微積總棟員 Calculus - Exercises

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The Definite Integral (定積分)

1. Evaluate the following definite integrals by interpreting each in terms of areas.

   (a) $\displaystyle\int_0^1 \sqrt{1-x^2}\,dx$

   (b) $\displaystyle\int_1^3 2x\,dx$

   (c) $\displaystyle\int_0^5 (1-x)\,dx$

2. Express $\displaystyle\int_0^1 x^2\,dx$ as a limit of sums and find its value.

Solution [註：本解答僅提示重點，請自行補足細節流程。]

1. (a) $\displaystyle\int_0^1 \sqrt{1-x^2}\,dx = \frac{1}{4}\times\pi\times 1^2 = \frac{\pi}{4}$

   (請自行作圖，為四分之一圓之面積。)

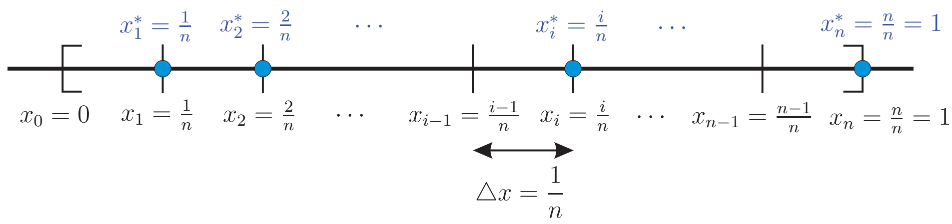
   (b) $\displaystyle\int_1^3 2x\,dx = \frac{(2+6)\times 2}{2} = 8$

   (請自行作圖，為梯形之面積。)

   (c) $\displaystyle\int_0^5 (1-x)\,dx = \frac{1\times 1}{2} - \frac{4\times 4}{2} = -\frac{15}{2}$

   (請自行作圖，為兩等腰直角三角形面積之差值。)

2. Define $f(x) = x^2$ on $[0,1]$. Let $x_0 = 0$ and $x_i = \dfrac{i}{n}$ for $i = 1, 2, \cdots, n$, and set $x_i^* = x_i = \dfrac{i}{n}$.



We get that

$$\int_0^1 x^2\,dx = \lim_{n\to\infty}\sum_{i=1}^n f(x_i^*)\triangle x = \lim_{n\to\infty}\sum_{i=1}^n \left(\frac{i}{n}\right)^2 \frac{1}{n}$$

$$= \lim_{n\to\infty}\frac{1}{n^3}\sum_{i=1}^n i^2 = \lim_{n\to\infty}\frac{1}{n^3}\times\frac{n(n+1)(2n+1)}{6} = \lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3} = \frac{1}{3}$$

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The Properties of the Definite Integral (定積分的基本性質)

1. Suppose that $\int_2^4 f(x)\,dx = 3$, $\int_2^7 f(x)\,dx = 6$, $\int_2^4 g(x)\,dx = 5$ and $\int_2^7 g(x)\,dx = -4$. Evaluate the following values.

   (a) $\int_4^2 f(x)\,dx$

   (b) $\int_7^7 g(x)\,dx$

   (c) $\int_4^7 f(x)\,dx$

   (d) $\int_2^4 [2f(x) - g(x)]\ dx$

   (e) $\int_2^7 [5g(x) + 2f(x)]\ dx$

   (f) $\int_2^4 [2g(x) + 5]\ dx$

2. Show that $4 \le \int_{-2}^2 \sqrt{1+x^4}\,dx \le 4\sqrt{17}$.

Solution [註：本解答僅提示重點，請自行補足細節流程。]

1. (a) $\displaystyle\int_4^2 f(x)\,dx = -\int_2^4 f(x)\,dx = -3$

   (b) $\displaystyle\int_7^7 g(x)\,dx = 0$

   (c) $\displaystyle\int_4^7 f(x)\,dx = \int_2^7 f(x)\,dx - \int_2^4 f(x)\,dx = 6 - 3 = 3$

