Symmetry Shape Analysis

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***Abstract*—Symmetry is a fundamental and pervasive property found in both natural and man-made objects, playing a key role in aesthetics, structure, and function. In computational domains, symmetry serves as a powerful cue for data compression, structure inference, and shape understanding. This work presents a comprehensive overview of symmetry analysis in 3D shapes, with a particular focus on computational methods for symmetry detection and their applications in diverse fields such as CAD, computer vision, medicine, archaeology, and 3D modeling. We provide formal definitions of exact, approximate, and partial symmetries in the context of rigid transformations, and we survey five major categories of detection approaches: transformation-based, correspondence-based, voting-based, optimization-based, and learning-based methods. Special emphasis is placed on recent deep learning techniques, which have significantly advanced the state of the art yet face challenges in generalization and robustness. Finally, we identify key open problems and future directions, including the need for richer and more varied datasets, better generalization of learning-based models, effective formulations for symmetry detection in incomplete data, and the integration of symmetry priors in generative modeling. Our analysis highlights both the progress and the limitations of current methods and aims to guide future research toward more principled and capable symmetry-aware systems.**

***Index Terms*—Symmetry analysis, shape analysis, geometry processing**

I. Introduction

Symmetry is a ubiquitous feature in the world. Its manifestation is present in natural objects (e.g., animals, plants, and even humans) and man-made artificial objects. Its almost generalized presence in man-made objects is due to the association between symmetry and beauty. Even more, symmetry has influenced everything from art to engineering. Given this importance, it is necessary to understand the concept of symmetry from different points of view and thus be able to use this understanding to solve problems. Symmetry is also a complexity-reducing concept that makes it appealing since it allows us to reduce the amount of information with which we represent an entity.

This reductive feature makes the analysis of symmetries interesting to search for more compact representations of objects in the computer, especially in computing. For example, suppose an image is symmetric. In that case, we do not need to store the whole image but only the part that is not repeated and thus be able to reproduce the original image with the knowledge of symmetries. The same applies to a 3D object. However, computational representations do not bring information about the possible symmetries they may contain, so it becomes necessary to analyze these representations and search for symmetries based on the available information. From the computational perspective, this task of symmetry detection is challenging and has received much attention recently by communities from different research areas such as computer vision and geometric processing. In particular, we are interested in proposals made for three-dimensional representations. For that reason, this section summarizes state of the art in symmetry analysis on 3D objects.

Countless applications have made use of the concept of symmetries in 3D objects to solve problems. The following are some applications:

- Computer-Aided Design (CAD) [1]–[5]: These applications seek to reduce the complexity of storing computer-designed parts.
- Computer vision [6]–[9]: These applications seek to take advantage of the symmetries to facilitate processes such as object and structure recognition from motion.
- Computational modeling [10]: This application uses 3D face symmetry to enhance the features of 3D modeled faces.
- Archaeology [11]–[21]: These applications use symmetries to reconstruct or improve the representation of 3D cultural heritage objects.
- Medicine [22]–[24]: These applications seek to leverage symmetries to segment the shape of prostates and thus improve the diagnosis of diseases.
- Modeling in graphics [25]–[29]: These applications use the concept of symmetry to enforce good modeling properties in 3D objects.

Given the broad relevance of symmetry across domains and its potential to simplify and enrich 3D shape understanding, it is essential to develop computational methods that can effectively detect and characterize symmetries from geometric data. This paper provides a comprehensive overview of existing approaches to symmetry analysis in three-dimensional representations, highlighting both classical and modern techniques. We organize these methods according to their underlying strategies and emphasize recent trends in learning-based approaches. By

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doing so, we aim to lay the groundwork for identifying current limitations and motivating future research directions in this area.

II. Background

In this section, we summarize the main definitions required to understand the paper. As we are devoted to analyzing symmetries in rigid bodies, we focus our attention on the definitions for this specific case.

Definition 1: A symmetry is a non-trivial transformation T such that when we apply this transformation to a 3D object $\mathcal{O}$, the result is the same object. Formally, T is a symmetry if $T(\mathcal{O}) = \mathcal{O}$.

By non-trivial, we mean that T is not the identity. Note also that the previous definition is strict concerning the required equality. In practice, $\mathcal{O}$ is a discrete representation of the continuous surface of a 3D object, so the sampling introduces errors that we should take into account when defining the symmetry. Next, we introduce an alternative approximated definition:

Definition 2: Let $\mathfrak{S}$ be the space of 3D objects and let $d : \mathfrak{S} \times \mathfrak{S} \to \mathbb{R}$ be a function that measures the congruence between two 3D objects. A transformation T is an α-approximated symmetry of an object $\mathcal{O}$ if $d(T(\mathcal{O}), \mathcal{O}) \leq \alpha$.

