Revisiting Bayesian Estimation in DSGE Models: A New Identification Strategy for Justiniano et al. (2010)

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Abstract

The objective of this paper is to present a different identification strategy for the Bayesian estimation of the DSGE model developed by Justiniano et al. (2010). I used a different set of exogenous variables for the estimation: the output gap, public spending, capital, and market price markups. The Bayesian estimation produces new parameter estimates that are largely consistent with those in the original paper, except for one key difference: the autoregressive coefficient in the monetary policy rule. In Justiniano et al. (2010), the posterior value of this parameter was estimated at 0.6, whereas I found a value of 0.2. As a result, the change in the exogenous variables significantly affects the estimation of monetary policy. In conclusion, the revised estimation highlights a substantial shift in the monetary policy rule, resulting in a different response pattern and producing notably different results when analyzing the impact of a monetary policy shock.

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1 Introduction

This paper presents a different approach in the identification strategy for the Bayesian estimation of the model used in Justiniano et al. (2010). I propose a new set of exogenous variables that are unconventional compared to those used in the original study. Specifically, instead of using variables such as consumption, production, investment, labor, inflation rate, and real wages, I employ a different set of variables for the United States to explore how unconventional choices affect the estimation of this model. The motivation behind this approach was to understand how Bayesian estimation changes when significant modifications are made to the observed variables and whether it remains feasible. The model achieved convergence, and the posterior parameters were estimated with small differences in most of the parameters.

The new set of exogenous variables includes the output gap, public spending, capital, and market price markups. These variables are in quarterly frequency and were transformed using log differences to ensure stationarity, with the exception of the output gap. All data were sourced from the Federal Reserve Bank of St. Louis database. These variables were incorporated into the model, and Bayesian estimation was performed, using the initial values from Justiniano et al. (2010) as a starting point. As is conventional, the Metropolis-Hastings algorithm was employed for estimation, and the model was simulated in Dynare/Matlab. Prior to this, I attempted to include other variables, but those attempts did not result in convergence.

The Bayesian estimation produces new parameter estimates that are largely consistent with those in the original paper, except for one key difference: the autoregressive coefficient in the monetary policy rule. In the new estimation, this coefficient decreases significantly from its prior value of 0.8 to 0.2. This is a noteworthy change because, in Justiniano et al. (2010), the posterior value of this parameter was estimated at 0.6, while I found a value of 0.2. As a result, the change in the exogenous variables significantly affects the estimation of monetary policy.

In conclusion, the revised estimation highlights a substantial shift in the monetary policy rule, resulting in a different response pattern and producing notably different results when analyzing the impact of a monetary policy shock. Specifically, the persistence parameter, which reflects how much the current interest rate depends on its previous value, is smaller in this estimation. Consequently, the central bank reacts more aggressively to changes in inflation or the output gap, allowing the interest rate to return to its steady-state value more quickly after a shock.

The paper is then structured as follows, Section 2 provides information on the identification problems in DSGE models. Section 3 presents the data and methodology that will be used, in relation to the JPT (2010) paper. Lastly, Section 4 discusses the conclusion and the recommendation for future studies.

2 Identification problems

The DSGE models, although studied extensively in the literature, are still vulnerable to some identification problems. One of the seminal works that have looked into this is provided by Canova and Sala (2009) where they showed weak identification, partial, and observational problems due to the difficulties in properly matching the structural parameters and the solutions. Even with larger samples observed, it does not diminish the bias in the estimates. Consecutively, Iskrev (2010) provided more evidences on the problems in identifying appropriate structural models by using the Smets and Wouters (2007) model. The authors provided some conditions in order to analyze which model parameters are identified. It is then important to use proper estimations and recognize identified parameters as not being able to do so will lead to wrong economic interpretations and incorrect policy decisions.

Other early works that investigated some of the identification problems in the DSGE models are Beyer and Farmer (2007); Cochrane (2011); Consolo et al. (2009). A more comprehensive and formal analysis followed, with an initial focus on addressing local identification challenges. This led to the formulation of rank conditions derived from a spectral domain perspective Qu and Tkachenko (2012) and by using properly defined Jacobian matrix Komunjer and Ng (2011). In a more recent work, Kocięcki and Kolasa (2018) introduced an alternative algorithm based on the conditions that establish links between observationally equivalent state space representations, as outlined by Komunjer and Ng (2011). This approach circumvents the requirement to solve the model for every possible parameter.

Identification problems in DSGE models refer to issues related to the ability to uniquely determine the parameters of the model using the available data. These problems arise when the model is not sufficiently constrained by the data to allow for a unique solution for its parameters.

Identification issues can make it difficult or even impossible to estimate the model accurately. One of the primary concerns is local identification, which occurs when the model's Jacobian matrix is degenerate or lacks full rank at the true parameter values, making it impossible to distinguish between different sets of parameter values that produce similar results (Iskrev, 2010). This is often compounded by over-parameterization, where the model contains more parameters than can be reliably estimated from the available data (Canova et al., 2014). Another major challenge is simultaneity and endogeneity, where the variables in the model are determined simultaneously within a system of equations, complicating the identification of causal relationships between them. This is particularly problematic when there is a lack of exogenous instruments—variables that influence the endogenous variables but are unaffected by them—as these are necessary to resolve endogeneity and simultaneity issues (Stock and Watson, 2007).

Additionally, DSGE models may suffer from multiple equilibria, where different sets of parameter values can generate multiple valid equilibrium solutions, making it difficult to pinpoint a unique set of parameters (Benhabib et al., 2001). This issue is exacerbated in nonlinear models, where complex relationships between variables can make the identification more sensitive to small changes in parameters. Finally, the quality and quantity of data play a critical role in identification, and if the data is sparse or does not capture enough variation in key economic variables, the model may become under-identified (Canova and Sala, 2009). Addressing these challenges requires careful attention to model specification, the use of appropriate estimation techniques, and ensuring that the data used is rich and comprehensive (Del Negro and Schorfheide, 2008).

3 Data and Methodology

This section provides description of the entirely new observable variables and where the data was sourced from. In addition to this, the methodology is also provided, discussing the equations provided in the Justiniano et al. (2010) paper.

3.1 Data

In the original paper, the authors used seven exogenous variables: production, consumption, investment, hours worked, wages, inflation, and interest rates. The idea behind this paragraph is to emphasize the selection of different variables as exogenous ones. I carefully ensured that the variables chosen for estimation are distinct from those already used by the authors, are available in time series format, and are included in the list of endogenous and exogenous variables in the theoretical model.

