L24: Least-Squares Linear Regression

Discussion and Applications; Lab 09; Other Topics

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E7 Spring 2017, University of California at Berkeley

March 17, 2017

Version: release

Announcements

Lab 09 is due on March 24 at 12 pm (noon)

Today:

- Least-squares linear regression: discussion and applications
- Introduction to lab 09
- E7 programming project
  - A first glance
  - Forming teams

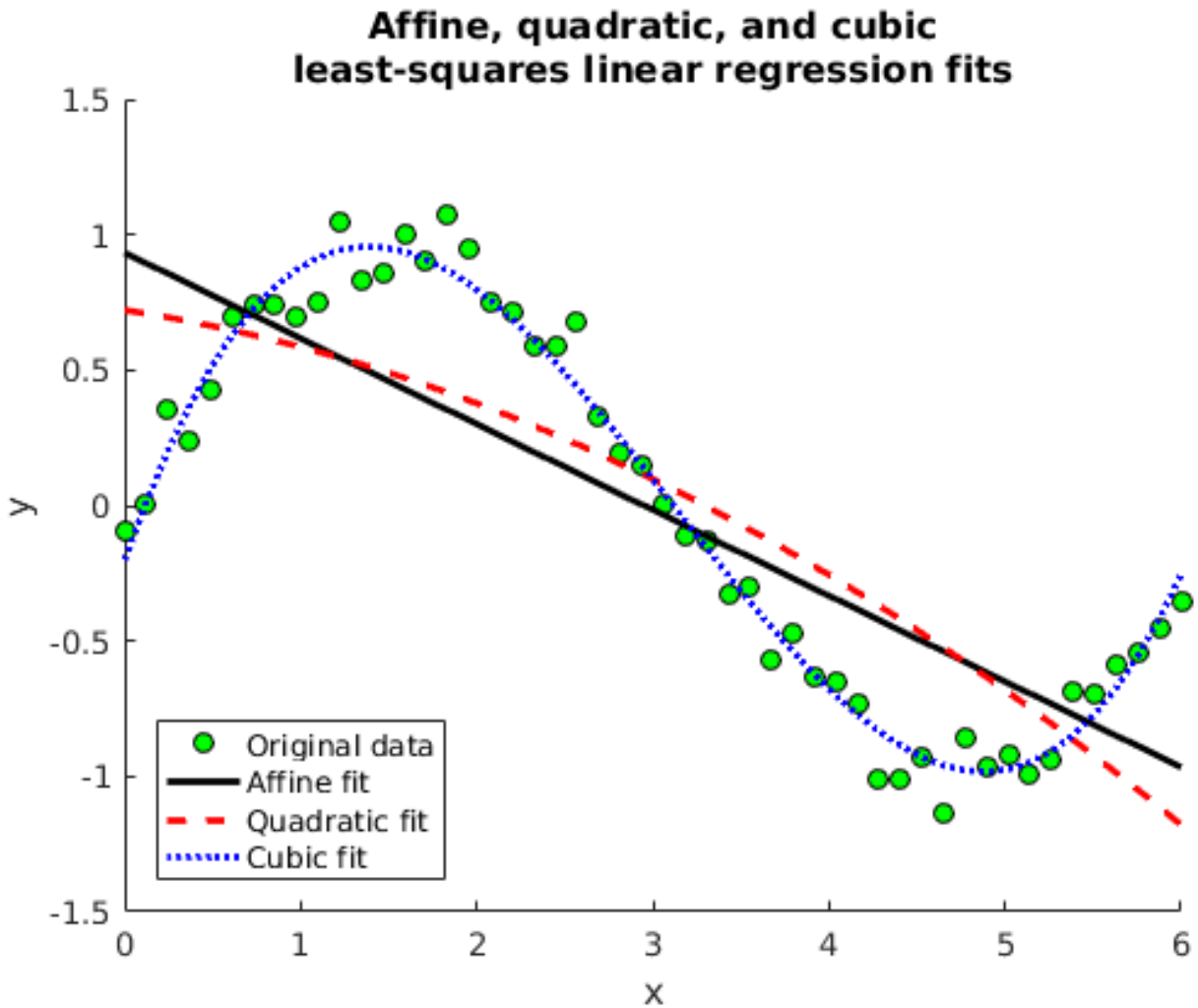
Next week:

- Monday: Interpolation (chapter 14)
- Wednesday: Series (chapter 15)
- Friday: Discussion
- Wednesday or Friday: presentation of the final programming project

