LLRF Theory

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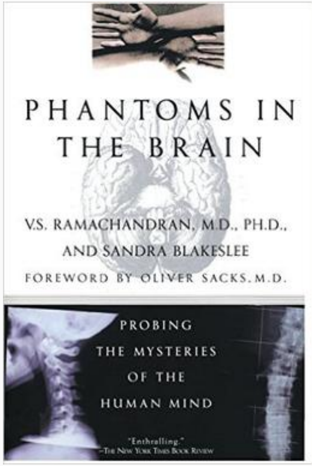
Meta

- Mental models described by by V.S. Ramachandran in his 1998 book **Phantoms in the Brain**

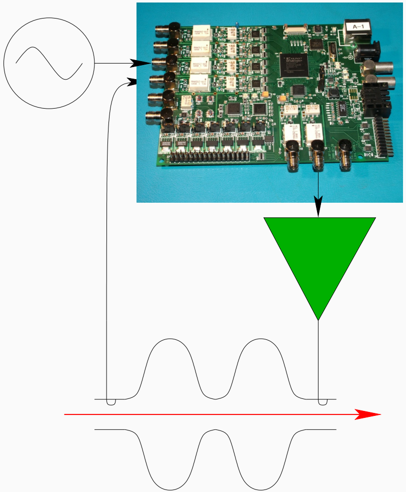


- Abstract theory is not enough:
  try to get (at least some of) this material deep into your mammalian brain
  - Yes, you should really be able to eat Laplace and z transforms for lunch
  - Yes, recite fundamental performance limits based on noise and delay
  - Yes, DSP, CORDICs, CIC filters, HDL, clock domains, etc.
  - Yes, field control loops, frequency control loops, calibration,
    and interactions with other accelerator control loops
    including global controls (software)

- want you to be receptive to and participate in efforts to apply LLRF to new accelerators, and help push the envelope to higher performance and/or lower (total) cost. Intrinsically a team sport, multidisciplinary problems and solutions, too big for a single individual

LLRF is short for Low-Level Radio Frequency - to distinguish it from the engineering of the high-powered RF drives, e.g., Klystron or SSA.

LLRF is an op-amp plus a digital storage 'scope. Well, a vector equivalent of an op-amp, and the LLRF also needs to compute cavity detuning.



Methods

Newton invented calculus for a reason!

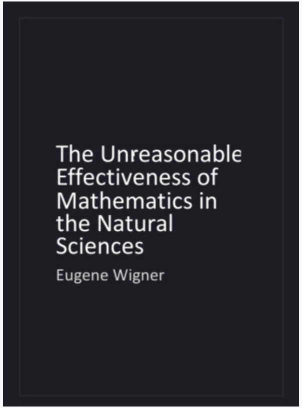
Differential equations are key to understanding anything dynamic.

State space formalism:

$$\frac{dx}{dt} = f(x) + \mathrm{excitation}$$

where x can be a real number, a complex number, or a vector.

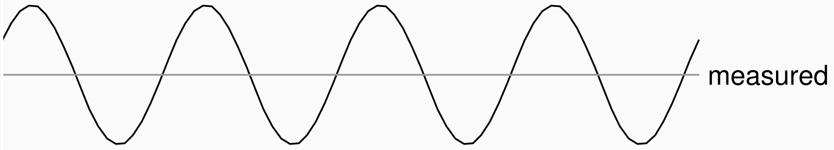
See **The Unreasonable Effectiveness of Mathematics**,
1960 essay by Eugene Wigner