The variables used in the model by Justiniano et al. (2010) include output (GDP) growth, consumption growth, investment growth, labor hours growth, wage growth, the inflation rate, and the nominal interest rate. The model-specific variables are key indicators such as output (GDP), consumption, investment, capital stock, capital utilization rate, hours worked, and labor productivity. Regarding prices and wages, the model incorporates marginal cost, real wage, price markup, and wage markup. Monetary policy is represented by the nominal interest rate and the real interest rate. The model also features a range of shocks, including permanent and temporary technology shocks, investment-specific technology shocks, capital quality shocks, preference shocks, labor supply shocks, monetary policy shocks, and government spending shocks.

I conducted a detailed review of the availability of these time series in official macroeconomic data sources, such as the Federal Reserve Economic Data (FRED) database. From this analysis, I identified variables that are available, measurable, have a quarterly frequency, are sufficiently long in duration, and differ from the exogenous variables already used by Justiniano et al. (2010). The variables that meet these criteria are: output gap, public spending, capital, and market price markups.

The four main observable variables used in the paper are output gap, government spending, capital, and price mark-ups. The paper will use a quarter data from 1960q1 to 2024q2. The following table summarizes the information about the variables and the source.

Table 1: Data and Source

Variable	Description	Source	Code
x	Output gap	FRED	GDPC1, GDPPOT
g	Federal Government: Current Expenditures	FRED	FGEXPND
k	All Sectors; Total Capital Expenditures	FRED	BOGZ1FA895050005Q
lambda_p	Price mark-ups	FRED	A053RC1Q027SBEA, NICUR

The output gap and price mark-ups are based on a combination of data. First, the output gap is measured based on the real gross domestic product (GDPC1), calculated by the U.S. Bureau of Economic Analysis (BEA), and the real potential gross domestic product (GDPPOT) from the U.S. Congressional Budget Office. The output gap is computed using the following equation based on the 2017 current-dollar value:

$$x = \frac{GDPC1 - GDPPOT}{GDPPOT} \times 100 \tag{1}$$

Consecutively, the price mark-ups are based on the the corporate profits before tax (A053RC1Q027SBEA) and the national income (NICUR), both calculated by BEA. The price mark-ups are formulated using the follwoing equation:

$$lambda\_p = \frac{A053RC1Q027SBEA}{NICUR} \times 100 \tag{2}$$

In summary, there will be a total of four (4) observable variables that will be used in the estimation of the paper. These variables are interpreted as growth rates by taking the log of the series. Consequently, there will also be a total of four (4) known shocks: technology shock, investment shock, monetary policy shock, and government spending shock.

3.2 Methodology

To begin with, the baseline DSGE model used was the one from Justiniano et al. (2010). I solved the model to find the steady state and used, as an initial guess for the parameters, the values provided by the authors. The following table presents the details of each equation from the theoretical model:

Table 2: Linear rational expectations of the JPT (2010) model: sticky price-wage economy

(1)	$\hat{y}_t = \frac{y+F}{y}\left[\alpha\hat{k}_t + (1-\alpha)\hat{L}_t\right]$
(2)	$\hat{\rho}_t = \hat{w}_t + \hat{L}_t - \hat{k}_t$
(3)	$\hat{s}_t = \alpha\hat{\rho}_t + (1-\alpha)\hat{w}_t$
(4)	$\hat{\pi}_t = \frac{\beta}{1+\iota_p\beta}E_t\hat{\pi}_{t+1} + \frac{\iota_p}{1+\iota_p\beta}\hat{\pi}_{t-1} + \kappa\hat{s}_t + \kappa\hat{\lambda}_{p,t}$
(5)	$\hat{\lambda}_t = \frac{h\beta e^{\gamma}}{(e^{\gamma}-h\beta)(e^{\gamma}-h)}E_t\hat{c}_{t+1} - \frac{e^{2\gamma}+h^2\beta}{(e^{\gamma}-h\beta)(e^{\gamma}-h)}\hat{c}_t$
	$+\frac{he^{\gamma}}{(e^{\gamma}-h\beta)(e^{\gamma}-h)}\hat{c}_{t-1} + \frac{h\beta e^{\gamma}\rho_z - he^{\gamma}}{(e^{\gamma}-h\beta)(e^{\gamma}-h)}\hat{z}_t$
	$+\frac{e^{\gamma}-h\beta\rho_b}{e^{\gamma}-h\beta}\hat{b}_t$
(6)	$\hat{\lambda}_t = \hat{R}_t + E_t\left(\hat{\lambda}_{t+1} - \hat{z}_{t+1} - \hat{\pi}_{t+1}\right)$
(7)	$\hat{\rho}_t = \chi\hat{u}_t$
(8)	$\hat{\phi}_t = (1-\delta)\beta e^{-\gamma}E_t\left(\hat{\phi}_{t+1} - \hat{z}_{t+1}\right) + \left(1-(1-\delta)\beta e^{-\gamma}\right)E_t\left[\hat{\lambda}_{t+1} - \hat{z}_{t+1} + \hat{\rho}_{t+1}\right]$
(9)	$\hat{\lambda}_t = \hat{\phi}_t + \hat{\mu}_t - e^{2\gamma}S''\left(\hat{\imath}_t - \hat{\imath}_{t-1} + \hat{z}_t\right) + \beta e^{2\gamma}S''E_t\left[\hat{\imath}_{t+1} - \hat{\imath}_t + \hat{z}_{t+1}\right]$
(10)	$\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t$
(11)	$\hat{\bar{k}}_t = (1-\delta)e^{-\gamma}\left(\hat{\bar{k}}_{t-1} - \hat{z}_t\right) + \left(1-(1-\delta)e^{-\gamma}\right)\left(\hat{\mu}_t + \hat{\imath}_t\right)$
(12)	$\hat{w}_t = \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} - \kappa_w\hat{g}_{w,t} +$
	$+\frac{\iota_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\iota_w}{1+\beta}\pi_t + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} +$
	$+\frac{\iota_w}{1+\beta}z_{t-1} - \frac{1+\beta\iota_w-\rho_z\beta}{1+\beta}z_t + \kappa_w\hat{\lambda}_{w,t}$
(13)	$\hat{g}_{w,t} = \hat{w}_t - \left(\nu\hat{L}_t + \hat{b}_t - \hat{\lambda}_t\right)$
(14)	$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[\phi_\pi\hat{\pi}_t + \phi_X\left(\hat{x}_t - \hat{x}_t^*\right)\right] + \phi_{dX}\left[\left(\hat{x}_t - \hat{x}_{t-1}\right) - \left(\hat{x}_t^* - \hat{x}_{t-1}^*\right)\right] + \hat{\eta}_{mp,t}$
(15)	$\hat{x}_t = \hat{y}_t - \frac{\rho k}{y}\hat{u}_t$
(16)	$\frac{1}{g}\hat{y}_t = \frac{1}{g}\hat{g}_t + \frac{c}{y}\hat{c}_t + \frac{\rho k}{y}\hat{u}_t$

