一、填空题

1.C
2.C
3.A

二、填空题

1.$\leq \frac{1}{n\lambda}$
2.$\frac{1}{12}$

三、解答与证明题

1. 由题可知 $X \sim \Gamma(m+1,1)$, 于是 $E(X)=m+1, D(X)=m+1$. 那么,

$$\begin{aligned} P(0<X<2(m+1)) &= P(-(m+1)<X-(m+1)<m+1) \\ &= P(|X-E(X)|<m+1) \end{aligned}$$

根据切比雪夫不等式有

$$\begin{aligned} P(|X-E(X)|<m+1) &\geq 1-\frac{D(X)}{(m+1)^2} \\ &= \frac{m}{m+1}. \end{aligned}$$

2. 随机变量序列 $\{X_k\}, X_k \sim U(0,\theta)$, 于是 $E(X_k)=\frac{\theta}{2}, D(X_k)=\frac{\theta^2}{12}$. 根据切比雪夫大数定律, 有

(1)

$$\begin{aligned} \overline{X} = \frac{1}{n}\sum_{k=1}^{n} X_k &\xrightarrow{P} \frac{1}{n}\sum_{k=1}^{n} E(X_k) \\ \Rightarrow \overline{X} &\xrightarrow{P} \frac{\theta}{2}. \end{aligned}$$

(2)

$$\begin{aligned} Y_n = \frac{1}{n}\sum_{k=1}^{n} X_k^4 &\xrightarrow{P} \frac{1}{n}\sum_{k=1}^{n} E(X_k^4) \\ \Rightarrow Y_n &\xrightarrow{P} \frac{\theta^4}{5}. \end{aligned}$$

(3)

$$\begin{aligned} Z_n = \frac{2}{n}\sum_{k=1}^{n} X_k &\xrightarrow{P} \frac{2}{n}\sum_{k=1}^{n} E(X_k) \\ \Rightarrow Z_n &\xrightarrow{P} \theta. \end{aligned}$$

3. 设 $X_i, i = 1, 2, \cdots, 200$ 表示灯泡的寿命, 则 $X_i \sim e(5)$. 设 Y 表示 200 个灯泡的平均寿命, 则

$$E(X_i) = \frac{1}{5}, \quad D(X_i) = \frac{1}{25}$$

$$Y = \frac{1}{200}\sum_{k=1}^{200} X_k.$$

于是根据中心极限定理,$Y \sim N(\frac{1}{5}, \frac{1}{200 \cdot 25})$.

$$\begin{aligned} P(Y > 0.21) &= 1 - P(Y \geq 0.21) \\ &= 1 - \Phi(\frac{0.21 - 0.2}{\sqrt{\frac{1}{200 \cdot 25}}}) \\ &\approx 0.2389. \end{aligned}$$

5.(1) 设

$$X_i = \begin{cases} 1, & \text{第 } i \text{ 个元件正常工作} \\ 0, & \text{第 } i \text{ 个元件不正常工作} \end{cases} \quad i = 1, 2, \cdots, 100.$$

那么 $X_i \sim B(1, 0.9)$. 则 $Y = \sum_{i=1}^{100} X_i$ 表示正常工作的元件个数, 那么 $E(Y) = 90, D(Y) = 9$. 则根据中心极限定理 $Y \sim N(90, 9)$.

$$\begin{aligned} P(Y \geq 85) &= 1 - \Phi(\frac{85 - 90}{3}) \\ &\approx 0.9515. \end{aligned}$$

(2) 由题意可知 $Y = \sum_{i=1}^{n} X_i$, 于是 $Y \sim N(0.9n, 0.09n)$.

$$\begin{aligned} &P(Y \geq 0.8n) \geq 0.95 \\ &\qquad\qquad \Rightarrow 1 - \Phi(\frac{0.8n - 0.9n}{\sqrt{0.09n}}) \geq 0.95 \\ &\qquad\qquad \Rightarrow n \geq 25. \end{aligned}$$