   (d) $\displaystyle\int_2^4 [2f(x) - g(x)]\ dx = 2\int_2^4 f(x)\,dx - \int_2^4 g(x)\,dx = 2\times 3 - 5 = 1$

   (e) $\displaystyle\int_2^7 [5g(x) + 2f(x)]\ dx = 5\int_2^7 g(x)\,dx + 2\int_2^7 f(x)\,dx$
   $= 5\times(-4) + 2\times 6 = -8$

   (f) $\displaystyle\int_2^4 [2g(x) + 5]\ dx = 2\int_2^4 g(x)\,dx + \int_2^4 5\,dx$
   $= 2\times 5 + 5\times(4-2) = 20$

2. $\because 1 \leq \sqrt{1+x^4} \leq \sqrt{17}$ for all $-2 \leq x \leq 2$.
   $\displaystyle\therefore \int_{-2}^2 dx \leq \int_{-2}^2 \sqrt{1+x^4}\,dx \leq \int_{-2}^2 \sqrt{17}\,dx$
   $\displaystyle\Longrightarrow 1\times 4 \leq \int_{-2}^2 \sqrt{1+x^4}\,dx \leq \sqrt{17}\times 4$

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First Fundamental Theorem of Calculus (微積分第一基本定理)

1. Find the derivative of the function $g(x)$.

   (a) $g(x) = \displaystyle\int_0^x \frac{1}{1+t^4}\,dt$

   (b) $g(x) = \displaystyle\int_1^x \frac{\sin t}{t}\,dt$

   (c) $g(x) = \displaystyle\int_x^\pi \cos(1+t^2)\,dt$

   (d) $g(x) = \displaystyle\int_0^{\sin x} \sqrt{t+\sqrt{t}}\,dt$

   (e) $g(x) = \displaystyle\int_{x^3}^{2x} \sqrt{2+t^3}\,dt$

2. Define $f(x) = \displaystyle\int_1^x t\sqrt[3]{1+t^2}\,dt$.

   (a) Find the interval(s) on which $f(x)$ is decreasing.

   (b) Find the interval(s) on which $f(x)$ is concave up.

Solution [註：本解答僅提示重點，請自行補足細節流程。]

1. (a) $g'(x) = \dfrac{d}{dx}\displaystyle\int_0^x \frac{1}{1+t^4}\,dt = \frac{1}{1+x^4}$

   (b) $g'(x) = \dfrac{d}{dx}\displaystyle\int_1^x \frac{\sin t}{t}\,dt = \frac{\sin x}{x}$

   (c) $g'(x) = -\dfrac{d}{dx}\displaystyle\int_\pi^x \cos(1+t^2)\,dt = -\cos(1+x^2)$

   (d) $g'(x) = \dfrac{d}{dx}\displaystyle\int_0^{\sin x} \sqrt{t+\sqrt{t}}\,dt = \sqrt{\sin x + \sqrt{\sin x}} \cdot \cos x$

   (e) $$\begin{aligned} g'(x) &= \frac{d}{dx}\int_0^{2x} \sqrt{2+t^3}\,dt - \frac{d}{dx}\int_0^{x^3} \sqrt{2+t^3}\,dt \\ &= \sqrt{2+(2x)^3}\cdot 2 - \sqrt{2+(x^3)^3}\cdot 3x^2 \end{aligned}$$

2. (a) $\because f'(x) = \dfrac{d}{dx}\displaystyle\int_1^x t\sqrt[3]{1+t^2}\,dt = x\sqrt[3]{1+x^2}$

   $\therefore f'(x) < 0$ on $(-\infty, 0)$ and $f'(x) > 0$ on $(0, \infty)$

   $\Longrightarrow f(x)$ is decreasing on $(-\infty, 0)$.

   (b) $\because f''(x) = (1+x^2)^{1/3} + x \cdot \dfrac{1}{3}(1+x^2)^{-2/3} \cdot 2x = \dfrac{3+5x^2}{3\sqrt[3]{(1+x^2)^2}} > 0$

   $\therefore f''(x) > 0$ on $(-\infty, \infty)$

   $\Longrightarrow f(x)$ is concave up on $(-\infty, \infty)$.