One more example of least-squares linear regression

See function `my_multiple_regressions`

Here: the cubic line fits the original data nicely, but the other lines do not



One more example of least-squares linear regression

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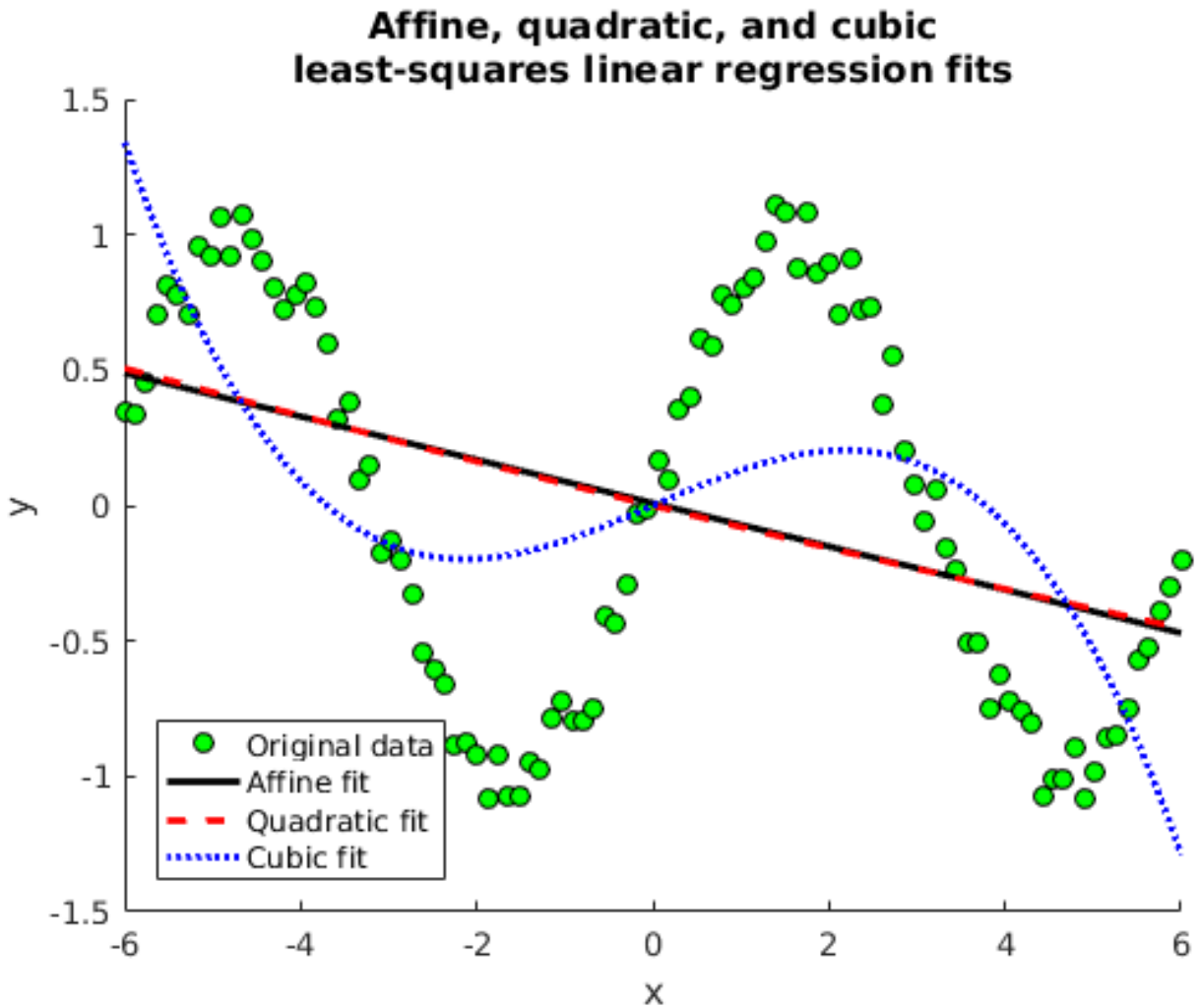
Here: none of the lines fit the original data nicely



“Unfittable” data

Sometimes, data just cannot be fitted in a way that makes sense...

