Homework 1 EE16A

① I, Wayne Li, affirm that I have read and understood the syllabus and I know that the exam dates are 7/11, 7/28, 8/11.

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② $A = \left[\begin{array}{cccc|c} 1 & -2 & 0 & -5 & 16 \\ 0 & 1 & 0 & 3 & -7 \\ -2 & -3 & 1 & -6 & 9 \\ 0 & 1 & 0 & 2 & -5 \end{array}\right] \qquad E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$

switch row 1 & 2. multiply r3 by −4 add 2x row2 to row4

b) We want some E that $EA = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & b_1 \\ 0 & 1 & 0 & 0 & b_2 \\ 0 & 0 & 1 & 0 & b_3 \\ 0 & 0 & 0 & 1 & b_4 \end{array}\right]$

Thus $E\begin{bmatrix} 1 & -2 & 0 & -5 \\ 0 & 1 & 0 & 3 \\ -2 & -3 & 1 & -6 \\ 0 & 1 & 0 & 2 \end{bmatrix} = I$. Let $\begin{bmatrix} 1 & -2 & 0 & -5 \\ 0 & 1 & 0 & 3 \\ -2 & -3 & 1 & -6 \\ 0 & 1 & 0 & 2 \end{bmatrix} = A'$

$EA' = I \Rightarrow [A' | I] \Rightarrow [I | E]$.

After row reduce $[A'|I]$ on the left side. we get $[I|E]$

where E is $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & 0 & 3 \\ 2 & 2 & 1 & 5 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

$EA = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & 0 & 3 \\ 2 & 2 & 1 & 5 \\ 0 & 1 & 0 & -1 \end{bmatrix} \left[\begin{array}{cccc|c} 1 & -2 & 0 & -5 & 16 \\ 0 & 1 & 0 & 3 & -7 \\ -2 & -3 & 1 & -6 & 9 \\ 0 & 1 & 0 & 2 & -5 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$

③ We can get $\begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} \text{Blue} \\ \text{Mango} \\ \text{Bana} \\ \text{Straw} \end{bmatrix} = \begin{bmatrix} 6\frac{2}{3} \\ 6\frac{2}{3} \\ 7\frac{2}{5} \\ 5\frac{2}{3} \end{bmatrix}$

a) Row reduce the system

$\left[\begin{array}{cccc|c} 5 & 0 & 5 & 5 & 100 \\ 0 & 5 & 5 & 5 & 100 \\ 0 & 9 & 6 & 0 & 111 \\ 0 & 0 & 5 & 10 & 85 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 20 \\ 0 & 1 & 1 & 1 & 20 \\ 0 & 3 & 2 & 0 & 37 \\ 0 & 0 & 1 & 2 & 17 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 20 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 3 & 2 & 0 & 37 \\ 0 & 0 & 1 & 2 & 17 \end{array}\right] \Rightarrow$

$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 14 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 3 & 2 & 0 & 37 \\ 0 & 0 & 1 & 0 & 5 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 1 & 0 & 0 & 9 \\ 0 & 0 & 1 & 0 & 5 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 9 \\ 0 & 1 & 0 & 0 & 9 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$

Thus Vasuki rates Blueberry: 9, Mango: 9

Banana: 5. Strawberry: 6

b) let's define the combination as $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, and let $A = [9.\ 9.\ 5.\ 6]$

So we're trying to maximize Ax. since 9 is the largest number, we ignore 5 and 6.
Thus, as long as $\begin{cases} x_1 + x_2 = 1 \\ x_3 = x_4 = 0 \end{cases}$ the score of the smoothie can be maximized.

→ One solution is blueberry: $\frac{1}{2}$. Mango: $\frac{1}{2}$. Banana: 0. Strawberry: 0.

→ The score is 9.

