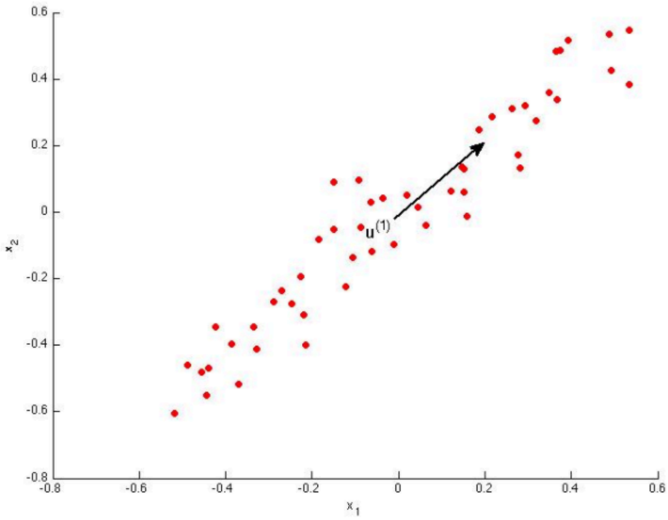
Principal Component Analysis

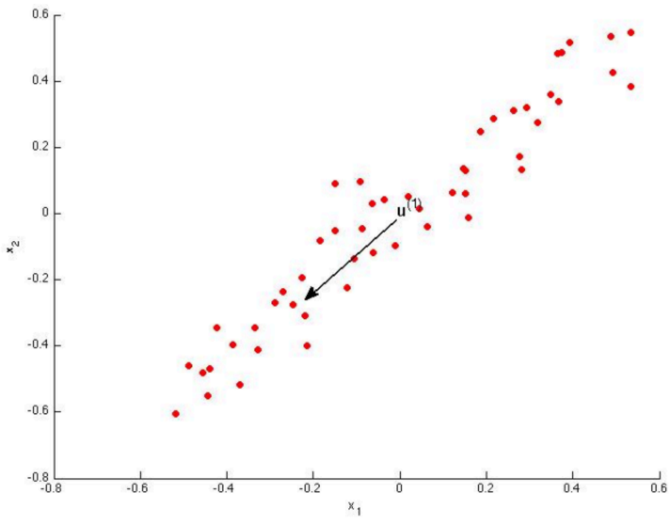
Question 1

Consider the following 2D dataset:

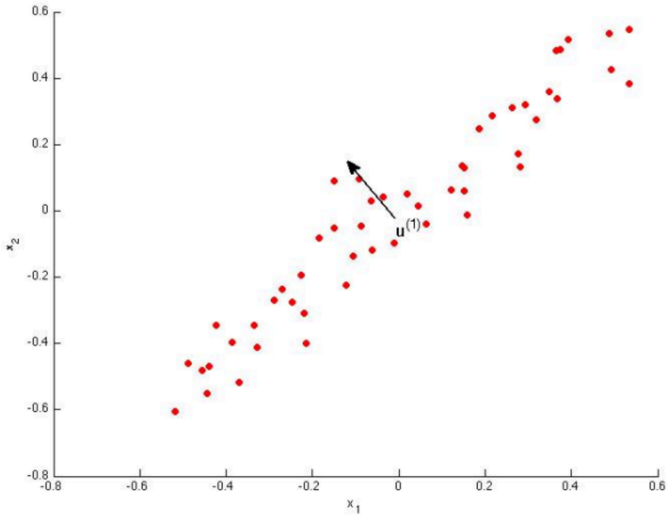
Which of the following figures correspond to possible values that PCA may return for $u^{(1)}$ (the first eigenvector / first principal component)? Check all that apply (you may have to check more than one figure).

