Øving 4

Made
by
Even
:)

① Oppgitt:

$$\alpha_{absorbent} = \underbrace{\frac{24 V \ln 10}{c S_{absorbent}}}_{\text{Defined as } K}\left(\frac{1}{T_{60,med}} - \frac{1}{T_{60,uten}}\right)$$

a) Vi har at

$$f\langle T_{60.med}, T_{60.uten}\rangle = \alpha_{absorption}$$

Utrykk for standardavvik

$$\sigma_f^2 = \underbrace{\left(\frac{\partial f}{\partial T_{60.med}}\right)^2}_{\frac{\partial F}{\partial T_{60.med}} = K\left(-\frac{1}{T_{60.med}^2}\right)} \sigma_{T_{60.med}}^2 + \underbrace{\left(\frac{\partial F}{\partial T_{60.uten}}\right)^2}_{\frac{\partial F}{\partial T_{60.uten}} = K\left(\frac{1}{T_{60.uten}^2}\right)} \sigma_{T_{60.uten}}^2$$

$$\sigma_f^2 = K^2\left(\frac{\sigma_{T_{60.med}}^2}{T_{60.med}^4} + \frac{\sigma_{T_{60.uten}}^2}{T_{60.uten}^4}\right)$$

$$\sigma_f = K\sqrt{\left(\frac{\sigma_{T_{60.med}}^2}{T_{60.med}^4} + \frac{\sigma_{T_{60.uten}}^2}{T_{60.uten}^4}\right)}$$

b)

```python
import numpy as np

V = 240  # m^3
S = 10 #m^2
c = 343  #m/s

#Calculate K
K = (24 * V * np.log(10))/(c * S)

#Defining reverberation times
T_60_uten = [4.28, 4.28, 4.13, 3.76, 4.14, 4.33, 4.10, 4.21]
T_60_med = [3.14, 3.15, 3.92, 3.65, 3.35, 3.89, 3.79, 3.32]

#Calculate means
mean_uten = np.mean(T_60_uten)
mean_med = np.mean(T_60_med)

#Calculate standard deviations
std_uten = np.std(T_60_uten)
std_med = np.std(T_60_med)

#Calculate absorption coefficient
alpha = K * (1/mean_med - 1/mean_uten)

#Calculate standard deviation of alpha
std_aplha = K * np.sqrt((std_med**2/mean_med**4) + (std_uten**2/mean_uten**4))

#Print results
print(f"Absorpsjonskoeffisient: {alpha:.4f} ± {std_aplha:.4f}")
```

```
Absorpsjonskoeffisient: 0.1657 ± 0.1016
```

c) 95% standardavvik for $\alpha_{absorbent}$:

Vi har

frihetsgrader $= n-1 = 7$

Tosidig 95% $\Rightarrow t_{95\%} = 2{,}365$

Regner ut konfidensintervallet

$$\mu_\alpha \in m_\alpha \pm t_{95\%}\frac{\sigma_\alpha}{\sqrt{n}}$$

$$\mu_\alpha \in 0.165 \pm 2.36\,\frac{0.108}{\sqrt{8}}$$

$$\mu_\alpha \in 0.165 \pm 0.09$$

② Gitt:

$$y = kt$$

Vi bruker nå andre kvadraters metode:

Vi har at residualene vil minske

$$r_i = y_i - kt$$

$$\Rightarrow J(k) = \sum_{i=1}^{n}(y_i - kt_i)^2$$

Dette gir:

$$\frac{\partial J}{\partial k} = \sum_{i=1}^{n} 2(y_i - kt_i)(-t_i) = 0$$

$$= -\sum_{i=1}^{n} y_i t_i + k\sum_{i=1}^{n} t_i^2$$

$$\Rightarrow k = \frac{\sum_{i=1}^{n} y_i t_i}{\sum_{i=1}^{n} t_i^2}$$

③ Gitt:

$$x = x_{sann} + \xi \qquad , \; \xi \sim N(0, \sigma_x)$$
$$y = y_{sann} + \eta \qquad , \; \eta \sim N(0, \sigma_x)$$

a) Vi har at

$$d = x_{sann} + \xi + y_{sann} + \eta$$

$$\Rightarrow E(d) = x_{sann} + y_{sann} = E(d_{sann})$$

<u>Dermed er forventet verdi av $d \to d_{sann}$</u>

b) Nå ser vi på standardavvik

$$Var(d) = Var(\xi) + Var(\eta)$$
$$\sigma_d^2 = \sigma_x^2 + \sigma_x^2$$
$$\sigma_d^2 = 2\sigma_x^2$$
$$\Rightarrow \underline{\underline{\sigma_d = \sqrt{2}\,\sigma_x}}$$

d) Vi prøver å beregne avstanden

$$d = \sqrt{(y-x)^2}$$

Standard avvik blir da:

$$\sigma_d^2 = \underbrace{\left(\frac{\partial d}{\partial x}\right)^2}\sigma_x^2 + \left(\frac{\partial d}{\partial y}\right)^2 \sigma_y^2 \quad , \qquad f = g^2, \; d = f, \; g = y - x$$
$$\frac{\partial d}{\partial x} = \frac{\partial d}{\partial f} \cdot \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x}$$

$$\frac{\partial d}{\partial x} = \frac{1}{2\sqrt{(y-x)^2}} 2(y-x)(-1) = -1$$

$$\Rightarrow \sigma_d^2 = \sigma_x^2 + \sigma_x^2$$
$$\underline{\underline{\sigma_d = \sqrt{2}\,\sigma_x}}$$

d) Forventet verdi for d^2

$$d^2 = (y-x)^2 \Rightarrow d = \underbrace{y_{sann} - x_{sann}}_{c \sim N(y_{sann} - x_{sann},\, 0)} + \overbrace{\eta - \xi}^{k \sim N(0, \sqrt{2}\,\sigma_x)})^2$$

$$\underline{\underline{d^2 = c^2 + 2ck + k^2}}$$

Estimator for d^2:

$$E(d^2) = (y_{sann} - x_{sann})^2 + 2(y_{sann} - x_{sann})\,E(k) + \underbrace{E(k^2)}_{Var(k)}$$
$$\Rightarrow \mu_{d^2} = (y_{sann} - x_{sann})^2 + 2\sigma_x^2$$
$$\underline{\underline{\mu_{d^2} = d_{sann}^2 + 2\sigma_x^2}}$$