Modeling & Simulations

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Introduction

Introduction

- Understanding the cavity Physics and the implications of connecting a cavity to a high-power RF source and a feedback controller is the first step in LLRF design
- Formalisms in literature provide accurate representations of the Physics involved, and one can find a good correspondence between modeling results and real measurements
- Both analytical and numerical models are available to us today and are helpful at every stage of the design and development process[1]
- From experience: Understanding your system prior to starting the LLRF system design is imperative (not a nice to have) and this is done with models & simulations

[1] This lecture is based on experience and tools developed at LBNL, many others are available out there.

LLRF modeling flow

Analytical studies

Determine feedback equations and apply control theory to analyze stability.

Software simulations

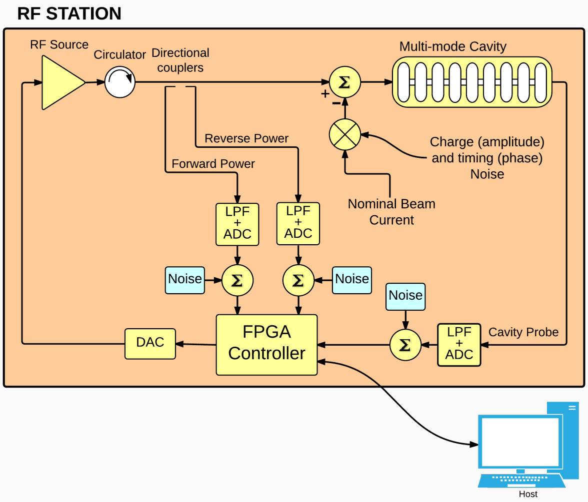
Discretize equations and run numerical simulations to analyze dynamics and behavior.

FPGA simulations

Implement state-space equations in FPGA for “live” simulations.

LLRF component models

RF station


RF STATION
RF Source
Circulator
Directional couplers
Multi-mode Cavity
Reverse Power
Forward Power
Charge (amplitude) and timing (phase) Noise
Nominal Beam Current
LPF + ADC
LPF + ADC
Noise
Noise
Noise
FPGA Controller
DAC
LPF + ADC
Cavity Probe
Host

RF Station

- The RF station is modeled at baseband, where up and down conversions in the real system are not considered, only slowly varying amplitude and phase modulations (or I and Q)
- Doing this allows for more computationally efficient numerical simulations, remember the RF is "just" the carrier, we are interested in amplitude and phase modulations in LLRF controls
- The RF station model includes all elements typically present in a LLRF system, including the cavity-beam interaction through beam-loading as well as different noise sources

Cavity field measurement

- From EM theory we know that $\vec{E}$ and $\vec{B}$ fields inside a cavity can be broken down into independent eigenmodes (independent solutions to Maxwell's equations inside a cavity or waveguide)
- Ideally, we would like to measure the EM fields from each mode present in the cavity in order to control them appropriately. However, the best we can do is measure the overall field in the cavity, designated here by $\vec{E}_{\mathrm{probe}}$. It is measured in practice using a probe antenna

$$\vec{E}_{\mathrm{probe}} = \sum_{\mu} \vec{V}_{\mu} / \sqrt{Q_{\mathrm{p}_{\mu}} (R/Q)_{\mu}}$$

where $\vec{V}_{\mu}$ is a representative measure of the energy stored in each electrical eigenmode μ, designated as mode cavity voltage, and where $Q_{\mathrm{p}_{\mu}}(R/Q)_{\mu}$ is the coupling impedance of the probe port for that mode.

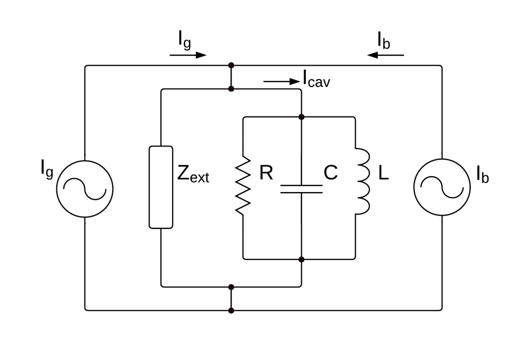
Reversed field

- Alternatively, the expression for reverse (a.k.a. reflected) wave traveling outward from the fundamental port includes a prompt reflection term, yielding

$$\vec{E}_{\mathrm{reverse}} = \sum_{\mu} \vec{V}_{\mu} / \sqrt{Q_{g_{\mu}} (R/Q)_{\mu}} - \vec{K}_{g}$$

