一、选择题

1.A
2.C
3.C

二、填空题

1.0.2
2.0.12
3.0

三、解答题

1.(1)

$$\begin{aligned}P(X<3)&=\Phi(\frac{3-(-2)}{3})\\&=\Phi(\frac{5}{3})\approx 0.9525\end{aligned}$$

(2)

$$\begin{aligned}P(X<-3)&=\Phi(\frac{-3-(-2)}{3})\\&=\Phi(-\frac{1}{3})\approx 0.3707\end{aligned}$$

(3)

$$\begin{aligned}P(|X|<1.5)&=\Phi(\frac{1.5-(-2)}{3})-\Phi(\frac{-1.5-(-2)}{3})\\&=\Phi(\frac{7}{6})-\Phi(\frac{1}{6})\approx 0.3134\end{aligned}$$

(4)

$$\begin{aligned}P(|X-2|\geq 2)&=1-P(|X-2|<2)\\&=1-[\Phi(\frac{4-(-2)}{3})-\Phi(\frac{0-(-2)}{3})]\approx 0.7682\end{aligned}$$

2. 设 A 表示事件" 新生儿体重小于 2719 克",Y 表示事件" 新生儿体重小于

2719 克的个数", 于是 $X \sim N(3315, 575^2), Y \sim B(100, p)$, 则

$$\begin{aligned} p = P(A) = P(X < 2719) &= \Phi(\frac{2719-3315}{575}) \approx 0.1515 \\ P(Y \geq 2) = 1 - \binom{100}{0}(1-p)^{100} &- \binom{100}{1}p(1-p)^{99} \\ \approx 1. \end{aligned}$$

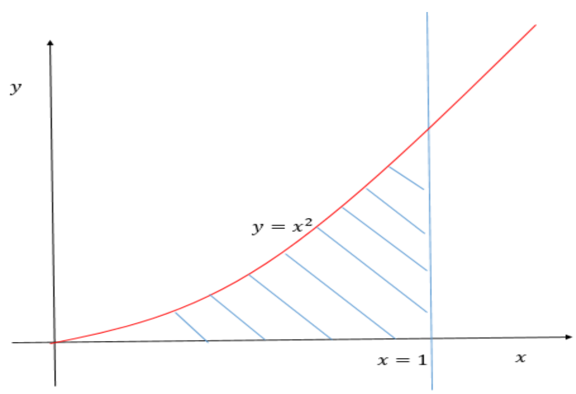
3. 取 $y \in (0, +\infty)$, 那么

$$\begin{aligned} F_Y(y) &= P(|X| \leq y) \\ &= \Phi(y) - \Phi(-y) \\ &= 2\Phi(y) - 1. \\ \Rightarrow f_Y(y) &= \begin{cases} \frac{\sqrt{2}}{\sqrt{\pi}} e^{-\frac{y^2}{2}}, & y > 0 \\ 0, & y \leq 0 \end{cases} \end{aligned}$$

4.(1) 因为 X 与 Y 相互独立, 且 $X \sim U(0,1), Y \sim e(\frac{1}{2})$, 那么

$$f(x,y) = \begin{cases} \frac{1}{2}e^{-\frac{1}{2}y}, & 0 < x < 1, y > 0 \\ 0, & \text{其它} \end{cases}$$

(2) 根据题目知 $\Delta = (2X)^2 - Y^2 \geq 0 \Rightarrow Y \leq X^2$. 如图



$$\begin{aligned} P(Y \leq X^2) &= \int_0^1 \int_0^{x^2} \frac{1}{2} e^{-\frac{1}{2}y} dy dx \\ &= 1 + \frac{\sqrt{\pi}}{2} - \sqrt{2\pi}\Phi(1) \\ &\approx 0.1445. \end{aligned}$$

一、选择题

1.B
2.C
3.B

二、填空题

1.$\frac{1}{3}, 7, N(\frac{1}{3}, 7)$.
3.$\frac{\sqrt{2\pi}}{2a}$

三、解答题

1. 取 $z \in (0, \infty)$, 其中使用极坐标 $x = r\cos(\theta), y = r\sin(\theta)$, 那么

$$\begin{aligned} F_Z(z) &= P(\frac{1}{2}m(X^2+Y^2) \le z) \\ &= P(r^2 \le \frac{2\pi}{m}) \\ &= \int_0^{2\pi}\int_0^{\sqrt{\frac{2\pi}{m}}} \frac{1}{2\pi\sigma^2}\exp(-\frac{r^2}{2\sigma^2}) r dr d\theta \\ &= 1 - \exp(-\frac{z}{m\sigma^2}). \\ \Rightarrow f_Z(z) &= \begin{cases} \frac{z}{m\sigma^2}\exp(-\frac{z}{m\sigma^2}), & z > 0 \\ 0, & z \le 0 \end{cases} \end{aligned}$$

2. 每个 X_i 的变异系数为 1, 说明 $\frac{\sqrt{D(X_i)}}{|E(X_i)|} = 1 \Rightarrow \sigma_i^2 = i^2, i = 1,2,3,4$. 因为 $X_i \sim N(i, \sigma_i^2)$, 且相互独立, 那么

$$\begin{aligned} X &= X_1 + X_2 + X_3 + X_4 \\ \Rightarrow X &\sim N(10, 30). \\ \Rightarrow P(2 < X < 18) &= \Phi(\frac{18-10}{\sqrt{30}}) - \Phi(\frac{2-10}{\sqrt{30}}) \\ &= 2\Phi(\frac{8}{\sqrt{30}}) - 1 \approx 0.8556. \end{aligned}$$

