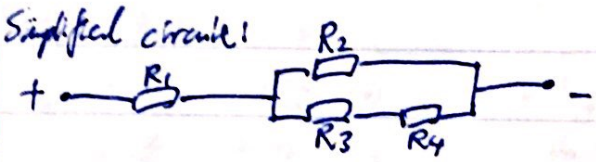
Homework #4.

1. Mechanical Circuits

Simplified circuit:



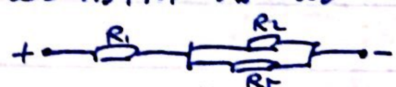
$V = 10V$. $R_1 = 3\Omega$, $R_2 = 3\Omega$.

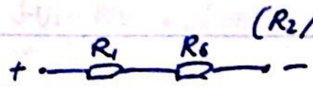
$R_3 = 4.5\Omega$. $R_4 = 1.5\Omega$

$\Downarrow$

Total $R = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3+R_4}\right)^{-1} = 3\Omega + \left(\frac{1}{3} + \frac{1}{6}\right)^{-1} = 5\Omega$

let $R_3 + R_4 = R_5 = 6\Omega$ simplify $\Rightarrow$ (R_2/R_5 system $= R_6 = 2\Omega$)





For R_1: $I_1 = \frac{V}{R_{total}} = \frac{10V}{5\Omega} = 2A$. $V_1 = \frac{R_1}{R_1+R_6} \cdot V = \frac{3}{3+2} \times 10 = 6V$

For R_2: $I_2 = \frac{R_5}{R_2+R_5} \cdot I_1 = \frac{6}{9} \cdot 2A = \frac{4}{3}A$. $V_2 = \frac{R_6}{R_{total}} \cdot V = \frac{2\Omega}{5\Omega} \cdot 10V = 4V$

For R_3: $I_3 = \frac{R_2}{R_2+R_5} \cdot I_1 = \frac{3}{9} \cdot 2A = \frac{2}{3}A$. $V_3 = \frac{R_3}{R_3+R_4} \cdot V_2 = \frac{4.5}{6} \times 4V = 3V$

For R_4: $I_4 = I_3 = \frac{2}{3}A$. $V_4 = \frac{R_4}{R_3+R_4} V_2 = \frac{1.5}{6} \times 4V = 1V$

2. Cell Phone Battery.

a) $Q = It \Rightarrow t = Q/I = 2770mAh / \frac{0.4W}{3.8V} = 26.315h$.

b) $1mAh = 3600mAs = 3.6C \Rightarrow 2770mAh = 2770 \times 3.6C = 9972C$

\# of Electrons $= 9972C / 1.602 \times 10^{-19} = 6.2247 \times 10^{22}$ electrons

c) $W = Pt = 0.4W \cdot 26.315h \times 3600 s/h = 37893.6J$

d) Cost $= W_{total} \times 0.16\$/kwh = 37893.6J / 3600 s/h \times 31 \times 0.16\$/kwh \doteq \$0.0522$

e) $P_{bat} = \frac{V^2}{R} = \left(\frac{R_{bat}}{200m\Omega + R_{bat}} \times 5V\right)^2 / R_{bat} = \left[\frac{25 R_{bat}}{(200m\Omega + R_{bat})^2}\right] W$

① $R_{bat} = 1m\Omega$. $P_{bat} = \frac{25 \times 1m\Omega}{(201 \times 10^{-3}\Omega)^2} = 0.6188W$. $t = \frac{37893.6Ws}{0.6188} \approx 17h$.

② $R_{bat} = 1\Omega$. $P_{bat} = \frac{25 \times 1\Omega}{(1.2)^2} = 17.36W$. $t = \frac{37893.6Ws}{17.36} \approx 2182$ seconds.

③ $R_{bat} = 10k\Omega$. $P_{bat} = \frac{25 \times 10^4}{(10000.2)^2} \approx 25 \times 10^{-4} W$. $t = \frac{37893.6Ws}{0.0025} = 15157804.6s \approx 175$ days

3. Nodal Analysis

a) KCL for Node 1 & 2

① $1 - \frac{V_1}{R_1} - \frac{V_1 - V_2}{R_2} = 0$

$1 - \frac{V_1}{10} - \frac{V_1 - V_2}{20} = 0$

$20 - 3V_1 + V_2 = 0$

$3V_1 - V_2 = 20$

② $\frac{V_1 - V_2}{R_2} + \frac{0 - V_2}{R_3} + (-3) = 0$

$\frac{V_1 - V_2}{20} + \frac{-V_2}{50} = 3$

$5V_1 - 5V_2 - 2V_2 = 300$

$5V_1 - 7V_2 = 300$

$$\begin{bmatrix} 3 & -1 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 100 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 3 & -1 & 20 \\ 5 & -7 & 300 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 16 & 0 & -160 \\ 5 & -7 & 300 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & -10 \\ 0 & 1 & -50 \end{array}\right]$$

⇒ Thus $V_1 = -10V$ $V_2 = -50V$

Verification by superposition.