Outline

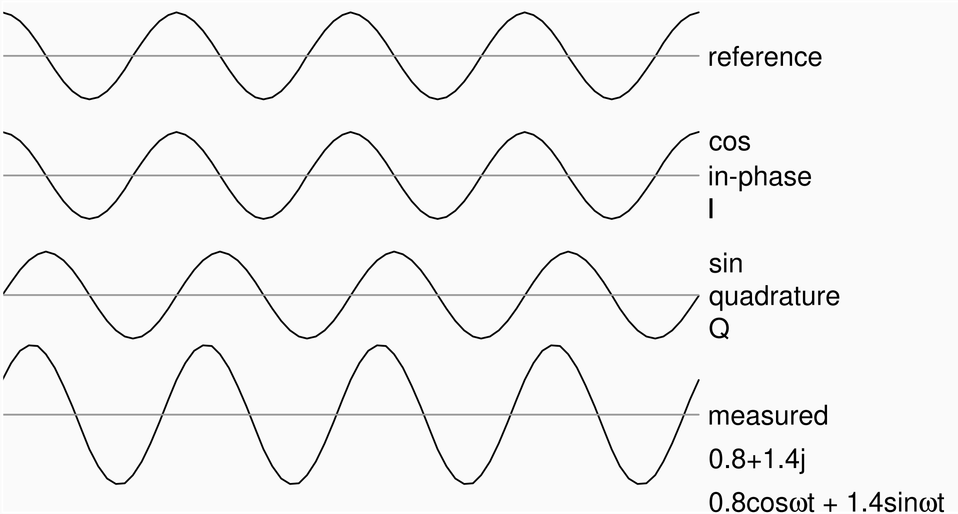
1. Definitions
2. Starting gate math
3. S-Parameters
4. Resonances
5. Cavity physics

RF Measurement



An RF waveform without a reference has no defined phase.

RF Measurement



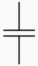
In the presence of a reference, represent an RF waveform as a complex number.

Starting gate math

Watch as I summarize results from a semester of college applied math into two slides.

Solving lumped-element Linear Time Invariant (LTI) Differential Equations

$$V = IR \qquad \text{Ohm's law}$$



$$I = C\frac{dV}{dt}$$

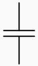
$$V = L\frac{dI}{dt}$$

Starting gate math

Watch as I summarize results from a semester of college applied math into two slides.

Solving lumped-element Linear Time Invariant (LTI) Differential Equations

$$V = IR \qquad Z = \frac{V}{I} = R \qquad Z \text{ is impedance}$$



$$I = C\frac{dV}{dt} \qquad Z = \frac{V}{I} = \frac{1}{sC} \qquad V = \frac{1}{C}\int I dt$$

$$V = L\frac{dI}{dt} \qquad Z = \frac{V}{I} = sL$$

Using s as the formal Laplace derivative operator.

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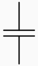

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Using s as the formal Laplace derivative operator. If $f(t) = ae^{j\omega t}$,

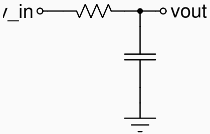
$$s \cdot f(t) = \frac{d}{dt}ae^{j\omega t} = j\omega ae^{j\omega t} = j\omega \cdot f(t)$$

showing “equivalence” between s and $j\omega$.

- Related to but more general than Phasor analysis.

Starting gate math continued

All (lumped element) transfer functions resolve to ratios of polynomials in s, that capture all of the dynamics. e.g., for a low-pass filter:



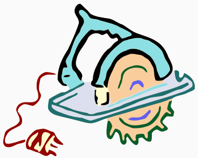
$$\frac{V_{\text{out}}}{V_{\text{in}}} = A(s) = \frac{1}{1+s\tau}, \quad \tau = RC$$

Can abstract further and just talk about poles and zeros in the complex s plane.

No particular limit on the order of those polynomials, or number of poles or zeros.

LTI assumptions give provable connection between time-domain and frequency-domain.

Core math dates to the 19th century, thanks to Laplace, Heaviside, and others.



LTI limitations

We really need the mathematical power of Laplace analysis to make progress understanding these complicated dynamics, which in turn requires them to be Linear Time Invariant (LTI).

And literally, nothing macroscopic in this world is perfectly linear or perfectly time-invariant. But as the saying goes, "Everything is linear to a first approximation."

Electrical engineers are used to analyzing a circuit "around its operating point."

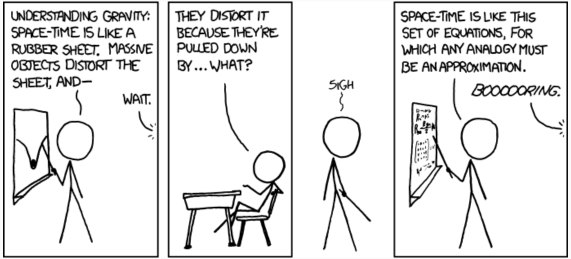


Image credit: xkcd

Also see **The Relativity of Wrong**, 1989 essay by Isaac Asimov

> *When people thought the earth was flat, they were wrong. When people thought the earth was spherical, they were wrong. But if you think that thinking the earth is spherical is just as wrong as thinking the earth is flat, then your view is wronger than both of them put together.*

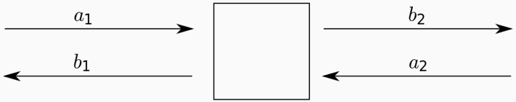
S-Parameters

In concept impedance, gain, and related circuit parameters can be determined based on V and I measurements, but that's not practical for RF. We need something that works with coaxial cables.