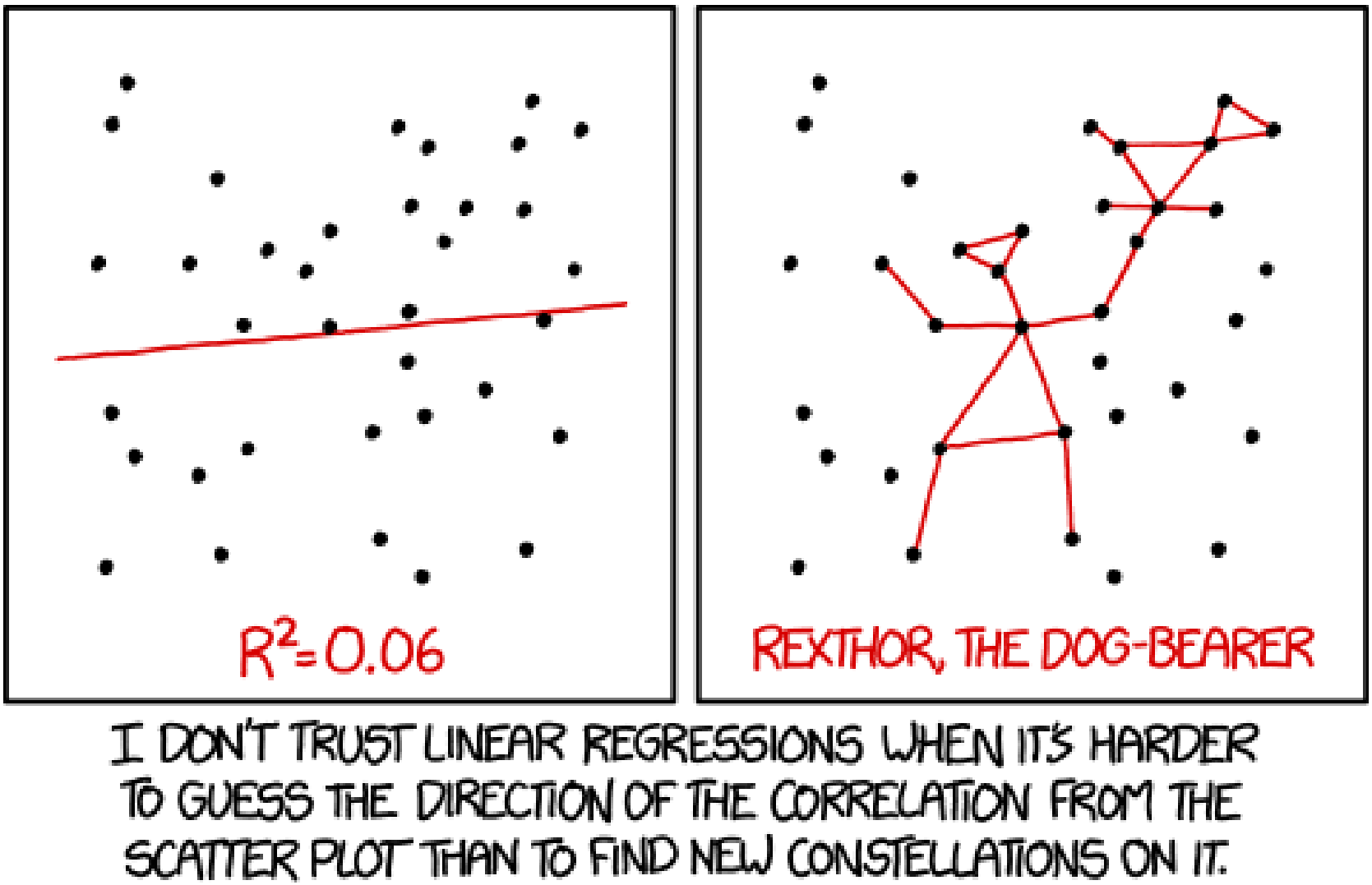


Image retrieved from https://xkcd.com/1725/ on March 17$^{\text{th}}$ 2017

Introduction to lab 09

Topic:

Least-squares linear regression

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Before working on the lab:

Review the material from lectures L22 and L23

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Review the material from lectures L22 and L23

As you work on the lab, think about what the results mean

Look at the figures in the instructions

(note: your functions should **not** create figures)

Q3: Fitting with sines and cosines

Objective: given a set of x- and y-data, fit a function of the form:

$$f(x) = k + \sum_{i=1}^{n} a_i \sin(ix) + \sum_{i=1}^{n} b_i \cos(ix)$$

where n is an integer that is greater than zero, and where the coefficients to fit are k, $a_i, i = \{1, 2, \ldots, n\}$, and $b_i, i = \{1, 2, \ldots, n\}$