Note: The observable variables are: output (y_t), consumption (c_t), investment (i_t), hours worked (L_t), real wage (w_t), inflation rate (π_T), and nominal interest rate (r_t). In this model, we have a total of 16 rational expectations equation

Table 3: Linear rational expectations of the JPT (2010) model: flexible-price-wage economy

(1)	$\hat{y}_t^* = \frac{y+F^*}{y^*}\left[\alpha\hat{k}_t^* + (1-\alpha)\hat{L}_t^*\right]$
(2)	$\hat{\rho}_t^* = \hat{w}_t^* + \hat{L}_t^* - \hat{k}_t^*$
(3)	$\hat{s}_t^* = \alpha\hat{\rho}_t^* + (1-\alpha)\hat{w}_t^*$
(4)	$\hat{\lambda}_t^* = \frac{h\beta e^{\gamma}}{(e^{\gamma}-h\beta)(e^{\gamma}-h)}E_t\hat{c}_{t+1}^* - \frac{e^{2\gamma}+h^2\beta}{(e^{\gamma}-h\beta)(e^{\gamma}-h)}\hat{c}_t^*$ $+\frac{he^{\gamma}}{(e^{\gamma}-h\beta)(e^{\gamma}-h)}\hat{c}_{t-1}^* + \frac{h\beta e^{\gamma}\rho_z - he^{\gamma}}{(e^{\gamma}-h\beta)(e^{\gamma}-h)}\hat{z}_t^*$ $+\frac{e^{\gamma}-h\beta\rho_b}{e^{\gamma}-h\beta}\hat{b}_t$
(5)	$\hat{\lambda}_t^* = \hat{R}_t^* + E_t\left(\hat{\lambda}_{t+1}^* - \hat{z}_{t+1} - \hat{\pi}_{t+1}\right)$
(6)	$\hat{\rho}_t^* = \chi\hat{u}_t^*$
(7)	$\hat{\phi}_t^* = (1-\delta)\beta e^{-\gamma}E_t\left(\hat{\phi}_{t+1}^* - \hat{z}_{t+1}\right) + \left(1-(1-\delta)\beta e^{-\gamma}\right)E_t\left[\hat{\lambda}_{t+1}^* - \hat{z}_{t+1} + \hat{\rho}_{t+1}^*\right]$
(8)	$\hat{\lambda}_t^* = \hat{\phi}_t^* + \hat{\mu}_t^* - e^{2\gamma}S''\left(\hat{\imath}_t^* - \hat{\imath}_{t-1}^* + \hat{z}_t\right) + \beta e^{2\gamma}S''E_t\left[\hat{\imath}_{t+1}^* - \hat{\imath}_t^* + \hat{z}_{t+1}\right]$
(9)	$\hat{k}_t^* = \hat{u}_t^* + \hat{\bar{k}}_{t-1}^* - \hat{z}_t$
(11)	$\hat{\bar{k}}_t^* = (1-\delta)e^{-\gamma}\left(\hat{\bar{k}}_{t-1}^* - \hat{z}_t\right) + \left(1-(1-\delta)e^{-\gamma}\right)\left(\hat{\mu}_t^* + \hat{\imath}_t^*\right)$
(12)	$\hat{w}_t^* = \frac{1}{1+\beta}\hat{w}_{t-1}^* + \frac{\beta}{1+\beta}E_t\hat{w}_{t+1}^* - \kappa_w\hat{g}_{w,t}^* +$ $+\frac{\iota_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\iota_w}{1+\beta}\pi_t + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} +$ $+\frac{\iota_w}{1+\beta}z_{t-1} - \frac{1+\beta\iota_w-\rho_z\beta}{1+\beta}z_t + \kappa_w\hat{\lambda}_{w,t}$
(13)	$\hat{g}_{w,t}^* = \hat{w}_t^* - \left(\nu\hat{L}_t^* + \hat{b}_t - \hat{\lambda}_t^*\right)$
(13)	$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[\phi_\pi\hat{\pi}_t + \phi_X\left(\hat{x}_t - \hat{x}_t^*\right)\right] + \phi_{dX}\left[\left(\hat{x}_t - \hat{x}_{t-1}\right) - \left(\hat{x}_t^* - \hat{x}_{t-1}^*\right)\right] + \hat{\eta}_{mp,t}$
(14)	$\hat{x}_t^* = \hat{y}_t^* - \frac{\rho k}{y}\hat{u}_t^*$
(15)	$\frac{1}{g}\hat{y}_t^* = \frac{1}{g}\hat{g}_t^* + \frac{c}{y}\hat{c}_t^* + \frac{\rho k}{y}\hat{u}_t^*$

Note: There will be a total of 15 equations for this flexible model given that there are no markup shocks, whose allocation is denoted as "".*

The Bayesian estimation was used to find the estimate values of the parameters. The Bayesian estimation is recommended by the literature were there is limited access to time series, as is these types of macroeconomic models. In mathematical terms, the set of equations presenters in table two can be represented as a system of equilibrium equations:

$$E_t\left[f(X_{t+1}, X_t, X_{t-1}, \varepsilon_t; \theta)\right] = 0 \tag{3}$$

where X_t denotes the vector of endogenous variables, ε_t represents the vector of shocks, and θ is the vector of structural parameters to be estimated.

The first step in the Bayesian estimation is to set the probability distribution of the parameters, for example, it can be a normal, gamma or beta distribution. For this case, I used the distribution stated in JPT (2010). The idea is that their distribution represents the econometrician's beliefs about the parameters before observing the data. The second step is to build the likelihood function, $p(Y|\theta)$, which describes the probability of the observed the exogenous data, Y, given the parameters θ. Using Bayes' theorem, the posterior distribution of the parameters is computed as:

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)} \tag{4}$$

where $p(Y)$ is the marginal likelihood, calculated as:

$$p(Y) = \int p(Y|\theta)p(\theta)d\theta \tag{5}$$

The posterior distribution then combines the information from the priors and the data, providing a complete probabilistic description of the parameters.

