$$a_b \equiv \frac{\partial a}{\partial b}$$

$$x_t = \frac{\partial x}{\partial t} = u$$

$$\mathbf{A} \equiv \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} \text{Continuity} \\ \text{x-Momentum} \\ \text{y-Momentum} \\ \text{z-Momentum} \\ \text{Energy} \end{bmatrix}$$

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{\partial \mathbf{G}}{\partial z} = 0$$

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \end{aligned}$$

$$\begin{aligned} x &= \rho\sin\theta\cos\phi \\ y &= \rho\sin\theta\sin\phi \\ z &= \rho\cos\theta \end{aligned}$$

$$\begin{aligned} x &= x \\ y &= y_c \left\{ 1 + \frac{\sinh[\tau(\bar{y} - B)]}{\sinh(\tau B)} \right\} \\ z &= z \end{aligned}$$

$$\text{where} \quad B = \frac{1}{2\tau} \ln\left[\frac{1 + (\exp^{\tau} - 1)(y_c/h)}{1 + (\exp^{-\tau} - 1)(y_c/h)}\right] \quad 0 < \tau < \infty$$

$$
\begin{aligned}
x &= x(\xi, \eta, \zeta) \\
y &= y(\xi, \eta, \zeta) \\
z &= z(\xi, \eta, \zeta)
\end{aligned}
$$

$$
\frac{\partial \mathbf{U}_1}{\partial t} + \frac{\partial \mathbf{E}_1}{\partial \xi} + \frac{\partial \mathbf{F}_1}{\partial \eta} + \frac{\partial \mathbf{G}_1}{\partial \zeta} = 0
$$

$$
\begin{aligned}
\mathbf{U}_1 &= \frac{\mathbf{U}}{J} \\
\mathbf{E}_1 &= \frac{1}{J}\left(\mathbf{E}\xi_x + \mathbf{F}\xi_y + \mathbf{G}\xi_z\right) \\
\mathbf{F}_1 &= \frac{1}{J}\left(\mathbf{E}\eta_x + \mathbf{F}\eta_y + \mathbf{G}\eta_z\right) \\
\mathbf{G}_1 &= \frac{1}{J}\left(\mathbf{E}\zeta_x + \mathbf{F}\zeta_y + \mathbf{G}\zeta_z\right)
\end{aligned}
$$

$$
\begin{aligned}
J &= \frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)} = \begin{vmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \zeta_x & \zeta_y & \zeta_z \end{vmatrix} \\
&= \xi_x(\eta_y\zeta_z - \eta_z\zeta_y) - \xi_y(\eta_x\zeta_z - \eta_z\zeta_x) + \xi_z(\eta_x\zeta_y - \eta_y\zeta_x)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{E} &= (\mathbf{E_i} - \mathbf{E_v}) \\
\mathbf{F} &= (\mathbf{F_i} - \mathbf{F_v}) \\
\mathbf{G} &= (\mathbf{G_i} - \mathbf{G_v})
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{\mathbf{U}}{J}\right) + \\
&\frac{\partial}{\partial \xi}\left\{\frac{1}{J}\left[\xi_x\left(\mathbf{E_i} - \mathbf{E_v}\right) + \xi_y\left(\mathbf{F_i} - \mathbf{F_v}\right) + \xi_z\left(\mathbf{G_i} - \mathbf{G_v}\right)\right]\right\} + \\
&\frac{\partial}{\partial \eta}\left\{\frac{1}{J}\left[\eta_x\left(\mathbf{E_i} - \mathbf{E_v}\right) + \eta_y\left(\mathbf{F_i} - \mathbf{F_v}\right) + \eta_z\left(\mathbf{G_i} - \mathbf{G_v}\right)\right]\right\} + \\
&\frac{\partial}{\partial \zeta}\left\{\frac{1}{J}\left[\zeta_x\left(\mathbf{E_i} - \mathbf{E_v}\right) + \zeta_y\left(\mathbf{F_i} - \mathbf{F_v}\right) + \zeta_z\left(\mathbf{G_i} - \mathbf{G_v}\right)\right]\right\} = 0
\end{aligned}
$$