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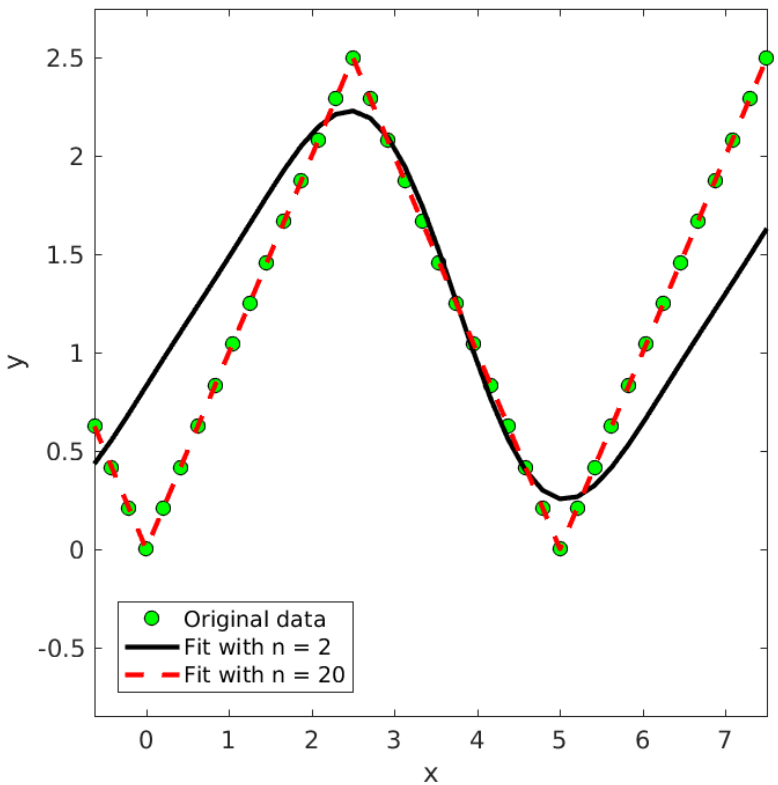

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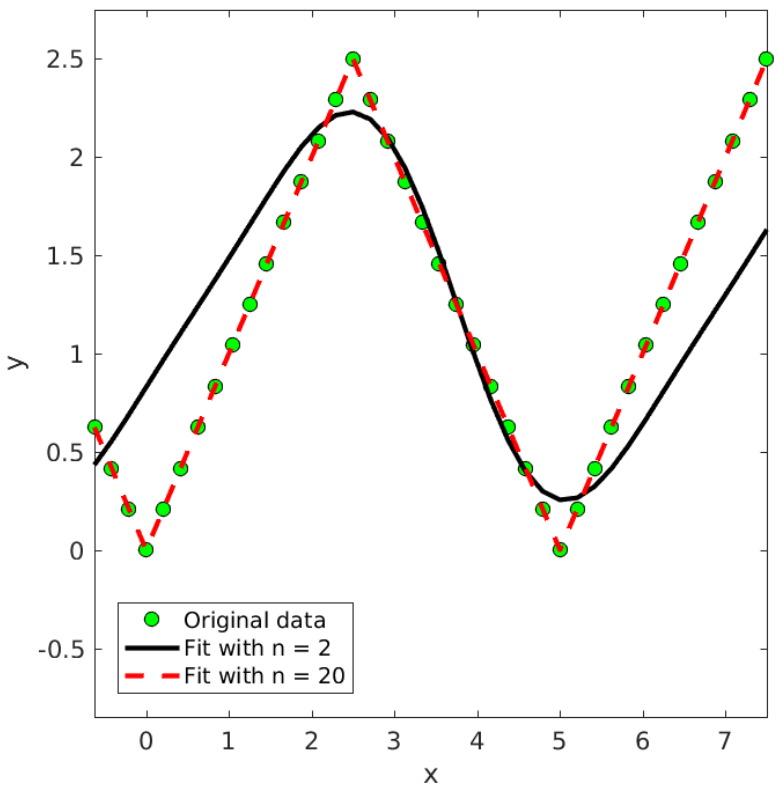

