Maxwell Equations

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Scalar and vector field

A function of space is known as a field. Let an arbitrary 3-D coordinate system be given.

Scalar field

If to each position $x = (x_1, x_2, x_3)$ of a region in space, it corresponds a scalar $\phi(x_1, x_2, x_3)$, then ϕ is called a *scalar field*. Like *density* field.

Vector field

If to each position $x = (x_1, x_2, x_3)$ of a region in space, it corresponds a vector $\vec{a}(x_1, x_2, x_3)$, then $\vec{a}$ is called a *vector field*. Like *velocity* field.

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How does scalar or vector field change along spatial dimensions?

Scalar and vector field

An example

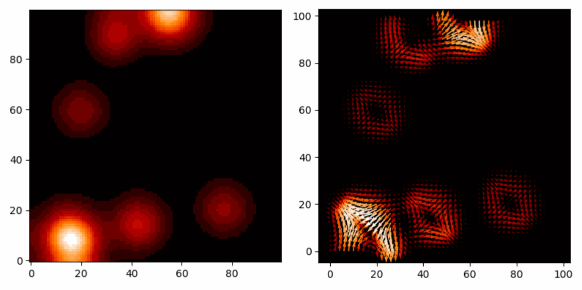

Figure: Heat Map

Nabla operator ∇

Partial derivatives is an useful tool to measure the changes along spatial dimensions.

For scalar field

$$\partial_i \phi = \frac{\partial}{\partial x_i} \phi \tag{1}$$

For vector field

$$(\partial_i \vec{a})(\vec{r}) = \lim_{\Delta x_i \to 0} \frac{\vec{a}(\vec{r} + \Delta x_i \vec{e}_i) - \vec{a}(\vec{r})}{\Delta x_i} \tag{2}$$

To simplify the partial derivatives, we imply *Nabla operator*.

Nabla operator

$$\nabla(\cdot) = \sum_i \vec{e}_i \partial_i (\cdot) \tag{3}$$

Gradient, diverence, curl

Gradient

$$grad\phi = \nabla\phi = \sum_i \vec{e}_i \partial_i \phi \tag{4}$$

Divergence

$$div\vec{a} = \nabla\vec{a} = \sum_i \partial_i \vec{a} \tag{5}$$

Curl

$$curl\vec{a} = \nabla \times \vec{a} = det \begin{pmatrix} \vec{e}_1 & \vec{e}_1 & \vec{e}_1 \\ \partial_1 & \partial_1 & \partial_1 \\ \vec{a}_1 & \vec{a}_1 & \vec{a}_1 \end{pmatrix} \tag{6}$$

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Definition

Maxwell Equations describe the dynamic of Electromagnetic Wave.

$$\nabla \cdot \vec{D} = \rho \tag{7}$$

$$\nabla \cdot \vec{B} = 0 \tag{8}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{9}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \tag{10}$$

Table: Meaning

$\vec{D}$	Electric Flux Density
$\vec{B}$	Magnetic Flux Density
$\vec{E}$	Electric Field
$\vec{H}$	Magnetic Field
$\vec{J}$	Current Density
ρ	Electric Charge Density

where $\vec{B} = \mu \vec{H}$, $\vec{D} = \epsilon \vec{E}$.

Example

In vacuum space, the Maxwell Equations are re-written as

$$\nabla \cdot \vec{E} = 0 \tag{11}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{12}$$

$$\nabla \cdot \vec{B} = 0 \tag{13}$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{14}$$

Example

$\vec{E}$ and $\vec{B}$ is almost symmetry.

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} \tag{15}$$

$$\nabla \times (\nabla \times \vec{B}) = -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{B} \tag{16}$$

use

$$\nabla \times (\nabla \times \vec{X}) = \nabla \cdot (\nabla \cdot \vec{X}) - \nabla^2 \vec{X} \tag{17}$$

we have

$$\nabla^2 \vec{X} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{X} \tag{18}$$

Solve the wave function, the wave speed is $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$.[1]

[1] $\mu_0 = 8.854187817 \times 10^{-12} F/m$, $\epsilon_0 = 4\pi \times 10^{-7} N/A^2$

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Forward process

Volume conductor

- Scalp
- Skull
- CSF
- Grey matter
- White matter
- Air pockets
- Conductivity tensor

