

一、选择题

1.B

2.D

二、填空题

1. $\frac{1}{e}$

2. $C_3^1 \cdot (18\%) \cdot (82\%)^2$

三、解答题

(1) 不放回抽样:

$$\begin{aligned} X=0 & \quad P = \frac{C_{16}^6}{C_{20}^6} \approx 20.66\% \\ X=1 & \quad P = \frac{C_{16}^5 C_4^1}{C_{20}^6} \approx 45.08\% \\ X=2 & \quad P = \frac{C_{16}^4 C_4^2}{C_{20}^6} \approx 28.17\% \\ X=3 & \quad P = \frac{C_{16}^3 C_4^3}{C_{20}^6} \approx 5.78\% \\ X=4 & \quad P = \frac{C_{16}^2 C_4^4}{C_{20}^6} \approx 3.1\% \end{aligned}$$

可得分布律为

X	0	1	2	3	4
P	20.66%	45.08%	28.18%	5.78%	3.1%

(2) 有放回抽取就是独立重复试验, 于是有

$$P(X=k) = C_6^k p^k (1-p)^{6-k}$$

可计算得 $p = \frac{C_4^1}{C_6^1} = 20\%$. 可的分布律为

X	0	1	2	3	4	5	6
P	26.21%	39.32%	24.58%	8.19%	1.54%	0.15%	0.01%

2. 这是一个独立重复试验.

(1)

$$P(X=3) = C_{15}^3 (20\%)^3 (80\%)^{12} \approx 25.01\%$$

(2)

$$\begin{aligned} P(X \geq 2) &= 1 - P(X=0) - P(X=1) \\ &= 1 - C_{15}^0 (20\%)^0 (80\%)^{15} - C_{15}^1 (20\%)^1 (80\%)^{14} \approx 83.29\% \end{aligned}$$

(3)

$$P(1 \leq X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) \approx 61.30\%$$

(4)

$$P(X \leq 1) = P(X = 0) + P(X = 1) \approx 16.71\%.$$

3. 问题是 300 台车床只有 4 台出故障, 于是由二项分布, 概率为

$$P(X = 4) = C_{300}^4 (0.01)^4 (0.99)^{(300-4)} \approx 16.89\%$$

用泊松分布近似, 于是知 $\lambda = 300 \times 0.01 = 3$.

$$P(X = 4) \approx \frac{3^4}{4!} e^{-3} \approx 16.80\%$$

相对误差为

$$\frac{|16.89\% - 16.80\%|}{16.89\%} \approx 0.53\%$$

4.(1)

$$\begin{aligned} P(X > r) &= \sum_{i=r+1}^{\infty} (1-p)^{i-1} p \\ &= p(1-p)^r \sum_{i=0}^{\infty} (1-p)^i \\ &= p(1-p)^r \frac{1}{p} = (1-p)^r. \end{aligned}$$

(2)

$$\begin{aligned} P(X > r+t | X > r) &= \frac{P(X > r+t, X > r)}{P(X > r)} = \frac{P(X > r+t)}{P(X > r)} \\ &= \frac{(1-p)^{r+t}}{(1-p)^r} = (1-p)^t = P(X > t). \end{aligned}$$