Base Station Positioning Using Statistical Averaging of Ray Intersection Points

Milos Borenovic and Aleksandar Neskovic

***Abstract* — This paper presents an algorithm for estimating a location of the base station in GSM system. The algorithm uses received signal strength measurements to detect and emphasize less reflected signal rays originating from the base station. The position estimate is obtained by averaging the intersection points of such rays. The performances of the algorithm have been verified by an extensive measurement campaign and the achieved median error was less than 50m.**

***Keywords* — Base station, GSM, Intersection points, Location, Positioning, Rays, Statistical.**

I. INTRODUCTION

POSITIONING algorithms in mobile communication networks have been a hot topic of many research studies. Although most of them focus on providing a location estimate of the mobile network nodes, there has been an identified need to find a position of network nodes as well.

The positioning of Base Stations (BSs) or network nodes in general can be used to address many burning issues. In wireless sensor networks this may be used to improve the routing performances. In Public Land Mobile Networks (PLMN), with the advancement in the hardware development, there are BSs in the market that can be used by an individual or an organisation as a rogue BS to intercept and eavesdrop network traffic. Finding these, along with other interfering emitters, may be a significant task for a regulatory agency. Also, procuring a location of a BS has been a known feature of some top-notch drive-test systems [1] enabling more comprehensive benchmarking of competitors' networks.

This paper presents initial analyses of the algorithm for GSM BS positioning. The position estimate is procured through a statistical process based on Received Signal Strength (RSS) of the BS measured at a number of Reference Points (RPs). The algorithm is favouring clearer paths of the signal originating from the BS. The BS position estimate is provided based on the Intersection Points (IPs) of such paths. The core algorithm is not, in any way, limited to GSM and can equally be used in a myriad of other radio technologies such as UMTS, LTE, TETRA, and WiMAX.

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To the extents of authors' knowledge, there are no relevant research papers concerning GSM BS positioning. However, research studies have been conducted in the field of general radio emitter positioning. Most commonly used approaches comprise Time Difference of Arrival (TDOA), Angle of Arrival (AoA) and Doppler shifted measurements [2], [3]. The reported median Distance Errors (DEs) are in the range of several tenths of meters. These results ought to be taken with caution because they were obtained by computer simulations and may not model the propagation in urban areas appropriately. Moreover, to verify the results in an actual environment, specialized hardware (apart from the receiver) might be required.

On the other hand, there are proprietary algorithms that are commercialized and, therefore, not available for the research community. For PLMN, [4] provides such features whereas, for IEEE 802.11 environments, there are a number of tools offering help in rogue access point tracking [5].

The reminder of the paper is organized as follows: Section II describes the positioning algorithm in details. In Section III the measurement campaign used to validate the performances of positioning algorithm is presented whereas, in Section IV, the obtained positioning performances are declared and discussed. Section V concludes the paper.

II. POSITIONING ALGORITHM

The positioning algorithm presented herein is statistical in nature. It uses a number of RPs where the RSS is measured along with the geographical position of the RP. Typically, this kind of measurement in PLMNs is performed by using a drive-test system.

The algorithm is executed in three phases. In the first phase, a number of intersection points are determined whereas, in the second phase, the IPs are further selected and the appropriate weight is associated with each IP. In the third phase, final BS position estimate is calculated as a weighted average of the previously selected IPs.

A. *Phase I*

Assume that the algorithm receives RSS measurements that have been performed in n locations. These georeferenced RSS measurements shall be referred to as the RP database or simply the database.

In the first phase of the algorithm, each set of four RPs from the database is taken, and a procedure in a two-dimensional Euclidean space is performed on them.

Assume that points A, B, C and D are a single set of four

RPs taken into calculation. Also, without loss of generality, assume that $RSS_A \le RSS_B \le RSS_C \le RSS_D$ and that no three points are lying on a single line. Fig. 1. shows the possible intersections of lines determined by these four points. There can be a maximum of three pairs of lines determined by the points: *AB* and *CD*, *AC* and *BD*, and *AD* and *BC*. Therefore, there can be a maximum of three IPs for each chosen set of four RPs. Not all of these IPs are of interest for the algorithm.