- where $Q_{g_{\mu}}(R/Q)_{\mu}$ is the coupling impedance of the drive port of mode μ

EM eigenmode equivalent circuit



$$\vec{I}_\mu = \vec{I}_{\mathrm{C}_\mu} + \vec{I}_{\mathrm{R}_\mu} + \vec{I}_{\mathrm{L}_\mu}$$

$$\frac{d\vec{I}_{\mathrm{C}_\mu}}{dt} = C_\mu \cdot \frac{d^2\vec{V}_\mu}{dt^2}, \qquad \frac{d\vec{I}_{\mathrm{R}_\mu}}{dt} = \frac{1}{R_{\mathrm{L}_\mu}}\frac{d\vec{V}_\mu}{dt} \qquad \text{and} \qquad \frac{d\vec{I}_{\mathrm{L}_\mu}}{dt} = \vec{V}_\mu / L_\mu$$

2nd order differential equation

The full vector (complex) differential equation for the cavity accelerating voltage $\vec{V}_\mu$ can be written as:

$$\frac{d^2\vec{V}_\mu}{dt^2} + \frac{1}{R_{\mathrm{L}_\mu} C_\mu}\frac{d\vec{V}_\mu}{dt} + \frac{1}{L_\mu C_\mu}\vec{V}_\mu = \frac{1}{C_\mu}\frac{d\vec{I}_\mu}{dt}$$

which can be expressed as a function of the mode's nominal resonance frequency ω_{0_μ} ($1/L_\mu C_\mu = \omega_{0_\mu}^2$) and loaded Q ($1/R_{\mathrm{L}_\mu} C_\mu = \omega_{0_\mu}/Q_{\mathrm{L}_\mu}$):

$$\frac{d^2\vec{V}_\mu}{dt^2} + \frac{\omega_{0_\mu}}{Q_{\mathrm{L}_\mu}}\frac{d\vec{V}_\mu}{dt} + \omega_{0_\mu}^2\vec{V}_\mu = \frac{\omega_{0_\mu}^2 R_{\mathrm{L}_\mu}}{Q_{\mathrm{L}_\mu}}\frac{d\vec{I}_\mu}{dt}$$

Cavity response at baseband

Approximations:

- Slowly varying envelope: $\omega_{f_\mu} \ll \omega_{0_\mu}$
- Small detune with respect to carrier frequency: $\omega_{d_\mu} \ll \omega_{0_\mu}$
- and $Q_{\mathrm{L}_\mu} \gg 1$

$$\left(1 - j\frac{\omega_{d_\mu}}{\omega_{f_\mu}}\right)\vec{V}_\mu + \frac{1}{\omega_{f_\mu}}\frac{d\vec{V}_\mu}{dt} = R_{\mathrm{L}_\mu}\vec{I}_\mu$$

where $\omega_{f_\mu} = \omega_{0_\mu}/2Q_{L_\mu}$ is the mode's bandwidth and $\omega_{d_\mu} = 2\pi\Delta f_\mu$ is the (time varying) detune frequency, given as $\omega_{d_\mu} = \omega_{0_\mu} - \omega_{ref}$, *i.e.*, the difference between actual eigenmode frequency ω_{0_μ} and the accelerator's time base ω_{ref}.

Driving the cavity from RF and beam

Transposing the cavity drive term into a combination of the RF source incident wave and beam loading (opposite sign indicating energy absorption by the beam):

$$\left(1 - j\frac{\omega_{d_\mu}}{\omega_{f_\mu}}\right)\vec{V}_\mu + \frac{1}{\omega_{f_\mu}}\frac{d\vec{V}_\mu}{dt} = 2\vec{K}_\mathrm{g}\sqrt{R_{\mathrm{g}_\mu}} - R_{\mathrm{b}_\mu}\vec{I}_\mathrm{beam}$$

where $\vec{K}_\mathrm{g}$ is the incident wave amplitude in $\sqrt{\mathrm{Watts}}$, $R_{\mathrm{g}_\mu} = Q_{\mathrm{g}_\mu}(R/Q)_\mu$ is the coupling impedance of the drive port, $\vec{I}_\mathrm{beam}$ is the beam current, and $R_{\mathrm{b}_\mu} = Q_{\mathrm{L}_\mu}(R/Q)_\mu$ is the coupling impedance to the beam.