Note that this definition is more general than the exact symmetry. The exact symmetry can be obtained when $\alpha = 0$. Note also that these definitions only hold when the symmetry applies to the overall object, commonly known as *global symmetry*. Nevertheless, objects often are not globally symmetric, but they contain symmetric parts. The next definition formalizes this idea:

Definition 3: Let $\mathcal{O}$ be a 3D object. A transformation T is a partial symmetry if there exist two regions $\mathcal{O}_i \subseteq \mathcal{O}$ y $\mathcal{O}_j \subseteq \mathcal{O}$, such that $d(T(\mathcal{O}_i), \mathcal{O}_j) \leq \alpha$.

The definition of partial symmetry is more general than a global symmetry. If we have $\mathcal{O}_i = \mathcal{O}_j = \mathcal{O}$, we have the definition of the global symmetry.

In our case, we are interested in transformations in the rigid world. In this scenario, the transformation T is extrinsic because it preserves the Euclidean metric in the 3D space. Given two points $p_i, p_j \in \mathcal{O}$, a transformation T is extrinsic if $\|p_i - p_j\|_2 = \|T(p_i) - T(p_j)\|_2$. In other words, transformation T preserves the pairwise Euclidean distances between points in $\mathcal{O}$. Examples of extrinsic transformations are rotations, reflections, and translations. In this work, we focus our attention on rotations and reflections. The rotational symmetry can be characterized by the vector of the rotational axis and a point within the vector's direction. The reflective symmetry can be characterized by the normal of the reflective plane and the plane's point. In both cases, the symmetry can be characterized by six floating-point numbers.

III. Methods for Symmetry Analysis

In this section, we show the main approaches proposed to detect symmetries in 3D objects. We classify the methods according to the approach used to tackle the problem of detection. Due to the limited space, we do not give details for every method. Instead, we describe the main characteristics of each approach and highlight the features of some methods. We do provide details about the learning-based methods, which are interesting in the context of this project.

1) Transformation-based methods: In this approach, the symmetry detection is formulated in a transformed domain, different from the original 3D space. The idea is to compute an intermediate representation for a 3D object such that the symmetries are easier to find. Table I lists methods in this approach.

For example, Martinet et al. [30] represent a shape with functions called *generalized moments*. A key property of the generalized moments is that the direction of these functions' vanishing gradients reveals the presence of symmetry. The method consists of evaluating candidate directions, transform the moments back to transformation matrices, and verify if the detected symmetry holds in 3D. On the other hand, Li et al. [31] detect symmetry by analyzing the entropy distribution of 3D viewpoints. The method computes a function over a sphere, where each point is the entropy related to observing the object from that point. The algorithm then checks the symmetric distribution over the sphere using an image-based method.

Reference	**Name**
Martinet et al. 2006 [30]	Generalized Moments
Grushko et al. 2012 [32]	Intrinsic Local Symmetries
Kakarala et al. 2013 [33]	Bilateral Symmetry in Phase Domain
Ovsjanikov et al. 2013 [34]	Quotient Spaces
Zhang et al. 2013 [35]	Symmetry Robust Descriptor
Jiang et al. 2014 [36]	Fourier-theoretic Approach
Wang et al. 2014 [37]	Global Intrinsic Symmetry Functions
Li et al. 2015 [31]	View-based Symmetry Detection
Liu et al. 2015 [38]	Orthonormal Functional Maps

TABLE I

Methods with the transformation-based approach.

2) Correspondence-based methods: In this approach, symmetry detection is based on determining symmetric candidate correspondences (points or regions) in the 3D objects. In general, this approach relies on solving the self-matching problem in a 3D shape. Table II lists some methods in this approach.

For example, Liu et al. [39] formulates the symmetry detection problem as finding the stationary points in the symmetric transformation. The idea to find some evidence of symmetric points over the surface is solved by applying the Blended Intrinsic Maps method [40]. The method detects potential key points in the 3D shape and computes a matrix of geodesic distances between the key points and the remaining points on the surface. The blended maps are computed using the geodesic distances as constraints. The zero-level curves of the blended maps are considered as the potential curve axis of the symmetry. On the other hand, Tevs et al. [41] rely on the analysis of edges in a 3D surface to find potential candidates of symmetric correspondences. The potential symmetric relationships are used to find high-level geometric relationships between objects in the same class.

Reference	Name
Mitra y Bronstein 2010 [42]	Intrinsic Regularity Detection
Raviv et al. 2010 [43]	Full and Partial Symmetries in Non-rigid
Raviv et al. 2010 [44]	Diffusion Symmetries
Berner et al. 2011 [45]	Subspace Symmetries
Wang et al. 2011 [46]	Symmetry Hierarchy
Liu et al. 2012 [39]	Symmetry Axis Curves
Shehu et al. 2014 [47]	Partial Intrinsic Symm.
Tevs et al. 2014 [41]	Geometric Symmetries and Regularities
Yoshiyasu et al.2014 [48]	Symmetry-aware Non-rigid Matching
Nagar et al.2025 [49]	Symmetry in point clouds
Tosyali et al.2025 [50]	Symmetry Axis Curve Generation

TABLE II
METHODS WITH THE CORRESPONDENCE-BASED APPROACH.