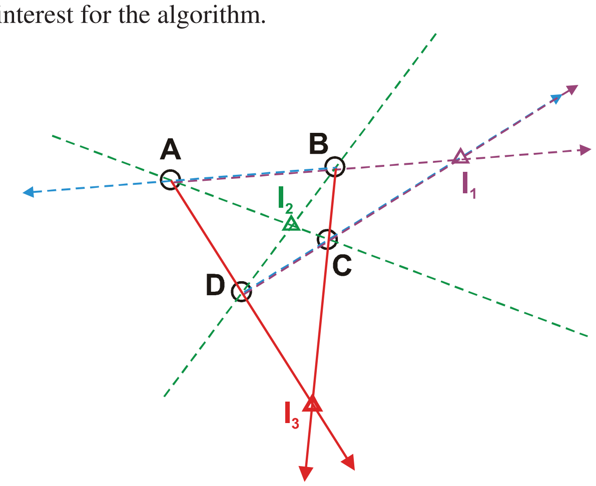

Fig. 1. Intersection points determination

IPs can be divided into four classes depending on side of the line on which they reside on. First off, the IP can be inside the ABCD polygon, e.g. I_2 in Fig. 1. Then, the IP can be formed by two rays starting at the points of higher (or equal) RSS value. The example for this case does not exist in Fig. 1. but it would be the IP of $\overrightarrow{BA}$ and $\overrightarrow{DC}$ if it existed. The IP can also be formed by two rays, one starting at the point of higher RSS value (e.g. $\overrightarrow{DC}$) and the other starting at the point of lower RSS value (e.g. $\overrightarrow{AB}$), e.g. I_1 in Fig. 1. Finally, the IP can be formed by two rays starting at the points of lower RSS value. In Fig. 1, this case is represented by $\overrightarrow{AD}$ and $\overrightarrow{BC}$ intersection, I_3.

The algorithm takes interest only in the latter class of IP (e.g. I_3 in Fig. 1.) which are forwarded to the second phase. The rationale behind this selection is that the algorithm in this phase is trying not to exclude the rays coming from the BS crossing through the RP of higher RSS and going towards the RPs of lower RSS.

B. *Phase II*

In the second phase, the algorithm is trying to select and favour the IPs that may have come from the BS with fewer reflections. To carry out the task, four criteria have been introduced:

- c_1 as the minimal difference of RSS values for each of the two rays forming an IP (e.g. $RSS_D - RSS_A$ and $RSS_C - RSS_B$ in Fig. 1.),
- c_2 as the minimal sum of RSS values for each of the two rays forming an IP (e.g. $RSS_D + RSS_A$ and $RSS_C + RSS_B$ in Fig. 1.),
- c_3 as the minimal distance between the points defining each of the two rays, and
- c_4 as the range of angles between the rays forming an IP (expressed as a maximum bias from π/2).

These criteria act twofold. First, the criteria are used as thresholds to further narrow the set of IPs that would be used to estimate the BS position. Then, the same criteria are used to assign the weights to each IP. Weights for each criterion are calculated as follows:

$$\begin{aligned} c_1^w &= \frac{\Delta RSS_1 \cdot \Delta RSS_2}{\left(c_1^t\right)^2} \\ c_2^w &= \frac{\Sigma RSS_1 \cdot \Sigma RSS_2}{\left(c_2^t\right)^2} \\ c_3^w &= \frac{\Delta L_1 \cdot \Delta L_2}{\left(c_3^t\right)^2} \\ c_4^w &= 1 - \frac{\Delta \theta}{c_4^t} \end{aligned} , \tag{1}$$

where ΔRSS_1 and ΔRSS_2 are the differences in RSS levels per rays, ΣRSS_1 and ΣRSS_2 are the sums of RSS levels per rays, ΔL_1 and ΔL_2 are the distances between the RPs per ray, $\Delta\Theta$ is the (absolute) bias of the angle between the rays and the π/2 angle, and c_i^t, $i = 1..4$, are the criteria thresholds.

