## Math 53 Final, 5/18/07, 12:30 PM - 3:30 PM

No calculators or notes. Each question is worth 10 points. Please write your solution to each of the 10 questions on a separate sheet with your name, SID#, and GSI on it. (If you are removing an incomplete for professor X, write "Incomplete/X" on each page. For Math 49, do questions 6-10 only.) To get full credit for a question, you must obtain the correct answer, put a box around it, and show correct work/justification. Please do not leave the exam between 3:00 and 3:30. Good luck!

1. Find the point on the sphere  $x^2 + y^2 + z^2 = 1$  that minimizes the function

$$f(x, y, z) = (x - 2)^{2} + (y - 2)^{2} + (z - 1)^{2}.$$

- Find the volume of the region consisting of all points that are inside the sphere  $x^2 + y^2 + z^2 = 4$ , above the plane z = 0, and below the plane z = x.
- Let R be the triangle with vertices (0,0), (1,0), and (1/2,1/2). Evaluate the integral

$$\iint_{R} \frac{e^{x+y}}{x+y} \, dA.$$

*Hint*: Use the change of variables u = x + y, v = x - y.

A particle moves along the intersection of the surfaces

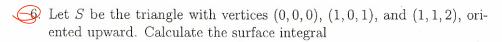
$$x^2 + y^2 + 2z^2 = 4$$
,  $z = xy$ .

Let  $\langle (x(t), y(t), z(t)) \rangle$  denote the location of the particle at time t. Suppose that  $\langle x(0), y(0), z(0) \rangle = \langle 1, 1, 1 \rangle$  and x'(0) = 1. Calculate y'(0) and z'(0).

5. Suppose f is a function on  $\mathbb{R}^2$  satisfying the following conditions on its directional derivatives:

$$D_{\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle} f(x, y) = \sqrt{2} x, \qquad D_{\left\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right\rangle} f(x, y) = \sqrt{2} y.$$

- (a) Find  $f_x(x,y)$  and  $f_y(x,y)$ .
- (b) Assuming also that f(0,0) = 0, find f(x,y).



$$\iint_{S} \langle 3, 4, 5 \rangle \cdot d\mathbf{S}.$$

7. Consider the vector field

$$\mathbf{F} = \sqrt{x^2 + y^2 + z^2} \langle x, y, z \rangle.$$

sphere  $\mathbb{H}$ pdOd0 (a) There is a constant c such that  $\operatorname{div} \mathbf{F} = c\sqrt{x^2 + y^2 + z^2}$ . Find c.

(b) Compute the outward flux of the vector field **F** through the boundary of the solid region  $x^2 + y^2 + z^2 \le 1$ ,  $z \ge \sqrt{x^2 + y^2}$ .

8. Calculate  $\int_C \mathbf{F} \cdot d\mathbf{r}$ , where C is the space curve  $\mathbf{r}(t) = \langle \cos t, 0, \sin t \rangle$  for  $0 \le t \le 2\pi$ , and

$$\mathbf{F} = \langle \sin(x^3) + z^3, \sin(y^3), \sin(z^3) - x^3 \rangle.$$

Hint: Use Stokes' Theorem.

Let  $S_1$  be the hemisphere  $x^2 + y^2 + z^2 = 1$ ,  $z \ge 0$ , oriented upward. Let

$$\mathbf{F} = \langle x + y^2 + z^2, x^2 - y + z^2, x^2 + y^2 \rangle$$
.

(a) Use the Divergence Theorem to show that there is an oriented surface  $S_2$  in the x, y plane such that  $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S} = \iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$ .

(b) Use part (a) to calculate  $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S}$ .

Let C be the semicircle  $x^2 + y^2 = 1$ ,  $y \ge 0$ , oriented counterclockwise. Calculate the line integral

$$\int_C (-y + \cos x) \, dx + (x + \sin y) \, dy.$$

*Hint:* Part of this integral can be evaluated directly from the definition, and the rest using the Fundamental Theorem of Line Integrals.