一、选择题

1.B
2.C
3.B

二、填空题

1.$\frac{1}{3}, 7, N(\frac{1}{3}, 7)$.
3.$\frac{\sqrt{2\pi}}{2a}$

三、解答题

1. 取 $z \in (0, \infty)$, 其中使用极坐标 $x = r\cos(\theta), y = r\sin(\theta)$, 那么

$$\begin{aligned} F_Z(z) &= P(\frac{1}{2}m(X^2+Y^2) \le z) \\ &= P(r^2 \le \frac{2\pi}{m}) \\ &= \int_0^{2\pi}\int_0^{\sqrt{\frac{2\pi}{m}}} \frac{1}{2\pi\sigma^2}\exp(-\frac{r^2}{2\sigma^2}) r dr d\theta \\ &= 1 - \exp(-\frac{z}{m\sigma^2}). \\ \Rightarrow f_Z(z) &= \begin{cases} \frac{z}{m\sigma^2}\exp(-\frac{z}{m\sigma^2}), & z > 0 \\ 0, & z \le 0 \end{cases} \end{aligned}$$

2. 每个 X_i 的变异系数为 1, 说明 $\frac{\sqrt{D(X_i)}}{|E(X_i)|} = 1 \Rightarrow \sigma_i^2 = i^2, i = 1, 2, 3, 4$. 因为 $X_i \sim N(i, \sigma_i^2)$, 且相互独立, 那么

$$\begin{aligned} X &= X_1 + X_2 + X_3 + X_4 \\ \Rightarrow X &\sim N(10, 30). \\ \Rightarrow P(2 < X < 18) &= \Phi(\frac{18-10}{\sqrt{30}}) - \Phi(\frac{2-10}{\sqrt{30}}) \\ &= 2\Phi(\frac{8}{\sqrt{30}}) - 1 \approx 0.8556. \end{aligned}$$