S-parameters must use defined cable impedance, written Z_0.

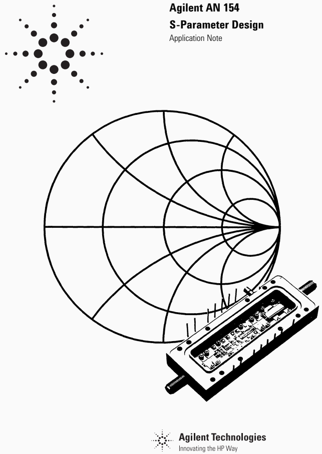
Attach a cable with characteristic impedance Z_0 to a circuit, launch a wave towards the circuit, and measure the response.

Most common example: 2-port device



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Notice how independent this equation is from units! The S-matrix itself is unitless. a and b can be in any (consistent) amplitude-based unit: amp, volt, or even $\sqrt{\mathrm{watt}}$.



Hewlett-Packard (Agilent) AN-154, 1972

S-Parameters

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Very concise mathematically. In words:

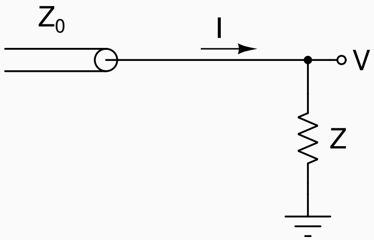
- S_{11} is the reflection from port 1
- S_{22} is the reflection from port 2
- S_{21} is how much output you get on port 2, excitation on port 1.
- S_{12} is how much output you get on port 1, excitation on port 2.

For amplifiers in particular, people use port 1 as the input, and port 2 as the output. That means S_{21} is gain, and S_{12} is reverse isolation.

This is RF, so every S parameter is assumed to be a complex number.

S-Parameters

Single-port reflection coefficient Γ is a special-case of S_{11}



$\Gamma = 1$ open circuit

$\Gamma = 0$ matched load, no reflection

$\Gamma = -1$ short circuit

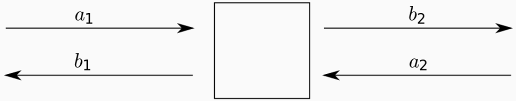
Generally

$\Gamma = \frac{Z - Z_0}{Z + Z_0}$

(you should be able to derive that)

S-Parameters

Chain multiple 2-port blocks together analytically using T matrices. Similar to S matrices, but with rearranged cause and effect. For a single 2-port block



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad \begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

Convert S to T:

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \frac{1}{S_{21}} \begin{pmatrix} -det(S) & S_{11} \\ -S_{22} & 1 \end{pmatrix}$$

Multiply T for a chain of blocks:

$$T_{\mathrm{tot}} = T_C \cdot T_B \ldots \cdot T_A$$

Convert T to S:

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \frac{1}{T_{22}} \begin{pmatrix} T_{12} & det(T) \\ 1 & -T_{21} \end{pmatrix}$$

All analytic; software does this easily. Various “toolboxes” for e.g., Python or Matlab, know how to do this.

S-Parameters

Calibration!

Modern computerized Vector Network Analyzers (VNAs) do the hard work for you.

Easy to understand how it calibrates S_{11}, given measurements under short, and open, and terminated conditions.

- with termination, measures the instrument's internal crosstalk, to be subtracted from all future measurements
- with open-circuit, determines phase shift and amplitude defined to be 1, sets a multiplicative factor
- with short, cross-checks that above process yields -1

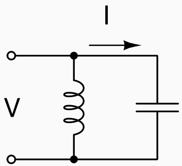
Calibrating S_{21} usually involves a through-device (of known electrical length). For precision work, measurements are made only on "insertable devices" and the through calibration standard disappears - there's simply a male-female cable connection.

Manufacturers refine all these steps for practical application with available parts.