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E_t \end{bmatrix}$$

$$E_t = \rho \left(e + \frac{u^2 + v^2 + w^2}{2} \right)$$

$$p = \rho R T$$

$$(p + a\rho^2) \left(\frac{1}{\rho} - b \right) = RT$$

$$\mathbf{E_i} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ (E_t + p)u \end{bmatrix} \qquad \mathbf{F_i} = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \\ (E_t + p)v \end{bmatrix} \qquad \mathbf{G_i} = \begin{bmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + p \\ (E_t + p)w \end{bmatrix}$$

$$\mathbf{E_v} = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \\ u\tau_{xx} + v\tau_{xy} + w\tau_{xz} - q_x \end{bmatrix}$$

$$\mathbf{F_v} = \begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{yz} \\ u\tau_{xy} + v\tau_{yy} + w\tau_{yz} - q_y \end{bmatrix}$$

$$\mathbf{G_v} = \begin{bmatrix} 0 \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{zz} \\ u\tau_{xz} + v\tau_{yz} + w\tau_{zz} - q_z \end{bmatrix}$$

$$
\begin{aligned}
\tau_{xx} = \frac{2}{3}\mu\big[&2(\xi_x u_\xi + \eta_x u_\eta + \zeta_x u_\zeta) \\
&-(\xi_y v_\xi + \eta_y v_\eta + \zeta_y v_\zeta) \\
&-(\xi_z w)\xi + \eta_z w_\eta + \zeta_z w_\zeta)\big] \\
\tau_{yy} = \frac{2}{3}\mu\big[&2(\xi_y v_\xi + \eta_y v_\eta + \zeta_y v_\zeta) \\
&-(\xi_x u_\xi + \eta_x u_\eta + \zeta_z w_\zeta) \\
&-(\xi_z w)\xi + \eta_z w_\eta + \zeta_z w_\zeta)\big] \\
\tau_{zz} = \frac{2}{3}\mu\big[&2(\xi_z w_\xi + \eta_z w_\eta + \zeta_z w_\zeta) \\
&-(\xi_x u_\xi + \eta_x u_\eta + \zeta_z w_\zeta) \\
&-(\xi_y v_\xi + \eta_y v_\eta + \zeta_y v_\zeta)\big]
\end{aligned}
$$

$$
\begin{aligned}
\tau_{xy} &= \mu\left(\xi_y u_\xi + \eta_y u_\eta + \zeta_y u_\zeta + \xi_x v_\xi + \eta_x v_\eta + \zeta_x v_\zeta\right) \\
\tau_{xz} &= \mu\left(\xi_z u_\xi + \eta_z u_\eta + \zeta_z u_\zeta + \xi_x w_\xi + \eta_x w_\eta + \zeta_x w_\zeta\right) \\
\tau_{yz} &= \mu\left(\xi_z v_\xi + \eta_z v_\eta + \zeta_z v_\zeta + \xi_y w_\xi + \eta_y w_\eta + \zeta_y w_\zeta\right)
\end{aligned}
$$

$$
\begin{aligned}
q_x &= -k(\xi_x T_\xi + \eta_x T_\eta + \zeta_x T_\zeta) + \rho\sum_{i=1}^{n} c_i h_i U_{x,i} \\
q_y &= -k(\xi_y T_\xi + \eta_y T_\eta + \zeta_y T_\zeta) + \rho\sum_{i=1}^{n} c_i h_i U_{y,i} \\
q_z &= -k(\xi_z T_\xi + \eta_z T_\eta + \zeta_z T_\zeta) + \rho\sum_{i=1}^{n} c_i h_i U_{z,i}
\end{aligned}
$$

$$
\begin{aligned}
&u, v, w, p, T \\
&u, v, w, p, T \text{(and species concentration)}
\end{aligned}
$$

$$
\sum_{i=1}^{\text{points}} \left\{ \frac{\partial \mathbf{U}_1}{\partial t} + \frac{\partial \mathbf{E}_1}{\partial \xi} + \frac{\partial \mathbf{F}_1}{\partial \eta} + \frac{\partial \mathbf{G}_1}{\partial \zeta} = 0 \right\}
$$

$$
\int_0^T \left[\sum_{i=1}^{\text{points}} \left\{ \frac{\partial \mathbf{U}_1}{\partial t} + \frac{\partial \mathbf{E}_1}{\partial \xi} + \frac{\partial \mathbf{F}_1}{\partial \eta} + \frac{\partial \mathbf{G}_1}{\partial \zeta} = 0 \right\} \right]
$$