3) Voting-based methods: The methods in this approach are based on the accumulation of evidence (votes) for symmetries. In general, these methods encode transformations generated from points with common geometric characteristics. Then, these transformations need to be verified to find the final set of symmetries. Table III shows some methods in this category.

For example, Lipman et al. [51] builds a matrix of correspondences that resembles a graph of symmetric relationships. The initial matrix is built by accumulating votes for potential transformations. Subsequently, the eigendecomposition of this matrix leads to a spectral formulation of symmetry detection. On the other hand, Jiang et al. [52] address the problem of symmetry detection by analyzing the skeletal representation of a 3D shape. The computation of symmetry support for the components of the skeleton is performed via voting.

Reference	Name
Mitra et al. 2006 [53]	Partial and Approximate Symmetries
Pauly et al. 2008 [54]	Structural Regularity Detection
Lipman et al. 2010 [51]	Symmetry Factored Embedding
Zheng et al. 2010 [55]	Non-local Scan Consolidation
Xu et al. 2012 [56]	Multi-scale Partial Intrinsic Symmetry
Jiang et al. 2013 [52]	Skeleton-based Intrinsic Symmetry
Sipiran et al. 2014 [15]	Approximate Symmetry in Meshes

TABLE III
METHOD WITH THE VOTING-BASED APPROACH.

4) Optimization-based methods: In this approach, the symmetry detection problem is an optimization problem where the symmetry is the best transformation that holds the optimization constraints. Table IV lists methods in this approach.

For example, Korman et al. [57] propose to characterize the space of solutions for reflective and rotational symmetries. Then, the authors formally prove that one can sample this space to make an exhaustive analysis of symmetries. The method assumes that the centroid of the object is a fixed point of symmetry, so it only works with global symmetries. On the other hand, Nagar et al. [58] formulates the problem as a linear programming problem where potential symmetric correspondences are used to build an affinity matrix with symmetric constraints.

5) Learning-based methods: In this approach, the methods take advantage of the recent progress in deep learning

Reference	Name
Korman et al. 2015 [57]	Provably Approximately Symmetries
Mavridis et al. 2015 [17]	K-sparse Optimization
Speciale et al. 2016 [59]	Convex Variational
Ecins et al. 2017 [60]	Symmetrical Fitting
Cicconet et al. 2017 [61]	Pairwise Alignment of Curves
Nagar et al. 2019 [58]	Symmetry by Manifold Optimization
Nagar et al. 2020 [62]	3DSymm: Reflection Symmetry Detection
Hruda et al. 2022 [63]	Rotational Symmetry Detection
Nguyen et al. 2023 [64]	Symmetric Semi-shape Signatures
Bizarri et al. 2024 [65]	Group Symmetry by Decomposition

TABLE IV
METHOD WITH THE OPTIMIZATION-BASED APPROACH.

approaches to formulate a learning problem's symmetry detection problem. In general, these methods rely on training a neural network that receives a 3D shape and delivers a symmetry representation.

Reference	Name
Bu et al. 2015 [66]	Local Deep Feature Learning
Ji et al. 2019 [67]	3D reflectional detector with neural network
Gao et al. 2020 [68]	PRS-Net
Shi et al. 2020 [69]	SymmetryNet
Shi et al. 2023 [70]	Detection with weak supervision
Li et al. 2023 [71]	Equivariant Symmetry Detection
Je et al. 2024 [72]	Langevin dynamics for symmetry detection
Aguirre et al. 2025 [73]	Self-prior symmetry detection
Aguirre et al. 2025 [74]	Training-free zero-shot symmetry detection

TABLE V
METHOD WITH THE LEARNING-BASED APPROACH.