一、填空题

1.$\frac{8}{9}$.
2.$\sqrt{\frac{m}{n+m}}$
3.$\frac{1}{2}$

二、解答题

1. 设 Y 表示收益,M(假设 $M \in [2000, 4000]$) 表示货源. 那么根据题意有

$$
\begin{aligned}
&Y = \begin{cases} 4X - M, & X \le M \\ 3M, & X > M \end{cases} \\
&\Rightarrow E(Y) = \int_{2000}^{M} (4x - M)\frac{1}{2000}dx + \int_{M}^{4000} 3M \cdot \frac{1}{2000}dx \\
&= -\frac{M^2}{1000} + 7M - 4000 \\
&\Rightarrow M = 3500.
\end{aligned}
$$

当 $M = 3500$ 时, 平均收益最大 8250 万美元.

2. 设

$$X_i = \begin{cases} 1, & \text{第 } i \text{ 个站有人下车} \\ 0, & \text{第 } i \text{ 个站没有人下车} \end{cases} \quad i = 1, 2, \cdots, 10.$$

那么,

$$p = P(\text{第 } i \text{ 站有人下车}) = 1 - \frac{9^{20}}{10^{20}} \approx 0.8784.$$

于是 $X_i \sim B(1, p)$.

设 $X \sum_{i=1}^{10} X_i$ 表示 10 个站停车的次数, 那么

$$E(X) = \sum_{i=1}^{10} E(X_i) = 10 \times 0.8784 = 8.784.$$

3.(1)

$$
\begin{aligned}
E(X) &= \int_{-1}^{1} x \cdot |x| dx = 0; \\
E(X^2) &= \int_{-1}^{1} x^2 \cdot |x| dx = \frac{1}{2}; \\
\Rightarrow D(X) &= E(X^2) - [E(X)]^2 = \frac{1}{2}.
\end{aligned}
$$

(2)

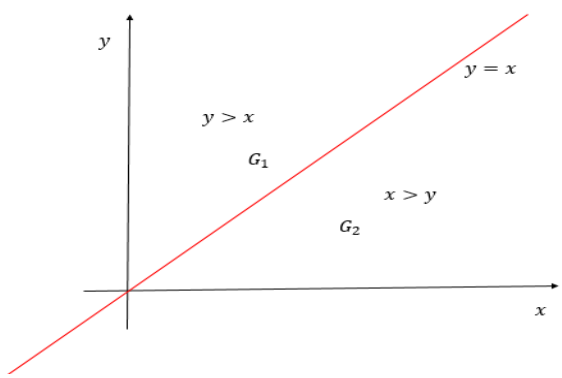
$$E(X|X|) = \int_{-1}^{1} x|x| \cdot |x| dx = 0$$
$$E(X) = 0;$$
$$\Rightarrow Cov(X, |X|) = E(X|X|) - E(X) \cdot E(|X|) = 0.$$

(3) 显然 $R(X, |X|) = 0$, 所以 X 与 $|X|$ 不相关. 取 $u, v \in (0, 1)$, 则

$$\begin{aligned} P(X \le u, |X| \le v) &= P(-v \le X \le \min(u, v)) \\ &= \int_{-v}^{\min(u,v)} |x| dx \\ &\Rightarrow \begin{cases} \frac{1}{2}(u^2 + v^2), & u \le v \\ v^2, & u > v \end{cases} \\ P(X \le u) &= \int_{-1}^{u} |x| dx \\ &= \frac{1}{2}(1 + u^2) \\ P(|X| \le v) &= \int_{-v}^{v} |x| dx \\ &= v^2. \end{aligned}$$

显然 $P(X \le u, |X| \le v) \ne P(X \le u) \cdot P(|X| \le v)$, 所以 X 与 $|X|$ 不独立.

4.



X, Y 独立同分布与 $N(0, 1), M = \max(X, Y), N = \min(X, Y)$. 那么, 联合

密度函数为 $f(x,y)=\frac{1}{2\pi}\exp(-\frac{1}{2}x^2-\frac{1}{2}y^2)$, 则

$$
\begin{aligned}
E(M) &= \iint_{G_1} \max(x,y)f(x,y)dxdy + \iint_{G_2} \max(x,y)f(x,y)dxdy \\
&= \int_{-\infty}^{\infty}\int_{x}^{\infty} yf(x,y)dydx + \int_{-\infty}^{\infty}\int_{-\infty}^{x} xf(x,y)dydx \\
&= \frac{1}{2\sqrt{\pi}} + \frac{1}{2\sqrt{\pi}} \\
&= \frac{1}{\sqrt{\pi}} \\
E(N) &= \iint_{G_1} \min(x,y)f(x,y)dydx + \iint_{G_2} \min(x,y)f(x,y)dydx \\
&= \int_{-\infty}^{\infty}\int_{x}^{\infty} xf(x,y)dydx + \int_{-\infty}^{\infty}\int_{-\infty}^{x} yf(x,y)dydx \\
&= -\frac{1}{2\sqrt{\pi}} - \frac{1}{2\sqrt{\pi}} \\
&= -\frac{1}{\sqrt{\pi}}
\end{aligned}
$$