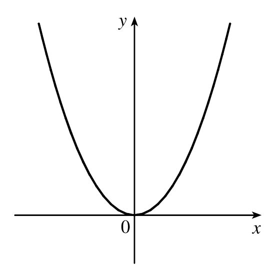
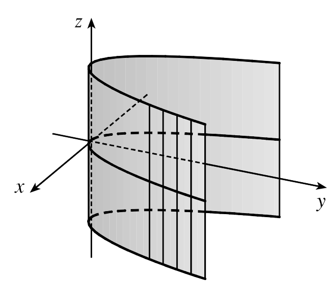
4. The vector from $(621, -147, 206)$ to $(563, 31, 242)$, $\mathbf{v}_1 = \langle -58, 178, 36 \rangle$, lies in the plane of the rectangle, as does the vector from $(621, -147, 206)$ to $(657, -111, 86)$, $\mathbf{v}_2 = \langle 36, 36, -120 \rangle$. A normal vector for the plane is $\mathbf{v}_1 \times \mathbf{v}_2 = \langle -1888, -142, -708 \rangle$ or $\langle 8, 2, 3 \rangle$, and an equation of the plane is $8x + 2y + 3z = 5292$. The line L intersects this plane when $8(230 + 630t) + 2(-285 + 390t) + 3(102 + 162t) = 5292 \quad \Rightarrow \quad t = \frac{1858}{3153} \approx 0.589$. The corresponding point is approximately $(601.25, -55.18, 197.46)$. Starting at this point, a portion of the line is hidden behind the rectangle. The line becomes visible again at the left edge of the rectangle, specifically the edge between the points $(621, -147, 206)$ and $(657, -111, 86)$. (This is most easily determined by graphing the rectangle and the line.) A plane through these two points and the camera's location, $(1000, 0, 0)$, will clip the line at the point it becomes visible. Two vectors in this plane are $\mathbf{v}_1 = \langle -379, -147, 206 \rangle$ and $\mathbf{v}_2 = \langle -343, -111, 86 \rangle$. A normal vector for the plane is $\mathbf{v}_1 \times \mathbf{v}_2 = \langle 10224, -38064, -8352 \rangle$ and an equation of the plane is $213x - 793y - 174z = 213{,}000$. L intersects this plane when $213(230 + 630t) - 793(-285 + 390t) - 174(102 + 162t) = 213{,}000 \quad \Rightarrow \quad t = \frac{44{,}247}{203{,}268} \approx 0.2177$. The corresponding point is approximately $(367.14, -200.11, 137.26)$. Thus the portion of L that should be removed is the segment between the points $(601.25, -55.18, 197.46)$ and $(367.14, -200.11, 137.26)$.

12.6 Cylinders and Quadric Surfaces

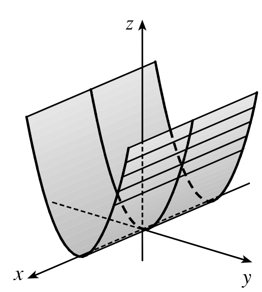
1. (a) In $\mathbb{R}^2$, the equation $y = x^2$ represents a parabola.



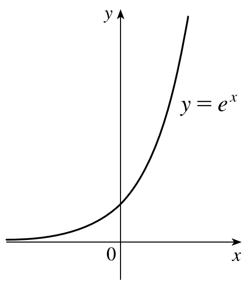
(b) In $\mathbb{R}^3$, the equation $y = x^2$ doesn't involve z, so any horizontal plane with equation $z = k$ intersects the graph in a curve with equation $y = x^2$. Thus, the surface is a parabolic cylinder, made up of infinitely many shifted copies of the same parabola. The rulings are parallel to the z-axis.



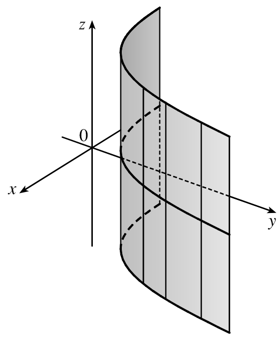
(c) In $\mathbb{R}^3$, the equation $z = y^2$ also represents a parabolic cylinder. Since x doesn't appear, the graph is formed by moving the parabola $z = y^2$ in the direction of the x-axis. Thus, the rulings of the cylinder are parallel to the x-axis.



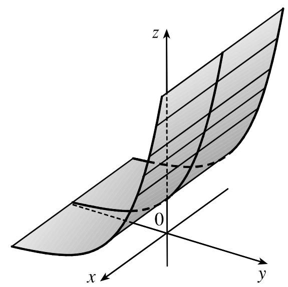
2. (a)



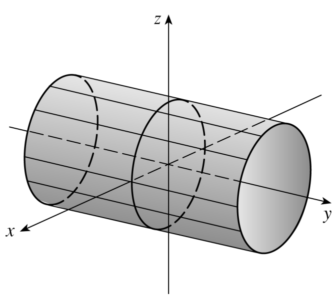
(b) Since the equation $y = e^x$ doesn't involve z, horizontal traces are copies of the curve $y = e^x$. The rulings are parallel to the z-axis.



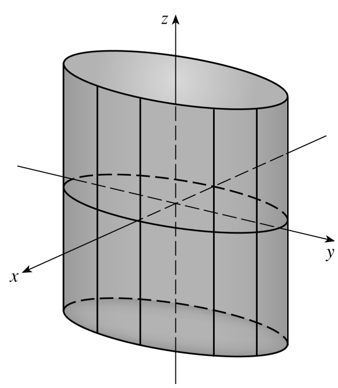
(c) The equation $z = e^y$ doesn't involve x, so vertical traces in $x = k$ (parallel to the yz-plane) are copies of the curve $z = e^y$. The rulings are parallel to the x-axis.



