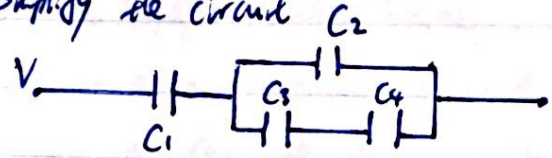
1. Simplify the circuit

a)



$C_1 = 3F,\ C_2 = 4F \quad V = 10v$
$C_3 = 6F,\ C_4 = 3F$

Let $C_5 = [C_3, C_4]$ system. thus $C_5 = \left(\frac{1}{C_3} + \frac{1}{C_4}\right)^{-1} = 2F$

$C_6 = [C_2, C_5]$ system, thus $C_6 = C_5 + C_2 = 6F$

$C_{total} = [C_1, C_6]$ system. $C_{total} = \left(\frac{1}{C_1} + \frac{1}{C_6}\right)^{-1} = 2F$

$Q_{total} = C_{total} \cdot V = 10v \cdot 2F$

$V_1 = \frac{Q}{C_1} = \frac{20}{3} v.$

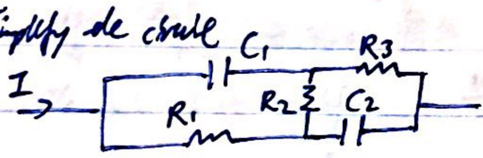
$V_2 = V - V_1 = 10 - \frac{20}{3} = \frac{10}{3} v.$

$V_3 + V_4 = V_2 = \frac{10}{3} v \Rightarrow Q_3 = Q_4 = C_3 \cdot V_3 = C_4 V_4 \Rightarrow \frac{V_3}{V_4} = \frac{C_4}{C_3} = \frac{3}{6} = \frac{1}{2}.$

Thus $V_3 = \frac{10}{3} \times \frac{1}{1+2} = \frac{10}{9} v \quad V_4 = \frac{10}{3} \times \frac{2}{3} = \frac{20}{9} v$

There is no current through the capacitors. $I_{C_i}\ [i \in [1,4]] = 0$

b) Simplify the circuit



$C_1 = 6F,\ C_2 = 6F \quad R_2 = 3\Omega$
$R_1 = 1.5\Omega \quad R_3 = 1.5\Omega \quad I = 4A$

There is no current through capacitors. $I_{C_1} = I_{C_2} = 0$

The resistors are in series. thus $R_{total} = R_T = R_1 + R_2 + R_3 = 6\Omega \quad V_{total} = I \cdot R_{total} = 24V.$

Thus $V_{R_1} = \frac{R_1}{R_T} \cdot V_T = \frac{1}{4} \cdot 24V = 6v. \quad I_{R_1} = I = 4v$

$V_{R_2} = \frac{R_2}{R_T} V_T = \frac{1}{2} \cdot 24V = 12v \quad I_{R_2} = I = 4v. \ /\ V_{R_3} = \frac{R_3}{R_T} \cdot V_T = \frac{1}{4} \times 24v = 6v. \quad I_{R_3} = I = 4v$

$V_{C_1} = \frac{R_1 + R_2}{R_T} \cdot V_T = V_{R_1} + V_{R_2} = 18v.$

$V_{C_2} = V_{R_2} + V_{R_3} = 18v$

2. Basic Amplifier Building Blocks.

a) Denote the node between R_1 and R_2 as node A.

Since the left side of R_1 is connected to ground.

we have voltage at node $A = \frac{R_1}{R_1+R_2} \cdot V_0$

Since the two inputs of the amplifier are equal. $V(\text{node } A) = V_S$.

thus $V_0 = V_S \cdot \frac{R_1+R_2}{R_1}$

Thus the gain of the amplifier is $\frac{V_0}{V_S} = \frac{R_1+R_2}{R_1} > 0$.

Name: non-inverting amplifier.

Origin: Gain of the amplifier is positive, which means that it doesn't invert the signal of input.

b) Denote the node on the right side of R_1 as A.

Since the inputs of the amplifier are equal (in voltage). $V(\text{node } A) = 0$.

