一、填空题

1.$\frac{8}{9}$.
2.$\sqrt{\frac{m}{n+m}}$
3.$\frac{1}{2}$

二、解答题

1. 设 Y 表示收益,M(假设 $M \in [2000, 4000]$) 表示货源. 那么根据题意有

$$Y = \begin{cases} 4X - M, & X \le M \\ 3M, & X > M \end{cases}$$
$$\Rightarrow E(Y) = \int_{2000}^{M} (4x - M) \frac{1}{2000} dx + \int_{M}^{4000} 3M \cdot \frac{1}{2000} dx$$
$$= -\frac{M^2}{1000} + 7M - 4000$$
$$\Rightarrow M = 3500.$$

当 $M = 3500$ 时, 平均收益最大 8250 万美元.

2. 设

$$X_i = \begin{cases} 1, & \text{第 } i \text{ 个站有人下车} \\ 0, & \text{第 } i \text{ 个站没有人下车} \end{cases} \quad i = 1, 2, \cdots, 10.$$

那么,

$$p = P(\text{第 } i \text{ 站有人下车}) = 1 - \frac{9^{20}}{10^{20}} \approx 0.8784.$$

于是 $X_i \sim B(1, p)$.

设 $X \sum_{i=1}^{10} X_i$ 表示 10 个站停车的次数, 那么

$$E(X) = \sum_{i=1}^{10} E(X_i) = 10 \times 0.8784 = 8.784.$$

3.(1)

$$E(X) = \int_{-1}^{1} x \cdot |x| dx = 0;$$
$$E(X^2) = \int_{-1}^{1} x^2 \cdot |x| dx = \frac{1}{2};$$
$$\Rightarrow D(X) = E(X^2) - [E(X)]^2 = \frac{1}{2}.$$

(2)

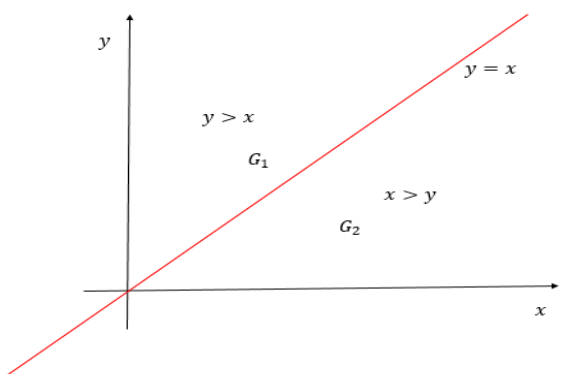
$$\begin{aligned}
E(X|X|) &= \int_{-1}^{1} x|x| \cdot |x| dx = 0 \\
E(X) &= 0; \\
\Rightarrow Cov(X, |X|) &= E(X|X|) - E(X) \cdot E(|X|) = 0.
\end{aligned}$$

(3) 显然 $R(X, |X|) = 0$, 所以 X 与 $|X|$ 不相关. 取 $u, v \in (0, 1)$, 则

$$\begin{aligned}
P(X \leq u, |X| \leq v) &= P(-v \leq X \leq \min(u, v)) \\
&= \int_{-v}^{\min(u,v)} |x| dx \\
&\Rightarrow \begin{cases} \frac{1}{2}(u^2 + v^2), & u \leq v \\ v^2, & u > v \end{cases} \\
P(X \leq u) &= \int_{-1}^{u} |x| dx \\
&= \frac{1}{2}(1 + u^2) \\
P(|X| \leq v) &= \int_{-v}^{v} |x| dx \\
&= v^2.
\end{aligned}$$

显然 $P(X \leq u, |X| \leq v) \neq P(X \leq u) \cdot P(|X| \leq v)$, 所以 X 与 $|X|$ 不独立.

4.



X, Y 独立同分布与 $N(0, 1), M = \max(X, Y), N = \min(X, Y)$. 那么, 联合

密度函数为 $f(x,y)=\frac{1}{2\pi}\exp(-\frac{1}{2}x^2-\frac{1}{2}y^2)$, 则

$$
\begin{aligned}
E(M) &= \iint_{G_1} \max(x,y)f(x,y)dxdy + \iint_{G_2} \max(x,y)f(x,y)dxdy \\
&= \int_{-\infty}^{\infty}\int_{x}^{\infty} yf(x,y)dydx + \int_{-\infty}^{\infty}\int_{-\infty}^{x} xf(x,y)dydx \\
&= \frac{1}{2\sqrt{\pi}} + \frac{1}{2\sqrt{\pi}} \\
&= \frac{1}{\sqrt{\pi}} \\
E(N) &= \iint_{G_1} \min(x,y)f(x,y)dydx + \iint_{G_2} \min(x,y)f(x,y)dydx \\
&= \int_{-\infty}^{\infty}\int_{x}^{\infty} xf(x,y)dydx + \int_{-\infty}^{\infty}\int_{-\infty}^{x} yf(x,y)dydx \\
&= -\frac{1}{2\sqrt{\pi}} - \frac{1}{2\sqrt{\pi}} \\
&= -\frac{1}{\sqrt{\pi}}
\end{aligned}
$$