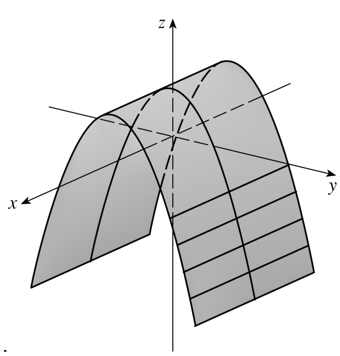
3. Since y is missing from the equation, the vertical traces $x^2 + z^2 = 1$, $y = k$, are copies of the same circle in the plane $y = k$. Thus the surface $x^2 + z^2 = 1$ is a circular cylinder with rulings parallel to the y-axis.



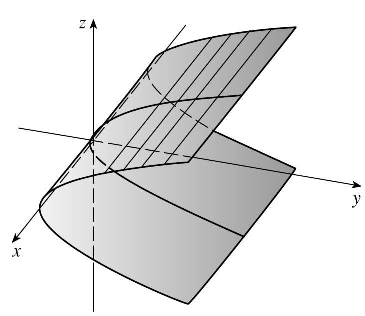
4. Since z is missing from the equation, the horizontal traces $4x^2 + y^2 = 4$, $z = k$, are copies of the same ellipse in the plane $z = k$. Thus the surface $4x^2 + y^2 = 4$ is an elliptic cylinder with rulings parallel to the z-axis.



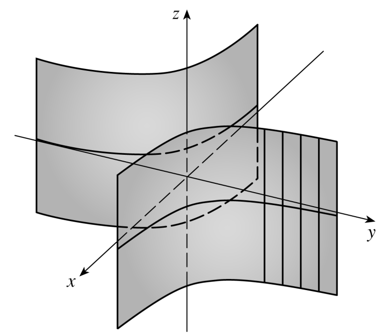
5. Since x is missing, each vertical trace $z = 1 - y^2$, $x = k$, is a copy of the same parabola in the plane $x = k$. Thus the surface $z = 1 - y^2$ is a parabolic cylinder with rulings parallel to the x-axis.



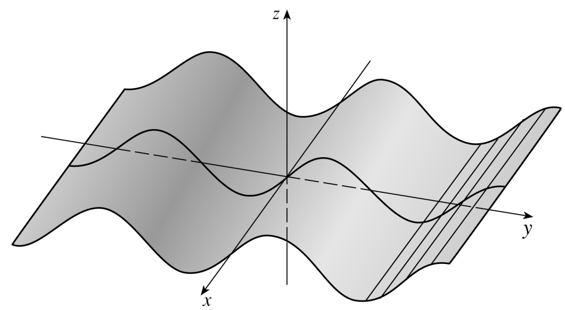
6. Since x is missing, each vertical trace $y = z^2$, $x = k$, is a copy of the same parabola in the plane $x = k$. Thus the surface $y = z^2$ is a parabolic cylinder with rulings parallel to the x-axis.



7. Since z is missing, each horizontal trace $xy = 1$, $z = k$, is a copy of the same hyperbola in the plane $z = k$. Thus the surface $xy = 1$ is a hyperbolic cylinder with rulings parallel to the z-axis.

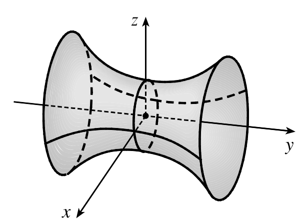


8. Since x is missing, each vertical trace $z = \sin y$, $x = k$, is a copy of a sine curve in the plane $x = k$. Thus the surface $z = \sin y$ is a cylindrical surface with rulings parallel to the x-axis.

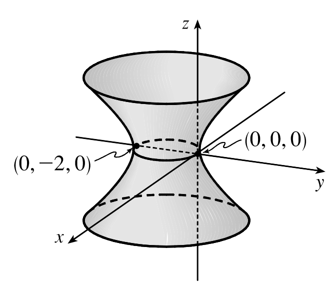


9. (a) The traces of $x^2 + y^2 - z^2 = 1$ in $x = k$ are $y^2 - z^2 = 1 - k^2$, a family of hyperbolas. (Note that the hyperbolas are oriented differently for $-1 < k < 1$ than for $k < -1$ or $k > 1$.) The traces in $y = k$ are $x^2 - z^2 = 1 - k^2$, a similar family of hyperbolas. The traces in $z = k$ are $x^2 + y^2 = 1 + k^2$, a family of circles. For $k = 0$, the trace in the xy-plane, the circle is of radius 1. As $|k|$ increases, so does the radius of the circle. This behavior, combined with the hyperbolic vertical traces, gives the graph of the hyperboloid of one sheet in Table 1.

(b) The shape of the surface is unchanged, but the hyperboloid is rotated so that its axis is the y-axis. Traces in $y = k$ are circles, while traces in $x = k$ and $z = k$ are hyperbolas.

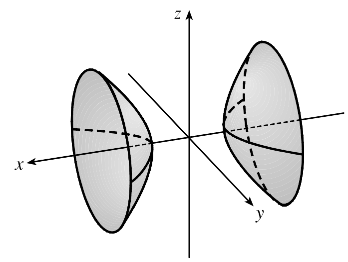


(c) Completing the square in y gives $x^2 + (y + 1)^2 - z^2 = 1$. The surface is a hyperboloid identical to the one in part (a) but shifted one unit in the negative y-direction.