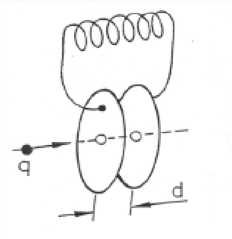
Cavities Outline

- Simple LC circuit
- 1-D transmission line
- Maxwell's equations
- TM and TE modes in cylindrical geometries
- Coupling and losses
- Perturbations

Zero-D lumped LC network



$$C\frac{dV}{dt} = I$$
$$L\frac{dI}{dt} = -V$$
$$\frac{d^2V}{dt^2} = -\frac{V}{LC}$$



Corresponding eigen-equation, label $-\omega^2$ as the eigenvalue

$$\frac{d^2V}{dt^2} = -\omega^2 V = -\frac{V}{LC}$$
$$\omega = 1/\sqrt{LC}$$

Can write explicit final solution in a few ways,

$$V(t) = A\sin\omega t + B\cos\omega t, \quad \text{or} \quad V(t) = e^{\pm j\omega t}$$

Like everything else here, this is a LTI (linear time-independent) differential equation.

Not conceptually so far from a cavity, all you need to do is picture a two-plate capacitor with a hole in it for the beam to go through.

1-D Helmholtz equation

The wave equation is

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

$\partial^2/\partial x^2$ is the (spatial) Laplace operator, which can also be written ∇^2.

With boundary conditions $u = 0$ at $x = 0$ and $x = D$, it equally well represents acoustic vibrations of a violin string or a doubly-shorted electrical transmission line. No extra credit for guessing which one was on d'Alembert's mind when he solved it in 1746.

"The eigenvalue problem for the Laplace operator is called the Helmholtz equation." We get there by assuming the same time dependence as before, frequency ω

$$\nabla^2 u = -\frac{\omega^2}{c^2} u$$

Solutions take the form $\sin(kx)$ where $kD = n\pi$ to satisfy the boundary condition, and therefore

$$k^2 = \frac{\omega^2}{c^2}$$

The standing wave frequencies are $\omega = n\pi c/D$, and linear superposition can be used to build up any solution to the wave equation from there.

Maxwell's equations

$\nabla \cdot \mathbf{D} = \rho$	$\mathbf{D}$ = electric flux density	coulomb/m^2
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\mathbf{E}$ = electric field	volt/m
$\nabla \cdot \mathbf{B} = 0$	$\mathbf{B}$ = magnetic flux density	weber/m^2
$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\mathbf{H}$ = magnetic field	amp/m
	ρ = charge density	coulomb/m^3
$\nabla \cdot \mathbf{J} = -\dfrac{\partial \rho}{\partial t}$	$\mathbf{J}$ = current density	amp/m^2

Nominal isotropic macroscopic relations

		vacuum value	
$\mathbf{B} = \mu \mathbf{H}$	μ = permeability	$4\pi \times 10^{-7}$	henry/m (no longer exact, unfortunately)
$\mathbf{D} = \epsilon \mathbf{E}$	ϵ = permittivity	8.854×10^{-12}	farad/m
$\mathbf{J} = \sigma \mathbf{E}$	σ = conductivity	0	ohm^{-1}/m
$c = 1/\sqrt{\mu\epsilon}$	c = speed of light	299792458	m/s

coulomb = amp $\cdot$ s   farad = coulomb/volt   ohm = volt/amp

weber = volt $\cdot$ s   henry = weber/amp

These equations are equivalent to those published in 1865 by James Clerk Maxwell, but in SI units; this modern vector notation by Oliver Heaviside dates to 1884.

Helmholtz equation

$$\nabla^2 \psi = -k^2 \psi$$

ψ is a scalar, $-k^2$ is the eigenvalue

1D version is easy; solutions were well understood by d'Alembert in 1746.

2-D version 'drumhead' well-studied in 19th century:

- rectangular coordinates by Poisson in 1829
- circular coordinates by Clebsch in 1862
- elliptical coordinates by Mathieu in 1890?

Clearly predates any interest in RF resonator cavities.

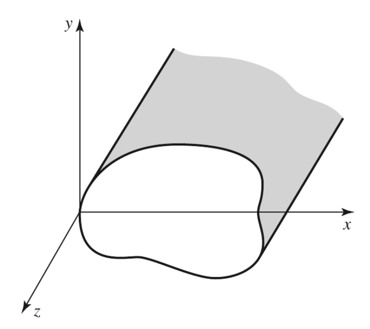
3D version now known to be separable in 11 orthogonal coordinate systems.

