REPORT

WEEK 9

ASSIGNMENT WEEK 9

<u>Pemodelan Dinamik Kolom Adsorber</u>

1. Cairan limbah yang mengandung polutan A akan dikurangi kadarnya dengan cara dilewatkan suatu kolom dengan panjang L yang berisi tumpukan butir penjerap. Aliran cairan masuk kolom berkonsentrasi Ca0. Kesetimbangan antara konsentrasi A di cairan dan di penjerap dinyatakan dengan hubungan :

$$C_A^* = X_A . H$$

Dengan X_A adalah konsentrasi A di padatan penjerap. Porositas campuran butir penjerap ε_b. **Ingin disusun persamaan matematis yang dapat digunakan untuk menentukan distribusi konsentrasi A di cairan (C_A) dan di penjerap (X_A) sepanjang kolom setiap saat**.

a. Susunlah model matematis yang menunjukkan distribusi CA di cairan dan padatan pada berbagai posisi dan waktu. Tunjukkan bahwa model yang berlaku:

$$\frac{dC_A}{dt} = \frac{D_e}{\varepsilon_b} \cdot \frac{d^2C_A}{dz^2} - \frac{F_0}{\left(\frac{\pi}{4} \cdot d^2\right) \cdot D_e} \cdot \frac{dC_A}{dz} - \frac{k_c \cdot a}{D_e} \cdot (C_A - x_A \cdot H)$$

$$\frac{dx_A}{dt} = \frac{k_c \cdot a}{\rho_b} \cdot (C_A - x_A \cdot H)$$

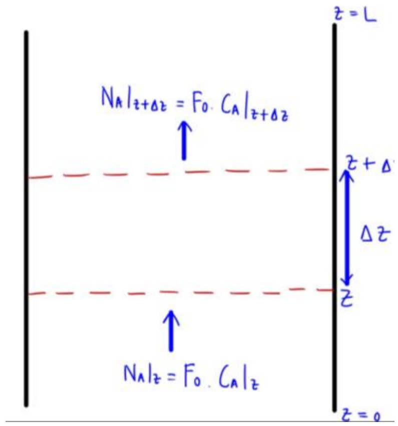
b. Selesaikanlah persamaan diferensial yang Saudara peroleh. Berikan plot kurva *breakthrough* dari hasil simulasi Saudara. Sajikan dinamika konsentrasi di cairan dan padatan sebagai video MP 4.

Gunakan data-data berikut untuk simulasi (semua satuan dianggap sudah sesuai):

		Initial condition:		Boundary condition:
$C_{A_i} = 1 \ \frac{mol}{m^3}$	$a = 100 \frac{m^2}{m^3}$			
$F_0 = 36.000 \ \frac{m^3}{jam}$	$k_c = 1.800 \frac{m}{jam}$	$z = 0$ $t = t$	: $C_A = C_{Ai}$ $x_A = 0$	$z = L$: $\frac{dC_A}{dz} = 0$ $t = t$
$L = 5 \ m$	$\rho_b = 800 \frac{kg}{m^3}$			
$d = 1{,}5 \ m$	$H = 0{,}005 \frac{m^3}{kg}$	$z = z$ $t = 0$	: $C_A = 0$ $x_A = 0$	
$\varepsilon_b = 0{,}7$	$D_e = 1{,}8x10^{-4} \ \frac{m^2}{jam}$			

Penyelesaian:

- **Ilustrasi Pemodelan Transfer Massa pada Kolom Adsorber**



Neraca Massa A pada Elemen Volume di Fase Cair

$$ROMI - ROMO = ROMA$$

$$\left(F_0 . C_A|_z - D_e \frac{\pi}{4} d^2 \frac{\partial C_A}{\partial z}|_z\right) - \left(F_0 . C_A|_{z+\Delta z} - D_e \frac{\pi}{4} d^2 \frac{\partial C_A}{\partial z}|_{z+\Delta z}\right) - k_c a (C_A - C_A^*) \frac{\pi}{4} d^2 \Delta z$$
$$= \frac{1}{4} \varepsilon_b \pi d^2 \Delta z \frac{\partial C_A}{\partial t}$$

