# EECS 16A Designing Information Devices and Systems I
# Spring 2017 Babak Ayazifar, Vladimir Stojanovic Midterm 2

Exam location: 145 Dwinelle, last SID# 2

PRINT your student ID: ________________________________________

PRINT AND SIGN your name: ________________, ________________ ____________________
(last) (first) (signature)

PRINT your Unix account login: ee16a-____________

PRINT your discussion section and GSI (the one you attend): ________________________________________

Name and SID of the person to your left: ________________________________________

Name and SID of the person to your right: ________________________________________

Name and SID of the person in front of you: ________________________________________

Name and SID of the person behind you: ________________________________________

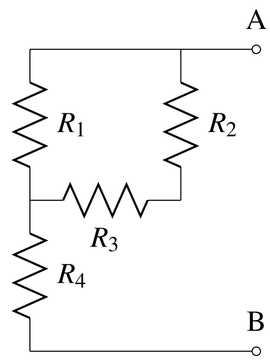
1. **What do you enjoy about 16A? (1 point)**

2. **What do you dislike about 16A? (1 point)**

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

**3. Simplifying Resistor Networks (6 Points)**

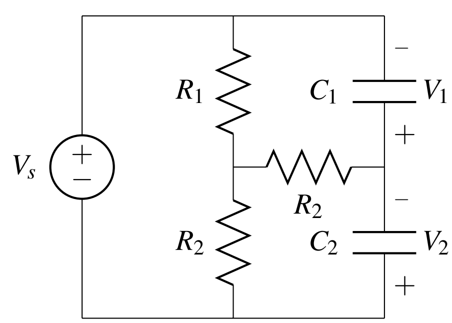
Determine the equivalent resistance between points A and B in the circuit shown below. You may use the parallel operator $\|$ in your final answer.



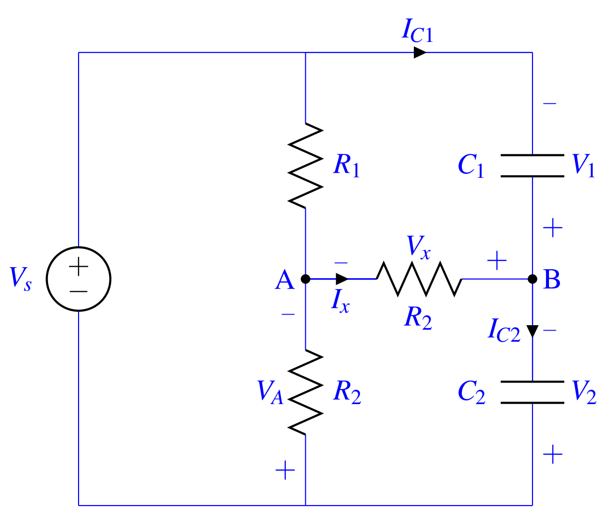

**Solutions:** $R_{eq} = R_1 \parallel (R_2 + R_3) + R_4$

**4. Capacitors (6 Points)**

For the circuit given below, determine the voltage across each capacitor at steady state. $R_2 = 2R_1$, $C_2 = C_1$, and $V_s = 3\,\mathrm{V}$.




**Solutions:** We label the circuit as follows:



In steady state,

$$I_{C1} = 0$$
$$I_{C2} = 0$$

By KCL at node $B$,

$$I_x = 0$$

which implies

$$V_x = 0$$
$$V_A = V_B$$

Since $I_x = 0$, then by the voltage divider equation.

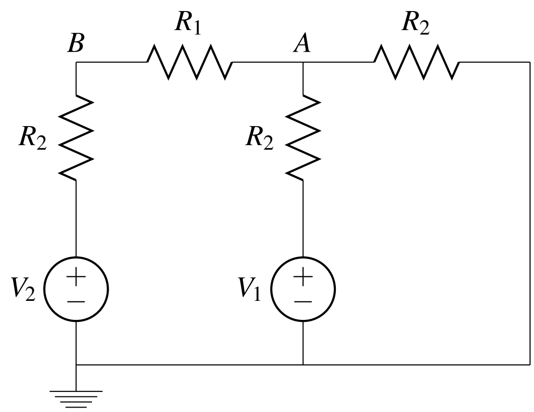
$$V_A = \frac{R_2}{R_1 + R_2} V_s$$

By KVL:

$$\begin{aligned} V_2 &= V_A - V_x \\ &= V_A \\ &= \frac{R_2}{R_1 + R_2} V_s \\ V_1 &= V_s - V_2 \\ &= \frac{R_1}{R_1 + R_2} V_s \end{aligned}$$

Therefore, $V_1 = 1\,\text{V}$ and $V_2 = 2\,\text{V}$.