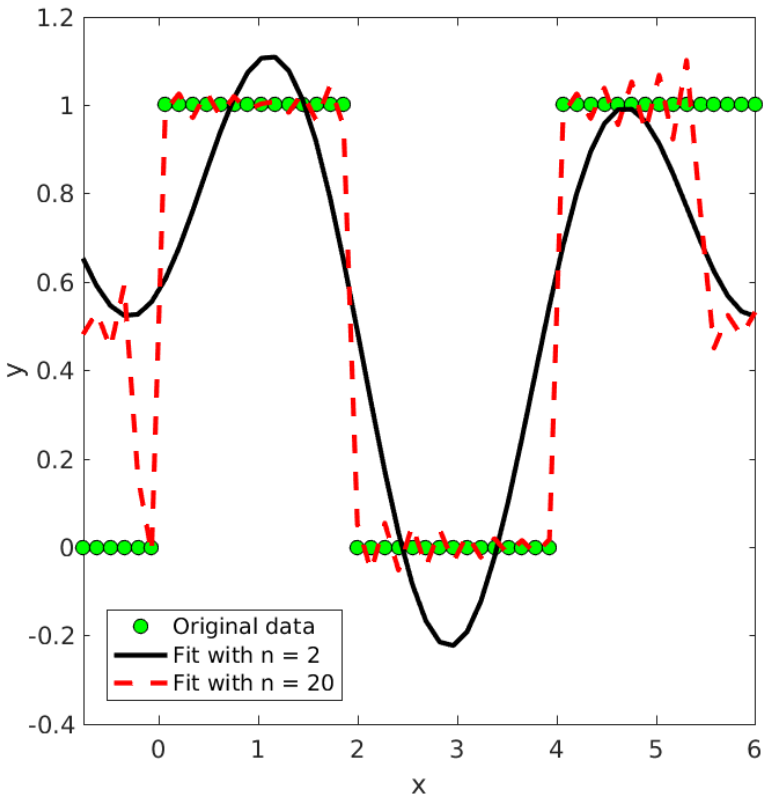

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Note:
Fitting polynomial of higher and higher degrees reduces the square error

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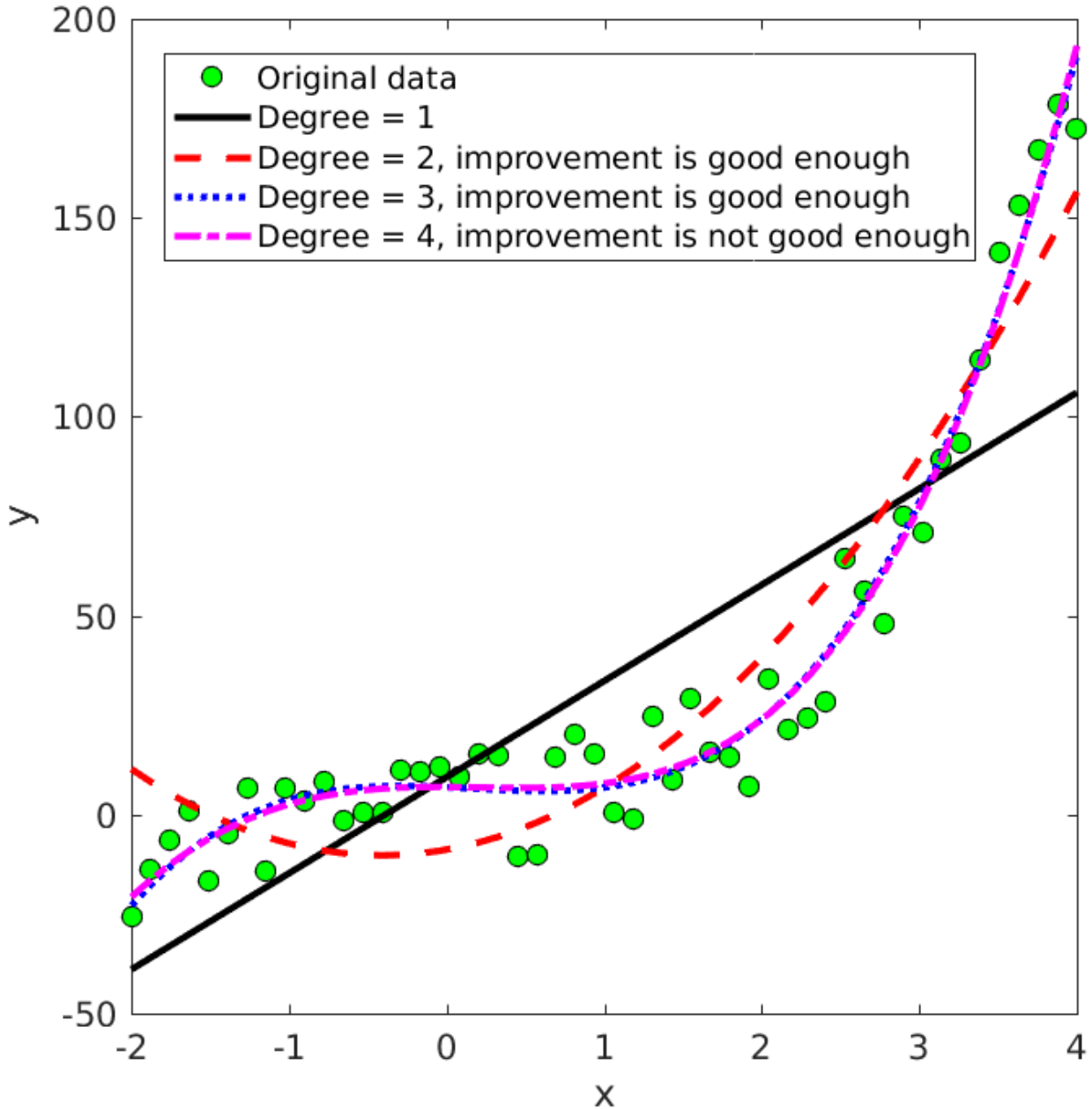