$$-\frac{F_0}{\varepsilon_b \frac{\pi}{4} d^2}\left(\frac{C_A|_{z+\Delta z} - C_A|_z}{\Delta z}\right) + \frac{D_e}{\varepsilon_b}\left(\frac{\frac{\partial C_A}{\partial z}|_{z+\Delta z} - \frac{\partial C_A}{\partial z}|_z}{\Delta z}\right) - \frac{k_c a}{\varepsilon_b}(C_A - C_A^*) = \frac{\partial C_A}{\partial t}$$

$$\frac{D_e}{\varepsilon_b}\left[\frac{\partial^2 C_A}{\partial z^2} - \frac{F_0}{\frac{\pi}{4} d^2 \mathrm{D_e}} \frac{\partial C_A}{\partial z} - \frac{k_c a}{D_e}(C_A - C_A^*)\right] = \frac{\partial C_A}{\partial t}$$

Dengan,

$C_A^* = X_A . H$, diperoleh

$$\frac{D_e}{\epsilon_b}\left[\frac{\partial^2 C_A}{\partial z^2} - \frac{F_0}{\frac{\pi}{4} d^2 \mathrm{D_e}} \frac{\partial C_A}{\partial z} - \frac{k_c a}{D_e}(C_A - X_A . H)\right] = \frac{\partial C_A}{\partial t}$$

Neraca Masssa A pada Elemen Volume di Penjerap

$$0 - 0 + k_c a \frac{\pi}{4} d^2 \Delta z (C_A - C_A^*) = \frac{1}{4} \rho_b \pi d^2 \Delta z \frac{\partial X_A}{\partial t}$$

$$\frac{\partial X_A}{\partial t} = \frac{k_c a}{\rho_b}(C_A - X_A . H)$$

Boundary Condition:

IC:

$z = z; t = 0 \ \rightarrow \ C_A = 0; x_A = 0$

BC:

$z = 0; t = t \ \rightarrow \ C_A = C_{Ai}; x_A = 0$

$z = L; t = t \ \rightarrow \ \frac{\partial C_A}{\partial z} = 0 \ (bernilai\ finite\ minimum)$

<u>Diskritisasi FDA + MOL</u>

Batas kiri:

$$C_A = C_{Ai}$$

$$C_A(0) = C_{Ai}$$

$$X_A = 0$$

Batas kanan *(Backward 2nd Order)*:

$$\frac{\partial C_A}{\partial z} = 0$$

$$\frac{3C_A(-1) - 4C_A(-2) + C_A(-3)}{2\Delta z} = 0$$

$$3C_A(-1) - 4C_A(-2) + C_A(-3) = 0$$

$$C_A(-1) = \frac{4C_A(-2) - C_A(-3)}{3}$$

$$\frac{\partial X_A}{\partial z} = 0$$

$$\frac{3X_A(-1) - 4X_A(-2) + X_A(-3)}{2\Delta z} = 0$$

$$3X_A(-1) - 4X_A(-2) + X_A(-3) = 0$$

$$X_A(-1) = \frac{4X_A(-2) - X_A(-3)}{3}$$

Persamaan *Overall*:

$$\frac{\partial C_A}{\partial t} = \frac{D_e}{\varepsilon_b}[\frac{C_A(i+1) - 2C_A(i) + C_A(i+1)}{\Delta z^2} - \frac{F_0}{\frac{\pi}{4}d^2\mathrm{D_e}}\frac{C_A(i+1) - C_A(i-1)}{2\Delta z}$$
$$- \frac{k_c a}{D_e}(C_A(i) - X_A(i).H)]$$

$$\frac{\partial X_A}{\partial t} = \frac{k_c a}{\rho_b}(C_A(i) - X_A(i).H)$$

Berikut ini adalah *hyperlink* untuk hasil kurva *breakthrough* versi .gif

https://drive.google.com/drive/folders/1WnlPiuPXf4U4yRtG-AhQTkorzr3r89UY?usp=sharing