General conversion of Maxwell's equations (3D, **E** and **H**) to Helmholtz (3D, scalar) is too much to ask.

Productive and understandable approach covers "cylindrical" (not necessarily round) geometries, 4 of the 11 possible coordinate systems. Builds on understanding of the 2-D Helmholtz equation, and leads to TE and TM solutions, useful to describe both waveguides and cavities.

Factor out one Cartesian coordinate

Initial Cartesian coordinate separation, separating dependence of solutions on z from that on x and y, thus handling "cylindrical" (not necessarily round) geometries.



The core assumption is

$$\mathbf{E}(x, y, z, t) = \mathbf{E}(x, y) \cdot e^{\pm jkz - j\omega t}$$

$$\mathbf{H}(x, y, z, t) = \mathbf{H}(x, y) \cdot e^{\pm jkz - j\omega t}$$

where k is a yet-unknown constant.

Perfectly-conducting boundary locations are independent of z, $\mathbf{E}_{\parallel} = 0$ and $\mathbf{B}_{\perp} = 0$

Factor out one Cartesian coordinate

Solutions fall in one of two categories, both of which make use of a function $\psi(x, y)$ which solves a 2-D Helmholtz equation:

$$\begin{array}{cc} \text{TM} & \text{TE} \\ \mathbf{B}_z = 0 & \mathbf{E}_z = 0 \end{array}$$

$$\nabla_t^2 \psi = -\gamma^2 \psi$$

$$\gamma^2 = \mu\varepsilon \frac{\omega^2}{c^2} - k^2$$

$$\begin{array}{cc} \psi = 0 \text{ on wall} & \psi' = 0 \text{ on wall} \\ \text{(Dirichlet boundary)} & \text{(Neumann boundary)} \\ \mathbf{E}_z = \psi e^{\pm jkt} & \mathbf{B}_z = \psi e^{\pm jkt} \\ \mathbf{E}_t = \pm(ik/\gamma^2)\nabla_t \psi & \mathbf{H}_t = \pm(ik/\gamma^2)\nabla_t \psi \\ \mathbf{H}_t = (\pm 1/Z)\mathbf{e}_z \times \mathbf{E}_t & \mathbf{E}_t = \mp Z\mathbf{e}_z \times \mathbf{B}_t \\ Z = (ck/\varepsilon\omega) & Z = (\mu\omega/ck) \end{array}$$

where the t subscript refers to transverse (e.g., x and y)

Fully general to any 2-D cross-sectional geometry. This version is based on Jackson. Rectangular, circular, and even elliptical shapes can be treated analytically. Others need numerical solutions; that's not normally considered a challenge for computers in this decade.

S-Parameters again

S-parameters can generalize far beyond cables to work with *any* cylindrical propagating (or even evanescent) mode. And remember that by "cylindrical" we mean an arbitrary 2-D shape extruded in a separable coordinate system.

An abstract $50\,\Omega$ cable and the reference plane of an SMA connector are special cases of a TEM propagation mode.

Also, multiple modes on a waveguide superimpose, and are treated as independent ports.

Circular cylinder

"Pillbox" with radius R and length L

Longer treatment at https://uspas.fnal.gov/materials/11ODU/Proton_4.pdf

Quantum numbers n, m, and l are connected with coordinates ϕ, ρ, and z. Parameter k from before is constrained to be $\pi N/L$, so z dependence is $\cos kz$. Helmholtz solution ψ is represented as

$$(A \sin n\phi + B \cos n\phi) \cdot J_n(k_C \rho)$$

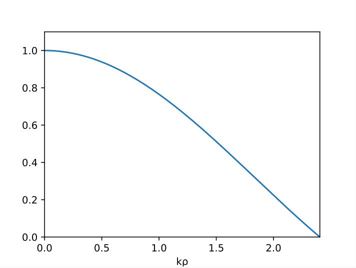
Model accelerating cavity is a pillbox operating in TM_{010} mode, $n = 0$, $m = 1$, $l = 0$. This means $\mathbf{B}_z = 0$, and $\mathbf{E}_z$ has no dependence on ϕ or z, only on ρ according to

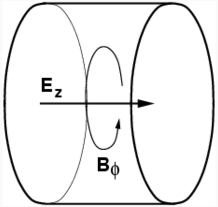
$$\mathbf{E}_z \propto J_0(k\rho)$$

and k takes on the smallest value such that $J_0(kR) = 0$.