The overall weight associated to an IP is calculated as:

$$w = \prod_{i=1}^{4} c_i^w \tag{2}$$

C. *Phase III*

The final BS position estimate is calculated as a weighted average of all the remaining IPs after the second phase. Assuming that there are *m* IPs remaining with their respective weights w_i, $i = 1..m$, the estimated BS coordinates, *x* and *y*, are calculated as:

$$\begin{aligned} x &= \left(\sum_{i=1}^{m} w_i\right)^{-1} \sum_{i=1}^{m} \left(x_i w_i\right) \\ y &= \left(\sum_{i=1}^{m} w_i\right)^{-1} \sum_{i=1}^{m} \left(y_i w_i\right) \end{aligned} . \tag{3}$$

The algorithm, according to Eq. (3), assigns greater weights to rays resembling a cleaner propagation (i.e. less reflections and nearer to LOS condition). Therefore, the intersections of such rays are favoured.

III. Measurement Campaign

To validate and verify the proposed algorithm, data from an extensive measurement campaign was taken. For the measurement campaign, Rohde & Schwarz (R&S) TSMQ network scanner was used in a drive-test setup. All the measurements were georeferenced using a differential

GPS receiver. This system can usually achieve less than 5m distance error [6] which is certainly better than the accuracy expected from the BS positioning algorithm. Therefore, it can be used for reference localization. The measurement and GPS location data were acquired by using a laptop equipped with "R&S Romes v4" software.

A site in a light urban environment was selected for the purpose of examining the performances of proposed algorithm. Regarding the chosen site, the RPs inside 1km radius from the BS were selected. By choosing one sector of the BS, the database was narrowed to 20,264 RPs where the broadcast channel of that sector was radio-visible. Fig. 2. shows the locations of the used RPs, relative to the position of the BS. RSS values are represented by the colour of the RPs. The BS, in Fig. 2, has (0, 0) coordinates and is denoted by a black square.

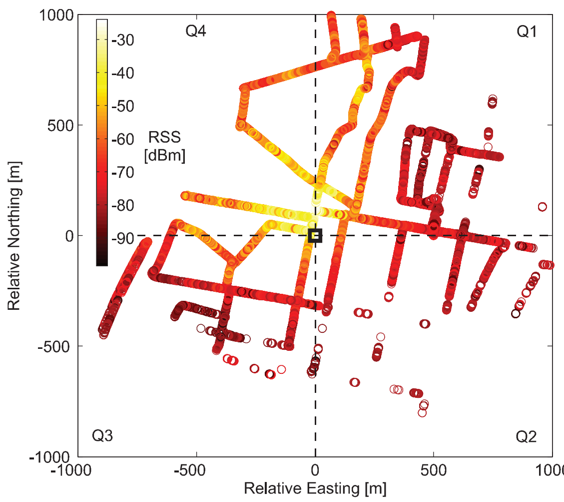

Fig. 2. Locations and RSS values of the RPs relative to the position of the BS

IV. Performances

Two series of tests have been performed in order to study the performances of BS positioning algorithm.

In the first series of tests, the aim was to check whether the positioning algorithm is capable of estimating a BS position based on RPs located only in one quadrant surrounding the BS. This property is important because, in the actual situation, there is no information on the location of the BS hence, there is no way to be sure the RPs are spread around the BS.

