.error(r_t)
}
t_mean <- mean(my_mean)
t_sd <- sqrt(var(my_mean))
#var is var(my_mean) = 6.445296e-05
t_stat <- t_mean/(t_sd/sqrt(numN))
plot(my_mean,type= "l")
```

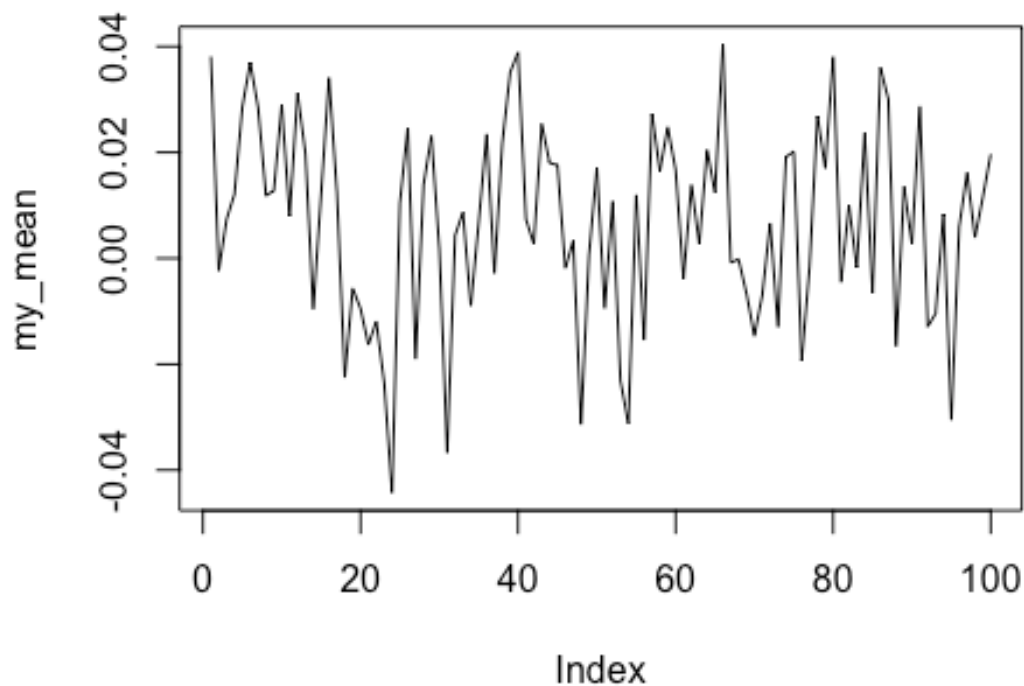


```
# it looks like a distribution from the plot
skewness(my_mean) #0.02659691
```

```
## [1] -0.2177322
kurtosis(my_mean)#-0.3752769
## [1] 0.0683155
jarque.bera.test(my_mean)
##
## Jarque Bera Test
##
## data: my_mean
## X-squared = 0.88539, df = 2, p-value = 0.6423
#p-value = 0.8009 so it could be a normal distribution
mean(my_error)
## [1] 0.007674494
#the mean of the standard errors are 0.007614783 which is slightly lower than
the standard deviation
```

t_stat is 6.69 which is greater than 1.96 so we reject the null hypothesis