Just add beampipes and a power coupler!

See more charts and modes in Pozar.





Orthogonal coordinates in 3D

"Helmholtz equation is separable in 11 orthogonal coordinate systems," but you should probably stick to Cartesian and circular cylindrical.

Elliptic cylindrical is also not so hard, and covers elliptical waveguide (not so common).

Spherical coordinates are harder but doable, lead to Schumann resonances. Follow a good reference.

Turn to software (FEM or FDTD) to (approximately) find eigenmodes of non-textbook geometries

- Urmel, Superfish, Microwave Studio, HFSS, ANSYS, MAFIA, ACE-3D, MEEP, OpenEMS, ...

And then we have

- "Analytical study of higher order modes of elliptical cavities using oblate spheroidal eigenvalue solution," Vikas Kumar Jain et al., April 2011, Review of Modern Physics, doi:10.1103/PhysRevSTAB.14.042002

very cool 12-page paper that's *very* heavy on the applied math and numerical techniques, but gives unique context to FEM results.

So far this discussion has covered the pure stored energy, no losses or coupling. From a mathematical perspective, the equations are singular at the resonant frequency ω_0.

Now add consideration of wall losses, so any energy in these fields will decay over time. This introduces a negative real component $-\omega_b$ to the pole location; ω_b is the half-bandwidth.

$$s_p = j\omega_0 - \omega_b$$

Accounting for the conjugate pole present in this physical system, the transfer functions constructed from here take the form

$$A = \frac{A_0 s}{(s - s_p)(s - s_p^*)}$$

The denominator perfectly matches the usual electrical engineering bandpass form

$$A = A_1 \frac{\frac{1}{Q}\frac{s}{\omega_1}}{1 + \frac{1}{Q}\frac{s}{\omega_1} + \left(\frac{s}{\omega_1}\right)^2}$$

if $\omega_1 = |s_p|$ and $\omega_b = \omega_0/2Q$. When working with high-Q cavities, we usually ignore the distinction between ω_0 and ω_1.

Circles!

For high-Q cavities and measurement frequencies ω near ω_0, we can ignore the effect of the conjugate pole and

$$A(\omega) \approx \frac{A_1}{1 + j\frac{\omega-\omega_0}{\omega_b}}$$

Now we have a universal curve of a complex number as function of a normalized (by the bandwidth) real-valued frequency offset χ:

