一、填空题

1.64%

2.$p^2+(1-p)^2$

二、解答题

1.$Z=X+Y$, 于是

$$Z\sim\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & \frac{C_3^2C_2^2}{C_7^4}+\frac{C_2^2C_2^2}{C_7^4}+\frac{C_3^1C_2^1}{C_7^4} & \frac{C_3^3C_2^1}{C_7^4}+\frac{C_3^2C_2^1C_2^1}{C_7^4} & \frac{C_3^2C_2^1}{C_7^4}+\frac{C_3^2C_2^2C_2^1}{C_7^4} & 0 \end{pmatrix}$$

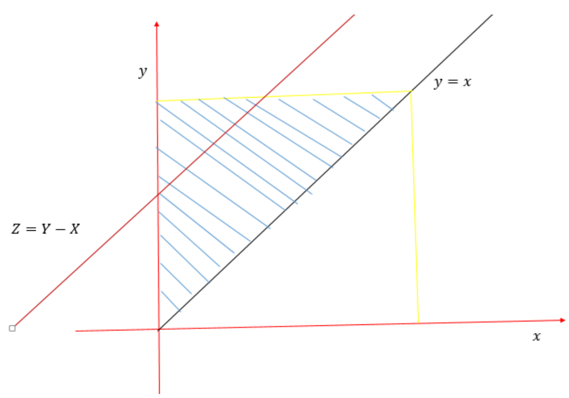
$U=\max(X,Y)$, 于是

$$U\sim\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & \frac{C_3^1C_2^1}{C_7^4} & \frac{C_2^2C_2^2}{C_7^4}+\frac{C_3^1C_2^2C_2^1}{C_7^4}+\frac{C_3^2C_2^2C_2^1}{C_7^4}+\frac{C_3^2C_2^2}{C_7^4}+\frac{C_3^2C_2^1C_2^1}{C_7^4} & \frac{C_3^3C_2^1}{C_7^4}+\frac{C_3^2C_2^1}{C_7^4} \end{pmatrix}$$

$V=\min(X,Y)$, 于是

$$V\sim\begin{pmatrix} 0 & 1 & 2 \\ 0 & \frac{C_3^1C_2^1}{C_7^4}+\frac{C_3^2C_2^1C_2^1}{C_7^4}+\frac{C_3^2C_2^1}{C_7^4}+\frac{C_2^2C_2^2}{C_7^4} & \frac{C_3^2C_2^2C_2^1}{C_7^4} \end{pmatrix}$$

2. 积分区域如图所示

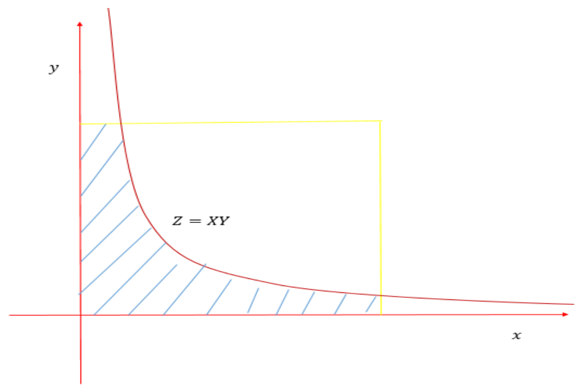


从图可知 $Z\in(0,1)$, 于是取 $z\in(0,1)$,

$$F_Z(z)=\int_0^{1-z}\int_x^{x+z}3ydydx+\int_{1-z}^1\int_x^1 3ydydx=-\frac{1}{2}z^3+\frac{3}{2}z.$$

$$\Rightarrow f_Z(z)=\begin{cases} -\frac{3}{2}z^2+\frac{3}{2}, & z\in(0,1) \\ 0, & 其它 \end{cases}$$

3.