Interpreting the Physics

- Loaded Q

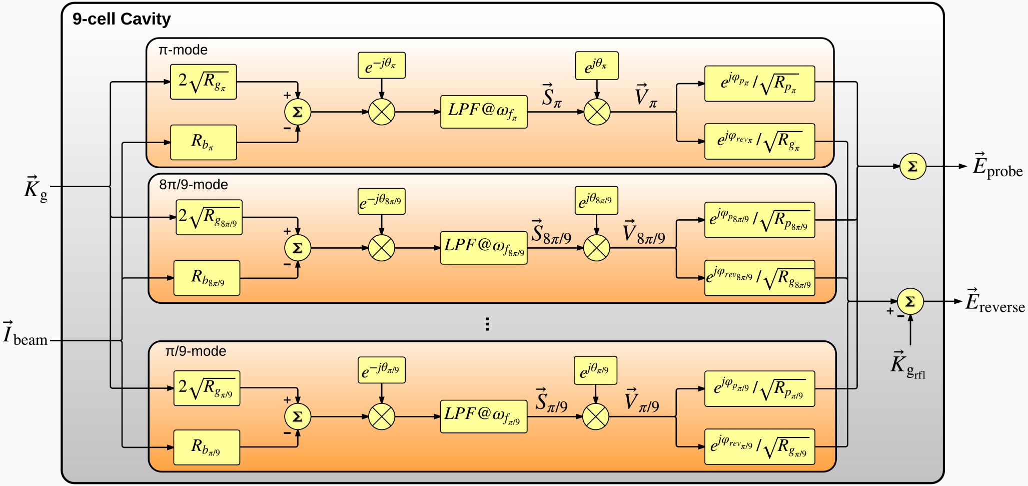
$$1/Q_{\mathrm{L}_\mu} = 1/Q_{0_\mu} + 1/Q_{g_\mu} + 1/Q_{p_\mu}$$

- $1/Q_{0_\mu}$ represents losses to the cavity walls
- $1/Q_{g_\mu}$ represents coupling to the input coupler
- $1/Q_{p_\mu}$ represents coupling to the field probe

$(R/Q)_\mu$ is the shunt impedance of the mode in Ohms, a pure geometry term. Physically, shunt impedance relates a mode's stored energy U_μ to the accelerating voltage it produces, according to

$$U_\mu = \frac{V_\mu^2}{(R/Q)_\mu \omega_{0_\mu}}$$

Computing a 9-cell cavity response



Going from Laplace to time domain

Start with the first order differential equation for a single-pole low pass filter. Its transfer function is expressed in Laplace form as

$$TF(s) = \frac{\vec{V}_{\mathrm{out}}(s)}{\vec{V}_{\mathrm{in}}(s)} = \frac{1}{s-p} \tag{1}$$

where $\vec{V}_{\mathrm{in}}$ and $\vec{V}_{\mathrm{out}}$ are the input and output signals, and p is the pole location. Make the differentiation explicit, and rearrange to get a form consistent with state-variable numerical ODE integration,

$$\frac{d\vec{V}_{\mathrm{out}}(t)}{dt} = \vec{V}_{\mathrm{in}}(t) + p \cdot \vec{V}_{\mathrm{out}}(t) \tag{2}$$

Going from continuous to discrete time domain

The simplest expression for a 'next' value at step n in discrete time is

$$\vec{V}^{n}_{\mathrm{out}} = (1 + \Delta t \cdot p)\, \vec{V}^{n-1}_{\mathrm{out}} + \Delta t \cdot V^{n}_{\mathrm{in}} \tag{3}$$

Using the Trapezoidal Formula, our rendition of the discrete time approximation to the above differential equation becomes