For the initial analyses included in this paper, a single set of criteria thresholds was adopted:

$$
\begin{aligned}
c_1^t &= 20\ dB \\
c_2^t &= -110\ dBm \\
c_3^t &= 70\ m \\
c_4^t &= \frac{3}{8}\pi
\end{aligned} \quad . \tag{4}
$$

To perform this test, the RPs database was divided into four smaller databases by quadrants, Q1 – Q4, as denoted in Fig. 2. and, for each quadrant, 20 RPs were randomly chosen and forwarded to the algorithm. To obtain statistically sound results, the experiment was repeated for 20 times. Results of this test are summarized in Table 1.

From Table 1. it can be seen that for a chosen set of threshold values in Eq. (4), there are experiments with no IPs. This is perhaps most visible in experiments performed for Q2 RPs where none of the 20 experiments yielded any IPs. The average x and y Distance Errors (DEs) seem to gravitate towards their respective quadrants which may indicate that the algorithm may be estimating the position of the BS slightly closer to the positions of the RPs. Also, comparing the average Distance Errors (DEs) in Q1 and Q4, and having in mind the larger number of IPs in Q4, may point to the idea that the higher number of IPs may yield a better BS estimate. This behaviour has been illustrated in Fig. 3. where the results from Table 1. have been presented and the number of IPs included in the calculation of each position estimate has been colour coded (i.e. the number of IPs used to obtain an estimate corresponds to a colour on the scale).

Table 1: Positioning results by quadrants.

Parameter	***Q1***	***Q2***	***Q3***	***Q4***
Database size [RPs]	9244	1790	4933	4287
Experiments with IP	6/20	0/20	2/20	20/20
Avg. no. of IPs	3302	-	2388	43k
Avg. DE [m]	167	-	31	60
Avg. DE x [m]	81	-	-21	51
Avg. DE y [m]	128	-	-23	3

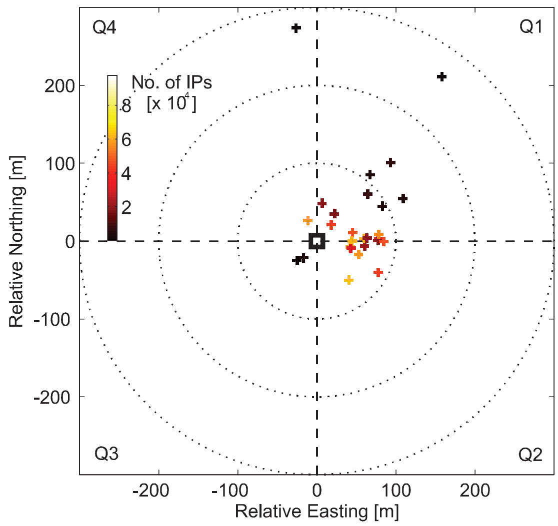

Fig. 3. BS position estimates from quadrant divided RP databases

To further verify the accuracy performances of the positioning algorithm, particularly with regards to the number of IPs, another series of experiments was conducted. In these experiments, the RP database from all quadrants was used. The number of randomly chosen RPs that were forwarded to the algorithm was varied from 20 to 60 in steps of 10 (i.e. 20, 30, 40 50, and 60 RPs). Again, all experiments were repeated 20 times.

The results of these tests have been plotted in Fig 4. Again, the number of IPs used to obtain an estimate corresponds to a colour on the scale. It can be seen that now that the RPs were selected from the entire database, the position estimates are more evenly spread around the position of the BS. Once more, the results indicate that the larger number of RPs induces a larger number of IPs and a better performing positioning algorithm. To further analyze this phenomenon, results of the second series of tests have been summed in Table 2.

TABLE 2: POSITIONING RESULTS BY NUMBER OF RPS.