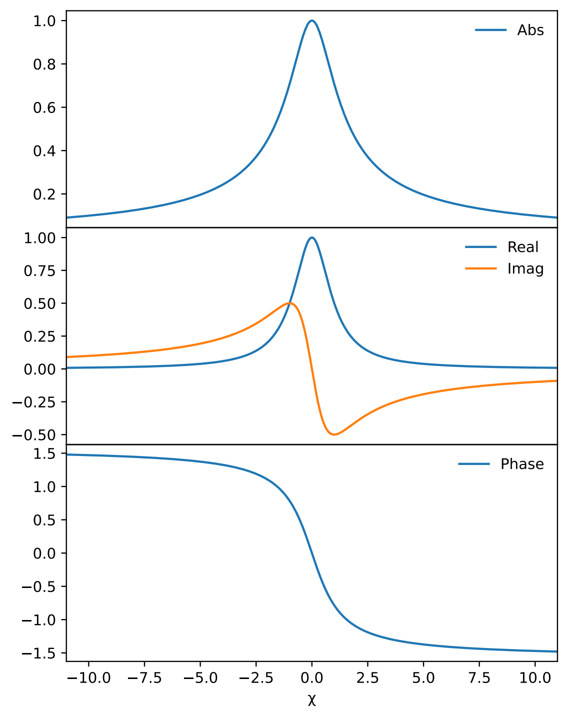
$$A(\chi) = \frac{1}{1 + j\chi}$$

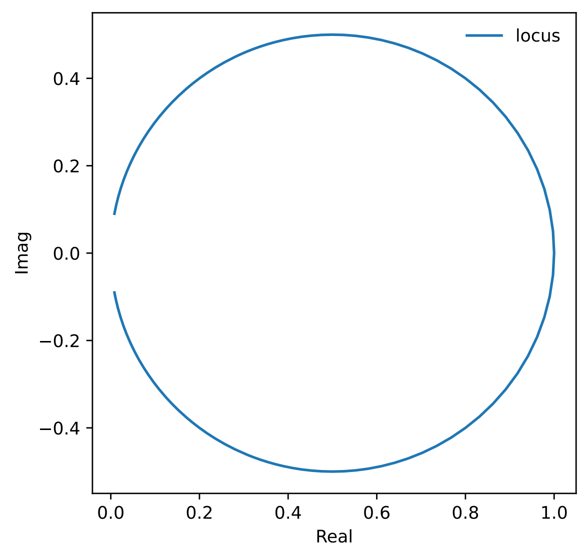
Usual default network analyzer settings display magnitudes, but the expression

$$|A(\chi)|^2 = \frac{1}{1 + \chi^2}$$

is maybe one of the least interesting derived quantities. See the plots on the next page.

Circles!





Ports

Coupling ports emit power, just as walls absorb it. Since absorbed or emitted power is related to stored energy according to

$$P = \frac{U\omega_0}{Q}$$

the addition relation for ports (and walls) with multiple Qs tied to the same eigenmode is

$$\frac{1}{Q_L} = \sum_n \frac{1}{Q_n}$$

Definition of coupling coefficient (Pozar's notation) is

$$\frac{Q_0}{Q_{\mathrm{EXT}}} = g$$

Despite the potential confusion with particle velocity, accelerator people traditionally call this quantity β. A setup with $g = 1$ means matched, no reflection in equilibrium, all the power going to the wall.

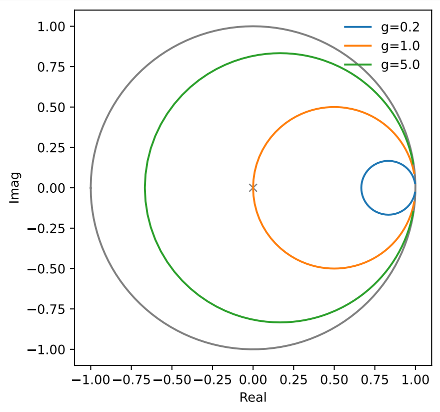
SRF cavities often have g values that are effectively infinite. I therefore look with suspicion at formulae involving g that make me think too hard about the limiting behavior for $g \to \infty$.

Reflections

When measuring S_{11}, a perfectly valid cavity out-coupling signal makes it to the network analyzer. Superimposed on that is a reflection from the coupler itself,
$S_{11} = 1$ for an E-field coupler and $S_{11} = -1$ for a B-field (loop) coupler.

With careful calibration you can see measurements that match

$$S_{11}(\chi) = \pm\left(1 - \frac{2g}{1+g}A(\chi)\right)$$



This S_{11} a.k.a. Γ locus is a Smith chart, unlike the S_{21} locus from before that is simply polar. The axes are the same, but the fancy labels of a Smith chart only apply to Γ.

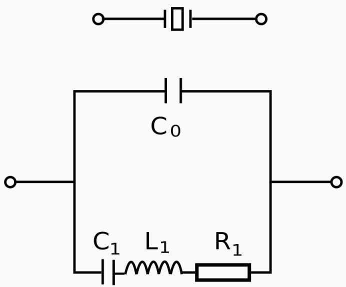
This is the topic of one lab. Calibration sometimes needs to be enhanced with *ad-hoc* adjustment of electrical delay (best done on a wide-band setup), or with curve-fitting.

Short detour for crystals

Quartz crystals have high-Q mechanical modes, that approximately correspond to the high-Q electromagnetic modes of a metal cavity.

Due to off-diagonal terms in quartz's Onsager matrix (specifically piezoelectricity) these modes can couple to the electrical world.

Yes, you can "model" the resulting impedance as an L-C-R circuit, and the parameters of the model are useful for plugging in to software that only understands electrical circuits (instead of a full-physics simulation). Just don't try to make physical sense of the *values* of the resulting parameters. IMHO it's the differential equations that are fundamental, and that are tied to both electrical and mechanical physics.



Perturbation of Boundaries

Interesting for multiple reasons: tuner design, bead-pull measurements, and understanding Lorentz detuning of SRF cavities.