**5. Ladder Nodal (8 Points)**




Use nodal analysis to write equations for each node $A$ and $B$.
Let $R_1 = 1\,\Omega$, $R_2 = 2\,\Omega$, and leave your expressions in terms of $V_1$, $V_2$, $R_1$, $R_2$, $V_A$ and $V_B$. Do not simplify.

**Solutions:**
For node $A$:

$$0 = \frac{V_A}{R_2} + \frac{(V_A - V_1)}{R_2} + \frac{(V_A - V_B)}{R_1}$$
$$\frac{1}{R_2}V_1 = V_A\left(\frac{1}{R_2} + \frac{1}{R_2} + \frac{1}{R_1}\right) - V_B(\frac{1}{R_1})$$
$$0.5V_1 = 2V_A - V_B$$

For node $B$:

$$0 = \frac{(V_B - V_2)}{R_2} + \frac{(V_B - V_A)}{R_1}$$
$$\frac{1}{R_2}V_2 = V_B\left(\frac{1}{R_2} + \frac{1}{R_1}\right) - V_A\left(\frac{1}{R_1}\right)$$
$$0.5V_2 = -V_A + 1.5V_B$$

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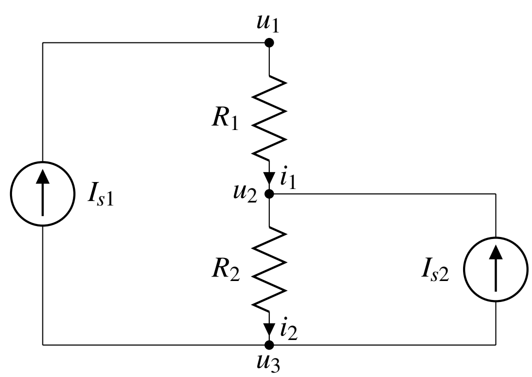
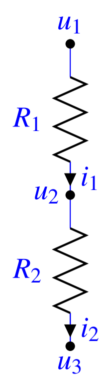
**6. Flows Through Circuits (12 Points)**




(a) (2 points) Redraw the circuit shown above with all current sources nulled.

**Solutions:**




(b) (5 points) Translate your circuit from part (a) into a graph and write its incidence matrix **F**.

**Solutions:**

$$\begin{tikzcd} u_1 \arrow[d, "i_1"] \\ u_2 \arrow[d, "i_2"] \\ u_3 \end{tikzcd}$$

The incidence matrix for the above graph is:

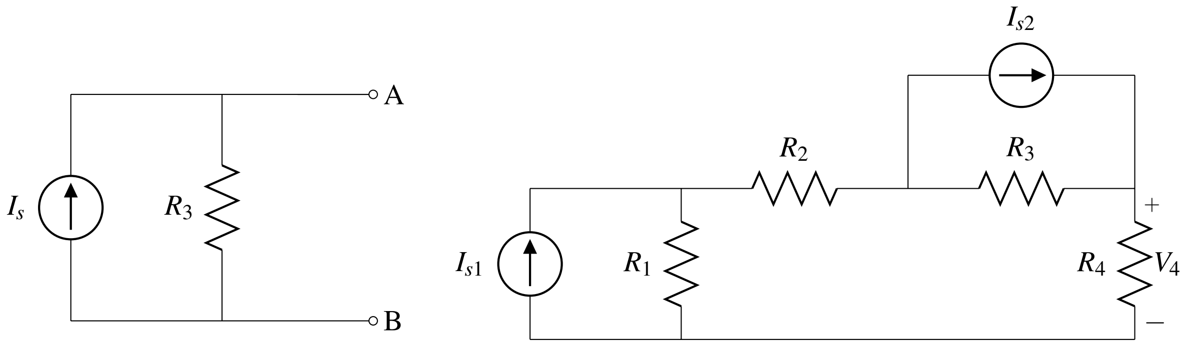
$$\mathbf{F} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