Thus. since current through R_1 = current through R_2

$\frac{V_S - 0}{R_1} = \frac{0 - V_0}{R_2} \Rightarrow V_0 = -\frac{R_2}{R_1} V_S$

The gain of the amplifier is $\frac{V_0}{V_S} = -\frac{R_2}{R_1} < 0$

Name: inverting amplifier.

Origin: voltage gain of the amplifier is negative. which means that it invert the signal of input.

3. Amplifier with Multiple Inputs.

a) Denote the node on the right side of R_1 as A.

Since the voltage at the two inputs of the amplifier must be equal.

$V_- = V_+ = 0V$ (since V_+ is connected to GND). thus $V_A = 0$

thus. no current nor voltage across R.

Denote the node on the left side of R_2 as B.

since. the right side of R_2 is 0V. $V_B = V_{o1}$

Thus. the line -GND-R_3-R_2- $\Rightarrow$ i_s is current though R_3 and R_2.

thus $i_s = \frac{V_B - 0}{R_2} = \frac{V_{o1}}{R_2}$

thus. $V_{o1} = i_s \cdot R_2$

b) Golden Rule: $V_- = V_+ = V_{S2}$

Denote the node on the right of R_1 as A. thus $V_A = V_{S2}$

since R_1 and R_2 are in series and the left of R_1 is GND.

Denote the left of R_2 as node B. thus $V_B = V_{o2}$.

thus. $\frac{R_1}{R_1+R_2} \cdot V_{o2} = V_{S2} \Rightarrow V_{o2} = V_{S2} \cdot \frac{R_1+R_2}{R_1} = V_{S2}(1+\frac{R_2}{R_1})$

c) Golden Rule. $V_- = V_+ = V_{S2}$. Denote the right of R_1 as node A. thus $V_A = V_{S2}$

By KCL: $\frac{V_A - V_{S1}}{R_1} + i_s + \frac{V_A - V}{R_2} = 0$ (Denote the left of R_2 as node B. thus $V_B = V_o$)

thus $\frac{V_{S2}-V_{S1}}{R_1} + i_s + \frac{V_{S2}-V_o}{R_2} = 0 \Rightarrow \frac{V_o - V_{S2}}{R_2} = \frac{-V_{S1}+V_{S2}}{R_1} + i_s$

$\Rightarrow V_o = \frac{R_2}{R_1}(V_{S2} - V_{S1}) + R_2 \cdot i_s + V_{S2}$

$= \frac{R_1+R_2}{V_{S2}} + R_2 \cdot i_s - \frac{R_2}{R_1} \cdot V_{S1}$

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d) ~~No. Vo doesn't change between part (c) and this part. [If Vo still mean~~

No, voltage $V_{o,new}$ doesn't depend on R_L.

If Vo still mean the voltage at the output (the node on the right side of R_2) then Vo do not change; if we compare Vo and $V_{o,new}$ then they are different.

Denote the node at the right side of R_2 as A.

Thus. $V_A = \frac{R_1+R_2}{V_{S2}} + R_2 \cdot i_s - \frac{R_2}{R_1} \cdot V_{S1}$

since $V_- = V_+$, for the second op-amp. denote the node on the top of R_4 as B.

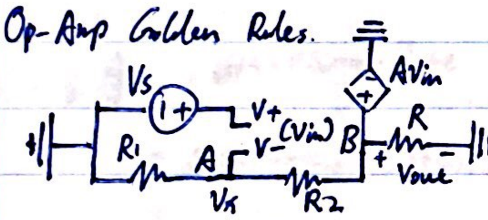
thus $V_B = V_A$

Denote the node on top of $R_3 = C$, then $V_C = V_{o,new}$.