Parameter	*20*	*30*	*40*	*50*	*60*
Experiments with IPs	14	16	19	20	19
Avg. no. of IPs	120	2k	6.5k	21k	36k
Avg. DE [m]	183	96	89	71	55
50th perc. DE [m]	138	92	73	64	44
67th perc. DE [m]	210	114	99	80	58
95th perc. DE [m]	485	171	214	150	107

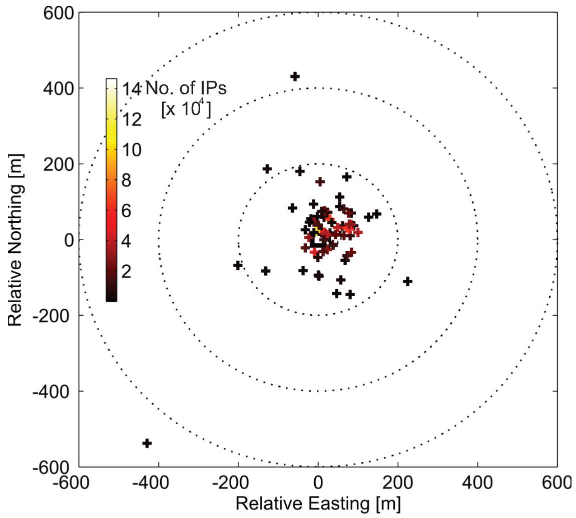

Fig. 4. BS position estimates using the complete RPs database

TABLE 3: POSITIONING RESULTS BY NUMBER OF IPS.

Parameter	*<100*	*<1k*	*<10k*	*>10k*
No. of experiments	15	17	23	33
Avg. DE [m]	188	129	70	58
50th perc. DE [m]	138	125	58	46
67th perc. DE [m]	198	154	74	78
95th perc. DE [m]	510	231	161	104

From Table 2. it can be seen that the performances of the positioning algorithm are improving with the increase in the number of RPs forwarded to the algorithm. On average, greater number of RPs induces a greater number of IPs. To examine the algorithm's accuracy versus the number of IPs, Table 3. has been created. In Table 3, the experiments have been grouped by the number of IPs into four groups: less than a 100 IPs, more than a 100 and less than 1000 IPs, more than a 1,000 and less than 10,000 IPs, and more than 10,000 IPs. Fig. 5. illustrates the accuracy improvement with the increase in the number of IPs.

Results in Table 3. collaborate to the earlier statements of better performances with the increase in the number of IPs. This ought to be considered as a good property of the positioning algorithm being that the number of IPs may well serve as confidence indicator of a position estimate.

The best performing class (more than 10,000 IPs) has an average DE of 58m, and a median error of 46m which is similar to the results of other research studies [2], [3] but without the need of specialized hardware for positioning (apart from the GPS receiver).

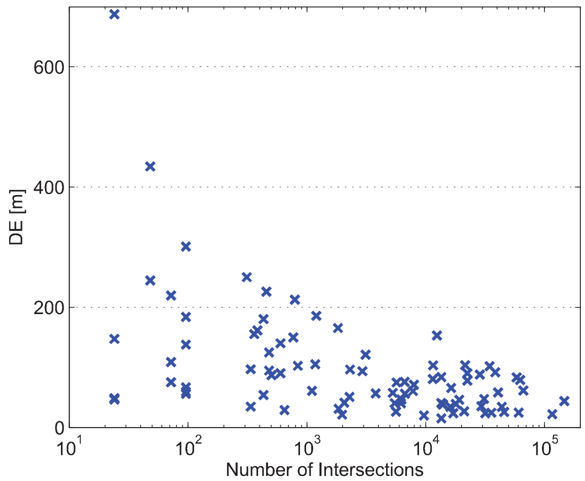

Fig. 5 DE vs. number of IPs

V. CONCLUSIONS

The paper presents an algorithm for estimating a GSM BS position by using the RSS values obtained by drive-test measurements.

To validate the performances of the algorithm, measurements from an extensive measurement campaign were used. It was shown that the algorithm is capable of estimating the base station position based on the measurements originating only from one quadrant surrounding the BS. Furthermore, when using the entire measurements database, the best performing class, with regards to the number of intersection points, achieves less than 50m median BS positioning error which is comparable with the results of other studies (verified only by computer simulations).

In future work, an optimization tool, such as neural network, may be employed to calculate the overall weight of an intersection point.

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