PREFACE

Richard von Mises' scientific work comprises two major fields: mechanics and probability-statistics; supplemented by numerical analysis, geometry, and philosophy of science they form the leading interests of his scientific life.

In 1931 he published a comprehensive textbook on probability consisting of four parts: the foundations (his frequency theory); limit theorems; statistics and theory of errors; and statistical problems in physics. While in the United States (1939–1953) he became deeply interested in "British-American" statistics (as did his former student A. Wald). Mises' aim was to understand this approach to statistical inference as a part of rigorous probability theory, its application. The results of his thinking were incorporated in Lectures on Probability and Statistics he gave repeatedly at Harvard University to advanced undergraduate and graduate students, and in lectures he gave in Rome (1951–1952) and finally in Zurich (summer 1952).

The Harvard Lectures were mimeographed. Brief but clear notes of the Zurich Lectures were kindly given to me after Mises' death by K. Schoeni, who attended them. Mises had planned to incorporate the various ideas in a comprehensive work on probability and statistics which, more than 20 years after his Wahrscheinlichkeitsrechnung, would have been a very different work, in many respects.

The present book is based on the material mentioned above as well as his papers and notebooks. It presents a unified mathematical theory of probability and statistics. In fact, for Mises there were never two different theories, one "pure" the other "applied," but *one* theory only, a frequency theory, mathematically rigorous and guided by an operational approach.

The fundamentals are presented in Chapters I and II. The mathematical foundations of a subject like probability can be laid in at least two ways. On the one hand one may decide to axiomatize the mathematics of probability, the connections to experience being left to the user. Of necessity, the basic field of such a theory will be to some extent indeterminate. On the other hand, one may wish to reproduce mathematically certain idealized experiences by formulating basic experience in a way that is reasonably realistic yet precise enough to provide the point of departure for the theoretical analysis of the subject. Either approach may one day prove too narrow or too general. To quote the later Bridgman, not the youthful all-out operationalist, "How could I confidently expect to exhaust the possibilities of a subject and to eliminate the possibility of a bright new idea against which I would be defenseless?" Indeed, whatever we propose relates to a certain state of our knowledge, factual knowledge as well as epistemology.

The frequency theory of objective probability presented here adopts the second approach (objective in contrast to subjective or personal). Chapter I gives the theory of discrete label spaces S_n. To each of countably many points corresponds a "probability" p_i with $\Sigma p_i = 1$. The collective forms a suggestive model; its consistency is proved in Appendix One. Probability is defined over the σ-field $\mathscr{S}_n$ of all subsets of S_n and, using the frequency definition, we *prove* that p is a σ-additive set function over $\mathscr{S}_n$. The aim of Chapter II, which is essentially new, is to derive the most general field F_1 of a frequency theory of probability, and this is done by explicit construction with $\mathscr{S}_n$ as starting point. The resulting field is different from Kolmogorov's; the probabilty over F_1 is completely additive but F_1 is not a σ-field; the probability of any set of F_1 admits a frequency interpretation while a Kolmogorov field will in general contain sets with no conceivable relation to observation. We think that this foundation which follows Mises' ideas is a rigorous mathematical construction; it is certainly very simple. Results and ideas of E. Tornier and A. Wald have helped decisively (Appendices One and Three).

Chapters III–VI contain standard material in probability theory (with some perhaps less familiar applications) presented from our standpoint. In the original draft there followed a long chapter on Markov Chains and Stochastic Processes. Upon the publisher's request to shorten the manuscript this material was (regretfully) omitted except for a few elementary explanations at the end of Chapter IV and in Appendix Four. Chapter VII introduces Bayesian inference (for Mises the basic tool of theoretical statistics) with some generality. This chapter provides preparation for Chapter X which formed the main subject of the Rome and Zurich lectures; it deals with Neyman-Pearson theory, confidence intervals, estimation, etc., in an original manner. Chapter VIII contains more on distributions including some elementary parts of the problem of moments. It is followed up by Chapter IX which develops among other things the chi-square method with unknown parameters, the omega-square method, and various deviation tests (Kolmogorov, Smirnov). Chapter XI introduces correlation theory, and the text concludes (Chapter XII) with Mises' far-reaching theory of Statistical Functions on which he had worked since 1936 and particularly during the last years of his life. The book has been written as an advanced textbook in the hope that both students and research workers will find it useful.

I am greatly indebted to many people for their interest and help. The crucial second chapter could not have been written without a study of E. Tornier's work, supplemented by discussions in an extensive correspondence. John Pratt read the entire manuscript and suggested important improvements. Keewhan Choi, who also read the entire manuscript, made many useful suggestions. A. B. Wilson became interested in Chapter I and made several contributions. W. Hoeffding read critically Chapters IX, X, and XII. I am, however, solely responsible for any remaining mistakes and for the content of the book as a whole.

The index was prepared by Keewhan Choi whom I thank cordially. He and Stephen Gill checked all problems independently. Thanks are also due to Hanna Szoeke and Stephen Gill who rendered valuable help in the editing of the manuscript.

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