An Introduction to Feedback Control Theory

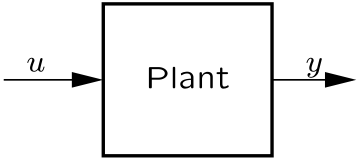
D. Teytelman

Dimtel, Inc., San Jose, CA, USA

January 24, 2023

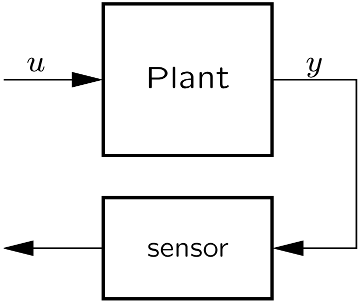
Outline

Closed-loop Feedback: Structure and Example



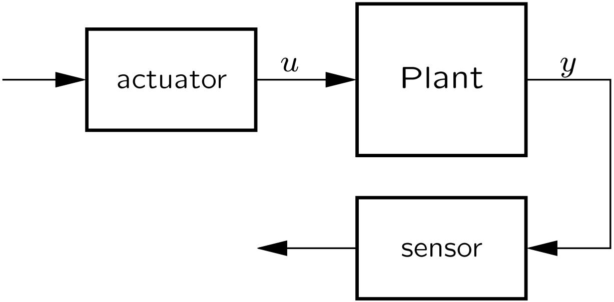
- Start with a physical system (plant).

Closed-loop Feedback: Structure and Example



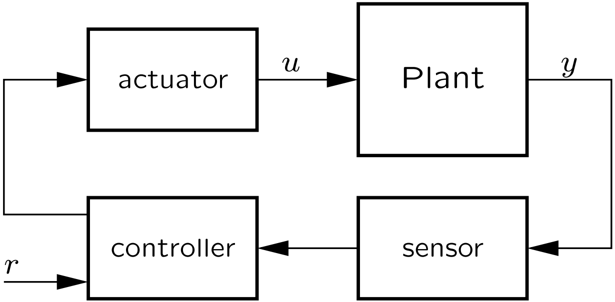
- Start with a physical system (plant).
- Measure some property of the plant with a sensor.

Closed-loop Feedback: Structure and Example



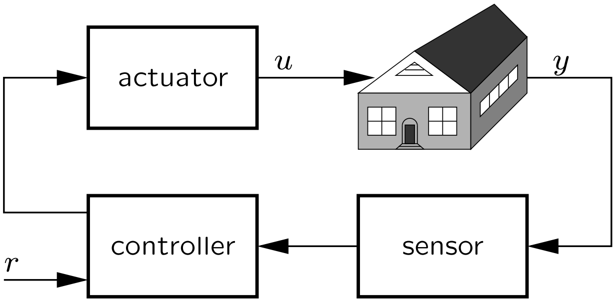
- Start with a physical system (plant).
- Measure some property of the plant with a sensor.
- Plant behavior (state) can be affected by an actuator.

Closed-loop Feedback: Structure and Example



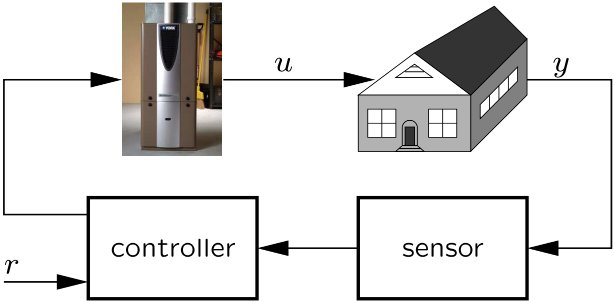
- Start with a physical system (plant).
- Measure some property of the plant with a sensor.
- Plant behavior (state) can be affected by an actuator.
- Feedback loop is completed by a controller.

Closed-loop Feedback: Structure and Example



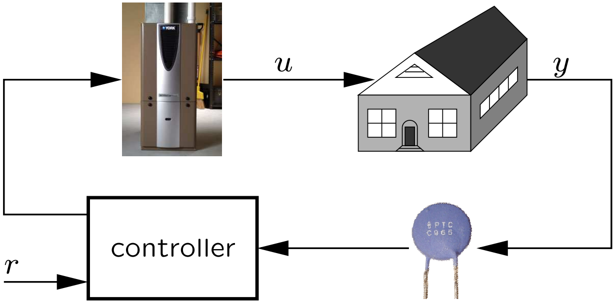
- Take a household heating system as an example.
  - Our plant is the house.

Closed-loop Feedback: Structure and Example



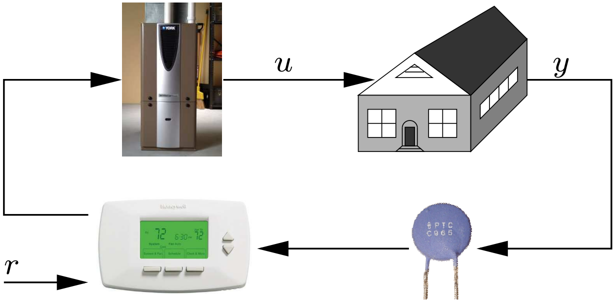
- Take a household heating system as an example.
  - Our plant is the house.
  - Actuator — heat pump.

Closed-loop Feedback: Structure and Example



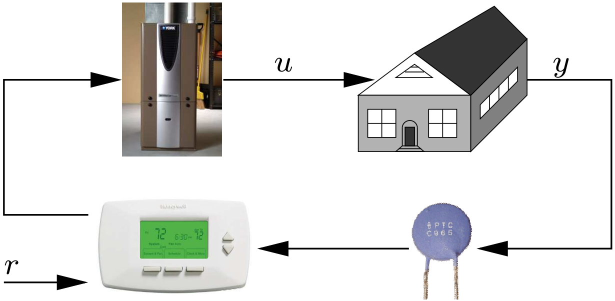
- Take a household heating system as an example.
  - Our plant is the house.
  - Actuator — heat pump.
  - Sensor — thermistor.

Closed-loop Feedback: Structure and Example



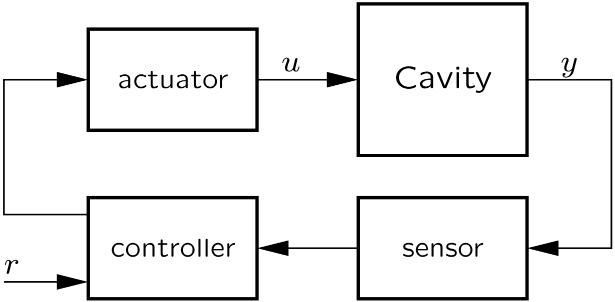
- Take a household heating system as an example.
  - Our plant is the house.
  - Actuator — heat pump.
  - Sensor — thermistor.
  - Controller — thermostat.

Closed-loop Feedback: Structure and Example



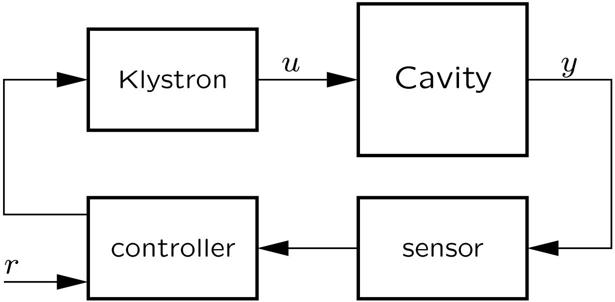
- Take a household heating system as an example.
  - Our plant is the house.
  - Actuator — heat pump.
  - Sensor — thermistor.
  - Controller — thermostat.
- Loop signals
  - Output y — temperature;
  - Input u — heated or cooled air from the heat pump;
  - Reference r — temperature setpoint.

Closed-loop Feedback: Structure and Example



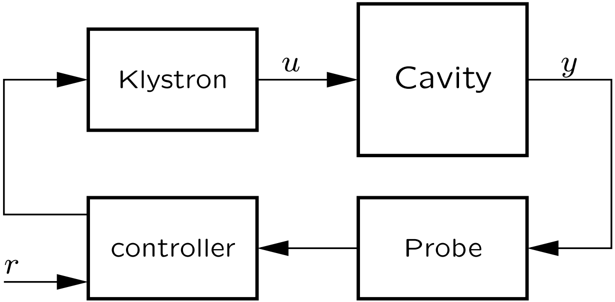
- For an accelerator RF system we have:
  - Our plant is an RF cavity.

Closed-loop Feedback: Structure and Example



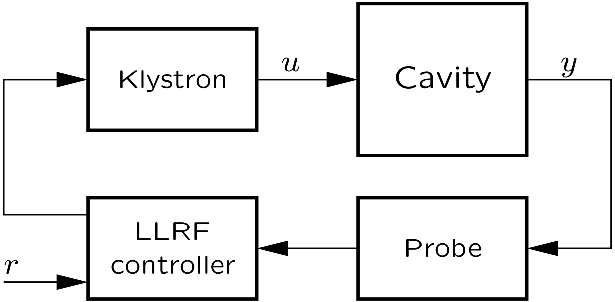
- For an accelerator RF system we have:
  - Our plant is an RF cavity.
  - Actuator — klystron/SSA/IOT.

Closed-loop Feedback: Structure and Example



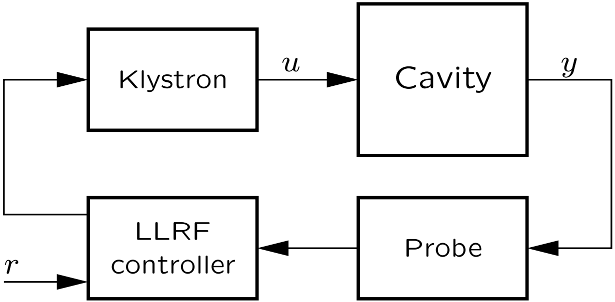
- For an accelerator RF system we have:
  - Our plant is an RF cavity.
  - Actuator — klystron/SSA/IOT.
  - Sensor — cavity probe.

Closed-loop Feedback: Structure and Example



- For an accelerator RF system we have:
  - Our plant is an RF cavity.
  - Actuator — klystron/SSA/IOT.
  - Sensor — cavity probe.
  - Controller — LLRF system.

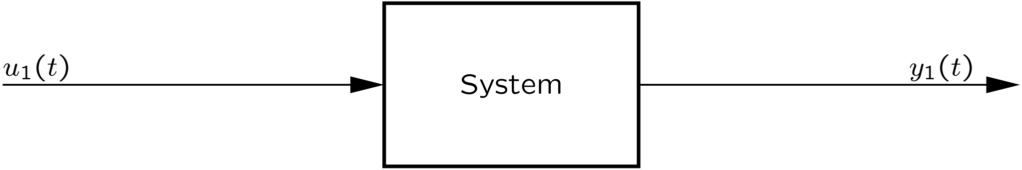
Closed-loop Feedback: Structure and Example



- For an accelerator RF system we have:
  - Our plant is an RF cavity.
  - Actuator — klystron/SSA/IOT.
  - Sensor — cavity probe.
  - Controller — LLRF system.
- Loop signals
  - Output y — cavity field;
  - Input u — klystron power;
  - Reference r — amplitude and phase.