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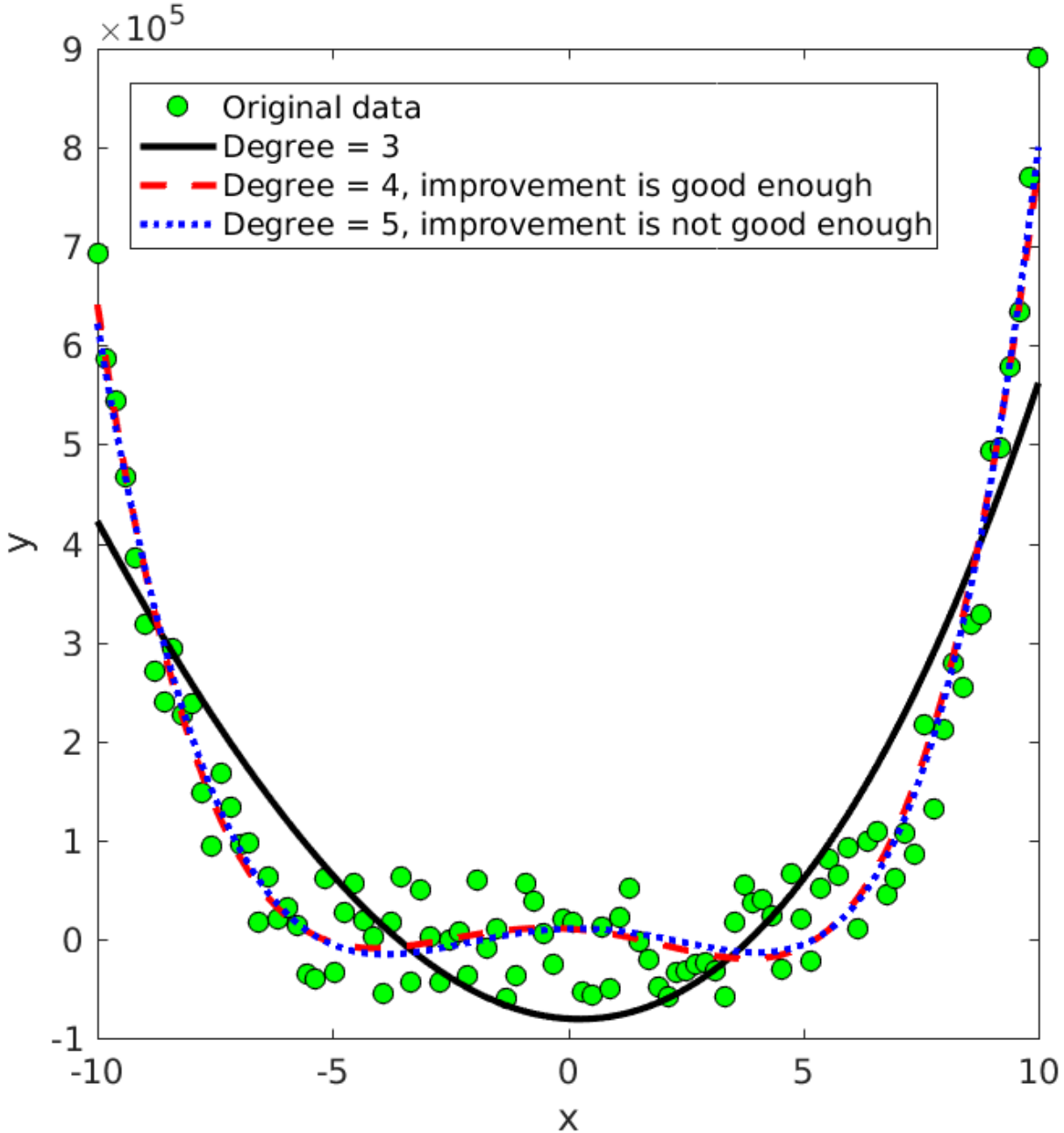