Loop A (consider else "1A" current on left) Loop B ("3A" on the right)

(denote current pass $R_1 \Rightarrow I_1$, $R_3 \Rightarrow I_3$)

Thus Loop A ($I_A = 1A$)

$I_1^A = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \cdot I_A$

$= \frac{70}{80} \cdot 1A = 0.875A$

$I_3^A = I_A - I_1^A = 0.125A$

$V_1^A = I_1^A \cdot R_1 = 0.875 \times 10\Omega = 8.75V$

$V_2^A = I_3^A R_3 = 0.125 \times 50 = 6.25V$

Loop B ($I_B = 3A$)

$I_1^B = \frac{R_3}{R_1 + R_2 + R_3} \cdot I_B$

$= \frac{50}{80} \cdot 3A = 1.875A$

$I_3^B = I_B - I_1^B = 1.125A$

$V_1^B = I_1^B \cdot R_1 = 0.625A \cdot 10\Omega = 18.75V$

$V_3^B = I_3^B \cdot R_3 = 1.125A \cdot 50\Omega = 56.25V$

Set direction of I_A as positive direction

Then $V_1 = V_1^A - V_1^B = 8.75V - 18.75V = -10V$

$V_2 = V_2^A - V_2^B = 6.25V - 56.25V = -50V$

⇒ Same as the previous answer.

b) KCL for node 1 & 2

① $\frac{V_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2 - 10V}{R_3 + R_4} - 10V_x = 0$

$\frac{V_1}{10} + \frac{V_1}{10} + \frac{V_1 - V_2 - 10}{105} - 10V_x = 0$

② $\frac{V_2 - V_1 + 10V}{R_3 + R_4} + \frac{V_2}{R_5} = 0$

$\frac{V_2 - V_1 + 10}{105} + \frac{V_2}{60} = 0$

From the node for V_x, and since one side of V_x is ground-connected. $V_2 = V_x$

Thus: ① $105V_1 + 105V_2 + 10V_1 - 10V_2 - 100 - 10500V_2 = 0$

$115V_1 - 10405V_2 = 100$

② $60V_2 - 60V_1 + 600 + 105V_2 = 0$

$60V_1 - 165V_2 = 600$

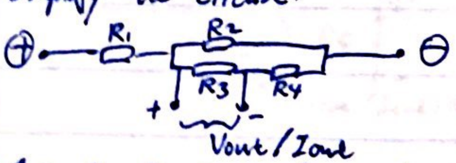
$$\begin{bmatrix} 115 & -10405 \\ 60 & -165 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 600 \end{bmatrix}$$

$\Downarrow$

$V_1 \cong 10.28621V$ $V_2 \cong 0.10408V$

4. Thévenin and Norton Equivalent Circuits

a) Simplify the circuit:



$R_1 = 3\Omega$, $R_2 = 3\Omega$ $R_3 = 4.5\Omega$ $R_4 = 1.5\Omega$, $V = 10V$

Let $R_5 = R_3 + R_4$ and R_6 = the $R_2/R_3/R_4$ system. then $R_5 = 6\Omega$. $R_6 = \left(\frac{1}{R_2} + \frac{1}{R_5}\right)^{-1} = 2\Omega$

Rewrite the circuit

(Define nodes A/B/C)

Thus $V_{AB} = \frac{R_3}{R_5} \times V_{AC} = \frac{R_3}{R_5} \times \frac{R_6}{R_1 + R_6} \times V = \frac{4.5}{6} \times \frac{2}{3+2} \times 10 = 3V = V_{out}$ (Same Circuit)