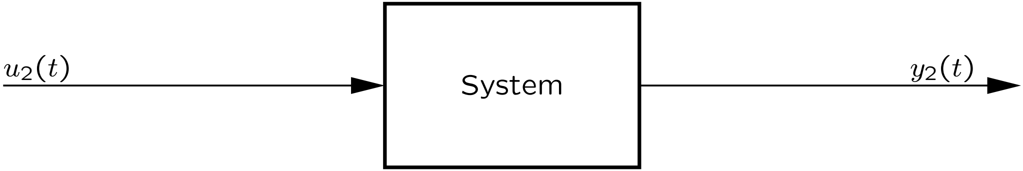
Outline

Definition of LTI



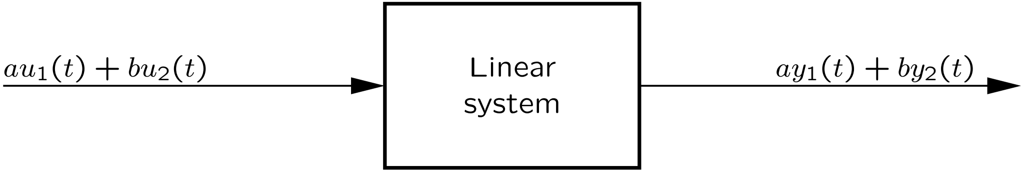
- Linearity
  - A system produces output $y_1(t)$ for input $u_1(t)$;

Definition of LTI



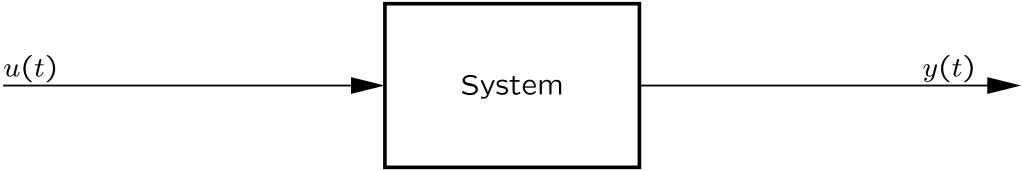
- Linearity
  - A system produces output $y_1(t)$ for input $u_1(t)$;
  - For input $u_2(t)$ the output is $y_2(t)$;

Definition of LTI



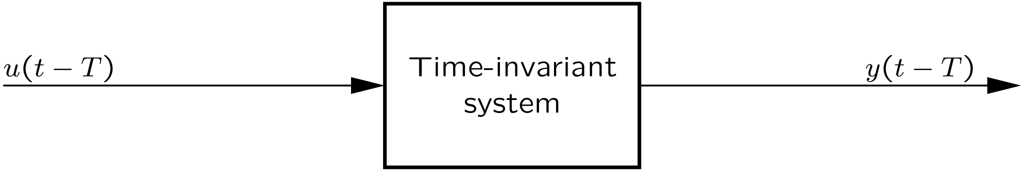
- Linearity
  - A system produces output $y_1(t)$ for input $u_1(t)$;
  - For input $u_2(t)$ the output is $y_2(t)$;
  - A system is linear if an input $au_1(t) + bu_2(t)$ results in an output $ay_1(t) + by_2(t)$ for all real constants a and b and all inputs $u_1(t)$, $u_2(t)$.

Definition of LTI



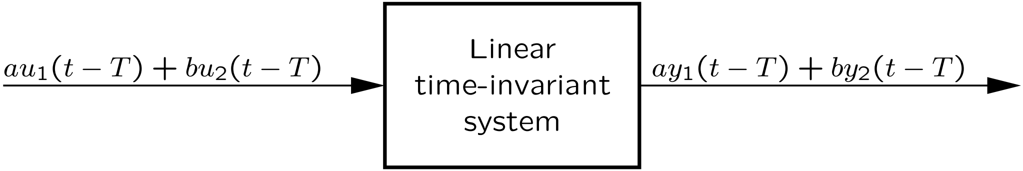
- Linearity
  - A system produces output $y_1(t)$ for input $u_1(t)$;
  - For input $u_2(t)$ the output is $y_2(t)$;
  - A system is linear if an input $au_1(t) + bu_2(t)$ results in an output $ay_1(t) + by_2(t)$ for all real constants a and b and all inputs $u_1(t)$, $u_2(t)$.
- Time invariance
  - Output $y(t)$ for input $u(t)$;

Definition of LTI



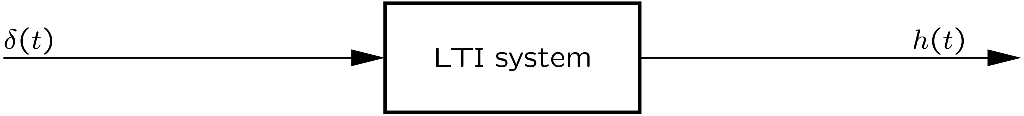
- Linearity
  - A system produces output $y_1(t)$ for input $u_1(t)$;
  - For input $u_2(t)$ the output is $y_2(t)$;
  - A system is linear if an input $au_1(t) + bu_2(t)$ results in an output $ay_1(t) + by_2(t)$ for all real constants a and b and all inputs $u_1(t)$, $u_2(t)$.
- Time invariance
  - Output $y(t)$ for input $u(t)$;
  - Output $y(t-T)$ for input $u(t-T)$.

Definition of LTI



- Linearity
  - A system produces output $y_1(t)$ for input $u_1(t)$;
  - For input $u_2(t)$ the output is $y_2(t)$;
  - A system is linear if an input $au_1(t) + bu_2(t)$ results in an output $ay_1(t) + by_2(t)$ for all real constants a and b and all inputs $u_1(t)$, $u_2(t)$.
- Time invariance
  - Output $y(t)$ for input $u(t)$;
  - Output $y(t-T)$ for input $u(t-T)$.
- Putting it all together — a linear, time-invariant (LTI) system.

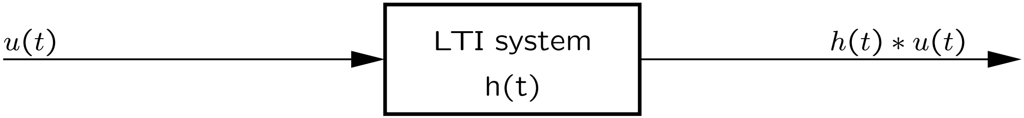
Impulse Response and Transfer Function



- Any LTI system is fully described by it's impulse response $h(t)$ (input signal is the Dirac's delta function);
- Output to an arbitrary input is defined by a convolution $y(t) = (u * h)(t) = \int_{-\infty}^{\infty} u(\tau)h(t-\tau)d\tau$;
- Analysis of the LTI systems is greatly simplified by the Laplace transform: $F(s) = \int_{0}^{\infty} e^{-st} f(t)dt$, where s is complex[1];
- Laplace transform $H(s)$ of the impulse response $h(t)$ defines the frequency response of the system.

[1] F(s) evaluated on the imaginary axis is, in most cases, the Fourier transform of $f(t)$.

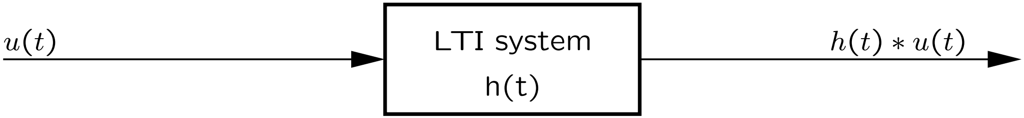
Impulse Response and Transfer Function



- Any LTI system is fully described by it's impulse response $h(t)$ (input signal is the Dirac's delta function);
- Output to an arbitrary input is defined by a convolution $y(t) = (u * h)(t) = \int_{-\infty}^{\infty} u(\tau)h(t-\tau)d\tau$;
- Analysis of the LTI systems is greatly simplified by the Laplace transform: $F(s) = \int_0^{\infty} e^{-st} f(t)dt$, where s is complex[1];
- Laplace transform $H(s)$ of the impulse response $h(t)$ defines the frequency response of the system.

[1] F(s) evaluated on the imaginary axis is, in most cases, the Fourier transform of $f(t)$.

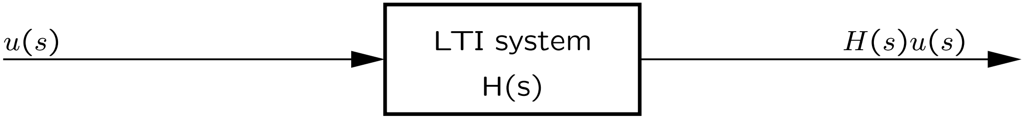
Impulse Response and Transfer Function



- Any LTI system is fully described by it's impulse response $h(t)$ (input signal is the Dirac's delta function);
- Output to an arbitrary input is defined by a convolution $y(t) = (u * h)(t) = \int_{-\infty}^{\infty} u(\tau)h(t-\tau)d\tau$;
- Analysis of the LTI systems is greatly simplified by the Laplace transform: $F(s) = \int_{0}^{\infty} e^{-st} f(t)dt$, where s is complex[1];
- Laplace transform $H(s)$ of the impulse response $h(t)$ defines the frequency response of the system.

[1] F(s) evaluated on the imaginary axis is, in most cases, the Fourier transform of $f(t)$

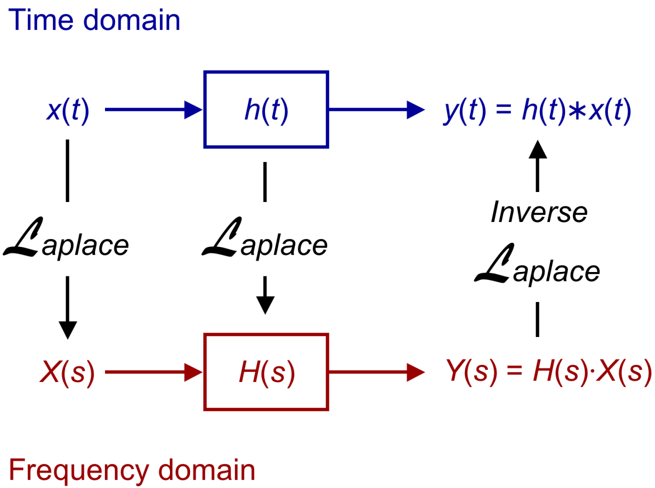
Impulse Response and Transfer Function



- Any LTI system is fully described by it's impulse response $h(t)$ (input signal is the Dirac's delta function);
- Output to an arbitrary input is defined by a convolution $y(t) = (u * h)(t) = \int_{-\infty}^{\infty} u(\tau)h(t-\tau)d\tau$;
- Analysis of the LTI systems is greatly simplified by the Laplace transform: $F(s) = \int_0^{\infty} e^{-st} f(t)dt$, where s is complex[1];
- Laplace transform $H(s)$ of the impulse response $h(t)$ defines the frequency response of the system.

[1] F(s) evaluated on the imaginary axis is, in most cases, the Fourier transform of $f(t)$

Time and Frequency Domain Relationships



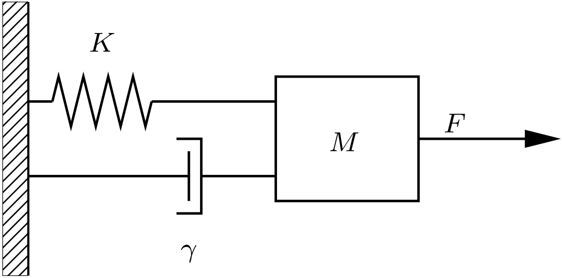
Some Properties of LTI Systems

Definition (Rational transfer function)

$$H(s) = \frac{B(s)}{A(s)} = \frac{\sum_{m=0}^{M} b_m s^m}{\sum_{n=0}^{N} a_n s^n},$$ where $M < N$ for physical systems.

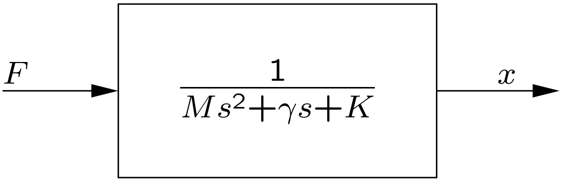
- Complex roots of the denominator polynomial $A(s)$ determine the stability of $H(s)$ — unstable roots (poles) have a non-negative real part.
- An LTI system cannot produce at its output frequencies that are not contained in the input signal;
- Many LTI systems can be described by linear differential equations with constant coefficients:
  - Mass with a spring and a damper;
  - R, L, C circuits;
  - Pendulum (for small displacements);
  - Inverted pendulum (for small displacements, unstable).
- Delay of τ is represented by $H(s) = e^{-s\tau}$, not rational.