```
numN <- 100
N<- 24
my_mean <- c()
my_error <-c()
for (i in 1:numN) {
  delta_t <- rnorm(N)
  epsilon_t <- rnorm(N)
  B_t <- rbinom(N, size = 1, prob = p)
  J_t <- B_t*(mu+sig*delta_t)
  r_t <- mu + sig*epsilon_t+J_t
  my_mean[i] <- mean(r_t)
  my_error[i] <- std.error(r_t)
}
t_mean <- mean(my_mean)
t_sd <- sqrt(var(my_mean))
#var is var(my_mean) = 0.0002818953
t_stat <- t_mean/(t_sd/sqrt(numN))
plot(my_mean,type= "l")
```



it looks like a distribution from the plot

`skewness(my_mean)` *#-0.1605593*

[1] -0.3708221

`kurtosis(my_mean)` *#-0.5593615*

[1] -0.3546094

`jarque.bera.test(my_mean)`

##

Jarque Bera Test

##

data: my_mean

X-squared = 2.7392, df = 2, p-value = 0.2542

#p-value = 0.4663 so it could be a normal distribution

`mean(my_error)`

[1] 0.01673394

*#the mean of the standard errors are 0.01727482 which is slightly higherer th
an the standard deviation*

t_stat is 3.8055 which is less than 1.96 so we reject the null hypothesis

prob 2:

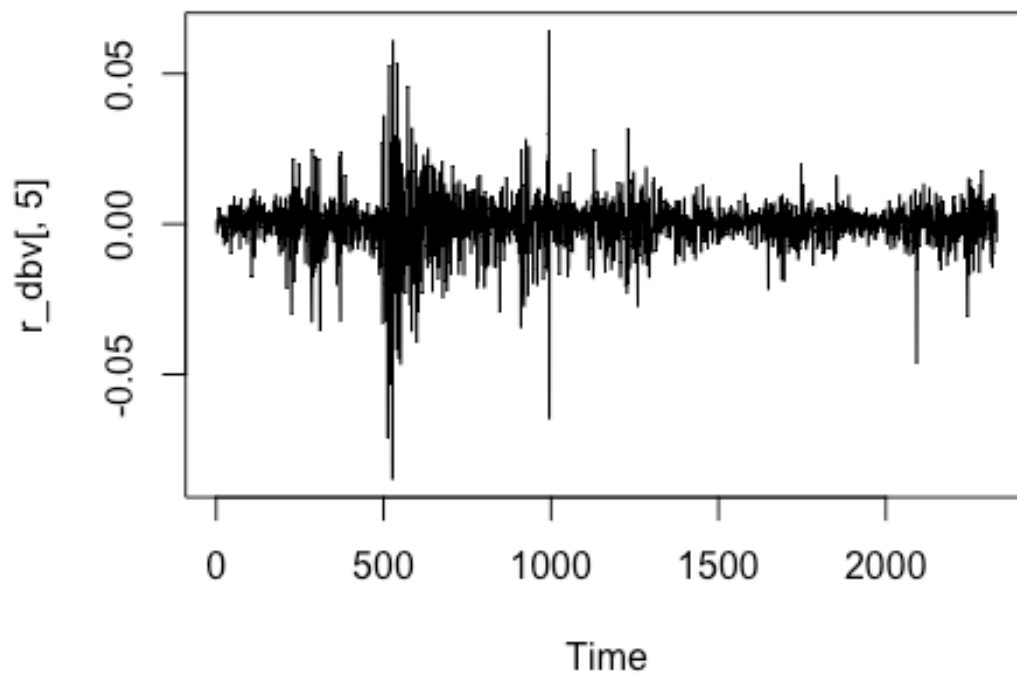
```
DBV <- read_excel("/Users/Lisa/Documents/Econometrics/ass1/DBV.xlsx")
names(DBV)= c("Date", "Open", "High", "Low", "Close", "Volume", "Adj")
attach(DBV)
dbv_ts<-ts(DBV,start = 1, frequency = 1)
# Growth rates:
log.dbv <- log(dbv_ts)
r_dbv <- diff(log.dbv)
head(log.dbv)

## Time Series:
## Start = 1
## End = 6
## Frequency = 1
##      Date      Open      High      \tLow      Close      Volume      Adj
## 1 20.87095 3.216874 3.218876 3.214868 3.218876  9.517825 3.174079
## 2 20.87102 3.222071 3.223266 3.218876 3.220075 11.203679 3.175278
## 3 20.87110 3.216874 3.217675 3.214466 3.216874  9.775654 3.172077
## 4 20.87117 3.217675 3.218476 3.216072 3.218076 12.333146 3.173279
## 5 20.87124 3.219276 3.220075 3.218476 3.218876 12.899220 3.174079
## 6 20.87147 3.218876 3.220075 3.216874 3.219276  9.622450 3.174479