Thus. (since no current enter op-amp). $\frac{R_4}{R_3+R_4} \cdot V_{o,new} = V_B = V_A$

Thus $V_{o,new} = \frac{R_3+R_4}{R_4}\left(\frac{R_1+R_2}{V_{S2}} + R_2 \cdot i_s - \frac{R_2}{R_1} \cdot V_{S1}\right)$

4. Op-Amp Golden Rules.



[Denote node A & B in circuit]

Thus $V_A = V_x$. $V_B = AV_{in} = V_{out} = A(V_+ - V_-) = (V_s - V_x)A$

Since $R_{in} = \infty$. no current between V_+ and V_-

Thus. $V_x = \frac{R_1}{R_1+R_2} \cdot V_B = \frac{R_1}{R_1+R_2} \cdot A(V_s - V_x)$

then $\frac{V_s - V_x}{V_x} = \frac{R_1+R_2}{AR_1} = \frac{V_s}{V_x} - 1 \Rightarrow \frac{V_s}{V_x} = \frac{AR_1+R_1+R_2}{AR_1} \Rightarrow V_x = \frac{AR_1 \cdot V_s}{AR_1+R_1+R_2}$

Since $\frac{V_x}{V_s} = \frac{AR_1}{AR_1+R_1+R_2} < 1$. V_x is smaller than V_s.

And $V_{out} = A(V_s - V_x) = AV_s\left(\frac{R_1+R_2}{AR_1+R_1+R_2}\right)$

The values doesn't depend on R.

b) When $A \to \infty$. $V_x = \frac{AR_1 \cdot V_s}{AR_1 + R_1 + R_2} = V_s \frac{1}{1 + \frac{R_1 + R_2}{AR_1}} \Rightarrow$ since $\frac{R_1 + R_2}{AR_1} \to 0. \Rightarrow V_x \to V_s$

$V_{out} = AV_s \left(\frac{R_1 + R_2}{AR_1 + R_1 + R_2}\right) = \frac{AV_s}{\frac{AR_1}{R_1 + R_2} + 1} = V_s \frac{1}{\frac{R_1}{R_1 + R_2} + \frac{1}{A}} \Rightarrow$ since $\frac{1}{A} \to 0$, $V_{out} \to V_s \frac{1}{\frac{R_1}{R_1 + R_2}} \to V_s \frac{R_1 + R_2}{R_1}$

By Golden Rule

$V_+ = V_s = V_- = V_x$. [same]

Since no current between V_+ and V_-. By KCL : $\frac{V_x - 0}{R_1} + \frac{V_x - V_{out}}{R_2} = 0$

Thus $V_x R_2 + V_x R_1 = V_{out} R_1 \Rightarrow V_{out} = \frac{R_1 + R_2}{R_1} V_x$. [same]

Thus answers are the same.

5. Dynamic Random Access Memory. (DRAM)

a) $Q_{initial} = Q_s = C_{bit} \cdot V_{dd} = 18 \times 10^{-15} \times 1.2 = 2.16 \times 10^{-14} C$

In order to hold $V > 0.8$ after 1ms. we must have $I_{leak} \cdot 1ms \leq 0.4V \cdot C_{bit}$

Thus in maximum, $I_{leak} = \frac{2.16 \times 10^{-14}}{3} / 1 \times 10^{-3} = 7.2 \times 10^{-12} A$

b) $C = \varepsilon \frac{A}{d} = \varepsilon \frac{1024 \times 0.5 \mu m \cdot l_{tube}}{s}$, where $\frac{l_{tube}}{s} = \frac{0.5 \mu m}{0.1 \mu m} = 5$

$= 5 \times 1024 \times 0.5 \times 8.854 \times 10^{-12} \times 10^{-6}$

$= 2.267 \times 10^{-14}$

c) ① 0V. [$V_{bit} = 0V$ initially]

② since $Q = C_{bit} V_{bit}$, after S_1 is switched $V_{tube} = V_{column} = V_{cell}$ [$V_{bit} = 1.2V$ initially]

thus : $V_{column} C_{bit} + V_{column} C_{tube} = C_{bit} V_{bit}$

$= V_{column} (C_{bit} + C_{tube})$

$$V_{column} = \frac{C_{bit} V_{bit}}{C_{bit} + C_{column}} = \frac{18 \times 10^{-15} \times 1.2}{(18 + 20) \times 10^{-15}} \cong 0.568842 V$$

d) $C_{tube} = \varepsilon \frac{A}{d} = 5 \times 0.5 \times 8.854 \times 10^{-12} \times 10^{-6} \times n \cong 2.3135 \times 10^{-17} n$