4 Results

In this paper, I developed a Bayesian estimation of the parameters of the DSGE model. The main idea is to assign prior values and probability distributions to these parameters. By incorporating exogenous data and applying Bayesian techniques, I estimated the posterior distributions. The following tables present the details of the priors, posterior means, standard deviations, and the proposed probability distributions for each parameter of the DSGE model.

Table 4: Estimation Results

Parameter	**Prior Mean**	**Posterior Mean**	**Prior Distribution**	**Prior Std. Dev.**
`aalpha`	0.330	0.0301	Beta	0.0500
`bbeta`	0.990	0.9908	Beta	0.0020
`nu`	2.000	0.0786	Gamma	0.5000
`zeta_p`	0.750	0.7616	Beta	0.0500
`zeta_w`	0.750	0.7227	Beta	0.0500
`phi_pi`	1.500	2.1528	Normal	0.2500
`phi_x`	0.125	0.1753	Normal	0.0500
`rho_R`	0.800	0.2864	Beta	0.1000
`rho_z`	0.850	0.4524	Beta	0.1000
`rho_g`	0.850	0.7737	Beta	0.1000
`rho_mu`	0.700	0.3819	Beta	0.1000
`rho_mp`	0.400	0.1125	Beta	0.1000
`rho_p`	0.500	0.3010	Beta	0.1000
`rho_w`	0.500	0.5199	Beta	0.1000
`rho_b`	0.700	0.8069	Beta	0.1000
`h`	0.700	0.9410	Beta	0.1000
`lambda_p_ss`	0.150	0.1485	Normal	0.0500
`lambda_w_ss`	0.150	0.1607	Normal	0.0500
`iw`	0.500	0.0645	Beta	0.1000
`ip`	0.250	0.0649	Beta	0.1000
`xi`	5.000	11.0904	Gamma	1.0000
`s2prime`	4.000	8.5190	Gamma	1.0000
`gammam`	0.005	-0.0245	Normal	0.0030
`L_ss`	0.300	0.3026	Normal	0.1000
`pi_ss`	1.005	1.0523	Normal	0.1000
`phi_dx`	0.130	0.1408	Normal	0.0200
`theta_p`	0.500	0.5462	Invg	0.1000
`theta_w`	0.500	0.5340	Invg	0.1000
`theta_mp`	0.100	0.0506	Invg	0.1000
`theta_g`	0.500	0.5670	Invg	0.1000
`theta_mu`	0.500	0.4911	Invg	0.1000
`theta_b`	0.100	0.0911	Invg	0.1000

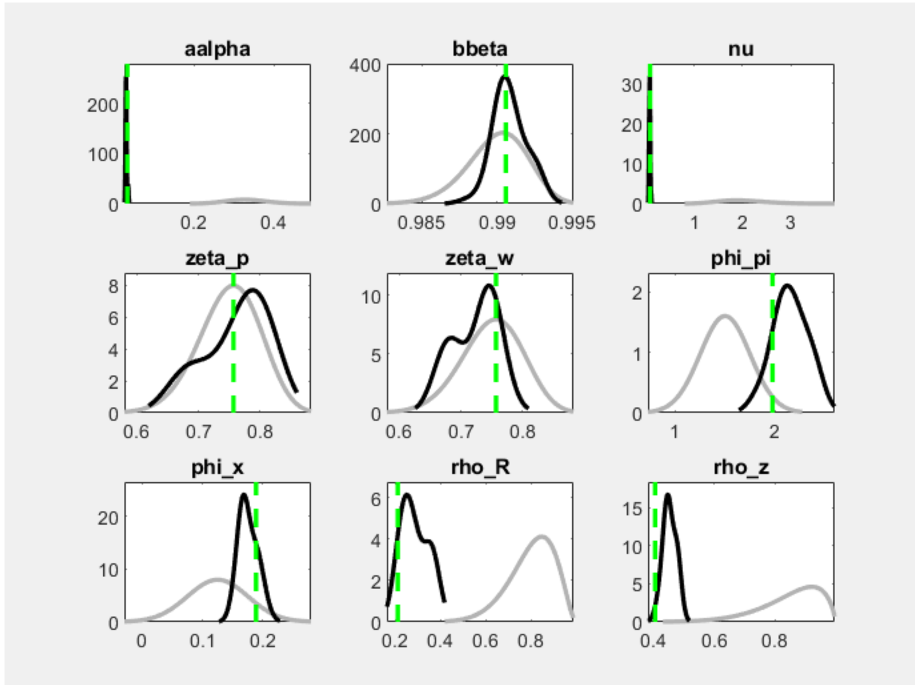

Figure 1: Prior and posterior distributions moments.

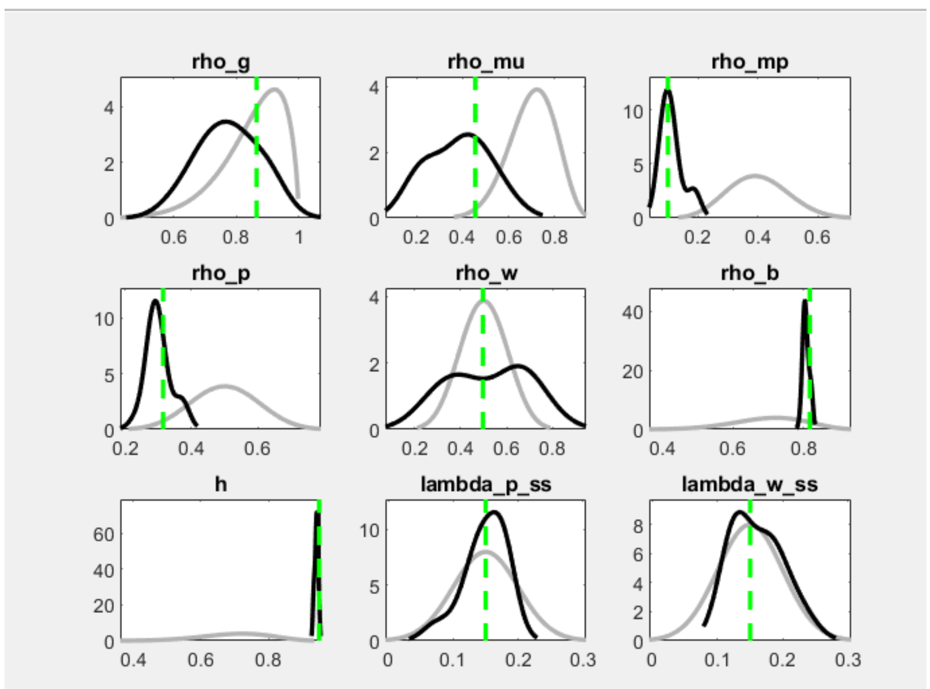

Figure 2: Prior and posterior distributions moments.