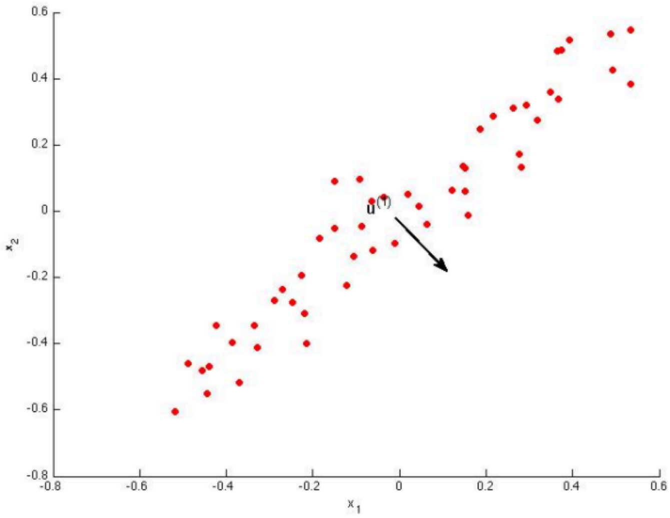
- [x] 

> The maximal variance is along the y = x line, so this option is correct.

- [x] 

> The maximal variance is along the y = x line, so the negative vector along that line is correct for the first principal component.

- [ ] 

- [ ] 

Question 2

Which of the following is a reasonable way to select the number of principal components k?

(Recall that n is the dimensionality of the input data and m is the number of input examples.)

- [ ] Choose the value of k that minimizes the approximation error $\frac{1}{m}\sum_{i=1}^{m}||x^{(i)} - x_{\text{approx}}^{(i)}||^2$.
- [x] **Choose k to be the smallest value so that at least 99% of the variance is retained.**

> This is correct, as it maintains the structure of the data while maximally reducing its dimension.

- [ ] Choose k to be the smallest value so that at least 1% of the variance is retained.
- [ ] Choose k to be 99% of n (i.e., $k = 0.99 * n$, rounded to the nearest integer).

Question 3

Suppose someone tells you that they ran PCA in such a way that "95% of the variance was retained." What is an equivalent statement to this?

- [ ] $\frac{\frac{1}{m}\sum_{i=1}^{m}||x^{(i)}-x_{\text{approx}}^{(i)}||^2}{\frac{1}{m}\sum_{i=1}^{m}||x^{(i)}||^2} \geq 0.05$
- [ ] $\frac{\frac{1}{m}\sum_{i=1}^{m}||x^{(i)}||^2}{\frac{1}{m}\sum_{i=1}^{m}||x^{(i)}-x_{\text{approx}}^{(i)}||^2} \geq 0.95$
- [ ] $\frac{\frac{1}{m}\sum_{i=1}^{m}||x^{(i)}-x_{\text{approx}}^{(i)}||^2}{\frac{1}{m}\sum_{i=1}^{m}||x^{(i)}||^2} \geq 0.95$
- [x] $\frac{\frac{1}{m}\sum_{i=1}^{m}||x^{(i)}-x_{\text{approx}}^{(i)}||^2}{\frac{1}{m}\sum_{i=1}^{m}||x^{(i)}||^2} \leq 0.05$

Question 4

Which of the following statements are true? Check all that apply.

- [x] **Given an input $x \in \mathbb{R}^n$, PCA compresses it to a lower-dimensional $z \in \mathbb{R}^k$.**

  > PCA compresses it to a lower dimensional vector by projecting it onto the learned principal components.

- [ ] Feature scaling is not useful for PCA, since the eigenvector calculation (such as using Octave's `svd(Sigma)` routine) takes care of this automatically.
- [ ] PCA can be used only to reduce the dimensionality of data by 1 (such as 3D to 2D, or 2D to 1D).
- [x] **If the input features are on very different scales, it is a good idea to perform feature scaling before applying PCA.**

  > Feature scaling prevents one feature dimension from becoming a strong principal component only because of the large magnitude of the feature values (as opposed to large variance on that dimension).

Question 5

Which of the following are recommended applications of PCA? Select all that apply.

- [x] **Data compression: Reduce the dimension of your data, so that it takes up less memory / disk space.**

  > If memory or disk space is limited, PCA allows you to save space in exchange for losing a little of the data's information. This can be a reasonable tradeoff.

- [x] **Data visualization: Reduce data to 2D (or 3D) so that it can be plotted.**

  > This is a good use of PCA, as it can give you intuition about your data that would otherwise be impossible to see.

- [ ] Preventing overfitting: Reduce the number of features (in a supervised learning problem), so that there are fewer parameters to learn.

- [ ] To get more features to feed into a learning algorithm.