Grinnell College

Heuristic Search (Lab 3)

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1 Designing a Heuristic

We tried several heuristics, but we chose the heuristic that was admissible.

1.1 Concept and Background

Consider a graph of jump size as a function of step number, $f : \mathbb{N} \to \mathbb{N}$. Starting from the first, highest jump size, $m_{t-1} + 2$, Dr. Smith must increase his jump size by two till the x_0th step. After this point, he must decrease the jump size of each subsequent step by 1, $m_{t-1} - 1$, till the last step n_f.

$$f(n) = \begin{cases} \lfloor 2n \rfloor & 0 \leq n \leq x_0 \\ \lfloor -n + 2x_0 \rfloor & x_0 < n \leq n_f \\ 0 & \text{otherwise.} \end{cases}$$

Since Dr. Smith needs to make a stop at the $N-1$th step, notice then that the area under this step-wise, piece-wise curve must be $N-1$. We determine the area under the curve for when Dr. Smith is increasing his jump size and decreasing his jump size to determine x_0.

We first, consider the problem for the initial position and momentum. For the increasing component, we have the original the area under the curve is

$$\begin{aligned} 2 + 4 + \cdots + 2 * x_0 &= 2(1 + 2 + \cdots + x_0) \\ &= 2 \cdot \sum_{k=1}^{x_0} k \\ &= 2 \cdot \frac{x_0(x_0+1)}{2} && \text{(sum of } \mathbb{N} \text{ till } x_0\text{)} \\ &= x_0(x_0+1). \end{aligned}$$

For the decreasing component, we have

$$\begin{aligned}
(2x_0 - 1) + (2x_0 - 2) + \cdots + 1 &= \sum_{j=1}^{2x_0-1} j && \text{(remove } 2x_0 \text{ to prevent double counting)} \\
&= \frac{(2x_0 - 1)(2x_0)}{2} && \text{(sum of } \mathbb{N} \text{ till } 2x_0 - 1\text{)} \\
&= x_0(2x_0 - 1).
\end{aligned}$$

Notice, however, that we double count the value $f(x_0)$. Thus, we subtract Thus, we have

$$\begin{aligned}
N - 1 &= x_0(x_0 + 1) + x_0(2x_0 - 1) \\
&= x_0^2 + x_0 + 2x_0^2 - x_0 \\
&= 3x_0^2.
\end{aligned}$$

By solving for the real, positive root of x_0, we get

$$\begin{aligned}
x_0 &= \frac{\sqrt{-4(-N+1)3}}{2 \cdot 3} \\
&= \frac{\sqrt{12N - 12}}{6}.
\end{aligned}$$

Notice then that if we expand the bounds of the decreasing function to $x = 0$, we have $f(x) = 2x_0$. Since the last step in the decreasing function is 0, we ignore the last step, so the number of steps from the start is $2x_0 - 1$.

Next, we need the heuristic to apply even at various momentum and positions. For the position, instead of $N - 1$, we determine the distance from the previous location to $N - 1$

$$l = N - 1 - p_{t-1}.$$

Lastly, for momentum, we start the positive curve at $m_{t-1} + 2$. Thus, we have

$$f(n) = \begin{cases}
\lfloor m_{t-1} + 2 + 2n \rfloor & 0 \le n \le x_0 \\
\lfloor -n + 2x_0 + m_{t-1} + 2 \rfloor & x_0 < n \le n_f \\
0 & \text{otherwise.}
\end{cases}$$

Instead of the area under the curve of the increasing component being $\sum_{k=1}^{x_0} k$, we have

$$\sum_{k=1}^{x_0} k + (m_{t-1} + 2)x_0.$$

For the decreasing component, the curve must intersect at $(x_0, m + 2)$, so we now have

$$\sum_{j=1}^{2x_0-1+m_{t-1}+2} j.$$

Thus, we have

$$
\begin{aligned}
l &= \sum_{k=1}^{x_0} k + (m_{t-1}+2)x_0 + \sum_{j=1}^{2x_0-1+m_{t-1}+2} j \\
&= x_0(x_0+1) + (m_{t-1}+2)x_0 + \frac{(m+1+2x_0)(m+2+2x_0)}{2} \\
&= 3x_0^2 + (6+3m_{t-1})x_0 + \left(1+\frac{3m_{t-1}}{2}+\frac{m_{t-1}^2}{2}\right).
\end{aligned}
$$

Solving for x_0, we have

$$
x_0 = \frac{-6-3m_{t-1}+\sqrt{9m_{t-1}^2+36m_{t-1}+36-4\cdot 3\cdot(1+\frac{3m_{t-1}}{2}+\frac{m_{t-1}^2}{2}-l)}}{2\cdot 3}
$$

Thus, the number of steps from p_{t-1} to the solution is $m_{t-1}+2+2x_0-1$.

1.2 Admissibility

Notice that this heuristic counts the number of steps Dr. Smith must take on the basis that Dr. Smith must take the longest jump he can make till the point at which he can slow down enough to reach the $N-1$ location. Suppose the optimal number of steps, n_0, is $n_0 < m_{t-1}+2+2x_0-1$. Since f is strictly positive, the integral of f is strictly monotonic increasing. Notice then that

$$
\begin{aligned}
\int_0^{n_0} f(n)dn &< \int_0^{m_{t-1}+2+2x_0-1} f(n)dn \\
&< l.
\end{aligned}
$$

Then with n_0 steps, Dr. Smith cannot reach the $N-1$ position. Thus, $n_0 \geq m_{t-1}+2+2x_0-1$.

2 Experimentation

We show the search cost and effective branching factor for each of the algorithms we tried and the *IDS* algorithm for comparison.

	Search Cost			Effective Branching Factor		
d	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	4	2	2	1.56	1	1
3	11	4	5	1.81	1.15	1.28
4	23	9	7	1.84	1.35	1.24
5	63	26	10	2.01	1.61	1.24
6	234	53	20	2.25	1.68	1.36
8	2771	287	34	2.53	1.84	1.32
10	40238	1255	73	2.76	1.89	1.35
12	655466	3974	165	2.95	1.87	1.38
14	10801946	10886	354	3.09	1.84	1.39

Table 2.1: Comparison of the search cost and effective branching factor for the Iterative-Deepening Search and A* algorithms with the aforementioned heuristic (h_1) and a heuristic that takes the maximum number of jumps from any given step (h_2).

The A* search with h_1 consistently outperforms IDS both in terms of the search cost and effective branching factor. The ratio of the search cost between IDS and A* with h_1 ranges from 2 to 992, indicating significantly higher performance by the A* with h_1. Similarly, for the effective branching factor, the ratio is consistently higher than 1. Since our heuristic is non-trivial and admissible, we expect these results.

In contrast, we find the comparison between the A* search with h_1 and h_2 strange and possibly inconsistent with Norvig & Russell (2010). Earlier, we proved that h_1 is admissible, but we can easily see that h_2 is not admissible. The h_2 heuristic consistently overestimates the cost to the solution as it minimizes the size of the jumps, in turn, maximizing the number of steps (cost) till the solution. We would expect as a rule of thumb that admissible heuristics perform better than non-admissible heuristic, but as the table shows, the effective branching factor and search cost is consistently higher for h_1 than h_2. Since the h_2 heuristic is not too different from having a constant 1 heuristic function, it is not unfathomable that uniform-cost search performs better than A* search with h_1.