一、选择题

1.A
2.C
3.C

二、填空题

1.0.2
2.0.12
3.0

三、解答题

1.(1)

$$\begin{aligned} P(X<3) &= \Phi(\frac{3-(-2)}{3}) \\ &= \Phi(\frac{5}{3}) \approx 0.9525 \end{aligned}$$

(2)

$$\begin{aligned} P(X<-3) &= \Phi(\frac{-3-(-2)}{3}) \\ &= \Phi(-\frac{1}{3}) \approx 0.3707 \end{aligned}$$

(3)

$$\begin{aligned} P(|X|<1.5) &= \Phi(\frac{1.5-(-2)}{3}) - \Phi(\frac{-1.5-(-2)}{3}) \\ &= \Phi(\frac{7}{6}) - \Phi(\frac{1}{6}) \approx 0.3134 \end{aligned}$$

(4)

$$\begin{aligned} P(|X-2|\geq 2) &= 1 - P(|X-2|<2) \\ &= 1 - [\Phi(\frac{4-(-2)}{3}) - \Phi(\frac{0-(-2)}{3})] \approx 0.7682 \end{aligned}$$

2. 设 A 表示事件” 新生儿体重小于 2719 克”,Y 表示事件” 新生儿体重小于

2719 克的个数”, 于是 $X \sim N(3315, 575^2), Y \sim B(100, p)$, 则

$$\begin{aligned} p = P(A) &= P(X < 2719) = \Phi(\frac{2719 - 3315}{575}) \approx 0.1515 \\ P(Y \geq 2) &= 1 - \binom{100}{0}(1-p)^{100} - \binom{100}{1}p(1-p)^{99} \\ &\approx 1. \end{aligned}$$

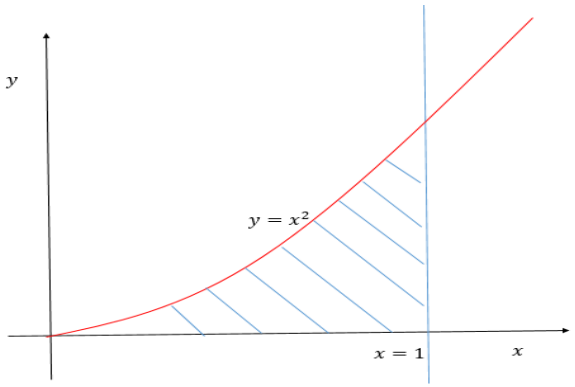
3. 取 $y \in (0, +\infty)$, 那么

$$\begin{aligned} F_Y(y) &= P(|X| \leq y) \\ &= \Phi(y) - \Phi(-y) \\ &= 2\Phi(y) - 1. \\ \Rightarrow f_Y(y) &= \begin{cases} \frac{\sqrt{2}}{\sqrt{\pi}} e^{-\frac{y^2}{2}}, & y > 0 \\ 0, & y \leq 0 \end{cases} \end{aligned}$$

4.(1) 因为 X 与 Y 相互独立, 且 $X \sim U(0,1), Y \sim e(\frac{1}{2})$, 那么

$$f(x,y) = \begin{cases} \frac{1}{2}e^{-\frac{1}{2}y}, & 0 < x < 1, y > 0 \\ 0, & \text{其它} \end{cases}$$

(2) 根据题目知 $\Delta = (2X)^2 - Y^2 \geq 0 \Rightarrow Y \leq X^2$. 如图



$$\begin{aligned} P(Y \leq X^2) &= \int_0^1 \int_0^{x^2} \frac{1}{2} e^{-\frac{1}{2}y} dy dx \\ &= 1 + \frac{\sqrt{\pi}}{2} - \sqrt{2\pi}\Phi(1) \\ &\approx 0.1445. \end{aligned}$$