$V_{column} = \frac{C_{bit} \cdot V_{bit}}{C_{bit} + C_{tube}} = \frac{1800 \times 1.2}{1800 + 2.3135 n} \geq 0.4 \Rightarrow 2160 \geq 720 + 0.927 n$

$\Rightarrow n \leq 1553.4$ thus (cell number) max = 1553

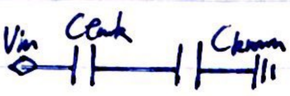
6. It's finally raining

a) Since $C = \frac{\varepsilon A}{d}$

when it's empty: $C_{empty} = \frac{\varepsilon(w \cdot h_{tot})}{w} = \varepsilon \cdot h_{tot}$

when it's full: $C_{full} = \frac{81\varepsilon \cdot (w \cdot h_{tot})}{w} = 81\varepsilon \cdot h_{tot}$

b) $C_{tank} = C_{H_2O} + C_{air} = \frac{81\varepsilon \cdot w \cdot h_{H_2O}}{w} + \frac{\varepsilon \cdot w \cdot (h_{tot} - h_{H_2O})}{w} = 81\varepsilon h_{H_2O} + \varepsilon(h_{tot} - h_{H_2O}) = \varepsilon(h_{tot} + 80 h_{H_2O})$

c) Phase 1: 

$C_{total} = C_T = \left(\frac{1}{C_{tank}} + \frac{1}{C_{known}}\right)^{-1} = \frac{C_{tank} C_{known}}{C_{tank} + C_{known}}$

Since in series. $Q_{tank} = Q_{known} = Q'_{total} = V_{in} \cdot C_T = \frac{C_{tank} \cdot C_{known}}{C_{tank} + C_{known}} \cdot V_{in}$

Phase 2:

$$Q_T = Q_{tank} + Q_{known} = 2 \times Q'_{total} = V_{out}(C^2_{Total}) = V_{out}(C_{tank} + C_{known}) = 2V_{in} \frac{C_{tank} \cdot C_{known}}{C_{tank} + C_{known}}$$

Thus $V_{out} = 2V_{in} \frac{C_{tank} \cdot C_{known}}{(C_{tank} + C_{known})^2}$

d) Done

e) $\frac{2V_{in}}{V_{out}} = \frac{(C_{tank} + C_{known})^2}{C_{tank} \cdot C_{known}} = (C_{tank} + C_{known})\left(\frac{1}{C_{tank}} + \frac{1}{C_{known}}\right) = 2 + \frac{C_{known}}{C_{tank}} + \frac{C_{tank}}{C_{known}}$

$C_{known} = C_{empty} = \varepsilon \cdot h_{tot}$. thus $\frac{C_{tank}}{C_{empty}} = \frac{h_{tot} + 80 h_{H_2O}}{h_{tot}}$

Take $\frac{C_{tank}}{C_{empty}} = x$, $x + \frac{1}{x} + 2 = \frac{2V_{in}}{V_{out}} \Rightarrow x + \frac{1}{x} = \frac{2V_{in}}{V_{out}} - 2 = k$ (define as)

$x + \frac{1}{x} = b. \Rightarrow x^2 - bx + 1 = 0 \Rightarrow x = \frac{b \pm \sqrt{b^2-4}}{2}$. since $C_{tank} \geq C_{empty}$ since $C_{tank} = 1 + \varepsilon \cdot 80 \cdot h_{H_2O}$

thus $x = \frac{b + \sqrt{b^2-4}}{2}$. $x = 1 + \frac{80 h_{H_2O}}{h_{tot}} \Rightarrow h_{H_2O} = \frac{(x-1) h_{tot}}{80}$