(c) (5 points) Recall that KCL can be stated as $\mathbf{F}^T\vec{i} = 0$ when all the sources are turned off. However, for circuits containing independent current sources, the equation is modified to be $\mathbf{F}^T\vec{i} = \vec{b}$. Find $\vec{b}$ such that $\mathbf{F}^T\vec{i} = \vec{b}$ represents the KCL constraint at every node in the above circuit.

**Solutions:** $I_{s1}$ flows into node 1 and out of node 3. $I_{s2}$ flows into node 2 and out of node 3. Therefore, the vector $\vec{b}$ is:

$$\vec{b} = \begin{bmatrix} I_{s1} \\ I_{s2} \\ -I_{s1} - I_{s2} \end{bmatrix}$$

## 7. One Norton At A Time! (12 Points)



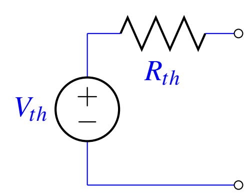

(a) (b)

(a) (2 points) Given the circuit labeled (a), draw the Thevenin equivalent circuit for $I_s = \frac{1}{5}$A and $R_3 = 20\,\Omega$ and determine $V_{th}$ and $R_{th}$.

**Solutions:**

Note that this is a Norton equivalent circuit. We can use $V_{th} = I_{no}R_{no}$ and $R_{th} = R_{no}$ to determine the Thevenin equivalent circuit.

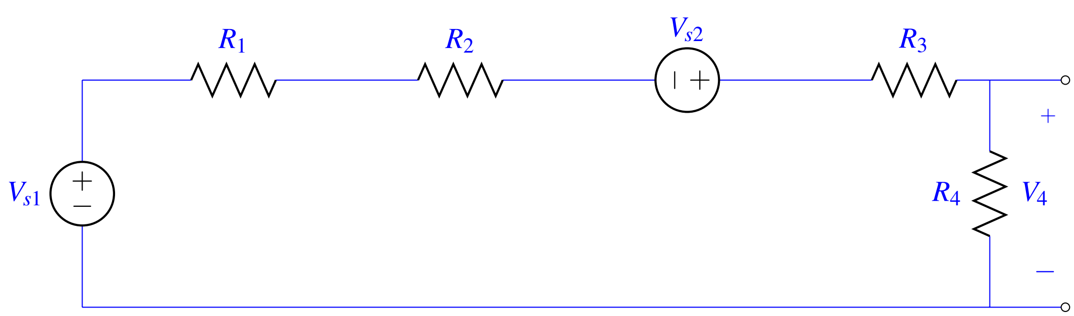
$$V_{th} = V_{AB} = I_s R_3 = 4\,\text{V}$$
$$R_{th} = R_3 = 20\,\Omega$$




(b) (5 points) Redraw the circuit labeled (b) with the following restrictions: you must use only voltage sources and the given four resistors. Determine the value of the voltage sources given $R_1 = 30\,\Omega$, $R_2 = 10\,\Omega$, $R_3 = 20\,\Omega$, $R_4 = 10\,\Omega$, $I_{s1} = \frac{1}{3}$A, and $I_{s2} = \frac{1}{5}$A. **Circuits drawn without $R_1, R_2, R_3,$ and $R_4$ will not be given full credit. Do not combine resistors.**

**Solutions:**

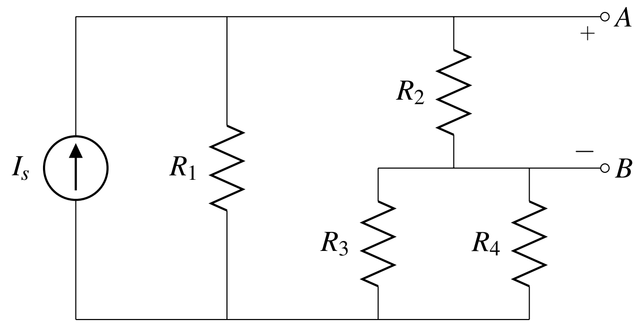
$I_{s1}$ and $R_1$ can be transformed from a Norton equivalent circuit to a Thevenin equivalent circuit. We can do the same for $I_{s2}$ and $R_3$.