A Mechanical System



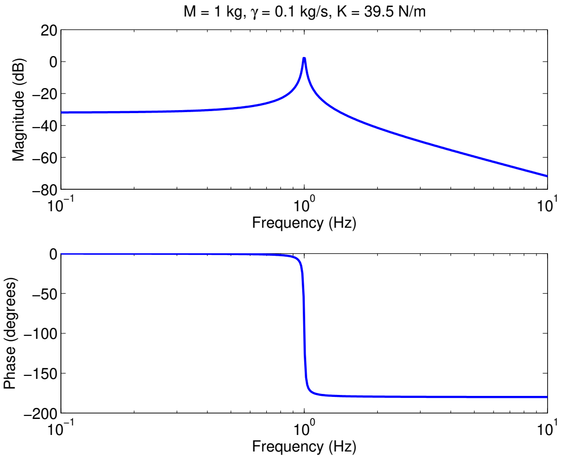
- Mechanical system: mass on a spring with a damper;
- Described by $M\ddot{x} + \gamma\dot{x} + Kx = F$.
- Differential equation is a time-domain description;
- Frequency domain - Laplace transform;
- Frequency response evaluated at $s = i\omega$;
- Plot of the transfer function magnitude and phase vs. frequency is known as Bode plot.

A Mechanical System



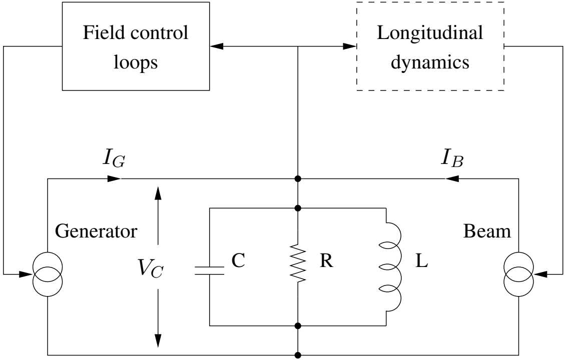
- Mechanical system: mass on a spring with a damper;
- Described by $M\ddot{x} + \gamma\dot{x} + Kx = F$.
- Differential equation is a time-domain description;
- Frequency domain - Laplace transform;
- Frequency response evaluated at $s = i\omega$;
- Plot of the transfer function magnitude and phase vs. frequency is known as Bode plot.

A Mechanical System



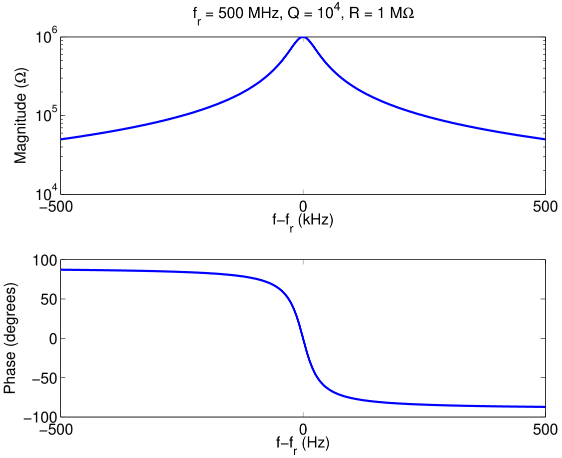
- Mechanical system: mass on a spring with a damper;
- Described by $M\ddot{x} + \gamma\dot{x} + Kx = F$.
- Differential equation is a time-domain description;
- Frequency domain - Laplace transform;
- Frequency response evaluated at $s = i\omega$;
- Plot of the transfer function magnitude and phase vs. frequency is known as Bode plot.

A Cavity



- One resonant mode;
- $P(s) = \frac{V_c}{I_g} = \frac{sRL}{s^2 RLC + sL + R}$;
- Rewrite as $P(s) = \frac{2\sigma Rs}{s^2 + 2\sigma s + \omega_r^2}$, where:
  - Resonant frequency $\omega_r = \frac{1}{\sqrt{LC}}$;
  - Quality factor $Q = R\sqrt{\frac{C}{L}}$;
  - Damping rate $\sigma = \omega_r/(2Q)$.
- Bandpass response, 180° phase shift across the resonance.

A Cavity

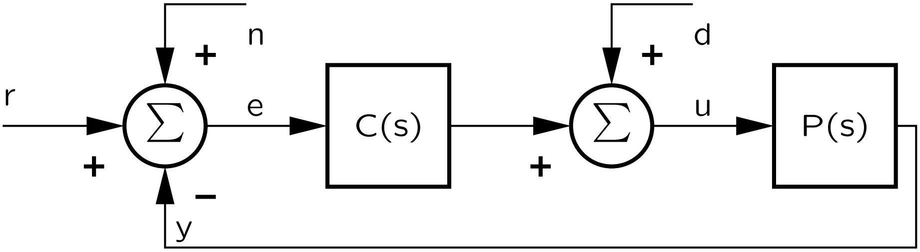


- One resonant mode;
- $P(s) = \frac{V_c}{I_g} = \frac{sRL}{s^2 RLC + sL + R}$;
- Rewrite as $P(s) = \frac{2\sigma Rs}{s^2 + 2\sigma s + \omega_r^2}$, where:
  - Resonant frequency $\omega_r = \frac{1}{\sqrt{LC}}$;
  - Quality factor $Q = R\sqrt{\frac{C}{L}}$;
  - Damping rate $\sigma = \omega_r/(2Q)$.
- Bandpass response, 180° phase shift across the resonance.

Outline

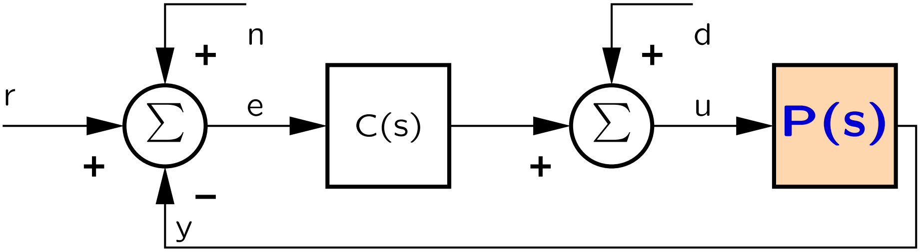
Outline

Feedback Loop Definitions



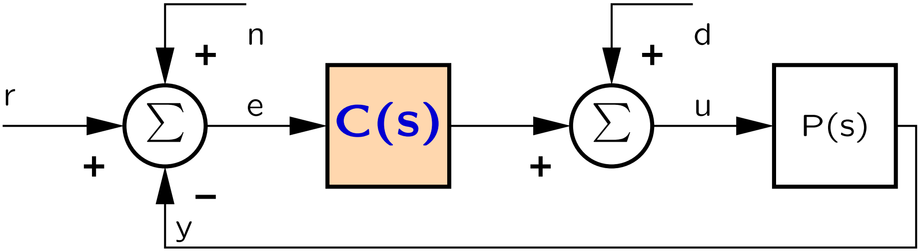
- Generalized loop block diagram;
- Plant response $P(s)$
- Feedback controller $C(s)$;
- Reference signal (setpoint) r;
- Plant output y;
- Error signal e;
- Measurement noise n;
- Disturbance d;
- Plant input u.

Feedback Loop Definitions



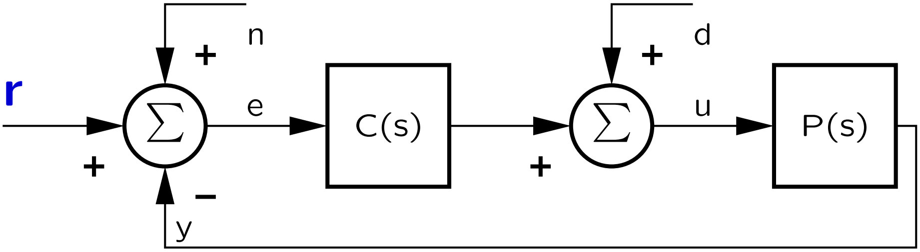
- Generalized loop block diagram;
- Plant response $P(s)$
- Feedback controller $C(s)$;
- Reference signal (setpoint) r;
- Plant output y;
- Error signal e;
- Measurement noise n;
- Disturbance d;
- Plant input u.

Feedback Loop Definitions



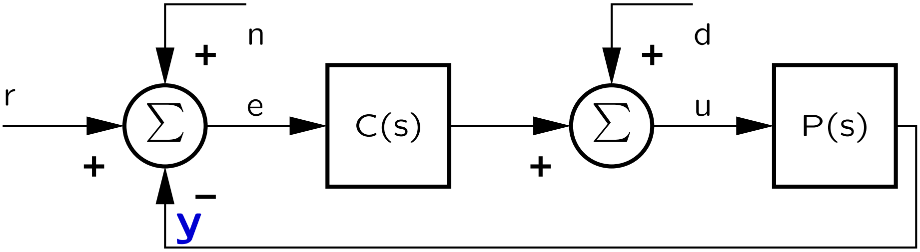
- Generalized loop block diagram;
- Plant response $P(s)$
- Feedback controller $C(s)$;
- Reference signal (setpoint) r;
- Plant output y;
- Error signal e;
- Measurement noise n;
- Disturbance d;
- Plant input u.

Feedback Loop Definitions



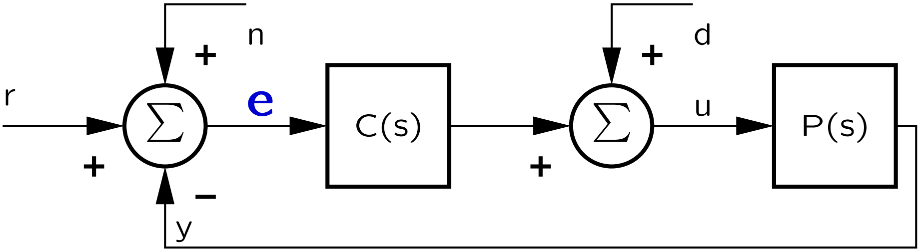
- Generalized loop block diagram;
- Plant response $P(s)$
- Feedback controller $C(s)$;
- Reference signal (setpoint) r;
- Plant output y;
- Error signal e;
- Measurement noise n;
- Disturbance d;
- Plant input u.

Feedback Loop Definitions



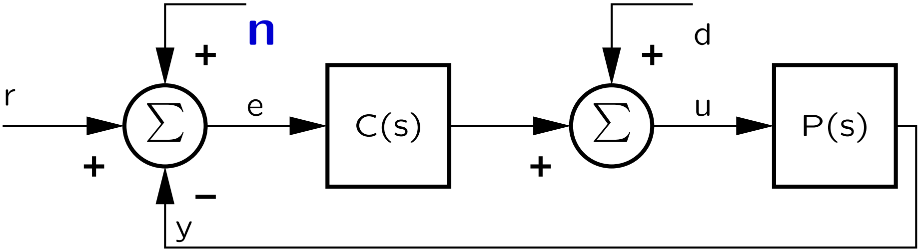
- Generalized loop block diagram;
- Plant response $P(s)$
- Feedback controller $C(s)$;
- Reference signal (setpoint) r;
- Plant output y;
- Error signal e;
- Measurement noise n;
- Disturbance d;
- Plant input u.

Feedback Loop Definitions



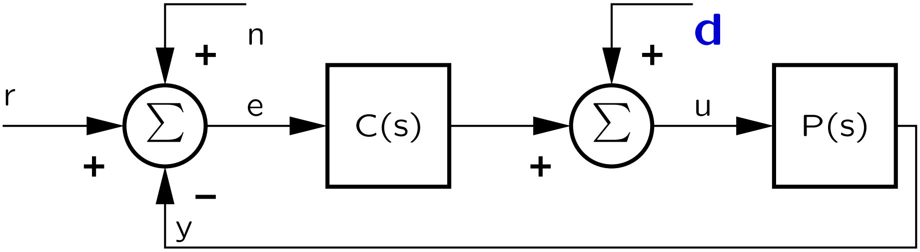
- Generalized loop block diagram;
- Plant response $P(s)$
- Feedback controller $C(s)$;
- Reference signal (setpoint) r;
- Plant output y;
- Error signal e;
- Measurement noise n;
- Disturbance d;
- Plant input u.