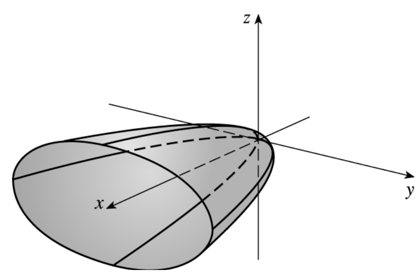


10. (a) The traces of $-x^2 - y^2 + z^2 = 1$ in $x = k$ are $-y^2 + z^2 = 1 + k^2$, a family of hyperbolas, as are the traces in $y = k$, $-x^2 + z^2 = 1 + k^2$. The traces in $z = k$ are $x^2 + y^2 = k^2 - 1$, a family of circles for $|k| > 1$. As $|k|$ increases, the radii of the circles increase; the traces are empty for $|k| < 1$. This behavior, combined with the vertical traces, gives the graph of the hyperboloid of two sheets in Table 1.

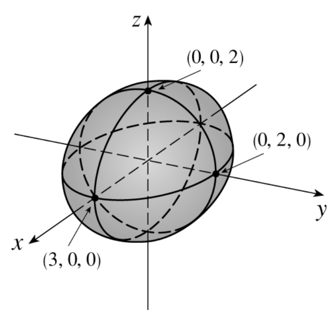
(b) The graph has the same shape as the hyperboloid in part (a) but is rotated so that its axis is the x-axis. Traces in $x = k$, $|k| > 1$, are circles, while traces in $y = k$ and $z = k$ are hyperbolas.



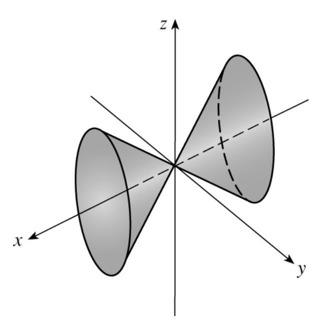
11. For $x = y^2 + 4z^2$, the traces in $x = k$ are $y^2 + 4z^2 = k$. When $k > 0$ we have a family of ellipses. When $k = 0$ we have just a point at the origin, and the trace is empty for $k < 0$. The traces in $y = k$ are $x = 4z^2 + k^2$, a family of parabolas opening in the positive x-direction. Similarly, the traces in $z = k$ are $x = y^2 + 4k^2$, a family of parabolas opening in the positive x-direction. We recognize the graph as an elliptic paraboloid with axis the x-axis and vertex the origin.



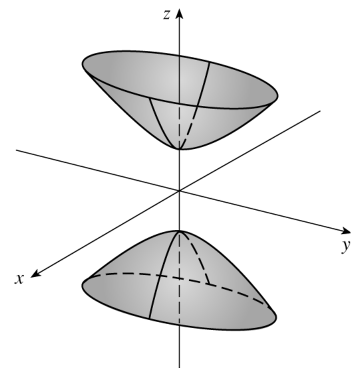
12. $4x^2 + 9y^2 + 9z^2 = 36$. The traces in $x = k$ are $9y^2 + 9z^2 = 36 - 4k^2 \Leftrightarrow y^2 + z^2 = 4 - \frac{4}{9}k^2$, a family of circles for $|k| < 3$. (The traces are a single point for $|k| = 3$ and are empty for $|k| > 3$.) The traces in $y = k$ are $4x^2 + 9z^2 = 36 - 9k^2$, a family of ellipses for $|k| < 2$. Similarly, the traces in $z = k$ are the ellipses $4x^2 + 9y^2 = 36 - 9k^2$, $|k| < 2$. The graph is an ellipsoid centered at the origin with intercepts $x = \pm 3$, $y = \pm 2$, $z = \pm 2$.



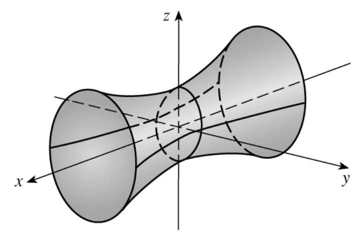
13. $x^2 = 4y^2 + z^2$. The traces in $x = k$ are the ellipses $4y^2 + z^2 = k^2$. The traces in $y = k$ are $x^2 - z^2 = 4k^2$, hyperbolas for $k \neq 0$ and two intersecting lines if $k = 0$. Similarly, the traces in $z = k$ are $x^2 - 4y^2 = k^2$, hyperbolas for $k \neq 0$ and two intersecting lines if $k = 0$. We recognize the graph as an elliptic cone with axis the x-axis and vertex the origin.



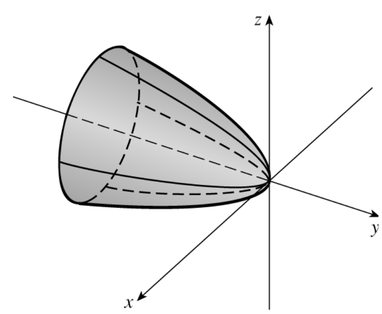
14. $z^2 - 4x^2 - y^2 = 4$. The traces in $x = k$ are the hyperbolas $z^2 - y^2 = 4 + 4k^2$, and the traces in $y = k$ are the hyperbolas $z^2 - 4x^2 = 4 + k^2$. The traces in $z = k$ are $4x^2 + y^2 = k^2 - 4$, a family of ellipses for $|k| > 2$. (The traces are a single point for $|k| = 2$ and are empty for $|k| < 2$.) The surface is a hyperboloid of two sheets with axis the z-axis.



