

# Linear Algebra

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## Question 1

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Let two matrices be

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 3 \\ 5 & 8 \end{bmatrix}$$

What is  $A + B$ ?

- ☒  $\begin{bmatrix} 1 & -1 \\ 3 & 9 \end{bmatrix}$
- ☐  $\begin{bmatrix} 1 & -1 \\ 7 & 9 \end{bmatrix}$
- ☐  $\begin{bmatrix} 1 & 7 \\ 7 & 9 \end{bmatrix}$
- ☐  $\begin{bmatrix} 1 & -7 \\ -7 & -7 \end{bmatrix}$

To add two matrices, add them element-wise.

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## Question 2

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Let  $x = \begin{bmatrix} 8 \\ 2 \\ 5 \\ 1 \end{bmatrix}$

What is  $2 * x$ ?

- ☐  $\begin{bmatrix} 4 \\ 1 \\ \frac{5}{2} \\ \frac{1}{2} \end{bmatrix}$
- ☒  $\begin{bmatrix} 16 \\ 4 \\ 10 \\ 2 \end{bmatrix}$
- ☐  $\begin{bmatrix} 4 & 1 & \frac{5}{2} & \frac{1}{2} \end{bmatrix}$
- ☐  $\begin{bmatrix} 16 & 4 & 10 & 2 \end{bmatrix}$

To multiply the vector  $x$  by 2, take each element of  $x$  and multiply that element by 2.

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## Question 3

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Let  $u$  be a 3-dimensional vector, where specifically

$$u = \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}$$

What is  $u^T$ ?

☐  $[8 \ 1 \ 2]$

☐  $\begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}$

☐  $\begin{bmatrix} 8 \\ 1 \\ 2 \end{bmatrix}$

☒  $[2 \ 1 \ 8]$

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## Question 4

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Let  $u$  and  $v$  be 3-dimensional vectors, where specifically

$$u = \begin{bmatrix} 4 \\ -4 \\ -3 \end{bmatrix}$$

and

$$v = \begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix}$$

What is  $u^T v$ ?

(Hint:  $u^T$  is a

$1 \times 3$  dimensional matrix, and  $v$  can also be seen as a  $3 \times 1$

matrix. The answer you want can be obtained by taking

the matrix product of  $u^T$  and  $v$ .) Do not add brackets to your answer.

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## Question 5

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Let  $A$  and  $B$  be  $3 \times 3$  (square) matrices. Which of the following

must necessarily hold true? Check all that apply.

- ☒ If  $B$  is the  $3 \times 3$  identity matrix, then  $A * B = B * A$

Even though matrix multiplication is not commutative in general ( $A * B \neq B * A$  for general matrices  $A, B$ ), for the special case where  $B = I$ , we have  $A * B = A * I = A$ , and also  $B * A = I * A = A$ . So  $A * B = B * A$ .

- ☐  $A * B = B * A$

- ☒ If  $C = B * A$ , then  $C$  is a  $3 \times 3$  matrix.

Since  $A$  and  $B$  are both  $3 \times 3$  matrices, their product is  $3 \times 3$ . More generally, if  $A$  were an  $m \times n$  matrix, and  $B$  a  $n \times o$  matrix, then  $C$  would be  $m \times o$ . (In our example,  $m = n = o = 3$ .)

- ☐  $A * B * A = B * A * B$