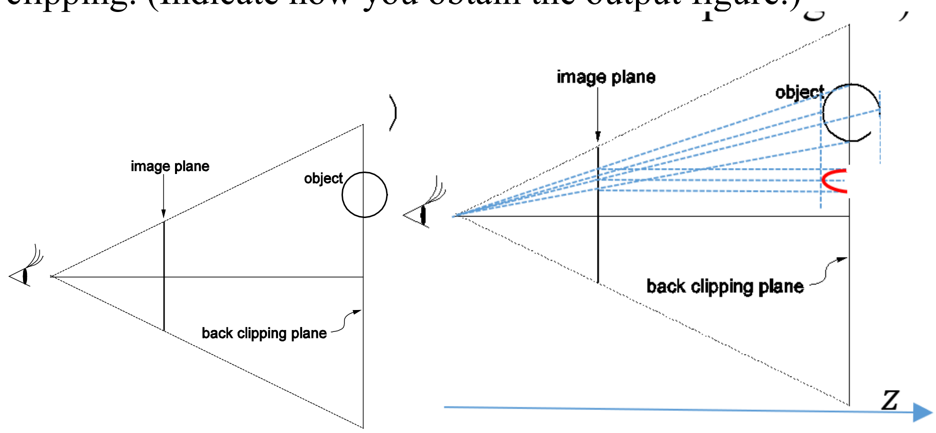
1. In the lecture, we have learned the line clipping algorithm, in which the clipping region is a rectangular in 2D space. Please show the **main** idea for extending this algorithm in 3D space, i.e., clipping a line against a 3D cubic. [15 points]

There are 6 borders (i.e., the 6 faces of the 3D cube), top, bottom, left, right, front, back, 27 sub-regions, and thus the **6-bit** code-words should be defined. Define the trivial accept and trivial reject criteria

2. Show the output of the following object after perspective **transformation** and clipping. (Indicate how you obtain the output figure.) [8 points]



Perspective transformation is computed as:

$x' = d * x / z$

$y' = d * y / z$

$z' = z$

The key point of this question is that the z dimension will retain after the perspective **transformation**, which is different from perspective **projection**

3. A 3D object is rotated by 90 degrees about an axis passing from (1, 0, 1) to (1, 2, 1). It is then uniformly scaled 4 times relative to the origin. Write in their proper order the individual matrices composing this transformation. [12 points]

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

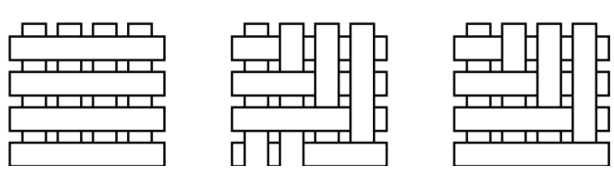
4. Describe what transformation the matrix **M** performs when applied to 3D points. [10 points]

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

First rotate 90 degree around the x-axis and then perform the non-uniform scaling around the x, y and z-axes with factors 1, 2, and 3, respectively.

5. Which of the following scenes in Figure 3 would cause problems for the painter's algorithm? Please justify your answer and suggest a method to



(a) (b) (c)

address the issue. (These drawing are in image space; each rectangle is a single primitive) [6 points]

(b) there is cyclic overlapping. We can use z-buffering algorithm.

6. Do parallel lines remain parallel in the axonometric projection? Justify your answer. [6 points]

yes

axonometric projection is parallel projection.

7. Please explain the **main** differences between Gouraud shading and Phone shading and show their advantages and disadvantages. [12 points]

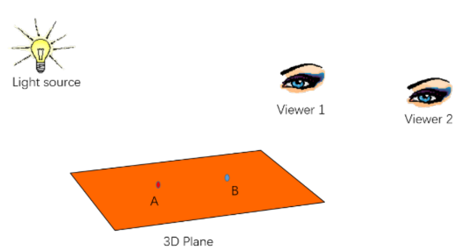
Gouraud shading: interpolate colors; difficult to handle specular reflection.

Phong shading: interpolate normal; render more realistic images but higher computational complexity

8. List all stages involved in the rendering pipeline. [8 points]

(1) world coordinate transformation
(2) perspective transformation
(3) back-face removal
(4) clipping
(5) rasterization
(6) hidden surface removal
(7) shading

9. As shown in the right figure,
(a) Assume the 3D plane has the uniform diffuse reflection property. If the light source is a directional light source, what is relationship between the diffuse reflections at points A and B? Please justify your answer [4 points]



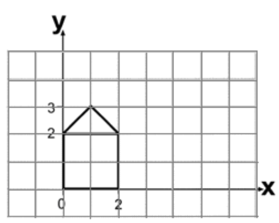
Let I_d^A and I_d^B be the diffuse reflections at points A and B, respectively. Let θ_A and θ_B are the incident angles of the light rays at points A and B, i.e., the angle between the light ray and the surface normal vector, respectively.
As it is a directional light source, the incoming light rays are parallel, i.e., θ_A=θ_B. Based on the formula of computing diffusion reflection, we can know $I_d^A = I_d^B$

(b) Assume the 3D plane has the uniform diffuse reflection property. If the light source is a point light source, what is relationship between the diffuse reflections at points A and B? Please justify your answer [4 points]

As it is a point light source, the incoming light rays are divergent, i.e., the incident angles of the light rays at points A and B are different. If your answer is I_d^A> I_d^B, your justification should be θ_A< θ_B; if your answer is I_d^A< I_d^B, your justification should be θ_A> θ_B.

(c) Assume the 3D plane is a glossy surface. If the light source is a point light source, what is the relationship between the specular reflections at point B observed by viewers 1 and 2? Please justify your answer. [7 points]

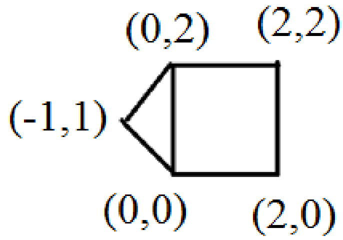
Let I_s^1 and I_s^2 be the specular reflections at viewpoints 1 and 2, respectively. Let θ_1 be the angle between the ideal reflected ray and viewing direction towards Viewer 1, and θ_2 the angle between the ideal reflected ray and viewing direction towards Viewer 1. If your answer is I_s^1>I_s^2, your justification is θ_1<θ_2. If your answer is I_s^1<I_s^2, your justification is θ_1>θ_2.
10. The following figure shows a 2D house model and a 2D affine transformation matrix $\boldsymbol{M}$. Sketch the transformed house and label the coordinates of all vertices. [8 points]



$$\boldsymbol{M} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Observe that $\boldsymbol{M} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So this transformation first rotates the house 90 degree around the origin and then moves it 2 units along the x-axis.



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