Feedback Loop Definitions



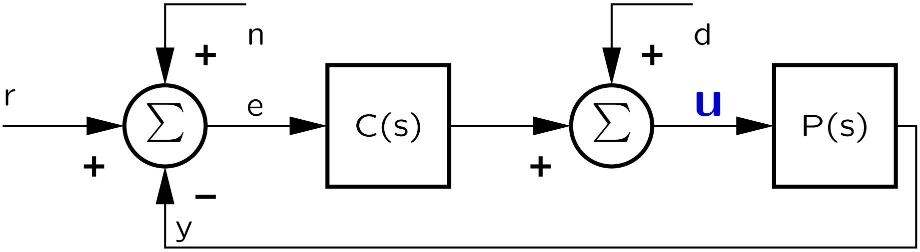
- Generalized loop block diagram;
- Plant response $P(s)$
- Feedback controller $C(s)$;
- Reference signal (setpoint) r;
- Plant output y;
- Error signal e;
- Measurement noise n;
- Disturbance d;
- Plant input u.

Feedback Loop Definitions



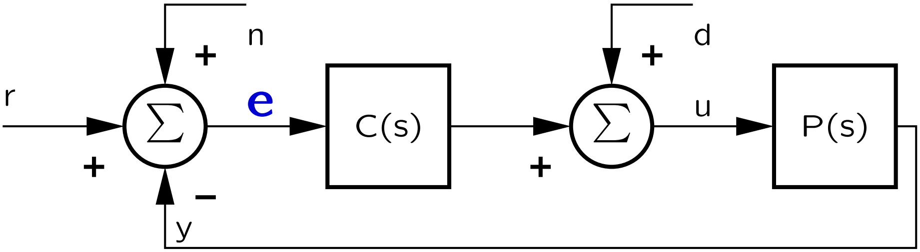
- Generalized loop block diagram;
- Plant response $P(s)$
- Feedback controller $C(s)$;
- Reference signal (setpoint) r;
- Plant output y;
- Error signal e;
- Measurement noise n;
- Disturbance d;
- Plant input u.

Feedback Loop Definitions



- Generalized loop block diagram;
- Plant response $P(s)$
- Feedback controller $C(s)$;
- Reference signal (setpoint) r;
- Plant output y;
- Error signal e;
- Measurement noise n;
- Disturbance d;
- Plant input u.

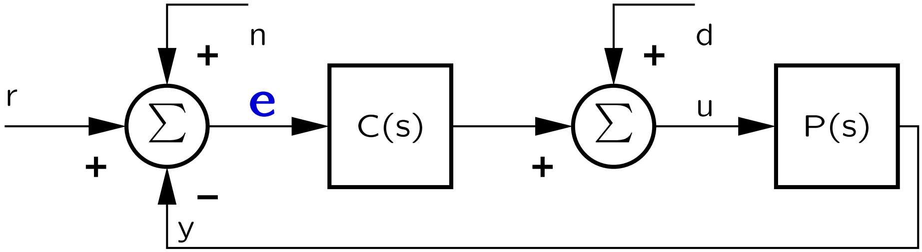
Feedback Loop Responses



Let's derive the response from the reference to the error signal (sensitivity function $S(s)$):

- $y = C(s)P(s)e = L(s)e$ where $L(s)$ is the open-loop transfer function;
- $e = r - y = r - L(s)e$;
- $S(s) = \frac{e}{r} = \frac{1}{1+L(s)} = \frac{u}{d}$.

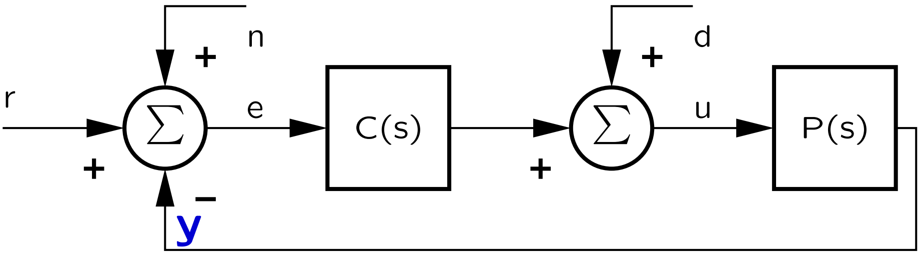
Feedback Loop Responses



Let's derive the response from the reference to the error signal (sensitivity function $S(s)$):

- $y = C(s)P(s)e = L(s)e$ where $L(s)$ is the open-loop transfer function;
- $e = r - y = r - L(s)e$;
- $S(s) = \frac{e}{r} = \frac{1}{1+L(s)} = \frac{u}{d}$.

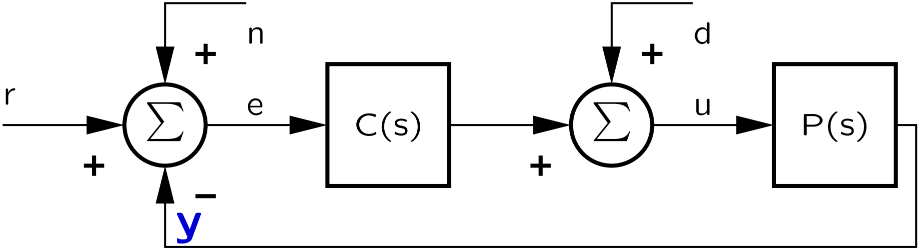
Feedback Loop Responses, Continued



The response from the reference to the measurement (complementary sensitivity function $T(s)$):

- $T(s) = \frac{y}{r} = \frac{L(s)}{1+L(s)}$;
- Obvious, but useful: $S(s) + T(s) = \frac{1}{1+L(s)} + \frac{L(s)}{1+L(s)} = 1$

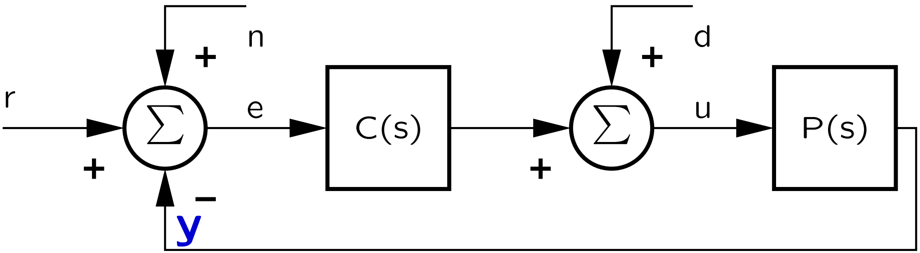
Feedback Loop Responses, Continued



The response from the reference to the measurement (complementary sensitivity function $T(s)$):

- $T(s) = \frac{y}{r} = \frac{L(s)}{1+L(s)}$;
- Obvious, but useful: $S(s) + T(s) = \frac{1}{1+L(s)} + \frac{L(s)}{1+L(s)} = 1$

Feedback Loop Responses, Continued

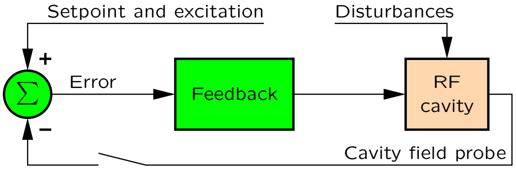


The response from the reference to the measurement (complementary sensitivity function $T(s)$):

- $T(s) = \frac{y}{r} = \frac{L(s)}{1+L(s)}$;
- Obvious, but useful: $S(s) + T(s) = \frac{1}{1+L(s)} + \frac{L(s)}{1+L(s)} = 1$

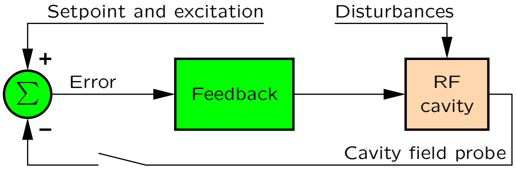
Outline

Open Loop Transfer Function



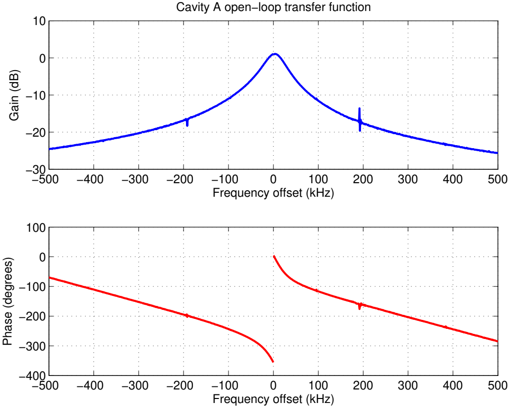
- Measured from the setpoint r to the cavity probe y;
- $C(s)$ in the open loop has no dynamics, just constant gain and phase shift, attenuated $L(s)$;
- Measured loop response;
- Fit a resonator model to extract gain, loaded Q, phase on resonance, detuning, and delay.

Open Loop Transfer Function



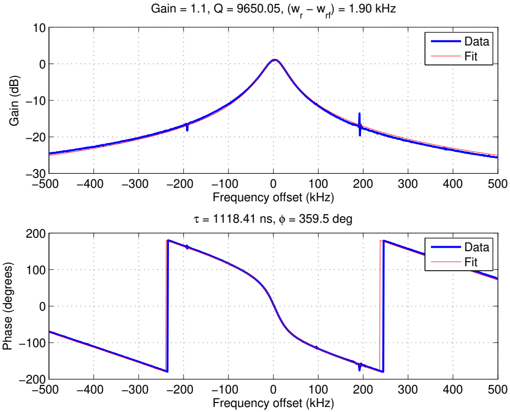
- Measured from the setpoint r to the cavity probe y;
- $C(s)$ in the open loop has no dynamics, just constant gain and phase shift, attenuated $L(s)$;
- Measured loop response;
- Fit a resonator model to extract gain, loaded Q, phase on resonance, detuning, and delay.

Open Loop Transfer Function



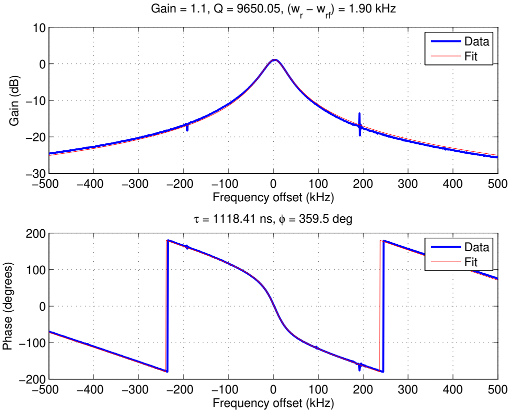
- Measured from the setpoint r to the cavity probe y;
- $C(s)$ in the open loop has no dynamics, just constant gain and phase shift, attenuated $L(s)$;
- Measured loop response;
- Fit a resonator model to extract gain, loaded Q, phase on resonance, detuning, and delay.

Open Loop Transfer Function



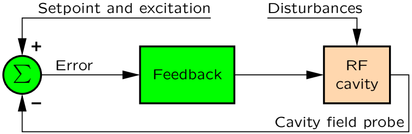
- Measured from the setpoint r to the cavity probe y;
- $C(s)$ in the open loop has no dynamics, just constant gain and phase shift, attenuated $L(s)$;
- Measured loop response;
- Fit a resonator model to extract gain, loaded Q, phase on resonance, detuning, and delay.

Open Loop Transfer Function



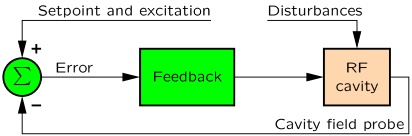
- Measured from the setpoint r to the cavity probe y;
- $C(s)$ in the open loop has no dynamics, just constant gain and phase shift, attenuated $L(s)$;
- Measured loop response;
- Fit a resonator model to extract gain, loaded Q, phase on resonance, detuning, and delay.

Closed Loop Transfer Functions



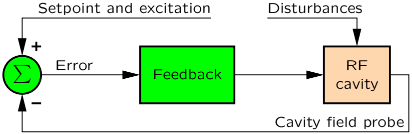
- Measured from the setpoint r to the error signal e;
- Shows attenuation at frequencies where feedback has gain;
- Perturbations at the input of the cavity are rejected with the same transfer function;
- Proportional only;
- Proportional and integral, much higher rejection at low frequencies;
- Easier to see with the logarithmic frequency scale.