Bu et al. [66] use a deep belief network to compute learned features for key points in a 3D shape. The neural network receives a vector quantization descriptor for every key point and computes a deep local feature as a new descriptor for the keypoint. These key points are thus evaluated in finding symmetric correspondences in non-rigid objects. Ji et al. [67] propose a neural network based on PointNet++ to classify points as symmetric or non-symmetric. Given an input point cloud, the neural network infers a binary label for each point: points with label one are considered to belong to the reflective plane. After the classification, the inferred positive points are used to compute the symmetric plane using RANSAC. More recently, Gao et al. [68] propose a neural network to predict the planar reflective symmetry of objects. The representation is voxelization. This method detects three potential reflective symmetries and three potential rotational symmetries, which are subsequently validated in a post-processing stage. The method is self-supervised because it uses symmetry distance loss that controls input shape congruence with the symmetric counterpart. A regularization term controls the similarity between output symmetries, adding variability to the result. The experiments use the ShapeNet dataset [75] and take advantage of the known alignment of objects to have the ground truth for evaluation. Similarly, Shi et al. [69] describe a multi-task approach to detect reflective and rotational symmetries in RGB-D images. The model learns to find the symmetries and symmetric correspondences to avoid overfitting and enable good generalization. The ground truth is built automatically by an optimization-based symmetry detection method, so the proposed method learns to simulate the ground method's answer. It is also notorious that RGB information provides essential information for the prediction, so the method is not purely geometric.

IV. Open Problems and Future Directions

We identify several promising avenues for future research in this area, which we detail below:

A. Datasets and Benchmarking

A major bottleneck in this domain is the lack of standardized datasets for evaluating the effectiveness of methods across diverse scenarios, including both synthetic and real-world settings. While recent approaches increasingly rely on deep learning techniques that achieve strong performance, they are often trained and evaluated on a limited set of popular benchmarks such as ShapeNet [75]. These datasets, although useful, may already exhibit saturation effects, making it difficult to assess performance on more challenging tasks or generalize to unseen domains. One potential direction is to leverage real-world data, such as point clouds from LIDAR sensors. However, the need for high-quality ground truth labels remains a significant hurdle, especially given the cost and complexity of manual annotation. An alternative lies in the generation of synthetic datasets. Initial efforts in this direction, such as Sipiran et al. [76], have demonstrated promise, but further research is needed to explore the scalability and variability of synthetically generated data, as well as their alignment with real-world distributions.

B. Generalization in Learning-Based Methods

State-of-the-art performance in symmetry detection and related tasks is currently driven by deep learning models. Despite their success, these models often struggle to generalize to unseen objects, limiting their robustness and applicability in open-world scenarios. We argue that self-supervised learning paradigms, in combination with geometry-aware regularization techniques, offer a promising path to overcoming this limitation. Embedding prior knowledge about geometric structure into the learning process could enable models to infer symmetry and other structural properties from minimal supervision. This fusion of data-driven and structure-driven approaches remains underexplored and may lead to more generalizable and interpretable models.

C. Formulating Solutions for Partial and Incomplete Data

Another open challenge lies in the formulation and resolution of tasks involving highly partial or incomplete data—cases where only small fragments of the object are available, often insufficient to reveal global symmetries. Even advanced learning-based methods fail to cope with such extreme cases of partiality. We hypothesize that a general problem formulation exists for this setting, one that could be tackled with an end-to-end learning framework designed specifically to infer symmetries under severe data incompleteness. Developing such methods could significantly broaden the applicability of symmetry detection algorithms in real-world conditions, such as robotics, archaeology, and autonomous mapping.

D. Symmetry-Aware Generative Models

3D generative models have recently gained substantial traction, producing increasingly high-quality results across various tasks [77]. However, symmetry—a pervasive and meaningful property in man-made and natural objects—is rarely used as an explicit inductive bias or quality criterion during training. We believe that incorporating symmetry awareness into the generative process could enhance the plausibility, regularity, and aesthetic quality of generated 3D shapes. There is precedent for this idea in crystallography [78], [79] and molecular design [80], where symmetry constraints are crucial for generating physically valid structures. Imposing such structural priors in generative modeling could pave the way for higher-quality 3D data synthesis, particularly in applications where geometric consistency is critical.

V. Final remarks

Symmetry remains a central concept in the understanding and representation of 3D shapes, offering both aesthetic and structural insights across a wide range of disciplines. Over the past two decades, the field has evolved from handcrafted geometric algorithms to learning-based methods capable of capturing complex symmetries from data. Despite these advances, several key challenges remain unresolved. The lack of standardized and diverse datasets continues to limit fair comparison and generalization. Deep learning models, while powerful, often struggle with out-of-distribution generalization and incomplete inputs. Moreover, symmetry—a highly structured and ubiquitous property—is still underutilized in generative modeling pipelines.

In this work, we have presented a unified view of the computational approaches for symmetry detection in 3D shapes, emphasizing their motivations, methodologies, and applications. We believe that future progress will depend on developing methods that are not only accurate but also data-efficient, interpretable, and robust to real-world conditions. In particular, bridging geometric structure and data-driven learning holds great promise for creating models that better understand and leverage symmetry.

Ultimately, symmetry is more than a geometric property—it is a powerful organizational principle. Harnessing it effectively may lead to more compact representations, better reconstructions, and more meaningful generations of 3D content. As such, it continues to offer fertile ground for fundamental research and impactful applications.

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