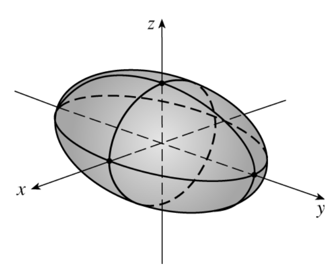
15. $9y^2 + 4z^2 = x^2 + 36$. The traces in $x = k$ are $9y^2 + 4z^2 = k^2 + 36$, a family of ellipses. The traces in $y = k$ are $4z^2 - x^2 = 9(4 - k^2)$, a family of hyperbolas for $|k| \neq 2$ and two intersecting lines when $|k| = 2$. (Note that the hyperbolas are oriented differently for $|k| < 2$ than for $|k| > 2$.) The traces in $z = k$ are $9y^2 - x^2 = 4(9 - k^2)$, a family of hyperbolas when $|k| \neq 3$ (oriented differently for $|k| < 3$ than for $|k| > 3$) and two intersecting lines when $|k| = 3$. We recognize the graph as a hyperboloid of one sheet with axis the x-axis.



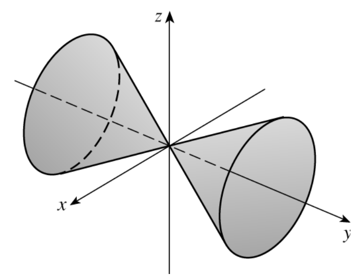
16. $3x^2 + y + 3z^2 = 0$. The traces in $x = k$ are the parabolas $y = -3z^2 - 3k^2$ which open to the left (in the negative y-direction). Traces in $y = k$ are $3x^2 + 3z^2 = -k \quad \Leftrightarrow \quad x^2 + z^2 = -\dfrac{k}{3}$, a family of circles for $k < 0$. (Traces are empty for $k > 0$ and a single point for $k = 0$.) Traces in $z = k$ are the parabolas $y = -3x^2 - 3k^2$ which open in the negative y-direction. The graph is a circular paraboloid with axis the y-axis, opening in the negative y-direction, and vertex the origin.



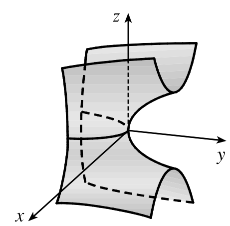
17. $\dfrac{x^2}{9} + \dfrac{y^2}{25} + \dfrac{z^2}{4} = 1$. The traces in $x = k$ are $\dfrac{y^2}{25} + \dfrac{z^2}{4} = 1 - \dfrac{k^2}{9}$, a family of ellipses for $|k| < 3$. (The traces are a single point for $|k| = 3$ and are empty for $|k| > 3$.) The traces in $y = k$ are the ellipses $\dfrac{x^2}{9} + \dfrac{z^2}{4} = 1 - \dfrac{k^2}{25}$, $|k| < 5$, and the traces in $z = k$ are the ellipses $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1 - \dfrac{k^2}{4}$, $|k| < 2$. The surface is an ellipsoid centered at the origin with intercepts $x = \pm 3$, $y = \pm 5$, $z = \pm 2$.



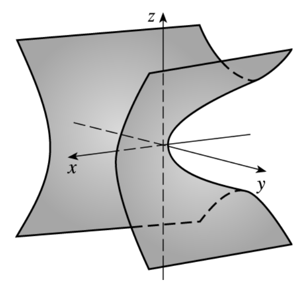
18. $3x^2 - y^2 + 3z^2 = 0$. The traces in $x = k$ are $y^2 - 3z^2 = 3k^2$, a family of hyperbolas for $k \neq 0$ and two intersecting lines if $k = 0$. Traces in $y = k$ are the circles $3x^2 + 3z^2 = k^2 \quad \Leftrightarrow \quad x^2 + z^2 = \frac{1}{3}k^2$. The traces in $z = k$ are $y^2 - 3x^2 = 3k^2$, hyperbolas for $k \neq 0$ and two intersecting lines if $k = 0$. We recognize the surface as a circular cone with axis the y-axis and vertex the origin.



19. $y = z^2 - x^2$. The traces in $x = k$ are the parabolas $y = z^2 - k^2$, opening in the positive y-direction. The traces in $y = k$ are $k = z^2 - x^2$, two intersecting lines when $k = 0$ and a family of hyperbolas for $k \neq 0$ (note that the hyperbolas are oriented differently for $k > 0$ than for $k < 0$). The traces in $z = k$ are the parabolas $y = k^2 - x^2$ which open in the negative y-direction. Thus the surface is a hyperbolic paraboloid centered at $(0, 0, 0)$.



20. $x = y^2 - z^2$. The traces in $x = k$ are $y^2 - z^2 = k$, two intersecting lines when $k = 0$ and a family of hyperbolas for $k \neq 0$ (oriented differently for $k > 0$ than for $k < 0$). The traces in $y = k$ are the parabolas $x = -z^2 + k^2$, opening in the negative x-direction, and the traces in $z = k$ are the parabolas $x = y^2 - k^2$ which open in the positive x-direction. The graph is a hyperbolic paraboloid centered at $(0, 0, 0)$.



21. This is the equation of an ellipsoid: $x^2 + 4y^2 + 9z^2 = x^2 + \dfrac{y^2}{(1/2)^2} + \dfrac{z^2}{(1/3)^2} = 1$, with x-intercepts ± 1, y-intercepts $\pm\frac{1}{2}$ and z-intercepts $\pm\frac{1}{3}$. So the major axis is the x-axis and the only possible graph is VII.

22. This is the equation of an ellipsoid: $9x^2 + 4y^2 + z^2 = \dfrac{x^2}{(1/3)^2} + \dfrac{y^2}{(1/2)^2} + z^2 = 1$, with x-intercepts $\pm\frac{1}{3}$, y-intercepts $\pm\frac{1}{2}$ and z-intercepts ± 1. So the major axis is the z-axis and the only possible graph is IV.

23. This is the equation of a hyperboloid of one sheet, with $a = b = c = 1$. Since the coefficient of y^2 is negative, the axis of the hyperboloid is the y-axis, hence the correct graph is II.