Closed Loop Transfer Functions



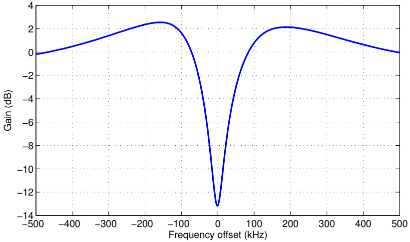
- Measured from the setpoint r to the error signal e;
- Shows attenuation at frequencies where feedback has gain;
- Perturbations at the input of the cavity are rejected with the same transfer function;
- Proportional only;
- Proportional and integral, much higher rejection at low frequencies;
- Easier to see with the logarithmic frequency scale.

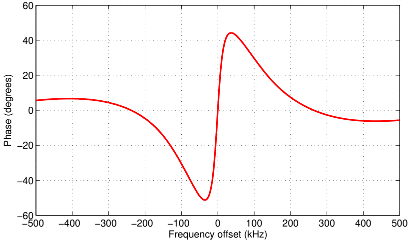
Closed Loop Transfer Functions



- Measured from the setpoint r to the error signal e;
- Shows attenuation at frequencies where feedback has gain;
- Perturbations at the input of the cavity are rejected with the same transfer function;
- Proportional only;
- Proportional and integral, much higher rejection at low frequencies;
- Easier to see with the logarithmic frequency scale.

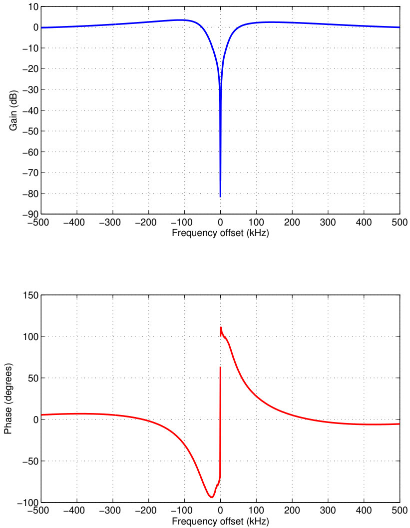
Closed Loop Transfer Functions





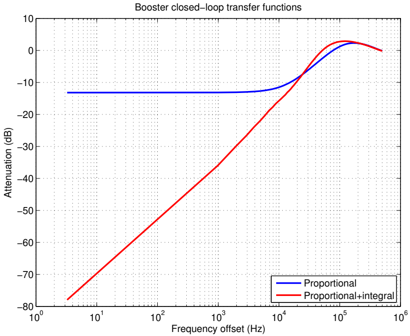
- Measured from the setpoint *r* to the error signal *e*;
- Shows attenuation at frequencies where feedback has gain;
- Perturbations at the input of the cavity are rejected with the same transfer function;
- Proportional only;
- Proportional and integral, much higher rejection at low frequencies;
- Easier to see with the logarithmic frequency scale.

Closed Loop Transfer Functions



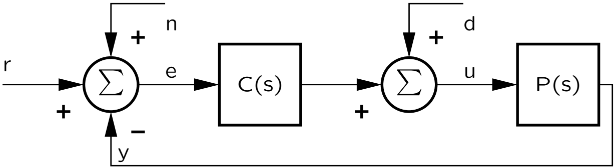
- Measured from the setpoint *r* to the error signal *e*;
- Shows attenuation at frequencies where feedback has gain;
- Perturbations at the input of the cavity are rejected with the same transfer function;
- Proportional only;
- Proportional and integral, much higher rejection at low frequencies;
- Easier to see with the logarithmic frequency scale.

Closed Loop Transfer Functions



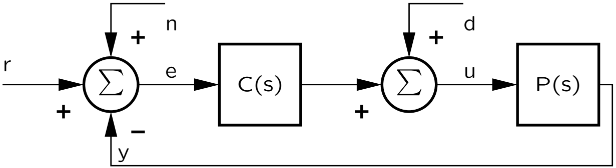
- Measured from the setpoint *r* to the error signal *e*;
- Shows attenuation at frequencies where feedback has gain;
- Perturbations at the input of the cavity are rejected with the same transfer function;
- Proportional only;
- Proportional and integral, much higher rejection at low frequencies;
- Easier to see with the logarithmic frequency scale.

Feedback Loop Transfer Functions Revisited



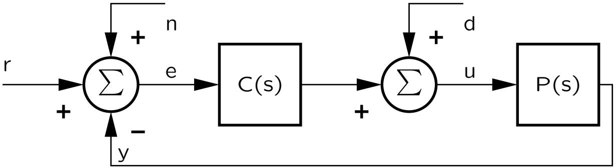
- $S(s)$ is small where the loop gain is high, good for tracking r;
- Same response to d — rejecting external disturbances;
- The loop does not care if the input came from y or n — tracking measurement noise;
- At frequencies where $L(s)$ is small, $S(s)$ settles near unity. If $C(s)$ has high proportional gain over a wide bandwidth, the transfer function from n to u — $C(s)S(s)$ — can amplify the measurement noise to a large fraction of the output power budget.

Feedback Loop Transfer Functions Revisited



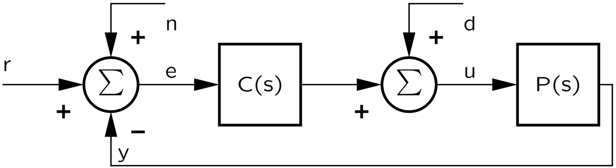
- $S(s)$ is small where the loop gain is high, good for tracking r;
- Same response to d — rejecting external disturbances;
- The loop does not care if the input came from y or n — tracking measurement noise;
- At frequencies where $L(s)$ is small, $S(s)$ settles near unity. If $C(s)$ has high proportional gain over a wide bandwidth, the transfer function from n to u — $C(s)S(s)$ — can amplify the measurement noise to a large fraction of the output power budget.

Feedback Loop Transfer Functions Revisited



- $S(s)$ is small where the loop gain is high, good for tracking r;
- Same response to d — rejecting external disturbances;
- The loop does not care if the input came from y or n — tracking measurement noise;
- At frequencies where $L(s)$ is small, $S(s)$ settles near unity. If $C(s)$ has high proportional gain over a wide bandwidth, the transfer function from n to u — $C(s)S(s)$ — can amplify the measurement noise to a large fraction of the output power budget.

Feedback Loop Transfer Functions Revisited



- $S(s)$ is small where the loop gain is high, good for tracking r;
- Same response to d — rejecting external disturbances;
- The loop does not care if the input came from y or n — tracking measurement noise;
- At frequencies where $L(s)$ is small, $S(s)$ settles near unity. If $C(s)$ has high proportional gain over a wide bandwidth, the transfer function from n to u — $C(s)S(s)$ — can amplify the measurement noise to a large fraction of the output power budget.

Outline

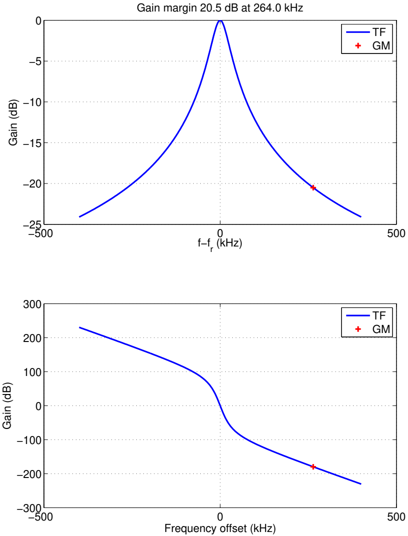
Introduction

Linear Time-Invariant Systems

Feedback Control Basics
- Loop Analysis
- A Real World Example
- Loop Stability
- Parasitic Mode Stability
- More Control Theory

Summary

Traditional Stability Margins: Gain Margin

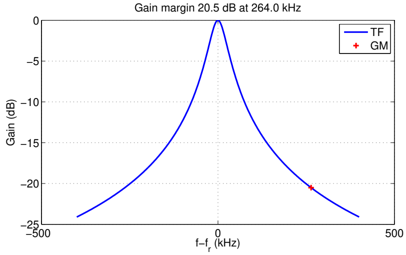


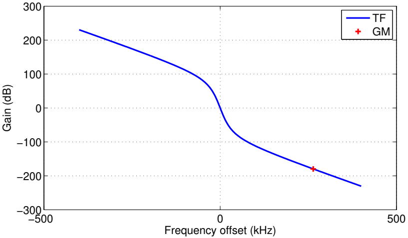
Definition

Gain margin of an open-loop transfer function is found by finding the frequency where the phase rotates by 180° and calculating the open-loop attenuation.

- Feedback loop goes unstable when positive feedback loop gain is above unity;
- Gain margin tells us how much the loop gain can be increased (or decreased for stabilizing feedback) before the system becomes unstable;
- A measure of system robustness under perturbations.

Traditional Stability Margins: Gain Margin



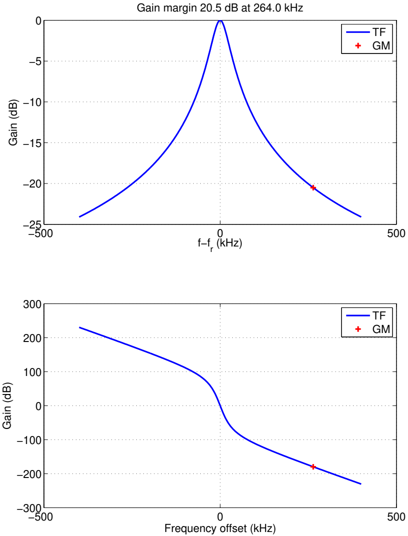


Definition

Gain margin of an open-loop transfer function is found by finding the frequency where the phase rotates by 180° and calculating the open-loop attenuation.

- Feedback loop goes unstable when positive feedback loop gain is above unity;
- Gain margin tells us how much the loop gain can be increased (or decreased for stabilizing feedback) before the system becomes unstable;
- A measure of system robustness under perturbations.

Traditional Stability Margins: Gain Margin

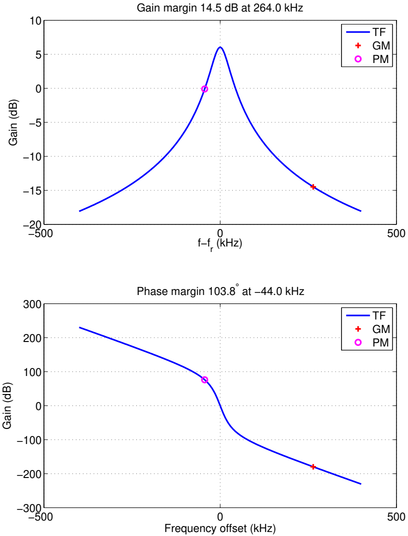


Definition

Gain margin of an open-loop transfer function is found by finding the frequency where the phase rotates by 180° and calculating the open-loop attenuation.

- Feedback loop goes unstable when positive feedback loop gain is above unity;
- Gain margin tells us how much the loop gain can be increased (or decreased for stabilizing feedback) before the system becomes unstable;
- A measure of system robustness under perturbations.

Traditional Stability Margins: Phase Margin

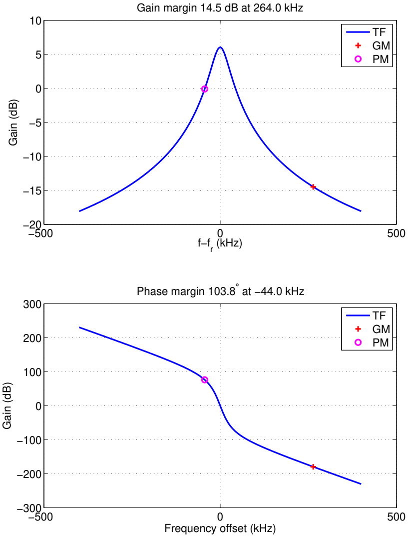


Definition

Phase margin of an open-loop transfer function is found by finding the frequency where the gain is unity and calculating the phase distance from 180°.

- If at unity gain the phase did not rotate by 180°, closed-loop response will be stable;
- Phase margin — how much can we offset the loop phase (think of a phase shifter in the path) before instability.

