一、选择题

1.C
2.B
3.B

二、填空题

1.$a=2,b=8.$
2.780
3.$E(X)=-\frac{67}{45},E(X^2)=\frac{143}{15}.$

三、解答题

1. 由题意可知, 分数 X 的取值为 $1,2,3,4,5,7,8,9,10,11,12.$. 显然,

$$\begin{array}{ll} P(X=i)=\frac{1}{6}, & i=1,\cdots,5. \\ P(X=i)=\frac{1}{36}, & i=7,\cdots,12. \end{array}$$

于是 X 的分布律为

$$X\sim\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 7 & 8 & 9 & 10 & 11 & 12 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} \end{pmatrix}$$

2.X 的分布函数为

$$F(x)=\begin{cases} 0, & x\le 5, \\ A-\frac{B}{x^2}, & x>5. \end{cases}$$

根据分布函数的性质: 右连续和规范性. 可得 $\lim_{x\to 5+0}F(x)=F(5)$ 和 $F(+\infty)=1$. 可得

$$\begin{cases} A-\frac{B}{25}=0, \\ \lim_{x\to+\infty}(A-\frac{B}{x^2})=1. \end{cases}$$

$$\Rightarrow A=1,B=25.$$

立即可得 X 的密度函数为

$$f(x)=\begin{cases} 0, & x\le 5, \\ \frac{50}{x^3}, & x>5. \end{cases}$$

于是

$$E(X)=\int_{-\infty}^{+\infty}xf(x)dx=\int_{5}^{+\infty}x\cdot\frac{50}{x^3}dx=10.$$

3. 设 Y 表示一台设备的可获得的利润, 由题可知

$$Y=\begin{cases}1000, & X>1,\\ 600, & X\le 1.\end{cases}$$

因为 $X\sim e(0.25)$, 于是 $P(X\le 1)=1-e^{-0.25}, P(X>1)=e^{-0.25}$.

则

$$E(Y)=1000\cdot P(X>1)+600\cdot P(X\le 1)=688.4797$$

4. 因为 $X\sim e(1), Y\sim \Gamma(2,1)$, 所以 $E(X)=1, D(X)=1, E(Y)=2$.

(1) $E(X-Y)=E(X)-E(Y)=1-2=-1$.

(2)

$$E(Y^3)=\int_{0}^{+\infty}\frac{1^2}{\Gamma(2)}y^3\cdot y^{2-1}e^{-1\cdot y}dy=24.$$

$$E(2X^2+3Y^3)=2E(X^2)+3E(Y^3)=2[(E(X))^2+D(X)]+3\cdot 24=76.$$

(3) 因为 X 和 Y 独立, 所以

$$E(XY)=E(X)E(Y)=2.$$

5. 由题可知, 调整设备的次数 X 的可能取值为 $0,1,2,3,4$. 设 p 表示每次检测需要调整的概率, 因此有

$$p=1-\binom{10}{0}(0.9)^{10}(0.1)^0-\binom{10}{1}(0.9)^9(0.1)^1=26.39\%.$$

于是 $X\sim B(4,p)$ 或 $X\sim B(4,26.39\%)$.

则 $E(X)=4\cdot p=4\cdot 26.39\%=1.0556$.

一、选择题

1.A

2.C

3.B

二、填空题

1.$\frac{9}{10}$.

2.$2a^2$.

3.$2^k \cdot k!$.

三、解答与证明题

1.

$$\begin{aligned} E(X) &= \int_{-1}^{1} \frac{1}{\pi} \frac{x}{\sqrt{1-x^2}} dx \\ &= 0. \\ D(X) &= E(X^2) + [E(X)]^2 = E(X^2). \\ D(X) &= \int_{-1}^{1} \frac{1}{\pi} \frac{x^2}{\sqrt{1-x^2}} dx \\ &= \frac{1}{2}. \end{aligned}$$

2. 因为 $X \sim U(0, \frac{1}{2})$, 所以 $E(X) = \int_0^{\frac{1}{2}} 2x dx = \frac{1}{4}, D(X) = \int_0^{\frac{1}{2}} 2(x-\frac{1}{4})^2 dx = \frac{1}{48}$.