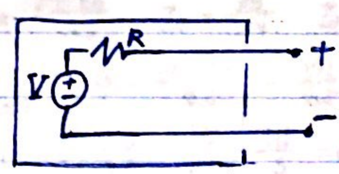
$I_{AB} = I_4 = \frac{V_{AC}'}{R_4} \Rightarrow R_6' = \left(\frac{1}{R_2} + \frac{1}{R_4}\right)^{-1} = \left(\frac{1}{3} + \frac{1}{1.5}\right)^{-1} = 1\Omega \Rightarrow V_{AC}' = \frac{R_6'}{R_1 + R_6'} \times 10V = 2.5V$ (short R_3)

$= 2.5V / 1.5\Omega = \frac{5}{3}A = I_{out}$

$R_{out} = \frac{V_{out}}{I_{out}} = \frac{3V}{5/3 A} = \frac{9}{5}\Omega$

Thus:

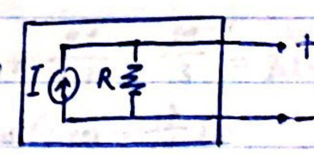
Thévenin equivalent circuit ⇒



$V = V_{out} = 3V$

$R = R_{out} = \frac{9}{5}\Omega$

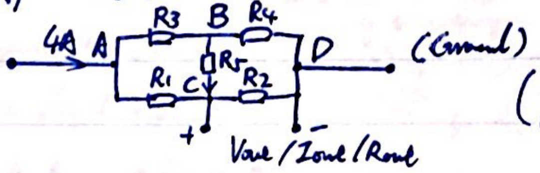
Norton equivalent circuit ⇒



$I = I_{out} = \frac{5}{3}A$

$R = R_{out} = \frac{9}{5}\Omega$

b) Simplify the circuit:



(Ground)

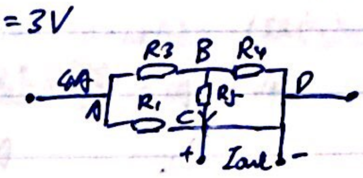
(Denote $R_j \Rightarrow I_j$ for $j \in [1,5]$ (correspondingly))

Thus we have $I = 4A$, $R_1 = 4.5\Omega$, $R_2 = 1.5\Omega$, $R_3 = 4.5\Omega$, $R_4 = 1.5\Omega$, $R_5 = 3\Omega$

By symmetry, since $R_3 + R_4 = R_1 + R_2 = 6\Omega$, $I_1 = I_2 = I_3 = I_4 = \frac{4A}{2} = 2A$.

Thus $V_{out} = I_2 \cdot R_2 = V_2 = 2A \cdot 1.5\Omega = 3V$

Now short $R_2 \Rightarrow$ Circuit same as



Now $V_C = V_D = 0$

And we have: Node A: $4A - I_3 - I_1 = 0$

Node B: $I_3 - I_4 - I_5 = 0$ Node C: $I_1 + I_5 - I_{out} = 0$

$I_1 = \frac{V_A}{R_1}$, $I_3 = \frac{V_A - V_B}{R_3}$, $I_4 = \frac{V_B}{R_4}$, $I_5 = \frac{V_B}{R_5}$

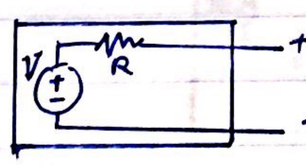
Thus $\begin{cases} 4 - \frac{V_A - V_B}{R_3} - \frac{V_A}{R_1} = 0 \\ \frac{V_A - V_B}{R_3} - \frac{V_B}{R_4} - \frac{V_B}{R_5} = 0 \end{cases} \Rightarrow \begin{cases} 4 - \frac{V_A - V_B}{4.5} - \frac{V_A}{4.5} = 0 \\ \frac{V_A - V_B}{4.5} - \frac{V_B}{1.5} - \frac{V_B}{3} = 0 \end{cases} \Rightarrow \begin{cases} V_A = 9.9V \\ V_B = 1.8V \end{cases}$

Thus $I_{out} = I_1 + I_5 = \frac{V_A}{R_1} + \frac{V_B}{R_5} = \frac{9.9}{4.5} + \frac{1.8}{3} = 2.2 + 0.6 = 2.8A$

