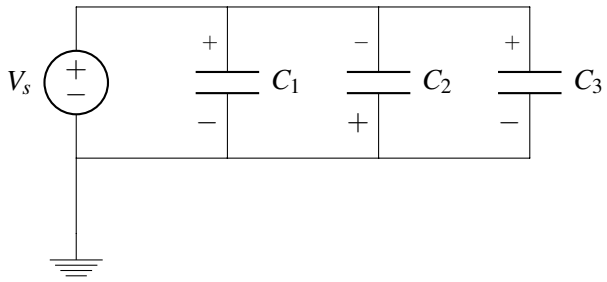
1. Pitfall Problem

(a) Consider the following circuit in ϕ_1. Assume that all capacitors are initially discharged. Find out the charge on each capacitor in this phase.



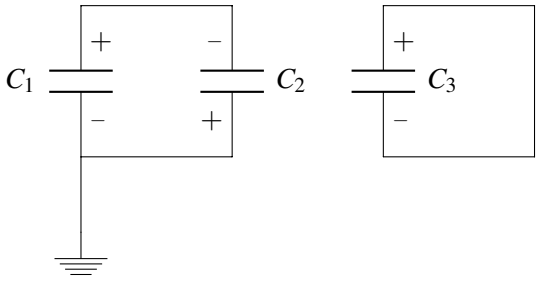
Answer:

$$Q_{C_1,\phi_1} = C_1 V_s$$
$$Q_{C_2,\phi_1} = -C_2 V_s$$
$$Q_{C_3,\phi_1} = C_3 V_s$$

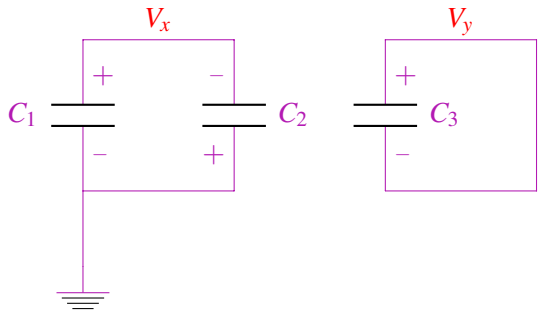
.

Note: What does it mean when we say that the charge on a capacitor is *negative*? It means that **on the positive plate of the capacitor, there is negative charge.** And since a capacitor has equal and opposite charge on each plate, the negative plate then has positive charge.

(b) Assume that ϕ_1 has taken place, and that the capacitors are then moved to the following configuration in ϕ_2. Calculate the charge across each capacitor in ϕ_2.



Answer: Step I: Write the voltage drop across each capacitor.
If it cannot be determined, create variables till it can be determined.
Circuit redrawn with unknown voltages marked.



$$V_{C_1,\phi_2} = V_x - 0 = V_x$$
$$V_{C_2,\phi_2} = 0 - V_x = -V_x$$
$$V_{C_3,\phi_2} = V_y - V_y = 0$$

Step 2: Write the charge on each capacitor.
Just use $Q = CV$ on the first step

$$Q_{C_1,\phi_2} = C_1 V_x$$
$$Q_{C_2,\phi_2} = -C_2 V_x$$
$$Q_{C_3,\phi_2} = 0$$

Step 3: Write the charge sharing equations on floating nodes.
Floating nodes are those where charges cannot escape or enter.
Node marked V_x is the only floating node

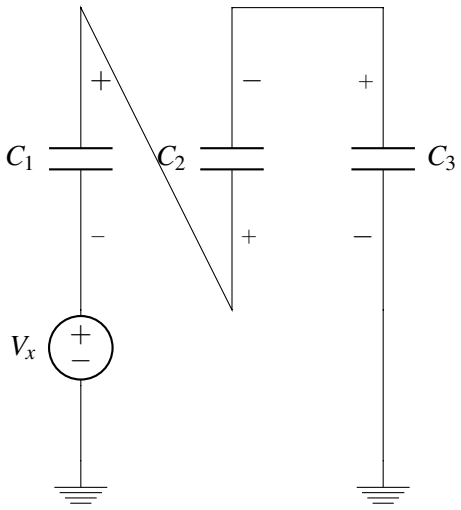
$$Q_{C_1,\phi_2} - Q_{C_2,\phi_2} = Q_{C_1,\phi_1} - Q_{C_2,\phi_1} \tag{1}$$

$$C_1 V_x - (-C_2 V_x) = C_1 V_s - (-C_2 V_2) \implies V_x = V_s$$

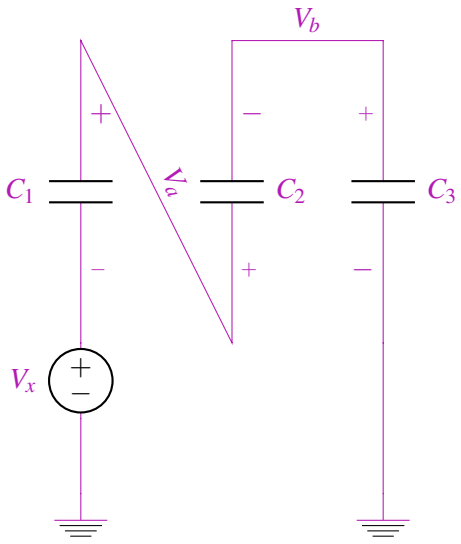
Final Step 4: Plug variable voltage into charge equations

$$Q_{C_1,\phi_2} = C_1 V_x = C_1 V_s$$
$$Q_{C_2,\phi_2} = -C_2 V_x = -C_2 V_s$$
$$Q_{C_3,\phi_2} = 0$$

(c) Assume that ϕ_2 has taken place, and that the capacitors are then moved to the following configuration in ϕ_3. Calculate the charge across each capacitor in ϕ_3.



Answer: Step I: Write the voltage drop across each capacitor.
If it cannot be determined, create variables till it can be determined.
Circuit redrawn with unknown voltages marked.



$$V_{C_1,\phi_3} = V_a - V_s$$
$$V_{C_2,\phi_3} = V_a - V_b$$
$$V_{C_3,\phi_3} = V_b$$

Step 2: Write the charge on each capacitor.
Just use $Q = CV$ on the first step

$$Q_{C_1,\phi_3} = C_1(V_a - V_s)$$
$$Q_{C_2,\phi_3} = C_2(V_a - V_b)$$
$$Q_{C_3,\phi_3} = C_3(V_b)$$

Step 3: Write the charge sharing equations on floating nodes.
Floating nodes are those where charges cannot escape or enter.
Node marked V_x is the only floating node

$$Q_{C_1,\phi_3} + Q_{C_2,\phi_3} = Q_{C_1,\phi_2} + Q_{C_2,\phi_2} \tag{2}$$

$$-Q_{C_2,\phi_3} + Q_{C_3,\phi_3} = -Q_{C_2,\phi_2} + Q_{C_3,\phi_2} \tag{3}$$

Plugging in values to Equation 2

$$C_1(V_a - V_s) + C_2(V_a - V_b) = C_1V_s + (-C_2V_s)$$

Plugging in values to Equation 3

$$-C_2(V_a - V_b) + C_3V_b = -(-C_2V_s) + 0$$

Final Step 4: Plug variable voltage into charge equations
We now have two equations in two variables (V_a, V_b), and so we can solve for them. After that, we can plug those into Step 2, and find the charges on each capacitor. There is no need to do this step as the expressions aren't very neat.