In [9]:

```python
#Nama : Liska Dewi Muktiarani
#NIM  : 21/477837/TK/52633

import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp as sol
from matplotlib.animation import FuncAnimation

#Data Perhitungan
Cai = 1 #mol/m3
F0 = 36000 #m3/jam
L = 5 #m
d = 1.5 #m
eps_b = 0.7
alfa = 100 #m2/m3
kc = 1800 #m/jam
rho_b = 800 #kg/m3
H = 0.005 #m3/kg
De = 1.8e-4 #m2/jam
Nz = 101
z = np.linspace(0,L,Nz)
dz = z[1]-z[0]
t_final = 25
Nt = 101
tspan = np.linspace(0,t_final,Nt)
dt = tspan[1]-tspan[0]
tbound = [0, t_final]

#Matriks Initial
Cinit = np.zeros(2*Nz)

#Subroutine
def fun(t, C):
    CA = C[0:Nz]
    XA = C[Nz:2*Nz]
    #Batas Kiri
    CA[0] = Cai
    XA[0] = 0
    #Batas Kanan
    CA[-1] = 1/3*(4*CA[-2]-CA[-3])
    XA[-1] = 1/3*(4*XA[-2]-XA[-3])
    dCdt = np.zeros(len(C))
    for i in range(1,Nz-1):
        dCdt[i] = De/eps_b*(CA[i+1]-2*CA[i]+CA[i-1])/(dz**2)-F0/(np.pi/4*d**2*De)*(CA[i+1]-CA[i-1])/2/dz-kc*alfa/
        dCdt[i+Nz] = kc*alfa/rho_b*(CA[i]-XA[i]*H)
    return dCdt

#Solver
solver = sol(fun, tbound, Cinit, t_eval=tspan, method='LSODA')
res = solver.y.T

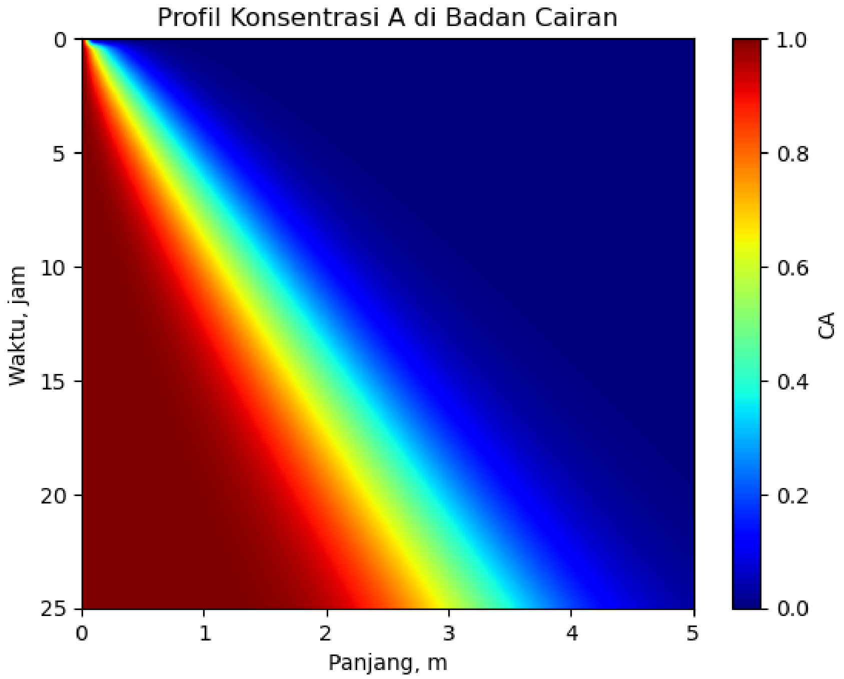
#Recalculation
#CA
CA = res[:,0:Nz]
CA[:,0] = Cai
CA[:,-1] = 1/3*(4*CA[:,-2]-CA[:,-3])
#XA
XA = res[:,Nz:2*Nz]
XA[:,0] = 0
XA[:,-1] = 1/3*(4*XA[:,-2]-XA[:,-3])

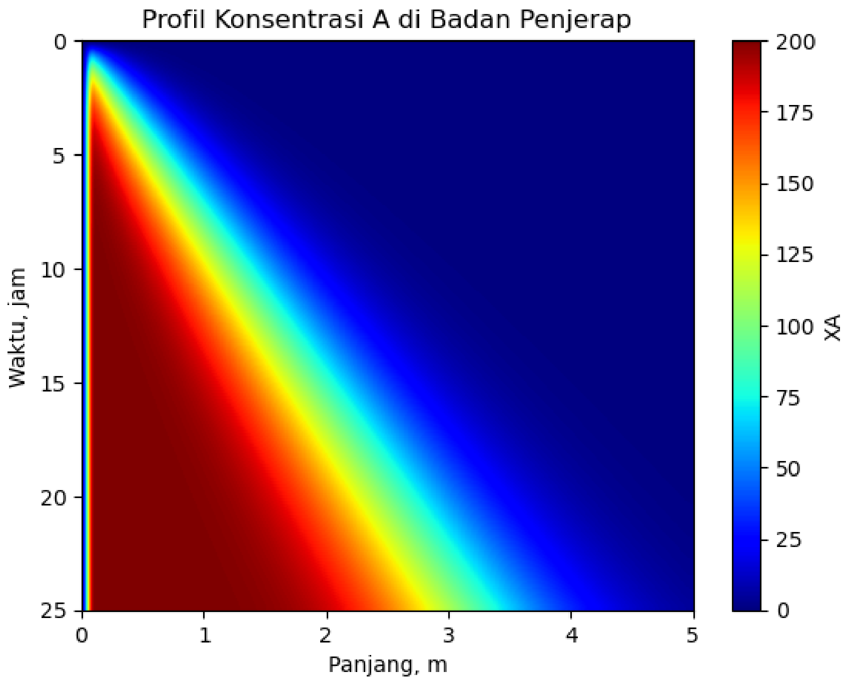
#Plotting
plt.figure(0)
plt.imshow(CA,cmap='jet',extent=(0,L,t_final,0),aspect='auto', interpolation='bicubic')
plt.ylabel('Waktu, jam')
plt.xlabel('Panjang, m')
plt.title('Profil Konsentrasi A di Badan Cairan')
plt.colorbar(label='CA')

plt.figure(1)
plt.imshow(XA, cmap='jet', extent=(0,L,t_final,0), aspect='auto', interpolation='bicubic')
plt.colorbar(label='XA')
plt.ylabel('Waktu, jam')
plt.xlabel('Panjang, m')
plt.title('Profil Konsentrasi A di Badan Penjerap')
```