Thus $R_{out} = V_{out} / I_{out} = 3V / 2.8A = \frac{15}{14}\Omega$

Thus

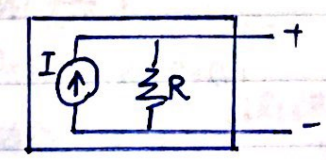
Thévenin equivalent circuit



$V = V_{out} = 3V$

$R = R_{out} = \frac{15}{14}\Omega$

Norton equivalent circuit



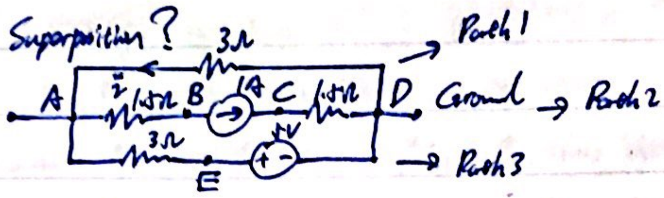
$I = I_{out} = 2.8A$

$R = R_{out} = \frac{15}{14}\Omega$

5. Nodal Analysis Or Superposition?

Simplify the circuit:

And define nodes



Node A: $i + I_E = 1A$; $V_{ED} = 5V \Rightarrow V_E = 5V$.

$V_{AB} = V_{CD} = 1.5\Omega \cdot 1A = 1.5V$

For current $i \Rightarrow$ since $1.5\Omega + 1.5\Omega = 3\Omega$. it is dived equally to Path 2/3.

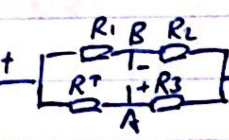
And since $V_{AD} = V_{AD} \Rightarrow \begin{cases} (1A - (-\frac{i}{2})) \times (1.5\Omega + 1.5\Omega) = -(3\Omega(-i) - \frac{i}{2}) \times 3\Omega & \Rightarrow i = -\frac{1}{3}A \\ \frac{V_{DA}}{3\Omega} + \frac{5V}{3\Omega} + \frac{V_{DA}}{3\Omega} = 1A \Rightarrow V_{DA} = -1V \Rightarrow i = \frac{V_{DA}}{3\Omega} = -\frac{1}{3}A \end{cases}$

$i + I_E = 1A$

Solve and we get $i = -\frac{1}{3}A$

6. Thermistor

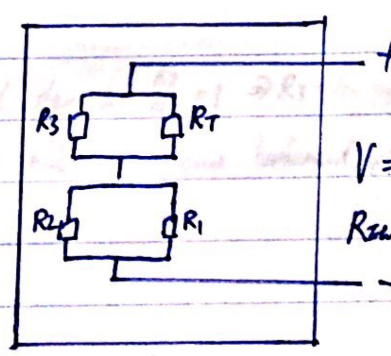
a) Simplify the original circuit: V_b+



V_b-

$V_{out} = V_A - V_B = V_b\left(\frac{R_3}{R_T + R_3}\right) - V_b\left(\frac{R_2}{R_1 + R_2}\right)$

Thus. Thevenin equivalent circuit:



$V = V_{out}$

$R_{Th} = \left(\frac{1}{R_T} + \frac{1}{R_3}\right)^{-1} + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$

b) $V_0 = V_{out} = V_b\left(\frac{R_3}{R_T + R_3}\right) - V_b\left(\frac{R_2}{R_1 + R_2}\right)$

$V_b(R_3(R_1 + R_2) - R_2(R_T + R_3)) = V_0(R_T + R_3)(R_1 + R_2)$

$\frac{V_b R_3 (R_1 + R_2)}{R_T + R_3} = V_0(R_1 + R_2) + V_b R_2 \Rightarrow R_T = \frac{V_b R_3 (R_1 + R_2) - V_0(R_1 + R_2) R_3 - V_b R_2 R_3}{V_0(R_1 + R_2) + V_b R_2}$

$= R_3 \frac{V_b R_1 - V_0(R_1 + R_2)}{V_b R_2 + V_0(R_1 + R_2)}$

c) If $R_T=R_1=R_2=R_3$. by symmetry. $V_{AB}=V_{out}=V_0=0$. if Temperature ↑ rise. R_T decrease ↓,
Then. $V_b(\frac{R_3}{R_T+R_3})-V_b(\frac{1}{2})>0$. which means $V_{AB}=V_{out}=V_0>0V$
if Temperature ↓ decrease. R_T increase ↑. thus. $\frac{R_3}{R_T+R_3}<\frac{1}{2}$. thus. $V_{AB}=V_{out}=V_0<0V$

d) $R_T(T)=R(T_0)\exp(\beta(\frac{1}{T}-\frac{1}{T_0}))$, $Q=\frac{dV_0}{dT}=R_T\frac{dV_0}{dR_T}\frac{1}{R_T}\frac{dR_T}{dT}$