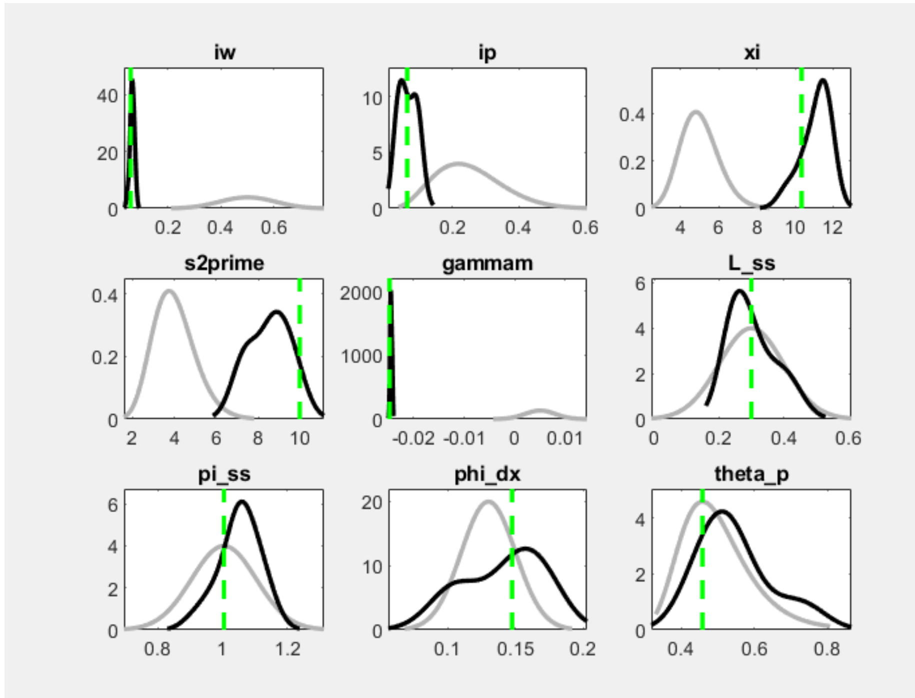

Figure 3: Prior and posterior distributions moments.

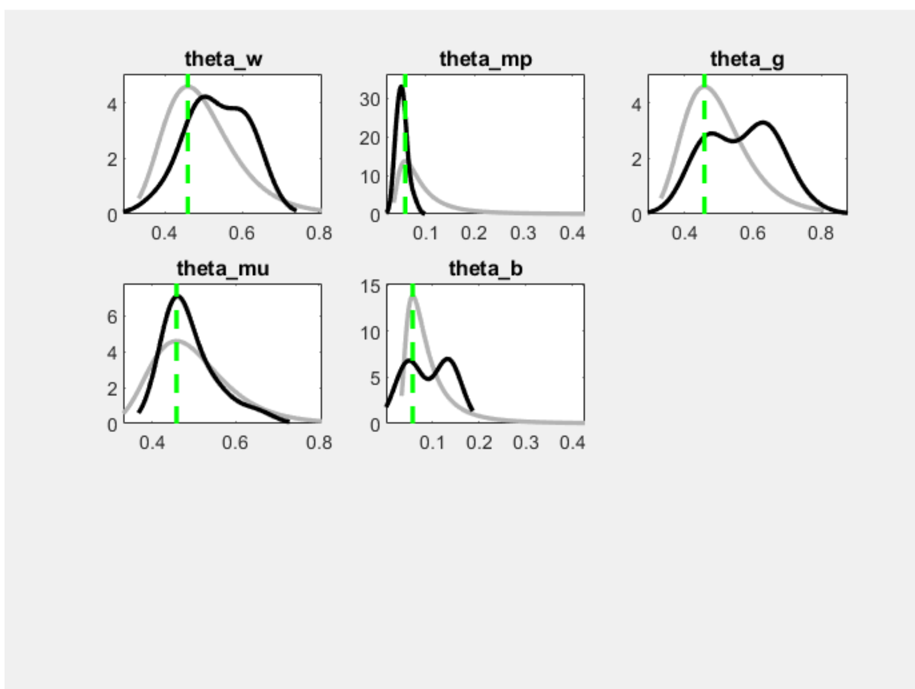

Figure 4: Prior and posterior distributions moments.

For the posterior values, these represent the estimated mean and standard deviation that best adjust to the observed data available in the model. The process of obtaining the solution involves a combination of maximum likelihood, Kalman filtering, and Metropolis-Hastings algorithms. Finally, the probability distributions (e.g., Beta, Gamma, Normal, Inverse Gamma) are also set by the author, following the propositions of Justiniano et al. (2010). The results of these estimations yield interesting insights.

To begin with, the posterior mean of the parameter α (0.0301) is much smaller than the prior mean (0.330), indicating that, based on the provided exogenous data, the mean value of α should be smaller. Another notable observation is the parameter β, where the prior mean was 0.990, and the posterior mean is 0.9908, suggesting that the initial guess for this parameter was very close to the value estimated using Bayesian techniques.

In general, most parameters exhibit changes (even minor ones, such as differences in the decimals) compared to their prior values. Significant changes are observed in the parameters related to the persistence of shocks. Most persistence parameters (ρ_R, ρ_z, ρ_g, ρ_μ, etc.) show posterior means lower than their prior means. Furthermore, the strong response to inflation (ϕ_π) suggests a model where the central bank prioritizes inflation stabilization.

Figure 5 presents the unique stable saddle paths and the non-unique stable saddle paths derived from the simulation of the DSGE model. As shown in the graph, the blue area represents the majority of points compared to the red area, indicating a higher likelihood of finding a stable saddle path that converges in the simulation. Similarly, Figure 6 illustrates the areas of no indeterminacy and indeterminacy. The area of no indeterminacy refers to the parameter regions where the DSGE simulation has a unique and stable solution. As can be observed, the areas of indeterminacy are consistent with the unique stable saddle paths shown in the previous plot.