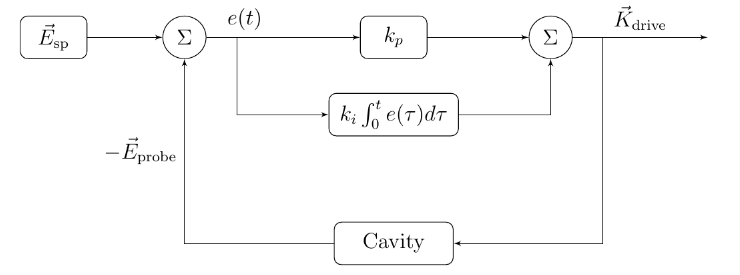
$$\vec{V}^{n}_{\mathrm{out}} = a \cdot \vec{V}^{n-1}_{\mathrm{out}} + \frac{1}{2} \cdot b \cdot (\vec{V}^{n-1}_{\mathrm{in}} + \vec{V}^{n}_{\mathrm{in}}) \tag{4}$$

$$\text{where} \quad a = \frac{1 + \frac{1}{2}\Delta t \cdot p}{1 - \frac{1}{2}\Delta t \cdot p}, \quad \text{and} \quad b = \frac{\Delta t}{1 - \frac{1}{2}\Delta t \cdot p}$$

Δt is the simulation step duration, and p is the pole location (a complex number).

Cavity controller

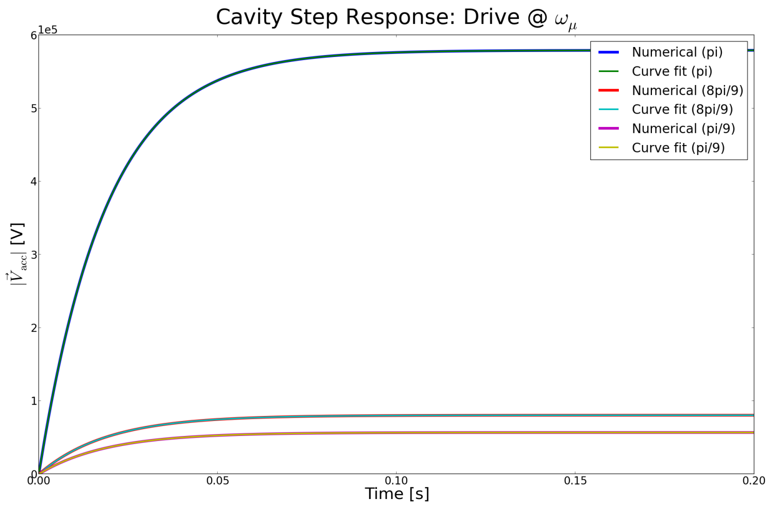
- A PI (proportional-integral) controller is close to the theoretically ideal controller for cavity field control



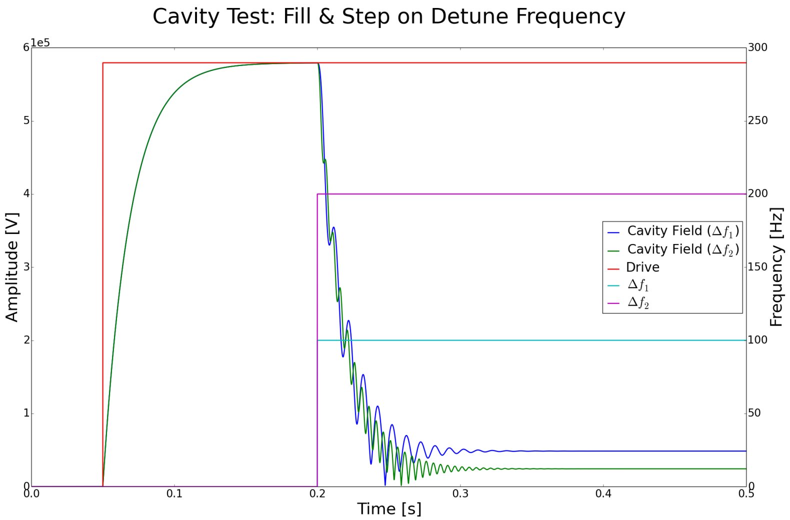
$$e(t) = \vec{E}_{\text{sp}} - \vec{E}_{\text{probe}}(t)$$