④ $U_1 = k\left(\frac{Q_1}{\sqrt{2}} + \frac{Q_2}{\sqrt{5}} + \frac{Q_3}{2}\right)$, $U_2 = k\left(\frac{Q_1}{1} + \frac{Q_2}{\sqrt{2}} + \frac{Q_3}{1}\right)$

$U_3 = k\left(\frac{Q_1}{2} + \frac{Q_2}{\sqrt{5}} + \frac{Q_3}{\sqrt{2}}\right)$

since k can be canceled, the matrix is

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{5}} & \frac{1}{2} \\ 1 & \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{2} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} \frac{4+3\sqrt{5}+\sqrt{10}}{2\sqrt{5}} \\ \frac{2+4\sqrt{2}}{\sqrt{2}} \\ \frac{4+\sqrt{5}+3\sqrt{10}}{2\sqrt{5}} \end{bmatrix}$$

Through Ipython, we get $U_1 \cong 1.0$ $U_2 \cong 2.0$. $U_3 \cong 3.0$

⑤ Since $P = 1 - 0.95^{\exp(R-25.66)}$. $R = \ln\left(\frac{\ln(1-P)}{\ln(0.95)}\right) + 25.66$

a) We have $P_1 = 0.1550$. $P_2 = 0.1108$. $P_3 = 0.094$. $P_4 = 0.0105$.

we have $R_1 = 26.8489$. $R_2 = 26.4883$ $R_3 = 26.3147$ $R_4 = 24.0791$

Since $R = a(\ln(age)) + b(\ln(choles)) + c(\ln(HDL)) + d(\ln(SBP))$

$$\begin{bmatrix} \ln(66) & \ln(198) & \ln(55) & \ln(132) \\ \ln(61) & \ln(180) & \ln(47) & \ln(124) \\ \ln(60) & \ln(180) & \ln(50) & \ln(120) \\ \ln(23) & \ln(172) & \ln(45) & \ln(130) \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 26.8489 \\ 26.4883 \\ 26.3147 \\ 24.0791 \end{bmatrix}$$

b) Through Ipython we get $a = 2.3096$. $b = 1.1700$ $c = -0.6944$ $d = 2.8195$

由 扫描全能王 扫描创建

⑥ $\vec{m}_1 = \cos(\theta)\cdot\vec{a} + \cos(\varphi)\vec{b}$. $\vec{m}_2 = \sin(\theta)\vec{a} + \sin(\varphi)\vec{b}$

a) Thus $\begin{bmatrix} \cos(\theta) & \cos(\varphi) \\ \sin(\theta) & \sin(\varphi) \end{bmatrix}\begin{bmatrix} \vec{a} \\ \vec{b} \end{bmatrix} = \begin{bmatrix} \vec{m}_1 \\ \vec{m}_2 \end{bmatrix}$ $\theta = 45°$, $\varphi = -30°$

Thus $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} \vec{a} \\ \vec{b} \end{bmatrix} = \begin{bmatrix} \vec{m}_1 \\ \vec{m}_2 \end{bmatrix}$

b) let $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} \end{bmatrix}$ be A. then $A\begin{bmatrix} \vec{a} \\ \vec{b} \end{bmatrix} = \begin{bmatrix} \vec{m}_1 \\ \vec{m}_2 \end{bmatrix} \Rightarrow \begin{bmatrix} \vec{a} \\ \vec{b} \end{bmatrix} = A^{-1}\begin{bmatrix} \vec{m}_1 \\ \vec{m}_2 \end{bmatrix}$

$$A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = -\frac{4}{\sqrt{6}+\sqrt{2}}\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{6}+\sqrt{2}} & \frac{2\sqrt{3}}{\sqrt{6}+\sqrt{2}} \\ \frac{2\sqrt{2}}{\sqrt{6}+\sqrt{2}} & -\frac{2\sqrt{2}}{\sqrt{6}+\sqrt{2}} \end{bmatrix}$$

$\vec{a} = u\cdot\vec{m}_1 + v\cdot\vec{m}_2$. $u = \frac{2}{\sqrt{6}+\sqrt{2}}$, $v = \frac{2\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

c) Done

⑦ Since $\{\vec{v}_1, \vec{v}_2, \dots \vec{v}_k\}$ is a set of dependent vectors. there exist some non-zero scalars $c_1 \sim c_n$ that $c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_k\vec{v}_k = 0$

Thus $A(c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_k\vec{v}_k) = c_1(A\vec{v}_1) + c_2(A\vec{v}_2) + \dots + c_k(A\vec{v}_k) = 0$

Thus the set $\{A\vec{v}_1, A\vec{v}_2, \dots A\vec{v}_k\}$ is a set of linearly dependent vectors.

