Question 1.

In the definition 122H functions in U must satisify

i. the domain of f is a coneglible subset of X and $f(x) \in [0, \infty[$ for each $x \in \operatorname{dom} f$

ii. there is a non-decreasing sequence of $\langle f_n \rangle_{n \in \mathbb{N}}$ of non-negative simple functions such that $\sup_{n \in \mathbb{N}} \int f_n < \infty$ and $\lim\limits_{n \to \infty} f_n(x) = f(x)$ for almost every $x \in X$

My question is this: Is condition (i) realy needed? For if $\lim\limits_{n \to \infty} f_n(x) = f(x)$ for almost every $x \in X$ we have that $\left\{x \in \operatorname{dom} f \,|\, \lim\limits_{n \to \infty} f_n(x) = f(x)\right\}$ is conegligible and as clearly $\left\{x \in \operatorname{dom} f \,|\, \lim\limits_{n \to \infty} f_n(x) = f(x)\right\} \subseteq \operatorname{dom} f$ it follows from the properties of conegligible sets tat $\operatorname{dom} f$ is also conegligible, so there is no need for (i) Is there a fault in reasoning here?

Question1.

In the proof of 122R Corollary (c)

The function $g = 2^{-k} \cdot \mathcal{X}^{F_k}$ is constructed based on $F_k = \{x \in E' | f(x) \geqslant 2^{-k}\}$ where $\mu F_k > 0$, this is then used further to reason that $g \leqslant_{a.e.} f$ so that $0 < 2^{-k} \mu F_k = \int g \leqslant f$. However I think that to be able to do this we must have that g is a simple function, which means that we also must prove that $\mu F_k < \infty$. If $0 \leqslant f(x)\ \forall x \in \operatorname{dom} f$ I think we can use 122J (a)(β) to do this but we have that $0 \leqslant_{a.e.} f$ and I fail to prove this in the more general case.