$$\vec{K}_{\text{drive}} = k_p e(t) + k_i \int_0^t e(\tau)d\tau$$

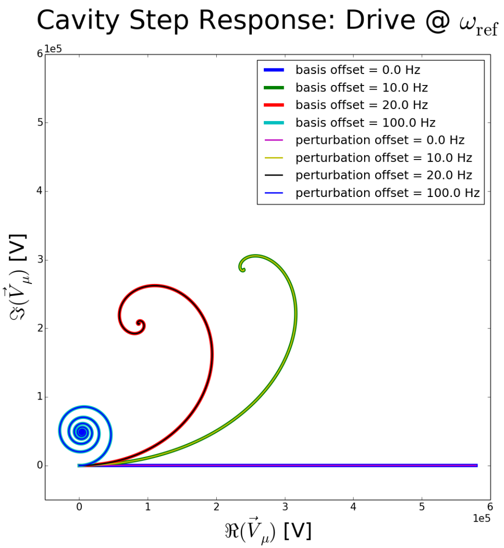
Cavity step response



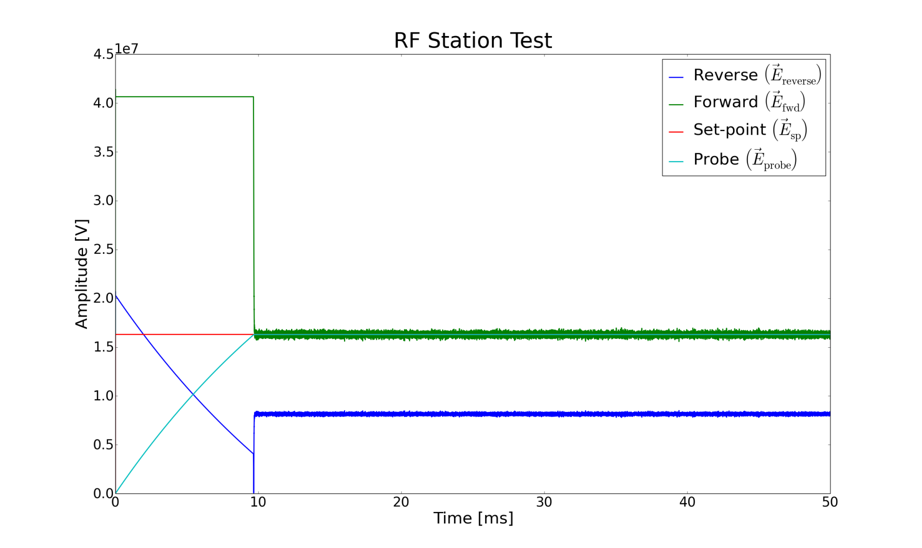
Cavity detuning



Cavity detuning (polar)