⇓

$$\frac{dR_T(T)}{dT}=-\beta\times\frac{1}{T^2}\times R(T_0)\exp(\beta(\frac{1}{T}-\frac{1}{T_0}))=\frac{-\beta}{T^2}R_T$$

$$V_0=V_b(\frac{R_3}{R_T+R_3}-\frac{R_2}{R_1+R_2})\Rightarrow\frac{dV_0}{dR_T}=\frac{-V_bR_3}{(R_3+R_T)^2}$$

$$Q=R_T\frac{dV_0}{dR_T}\frac{1}{R_T}\frac{dR_T}{dT}=\frac{-V_bR_3R_T}{(R_3+R_T)^2}\times\frac{-\beta}{T^2}\Rightarrow\alpha=\frac{R_3}{R_T}=\frac{V_b\alpha\beta}{(1+\alpha)^2T^2}$$

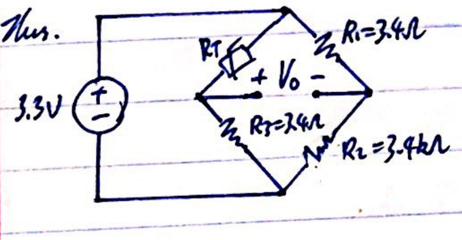
$$\frac{dQ}{d\alpha}=0=\frac{d}{d\alpha}(\frac{V_b\beta}{T^2}(1+\alpha)^{-1}(\frac{1}{\alpha}+1)^{-1}))=\frac{V_b\beta}{T^2}(-\frac{1}{(1+\alpha)^2}\frac{\alpha}{(1+\alpha)}+\frac{1}{(1+\alpha)}\frac{\frac{1}{\alpha^2}}{(\frac{1}{\alpha}+1)^2})$$

$$=\frac{V_b\beta}{T^2}(-\frac{\alpha}{(1+\alpha)^3}+\frac{1}{1+\alpha}\cdot(\frac{1}{\alpha^2}\times\frac{\alpha^2}{(1+\alpha)^2}))=\frac{V_b\beta}{T^2}\times\frac{(1-\alpha)}{(1+\alpha)^3}=0$$

Thus $\alpha=1$ if $\frac{dQ}{d\alpha}=0$

e) $R_T=3.4k\Omega$ when $T=30°C$.
Since $\frac{dQ}{d\alpha}=\frac{V_b\beta}{T^2}\frac{(1-\alpha)}{(1+\alpha)^3}=0$ is max, and then $\alpha=\frac{R_3}{R_T}=1\Rightarrow R_3=R_T=3.4k\Omega$ (while $R_2=R_3$)
Now $R_2=R_3=R_T$, we can set $R_1=R_2$ to make the circuit balanced thus $V_0=0V$ and can show variance in a more clear way.

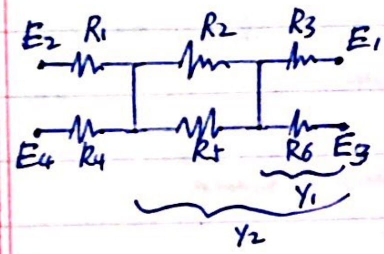
Thus.



7. Multitouch Resistive Touchscreen.

a) $R=\rho\frac{L}{A}=\rho\left(\frac{H}{TW}\right)=1\times\left(\frac{0.12}{0.03\times0.0005}\right)=8\times10^{3}\Omega=R_{E12}$

b)



$R_1=R_4=\frac{12-Y_2}{12}\times R_{E12}=\frac{5}{12}\times5\cong2.0833k\Omega$

$R_2=R_5=\frac{Y_2-Y_1}{12}\times R_{E12}=\frac{7-3}{12}\times5\cong1.6667k\Omega$

$R_3=R_6=\frac{Y_1}{12}\times R_{E12}=\frac{3}{12}\times5=1.25k\Omega$

c) Thus $R_{total}=R_4+\left(\frac{1}{R_2}+\frac{1}{R_5}\right)^{-1}+R_6=2.0833+\left(\frac{1}{1.6667}+\frac{1}{1.6667}\right)^{-1}+1.25\cong4.1667k\Omega$

Thus $V_{43}=R_{total}\times1mA=4.1667\times10^{3}\times1\times10^{-3}\cong4.1667V$

d) $R_1=R_4=\frac{12-Y_2}{12}R_{E12}=\frac{12-Y_2}{12}\times5\,(k\Omega)$ $\quad R_2=R_5=\frac{Y_2-Y_1}{12}\times5\,(k\Omega)$ $\quad R_3=R_6=\frac{Y_1}{12}\times5\,(k\Omega)$