24. This is a hyperboloid of two sheets, with $a = b = c = 1$. This surface does not intersect the xz-plane at all, so the axis of the hyperboloid is the y-axis and the graph is III.

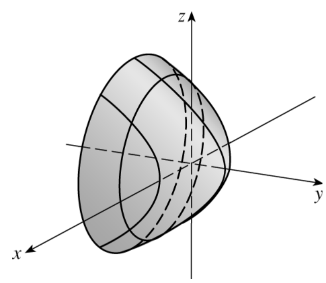
25. There are no real values of x and z that satisfy this equation for $y < 0$, so this surface does not extend to the left of the xz-plane. The surface intersects the plane $y = k > 0$ in an ellipse. Notice that y occurs to the first power whereas x and z occur to the second power. So the surface is an elliptic paraboloid with axis the y-axis. Its graph is VI.

26. This is the equation of a cone with axis the y-axis, so the graph is I.

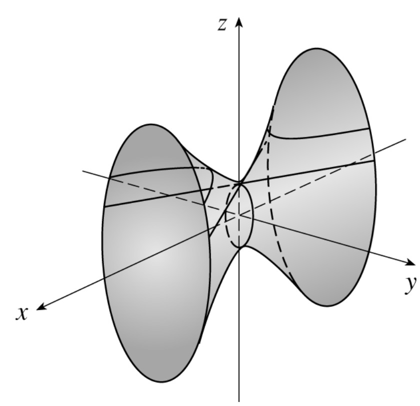
27. This surface is a cylinder because the variable y is missing from the equation. The intersection of the surface and the xz-plane is an ellipse. So the graph is VIII.

28. This is the equation of a hyperbolic paraboloid. The trace in the xy-plane is the parabola $y = x^2$. So the correct graph is V.

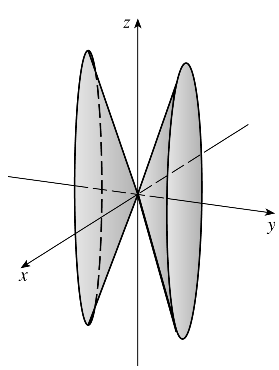
29. Vertical traces parallel to the xz-plane are circles centered at the origin whose radii increase as y decreases. (The trace in $y = 1$ is just a single point and the graph suggests that traces in $y = k$ are empty for $k > 1$.) The traces in vertical planes parallel to the yz-plane are parabolas opening to the left that shift to the left as $|x|$ increases. One surface that fits this description is a circular paraboloid, opening to the left, with vertex $(0, 1, 0)$.



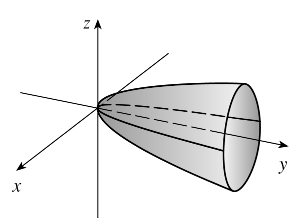
30. The vertical traces parallel to the yz-plane are ellipses that are smallest in the yz-plane and increase in size as $|x|$ increases. One surface that fits this description is a hyperboloid of one sheet with axis the x-axis. The horizontal traces in $z = k$ (hyperbolas and intersecting lines) also fit this surface, as shown in the figure.



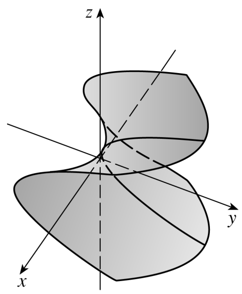
31. $y^2 = x^2 + \frac{1}{9}z^2$ or $y^2 = x^2 + \dfrac{z^2}{9}$ represents an elliptic cone with vertex $(0, 0, 0)$ and axis the y-axis.



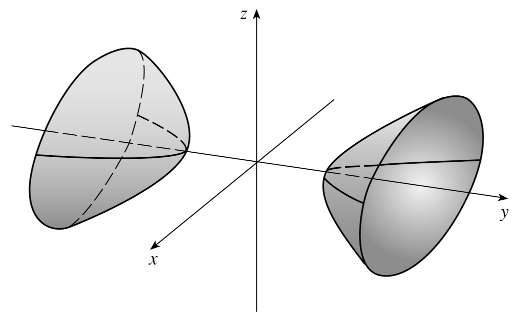
32. $4x^2 - y + 2z^2 = 0$ or $y = \dfrac{x^2}{1/4} + \dfrac{z^2}{1/2}$ or $\dfrac{y}{4} = x^2 + \dfrac{z^2}{2}$ represents an elliptic paraboloid with vertex $(0, 0, 0)$ and axis the y-axis.



33. $x^2 + 2y - 2z^2 = 0$ or $2y = 2z^2 - x^2$ or $y = z^2 - \dfrac{x^2}{2}$ represents a hyperbolic paraboloid with center $(0, 0, 0)$.



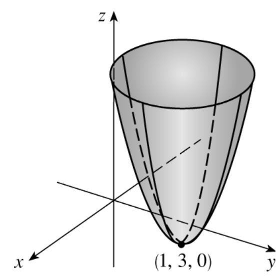
34. $y^2 = x^2 + 4z^2 + 4$ or $-x^2 + y^2 - 4z^2 = 4$ or $-\dfrac{x^2}{4} + \dfrac{y^2}{4} - z^2 = 1$ represents a hyperboloid of two sheets with axis the y-axis.



35. Completing squares in x and y gives

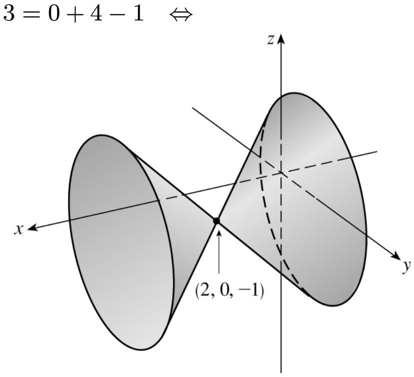
$\left(x^2 - 2x + 1\right) + \left(y^2 - 6y + 9\right) - z = 0 \quad \Leftrightarrow$

$(x-1)^2 + (y-3)^2 - z = 0$ or $z = (x-1)^2 + (y-3)^2$, a circular paraboloid opening upward with vertex $(1, 3, 0)$ and axis the vertical line $x = 1$, $y = 3$.