⑧ We can get

a)
$$\underbrace{\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}}_{A} \begin{bmatrix} r_1 \\ \vdots \\ r_6 \end{bmatrix} = \begin{bmatrix} p_1 \\ \vdots \\ p_6 \end{bmatrix}$$

$A \to$ let A be represented as $\begin{bmatrix} - r_1 - \\ - r_2 - \\ \vdots \\ - r_6 - \end{bmatrix}$

Since $r_1 - r_2 + r_3 - r_4 + r_5 = r_6$. A can't have pivot in every row/column.
Thus the system $\left[\begin{array}{c|c} A & \begin{matrix} p_1 \\ \vdots \\ p_6 \end{matrix} \end{array}\right]$ has infinite/zero solutions.

$\Rightarrow$ Thus we cannot determine individual tips.

Suppose $p_1 = p_2 = p_3 = \cdots = p_6 = 1$.

Then
$$\left[\begin{array}{cccccc|c} 1 & 1 & & & & & 2 \\ & 1 & 1 & & & & 2 \\ & & 1 & 1 & & & 2 \\ & & & 1 & 1 & & 2 \\ & & & & 1 & 1 & 2 \\ 1 & & & & & 1 & 2 \end{array}\right] \Rightarrow ① \begin{bmatrix} r_1 \\ \vdots \\ r_6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad ② \begin{bmatrix} r_1 \\ \vdots \\ r_6 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

b) We can get

$$\underbrace{\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & & & \\ & \frac{1}{2} & \frac{1}{2} & & \\ & & \frac{1}{2} & \frac{1}{2} & \\ & & & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & & & & \frac{1}{2} \end{bmatrix}}_{A} \begin{bmatrix} r_1 \\ \vdots \\ r_5 \end{bmatrix} = \begin{bmatrix} p_1 \\ \vdots \\ p_5 \end{bmatrix}$$

row reduce A: $\begin{bmatrix} 1 & 1 & & & -1 \\ & 1 & 1 & & \\ & & 1 & 1 & \\ & & & 1 & 1 \\ 1 & & & & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & & & & \\ & 1 & 1 & & \\ & & 1 & 1 & \\ & & & 1 & 1 \\ & & & & 1 \end{bmatrix}$

Thus there is pivot in every row/column and A is invertible.

$\Rightarrow$ Thus there is determined individual tips.

c) $\{n \mid n = 2k+1,\ k \in N^+\}$

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⑩ $A=\begin{bmatrix}0&2&1&3\\0&0&1&-2\\0&0&0&3\\0&0&0&0\end{bmatrix}$, $A^2=\begin{bmatrix}0&2&1&3\\0&0&1&-2\\0&0&0&3\\0&0&0&0\end{bmatrix}\begin{bmatrix}0&2&1&3\\0&0&1&-2\\0&0&0&3\\0&0&0&0\end{bmatrix}=\begin{bmatrix}0&0&2&-1\\0&0&0&3\\0&0&0&0\\0&0&0&0\end{bmatrix}$

a) $A^3=\begin{bmatrix}0&0&2&-1\\0&0&0&3\\0&0&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}0&2&1&3\\0&0&1&-2\\0&0&0&3\\0&0&0&0\end{bmatrix}=\begin{bmatrix}0&0&0&6\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$ $A^4=\begin{bmatrix}0&0&0&6\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}0&2&1&3\\0&0&1&-2\\0&0&0&3\\0&0&0&0\end{bmatrix}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$

b) $B=\begin{bmatrix}3&4&2&1\\-5&-6&-3&-1\\6&7&3&2\\2&2&1&0\end{bmatrix}$ $B^2=\begin{bmatrix}3&4&2&1\\-5&-6&-3&-1\\6&7&3&2\\2&2&1&0\end{bmatrix}\begin{bmatrix}3&4&2&1\\-5&-6&-3&-1\\6&7&3&2\\2&2&1&0\end{bmatrix}=\begin{bmatrix}3&4&1&3\\-5&-7&-2&-5\\5&7&2&5\\2&3&1&2\end{bmatrix}$