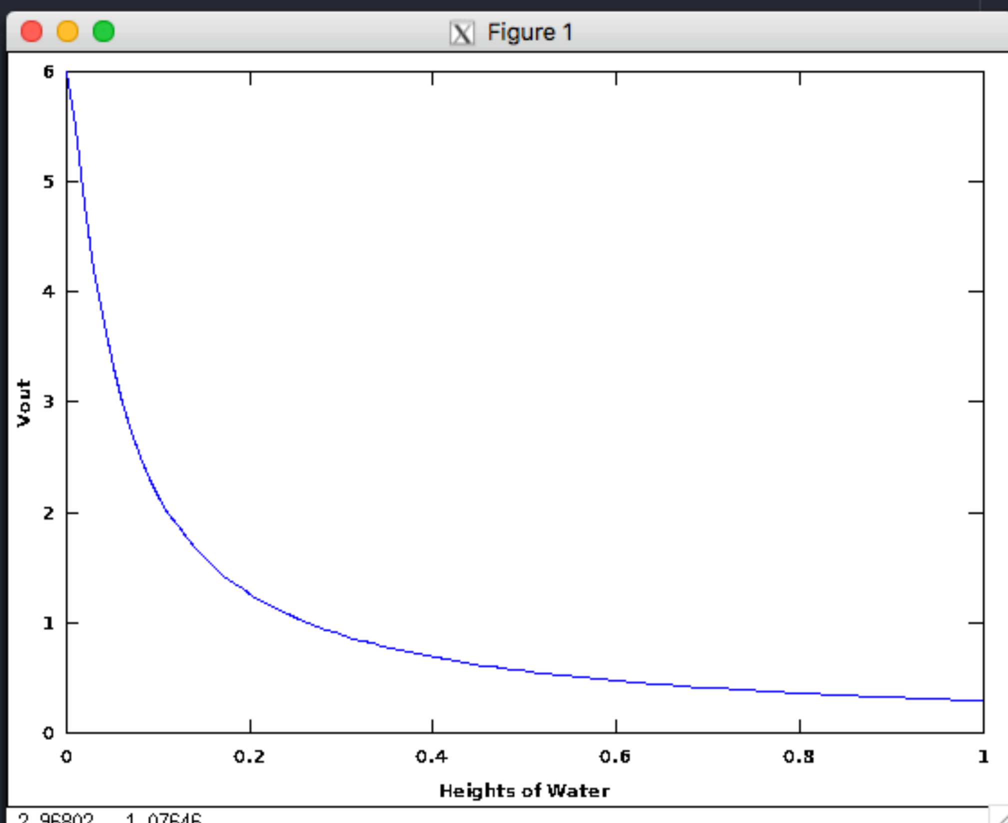
thus $h_{H_2O} = \frac{h_{tot}}{80} \times \left(\frac{\frac{2V_{in}}{V_{out}} - 2 + \sqrt{(\frac{2V_{in}}{V_{out}} - 2)^2 - 4}}{2} - 1\right)$

$$= \frac{h_{tot}}{80} \times \left(\frac{V_{in}}{V_{out}} + \sqrt{(\frac{V_{in}}{V_{out}} - 1)^2 - 1} - 2\right)$$

f) $unit(V_{out}) = unit(V_{in}) \times unit\left(\frac{unit(C_{tank}) \cdot unit(C_{known})}{(unit(C_{tank}) + unit(C_{known}))^2}\right) = V \cdot \left(\frac{F \cdot F}{(F + F)^2}\right) \Rightarrow V \cdot \frac{F^2}{F^2} = Volts$

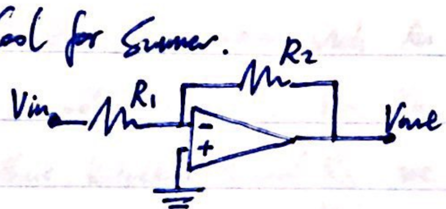
unit $h_{H_2O} = \frac{m}{80} \times \left(\frac{Vol}{Vol} + \sqrt{(\frac{Vol}{Vol} - 1)^2 - 1} - 2\right) \Rightarrow \frac{m}{80} \times (1 + \sqrt{(1-1)^2 - 1} - 2) \Rightarrow m \Rightarrow$ meters.

```
function Vout = the_plot(heights)
  Vin = 12;
  w = 0.5;
  htot = 1;
  c = 8.854 * 1e-12;
  Cknown = c * htot;
  Vout = zeros(size(heights));
  for i = 1:length(heights);
    height = heights(i);
    Ctank = c * (htot + 80 * height);
    Vout(i) = 2 * Vin * (Ctank * Cknown)/(Ctank + Cknown)^2;
  end
  plot(heights, Vout)
  xlabel('Heights of Water')
  ylabel('Vout')
end
```