36. Completing squares in x and z gives $\left(x^2 - 4x + 4\right) - y^2 - \left(z^2 + 2z + 1\right) + 3 = 0 + 4 - 1 \quad \Leftrightarrow$

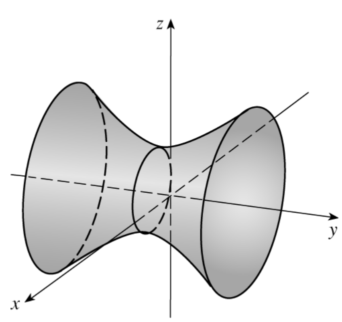
$(x-2)^2 - y^2 - (z+1)^2 = 0$ or $(x-2)^2 = y^2 + (z+1)^2$, a circular cone with vertex $(2, 0, -1)$ and axis the horizontal line $y = 0$, $z = -1$.



37. Completing squares in x and z gives

$\left(x^2 - 4x + 4\right) - y^2 + \left(z^2 - 2z + 1\right) = 0 + 4 + 1 \quad \Leftrightarrow$

$(x-2)^2 - y^2 + (z-1)^2 = 5$ or $\dfrac{(x-2)^2}{5} - \dfrac{y^2}{5} + \dfrac{(z-1)^2}{5} = 1$, a hyperboloid of one sheet with center $(2, 0, 1)$ and axis the horizontal line $x = 2$, $z = 1$.



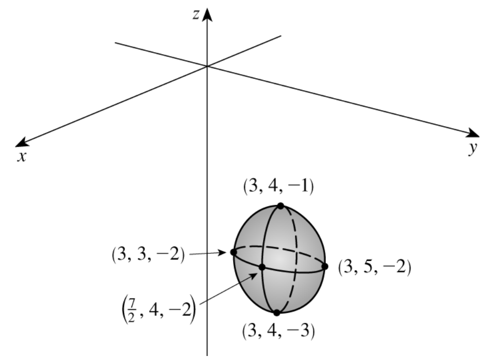
38. Completing squares in all three variables gives

$$4(x^2 - 6x + 9) + (y^2 - 8y + 16) + (z^2 + 4z + 4) = -55 + 36 + 16 + 4 \quad \Leftrightarrow$$

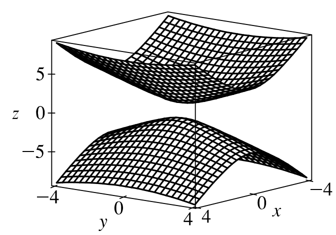
$4\,(x-3)^2 + (y-4)^2 + (z+2)^2 = 1$ or

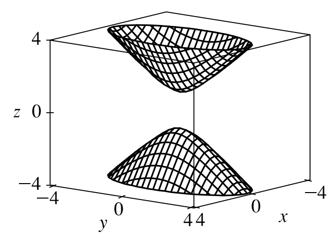
$\dfrac{(x-3)^2}{1/4} + (y-4)^2 + (z+2)^2 = 1$, an ellipsoid with

center $(3, 4, -2)$.



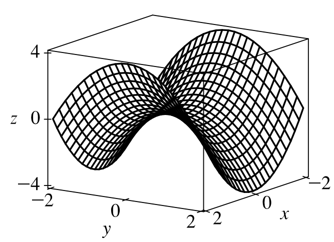
39. Solving the equation for z we get $z = \pm\sqrt{1 + 4x^2 + y^2}$, so we plot separately $z = \sqrt{1 + 4x^2 + y^2}$ and $z = -\sqrt{1 + 4x^2 + y^2}$.



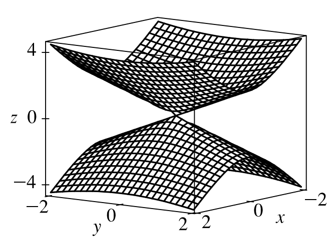


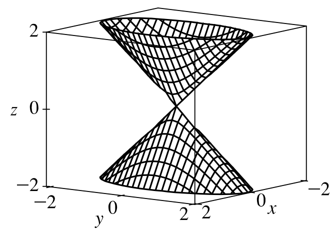
To restrict the z-range as in the second graph, we can use the option `view = -4..4` in Maple's `plot3d` command, or `PlotRange -> {-4,4}` in Mathematica's `Plot3D` command.

40. We plot the surface $z = x^2 - y^2$.

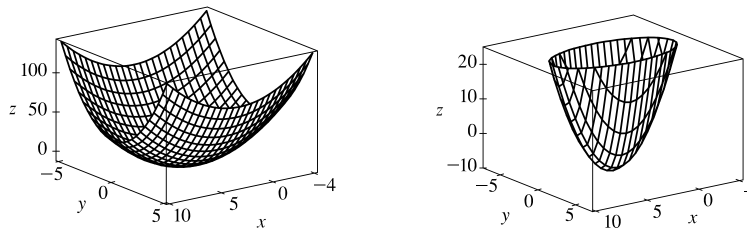


41. Solving the equation for z we get $z = \pm\sqrt{4x^2 + y^2}$, so we plot separately $z = \sqrt{4x^2 + y^2}$ and $z = -\sqrt{4x^2 + y^2}$.