$B^3=\begin{bmatrix}3&4&1&3\\-5&-7&-2&-5\\5&7&2&5\\2&3&1&2\end{bmatrix}\begin{bmatrix}3&4&2&1\\-5&-6&-3&-1\\6&7&3&2\\2&2&1&0\end{bmatrix}=\begin{bmatrix}1&1&0&1\\-2&-2&0&-2\\2&2&0&2\\1&1&0&1\end{bmatrix}$ $B^4=\begin{bmatrix}1&1&0&1\\-2&-2&0&-2\\2&2&0&2\\1&1&0&1\end{bmatrix}\begin{bmatrix}3&4&2&1\\-5&-6&-3&-1\\6&7&3&2\\2&2&1&0\end{bmatrix}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$

prob1

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1 EE16A: Homework 1

1.1 Problem 4: Finding Charges from Potential Measurements

```
In [1]: import numpy as np
        from math import sqrt
        a = np.array([
            [1/np.sqrt(2), 1/np.sqrt(5), 1/2],
            [1, 1/np.sqrt(2), 1],
            [1/2,1/np.sqrt(5),1/np.sqrt(2)]
        ])
        b = np.array([(4+3*np.sqrt(5)+np.sqrt(10))/(2*np.sqrt(5)), (2+4*np.sqrt(2))/np.sqrt(2),
        x = np.linalg.solve(a, b)
        print(x)
```

```
[ 1.  2.  3.]
```

```
In [ ]:
```

1.2 Problem 5: The Framingham Risk Score

```
In [5]: # Tip: np.log works element-wise on an np.array
        import numpy as np
        from math import log
        a = np.array([
            [log(66), log(198), log(55), log(132)],
            [log(61), log(180), log(47), log(124)],
            [log(60), log(180), log(50), log(120)],
            [log(23), log(132), log(45), log(132)]
        ])
        b = np.array([26.8489, 26.4883, 26.3147, 24.0791])
        x = np.linalg.solve(a, b)
        print(x)
```

```
[ 2.30963351  1.17009393 -0.69445483  2.81958515]
```

1.3 Problem 6: Filtering Out The Troll

```
In [9]: import numpy as np
        import matplotlib.pyplot as plt
        import wave as wv
        import scipy
        from scipy import io
        import scipy.io.wavfile
        from scipy.io.wavfile import read
        from IPython.display import Audio
        from math import sqrt
        import warnings
        warnings.filterwarnings('ignore')
        sound_file_1 = 'm1.wav'
        sound_file_2 = 'm2.wav'
```

Let's listen to the recording of the first microphone (it can take some time to load the sound file).

```
In [10]: Audio(url='m1.wav', autoplay=False)

Out[10]: <IPython.lib.display.Audio object>
```

And this is the recording of the second microphone (it can take some time to load the sound file).

```
In [11]: Audio(url='m2.wav', autoplay=False)

Out[11]: <IPython.lib.display.Audio object>
```

We read the first recording to the variable `corrupt1` and the second recording to `corrupt2`.

```
In [12]: rate1,corrupt1 = scipy.io.wavfile.read('m1.wav')
         rate2,corrupt2 = scipy.io.wavfile.read('m2.wav')
```

Enter the gains of the two recordings to get the clean speech.
Note: The square root of a number a can be written as `np.sqrt(a)` in IPython.

```
In [16]: # enter the gains u (recording 1) and v (recording 2)
         u = 2/(sqrt(6)+sqrt(2))
         v = (2*sqrt(3))/(sqrt(6)+sqrt(2))
```

Weighted combination of the two recordings:

```
In [17]: s1 = u*corrupt1 + v*corrupt2
```

Let's listen to the resulting sound file (make sure your speaker's volume is not very high, the sound may be loud if things go wrong).

```
In [19]: Audio(data=s1, rate=rate1)

Out[19]: <IPython.lib.display.Audio object>

In [ ]:
```