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Second Fundamental Theorem of Calculus (微積分第二基本定理)

1. Evaluate the integrals.

   (a) $\displaystyle\int_0^2 (3x^2 + 4x + 1)\,dx$

   (b) $\displaystyle\int_1^4 \left(\sqrt{s} + \frac{1}{\sqrt{s}}\right) ds$

   (c) $\displaystyle\int_1^3 \frac{2t^3 - 5t^2 + 1}{t^2}\,dt$

   (d) $\displaystyle\int_{\pi/3}^{\pi/2} \cos x\,dx$

   (e) $\displaystyle\int_0^{\pi/4} (\sec^2\theta - \sin\theta)\,d\theta$

   (f) $\displaystyle\int_{-1}^2 |x - 1|\,dx$

   (g) $\displaystyle\int_0^{\pi} f(x)\,dx$, where $f(x) = \begin{cases} \sin x & \text{if } 0 \le x < \pi/2, \\ \cos x & \text{if } \pi/2 \le x \le \pi. \end{cases}$

2. Find the area under the parabola $y = x^2 + 1$ from 0 to 2.

3. Evaluate $\displaystyle\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$.

Solution [註：本解答僅提示重點，請自行補足細節流程。]

1. (a) $\displaystyle\int_0^2 (3x^2+4x+1)\,dx = \left[x^3+2x^2+x\right]_0^2 = 18-0 = 18$

   (b) $\displaystyle\int_1^4 \left(\sqrt{s}+\frac{1}{\sqrt{s}}\right)\,ds = \int_1^4 \left(s^{1/2}+s^{-1/2}\right)\,ds = \left[\frac{2}{3}s^{3/2}+2s^{1/2}\right]_1^4$

   $\displaystyle= \frac{28}{3}-\frac{8}{3} = \frac{20}{3}$

   (c) $\displaystyle\int_1^3 \frac{2t^3-5t^2+1}{t^2}\,dt = \int_1^3 (2t-5+t^{-2})\,dt = \left[t^2-5t-t^{-1}\right]_1^3$

   $\displaystyle= \left(-\frac{19}{3}\right)-(-5) = -\frac{4}{3}$

   (d) $\displaystyle\int_{\pi/3}^{\pi/2} \cos x\,dx = [\sin x]_{\pi/3}^{\pi/2} = 1-\frac{\sqrt{3}}{2}$

   (e) $\displaystyle\int_0^{\pi/4} (\sec^2\theta-\sin\theta)\,d\theta = [\tan\theta+\cos\theta]_0^{\pi/4} = \left(1+\frac{1}{\sqrt{2}}\right)-1$

   $\displaystyle= \frac{1}{\sqrt{2}}$

   (f) $\displaystyle\int_{-1}^2 |x-1|\,dx = \int_{-1}^1 (1-x)\,dx + \int_1^2 (x-1)\,dx$

   $\displaystyle= \left[x-\frac{1}{2}x^2\right]_{-1}^1 + \left[\frac{1}{2}x^2-x\right]_1^2 = \left(\frac{1}{2}-\left(-\frac{3}{2}\right)\right)+\left(0-\left(-\frac{1}{2}\right)\right)$

   $\displaystyle= \frac{5}{2}$

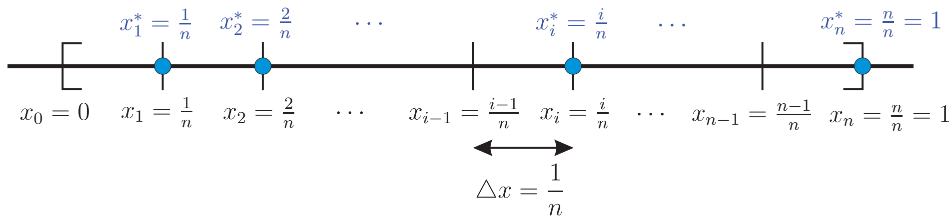
   (g) $\displaystyle\int_0^\pi f(x)\,dx = \int_0^{\pi/2} \sin x\,dx + \int_{\pi/2}^\pi \cos x\,dx = [-\cos x]_0^{\pi/2}+[\sin x]_{\pi/2}^\pi$