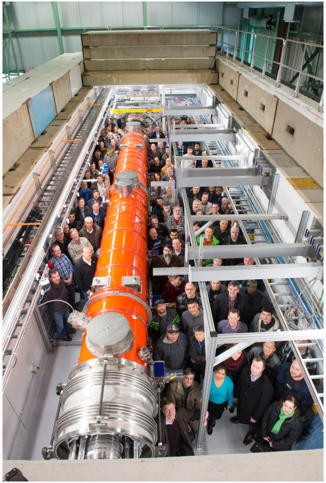


RF Station pulse

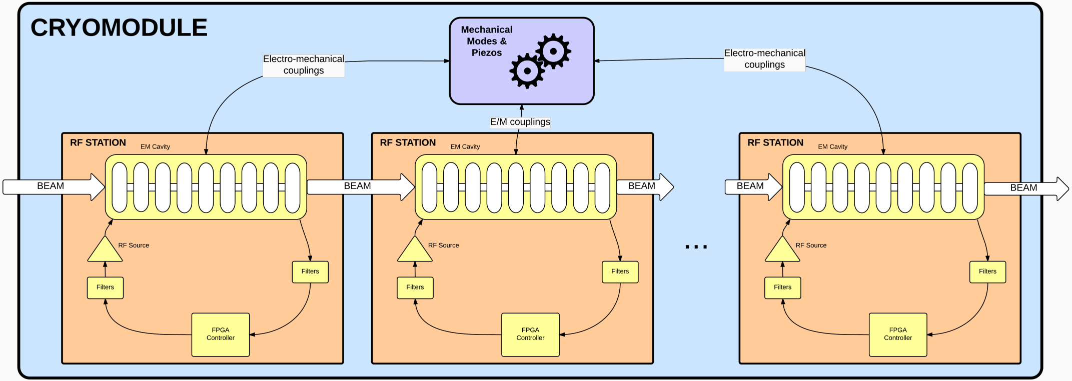


Modeling a cryomodule

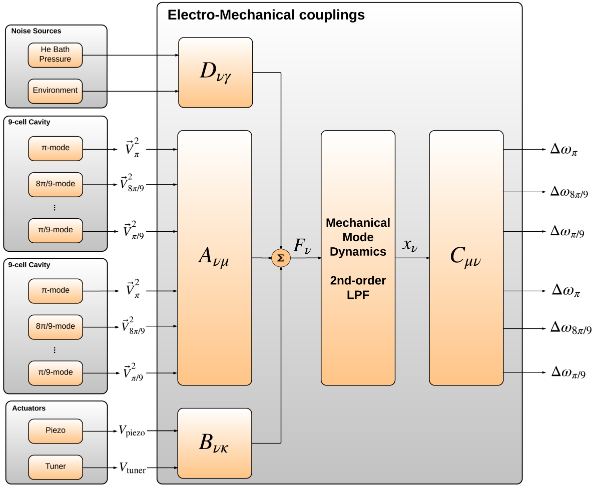
LCLS-II Cryomodule



Cryomodule Model



Electro-mechanical interactions



Mechanical eigenmodes

The differential equation governing the dynamics of each mechanical eigenmode is that of a textbook second order low-pass filter. In Laplace form:

$$k_\nu x_\nu = \frac{F_\nu}{1 + \frac{1}{Q_\nu}\frac{s}{\omega_\nu} + \left(\frac{s}{\omega_\nu}\right)^2} \quad ,$$

where k_ν is the spring constant.

Relating electrical and mechanical

Relating forces and responses of a single electrical eigenmode μ of the cavity comes from Slater's perturbation theory. For an eigenmode solution to Maxwell's equations in a closed conducting cavity (volume V), the stored energy U_μ is given by

$$U_\mu = \int_V \left[\frac{\mu_0}{4} H_\mu^2(\vec{r}) + \frac{\varepsilon_0}{4} E_\mu^2(\vec{r})\right] dv$$

Both the force on the mode and the response to a deflection x_ν are given in terms of the Slater integral

$$F_\mu = \int_S \left[\frac{\mu_0}{4} H^2(\vec{r}) - \frac{\varepsilon_0}{4} E^2(\vec{r})\right] \vec{n}(\vec{r}) \cdot \vec{\xi}(\vec{r}) dS$$

