WP4: Netzbooster

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1 The problem for a single line and single time

We have N nodes $i, j = 0, ...N-1$ and L lines $k, \ell = 0, ...L-1$.

Suppose we have nodal power imbalances p_i at each node (positive is net generation, negative is net load). By energy conservation in a lossless network we have

$$\sum_{i=0}^{N-1} p_i = 0 \tag{1}$$

so only $N-1$ of the values are independent.

The power transfer distribution factor $PTDF_{\ell i}$ relates the flows f_ℓ to the nodal imbalances:

$$f_\ell = \sum_{i=0}^{N-1} PTDF_{\ell i} p_i \qquad \forall \ell \tag{2}$$

As a result of the LOPF, the flows will respect the thermal limits, possibly also with a buffer c_ℓ:

$$|f_\ell| \leq c_\ell F_\ell \qquad \forall \ell \tag{3}$$

If a line k fails, the flows after the failure of k, $f_\ell^{(k)}$, are related to the flows f_ℓ before the failure by the line (or branch if we include transformers) outage distribution factor $LODF_{\ell k}$

$$f_\ell^{(k)} = f_\ell + LODF_{\ell k} f_k \qquad \forall \ell \neq k \tag{4}$$

Nota Bene: no summation over k.

The idea of the Netzbooster is to allow line k to be loaded right up to its thermal limit, i.e. $c_k = 1$ so that $|f_k| \leq F_k$. Then if the line fails, we can de-load any line by using flexibility options.

For example, suppose line k starts at node i and ends at node j, we can change the power injections at i and j while maintaining the energy balance, i.e. inject power p MW at node i and withdraw p MW at node j, so that the flow in line ℓ after the failure of k is:

$$f_\ell^{(k)}(p) = f_\ell + LODF_{\ell k} f_k + PTDF_{\ell i} p - PTDF_{\ell j} p \qquad \forall \ell \neq k \tag{5}$$

The factor $PTDF_{\ell i} - PTDF_{\ell j}$ is nothing other than the line-PTDF:

$$LPTDF_{\ell k} = \sum_i PTDF_{\ell i} K_{ik} \tag{6}$$

where K_{ik} is the incidence matrix.

So now we have a well-defined problem: minimise $|p|$ such that

$$f_\ell^{(k)}(p) = f_\ell + LODF_{\ell k} f_k + LPTDF_{\ell k} p \qquad \forall \ell \neq k \tag{7}$$

is within the thermal limits, i.e.

$$|f_\ell^{(k)}(p)| = |f_\ell + LODF_{\ell k} f_k + LPTDF_{\ell k} p| \leq F_\ell \qquad \forall \ell \neq k \tag{8}$$

This can be written as an optimization problem with objective function

$$\min_{p^+, p^-} p^+ + p^- \tag{9}$$

such that

$$p^+ \geq 0 \tag{10}$$

$$p^- \geq 0 \tag{11}$$

$$\left| f_\ell + LODF_{\ell k} f_k + LPTDF_{\ell k}(p^+ - p^-) \right| \leq F_\ell \qquad \forall \ell \neq k \tag{12}$$

2 Problem for dimensioning Netzbooster for a single line over many times

Find the required Netzbooster power $p_{k,t}$ for the failure of line k in all snapshots t.

Then the required size of the Netzbooster is $P_k = \max_t p_{k,t}$. We might need to do this both as a generator and as a demand and the two nodes, e.g. also find $\min_t p_{k,t}$ depending on the direction of the flow.

3 Problem for dimensioning Netzbooster for all lines over many times

We could abandon $N - 1$ for all lines. Then we have everywhere $c_\ell = 1$.

What is the required size of the Netzbooster at each node so that we can catch all possible line failures?

We can define this as an optimization problem, where we optimise the capacity P_i^+ to increase power at node i (or reduce demand) and the capacity P_i^- to decrease power at node i (or increase demand).

We take the flows $f_{\ell,t}$ from an existing LOPF where everything has been optimized without $N-1$ so that:

$$|f_{\ell,t}| \leq F_\ell \qquad \forall \ell, t \tag{13}$$

These are treated as constants for the Netzbooster analysis.

The objective function is:

$$\min_{P_i^+, P_i^-, p_{i,t,k}^+, p_{i,t,k}^-} \sum_i \left(c^+ P_i^+ + c^- P_i^-\right) + \sum_{i,t,k} \left(o^+ p_{i,t,k}^+ + o^- p_{i,t,k}^-\right) \tag{14}$$

where we minimise the Netzbooster capacities P_i^+, P_i^-, while $p_{i,t,k}^+, p_{i,t,k}^-$ are the dispatch of the Netzbooster at node i in time t for the outage of line k. Initially we can set the costs for upward and downward capacity the same $c^+ = c^- = 1$. The costs for dispatch o^+ and o^- could correspond to compensation paid to DSM consumers, or running costs for storage. Initially we can also set them to zero or some small amount $o^+ = o^- = \varepsilon$. A small positive value will guarantee that $p_{i,t,k}^+$ and $p_{i,t,k}^-$ are not both simultaneously positive for a given i, t, k.

The constraints are:

$$P_i^+ \geq 0 \qquad \forall i \tag{15}$$

$$P_i^- \geq 0 \qquad \forall i \tag{16}$$

$$0 \leq p_{i,t,k}^+ \leq P_i^+ \qquad \forall i, t, k \tag{17}$$

$$0 \leq p_{i,t,k}^- \leq P_i^- \qquad \forall i, t, k \tag{18}$$

$$\sum_i \left(p_{i,t,k}^+ - p_{i,t,k}^-\right) = 0 \qquad \forall t, k \tag{19}$$

$$\left| f_{\ell,t} + LODF_{\ell k} f_{k,t} + \sum_i PTDF_{\ell i} \left(p_{i,t,k}^+ - p_{i,t,k}^-\right) \right| \leq F_\ell \qquad \forall t, k, \ell \neq k \tag{20}$$

4 Further steps

We could integrate the above constraints into the full electricity system investment problem, where the generators and storage dispatch are cooptimised. The problem remains linear. Then it is quite similar to how operating reserves are optimised. It is already quite similar to the SCLOPF formulation, just with an alternative option for relieving overloading.

We could also consider implementing the Netzbooster only on a subset of lines, while doing passive $N-1$ on the rest. We could optimise which lines we use it on based on a cost-benefit analysis.