$V_{s1} = 10\,\text{V}$ and $V_{s2} = 4\,\text{V}$

(c) (5 points) Calculate the voltage drop across $R_4$.
**Solutions:** $V_4 = (10V + 4V)\frac{R_4}{R_1+R_2+R_3+R_4} = \frac{140V}{70} = 2V$

**8. Equivalent Circuits (13 Points)**



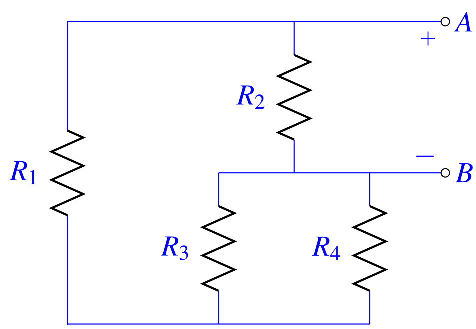

Find $V_{th}$ and $R_{th}$ with respect to nodes $A$ and $B$ for the following circuit in terms of the variables $I_s, R_1, R_2, R_3$, and $R_4$. You may leave the parallel operator $\|$ in your final answer.

**Solutions:** The Thevenin voltage is the voltage across $R_2$. To find the $V_{R2}$ we begin by finding the current through $R_2$ using the current divider formula.

$$I_{R2} = \frac{R_1}{R_1 + R_2 + (R_3 \parallel R_4)} I_s$$

$$V_{th} = V_{R2} = R_2 I_{R2} = \frac{R_1 R_2}{R_1 + R_2 + (R_3 \parallel R_4)} I_s$$

To find the Thevenin resistance, we begin by removing all independent source. The circuit has been drawn below.




From the above circuit, the Thevenin Resistance $R_{th} = R_2 \parallel (R_1 + (R_3 \parallel R_4))$

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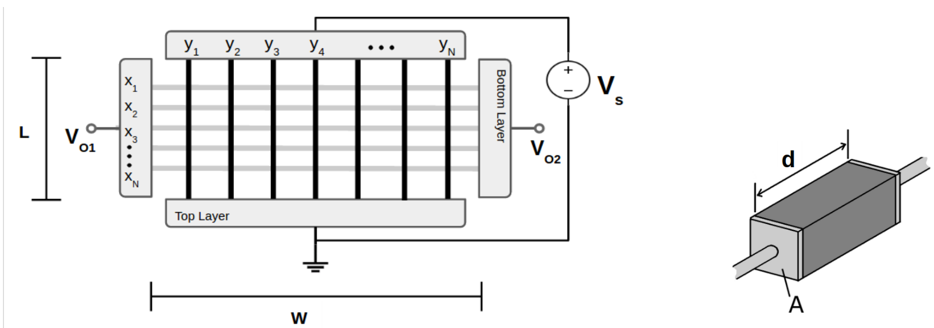
**9. Resist the Touch (16 Points)**

In this question, we will be re-examining the 2-dimensional resistive touchscreen previously discussed in both lecture and lab. The general touch screen is shown in Figure 1 (a). The touchscreen has length $L$ and width $W$ and is composed of a rigid bottom layer and a flexible upper layer. The strips of a single layer are all connected by an ideal conducting plate on each side. The upper left corner is position $(1,1)$.

The top layer has $N$ vertical strips of denoted by $y_1, y_2, ..., y_N$. These vertical strips have cross sectional area $A$, and resistivity $\rho_y$.

The bottom layer has $N$ horizontal strips denoted by $x_1, x_2, ..., x_N$. These horizontal strips all have cross sectional area $A$ as well, and resistivitiy $\rho_x$.

Assume that all top layer resistive strips and bottom layer resistive strips are spaced apart equally. Also assume that all resistive strips are rectangular as shown by Figure 1 (b).




