

Source Location

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Scalar and vector field

A function of space is known as a field. Let an arbitrary 3-D coordinate system be given.

Scalar field

If to each position $x = (x_1, x_2, x_3)$ of a region in space, it corresponds a scalar $\phi(x_1, x_2, x_3)$, then ϕ is called a *scalar field*. Like *density* field.

Vector field

If to each position $x = (x_1, x_2, x_3)$ of a region in space, it corresponds a vector $\vec{a}(x_1, x_2, x_3)$, then \vec{a} is called a *vector field*. Like *velocity* field.

How does scalar or vector field change along spatial dimensions?

Scalar and vector field

An example of scalar (*temperature*) and vector (*heat transfer*) field.

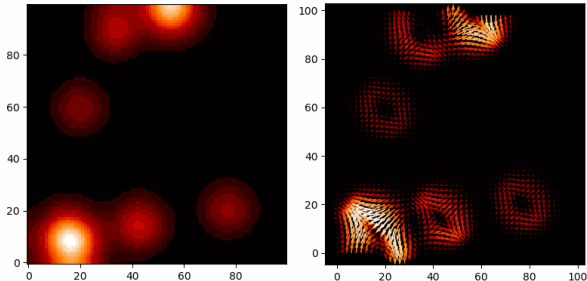


Figure: Heat Map. left: temperature, right: heat transfer

Nabla operator ∇

Partial derivatives is an useful tool to measure the changes along spatial dimensions.

For scalar field

$$\partial_i \phi = \frac{\partial}{\partial x_i} \phi \quad (1)$$

For vector field

$$(\partial_i \vec{a})(\vec{r}) = \lim_{\Delta x_i \rightarrow 0} \frac{\vec{a}(\vec{r} + \Delta x_i \vec{e}_i) - \vec{a}(\vec{r})}{\Delta x_i} \quad (2)$$

To simplify the partial derivatives, we imply *Nabla operator*.

Nabla operator

$$\nabla(\cdot) = \sum_i \vec{e}_i \partial_i(\cdot) \quad (3)$$

Gradient, divergence, curl

Gradient

$$\text{grad}\phi = \nabla\phi = \sum_i \vec{e}_i \partial_i \phi \quad (4)$$

Divergence

$$\text{div}\vec{a} = \nabla\vec{a} = \sum_i \partial_i \vec{a} \quad (5)$$

Curl

$$\text{curl}\vec{a} = \nabla \times \vec{a} = \det \begin{pmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \partial_1 & \partial_2 & \partial_3 \\ \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \end{pmatrix} \quad (6)$$

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Definition

Maxwell Equations describe the dynamic of Electromagnetic Wave.

$$\nabla \cdot \vec{D} = \rho \quad (7)$$

$$\nabla \cdot \vec{B} = 0 \quad (8)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (9)$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \quad (10)$$

Table: Meaning

\vec{D}	Electric Flux Density
\vec{B}	Magnetic Flux Density
\vec{E}	Electric Field
\vec{H}	Magnetic Field
\vec{J}	Current Density
ρ	Electric Charge Density

where $\vec{B} = \mu \vec{H}$, $\vec{D} = \epsilon \vec{E}$.

Explaining

The meaning of Maxwell equations is quite intuitive

- Electric Charge Density produces Electric Flux Density (7)
- Magnetic Flux Density is non-source field (8)
- Changing Magnetic Flux Density produces Electric Field (9)
- Changing Electric Flux Density or Current Density produces Magnetic Field (10)

Example

In vacuum space, the Maxwell Equations are re-written as

$$\nabla \cdot \vec{E} = 0 \quad (11)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (12)$$

$$\nabla \cdot \vec{B} = 0 \quad (13)$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (14)$$

Example

\vec{E} and \vec{B} is almost symmetry.

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} \quad (15)$$

$$\nabla \times (\nabla \times \vec{B}) = -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{B} \quad (16)$$

use

$$\nabla \times (\nabla \times \vec{X}) = \nabla \cdot (\nabla \cdot \vec{X}) - \nabla^2 \vec{X} \quad (17)$$

we have

$$\nabla^2 \vec{X} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{X} \quad (18)$$

Solve the wave function, the wave speed is $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$.¹


¹ $\mu_0 = 8.854187817 \times 10^{-12} \text{ F/m}$, $\epsilon_0 = 4\pi \times 10^{-7} \text{ N/A}^2$ 

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Forward process

Volume conductor

- Scalp
- Skull
- CSF
- Grey matter
- White matter
- Air pockets
- Conductivity tensor

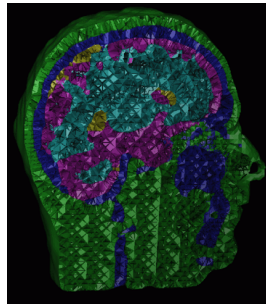


Figure: Skull

Forward process

Source model

- Neurons – brain cells, building blocks
- Pyramidal cells 10^{-5} per mm in cortex
- Send electric impulse (action potential 20fA per synapse)
- 40mm^2 of active cortex
- Current $I_0 = 10\text{nA}$
- Source decays $d = 0.1\text{mm}$
- Current dipole $P = I_0 * d$