$$\begin{aligned} E(Y) &= E(2X^2) = 2E(X^2) \\ &= 2\{[E(X)]^2 + D(X)\} \\ &= \frac{1}{6}. \\ E(Y^2) &= E[(2X^2)^2] \\ &= \int_0^{\frac{1}{2}} 2(2x^2) dx \\ &= \frac{1}{20}. \\ D(Y) &= E(Y^2) - [E(Y)]^2 \\ &= \frac{1}{45}. \end{aligned}$$

3. 因为 $X \sim U(-1,3), Y \sim e(2), Z \sim \Gamma(2,2)$. 所以

$$\begin{aligned} E(X) = 1, \quad D(X) = \frac{4}{3} \\ E(Y) = \frac{1}{2}, \quad D(Y) = \frac{1}{4} \\ E(Z) = 1, \quad D(Z) = \frac{1}{2}. \end{aligned}$$

则

(1)

$$\begin{aligned} E(U) &= E(3X - 2XY + 4YZ - 2) \\ &= 3E(X) - 2E(X)E(Y) + 4E(Y)E(Z) - 2 \\ &= 2. \end{aligned}$$

(2)

$$\begin{aligned} D(V) &= D(X - 2Y + 3Z - 2) \\ &= D(X) + 4D(Y) + 9D(Z) \\ &= \frac{41}{6}. \end{aligned}$$

4.

证明. 对任意常数 C, 设

$$f(C) = E(X-C)^2 = C^2 - 2[E(X)]C + E(X^2).$$

那么由二次函数的性质可知, 当 $C = -\frac{-2E(X)}{2\cdot 1} = E(X)$ 时,$f(C)$ 取最小值. 即

$$\begin{aligned} f(E(X)) = E[X - E(X)]^2 \le E(X-C)^2 \\ \Rightarrow D(X) \le E(X-C)^2. \end{aligned}$$

5. 设随机变量

$$X_i = \begin{cases} 1, & \text{第 } i \text{ 个部件需调整} \\ 0, & \text{第 } i \text{ 个部件不需调整} \end{cases}$$

$i = 1,2,3,4,5$. 则

$$\begin{aligned} E(X_i) &= P(X_i = 1) = \frac{i}{10}, \\ D(X_i) &= [1 - \frac{i}{10}]^2 \cdot P(X_i = 1) + [0 - \frac{i}{10}]^2 \cdot P(X_i = 0), \\ &= (1 - \frac{i}{10})^2 \cdot \frac{i}{10} + (0 - \frac{i}{10})(1 - \frac{i}{10}). \end{aligned}$$

于是 $X = \sum_{i=1}^{5} X_i$,

$$E(X) = E(\sum_{i=1}^{5} X_i) = \sum_{i=1}^{5} E(X_i) = \frac{3}{2},$$
$$D(X) = D(\sum_{i=1}^{5} X_i) = \sum_{i=1}^{5} D(X_i) = \frac{19}{20}.$$

一、选择题

1.B
2.A
3.C

二、填空题

1.$\frac{4}{3}+\frac{\sqrt{3}}{6}$.
2.9
3.$\frac{2}{3}\sigma^2$.

三、解答与证明题

1. 已知

$$V=\begin{pmatrix} DX & Cov(X,Y) \\ Cov(X,Y) & DY \end{pmatrix}$$
$$R(X,Y)=\frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}}$$

可以求得

$$f_X(x)=\frac{3}{2}-x \quad f_Y(y)=\frac{3}{2}-y$$

于是

$$EX=\int_0^1 x(\frac{3}{2}-x)dx=\frac{5}{12}$$
$$EY=\int_0^1 y(\frac{3}{2}-y)dy=\frac{5}{12}$$
$$DX=\int_0^1 (x-\frac{5}{12})^2(\frac{3}{2}-x)dx=\frac{11}{144}$$
$$DY=\int_0^1 (y-\frac{5}{12})^2(\frac{3}{2}-y)dy=\frac{11}{144}$$
$$EXY=\int_0^1 xy(2-x-y)dxdy=\frac{1}{6}$$
$$Cov(X,Y)=E(XY)-(EX)(EY)=-\frac{1}{144}$$