7. Cool for Summer.

a)

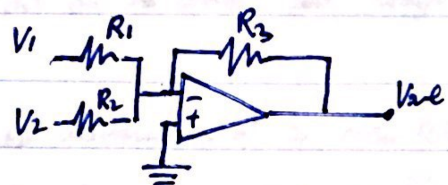


$V_+ = V_- = 0$

Thus. $\frac{0-V_{in}}{R_1} + \frac{0-V_{out}}{R_2} = 0 \Rightarrow V_{out} = -\frac{R_2}{R_1} \cdot V_{in}$

Voltage gain: $\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$

b)



$V_+ = V_- = 0$

Thus: $\frac{0-V_1}{R_1} + \frac{0-V_2}{R_2} + \frac{0-V_{out}}{R_3} = 0$

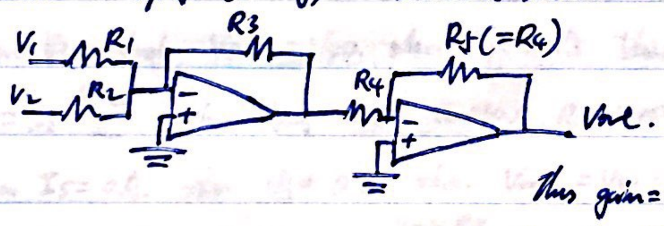
thus $V_{out} = -R_3\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$

c) $V_{out} = -(\frac{1}{4}V_{s1} + 2V_{s2}) \Rightarrow \frac{R_3}{R_1} = \frac{1}{4}$ & $\frac{R_3}{R_2} = 2$.

Thus. possibly, $R_3 = 1k\Omega$. thus $R_1 = 4k\Omega$. thus $R_2 = 500\,\Omega$.

d) Since the first op-amp is an inverting amplifier, thus Vout is negative. thus we need to add another amplifier (inverting) to balance it back.

The.



The signs are inverted, but the value do not change if we add the other op-amp

Thus gain $= \frac{V_{out2}}{V_{in2}} = -1 \Rightarrow |gain| = 1$

Thus $\frac{V_{out}}{V_{in2}} = -\frac{R_5}{R_4} = -1 \Rightarrow R_5 = R_4$

8. Wireless Communication with An LED.

a) By Golden Rule: $V_- = V_+$. Thus $V_0 = V_2 = 0$. and since there's no current between V_+ and V_-

Thus. between. R_1 and R_2. we have $0V \xrightarrow{I_1} R_1 \xrightarrow{I_2} R_2 \; 0V$ (V_1 between R_1 and R_2). and $V_1 \neq 0$

Thus $|I_1| = |\frac{V_1}{R_1}|$, $|I_2| = |\frac{V_1}{R_2}|$. when $R_1 = R_2$. the value of $i_1 = i_2$. if not then $i_1 \neq i_2$

and as no current pass through V_+ and V_-, $I_s = i_1$. $i_2 = i_3$ (if we take the direction of i currents. then $i_1 \neq i_2$ anyways)

b) As discussed in a). by Golden Rule. $V_0 = V_2 = 0$. and $i_1 = \frac{0 - V_1}{R_1} = -\frac{V_1}{R_1}$

thus $V_1 = -i_1 R_1$. and $i_3 = \frac{V_2 - V_3}{R_3} \Rightarrow V_3 = -i_3 R_3 + V_2 = -i_3 R_3 = -i_2 R_3 = -R_3 \left(\frac{V_1 - V_2}{R_2}\right)$

$= -I_s R_1$

$= -R_3 \frac{V_1}{R_2} = -R_3 \frac{-I_s R_1}{R_2} = \frac{I_s R_1 R_3}{R_2}$

c) Same as problem [6]. we can check units.

unit(V_1) = $-(A)\cdot(\Omega)$ = Volts. [correct].