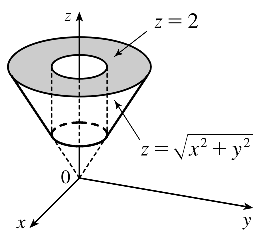




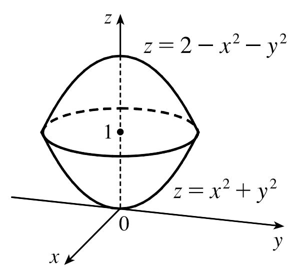
42. We plot the surface $z = x^2 - 6x + 4y^2$.



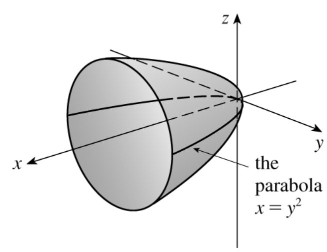
43.



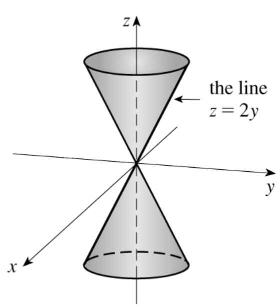
44.



45. The curve $y = \sqrt{x}$ is equivalent to $x = y^2$, $y \geq 0$. Rotating the curve about the x-axis creates a circular paraboloid with vertex at the origin, axis the x-axis, opening in the positive x-direction. The trace in the xy-plane is $x = y^2$, $z = 0$, and the trace in the xz-plane is a parabola of the same shape: $x = z^2$, $y = 0$. An equation for the surface is $x = y^2 + z^2$.



46. Rotating the line $z = 2y$ about the z-axis creates a (right) circular cone with vertex at the origin and axis the z-axis. Traces in $z = k$ ($k \neq 0$) are circles with center $(0, 0, k)$ and radius $y = z/2 = k/2$, so an equation for the trace is $x^2 + y^2 = (k/2)^2$, $z = k$. Thus an equation for the surface is $x^2 + y^2 = (z/2)^2$ or $4x^2 + 4y^2 = z^2$.



47. Let $P = (x, y, z)$ be an arbitrary point equidistant from $(-1, 0, 0)$ and the plane $x = 1$. Then the distance from P to $(-1, 0, 0)$ is $\sqrt{(x+1)^2 + y^2 + z^2}$ and the distance from P to the plane $x = 1$ is $|x - 1| / \sqrt{1^2} = |x - 1|$ (by Equation 12.5.9). So $|x - 1| = \sqrt{(x+1)^2 + y^2 + z^2} \quad \Leftrightarrow \quad (x-1)^2 = (x+1)^2 + y^2 + z^2 \quad \Leftrightarrow$ $x^2 - 2x + 1 = x^2 + 2x + 1 + y^2 + z^2 \quad \Leftrightarrow \quad -4x = y^2 + z^2$. Thus the collection of all such points P is a circular paraboloid with vertex at the origin, axis the x-axis, which opens in the negative x-direction.

48. Let $P = (x, y, z)$ be an arbitrary point whose distance from the x-axis is twice its distance from the yz-plane. The distance from P to the x-axis is $\sqrt{(x-x)^2 + y^2 + z^2} = \sqrt{y^2 + z^2}$ and the distance from P to the yz-plane ($x = 0$) is $|x|/1 = |x|$. Thus $\sqrt{y^2 + z^2} = 2|x| \quad \Leftrightarrow \quad y^2 + z^2 = 4x^2 \quad \Leftrightarrow \quad x^2 = (y^2/2^2) + (z^2/2^2)$. So the surface is a right circular cone with vertex the origin and axis the x-axis.

49. (a) An equation for an ellipsoid centered at the origin with intercepts $x = \pm a$, $y = \pm b$, and $z = \pm c$ is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

Here the poles of the model intersect the z-axis at $z = \pm 6356.523$ and the equator intersects the x- and y-axes at $x = \pm 6378.137$, $y = \pm 6378.137$, so an equation is

$$\frac{x^2}{(6378.137)^2} + \frac{y^2}{(6378.137)^2} + \frac{z^2}{(6356.523)^2} = 1$$

(b) Traces in $z = k$ are the circles $\dfrac{x^2}{(6378.137)^2} + \dfrac{y^2}{(6378.137)^2} = 1 - \dfrac{k^2}{(6356.523)^2} \quad \Leftrightarrow$

$x^2 + y^2 = (6378.137)^2 - \left(\dfrac{6378.137}{6356.523}\right)^2 k^2$.

(c) To identify the traces in $y = mx$ we substitute $y = mx$ into the equation of the ellipsoid:

$$\begin{aligned} \frac{x^2}{(6378.137)^2} + \frac{(mx)^2}{(6378.137)^2} + \frac{z^2}{(6356.523)^2} &= 1 \\ \frac{(1+m^2)x^2}{(6378.137)^2} + \frac{z^2}{(6356.523)^2} &= 1 \\ \frac{x^2}{(6378.137)^2/(1+m^2)} + \frac{z^2}{(6356.523)^2} &= 1 \end{aligned}$$

As expected, this is a family of ellipses.