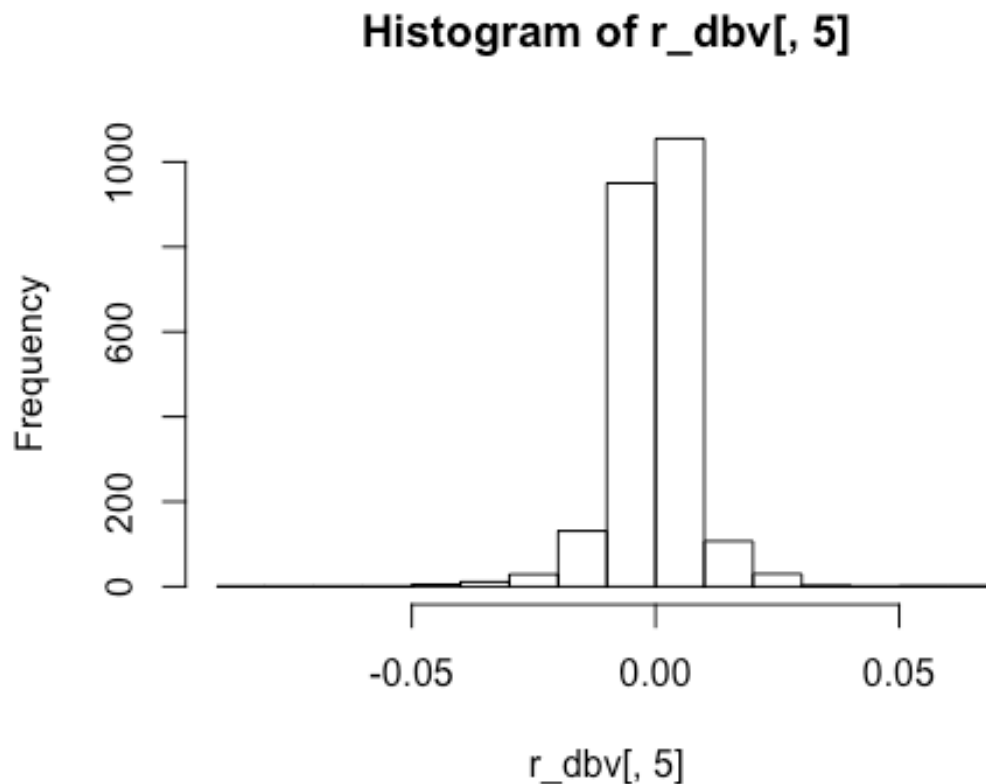
head(r_dbv)

## Time Series:
## Start = 2
## End = 7
## Frequency = 1
##      Date      Open      High      \tLow      Close
## 2 7.453509e-05  0.005196853  0.0043903881  0.004008021  0.0011993205
## 3 7.452953e-05 -0.005196853 -0.0055911488 -0.004409749 -0.0032012831
## 4 7.452398e-05  0.000801202  0.0008006806  0.001605821  0.0012016424
## 5 7.451842e-05  0.001600681  0.0015994005  0.002403847  0.0008003202
## 6 2.235220e-04 -0.000399920  0.0000000000 -0.001601883  0.0003999200
## 7 7.449622e-05  0.001598761  0.0003994408  0.001601883 -0.0003999200
##      Volume      Adj
## 2  1.6858541  0.001199327
## 3 -1.4280250 -0.0032012688
## 4  2.5574915  0.0012016090
## 5  0.5660741  0.0008003271
```

```
## 6 -3.2767698  0.0003999653  
## 7  0.2709872 -0.0003999653  
  
dim(r_dbv)  
## [1] 2330    7  
  
plot(r_dbv[,5])
```



```
hist(r_dbv[,5])
```



```
my_T<- 2330
my_skew<-skewness(r_dbv[,5])
skew_t <- my_skew/sqrt(6/my_T)
#skew_t is -16.9339 whose absolute value is greater than 1.96, so we reject the null hypothesis

my_kurt <-kurtosis(r_dbv[,5])
kurt_t <-(my_kurt)/sqrt(24/my_T)
#kurt_t is 131.615 whose absolute value is greater than 1.96, so we reject the null hypothesis
jarque.bera.test(r_dbv[,5])

##
##  Jarque Bera Test
##
## data:  r_dbv[, 5]
## X-squared = 17646, df = 2, p-value < 2.2e-16
#p-value < 2.2e-16 so reject the null
```

Including Plots

You can also embed plots, for example:


```

GSPC <- read_excel("/Users/Lisa/Documents/Econometrics/ass1/GSPC.xlsx")
names(GSPC)= c("Date", "Open", "High", "Low", "Close", "Volume", "Adj")
attach(GSPC)

## The following objects are masked from DBV:
##
##      Low, Adj, Close, Date, High, Open, Volume

GSPC_ts<-ts(GSPC,start = 1, frequency = 1)
# Growth rates:
log.GSPC <- log(GSPC_ts)
r_GSPC <- diff(log.GSPC)
head(log.GSPC)

## Time Series:
## Start = 1
## End = 6
## Frequency = 1
##      Date      Open      High      \tLow      Close      Volume      Adj
## 1 20.87095 7.181425 7.192445 7.178988 7.190201 21.72030 7.190201
## 2 20.87102 7.190186 7.197884 7.189394 7.197697 21.70660 7.197697
## 3 20.87110 7.197525 7.200485 7.195592 7.197877 21.73457 7.197877
## 4 20.87117 7.197854 7.200634 7.195750 7.199589 21.59783 7.199589
## 5 20.87124 7.199790 7.200335 7.197166 7.197323 21.54456 7.197323
## 6 20.87147 7.197301 7.199335 7.193145 7.193926 21.49082 7.193926

head(r_GSPC)

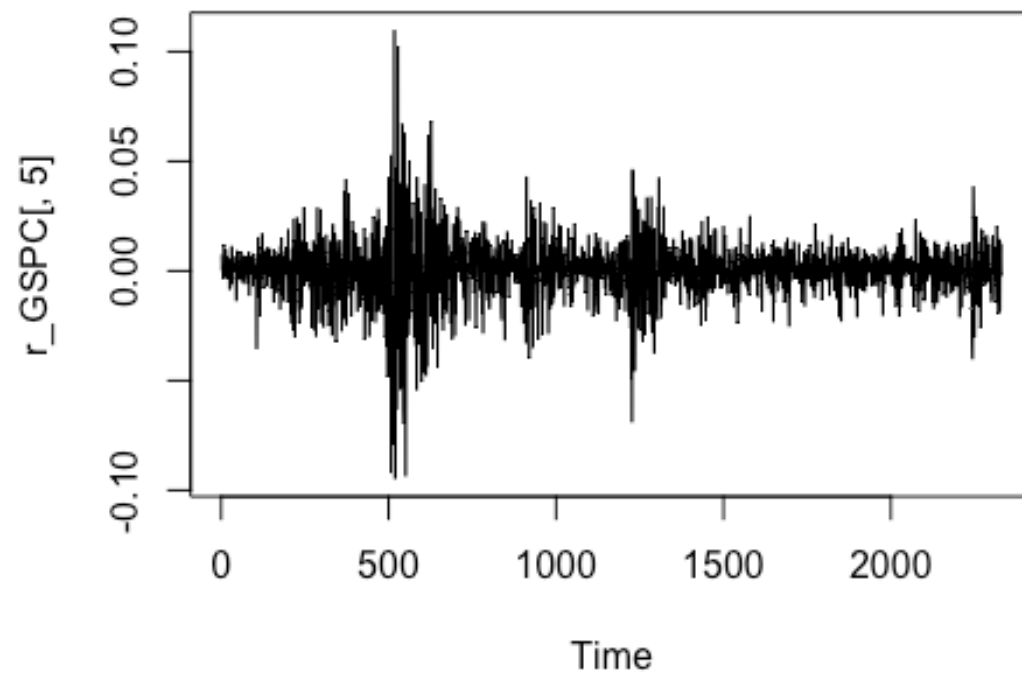
## Time Series:
## Start = 2
## End = 7
## Frequency = 1
##      Date      Open      High      \tLow      Close
## 2 7.453509e-05 0.0087614173 0.0054389752 0.0104064021 0.0074961151
## 3 7.452953e-05 0.0073390971 0.0026002227 0.0061982038 0.0001795701
## 4 7.452398e-05 0.0003293054 0.0001492882 0.0001574339 0.0017118783
## 5 7.451842e-05 0.0019359091 -0.0002985076 0.0014160653 -0.0022656719
## 6 2.235220e-04 -0.0024898072 -0.0010005646 -0.0040211208 -0.0033968847
## 7 7.449622e-05 -0.0033744044 -0.0001718288 -0.0023933758 0.0020935009
##      Volume      Adj
## 2 -0.01370482 0.0074961151
## 3 0.02797396 0.0001795701
## 4 -0.13674633 0.0017118783
## 5 -0.05327029 -0.0022656719
## 6 -0.05374031 -0.0033968847
## 7 0.21927063 0.0020935009

dim(r_GSPC)