Then $R_{43}=R_4+\left(\frac{1}{R_2}+\frac{1}{R_5}\right)^{-1}+R_6=\frac{12-Y_2}{12}\times5+\left(\frac{12\times2}{5\times(Y_2-Y_1)}\right)^{-1}+\frac{Y_1}{12}\times5$

$=\frac{5}{12}\left(12-Y_2+\frac{Y_2-Y_1}{2}+Y_1\right)=\frac{5}{12}\left(\frac{1}{2}Y_1-\frac{1}{2}Y_2+12\right)(k\Omega)$

Thus $V_{43}=R_{43}\cdot1mA=\frac{5}{12}\left(\frac{1}{2}Y_1-\frac{1}{2}Y_2+12\right)V$, thus we can determine $V_{E4\text{-}E3}$ by measuring Y_1 and Y_2.

e) $V_{E4\text{-}E2}=1mA\times R_4=1\times10^{-3}\times\frac{12-Y_2}{12}\times5\times10^{3}=\frac{12-Y_2}{12}\times5\,(V)$

$V_{E1\text{-}E3}=1mA\times R_6=1\times10^{-3}\times\frac{Y_1}{12}\times5\times10^{3}=\frac{5}{12}Y_1\,(V)$

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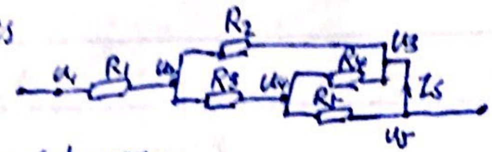
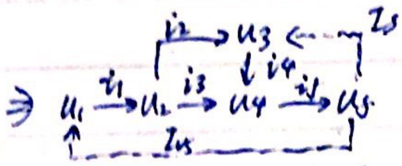
8. SPICE-y Circuits

a) Simplify circuit:



$\Rightarrow$



There are 5 nodes and 5 currents.

The $F = \begin{bmatrix} \downarrow \downarrow \end{bmatrix}$ → row: currents, column: balance of nodes

$\Rightarrow F = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$ (Without independent source) (Take input as (-1) and output as (1))

$$F(\text{with independent sources}) = \begin{array}{c} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right\} i_1 \sim i_5 \\ \rightarrow i_{vs} \\ \rightarrow I_s \end{array}$$

b) $$F^T \vec{i} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_{vs} \\ I_s \end{bmatrix} = \begin{bmatrix} i_1 - I_{vs} \\ -i_1 + i_2 + i_3 \\ -i_2 + i_4 - I_s \\ -i_3 - i_4 + i_5 \\ -i_5 + I_{vs} + I_s \end{bmatrix}$$ $\Rightarrow$ This shows the KCL equations at each node

c) $$\tilde{F}^T \cdot \tilde{i} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ I_{vs} \end{bmatrix} = \begin{bmatrix} i_1 - I_{vs} \\ -i_1 + i_2 + i_3 \\ -i_2 + i_4 \\ -i_3 - i_4 + i_5 \\ -i_5 + I_{vs} \end{bmatrix} = -I_s \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

d) $$F \cdot \vec{u} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} u_1 - u_2 \\ u_2 - u_3 \\ u_2 - u_4 \\ u_3 - u_4 \\ u_4 - u_5 \\ u_5 - u_1 \\ u_5 - u_3 \end{bmatrix} = \vec{v}$$

$u_1 - u_2, \dots, u_4 - u_5$: correspond to $R_1 \sim R_5$ (the voltage across which)

$u_5 - u_1$ → correspond to V_s (the voltage across which)

$u_5 - u_3$ → correspond to I_s (the voltage across which)

e) $$\vec{d} = \vec{v} = \begin{bmatrix} i_1 \cdot R_1 \\ i_2 \cdot R_2 \\ i_3 \cdot R_3 \\ i_4 \cdot R_4 \\ i_5 \cdot R_5 \\ V_s \\ V_{cs} \end{bmatrix}$$

f) $\vec{d} = R\tilde{i} + \vec{V_s} = \begin{bmatrix} R_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & V_{cs} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_s \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_s \\ 0 \end{bmatrix} = \begin{bmatrix} i_1R_1 \\ i_2R_2 \\ i_3R_3 \\ i_4R_4 \\ i_5R_5 \\ V_s \\ V_{cs} \end{bmatrix}$