Sometimes called Slater's theorem or Slater's integral, published in 1946 (Delayen claims the idea is much older than that).

$$\frac{\omega - \omega_0}{\omega_0} \approx \frac{\int_{\Delta V} \left[\mu H^2(\vec{r}) - \varepsilon E^2(\vec{r})\right] dv}{\int_V \left[\mu H^2(\vec{r}) + \varepsilon E^2(\vec{r})\right] dv}$$

Signs are important!
An inward deflection of the cavity wall at a point of high

- magnetic field will increase the resonant frequency
- electric field will decrease the resonant frequency

Holds true anywhere, not just on walls; a metallic bead will shift the frequency up when it's at a point of high magnetic field, or down at a point of high electric field.

Bead-pull measurements are a big part of the USPAS Microwave Measurements course.

Units

$H \cdot B = \mu H^2$ and $E \cdot D = \varepsilon E^2$ terms found in those integrals have units of energy density:

$$\mathrm{henry/m} \cdot (\mathrm{amp/m})^2 = \mathrm{volt} \cdot \mathrm{amp} \cdot \mathrm{s/m}^3 = \mathrm{J/m}^3$$

$$\mathrm{farad/m} \cdot (\mathrm{volt/m})^2 = \mathrm{volt} \cdot \mathrm{amp} \cdot \mathrm{s/m}^3 = \mathrm{J/m}^3$$

Factors of 2 abound! Local, instantaneous energy density terms are really $\frac{1}{2}\mu H^2$ and $\frac{1}{2}\varepsilon E^2$. Integrating over space and averaging over time gives the Slater denominator

$$U = \frac{1}{4}\int_V \left[\mu_0 H^2(\vec{r}) + \varepsilon_0 E^2(\vec{r})\right] dv$$

The coefficient $\frac{1}{4}$ isn't normally written in Slater's equation because two of them (one each in the numerator and denominator) cancel.

Energy is conserved: it sloshes back and forth between electric and magnetic forms twice per cycle, so it's also valid to write

$$U = \int_V \tfrac{1}{2}\mu_0 H^2(\vec{r})\, dv = \int_V \tfrac{1}{2}\varepsilon_0 E^2(\vec{r})\, dv$$

Slater's works are written in terms of $(\omega^2 - \omega_0^2)/\omega_0^2$, not $(\omega - \omega_0)/\omega_0$, that also introduces a factor of 2.

Coupling to beam

RF measurements can only tell you Watts of drive and Joules of cavity stored energy (or take the square root of those for amplitudes).

A well-calibrated bead pull measurement and/or electromagnetic solver software are needed to find $E(z)/\sqrt{U}$ for that mode, along the beam path.

For a beam bunch traveling at velocity βc, the total energy imparted by the cavity is

$$V(\beta) = \int E(z) \exp(j\omega z/\beta c)\ dz$$

where the dependence on normalized beam velocity β is made explicit for the benefit of anyone working with hadron beams.

A useful distillation of the results that only depends on cavity geometry is the conversion factor

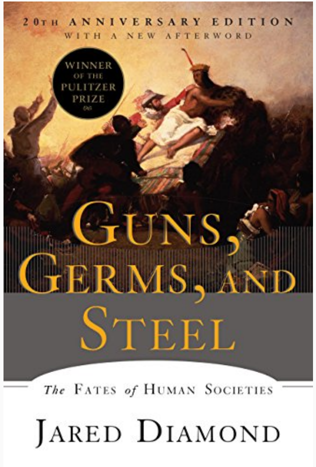
$$K_c(\beta) = \frac{V(\beta)}{\sqrt{U}} = \sqrt{\omega_0 (R/Q)}$$

where this conversion constant is also connected to the usually-published value of normalized shunt impedance (R/Q). Units are simply volt/$\sqrt{\text{joule}}$, which is the same as $\sqrt{\text{ohm/s}}$.

Closing Meta

I've learned this past couple of decades to place tremendous value on a functioning team.
I have also noticed that it takes a long time to build up such a team, and that one can evaporate almost instantaneously if funding or other institutional conditions fall short.

One central lesson of Jared Diamond's 1997 book
Guns, Germs, and Steel: The Fates of Human Societies
is that people need people: a larger community succeeds where a smaller one won't.



Thanks for your patience!

See you in the lab

I'll be here all week! ;-)