一、选择题

1.B
2.A
3.C

二、填空题

1.$\frac{4}{3}+\frac{\sqrt{3}}{6}$.
2.9
3.$\frac{2}{3}\sigma^2$.

三、解答与证明题

1. 已知

$$V=\begin{pmatrix} DX & Cov(X,Y) \\ Cov(X,Y) & DY \end{pmatrix}$$
$$R(X,Y)=\frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}}$$

可以求得

$$f_X(x)=\frac{3}{2}-x \quad f_Y(y)=\frac{3}{2}-y$$

于是

$$EX=\int_0^1 x(\frac{3}{2}-x)dx=\frac{5}{12}$$
$$EY=\int_0^1 y(\frac{3}{2}-y)dy=\frac{5}{12}$$
$$DX=\int_0^1 (x-\frac{5}{12})^2(\frac{3}{2}-x)dx=\frac{11}{144}$$
$$DY=\int_0^1 (y-\frac{5}{12})^2(\frac{3}{2}-y)dy=\frac{11}{144}$$
$$EXY=\int_0^1 xy(2-x-y)dxdy=\frac{1}{6}$$
$$Cov(X,Y)=E(XY)-(EX)(EY)=-\frac{1}{144}$$

即得

$$V = \begin{pmatrix} \frac{11}{144} & -\frac{1}{144} \\ -\frac{1}{144} & \frac{11}{144} \end{pmatrix}$$

$$R(X,Y) = \frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}} = -\frac{1}{11}.$$

2. 由题可知密度函数为

$$f(x,y) = \begin{cases} 1, & (x,y) \in G \\ 0, & \text{其它} \end{cases}$$

求得边缘密度函数为

$$f_X(x) = 2 - 2x$$
$$f_Y(y) = 1 - \frac{y}{2}$$

(1)

$$EX = \int_0^1 x(2-2x)dx = \frac{1}{3}$$
$$EY = \int_0^2 y(1-\frac{y}{2})dy = \frac{2}{3}$$

(2)

$$DX = \int_0^1 (x-\frac{1}{3})^2(2-2x)dx = \frac{1}{18}$$
$$DY = \int_0^2 (y-\frac{2}{3})^2(1-\frac{y}{2}) = \frac{2}{9}$$

(3)

$$E(XY) = \iint_G xyf(x,y)dxdy = \int_0^1 \int_0^{2-2x} xy \cdot 1dydx = \frac{1}{6}$$

$$\Rightarrow Cov(X,Y) = E(XY) - EXEY = -\frac{1}{18}$$

$$R(X,Y) = \frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}} = -\frac{1}{2}$$

(4) 因为 $R(X,Y) \neq 0$, 所以 X, Y 相关, 且不独立.

3. 根据协方差的性质展开可证.

4. 已知

$$EX = EY = \mu, \quad DX = DY = \sigma^2.$$

$$\begin{aligned}
DU &= D(\alpha X + \beta Y) \\
&= \alpha^2 DX + \beta^2 DY = \alpha^2\sigma^2 + \beta^2\sigma^2; \\
DV &= D(\alpha X - \beta Y) \\
&= \alpha^2 DX + \beta^2 DY = \alpha^2\sigma^2 + \beta^2\sigma^2. \\
Cov(U,V) &= D(\alpha X) - D(\beta Y) \\
&= (\alpha^2 - \beta^2)\sigma^2 \\
\Rightarrow R(X,Y) &= \frac{\alpha^2 - \beta^2}{\alpha^2 + \beta^2}.
\end{aligned}$$

5 由题可得 X, Y 的分布律为

$$X \sim \begin{pmatrix} 1 & 2 & 3 \\ \frac{4}{9} & \frac{3}{9} & \frac{2}{9} \end{pmatrix}$$

$$Y \sim \begin{pmatrix} 0 & 1 & 2 \\ \frac{5}{9} & \frac{3}{9} & \frac{1}{9} \end{pmatrix}$$

(1)

$$\begin{aligned}
EX &= 1 \times \frac{4}{9} + 2 \times \frac{3}{9} + 3 \times \frac{2}{9} = \frac{16}{9} \\
EY &= 0 \times \frac{5}{9} + 1 \times \frac{3}{9} + 2 \times \frac{1}{9} = \frac{5}{9} \\
DX &= (1 - \frac{16}{9})^2 \times \frac{4}{9} + (2 - \frac{16}{9})^2 \times \frac{3}{9} + (3 - \frac{16}{9})^2 \times \frac{2}{9} = \frac{50}{81} \\
DY &= (0 - \frac{5}{9})^2 \times \frac{5}{9} + (1 - \frac{5}{9})^2 \times \frac{3}{9} + (2 - \frac{5}{9})^2 \times \frac{1}{9} = \frac{38}{81}
\end{aligned}$$

(2)

$$\begin{aligned}
E(XY) &= 1 \times 1 \times \frac{2}{9} + 1 \times 2 \times \frac{1}{9} + 2 \times 1 \times \frac{1}{9} = \frac{2}{3} \\
Cov(X,Y) &= E(XY) - EXEY = -\frac{26}{81} \\
R(X,Y) &= \frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}} = -\frac{13}{5\sqrt{19}}
\end{aligned}$$

(3)

$$D(X - 3Y) = DX + 9DY - 6Cov(X,Y) = \frac{548}{81}.$$