I'm trying to prove exercise 113Y(b) of Measure Theory from Fremlin (see https://www1.essex.ac.uk/maths/people/fremlin/chap11.pdf page 39).

Given a measure space $(X, \sum, \mu)$ and $D \subseteq X$ prove that $\sum_D = \{E \bigcap D | E \in \sum\}$ is a σ-algebra of subsets of D and that μ_D defined by $\mu_D(A) = \inf\{\mu(E) | E \in \sum, A \subseteq E\}$ is a measure on $\sum_D$. I have already proved that Σ_D is a σ-algebra and that $\mu^* : \mathcal{P}(X) \to [0, \infty]$ defined by $\mu^*(A) = \inf\{\mu(E) | E \in \sum, A \subseteq E\}$ is a outer-measure such that $\forall A \in \mathcal{P}(X)$ there exists a $E \in \sum$ with $\mu(A) = \mu^*(A)$ (exercise 113Y(a)). I want then to apply the Caratheodory theorem for the rest of the proof and must then show that $\sum_D \subseteq \{E \subseteq X | \forall A \subseteq X$ we have $\mu^*(A \bigcap E) + \mu^*(A \backslash E) \leqslant \mu^*(A)\}$ but fails to do so. Also is this the correct way to solve the exercise?

Any help in the last part is appreciated