$F\vec{u} - R\tilde{i} = \vec{V_s} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} - \begin{bmatrix} R_1 & & & & & & \\ & R_2 & & & & 0 & \\ & & R_3 & & & & \\ & & & R_4 & & & \\ & 0 & & & R_5 & & \\ & & & & & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_s \\ i_{cs} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -V_s \\ V_{cs} \end{bmatrix}$

g) $A\vec{x} = \vec{b} = \begin{bmatrix} F & -R \\ 0 & F^T \end{bmatrix} \begin{bmatrix} \vec{u} \\ \vec{i} \end{bmatrix}$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -R_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & -R_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & -R_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & -R_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & -R_5 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -V_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_s \\ i_{cs} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -V_s \\ 0 \\ 0 \\ 0 \\ I_s \\ 0 \\ -I_s \end{bmatrix}$$

h) $\hat{A}\hat{x} = \vec{b}$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -R_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & -R_2 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & -1 & 0 & 0 & -R_3 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & -R_4 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -R_5 & 0 \\ -3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_{cs} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -V_s \\ 0 \\ 0 \\ 0 \\ I_s \\ 0 \\ -I_s \end{bmatrix}$$

i) $$\begin{bmatrix} 1 & -1 & 0 & 0 & -R_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & -R_2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & -R_3 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & -R_4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -R_5 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_{cs} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -V_s \\ 0 \\ I_s \\ 0 \\ -I_s \end{bmatrix}$$

j) Use Ipython. to calculate

$u_1 = 5V$
$u_2 = 14.6V$
$u_3 = 21.2V$
$u_4 = 15.6V$
$i_1 = -2.4 \times 10^{-3} A$
$i_2 = -2.2 \times 10^{-3} A$
$i_3 = -2 \times 10^{-4} A$
$i_4 = 2.8 \times 10^{-3} A$
$i_5 = 2.6 \times 10^{-3} A$
$i_{cs} = -2.4 \times 10^{-3} A$

9. Resistive Voltage "Regulator"

a) $V_{out} = \frac{R}{R+10k\Omega} = 4V \quad \frac{R}{10k\Omega} = \frac{4V}{(6-4)V} = 2. \quad R = 20k\Omega$

b) let R' denote the system of R and $1k\Omega$. $R' = \left(\frac{1}{R} + \frac{1}{1k\Omega}\right)^{-1} = \frac{20}{21} k\Omega$

$V_{out} = \frac{R'}{R'+10k\Omega} \times 6V = \frac{\frac{20}{21}}{\frac{20}{21}+10} \times 6 \approxeq 0.52174V$

c) let R' denote $[R \mid 100k\Omega]$ system $R' = \left(\frac{1}{R} + \frac{1}{100k\Omega}\right)^{-1} = \frac{50}{3}\Omega$

$V_{out} = \frac{R'}{R'+10k\Omega} \times 6V = \frac{\frac{50}{3}}{\frac{50}{3}+10} \times 6 = 3.75V$

d) $I_{short} = I_{NO} = \frac{6V}{R_1} = (6V) \cdot G_1, \quad V_{short} = \frac{R_2}{R_1+R_2} 6V = \frac{\frac{1}{G_2}}{\frac{1}{G_1}+\frac{1}{G_2}}(6V) = \frac{G_1}{G_1+G_2}(6V)$

$R_{eff} = V_{short}/I_{NO} = \frac{\cancel{G_1}}{G_1+G_2}\cancel{(6V)} \times \frac{1}{\cancel{(6V)G_1}} = \frac{1}{G_1+G_2}$

$G_{eff} = \frac{1}{R_{eff}} = G_1 + G_2$

e) $V_{out} = I_{NO} \times \frac{R_{eff}}{R_{eff}+R_e} \times R_e \in [3.8, 4.2]$

$\frac{1}{I_{NO}R_e} \times \frac{R_{eff}+R_e}{R_{eff}} \in \left[\frac{1}{4.2}, \frac{1}{3.8}\right] \Rightarrow 1 + \frac{R_e}{R_{eff}} \in \left[\frac{I_{NO}R_e}{4.2}, \frac{I_{NO}R_e}{3.8}\right]$

$R_{eff} \in \left[\frac{4.2R_e}{I_{NO}R_e - 4.2}, \frac{3.8R_e}{I_{NO}R_e - 3.8}\right] \Rightarrow \left[\frac{4.2}{I_{NO} - 4.2G_e}, \frac{3.8}{I_{NO} - 3.8G_e}\right]$