## [1] 2330      7

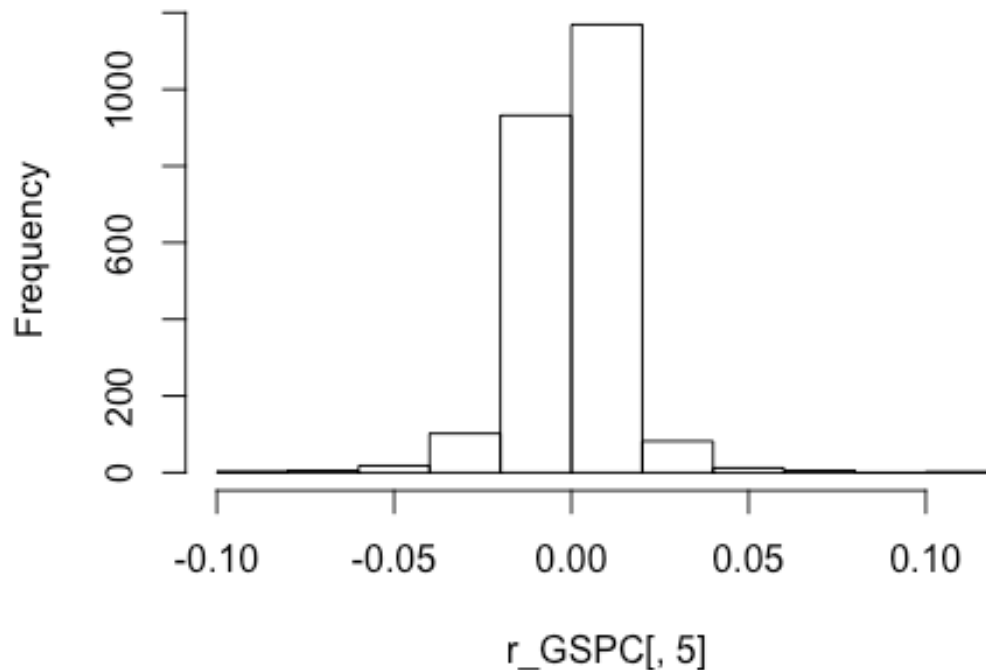
plot(r_GSPC[,5])

```



```
hist(r_GSPC[,5])
```

Histogram of r_GSPC[, 5]



```
my_T<- 2330
my_skew2<-skewness(r_GSPC[,5])
skew_t2 <- my_skew2/sqrt(6/my_T)
#skew_t2 is -6.38153 whose absolute value is greater than 1.96, so we reject
the null hypothesis

my_kurt2 <-kurtosis(r_GSPC[,5])
kurt_t2 <-(my_kurt2)/sqrt(24/my_T)
#kurt_t2 is 96.4317 whose absolute value is greater than 1.96, so we reject t
he null hypothesis

jarque.bera.test(r_GSPC[,5])

##
##  Jarque Bera Test
##
## data:  r_GSPC[, 5]
## X-squared = 9360.7, df = 2, p-value < 2.2e-16
#p-value < 2.2e-16 so reject the null

logrDBV <-r_dbv[,5]
logrGSPC <-r_GSPC[,5]
```

```

my_model<-lm(logrDBV~logrGSPC)
summary(my_model)

##
## Call:
## lm(formula = logrDBV ~ logrGSPC)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.069545 -0.003080  0.000239  0.003311  0.058668
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.0001153  0.0001367  -0.843    0.399
## logrGSPC      0.4323363  0.0101522  42.585 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.006598 on 2328 degrees of freedom
## Multiple R-squared:  0.4379, Adjusted R-squared:  0.4376
## F-statistic: 1814 on 1 and 2328 DF, p-value: < 2.2e-16

g_model <-glm(logrDBV~logrGSPC)
summary(g_model)