unit(V_2) = 0 volts [correct] $\quad V_3 = \frac{(A)(\Omega)(\Omega)}{(\Omega)}$ = volts [correct]

d) when $V_3 > V_{ref}$, $V_{out} = V_{DD}$. when $V_{ref} > V_3$, $V_{out} = V_{SS}$

$V_3 = I_s \frac{R_1 R_3}{R_2}$, since $V_{ref} = 2V$. $R_1 = 10\Omega$. $R_2 = 1000\Omega$. I_s = 0A or 0.1A

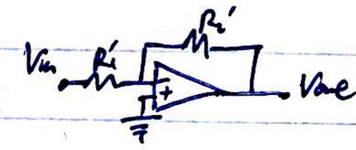
when $I_s = 0A$. then $V_3 = 0v$. then $V_{out} = V_{SS} = -5V$

when $I_s = 0.1A$. then $V_3 = 0.1 \times \frac{10 \times R_3}{1000} = 0.001 R_3$.

in order to let $V_{out} = V_{DD}$. $V_3 > V_{ref} \Rightarrow 0.001 R_3 > 2 \Rightarrow R_3 > 2000\Omega$

Thus we can for example set $R_3 = 3000\Omega$.

e) ~~We can check the units~~ $V_3 = -\frac{R_3}{R_2} V_1$



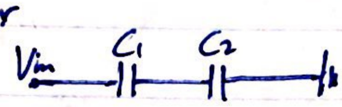
since O_1 and O_2 are invert amplifiers. we must ensure that for small or large

some input. we ~~have~~ can amplify the input be larger than R_{ref}. at least we can amplify

it. thus $|V_3| > |V_1|$. thus $R_3 > R_2$. and it works.

9. DC-DC Voltage Divider

a) Simplify the circuit:



$Q_1 = Q_2 = V_1 C_1 = V_2 C_2 \Rightarrow Q_{total} = Q_1 = Q_2 = C_{total} \cdot V_{in} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} V_{in} = V_{in} \frac{C_1 C_2}{C_1 + C_2}$

Thus $C_1 = \frac{Q_1}{V_1} = \frac{V_{in} C_1 C_2}{(C_1+C_2)V_1}$, $C_2 = \frac{V_{in} C_1 C_2}{(C_1+C_2)V_2}$ $\quad C'_{total} = \frac{C_1 C_2}{C_1 + C_2}$

$$\begin{cases} V_1 + V_2 = V_{in} \\ V_1 C_1 = Q_1 = Q_{total} = V_{in} \frac{C_1 C_2}{C_1 + C_2} \\ V_2 C_2 = Q_2 = \circlearrowright \end{cases} \Rightarrow V_1 = V_{in} \frac{C_2}{C_1 + C_2} \Rightarrow V_2 = V_{in} \frac{C_1}{C_1 + C_2}$$

b) $Q_{total}^2 = Q_1 + Q_2 = 2 \times Q_{total} = 2 V_{in} \frac{C_1 C_2}{C_1 + C_2} = V_{out} \times C_{total}^2 = V_{out} \times (C_1 + C_2)$

thus $V_{out} = 2 V_{in} \frac{C_1 C_2}{(C_1 + C_2)^2}$

c) when $C_1 = C_2$. $V_{out} = 2 V_{in} \frac{C_1^2}{(2C_1)^2} = \frac{1}{2} V_{in}$

Phase 1: $Q_{total} = V_{in} \frac{C_1 C_2}{C_1 + C_2}$ $\quad E_1 = \frac{1}{2} C V^2 = \frac{1}{2} \times C'_{total} V_{in}^2 = \frac{1}{2} \times \frac{C_1}{2} \cdot V_{in}^2 = \frac{C_1}{4} V_{in}^2$

$C_{total} = \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1}{2}$

Phase 2: $E_2 = \frac{1}{2} C V^2 = \frac{1}{2} \times (C_1 + C_2) \times (\frac{1}{2} V_{in})^2 = \frac{C_1}{4} V_{in}^2$

$e = \frac{E_2}{E_1} = 100\%$

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