(a) 2-D Resistive Touch Screen

(b) 3D Model of a Single Resistive Strip

Figure 1:

(a) (3 points) Figure 1(b) shows a model for a single resistive strip. Find the equivalent resistance $R_y$ for the vertical strips and $R_x$ for the horizontal strips, as a function of the screen dimensions W and L, the respective resistivities, and the cross-sectional area $A$.

**Solutions:** The equation for resistance is $R = \frac{\rho l}{A}$

Therefore, $R_y = \frac{\rho_y L}{A}$.

For the bottom, $R_x = \frac{\rho_x W}{A}$.

(b) (5 points) Consider a $2 \times 2$ example for the touchscreen circuit.

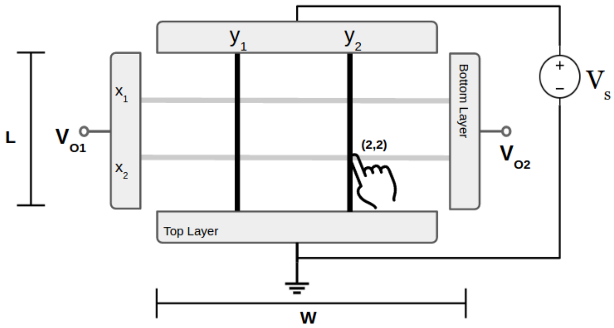


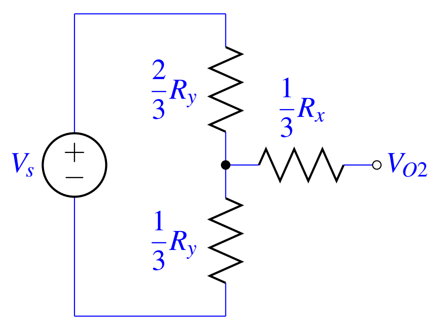

Figure 2: $2 \times 2$ Case of the Resistive Touchscreen

Given that $V_s = 3\,\text{V}$, $R_x = 2000\,\Omega$, and $R_y = 2000\,\Omega$, draw the equivalent circuit for when the point $(2,2)$ is pressed and solve for the voltage at terminal $V_{O2}$ with respect to ground.

**Solutions:**

Since all of the resistive strips are equally spaced, the resistor above point $(2,2)$ on strip $y_2$ becomes $\frac{2}{3}R_y$ and the resistor below point $(2,2)$ on strip $y_2$ becomes $\frac{1}{3}R_y$.

The bottom layer resistors, although they must be drawn in the equivalent circuit, do not affect the voltage at $V_{O1}$ as they are open circuits.




Observing that the resistive strips form a voltage divider, we can determine $V_{O2}$ using the voltage divider equation.

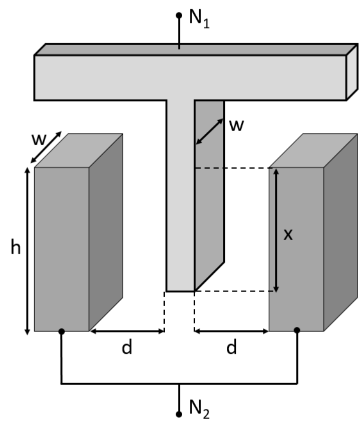
Therefore, $V_{O2} = V_{(2,2)} = V_s \dfrac{\frac{1}{3}R_y}{\frac{1}{3}R_y + \frac{2}{3}R_y} = \dfrac{1}{3}V_s = 1\text{V}.$

(c) (8 points) Suppose a touch occurs at coordinates $(i, j)$ in Figure 1(a). Find an expression for $V_{O2}$ as a function of $V_s$, $N$, $i$, and $j$.