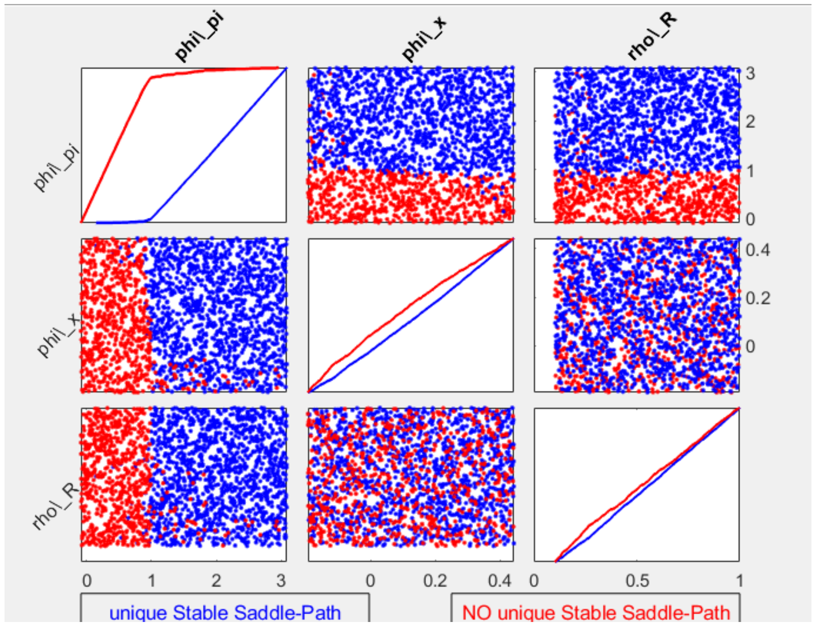

Figure 5: Prior and posterior distributions moments.

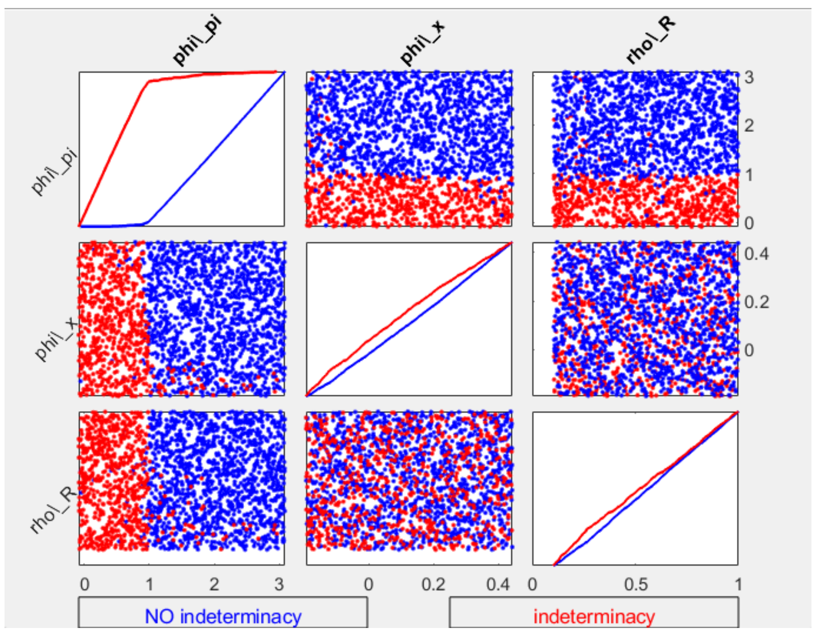

Figure 6: Prior and posterior distributions moments.

Figure 7 presents the No explosive and explosive solutions from the DSGE simulation.The scatterplots of points (blue) represent non-explosive solutions, showing that for certain parameter combinations, the model remains stable. At the same time, the red lines show the possible explosive solutions (when the Blanchard-Khan assumption breaks). As I am introducing non-conventional variables for the estimation of the set of parameters, it is more likely to find possible explosive solutions, as shown in the plot.

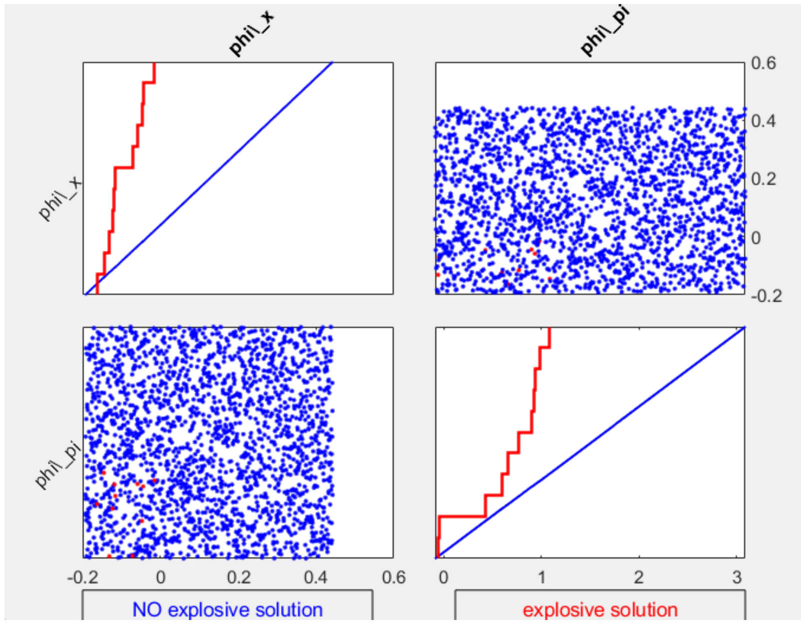

Figure 7: Prior and posterior distributions moments.