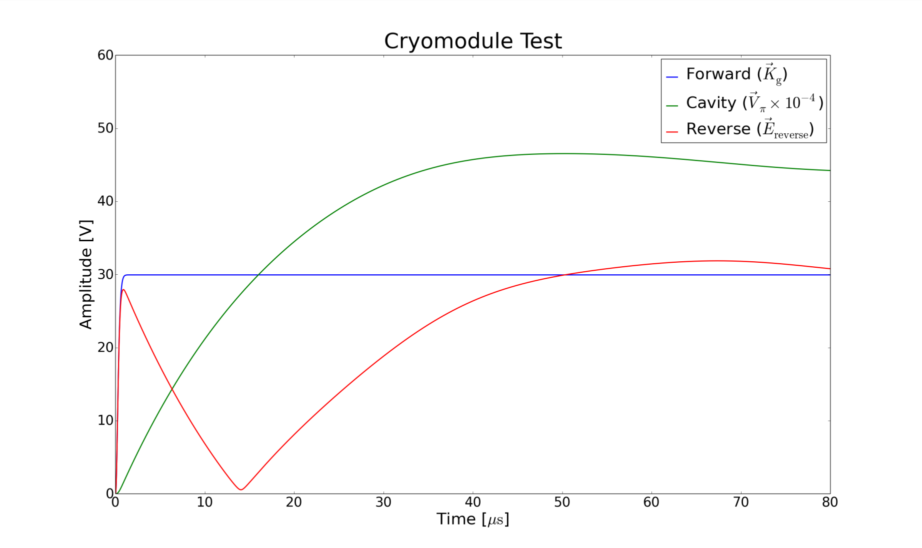
Lorentz response

Slater's analysis lets us express the static Lorentz response as

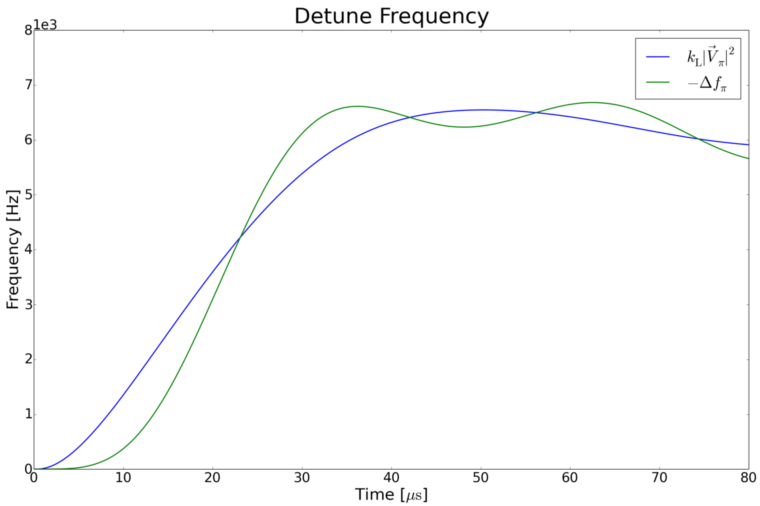
$$\left(\frac{\Delta\omega}{V^2}\right)_{\nu\mu} = \frac{C_{\mu\nu}A_{\nu\mu}}{k_\nu} = -\left(\frac{F}{U}\right)_\mu^2 \cdot \frac{1}{k_\nu (R/Q)_\mu}$$

where $(F/U)_\mu$ is a property of the electrical eigenmode, independent of amplitude, with units of m^{-1}. The expression correctly shows that this constant is always negative: the mode's static resonance frequency gets lower as it is filled. Summing over all mechanical modes ν gives the total DC response, often quoted in units of $\mathrm{Hz}/(\mathrm{MV/m})^2$.

Step with Lorentz force detuning



Lorentz force detuning

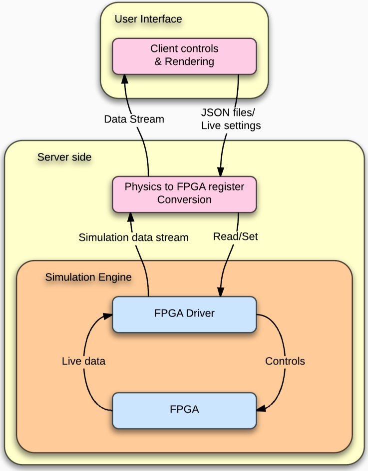


Simulation engines

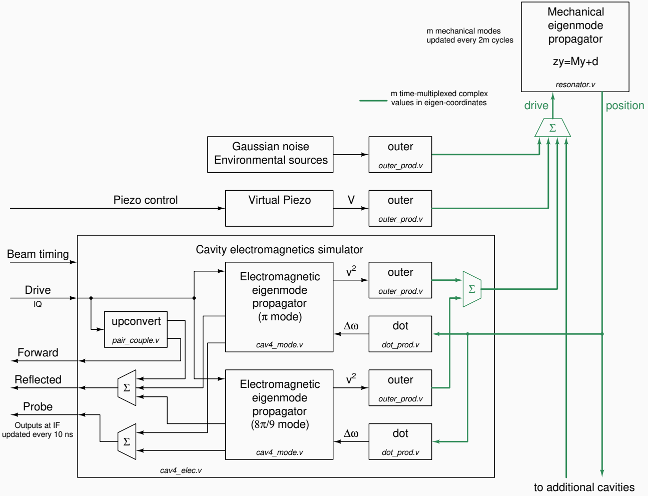
Simulation engines

- After the Physics analysis and the transitions from Laplace, to continuous time, to discrete time, one can implement the state-space equations
- In floating point (like C) the implementation is simpler and the simulation results satisfying during the analysis/design phase
- With those same equations, one can imagine to implement them in an FPGA for a different interaction experience with the models (fixed point arithmetic makes it harder)
- An FPGA-based simulation engine provides the sense of "live" simulations, which can be used in the development process as well as after (e.g. operator training)