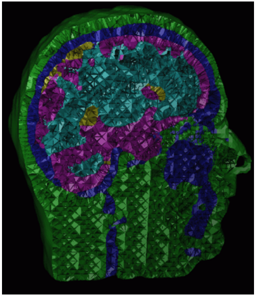

Figure: Skull

Forward process

Source model

- Neurons – brain cells, building blocks
- Pyramidal cells $10^{-5} permm$ in cortex
- Send electric impulse (action potential $20fA$ per synapse)
- $40mm^2$ of active cortex
- Current $I_0 = 10nA$
- Source decays $d = 0.1mm$
- Current dipole $P = I0 * d$

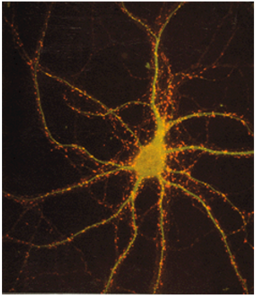

Figure: Source model

Forward process

Physical model

- Head is a volume conductor
- Concentrated current source (intra-cellular currents)
- Passive return volume currents (extra-cellular currents)

$$\vec{J} = \vec{J}_i + \vec{J}_e$$

- No flux boundary conditions

$$(\vec{J}_1 - \vec{J}_2) \cdot \vec{n} = 0$$

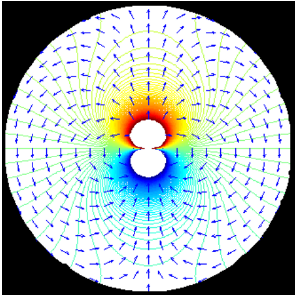

Figure: Physical model

Forward process

Forward problem solution

- Estimate the potential on the skull surface
- Knowing the currents beneath the skull

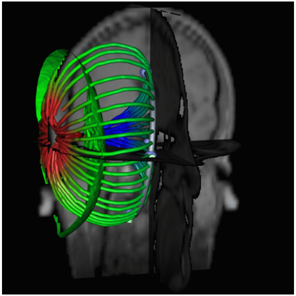

Figure: Forward solution

Mathematical model

Poisson equation

$$\nabla \cdot (\sigma \nabla \Phi) = \sum \vec{I_s}, \vec{I_s} in \Omega \tag{19}$$

Neumann boundary conditions

$$\sigma(\nabla \Phi) \cdot n = 0 \tag{20}$$

Dipole current source

$$I_s(\vec{r}) = \lim_{d \to 0} \vec{I_0}(\delta(\vec{r} - \vec{r_s} - \frac{d}{2}) - \delta(\vec{r} - \vec{r_s} + \frac{d}{2})) \tag{21}$$

Forward estimation

The forward solution is exampled as following

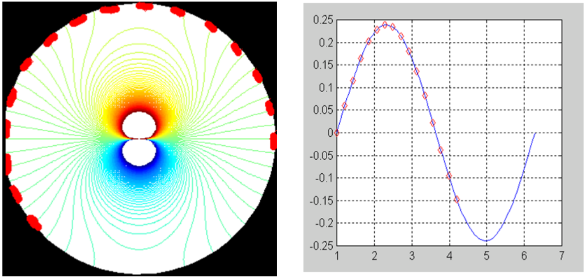

Figure: Forward estimation

Backward process

The reverse process of forward process is Backward process.
EEG measures the potential on the skull surface

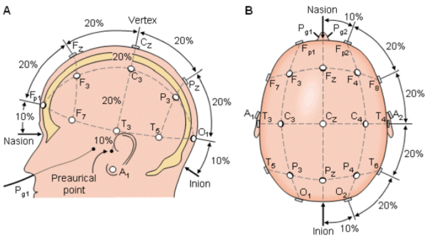

Figure: EEG hat

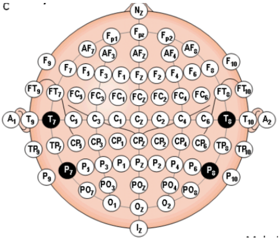

Figure: EEG layout

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Formulation

Solution

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Items

- Text visible on slide 1

Items

- Text visible on slide 1
- Text visible on slide 2

Items

- Text visible on slide 1
- Text visible on slide 2
- Text visible on slide 3

Items

- Text visible on slide 1
- Text visible on slide 2
- Text visible on slide 4

Pause

In this slide

Pause

In this slide the text will be partially visible

Pause

In this slide the text will be partially visible And finally everything will be there

Sample frame title

In this slide, some important text will be highlighted because it's important. Please, don't abuse it.

Remark

Sample text

Important theorem

Sample text in red box

Examples

Sample text in green box. The title of the block is "Examples".

Two-column slide

This is a text in first column.

$$E = mc^2$$

- First item
- Second item

This text will be in the second column and on a second through this is a nice looking layout in some cases.