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by finding a such that:

$$E_p'(a) = \sum_{i=1}^{m} p x_i (ax_i - y_i)^{p-1} = 0$$

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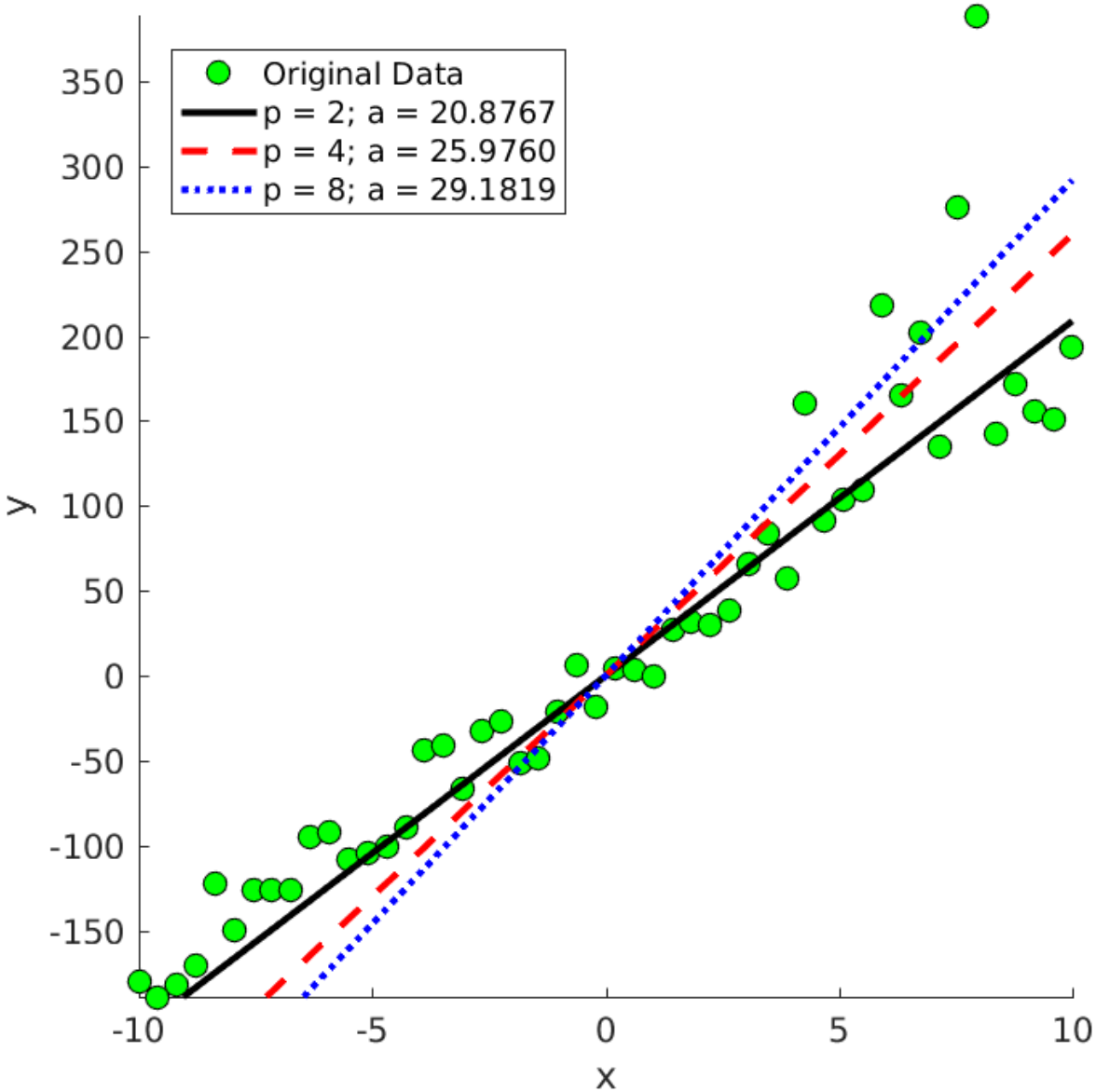
The larger the p, the most importance the "outliers" have