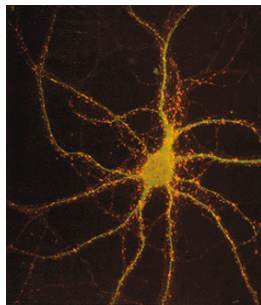


Figure: Source model

Forward process

Physical model

- Head is a volume conductor
- Concentrated current source (intra-cellular currents)
- Passive return volume currents (extra-cellular currents)

$$\vec{J} = \vec{J}_i + \vec{J}_e$$

- No flux boundary conditions

$$(\vec{J}_1 - \vec{J}_2) \cdot \vec{n} = 0$$

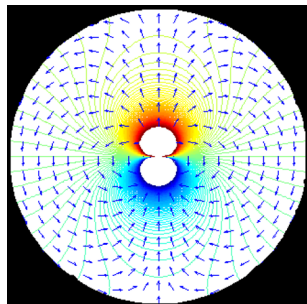


Figure: Physical model

Forward process

Forward problem solution

- Estimate the potential on the skull surface
- Knowing the currents beneath the skull

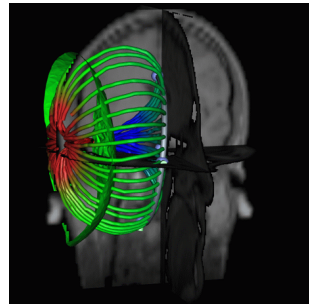


Figure: Forward solution

Mathematical model

Poisson equation

$$\nabla \cdot (\sigma \nabla \Phi) = \sum \vec{l}_s, \vec{l}_s \text{ in } \Omega \quad (19)$$

Neumann boundary conditions

$$\sigma(\nabla \Phi) \cdot n = 0 \quad (20)$$

Dipole current source

$$I_s(\vec{r}) = \lim_{d \rightarrow 0} \vec{l}_0 \left(\delta(\vec{r} - \vec{r}_s - \frac{d}{2}) - \delta(\vec{r} - \vec{r}_s + \frac{d}{2}) \right) \quad (21)$$

Forward estimation

The forward solution is exemplified as following

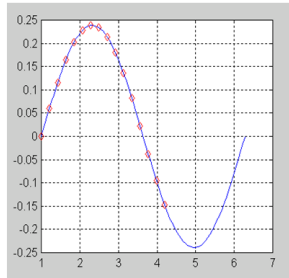
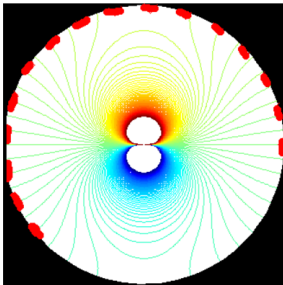


Figure: Forward estimation

Backward process

The reverse process of forward process is Backward process.
EEG measures the potential on the skull surface

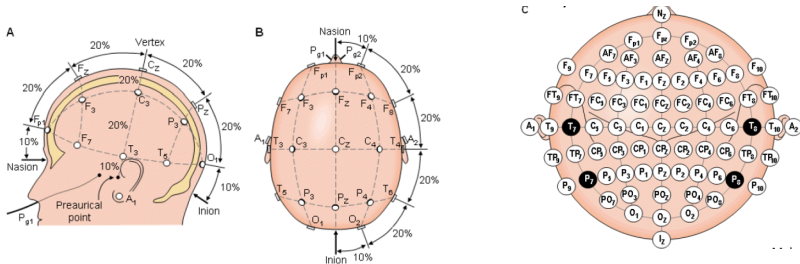


Figure: EEG hat

Figure: EEG layout

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Co-register

The first thing is to co-register the two coordinate

- The cortex coordinate, where the source currents are
 - Sphere model
 - Structure MRI model
 - ...
- The EEG cap coordinate, where the EEG sensors are

Gain matrix

Calculate a *Gain Matrix* using the forward model

$$\text{GainMatrix} \quad G \in \mathbb{R}^{m,n} \quad (22)$$

where m is the number of EEG sensors, n is the number of source currents.

$G_{i,j}$ refers the \vec{E} detected by i^{th} EEG sensor caused by j^{th} source current.

In another word, the EEG signal were considered as a *linear combination* of source currents.

Solution

The *Source location* problem can be formulated as

$$\arg \min_{\vec{W}} \left\| \vec{Y} - G \cdot \vec{W} \right\|_2^2 + \text{loss}(\vec{W}) \quad (23)$$

where $\vec{Y} \in \Re^{m,1}$ is the EEG signal, $\vec{W} \in \Re^{n,1}$ is the strong of source currents.

The problem is \vec{G} has more rows than columns, it makes the solution can be variable.

So, using proper *loss* function, like *sparse loss* is necessary.