$G_{eff} \in \left[\frac{I_{NO}}{4.2} - G_e, \frac{I_{NO}}{3.8} - G_e\right]$

f) $G_{eff} = G_1 + G_2 \Rightarrow G_2 \in \left[\frac{I_{NO}}{4.2} - G_e - G_1, \frac{I_{NO}}{3.8} - G_e - G_1\right] \Rightarrow \left[\frac{3}{7}G_1 - G_e, \frac{11}{19}G_1 - G_e\right]$

g) $R_e \in [1k\Omega, 100k\Omega]$

$G_2 \in [\frac{3}{7}G_1 - 1mS, \frac{11}{19}G_1 - 1mS]$ $\quad$ $G_2 \in [\frac{3}{7}G_1 - \frac{1}{100}mS, \frac{11}{19}G_1 - \frac{1}{100}mS]$

$G_2 \in [\frac{3}{7}G_1 - \frac{1}{100}mS, \frac{11}{19}G_1 - 1mS]$.

$\frac{1}{R_2} \in [\frac{3}{7R_1} - \frac{1}{100}mS, \frac{11}{19R_1} - 1mS] \Rightarrow R_2 \in [\frac{1}{\frac{11}{19}R_1 - 1mS}, \frac{1}{\frac{3}{7R_1} - \frac{1}{100}mS}]$

h) $\begin{cases} \frac{3}{7}G_1 - \frac{1}{100}mS \geq 0 \Rightarrow G_1 \geq \frac{7}{300}mS \\ \frac{3}{7}G_1 - \frac{1}{100}mS \leq \frac{11}{19}G_1 - 1mS \Rightarrow (\frac{11}{19} - \frac{3}{7})G_1 \geq \frac{99}{100}mS \end{cases}$

$\Rightarrow G_1 \geq \frac{99}{100} \times \frac{133}{20} \Rightarrow G_1 \geq \frac{13167}{2000} \Rightarrow G_1 \geq 6.5835mS$

$R_1 \leq \frac{1}{G_1} \Rightarrow R_1 \leq 0.15189$

i) $R_1(max) = 0.15189k\Omega$ $R_2(max) = \frac{1}{\frac{3}{7 \times 0.15189} - \frac{1}{100}mS} = 0.35567k\Omega$

$P = IV = \frac{V^2}{R^2} \Rightarrow P = \frac{V^2}{R_1 + R_2}(min) \Rightarrow R_1 + R_2(max)$

$P(min) = \frac{(6V)^2}{(0.15189 + 0.35567)k\Omega} = 70.928 \times 10^{-3} W$

$R_{eff} = (\frac{1}{R_1} + \frac{1}{R_2})^{-1} = 0.10644k\Omega$, $I_{no} = 6V \cdot G_1 = 6V/R_1 = 39.502 \times 10^{-3} A$

$P_N = I^2 \cdot R = I_{no}^2 \cdot R_{eff} = 0.16609 W$

The power of original circuit and Norton circuit are not equal

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Untitled

July 16, 2017

```
In [6]: import numpy as np

In [11]: a = np.array([[1, -1, 0, 0, -4000, 0, 0, 0, 0, 0],
                       [0, 1, -1, 0, 0, -3000, 0, 0, 0, 0],
                       [0, 1, 0, -1, 0, 0, -5000, 0, 0, 0],
                       [0, 0, 1, -1, 0, 0, 0, -2000, 0, 0],
                       [0, 0, 0,  1, 0, 0, 0, 0, -6000, 0],
                       [-1, 0, 0, 0, 0, 0, 0, 0, 0, 0],
                       [0, 0, 0, 0, -1, 1, 1, 0, 0, 0],
                       [0, 0, 0, 0, 0, -1, 0, 1, 0, 0],
                       [0, 0, 0, 0, 0, 0, -1, -1, 1, 0],
                       [0, 0, 0, 0, 0, 0, 0, 0, -1, 1]])
         b = np.array([0, 0, 0, 0, 0, -5, 0, 0.005, 0, -0.005])

In [12]: np.linalg.solve(a,b)

Out[12]: array([  5.00000000e+00,   1.46000000e+01,   2.12000000e+01,
                  1.56000000e+01,  -2.40000000e-03,  -2.20000000e-03,
                 -2.00000000e-04,   2.80000000e-03,   2.60000000e-03,
                 -2.40000000e-03])

In [ ]:
```