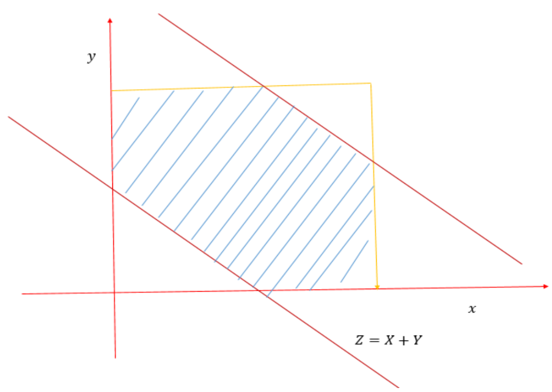


知 $Z\in(0,1)$, 取 $z\in(0,1).X,Y$ 独立同分布可得 $f(x,y)=1, 0<x<1, 0<y<1$.

$$F_Z(x)=\int_0^z\int_0^1 1dydx+\int_z^1\int_0^{\frac{z}{x}} 1dydx=z-z\ln z.$$

$$\Rightarrow f_Z(z)=\begin{cases} -\ln z, & z\in(0,1) \\ 0, & 其它 \end{cases}$$

4.



从图可知 $Z\in(0,2)$, 取 $z\in(0,1)$.

$$F_Z(z)=\int_0^z\int_0^{z-x} 1dydx=\frac{1}{2}x^2.$$

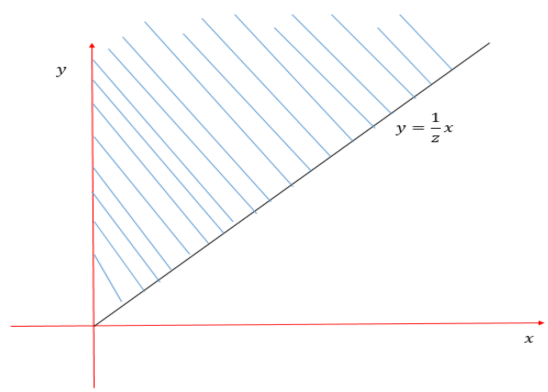
取 $z \in (1,2)$.

$$F_Z(z) = \int_0^{z-1}\int_0^1 1dydx + \int_{z-1}^1\int_0^{z-x} 1dydx = 2z - \frac{1}{2}z^2 - 1.$$

$$\Rightarrow f_Z(z) = \begin{cases} z, & z \in (0,1) \\ 2-z, & z \in (1,2) \\ 0, & \text{其它} \end{cases}$$

5.$Z = \frac{X}{Y} \in (0,+\infty)$. 取 $z \in (0,+\infty)$.

$$F_Z(z) = P(\frac{X}{Y} \le z) = P(Y \ge \frac{1}{z}X)$$



$$\Rightarrow F_Z(z) = \int_0^{+\infty}\int_{\frac{1}{z}x}^{+\infty} e^{-x-y}dydx = \frac{z}{z+1}.$$

$$\Rightarrow f_Z(z) = \begin{cases} \frac{1}{(z+1)^2}, & z \in (0,+\infty) \\ 0, & \text{其它} \end{cases}$$

6.(1) 串联. 设 T 为系统的使用寿命, 那么 $t \in (0,+\infty)$,

$$F(t) = P(T \le t) = P(\min(T_1, T_2, \cdot, T_5) \le t) = 1 - \Pi_{i=1}^5 P(T_i \ge t) = 1 - (e^{\frac{-t}{5}})^5 = 1 - e^{-t}.$$

$$\Rightarrow f(t) = \begin{cases} e^{-t}, & t \in (0,+\infty) \\ 0, & \text{其它} \end{cases}$$

$$P(T \geq 1) = 1 - F(1) = e^{-1}.$$

(2) 并联.

$$F(t) = P(\max(T_1, T_2, \cdots, T_5) \leq t) = \Pi_{i=1}^{5} P(T_i \leq t) = (1 - e^{-0.2t})^5.$$

$$\Rightarrow f(t) = \begin{cases} e^{-0.2t}(1 - e^{-0.2t})^4, & t \in (0, +\infty) \\ 0, & \text{其它} \end{cases}$$

$$P(T \geq 1) = 1 - F(1) = 1 - (1 - e^{-0.2})^5 \approx 99.98\%.$$