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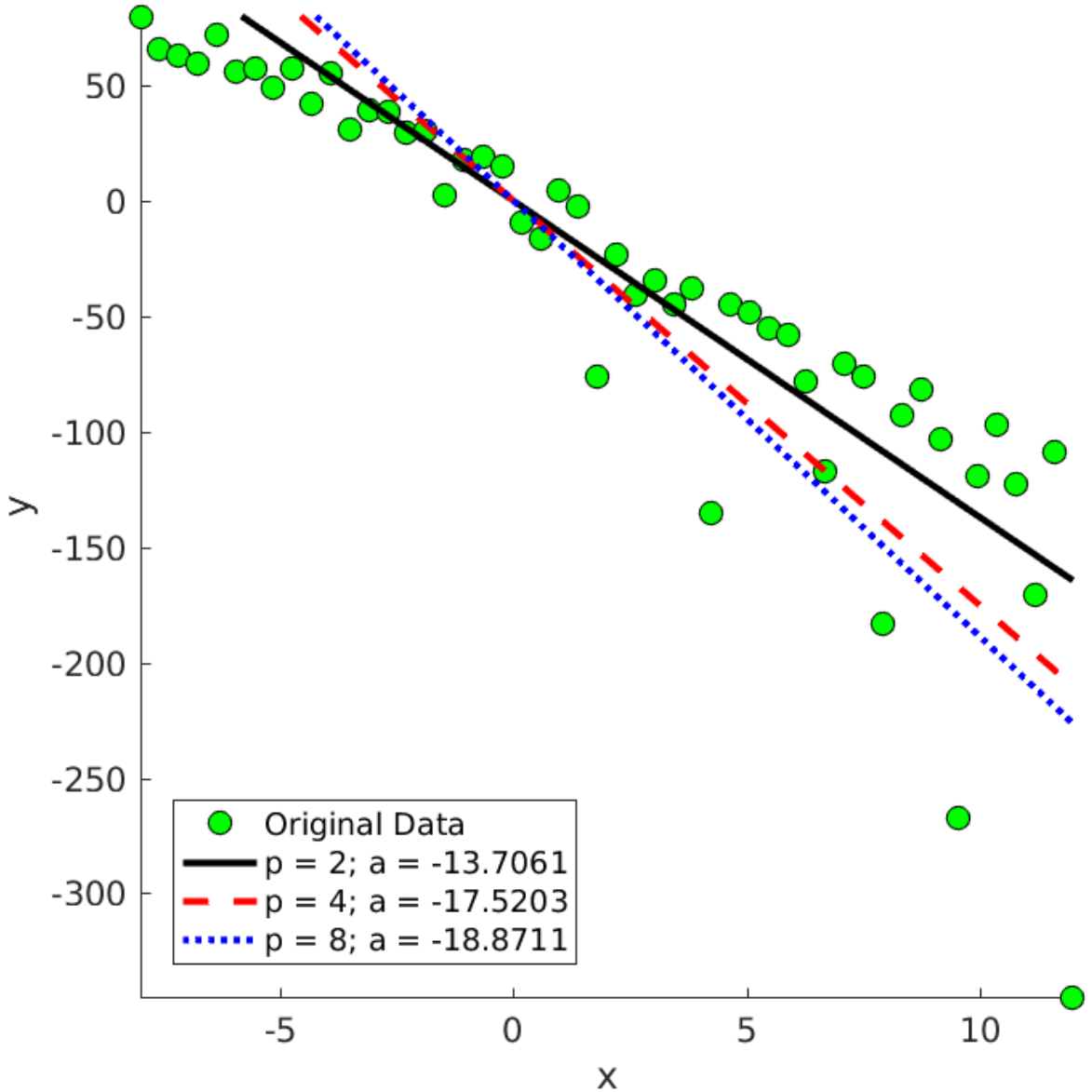

Useful functions: find and arrayfun

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You can code anything without using these functions (*e.g.*, by using `for` loops instead), but using them might make your code shorter and/or more readable

Note: you do not need to use these functions for lab 09

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The grading will be based on various criteria, such as:

- Collecting scrap efficiently
- Avoiding (moving), asteroids, fixed obstacles, and/or nebulae

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By next Wednesday (March 22^{nd}): **email/bCourses message your primary GSI with the name(s) of your teammate(s)**

If you cannot find teammates, let your GSI know, they will help you form a team