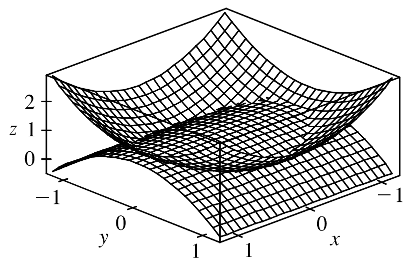
50. If we position the hyperboloid on coordinate axes so that it is centered at the origin with axis the z-axis then its equation is given by $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$. Horizontal traces in $z = k$ are $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 + \dfrac{k^2}{c^2}$, a family of ellipses, but we know that the traces are circles so we must have $a = b$. The trace in $z = 0$ is $\dfrac{x^2}{a^2} + \dfrac{y^2}{a^2} = 1 \quad \Leftrightarrow \quad x^2 + y^2 = a^2$ and since the minimum radius of 100 m occurs there, we must have $a = 100$. The base of the tower is the trace in $z = -500$ given by $\dfrac{x^2}{a^2} + \dfrac{y^2}{a^2} = 1 + \dfrac{(-500)^2}{c^2}$ but $a = 100$ so the trace is $x^2 + y^2 = 100^2 + 50{,}000^2\,\dfrac{1}{c^2}$. We know the base is a circle of radius 140, so we must have $100^2 + 50{,}000^2\,\dfrac{1}{c^2} = 140^2 \quad \Rightarrow \quad c^2 = \dfrac{50{,}000^2}{140^2 - 100^2} = \dfrac{781{,}250}{3}$ and an equation for the tower is $\dfrac{x^2}{100^2} + \dfrac{y^2}{100^2} - \dfrac{z^2}{(781{,}250)/3} = 1$ or $\dfrac{x^2}{10{,}000} + \dfrac{y^2}{10{,}000} - \dfrac{3z^2}{781{,}250} = 1$, $-500 \le z \le 500$.

51. If (a, b, c) satisfies $z = y^2 - x^2$, then $c = b^2 - a^2$. L_1: $x = a + t$, $y = b + t$, $z = c + 2(b - a)t$, L_2: $x = a + t$, $y = b - t$, $z = c - 2(b + a)t$. Substitute the parametric equations of L_1 into the equation of the hyperbolic paraboloid in order to find the points of intersection: $z = y^2 - x^2 \quad \Rightarrow$
$c + 2(b - a)t = (b + t)^2 - (a + t)^2 = b^2 - a^2 + 2(b - a)t \quad \Rightarrow \quad c = b^2 - a^2$. As this is true for all values of t,

L_1 lies on $z = y^2 - x^2$. Performing similar operations with L_2 gives: $z = y^2 - x^2 \quad \Rightarrow$ $c - 2(b+a)t = (b-t)^2 - (a+t)^2 = b^2 - a^2 - 2(b+a)t \quad \Rightarrow \quad c = b^2 - a^2$. This tells us that all of L_2 also lies on $z = y^2 - x^2$.

52. Any point on the curve of intersection must satisfy both $2x^2 + 4y^2 - 2z^2 + 6x = 2$ and $2x^2 + 4y^2 - 2z^2 - 5y = 0$. Subtracting, we get $6x + 5y = 2$, which is linear and therefore the equation of a plane. Thus the curve of intersection lies in this plane.

53.



The curve of intersection looks like a bent ellipse. The projection of this curve onto the xy-plane is the set of points $(x, y, 0)$ which satisfy $x^2 + y^2 = 1 - y^2 \quad \Leftrightarrow \quad x^2 + 2y^2 = 1 \quad \Leftrightarrow$ $x^2 + \dfrac{y^2}{\left(1/\sqrt{2}\right)^2} = 1$. This is an equation of an ellipse.

12 Review

TRUE-FALSE QUIZ

1. This is false, as the dot product of two vectors is a scalar, not a vector.

2. False. For example, if $\mathbf{u} = \mathbf{i}$ and $\mathbf{v} = -\mathbf{i}$ then $|\mathbf{u} + \mathbf{v}| = |\mathbf{0}| = 0$ but $|\mathbf{u}| + |\mathbf{v}| = 1 + 1 = 2$.

3. False. For example, if $\mathbf{u} = \mathbf{i}$ and $\mathbf{v} = \mathbf{j}$ then $|\mathbf{u} \cdot \mathbf{v}| = |0| = 0$ but $|\mathbf{u}|\,|\mathbf{v}| = 1 \cdot 1 = 1$. In fact, by Theorem 12.3.3, $|\mathbf{u} \cdot \mathbf{v}| = \big||\mathbf{u}|\,|\mathbf{v}| \cos\theta\big|$.

4. False. For example, $|\mathbf{i} \times \mathbf{i}| = |\mathbf{0}| = 0$ (see Example 12.4.2) but $|\mathbf{i}|\,|\mathbf{i}| = 1 \cdot 1 = 1$. In fact, by Theorem 12.4.9, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| \sin\theta$.

5. True, by Theorem 12.3.2, property 2.

6. False. Property 1 of Theorem 12.4.11 says that $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$.

7. True. If θ is the angle between $\mathbf{u}$ and $\mathbf{v}$, then by Theorem 12.4.9, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| \sin\theta = |\mathbf{v}|\,|\mathbf{u}| \sin\theta = |\mathbf{v} \times \mathbf{u}|$. (Or, by Theorem 12.4.11, $|\mathbf{u} \times \mathbf{v}| = |-\mathbf{v} \times \mathbf{u}| = |-1|\,|\mathbf{v} \times \mathbf{u}| = |\mathbf{v} \times \mathbf{u}|$.)

8. This is true by Theorem 12.3.2, property 4.

9. Theorem 12.4.11, property 2 tells us that this is true.

10. This is true by Theorem 12.4.11, property 4.

11. This is true by Theorem 12.4.11, property 5.

12. In general, this assertion is false; a counterexample is $\mathbf{i} \times (\mathbf{i} \times \mathbf{j}) \neq (\mathbf{i} \times \mathbf{i}) \times \mathbf{j}$. (See the paragraph preceding Theorem 12.4.11.)

13. This is true because $\mathbf{u} \times \mathbf{v}$ is orthogonal to $\mathbf{u}$ (see Theorem 12.4.8), and the dot product of two orthogonal vectors is 0.