**Solutions:**

$$V_{O2} = \frac{\frac{N+1-i}{N+1}R_y}{R_y}Vs$$
$$= \frac{N+1-i}{N+1}V_s$$

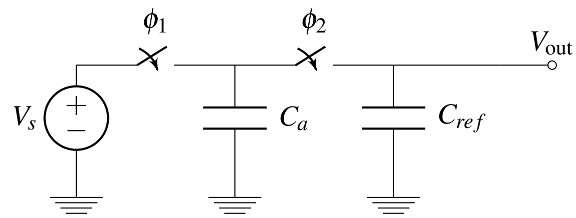
**10. MEMS Sensor (20 Points)**



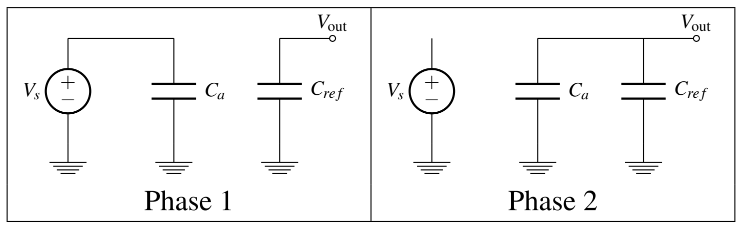

Above is the cross section of a MEMS accelerometer, which consists of a T-shaped mass between two sidewalls. When the device accelerates, the T structure, which is held up by springs, displaces up or down. Assume that all the structures are made of metal separated by air, and that the only relevant capacitances in this problem are parallel plate capacitances.

(a) (4 points) Suppose both sidewalls are connected together with an ideal wire. Find the capacitance between the stem of the T mass and a sidewall as a function of $x$, the displacement of the T mass. Find the net capacitance between the two terminals $N_1$ and $N_2$ assuming the only parallel plate capacitance is that of the T mass stem and sidewall.

**Solutions:** The capacitance between the T and a sidewall is found using the equation for physical capacitance.
$C = \varepsilon_o \frac{Wx}{d}$
There are two such capacitors in the above structure, and the net capacitance is $2C$.




The above circuit is designed to measure the displacement of the T structure. It cycles through two phases, which are depicted in the diagram below. For this problem, you may assume that switching is controlled through a microcontroller and that all capacitors reach steady state during each phase. Furthermore, assume that the T mass does not move during the measurement cycle and all capacitors are initially uncharged.




(b) (8 points) The circuit above is designed to measure the change in capacitance and to output a voltage. The phases of its operation are shown above. Find $V_{\text{out}}$ in Phase 2 as a function of $C_a$ and $C_{ref}$.

**Solutions:** In Phase 1, the charge on $C_{ref}$ is not connected to a voltage source, so it has no additional charge on it. Since initially all capacitors were uncharged, the charge on $C_{ref}$ is 0. $C_a$ is connected to a voltage source, and has charge $C_aV_s$ on it. We use $Q_{tot}$ to denote the total charge on the nodes that will be shared.

$$Q_{tot,1} = Q_a + Q_{ref} = C_aV_s + 0$$

In Phase 2, the two capacitor charge share. Since they are now connected in parallel, they have the same voltage, $V_{out}$ across them.

$$\begin{aligned} Q_{tot,2} = Q_a + Q_{ref} &= (C_a + C_{ref})V_{out} \\ Q_{tot,2} &= Q_{tot,1} \\ (C_a + C_{ref})V_{out} &= C_aV_s \\ V_{out} &= \frac{C_a}{C_a + C_{ref}}V_s \end{aligned}$$

(c) (8 points) Let $V_{out}[i]$ denote $V_{out}$ at the $i$th measurement cycle. Suppose the circuit had been running for $k$ cycles. What is $V_{out}[k]$ in terms of $V_{out}[k-1]$, $C_a$ and $C_{ref}$?

**Solutions:** Here, unlike before there is some charge on $C_{ref}$ because $C_{ref}$ is not being discharged.

The charge on $C_{ref}$ in Phase 1 can be found using the voltage at cycle k-1.

$$
\begin{aligned}
Q_{tot,1} &= C_a V_s + C_{ref} V_{out}[k-1] \\
Q_{tot,2} &= C_a V_{out}[k] + C_{ref} V_{out}[k] \\
Q_{tot,2} &= Q_{tot,1} \\
C_a V_{out}[k] + C_{ref} V_{out}[k] &= C_a V_s + C_{ref} V_{out}[k-1] \\
V_{out}[k] &= \frac{C_a}{C_a + C_{ref}} V_s + \frac{C_{ref}}{C_a + C_{ref}} V_{out}[k-1]
\end{aligned}
$$