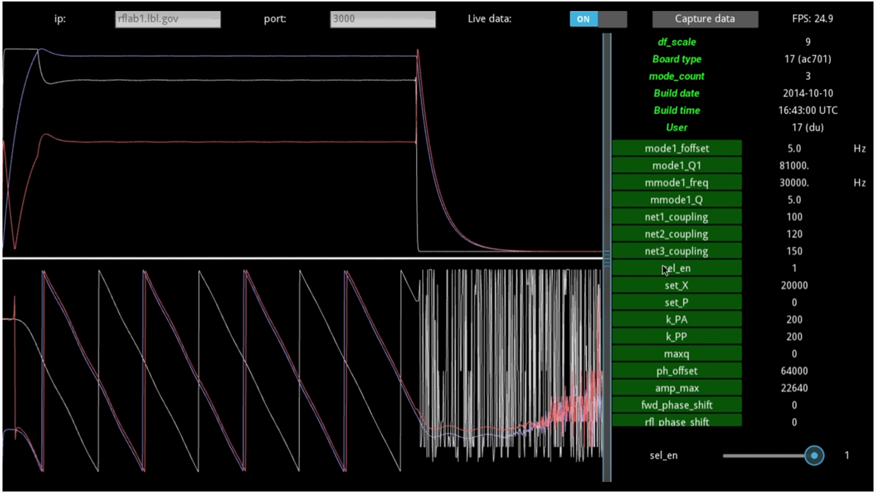
FPGA Model Architecture



FPGA Model Implementation



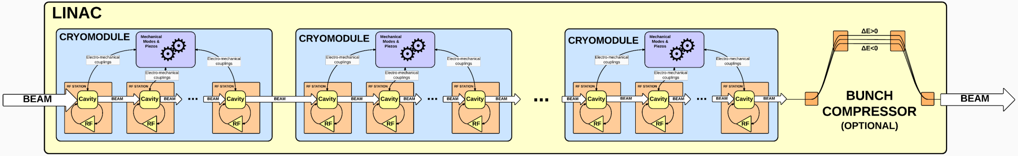
User interface



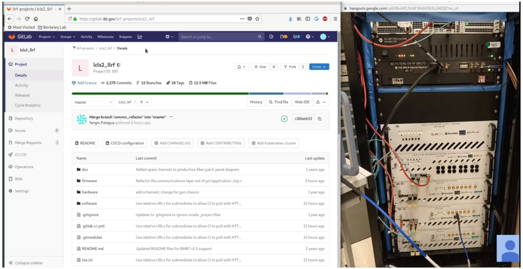
Beyond LLRF simulations

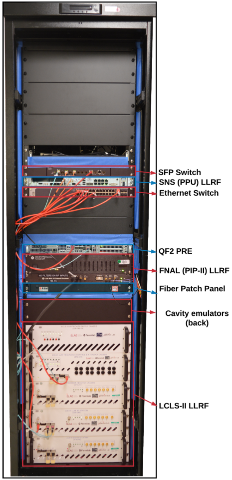
Accelerator level simulations

- Simulators like Litrack for Linacs, or Elegant for Synchrotrons include accelerator components (also RF) to simulate beam characteristics
- If combined, one can make extrapolate the RF system sensitivities to the beam figures of merit, the ultimate goal in accelerator design



From simulators to hardware emulators





Final remarks

- Cavity Physics, as well as its interaction with LLRF system components, is well known in literature and allows to understand systems sensitivities and engineer LLRF systems accordingly prior to their implementation
- Modeling, analysis and simulations are imperative during the design phase, as the analysis results feed design choices during the engineering phase
- Implementations of models in the FPGA, and the use of cavity emulators provide a powerful development (and verification) framework beyond the design phase