即得

$$V = \begin{pmatrix} \frac{11}{144} & -\frac{1}{144} \\ -\frac{1}{144} & \frac{11}{144} \end{pmatrix}$$

$$R(X,Y) = \frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}} = -\frac{1}{11}.$$

2. 由题可知密度函数为

$$f(x,y) = \begin{cases} 1, & (x,y) \in G \\ 0, & \text{其它} \end{cases}$$

求得边缘密度函数为

$$f_X(x) = 2 - 2x$$
$$f_Y(y) = 1 - \frac{y}{2}$$

(1)

$$EX = \int_0^1 x(2-2x)dx = \frac{1}{3}$$
$$EY = \int_0^2 y(1-\frac{y}{2})dy = \frac{2}{3}$$

(2)

$$DX = \int_0^1 (x-\frac{1}{3})^2(2-2x)dx = \frac{1}{18}$$
$$DY = \int_0^2 (y-\frac{2}{3})^2(1-\frac{y}{2}) = \frac{2}{9}$$

(3)

$$E(XY) = \iint_G xyf(x,y)dxdy = \int_0^1 \int_0^{2-2x} xy \cdot 1dydx = \frac{1}{6}$$

$$\Rightarrow Cov(X,Y) = E(XY) - EXEY = -\frac{1}{18}$$

$$R(X,Y) = \frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}} = -\frac{1}{2}$$

(4) 因为 $R(X,Y) \neq 0$, 所以 X, Y 相关, 且不独立.

3. 根据协方差的性质展开可证.

4. 已知

$$EX = EY = \mu, \quad DX = DY = \sigma^2.$$

$$\begin{aligned} DU &= D(\alpha X + \beta Y) \\ &= \alpha^2 DX + \beta^2 DY = \alpha^2\sigma^2 + \beta^2\sigma^2; \\ DV &= D(\alpha X - \beta Y) \\ &= \alpha^2 DX + \beta^2 DY = \alpha^2\sigma^2 + \beta^2\sigma^2. \\ Cov(U, V) &= D(\alpha X) - D(\beta Y) \\ &= (\alpha^2 - \beta^2)\sigma^2 \\ \Rightarrow R(X, Y) &= \frac{\alpha^2 - \beta^2}{\alpha^2 + \beta^2}. \end{aligned}$$

5 由题可得 X, Y 的分布律为

$$X \sim \begin{pmatrix} 1 & 2 & 3 \\ \frac{4}{9} & \frac{3}{9} & \frac{2}{9} \end{pmatrix}$$

$$Y \sim \begin{pmatrix} 0 & 1 & 2 \\ \frac{5}{9} & \frac{3}{9} & \frac{1}{9} \end{pmatrix}$$

(1)

$$\begin{aligned} EX &= 1 \times \frac{4}{9} + 2 \times \frac{3}{9} + 3 \times \frac{2}{9} = \frac{16}{9} \\ EY &= 0 \times \frac{5}{9} + 1 \times \frac{3}{9} + 2 \times \frac{1}{9} = \frac{5}{9} \\ DX &= (1 - \frac{16}{9})^2 \times \frac{4}{9} + (2 - \frac{16}{9})^2 \times \frac{3}{9} + (3 - \frac{16}{9})^2 \times \frac{2}{9} = \frac{50}{81} \\ DY &= (0 - \frac{5}{9})^2 \times \frac{5}{9} + (1 - \frac{5}{9})^2 \times \frac{3}{9} + (2 - \frac{5}{9})^2 \times \frac{1}{9} = \frac{38}{81} \end{aligned}$$

(2)

$$\begin{aligned} E(XY) &= 1 \times 1 \times \frac{2}{9} + 1 \times 2 \times \frac{1}{9} + 2 \times 1 \times \frac{1}{9} = \frac{2}{3} \\ Cov(X, Y) &= E(XY) - EXEY = -\frac{26}{81} \\ R(X, Y) &= \frac{Cov(X, Y)}{\sqrt{DX}\sqrt{DY}} = -\frac{13}{5\sqrt{19}} \end{aligned}$$

(3)

$$D(X - 3Y) = DX + 9DY - 6Cov(X, Y) = \frac{548}{81}.$$