

一、选择题

1.D

2.C

二、填空题

1. $\frac{1}{4\sqrt{y}}, 0 < y < 4$.

2.0

三、解答题

1.(1) 从分布函数可得

$$X = -2 \quad p = 0.2$$

$$X = -1 \quad p = 0.3$$

$$X = 2 \quad p = 0.2$$

$$X = 3 \quad p = 0.3$$

X	-2	-1	2	3
P	0.2	0.3	0.2	0.3

(2)

Y	1	2	3
P	0.3	0.4	0.3

2.(1) 取 $y > -1$. 那么

$$F_Y(y) = P(Y \leq y) = P(2X - 1 \leq y) = P(X \leq \frac{1}{2}y + \frac{1}{2})$$

$$= \int_0^{\frac{1}{2}y + \frac{1}{2}} e^x dx$$

$$= 1 - e^{-\frac{1}{2}y - \frac{1}{2}}$$

$$\Rightarrow f_Y(y) = \frac{1}{2}e^{-\frac{1}{2}y - \frac{1}{2}}.$$

(2) 取 $y > 1$. 那么

$$F_Y(y) = P(Y \leq y) = P(e^X \leq y) = P(X \leq \ln y) = F_X(\ln y)$$

$$= \int_0^{\ln y} e^{-x} dx \Rightarrow f_Y(y) = \frac{1}{y^2}.$$

(3) 取 $y > 0$. 那么

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P(X^2 \leq y) = P(-\sqrt{y} \leq X \leq \sqrt{y}) \\ &= \int_0^{\sqrt{y}} e^{-x} dx \\ \Rightarrow f_Y(y) &= \frac{1}{2\sqrt{y}} e^{-\sqrt{y}}. \end{aligned}$$

3.

$$F_Y(y) = P(Y \leq y) = P(X^2 \leq y) = P(-\sqrt{y} \leq X \leq \sqrt{y})$$

当 $0 \leq y \leq 1$ 时,

$$\begin{aligned} F_Y(y) &= \frac{\sqrt{y}}{2} \\ \Rightarrow f_Y(y) &= \frac{1}{4\sqrt{y}}. \end{aligned}$$

当 $1 < y < 9$ 时,

$$\begin{aligned} F_Y(y) &= \frac{1 + \sqrt{y}}{4} \\ \Rightarrow f_Y(y) &= \frac{1}{8\sqrt{y}}. \end{aligned}$$

综上,

$$f_Y(y) = \begin{cases} \frac{1}{4\sqrt{y}} & 0 \leq y \leq 1 \\ \frac{1}{8\sqrt{y}} & 1 < y < 9 \\ 0 & \text{其它.} \end{cases}$$