   $= (0-(-1))+(0-1) = 0$

2. area $\displaystyle= \int_0^2 (x^2+1)\,dx = \left[\frac{1}{3}x^3+x\right]_0^2 = \frac{14}{3}-0 = \frac{14}{3}$

3. Define $f(x)=\sqrt{x}$ on $[0,1]$. Let $x_0=0$ and $x_i=\dfrac{i}{n}$ for $i=1,2,\cdots,n$,

and set $x_i^* = x_i = \dfrac{i}{n}$.



We get that

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \cdots + \sqrt{\frac{n}{n}}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{i}{n}}\,\frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\triangle x = \int_0^1 \sqrt{x}\,dx = \left[\frac{2}{3}x^{3/2}\right]_0^1 = \frac{2}{3}$$

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The Substitution Rule (代換積分法)

1. Evaluate the indefinite integrals.

   (a) $\displaystyle\int \sqrt[3]{1+4x}\,dx$

   (b) $\displaystyle\int x^2\sin(x^3)\,dx$

   (c) $\displaystyle\int x^3(2+3x^4)^6\,dx$

   (d) $\displaystyle\int \frac{\sqrt{1+\tan t}}{\cos^2 t}\,dt$

   (e) $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

   (f) $\displaystyle\int x^5\sqrt{1+x^2}\,dx$

2. Evaluate the definite integrals.

   (a) $\displaystyle\int_0^1 (3x+1)^3\,dx$

   (b) $\displaystyle\int_0^{\pi/4} \sin^3\theta\cos\theta\,d\theta$

   (c) $\displaystyle\int_0^2 \frac{3x}{\sqrt{1+2x^2}}\,dx$

   (d) $\displaystyle\int_{-1}^1 (x^3+1)(x^4+4x)^2\,dx$

3. If $f(x)$ is continuous and $\displaystyle\int_0^4 f(x)\,dx = 7$, find $\displaystyle\int_0^2 xf(x^2)\,dx$.

4. Suppose that $f(x)$ is continuous on $(-\infty,\infty)$ and is periodic with period L, that is, $f(x+L)=f(x)$ for all real number x. Show that

$$\int_a^b f(x)\,dx = \int_{a+L}^{b+L} f(x)\,dx$$

   for all real numbers a and b.

Solution [註：本解答僅提示重點，請自行補足細節流程。]

1. (a) Let $u = 1 + 4x \Longrightarrow du = 4\,dx$. We get
$$\int \sqrt[3]{1+4x}\,dx = \frac{1}{4}\int u^{1/3}\,du = \frac{3}{16}u^{4/3} + C = \frac{3}{16}(1+4x)^{4/3} + C$$
(b) Let $u = x^3 \Longrightarrow du = 3x^2\,dx$. We get
$$\int x^2\sin(x^3)\,dx = \frac{1}{3}\int \sin u\,du = -\frac{1}{3}\cos u + C = -\frac{1}{3}\cos(x^3) + C$$
(c) Let $u = 2 + 3x^4 \Longrightarrow du = 12x^3\,dx$. We get
$$\int x^3(2+3x^4)^6\,dx = \frac{1}{12}\int u^6\,du = \frac{1}{84}u^7 + C = \frac{1}{84}(2+3x^4)^7 + C$$
(d) Let $u = 1 + \tan t \Longrightarrow du = \sec^2 t\,dt$. We get
$$\int \frac{\sqrt{1+\tan t}}{\cos^2 t}\,dt = \int \sec^2 t\sqrt{1+\tan t}\,dt = \int \sqrt{u}\,du$$
$$= \frac{2}{3}u^{3/2} + C = \frac{2}{3}(1+\tan t)^{3/2} + C$$
(e) Let $u = \sqrt{x} = x^{1/2} \Longrightarrow du = \frac{1}{2}x^{-1/2}\,dx = \frac{1}{2\sqrt{x}}\,dx$. We get
$$\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = 2\int \sin u\,du = -2\cos u + C = -2\cos(\sqrt{x}) + C$$
(f) Let $u = 1 + x^2 \Longrightarrow du = 2x\,dx$ and $x^2 = u - 1$. We get
$$\int x^5\sqrt{1+x^2}\,dx = \int \sqrt{1+x^2}\cdot(x^2)^2\cdot x\,dx = \frac{1}{2}\int \sqrt{u}(u-1)^2\,du$$
$$= \frac{1}{2}\int \left(u^{5/2} - 2u^{3/2} + u^{1/2}\right)\,du$$
$$= \frac{1}{2}\left(\frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right) + C$$
$$= \frac{1}{7}(1+x^2)^{7/2} - \frac{2}{5}(1+x^2)^{5/2} + \frac{1}{3}(1+x^2)^{3/2} + C$$

