一、选择题

1.A

2.C

3.B

二、填空题

1.$\frac{9}{10}$.

2.$2a^2$.

3.$2^k\cdot k!$.

三、解答与证明题

1.

$$\begin{aligned}E(X)&=\int_{-1}^{1}\frac{1}{\pi}\frac{x}{\sqrt{1-x^2}}dx\\&=0.\\D(X)&=E(X^2)+[E(X)]^2=E(X^2).\\D(X)&=\int_{-1}^{1}\frac{1}{\pi}\frac{x^2}{\sqrt{1-x^2}}dx\\&=\frac{1}{2}.\end{aligned}$$

2. 因为 $X\sim U(0,\frac{1}{2})$, 所以 $E(X)=\int_0^{\frac{1}{2}}2xdx=\frac{1}{4}, D(X)=\int_0^{\frac{1}{2}}2(x-\frac{1}{4})^2dx=\frac{1}{48}$.

$$\begin{aligned}E(Y)&=E(2X^2)=2E(X^2)\\&=2\{[E(X)]^2+D(X)\}\\&=\frac{1}{6}.\\E(Y^2)&=E[(2X^2)^2]\\&=\int_0^{\frac{1}{2}}2(2x^2)dx\\&=\frac{1}{20}.\\D(Y)&=E(Y^2)-[E(Y)]^2\\&=\frac{1}{45}.\end{aligned}$$

3. 因为 $X \sim U(-1,3), Y \sim e(2), Z \sim \Gamma(2,2)$. 所以

$$E(X) = 1, \quad D(X) = \frac{4}{3}$$
$$E(Y) = \frac{1}{2}, \quad D(Y) = \frac{1}{4}$$
$$E(Z) = 1, \quad D(Z) = \frac{1}{2}.$$

则

(1)

$$\begin{aligned} E(U) &= E(3X - 2XY + 4YZ - 2) \\ &= 3E(X) - 2E(X)E(Y) + 4E(Y)E(Z) - 2 \\ &= 2. \end{aligned}$$

(2)

$$\begin{aligned} D(V) &= D(X - 2Y + 3Z - 2) \\ &= D(X) + 4D(Y) + 9D(Z) \\ &= \frac{41}{6}. \end{aligned}$$

4.

证明. 对任意常数 C, 设

$$f(C) = E(X-C)^2 = C^2 - 2[E(X)]C + E(X^2).$$

那么由二次函数的性质可知, 当 $C = -\frac{-2E(X)}{2\cdot 1} = E(X)$ 时,$f(C)$ 取最小值. 即

$$\begin{aligned} f(E(X)) = E[X - E(X)]^2 \le E(X-C)^2 \\ \Rightarrow D(X) \le E(X-C)^2. \end{aligned}$$

5. 设随机变量

$$X_i = \begin{cases} 1, & \text{第 } i \text{ 个部件需调整} \\ 0, & \text{第 } i \text{ 个部件不需调整} \end{cases}$$

$i = 1,2,3,4,5$. 则

$$\begin{aligned} E(X_i) &= P(X_i = 1) = \frac{i}{10}, \\ D(X_i) &= [1 - \frac{i}{10}]^2 \cdot P(X_i = 1) + [0 - \frac{i}{10}]^2 \cdot P(X_i = 0), \\ &= (1 - \frac{i}{10})^2 \cdot \frac{i}{10} + (0 - \frac{i}{10})(1 - \frac{i}{10}). \end{aligned}$$

于是 $X = \sum_{i=1}^{5} X_i$,

$$E(X) = E(\sum_{i=1}^{5} X_i) = \sum_{i=1}^{5} E(X_i) = \frac{3}{2},$$

$$D(X) = D(\sum_{i=1}^{5} X_i) = \sum_{i=1}^{5} D(X_i) = \frac{19}{20}.$$