Traditional Stability Margins: Phase Margin

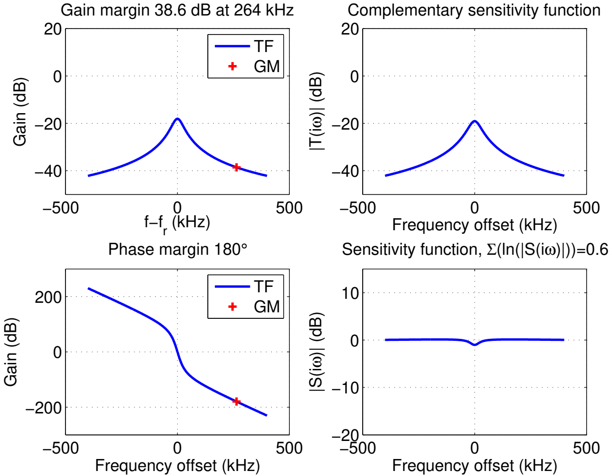


Definition

Phase margin of an open-loop transfer function is found by finding the frequency where the gain is unity and calculating the phase distance from 180°.

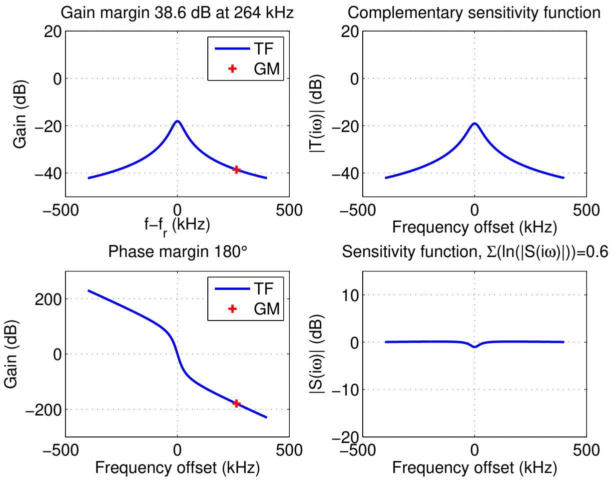
- If at unity gain the phase did not rotate by 180°, closed-loop response will be stable;
- Phase margin — how much can we offset the loop phase (think of a phase shifter in the path) before instability.

Exploring the Margins



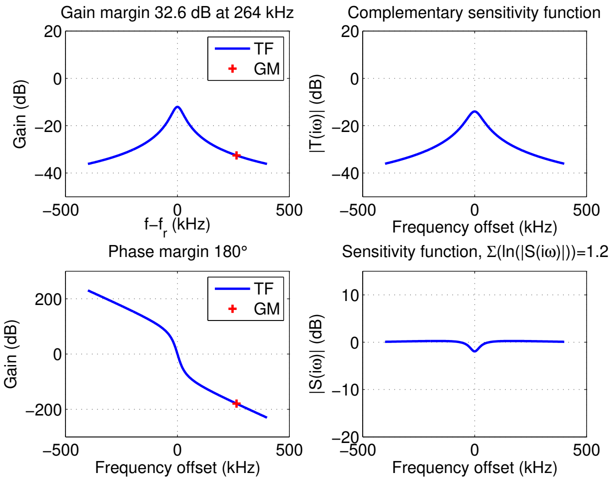
- Plotting $L(i\omega)$, $|T(i\omega)|$, and $|S(i\omega)|$ vs. loop gain, delay of 1 μs;
- Gain of 0.125;
- Gain of 0.25;
- Gain of 0.5;
- Gain of 1;
- Gain of 2;
- Gain of 4;
- Gain of 8.

Exploring the Margins



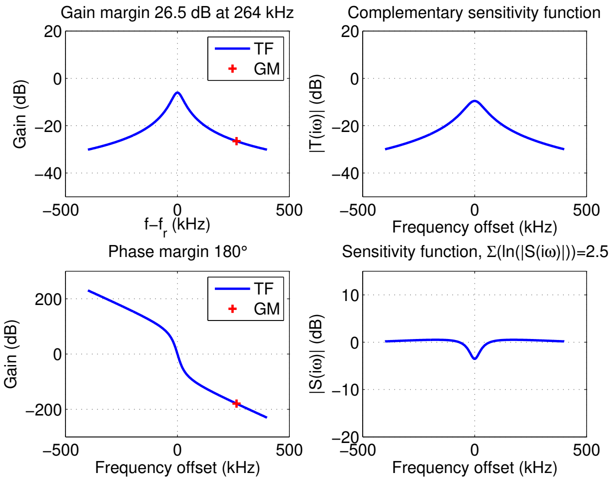
- Plotting $L(i\omega)$, $|T(i\omega)|$, and $|S(i\omega)|$ vs. loop gain, delay of 1 μs;
- Gain of 0.125;
- Gain of 0.25;
- Gain of 0.5;
- Gain of 1;
- Gain of 2;
- Gain of 4;
- Gain of 8.

Exploring the Margins



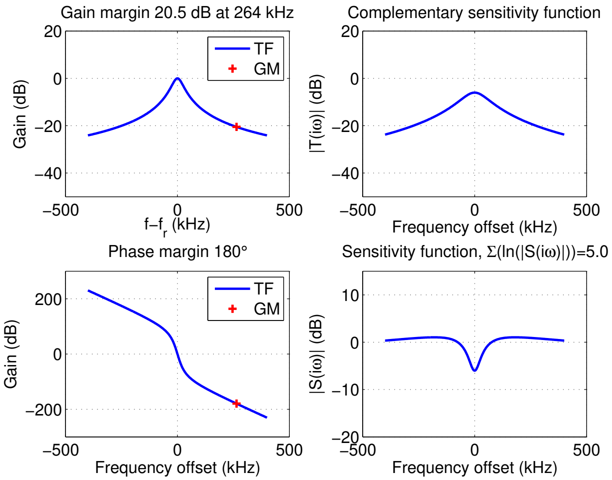
- Plotting $L(i\omega)$, $|T(i\omega)|$, and $|S(i\omega)|$ vs. loop gain, delay of 1 μs;
- Gain of 0.125;
- Gain of 0.25;
- Gain of 0.5;
- Gain of 1;
- Gain of 2;
- Gain of 4;
- Gain of 8.

Exploring the Margins



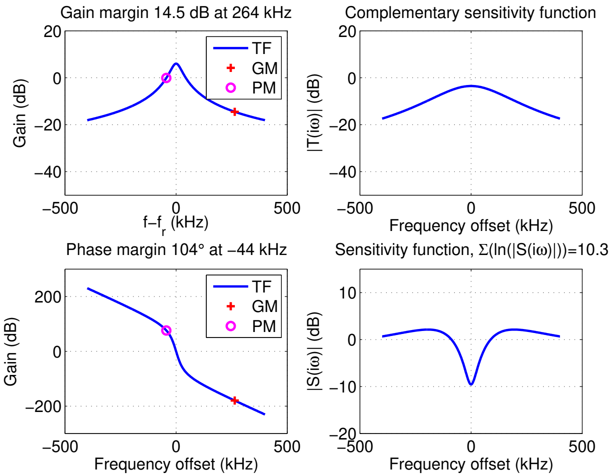
- Plotting $L(i\omega)$, $|T(i\omega)|$, and $|S(i\omega)|$ vs. loop gain, delay of 1 μs;
- Gain of 0.125;
- Gain of 0.25;
- Gain of 0.5;
- Gain of 1;
- Gain of 2;
- Gain of 4;
- Gain of 8.

Exploring the Margins



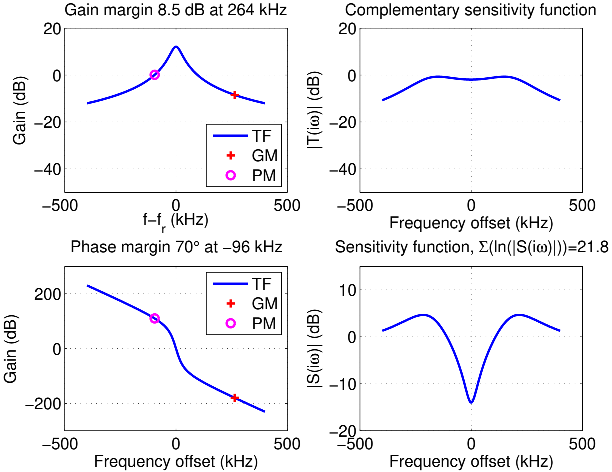
- Plotting $L(i\omega)$, $|T(i\omega)|$, and $|S(i\omega)|$ vs. loop gain, delay of 1 μs;
- Gain of 0.125;
- Gain of 0.25;
- Gain of 0.5;
- Gain of 1;
- Gain of 2;
- Gain of 4;
- Gain of 8.

Exploring the Margins



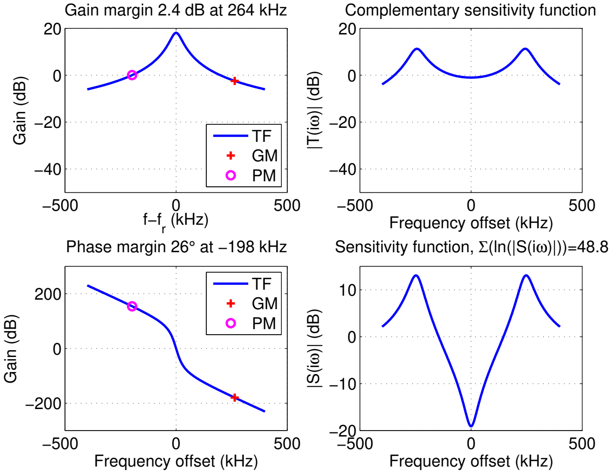
- Plotting $L(i\omega)$, $|T(i\omega)|$, and $|S(i\omega)|$ vs. loop gain, delay of 1 μs;
- Gain of 0.125;
- Gain of 0.25;
- Gain of 0.5;
- Gain of 1;
- Gain of 2;
- Gain of 4;
- Gain of 8.

Exploring the Margins



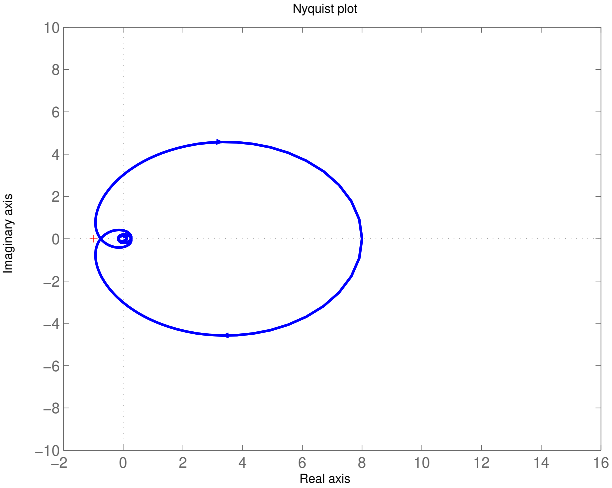
- Plotting $L(i\omega)$, $|T(i\omega)|$, and $|S(i\omega)|$ vs. loop gain, delay of 1 μs;
- Gain of 0.125;
- Gain of 0.25;
- Gain of 0.5;
- Gain of 1;
- Gain of 2;
- Gain of 4;
- Gain of 8.

Exploring the Margins



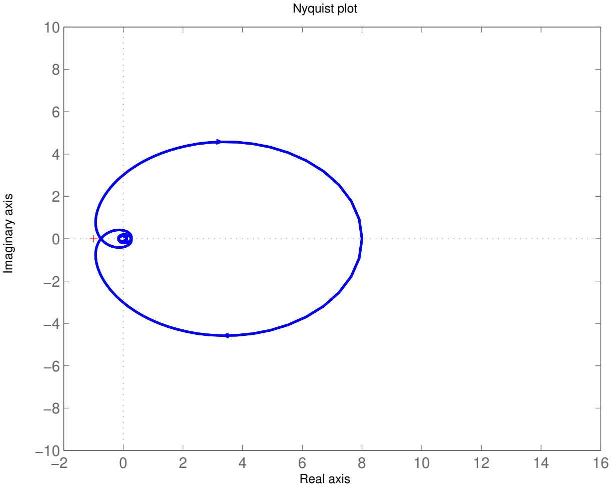
- Plotting $L(i\omega)$, $|T(i\omega)|$, and $|S(i\omega)|$ vs. loop gain, delay of 1 µs;
- Gain of 0.125;
- Gain of 0.25;
- Gain of 0.5;
- Gain of 1;
- Gain of 2;
- Gain of 4;
- Gain of 8.