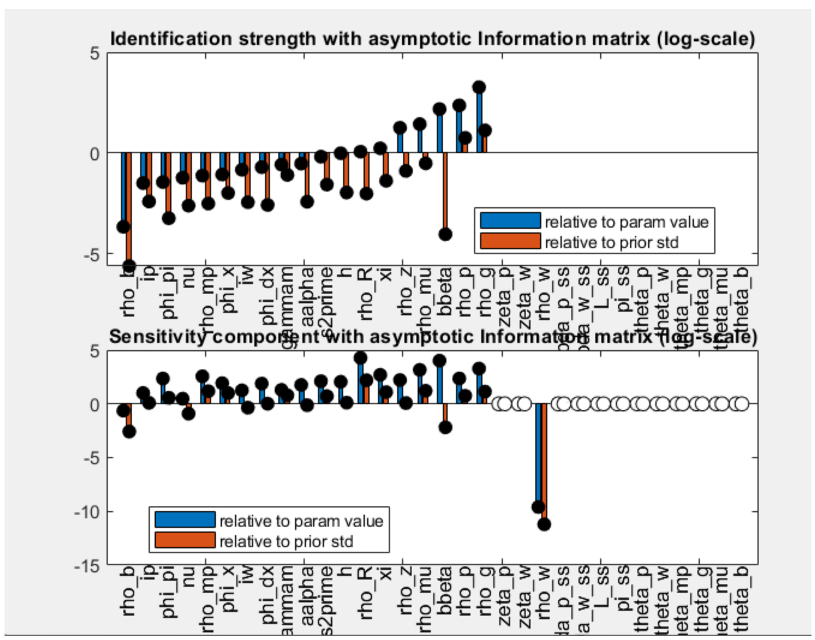

Figure 8: Prior and posterior distributions moments.

An interesting result is presented in Figure 8, which shows a relatively strong identification in a set of parameters. A large value (indicated by the blue or orange lines) means that the data is useful in the estimation and has enough variation to find different estimated values in comparison to the initial values. In other words, the larger the bars, the more reliable the estimation is based on the exogenous variables. Also, the box below (Figure 9) shows the sensitivity component, where there are some parameters with high sensitivity.

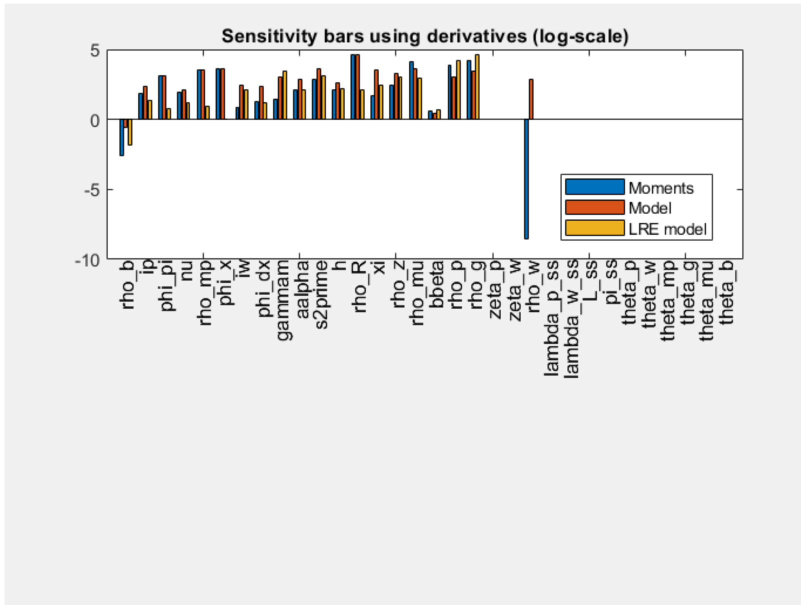

Figure 9: Prior and posterior distributions moments.

Graphs 10 and 11 show evidence of collinearity between parameters in the case of prior means. This can eventually affect the estimation, but the estimation with four exogenous variables is still finding stable and unique convergent solutions. Finally, in the last plot, the evolution of the time series used for the Bayesian estimation is presented. The key observation is that they are stationary, mainly because they track the logarithmic difference of the level values

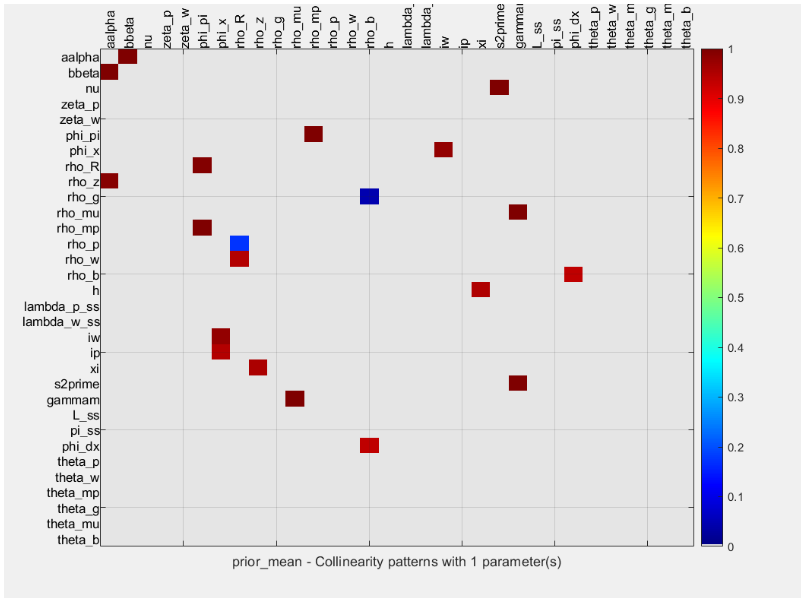

Figure 10: Prior and posterior distributions moments.

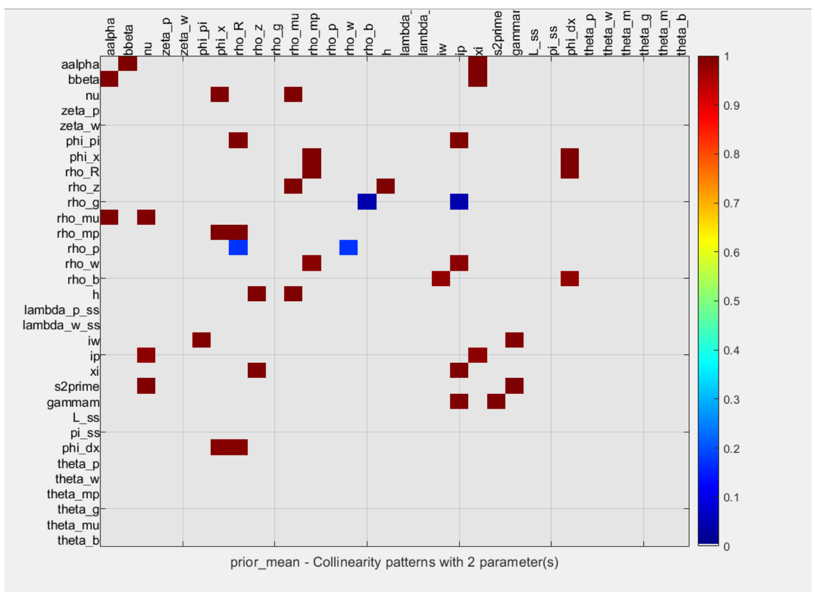

Figure 11: Prior and posterior distributions moments.