2. (a) Let $u = 3x + 1 \Longrightarrow du = 3\,dx$ & $\begin{cases} u(1) = 4 \\ u(0) = 1 \end{cases}$. We get
$$\int_0^1 (3x+1)^3\,dx = \frac{1}{3}\int_1^4 u^3\,du = \frac{1}{3}\left[\frac{1}{4}u^4\right]_1^4 = \frac{1}{3}\left(64 - \frac{1}{4}\right) = \frac{85}{4}$$
(b) Let $u = \sin\theta \Longrightarrow du = \cos\theta\,d\theta$ & $\begin{cases} u(\pi/4) = 1/\sqrt{2} \\ u(0) = 0 \end{cases}$. We get
$$\int_0^{\pi/4} \sin^3\theta\cos\theta\,d\theta = \int_0^{1/\sqrt{2}} u^3\,du = \left[\frac{1}{4}u^4\right]_0^{1/\sqrt{2}} = \frac{1}{16} - 0 = \frac{1}{16}$$

(c) Let $u = 1 + 2x^2 \Longrightarrow du = 4x\,dx$ & $\begin{cases} u(2) = 9 \\ u(0) = 1 \end{cases}$. We get

$$\int_0^2 \frac{3x}{\sqrt{1+2x^2}}\,dx = \frac{3}{4}\int_1^9 \frac{1}{\sqrt{u}}\,du = \frac{3}{4}\left[2u^{1/2}\right]_1^9 = \frac{3}{4}(6-2) = 3$$

(d) Let $u = x^4 + 4x \Longrightarrow du = (4x^3 + 4)\,dx$ & $\begin{cases} u(1) = 5 \\ u(-1) = -3 \end{cases}$. We get

$$\int_{-1}^1 (x^3+1)(x^4+4x)^2\,dx = \frac{1}{4}\int_{-3}^5 u^2\,du = \frac{1}{4}\left[\frac{1}{3}u^3\right]_{-3}^5$$

$$= \frac{1}{4}\left(\frac{125}{3} - (-9)\right) = \frac{152}{12}$$

3. Let $u = x^2 \Longrightarrow du = 2x\,dx$ & $\begin{cases} u(2) = 4 \\ u(0) = 0 \end{cases}$. We get

$$\int_0^2 xf(x^2)\,dx = \frac{1}{2}\int_0^4 f(u)\,du = \frac{1}{2}\times 7 = \frac{7}{2}$$

4. Let $u = x + L \Longrightarrow du = dx$ & $\begin{cases} u(b) = b + L \\ u(a) = a + L \end{cases}$. We get

$$\int_a^b f(x)\,dx = \int_a^b f(x+L)\,dx = \int_{a+L}^{b+L} f(u)\,du = \int_{a+L}^{b+L} f(x)\,dx.$$

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The Definite Integrals of Odd and Even Functions (奇偶函數的定積分)