##
## Call:
## glm(formula = logrDBV ~ logrGSPC)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.069545 -0.003080  0.000239  0.003311  0.058668
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.0001153  0.0001367  -0.843    0.399
## logrGSPC      0.4323363  0.0101522  42.585 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 4.3538e-05)
##
##      Null deviance: 0.18031  on 2329  degrees of freedom
## Residual deviance: 0.10136  on 2328  degrees of freedom
## AIC: -16781
##
## Number of Fisher Scoring iterations: 2

#rob_model <- lmrob(LogrDBV~LogrGSPC)
#summary(rob_model)
my_model %>%

```

```
vcovHC() %>%
diag() %>%
sqrt()

## (Intercept)      logrGSPC
## 0.0001375697 0.0182961682
```

Under OLS assumptions, the standard error of the intercept is 0.0001367 and that of the slope is 0.0101522. Allowing for non-normalities, the standard error of the intercept is 0.0001367 and that of the slope is 0.0101522. the heteroskedastic standard error of the intercept is 0.0001375697 and that of the slope is 0.0182961682.

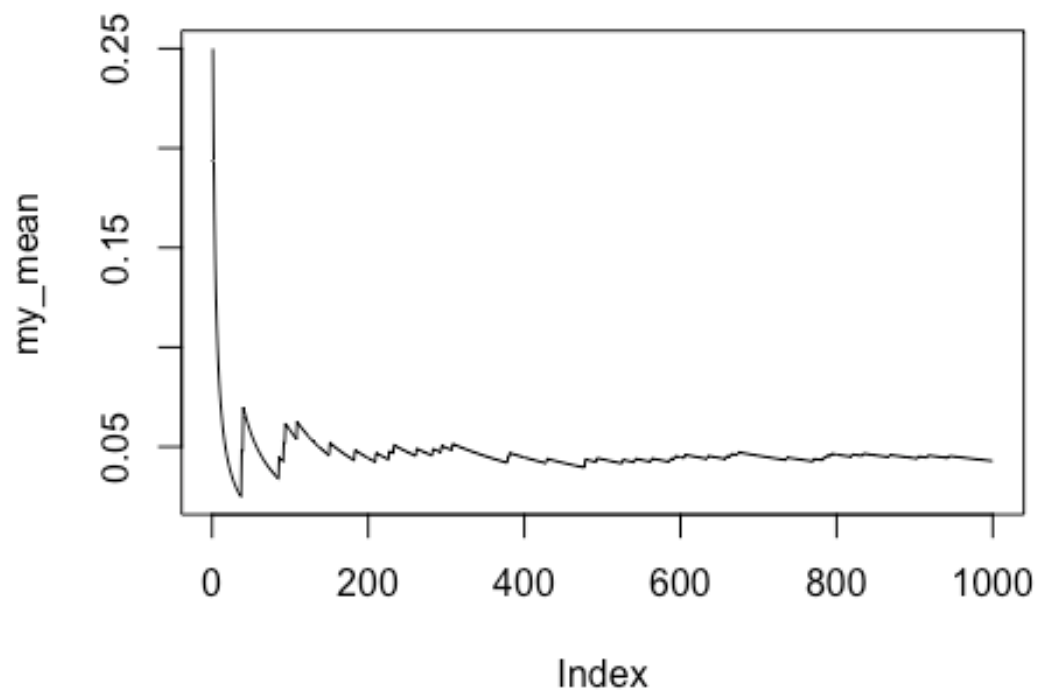
The White standard errors are larger than the classic OLS standard errors because the homoskedasticity-only standard errors are only valid if the errors are homoskedastic and that the White standard errors are valid whether or not the errors are heteroskedastic. Therefore, the White standard errors are more general and more conservative.

Prob 3:

```
N<-1000
a<-1
A<-3
pois <- rpois(N, 0.05)
sumpois <- cumsum(pois)
my_mean <- rep(0, N)

for (i in 1:N){
  my_mean[i] <- (a+sumpois[i])/(A+i)
}

plot(my_mean, type = "l")
```



It doesn't look odd to me. The mean start off close to the mean i set which is $1/A = 1/3$ and then it jumps up and slowly goes down and eventually converges to $\lambda = 0.05$