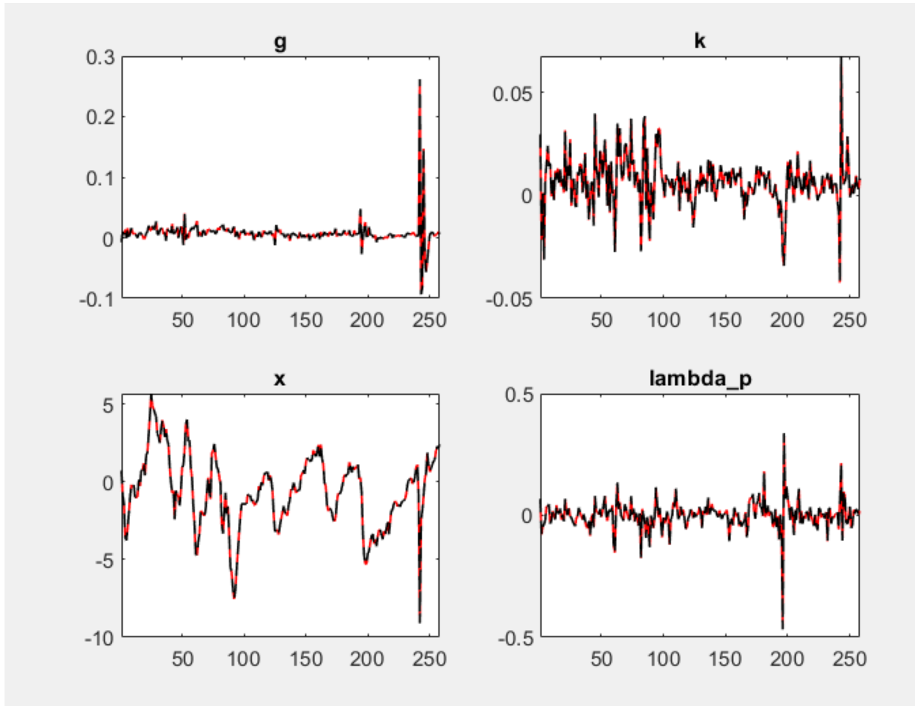

Figure 12: Prior and posterior distributions moments.

The autoregressive coefficient in the monetary policy rule is the only significant difference between the new parameter estimates produced by the Bayesian estimation and those in the original publication. This coefficient drops dramatically from its previous value of 0.8 to 0.2 in the updated estimation. This is a significant difference since I discovered a value of 0.2 for this parameter, but Justiniano et al. (2010) calculated its posterior value to be 0.6. Consequently, the estimation of monetary policy is greatly impacted by changes in the exogenous variables.

5 Conclusion

This paper presents a new identification strategy for the Bayesian estimation of the model used in Justiniano et al. (2010). I propose a new set of exogenous variables to estimate the same parameters and, finally, to analyze the differences. I used a different set of exogenous variables for the estimation: the output gap, public spending, capital, and market price markups. In addition, I consider the theoretical model from the paper Justiniano et al. (2010). The solution was stable and unique; nevertheless, there is still a region of indeterminacy and no stable solution for some sets of parameters. However, I found a unique solution for a set of parameter values, as presented in Graph 8. There is evidence of good identification, even when using these four exogenous and non-conventional variables.

The four exogenous time series included in the estimation were able to find the final solution of the model and, in addition, created some variation in the estimated values of the parameters with respect to the original paper. This coefficient drops dramatically from its previous value of 0.8 to 0.2 in the current calculation. This is a significant difference, as I found a value of 0.2 for this parameter, while Justiniano et al. (2010) estimated its posterior value to be 0.6. Consequently, the estimation of monetary policy is greatly impacted by changes in the exogenous variables.

In conclusion, the updated estimator reveals a significant change in the monetary policy rule, which leads to a distinct response pattern and noticeably different outcomes when examining the effects of a monetary policy shock. In particular, this estimation results in a decreased persistence parameter, which measures the degree to which the present interest rate depends on its historical value. As a result, the central bank responds more forcefully to shifts in the production gap or inflation, speeding up the interest rate's recovery from a shock.

References

Benhabib, J., Schmitt-Grohé, S., and Uribe, M. (2001). The perils of taylor rules. *Journal of Economic Theory*, 96(1-2):40–69.

Beyer, A. and Farmer, R. E. A. (2007). Testing for indeterminacy: An application to us monetary policy: Comment. *American Economic Review*, 97(1):524–529.

Canova, F., Ferroni, F., and Matthes, C. (2014). Choosing the variables to estimate singular dsge models. *Journal of Applied Econometrics*, 29(7):1099–1117.

Canova, F. and Sala, L. (2009). Back to square one: Identification issues in dsge models. *Journal of Monetary Economics*, 56(4):431–449.

Cochrane, J. H. (2011). Determinacy and identification with taylor rules. *Journal of Political economy*, 119(3):565–615.

Consolo, A., Favero, C. A., and Paccagnini, A. (2009). On the statistical identification of dsge models. *Journal of Econometrics*, 150(1):99–115.

Del Negro, M. and Schorfheide, F. (2008). Forming priors for dsge models (and how it affects the assessment of nominal rigidities). *Journal of Monetary Economics*, 55(7):1191–1208.

Iskrev, N. (2010). Local identification in dsge models. *Journal of Monetary Economics*, 57(2):189–202.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2010). Investment shocks and business cycles. *Journal of Monetary Economics*, 57(2):132–145.

Kocięcki, A. and Kolasa, M. (2018). Global identification of linearized dsge models. *Quantitative Economics*, 9(3):1243–1263.

Komunjer, I. and Ng, S. (2011). Dynamic identification of dynamic stochastic general equilibrium models. *Econometrica*, 79(6):1995–2032.

Qu, Z. and Tkachenko, D. (2012). Identification and frequency domain quasi-maximum likelihood estimation of linearized dynamic stochastic general equilibrium models. *Quantitative Economics*, 3(1):95–132.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American economic review*, 97(3):586–606.

Stock, J. H. and Watson, M. W. (2007). Econometrics.