Nyquist Stability Criterion



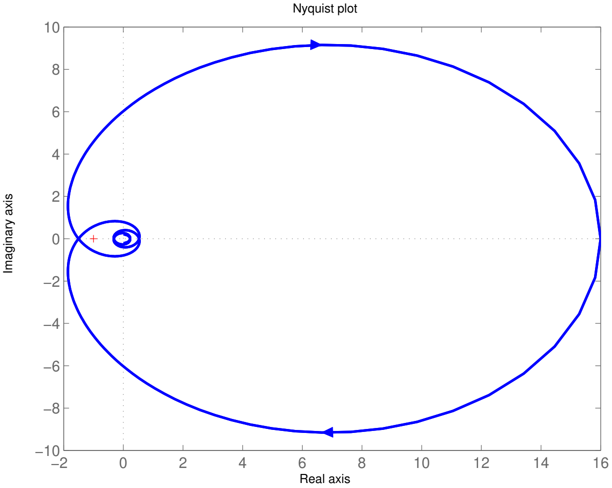
- Plot of $L(i\omega)$ on the complex plane is called the Nyquist plot. The number of clockwise encirclements of the point $-1 + 0i$ corresponds to the difference between the number of the closed-loop poles and closed-loop zeros in the right half plane.
- Gain of 8 (stable);
- Gain of 16 (unstable);

Nyquist Stability Criterion



- Plot of $L(i\omega)$ on the complex plane is called the Nyquist plot. The number of clockwise encirclements of the point $-1+0i$ corresponds to the difference between the number of the closed-loop poles and closed-loop zeros in the right half plane.
- Gain of 8 (stable);
- Gain of 16 (unstable);

Nyquist Stability Criterion



- Plot of $L(i\omega)$ on the complex plane is called the Nyquist plot. The number of clockwise encirclements of the point $-1 + 0i$ corresponds to the difference between the number of the closed-loop poles and closed-loop zeros in the right half plane.
- Gain of 8 (stable);
- Gain of 16 (unstable);

Outline

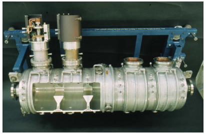
PETRA 5-cell Cavity

DATA SHEET

500 MHz, 5-Cell Cavity

DESY-MHFe, Vers. 2.1 October 2007

Type: PETRA
Manufacturer: **DESY / Balzers Hochvakuum GmbH**



Technical Data:

	Unit	Min.	Nom.	Max.	Remarks
π-mode frequency @ 35°C	MHz		499.67		Plungers flat
Tuning range	MHz			501	Plungers s = +40 mm
Tuning range	MHz	499			Plungers s = -20 mm
Unloaded quality factor	-	29,000		36,000	
R/(Q*l)	Ω/m		370		± 5%
Shunt impedance	MΩ		15		
Coupling factor	-			3.0	
Bandwidth	kHz			74	Coupling factor 3.0
Beam tube cut-off frequency	GHz		1.46		H_{11}
Field flatness	%	± 25			@ maximum power & cooling flow not adjusted to dissipated power
Coupling between cells	%		0.67		$k = \frac{1}{2}\frac{\omega_0^2 - \omega_\pi^2}{2\omega_\pi^2 - \omega_0^2 \cdot (1-\cos\left(\frac{\pi}{N}\right))}$
Detuning due to temperature	kHz/°C		8,5		
Detuning due to plunger pos.	kHz/mm	10	20	40	Both plungers moved
Accelerating voltage	MV		1.34	1,94	
Accelerating gradient	MV/m		0.89	1.29	
Dissipated cavity power	kW		60	125	
Water flow rate of single cooling circuits	l/h	1600		2000	@ pressure drop 3 bar
Water flow rate (total)	m³/h		8		Cooling circuits parallel. No orifice plates. Pressure drop 1.2 bar
Pressure drop	bar			4	Cooling circuits parallel and flow rates adjusted by orifice plates
Test pressure	bar			8	15 minutes
Total length	mm		1800		(Flange to flange)
Cell length	mm		5*300		
Outside diameter	mm		445		Without water installation
Beam tube aperture	mm		120		
Weight	kg		500		Accessories and blind flanges excluded

- An inexpensive cavity at 500 MHz;
- Five cells, three probes, two tuners;
- Main accelerating mode: π;
- Input coupler and probe are well coupled to the $\pi/2$ mode, typically 2.8 MHz above the π mode.

PETRA 5-cell Cavity

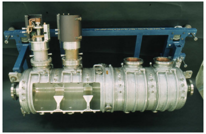
DATA SHEET

500 MHz, 5-Cell Cavity

DESY-MHFe, Vers. 2.1 October 2007

Type: PETRA

Manufacturer: **DESY / Balzers Hochvakuum GmbH**



Technical Data:

	Unit	Min.	Nom.	Max.	Remarks
π-mode frequency @ 35°C	MHz		499.67		Plungers flat
Tuning range	MHz			501	Plungers s = +40 mm
Tuning range	MHz	499			Plungers s = -20 mm
Unloaded quality factor	-	29,000		36,000	
R/(Q*l)	Ω/m		370		± 5%
Shunt impedance	MΩ		15		
Coupling factor	-			3.0	
Bandwidth	kHz			74	Coupling factor 3.0
Beam tube cut-off frequency	GHz		1.46		H_{11}
Field flatness	%	± 25			@ maximum power & cooling flow not adjusted to dissipated power
Coupling between cells	%		0.67		$k = \frac{1}{2}\frac{\omega_0^2 - \omega_\pi^2}{2\omega_\pi^2 - \omega_0^2 \cdot (1 - \cos\left(\frac{\pi}{N}\right))}$
Detuning due to temperature	kHz/°C		8,5		
Detuning due to plunger pos.	kHz/mm	10	20	40	Both plungers moved
Accelerating voltage	MV		1.34	1,94	
Accelerating gradient	MV/m		0.89	1.29	
Dissipated cavity power	kW		60	125	
Water flow rate of single cooling circuits	l/h	1600		2000	@ pressure drop 3 bar
Water flow rate (total)	m³/h		8		Cooling circuits parallel. No orifice plates. Pressure drop 1.2 bar
Pressure drop	bar			4	Cooling circuits parallel and flow rates adjusted by orifice plates
Test pressure	bar			8	15 minutes
Total length	mm		1800		(Flange to flange)
Cell length	mm		5*300		
Outside diameter	mm		445		Without water installation
Beam tube aperture	mm		120		
Weight	kg		500		Accessories and blind flanges excluded

- An inexpensive cavity at 500 MHz;
- Five cells, three probes, two tuners;
- Main accelerating mode: π;
- Input coupler and probe are well coupled to the $\pi/2$ mode, typically 2.8 MHz above the π mode.

PETRA 5-cell Cavity

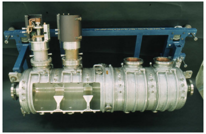
DATA SHEET

500 MHz, 5-Cell Cavity

DESY-MHFe, Vers. 2.1 October 2007

Type: PETRA

Manufacturer: **DESY / Balzers Hochvakuum GmbH**



Technical Data:

	Unit	Min.	Nom.	Max.	Remarks
π-mode frequency @ 35°C	MHz		499.67		Plungers flat
Tuning range	MHz			501	Plungers s = +40 mm
Tuning range	MHz	499			Plungers s = -20 mm
Unloaded quality factor	-	29,000		36,000	
R/(Q*l)	Ω/m		370		± 5%
Shunt impedance	MΩ		15		
Coupling factor	-			3.0	
Bandwidth	kHz			74	Coupling factor 3.0
Beam tube cut-off frequency	GHz		1.46		H_{11}
Field flatness	%	± 25			@ maximum power & cooling flow not adjusted to dissipated power
Coupling between cells	%		0.67		$k = \frac{1}{2} \frac{\omega_0^2 - \omega_\pi^2}{2\omega_\pi^2 - \omega_0^2 \cdot (1 - \cos\left(\frac{\pi}{N}\right))}$
Detuning due to temperature	kHz/°C		8,5		
Detuning due to plunger pos.	kHz/mm	10	20	40	Both plungers moved
Accelerating voltage	MV		1.34	1,94	
Accelerating gradient	MV/m		0.89	1.29	
Dissipated cavity power	kW		60	125	
Water flow rate of single cooling circuits	l/h	1600		2000	@ pressure drop 3 bar
Water flow rate (total)	m³/h		8		Cooling circuits parallel. No orifice plates. Pressure drop 1.2 bar
Pressure drop	bar			4	Cooling circuits parallel and flow rates adjusted by orifice plates
Test pressure	bar			8	15 minutes
Total length	mm		1800		(Flange to flange)
Cell length	mm		5*300		
Outside diameter	mm		445		Without water installation
Beam tube aperture	mm		120		
Weight	kg		500		Accessories and blind flanges excluded

- An inexpensive cavity at 500 MHz;
- Five cells, three probes, two tuners;
- Main accelerating mode: π;
- Input coupler and probe are well coupled to the $\pi/2$ mode, typically 2.8 MHz above the π mode.

PETRA 5-cell Cavity

DATA SHEET

500 MHz, 5-Cell Cavity

DESY-MHFe, Vers. 2.1 October 2007

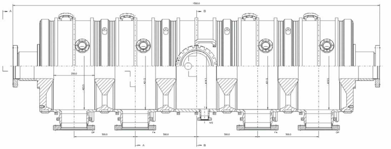
Cooling water flow for maximum field flatness

Cooling circuit	Q [l/h]
end discs 1 & 6	250
cells 1 - 5	740
discs 2 - 5	980
Sum	8000

Mode frequencies

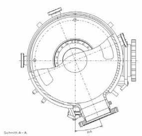
Mode	Frequency [MHz]
π	499,0
$3/4\ \pi$	500,3
$\pi/2$	501,9
$\pi/4$	503,4
0	504,8

Disc #: 1 2 3 4 5 6
Cell #: 1 2 3 4 5

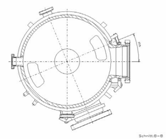


Flange neck diameters:	2x DN35, 1x DN125	2x DN35, 1x DN125	2x DN35, 1x DN144	2x DN35, 1x DN125	2x DN35, 1x DN125
Flange types:	2x DN35 CF 1x DN150 CA	2x DN35 CF 1x DN150 CA	2x DN35 CF 1x DN150 SF	2x DN35 CF 1x DN150 CA	2x DN35 CF 1x DN150 CA

Sectional drawing A-A
Disk 1 to Cell 2 (Plunger cell)



Sectional drawing B-B
Cell 3 (Input coupler cell)

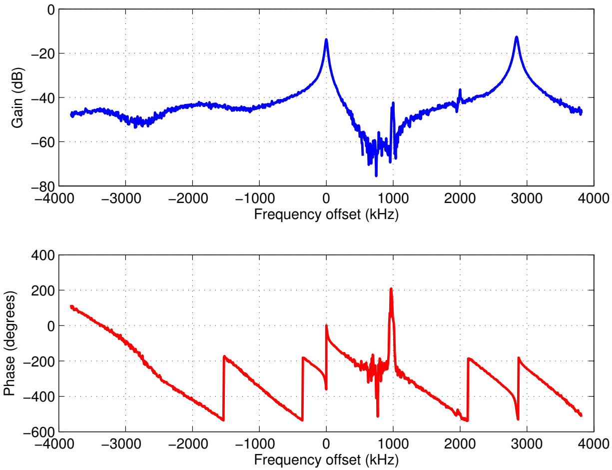


Beam tube flanges:
Flange neck diameters: DN120

Flange types: DN150 SF

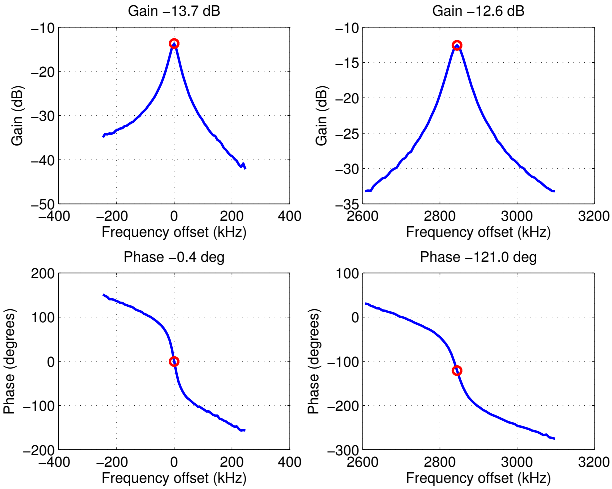
- An inexpensive cavity at 500 MHz;
- Five cells, three probes, two tuners;
- Main accelerating mode: π;
- Input coupler and probe are well coupled to the $\pi/2$ mode, typically 2.8 MHz above the π mode.

Wideband Open Loop Transfer Function



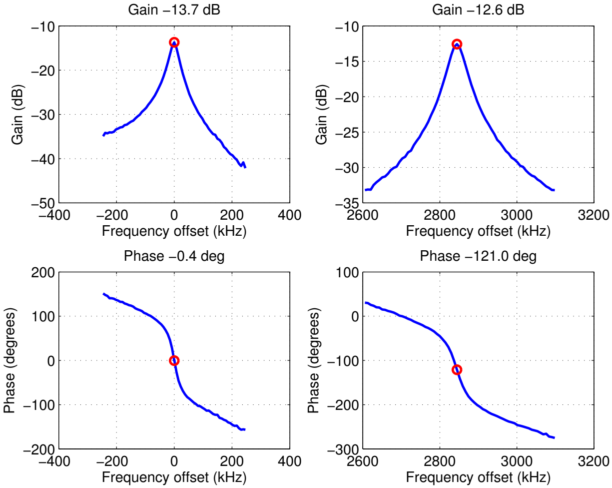
- Two modes are clearly seen: π mode at f_{RF} and $\pi/2$ roughly 3 MHz above it;
- Negative feedback for the π mode is positive for the parasitic mode;
- This positive feedback limits direct loop gain;
- The simplest way around the issue is to use digital delay to equalize the modal phase shifts (230 ns).

Wideband Open Loop Transfer Function



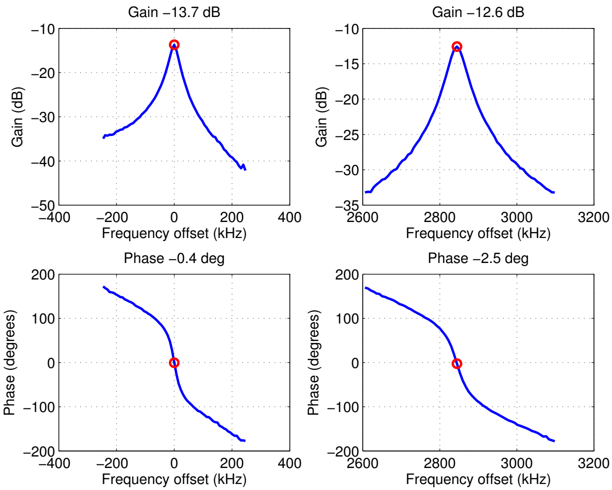
- Two modes are clearly seen: π mode at f_{RF} and $\pi/2$ roughly 3 MHz above it;
- Negative feedback for the π mode is positive for the parasitic mode;
- This positive feedback limits direct loop gain;
- The simplest way around the issue is to use digital delay to equalize the modal phase shifts (230 ns).

Wideband Open Loop Transfer Function



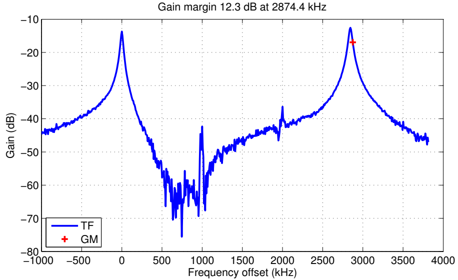
- Two modes are clearly seen: π mode at f_{RF} and $\pi/2$ roughly 3 MHz above it;
- Negative feedback for the π mode is positive for the parasitic mode;
- This positive feedback limits direct loop gain;
- The simplest way around the issue is to use digital delay to equalize the modal phase shifts (230 ns).

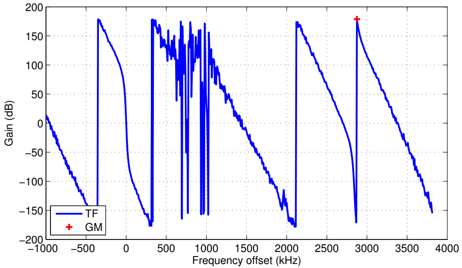
Wideband Open Loop Transfer Function



- Two modes are clearly seen: π mode at f_{RF} and $\pi/2$ roughly 3 MHz above it;
- Negative feedback for the π mode is positive for the parasitic mode;
- This positive feedback limits direct loop gain;
- The simplest way around the issue is to use digital delay to equalize the modal phase shifts (230 ns).

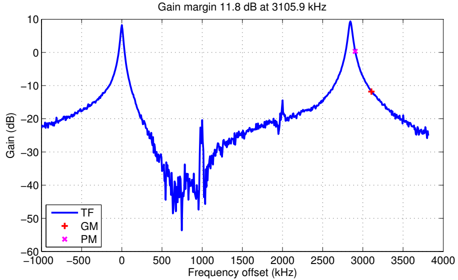
Proportional Loop Gain and Delay

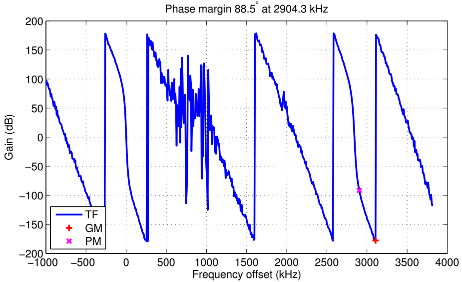




- Set up minimum delay and equalized transfer functions for identical 3 dB closed-loop peaking.
  - Minimum delay: peak gain at RF is -9 dB, gain margin 12 dB;
  - Equalized: peak gain at RF is $+8$ dB, gain margin 12 dB, phase margin 88°.
- More sophisticated parasitic mode suppression methods can improve the performance only slightly, around 2-3 dB.

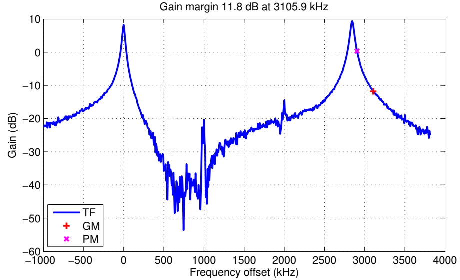
Proportional Loop Gain and Delay

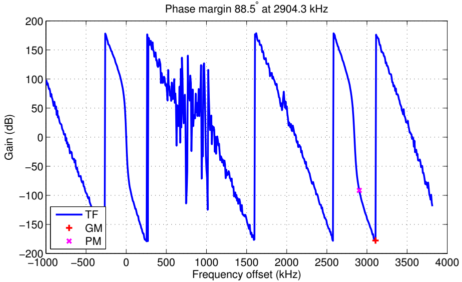




- Set up minimum delay and equalized transfer functions for identical 3 dB closed-loop peaking.
  - Minimum delay: peak gain at RF is -9 dB, gain margin 12 dB;
  - Equalized: peak gain at RF is $+8$ dB, gain margin 12 dB, phase margin 88°.
- More sophisticated parasitic mode suppression methods can improve the performance only slightly, around 2-3 dB.

Proportional Loop Gain and Delay





- Set up minimum delay and equalized transfer functions for identical 3 dB closed-loop peaking.
  - Minimum delay: peak gain at RF is -9 dB, gain margin 12 dB;
  - Equalized: peak gain at RF is $+8$ dB, gain margin 12 dB, phase margin 88°.
- More sophisticated parasitic mode suppression methods can improve the performance only slightly, around 2-3 dB.

Outline

Some Buzzwords

- State-space analysis of LTI:

$$
\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{aligned}
$$

- For single-input single-output (SISO) systems, x, $\dot{x}$, and B are $n \times 1$, state matrix A is $n \times n$, C is $1 \times n$, and D is a constant.
- Controllability, observability;
- Multi-input, multi-output (MIMO) systems;
- Robust Control;
- Optimal Control;
- Nonlinear Control.

Some Buzzwords

- State-space analysis of LTI:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

- For single-input single-output (SISO) systems, x, $\dot{x}$, and B are $n \times 1$, state matrix A is $n \times n$, C is $1 \times n$, and D is a constant.
- Controllability, observability;
- Multi-input, multi-output (MIMO) systems;
- Robust Control;
- Optimal Control;
- Nonlinear Control.

Some Buzzwords

- State-space analysis of LTI:

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$$

- For single-input single-output (SISO) systems, x, $\dot{x}$, and B are $n \times 1$, state matrix A is $n \times n$, C is $1 \times n$, and D is a constant.
- Controllability, observability;
- Multi-input, multi-output (MIMO) systems;
- Robust Control;
- Optimal Control;
- Nonlinear Control.

Some Buzzwords

- State-space analysis of LTI:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

- For single-input single-output (SISO) systems, x, $\dot{x}$, and B are $n \times 1$, state matrix A is $n \times n$, C is $1 \times n$, and D is a constant.
- Controllability, observability;
- Multi-input, multi-output (MIMO) systems;
- Robust Control;
- Optimal Control;
- Nonlinear Control.

Some Buzzwords

- State-space analysis of LTI:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

- For single-input single-output (SISO) systems, x, $\dot{x}$, and B are $n \times 1$, state matrix A is $n \times n$, C is $1 \times n$, and D is a constant.
- Controllability, observability;
- Multi-input, multi-output (MIMO) systems;
- Robust Control;
- Optimal Control;
- Nonlinear Control.

Some Buzzwords

- State-space analysis of LTI:

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$$

- For single-input single-output (SISO) systems, x, $\dot{x}$, and B are $n \times 1$, state matrix A is $n \times n$, C is $1 \times n$, and D is a constant.
- Controllability, observability;
- Multi-input, multi-output (MIMO) systems;
- Robust Control;
- Optimal Control;
- Nonlinear Control.

Outline

Summary

- Feedback control relies heavily on the LTI analysis tools;
- Mathematical descriptions are great, but it is good to have some intuitive understanding of system behavior;
- Transfer functions and corresponding impulse responses help (in my view) to gain such intuition;
- Once you master linear, time-invariant systems, you can start thinking about controlling real-world systems, often non-linear and time-varying.

Summary

- Feedback control relies heavily on the LTI analysis tools;
- Mathematical descriptions are great, but it is good to have some intuitive understanding of system behavior;
- Transfer functions and corresponding impulse responses help (in my view) to gain such intuition;
- Once you master linear, time-invariant systems, you can start thinking about controlling real-world systems, often non-linear and time-varying.

Summary

- Feedback control relies heavily on the LTI analysis tools;
- Mathematical descriptions are great, but it is good to have some intuitive understanding of system behavior;
- Transfer functions and corresponding impulse responses help (in my view) to gain such intuition;
- Once you master linear, time-invariant systems, you can start thinking about controlling real-world systems, often non-linear and time-varying.

Summary

- Feedback control relies heavily on the LTI analysis tools;
- Mathematical descriptions are great, but it is good to have some intuitive understanding of system behavior;
- Transfer functions and corresponding impulse responses help (in my view) to gain such intuition;
- Once you master linear, time-invariant systems, you can start thinking about controlling real-world systems, often non-linear and time-varying.

References

- Gene F. Franklin, J. David Powell, and Abbas Emami-Naeini. *Feedback Control of Dynamic Systems*. Reading, Massachusetts: Addison-Wesley, 1991.
- Thomas Kailath. *Linear Systems*. Englewood Cliffs, New Jersey: Prentice Hall, 1980.
- John Doyle, Bruce Francis, and Allen Tannenbaum. *Feedback Control Theory*. Macmillan Publishing Co., 1992. URL: https://www.control.utoronto.ca/people/profs/francis/dft.html.