1. Evaluate the definite integrals.

   (a) $\displaystyle\int_{-1}^{1} \frac{x}{1+x^4}\,dx$

   (b) $\displaystyle\int_{-\pi/3}^{\pi/3} x^2 \sin x\,dx$

   (c) $\displaystyle\int_{-\pi/4}^{\pi/4} \left(x^2 + x^6 \tan x\right)\,dx$

   (d) $\displaystyle\int_{-1}^{1} \frac{3x^6+6x^2+5x}{x^4+2}\,dx$

Solution [註：本解答僅提示重點，請自行補足細節流程。]

1. (a) $\because f(x) = \dfrac{x}{1+x^4}$ is an odd function. $\therefore \displaystyle\int_{-1}^{1} \frac{x}{1+x^4}\,dx = 0$

   (b) $\because f(x) = x^2 \sin x$ is an odd function. $\therefore \displaystyle\int_{-\pi/3}^{\pi/3} x^2 \sin x\,dx = 0$

   (c) $\displaystyle\int_{-\pi/4}^{\pi/4} \left(x^2 + x^6 \tan x\right)\,dx = \int_{-\pi/4}^{\pi/4} x^2\,dx + \int_{-\pi/4}^{\pi/4} x^6 \tan x\,dx = 2\int_{0}^{\pi/4} x^2\,dx$

   $[\because f(x) = x^2$ is an even function and $g(x) = x^6 \tan x$ is an odd function.$]$

   $= 2\left[\dfrac{1}{3}x^3\right]_0^{\pi/4} = \dfrac{\pi^3}{96}$

   (d) $\displaystyle\int_{-1}^{1} \frac{3x^6+6x^2+5x}{x^4+2}\,dx = \int_{-1}^{1} \left(3x^2 + \frac{5x}{x^4+2}\right)\,dx = 2\int_{0}^{1} 3x^2\,dx$

   $\left[\because f(x) = 3x^2 \text{ is an even function and } g(x) = \dfrac{5x}{x^4+2} \text{ is an odd function.}\right]$

   $= 2\left[x^3\right]_0^1 = 2$

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The Integral (積分) [綜合練習]

1. Suppose that $\int_2^4 f(x)\,dx = 3$, $\int_2^7 f(x)\,dx = 6$, $\int_2^4 g(x)\,dx = 5$ and $\int_2^7 g(x)\,dx = -4$. Evaluate $\int_4^7 f(x)\,dx$, $\int_2^4 [2g(x)+5]\,dx$ and $\int_2^7 [5g(x)+2f(x)]\,dx$.

2. Find the derivative of the function $g(x)$.

   (a) $g(x) = \int_1^x \frac{\sin t}{t}\,dt$

   (b) $g(x) = \int_0^{\sin x} \sqrt{t+\sqrt{t}}\,dt$

   (c) $g(x) = \int_{x^3}^{2x} \sqrt{2+t^3}\,dt$

3. Evaluate the definite integrals.

   (a) $\int_{-1}^1 \frac{x}{1+x^4}\,dx$

   (b) $\int_0^2 (3x^2+4x+1)\,dx$

   (c) $\int_{\pi/3}^{\pi/2} \cos x\,dx$

   (d) $\int_0^2 \frac{3x}{\sqrt{1+2x^2}}\,dx$

   (e) $\int_{-\pi/4}^{\pi/4} \left(x^2+x^6\tan x\right)\,dx$

   (f) $\int_{-1}^2 |x-1|\,dx$

   (g) $\int_0^{\pi/4} \sin^3\theta\cos\theta\,d\theta$

4. Evaluate the indefinite integrals.

(a) $\displaystyle\int \sqrt[3]{1+4x}\,dx$

(b) $\displaystyle\int x^2\sin(x^3)\,dx$

(c) $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

(d) $\displaystyle\int x^5\sqrt{1+x^2}\,dx$

5. Evaluate $\displaystyle\lim_{n\to\infty}\frac{1}{n}\left(\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+\sqrt{\frac{3}{n}}+\cdots+\sqrt{\frac{n}{n}}\right)$.