Out[9]:

```
Text(0.5, 1.0, 'Profil Konsentrasi A di Badan Penjerap')
```





Pembahasan

Konsentrasi A di badan cairan pada setiap posisi akan terus menerus meningkat seiring waktu berjalan karena adanya proses adsorbsi atau perpindahan massa A dari badan cairan menuju bahan padatan. Begitu juga halnya dengan grafik konsentrasi A di penjerap. Pada awalnya (t = 0 jam), konsentrasi A di seluruh posisi penjerap bernilai 0. Seiring waktu berjalan terjadi proses adsorbsi komponen A dari bacan cairan menuju penjerap hingga tercapai kondisi jenuhnya.

In [13]:

```
#CA
plt.figure(2)
fig, ax = plt.subplots()
ax.set_title('Grafik Breakthrough Konsentrasi A di Badan Cairan')
ax.set_xlabel('Panjang, m')
ax.set_ylabel('Konsentrasi, mol/m^3')
ln, = plt.plot([], [])
def init():
    ax.set_xlim(0,L)
    ax.set_ylim(0,1)
    return ln,
def update(i):
    xdata =(np.linspace(0,L,Nz-1))
    ydata =(CA[i,1:])
    ln.set_data(xdata, ydata)
```

```
    return ln,
ani = FuncAnimation(fig,update,frames=100,interval=20,init_func=init,blit=True)
ani.save('CA Breakthorugh.gif',writer='ffmpeg')
plt.show()

#XA
plt.figure(3)
fig1,ax1=plt.subplots()
ax1.set_title('Grafik Breakthrough Konsentrasi A di Penjerap')
ax1.set_xlabel('Panjang,m')
ax1.set_ylabel('Konsentrasi, mol/m^3')
ln1,=plt.plot([],[])
def init1():
    ax1.set_xlim(0,L)
    ax1.set_ylim(0,200)
    return ln,
def update1(i):
    xdata1 =(np.linspace(0,L,Nz-1))
    ydata1 =(XA[i,1:])
    ln1.set_data(xdata1,ydata1)
    return ln1,
ani=FuncAnimation(fig1,update1,frames=100,interval=20,init_func=init1,blit=True)
ani.save('XA Breakthrough.gif', writer = 'ffmpeg')
plt.show()
```

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MovieWriter ffmpeg unavailable; using Pillow instead.
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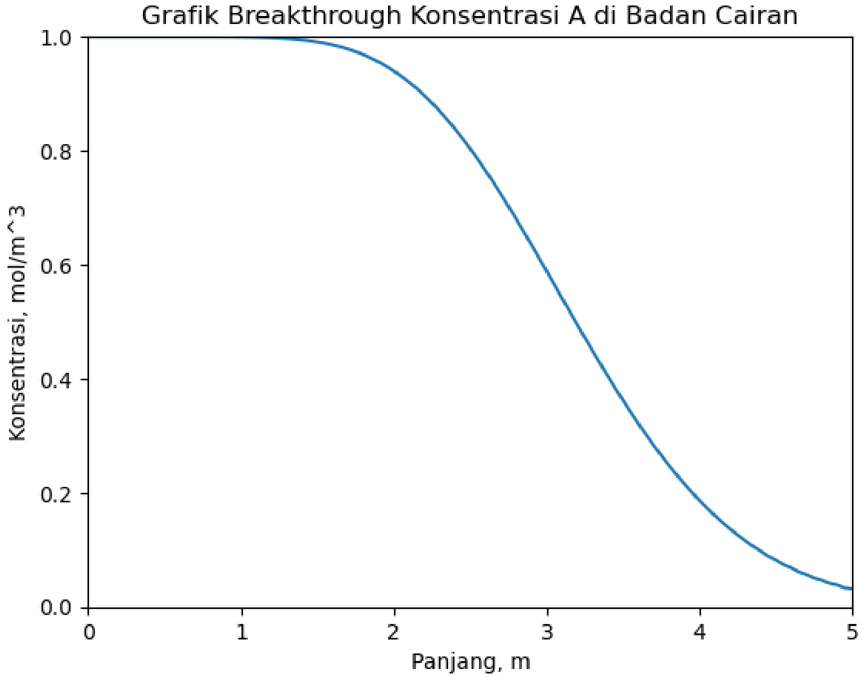
```

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MovieWriter ffmpeg unavailable; using Pillow instead.
<Figure size 640x480 with 0 Axes>
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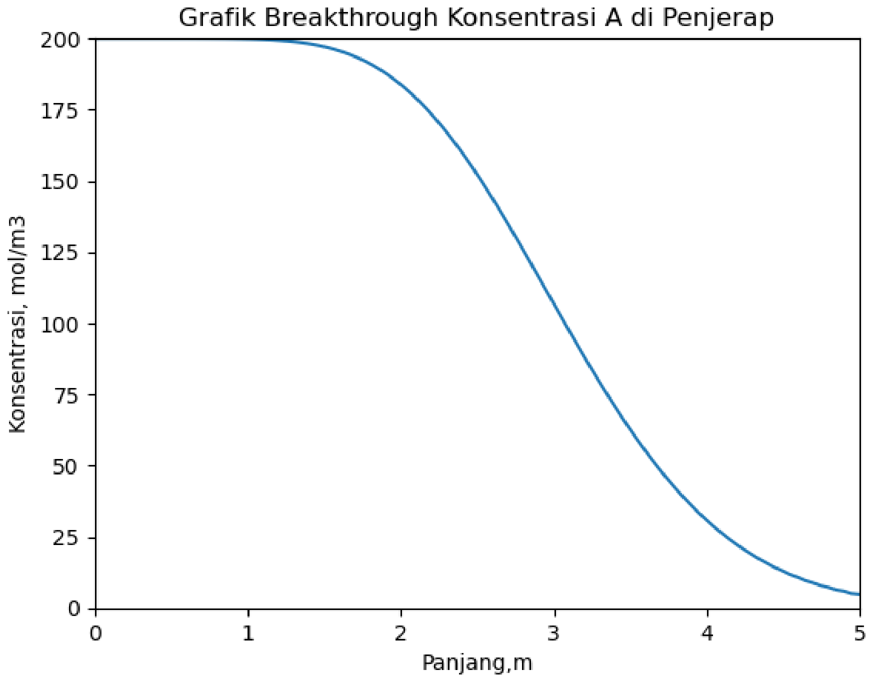
Grafik Breakthrough Konsentrasi A di Penjerap
Konsentrasi, mol/m3
Panjang,m
200
175
150
125
100
75
50
25
0
0
1
2
3
4
5

In [ ]: