DETERMINING THE WEATHER ON AN EXTRA-SOLAR PLANET

Sharif Khan-Bennett

Student at the School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham, B15 2TT, UK

Abstract

We present an analysis of Extra-Solar Planets (Exoplanets) to determine their albedos and hence provide predictions for their temperatures. The original, complete analysis took place with Kepler 2b, obtaining a temperature of 2149.22 ± 128.77K, which is within 0.4% error of previously quoted values.

Introduction

The data for this experiment was collected by the Kepler space telescope and compiled by NASA. The "STScI" database [1] which we used, contains records from over 4770 [2] exoplanets, gathered by Kepler since its launch in 2009 [3]. Kepler's field of view covers 0.035 steradians across the constellations of Cygnus, and Lyra [4], see figure 1.

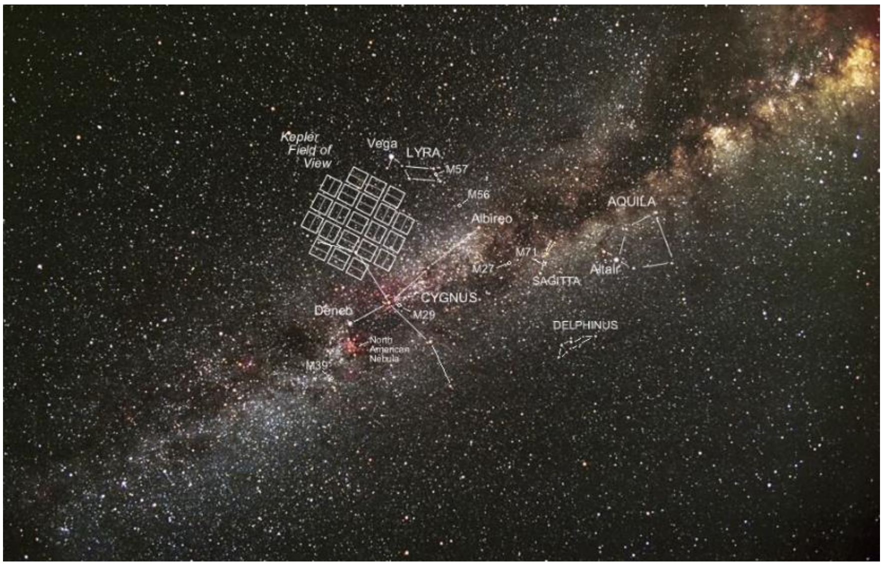

Figure 1: Field of View of Kepler [5].

We selected exoplanets for analysis based on which had been quoted from online sources to have relatively large albedos as this made prospective analysis easier. In particular, we were looking for "Hot Jupiters" as these are the planets which are the largest and therefore cause the most significant intensity change during transit. They also generally have short periods (<10 days) meaning that there would be plenty of data to examine, this will be discussed in further detail in the coding section. The Albedo is a ratio of light incident upon an Exoplanet, which is reflected; therefore, planets with larger albedos have whiter surfaces. In this report, the albedo will be used as a proxy for cloud abundance in the exoplanet's atmosphere. The albedo can also be introduced into the Stefan-Boltzmann's law to estimate the temperature on the exoplanet as a result of stellar irradiation.

Theory

The Kepler Space telescope utilises the transit method of observing exoplanets. This involves the continuous monitoring of the intensity received from a star. If such star is the host to a planetary system whose plane lies parallel to our line of sight, dips in intensity will be observed as the planet crosses between the star and the observer. These characteristic lightcurves reveal a wealth of information about the system. To predict the temperature on the exoplanet, its albedo must initially be found so that only the absorbed light is considered to heat the planet. The process of calculations leading to this has been outlined in the remainder of this section.

From the primary transit, that is the one in which the exoplanet passes between the star and observer, the radius of the exoplanet, r, can be determined. The squared ratio of radii equals the ratio of intensities before and during transit:

$$\left(\frac{\mathrm{r}}{\mathrm{R}}\right)^2 = \frac{\mathrm{I_E}}{\mathrm{I_s}} \tag{1}$$

where R is the stellar radius (taken from scientific literature,) $\mathrm{I_s}$ is the intensity of the star alone, and $\mathrm{I_E}$ is the intensity of the exoplanet alone, i.e. $\mathrm{I_E} = \mathrm{I_s} - \mathrm{T}$, where T is the intensity during transit. The square is present due to the intensity being proportional to the surface area of the bodies. While processing the data, the intensity of the star was normalised about one such that equation (1) reduces to:

$$\mathrm{r} = \mathrm{R}\sqrt{1 - \mathrm{T}}. \tag{2}$$

The overriding periodic nature of the transit dips reveals the orbital period of the exoplanet, P, this can be used in tandem with the host star's mass, M (taken from scientific literature) to find the orbital radius of the exoplanet, a, by use of a rearrangement of Kepler's third law. In this equation, the mass of the exoplanet has been neglected as it is generally 0.001% of the stellar mass.

$$\mathrm{a} = \sqrt[3]{\frac{\mathrm{GM}}{4\pi^2}\mathrm{P}^2}. \tag{3}$$

In addition to the primary transit, careful inspection of the light curve reveals a secondary dip in intensity half a period later. In fact, the intensity gradually rises between these two phenomena, towards what is called the secondary transit, in which the host star occults the exoplanet. Similar to the crescents of the moon, the exoplanet begins to reflect light towards us as it sweeps through the phase angles of one period, where maximum lies at $\theta \approx 0°$, equivalent to a full moon. Once the host star occults the exoplanet, this reflected light is blocked, causing the secondary transit. The proportion of light which is reflected off the exoplanet at a phase angle of zero describes the Geometric Albedo of the Exoplanet, $\mathrm{A_G}$.

$$\mathrm{A_G} = \frac{\mathrm{f_E}}{\mathrm{f_s}} \tag{4}$$

where f_s is the flux from the star, at a distance equal to the orbital radius. f_E is the flux from the exoplanet alone, found from the depth of the secondary transit, I_0, where the flux of the exoplanet is completely obscured, see figure 2 for an exaggerated illustration of this.

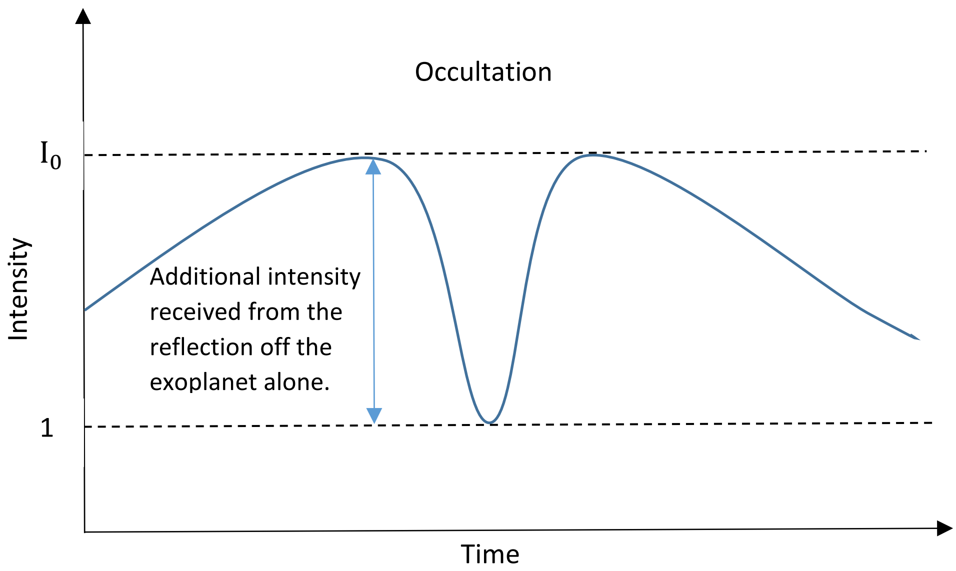

Figure 2: Exaggerated view of the secondary transit to show.

As a result, equation 4 can be written in terms of intensities as:

$$A_G = \frac{I_0}{r^2} \div \frac{I_s}{a^2} \tag{5}$$

which taking into account the normalisation of I_s, reduces to:

$$A_G = I_0 \left(\frac{a}{r}\right)^2. \tag{6}$$

The geometric albedo only describes the albedo during a very particular phase of the orbit, α. The geometric albedo refers to a phase angle of zero, while the exoplanet, host star and observer make a 180-degree angle such that the exoplanet is fully illuminated in the perspective of the observer. The Bond albedo, A_B, however, is constant, found from an average of all reflected intensities during half a period and hence is always smaller than the geometric albedo. It is found from the geometric albedo from;

$$A_B = qA_G \tag{7}$$

where q is the phase integral given by:

$$q = 2\int_0^{\pi} \frac{I(\alpha)}{I_s} \sin(\alpha)\, d\alpha. \tag{8}$$

In practice, this equation was reduced to the following sum:

$$q = 2\sum_{n}^{\pi} I(n)\sin(n) \tag{9}$$

where $n = 10\alpha$ to increase processing time.

In addition to the bond albedo, the luminosity of the star, L, is also required for calculation of the temperature by use of Stefan Boltzmann's law. This was found using the flux-magnitude relation in which Vega was taken as the reference due to its small magnitude, approximated as zero. The distance of each star from Earth was taken from online sources.

$$L = L_v\left(\frac{D}{D_v}\right)^2 10^{-\frac{m}{2.5}} \tag{10}$$

where L_v is the luminosity of vega, D is the distance to the host star, D_v is the distance to vega and m is the magnitude of the host star.

Knowledge of all these parameters allows for calculations of the temperature of the exoplanet by rearranging Stefan Boltzmann's Law.

$$Temp = \left[\frac{L(1 - A_B)}{4\pi\sigma a^2}\right]^{\frac{1}{4}} \tag{11}$$

where the factor of $1 - A_B$ has been included only to consider the light that has been absorbed by the exoplanet. But, in reality, the temperature on the exoplanet generally exceeds this due to greenhouse effects which are not taken into account by this equation due to its blackbody assumptions. See figure 3 for a flowchart of these calculations.

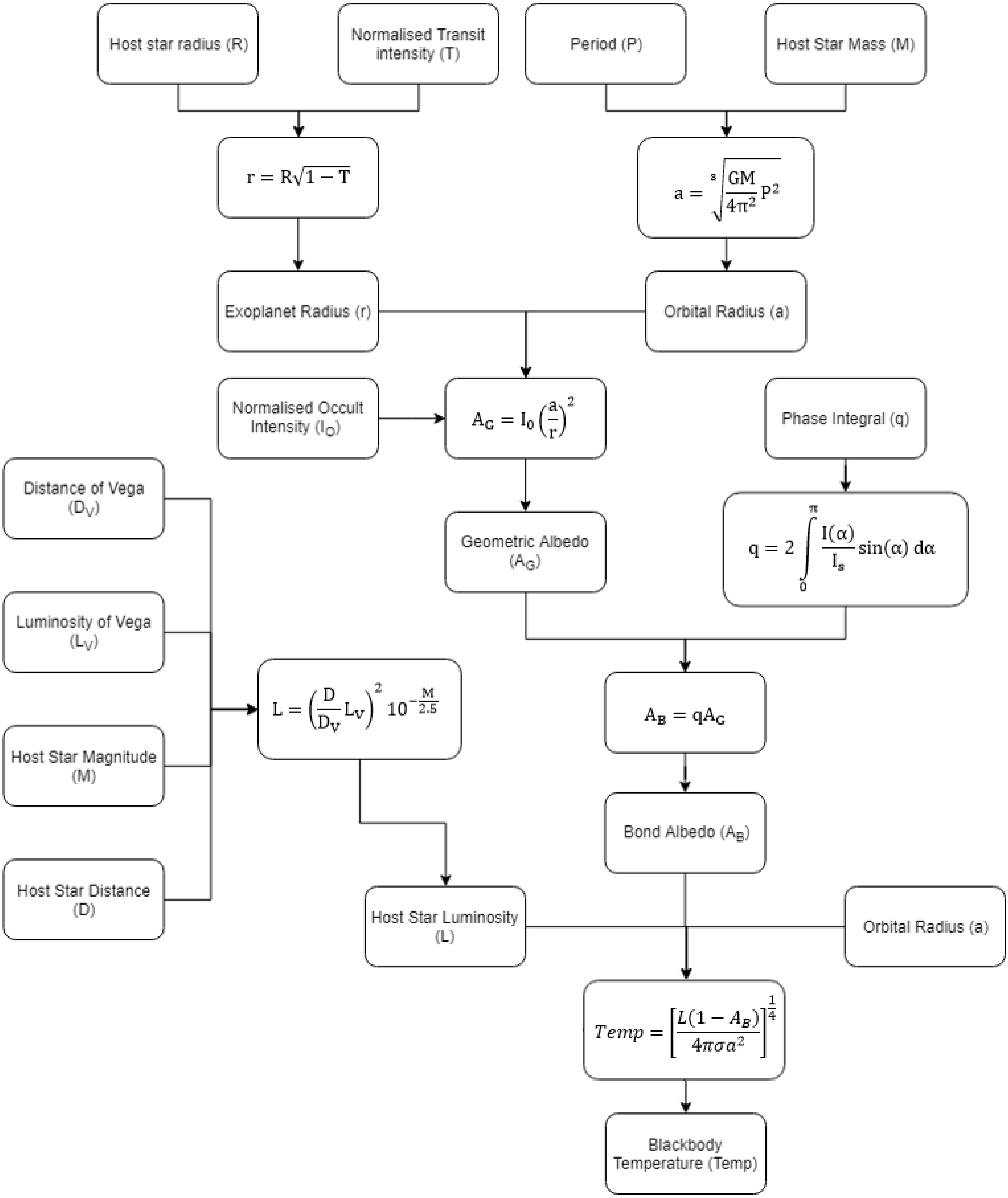

Figure 3: Flow chart showing the process of equations to obtain the Blackbody Temperature of an exoplanet.

Coding and Results

Initially, Kepler 7b was analysed as suggested by our demonstrator. The relevant flux data was downloaded from the STSci database as one .FITS file which spanned 1581 days. The data was available in two formats on the STSci website, Simple Aperture Photometry Flux (SAP FLUX) and Pre-search Data Conditioned SAPFLUX (PDCSAP FLUX). The PDCSAP FLX was used as it had undergone conditioning to remove noise which was inherent to the detectors on the telescope. This could be done by removing signals which are always present, despite using different targets. This data was converted into an excel spreadsheet using the pandas module and processed by the Lightkurve [6] module. The processing time to plot flux against time for the complete data set was very long, and so it was decided to initially focus on a small time interval for which we would develop the code, and then apply it to the complete data set. This small interval can be seen in figure 4 where the lower plot displays the flux, normalised about 0.

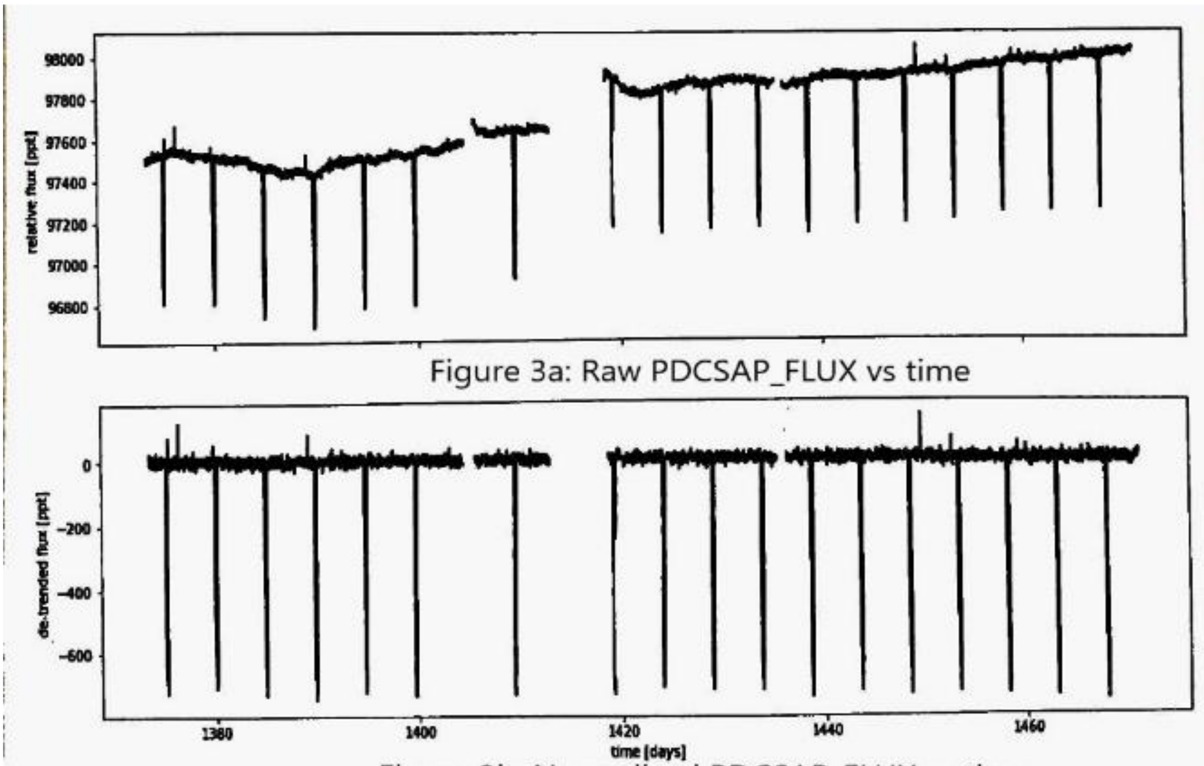

Figure 4: Section of flux data for Kepler 7b showing several discontinuities.

It can be seen from the graph that there are several discontinuities, it was assumed that this absence of data was a result of the telescope not taking data to realign itself. It was discovered that failure to remove the NAN values (not a number) resulted in an incoherent folded lightcurve due to the Lightkurve module on Spyder being unable to find a strong period. This is shown in figure 5.

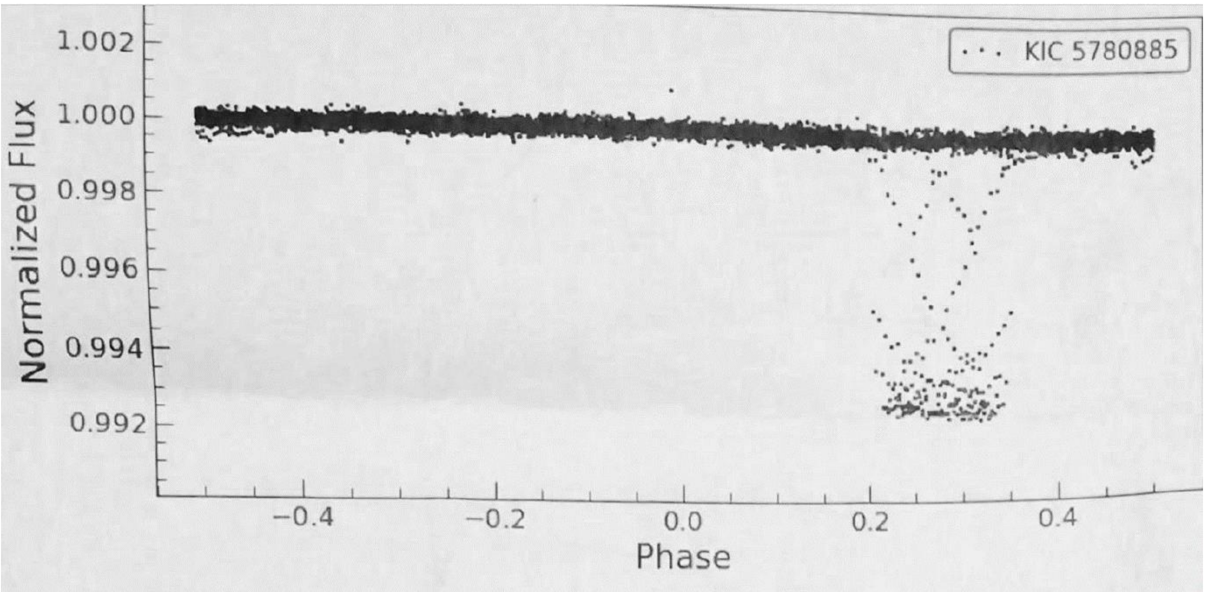

Figure 5: Result of folding the normalised flux data in which the NAN values skew the period.

The NAN values were replaced with their local median so that data points maintained a constant time separation to fix this. The Lightkurve module provides a selection of data cleaning functions which were also used, these include remove_nans() and flatten() which helped in removing low-frequency noise.

The data had to be folded upon itself every period so that a clearer trendline could be discovered. We were expecting a drop in intensity for the secondary transit of approximately 0.003%, and so a vast amount of data would be required to detect this behind the noise. Folding 1581 days of data into one period makes this possible. The periodogram() function was used to do this; it does this by plotting relative power of each detected frequency and selects the one with the most significant power, as can be seen by figure 6

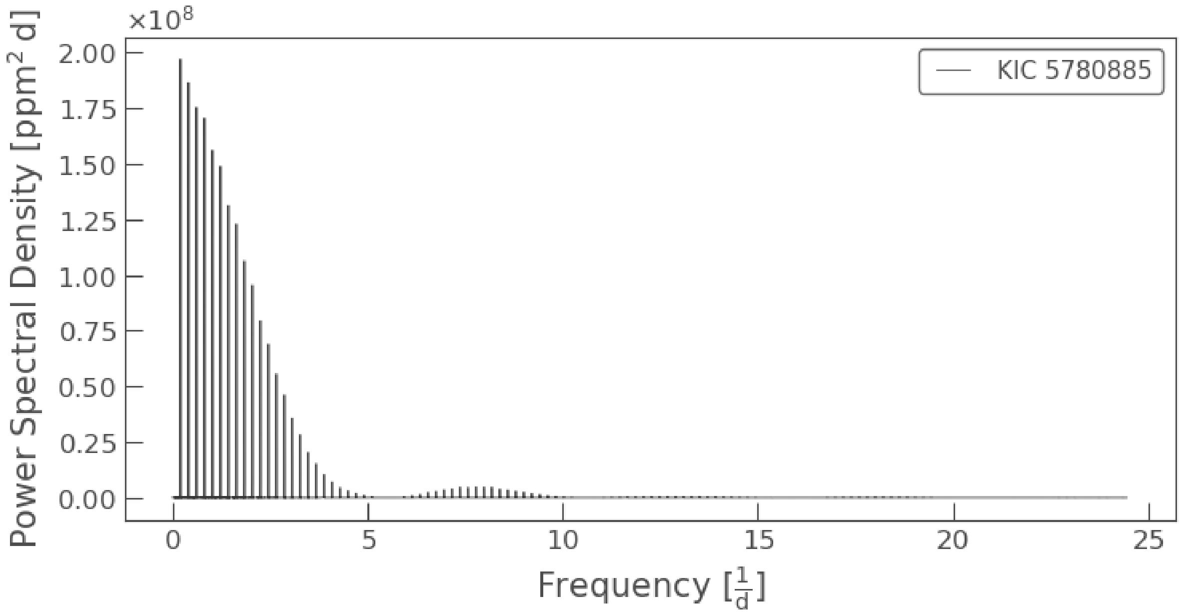

Figure 6: Periodogram for Kepler 7b.

Upon removal of the NAN values, replotting obtained figure 7.

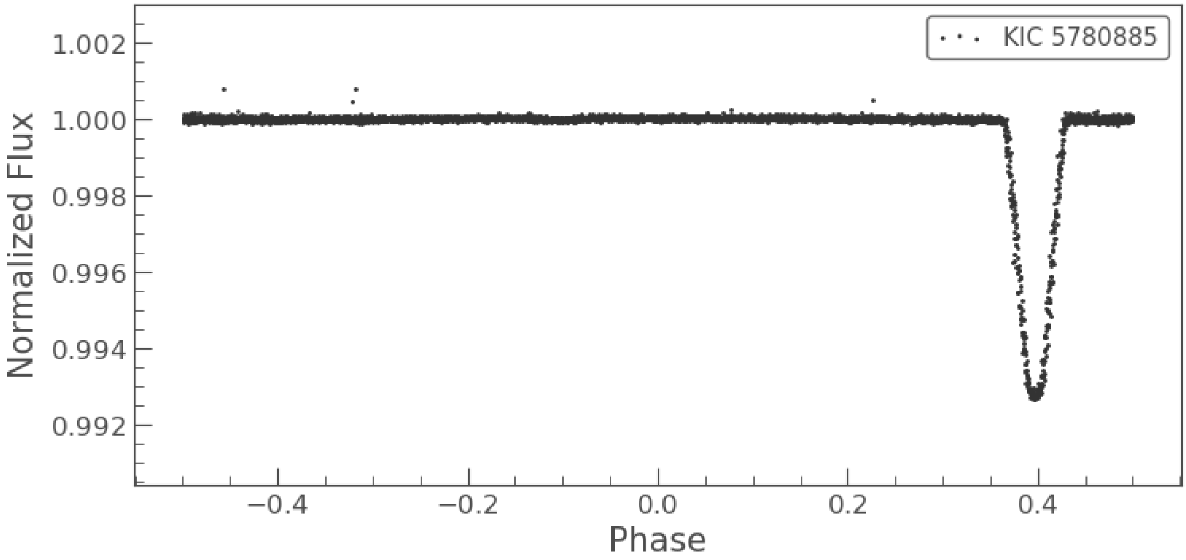

Figure 7: Result of folding the normalised flux data with the NAN values removed

This fold was retrieved from a period of 4.885 days which agreed with what was found by Latham, David W et al. to be 4.886 days [7]. Due to the nature of how the period is retrieved from the data, the spyder module is unable to provide an error on the period

Despite retrieving a promising period, a significant secondary transit was unable to be found; the secondary dip was less than 0.02% of the stellar output which was too small in comparison to noise. The same process was excecuted with Kepler 2b, and in such case, the secondary transit was much more significant. Figure 8 shows the periodogram and accompanying folded lightcurve below and on the next page in which the red box marks the secondary transit.

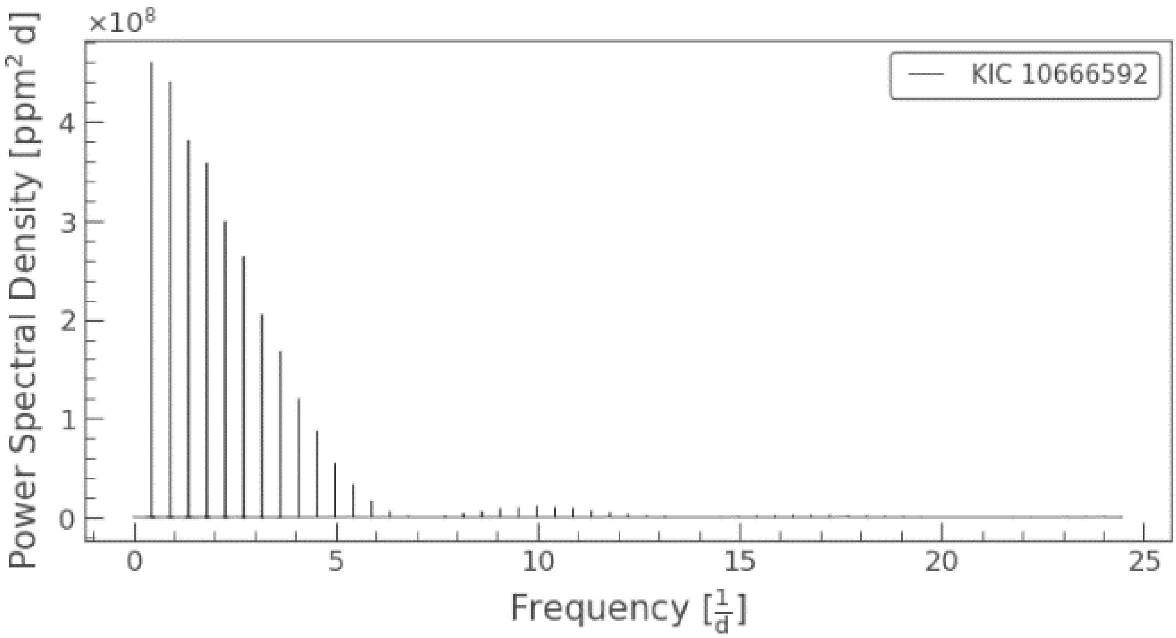

Figure 8: Periodogram for Kepler 2b.

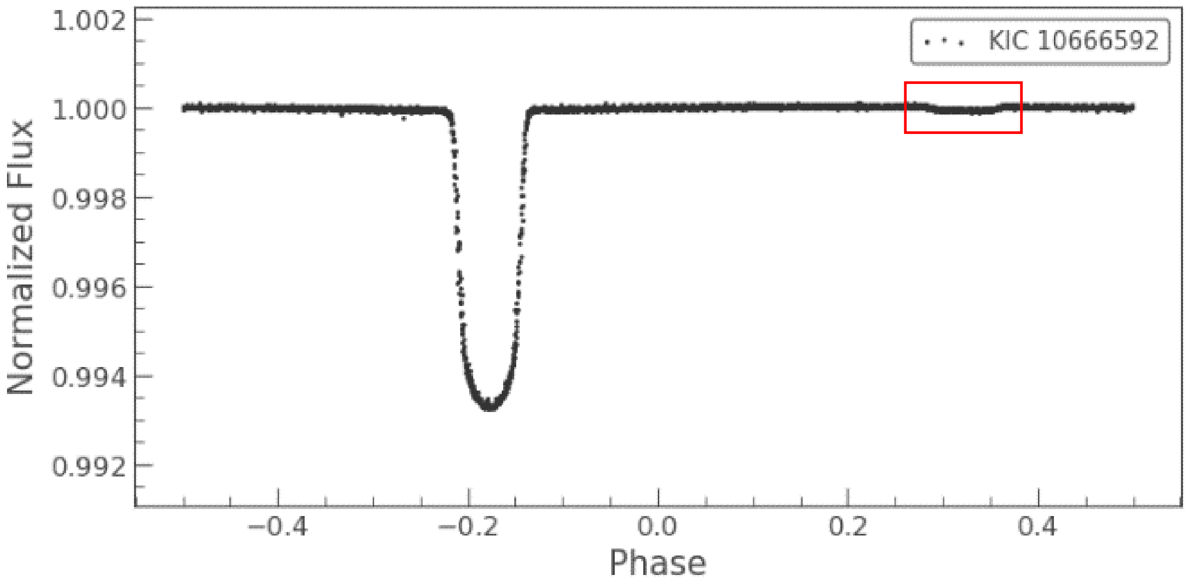

Figure 9: Result of folding the normalised flux data for Kepler 2b.

The Periodogram on the previous page obtained the strongest period as 2.204737 days. This was then used to fold the flux data upon itself to produce the lightcurve above. Kepler 2b produced a much more promising lightcurve; both transits could be seen as well as the increasing intensity from reflection as seen in figure 10. It was here that it was decided to focus on Kepler 2b.

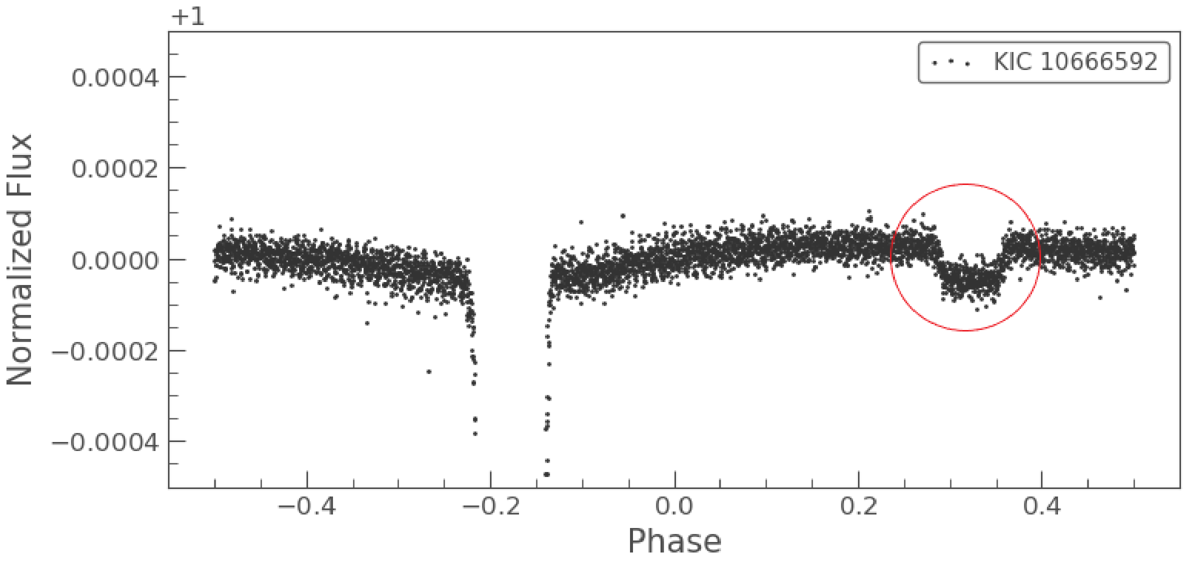

Figure 10: Lightcurve of Kepler 2b with an enlarged y-axis.

From this lightcurve, the values for the period, primary transit and secondary transit depth could be found. Then, by employing the series of calculations described in the theory section, the parameters of this system were calculated and are written alongside parameters obtained from online sources in the table on the following page. The first six entries are the starting properties which were unobtainable from our data, but crucial for calculations, and so they have been quoted from online sources. Equations for calculating errors have been included in the appendix.

Parameter	Obtained Value	Accepted value
Mass of Kepler 2 (M) / $M_\odot$		1.47 ± 0.80 [8]
Radius of Kepler 2 (R) / $R_\odot$		1.84 ± 0.23 [8]
Magnitude of Kepler 2 (m)		10.46 ± 0.03 [8]
Distance to Kepler 2 (D) / ly		1100 ± 160 [8]
Luminosity of Vega (L_v) / $L_\odot$		40.12 ± 0.45 [9]
Distance to Vega (D_v) / ly		25.04 ± 0.07 [10]
Transit Intensity (T) / %	0.99325 ± 0.00003	
Radius of Exoplanet (r) / km	$(1.05 \pm 0.13)\ 10^5$	$(0.97 \pm 0.12) *10^5$ [11]
Period (P) / days	2.204737	2.204737 ± 0.000017 [12]
Orbital Radius (a) / km	$(5.64 \pm 1.02)\ 10^6$	$(5.67 \pm 0.06)\ 10^6$ [13]
Occult Intensity (I_0) / %	$(4.9 \pm 1.0)\ 10^{-6}$	
Geometric Albedo (A_G)	$(1.41 \pm 0.69)\ 10^{-2}$	
Bond Albedo (A_B)	$(3.12 \pm 0.75)\ 10^{-3}$	<0.03 [14]
Host Star Luminosity (L) / J	$(1.94 \pm 0.57)\ 10^{27}$	$(2.01 \pm 0.09)\ 10^{27}$ [15]
Temperature (Temp) / K	2149.2 ± 281.76	2121.0 [16]

These results describe a very large and hot planet. Kepler 2b is approximately 1.4 times the size as Jupiter. It has a very small albedo and therefore absorbs the majority of the light incident upon it; consequently, it is very hot. This heat would not support the presence of clouds of water vapour like on earth as water is too light and would have escaped Kepler 2b's atmosphere. It is suggested that Hot Jupiters typically have atmospheres have containing titanium dioxide (TiO_2) and corundum (Al_2O_3) [17]. It was attempted to obtain spectra from the exoplanet using spritzer data. The motivation for this is that a better understanding of the chemical composition would allow for more informed predictions of the temperature on the planet. Equation 11 assumes blackbody behaviour and so negated the effect from greenhouse gasses. But, we were unable to manipulate this data and so could not produce results.

Modelling

These parameters allowed for the construction of a computer model of the system. It was the aim to make a model which would make it clearer to see the secondary transit, especially for stars with small secondary dips. A model was made for the primary and secondary transit separately. For the primary transit, the star and exoplanet were modelled by one bright and one semi-circle of proportionate radii. For the secondary transit, the bright semi-circle remained the same; however, the semi-circle representing the exoplanet was no longer dark. It was assigned a brightness equal to its geometric albedo multiplied by its surface area (the intensity of the star = 1). To ensure that the transit time is correct in the model, the impact parameter, b, would have to be considered, this being a fraction of the stellar radius at which the exoplanet's projection sits from the centre of the star during transit. Exoplanets at a greater impact parameter would spend a shorter time blocking starlight during transit. The method of this calculation has been outlined in the following pages.

To find the transit time, first, the impact parameter must be found. This can be found geometrically by considering figure 11 [18].

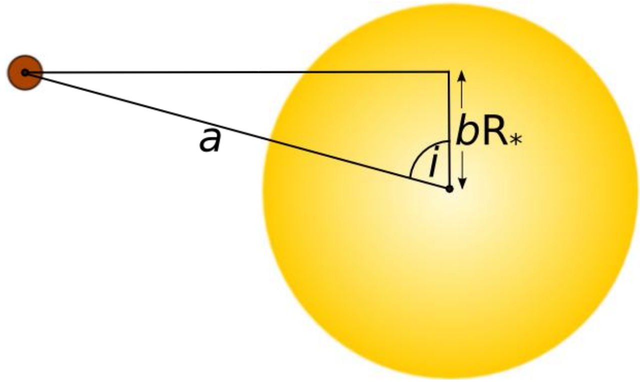

Figure 11: View of orbit at $\pi/2$ phase to show the trigonometric relationship between b, a and the angle of inclination.

Using trigonometry, one can show that

$$b = \frac{a\cos(i)}{R}. \tag{12}$$

To find the transit time the total period will be multiplied by the proportion of arc length during transit to the entire circumference. The arc length can be expressed in terms of the radii and impact parameter, detailed below. Using Pythagoras's theorem, the chord that the exoplanet traverses can be found, as shown by figure 12 [18].

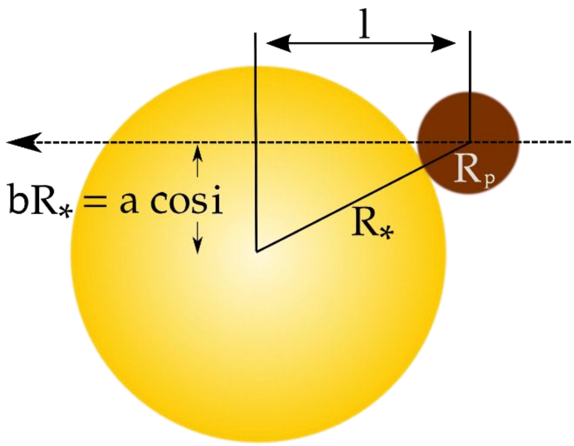

Figure 12: View of orbit at the beginning of transit to show the length of the chord travelled during transit.

$$2l = 2\sqrt{\left(R_* + R_p\right)^2 - (bR_*)^2}. \tag{13}$$

This can be used to express the angle which subtends the chord as seen in figure 13 and by equation (14) below [18].

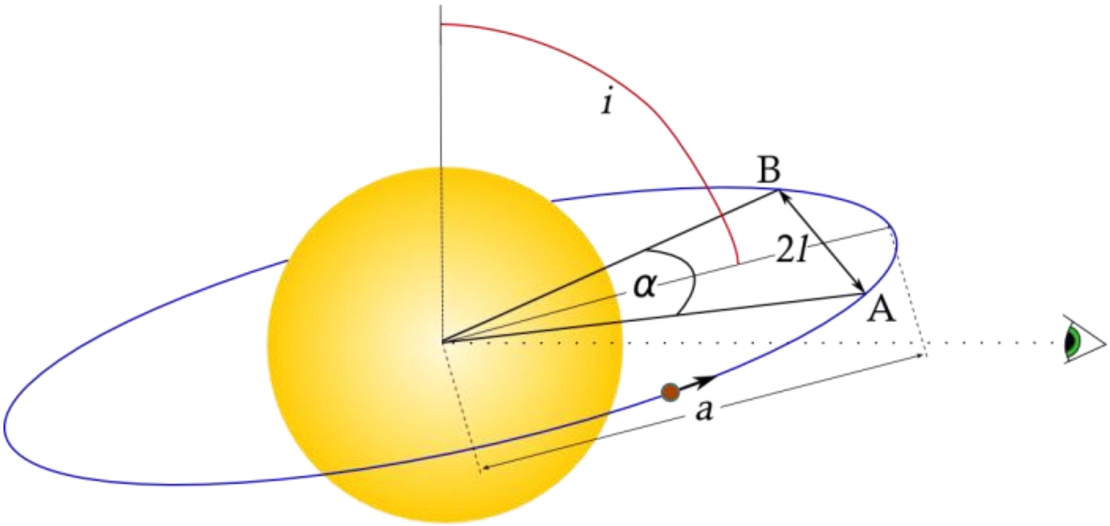

Figure 13: Relating α to a and l.

With the use of trigonometry, one can show that

$$\sin\left(\frac{\alpha}{2}\right) = \frac{l}{a}. \tag{14}$$

Finally, the transit time can be found as shown in the equation below in which a rearrangement of equation (15) has been substituted in for α.

$$\text{Transit time} = P\frac{a\alpha}{2\pi a} = \frac{P}{\pi}\sin^{-1}\left(\frac{l}{a}\right) = \frac{p}{\pi}\sin^{-1}\left(\frac{\sqrt{\left(R_* + R_p\right)^2 - (bR_*)^2}}{a}\right) \tag{15}$$

This model neglects the effects of limb darkening towards the edges of the star which would cause a less steep decline of intensity and a shallower dip during transit. Kepler 2b was found to have an impact parameter of 0.497±0.248 and an orbital inclination of 83.5±7.93 degrees. Figures 13 and 14 show the models obtained for the primary and secondary transits, overlaid onto the data. Figure 13 is skewed due to imperfect phase alignment.

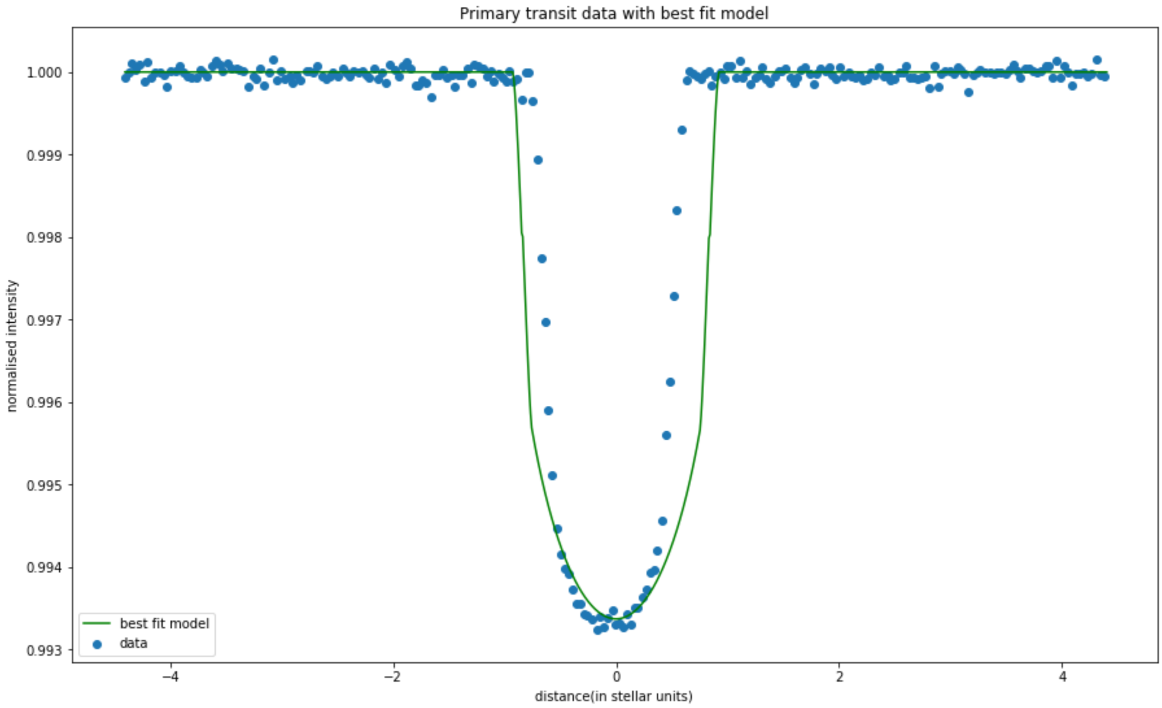

Figure 14: Model compared to data for the primary transit

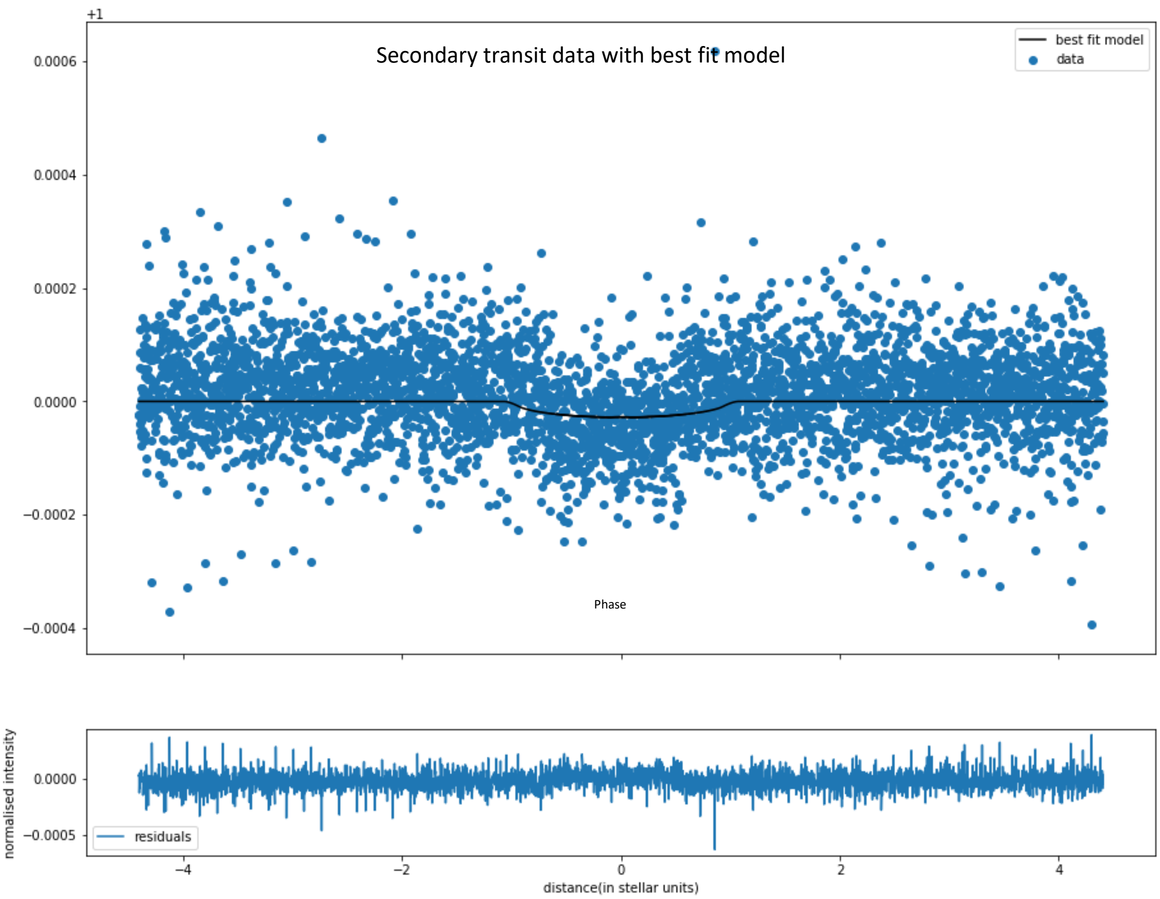

Figure 15: Model compared to data for the secondary transit

Conclusion

The aim of this project was determining the weather on an extrasolar planet. The planet we chose was Kepler 2b as it is classed as a "Hot Jupiter" The reason for this is that its lightcurve has significant primary and secondary transits. It has also got a very short orbital period which makes data analysis easier since there are more periods available to fold. This reduces the error in finding the intensity the transit and occultation intensity.

The majority of the time was spent trying to attain a lightcurve with errors that could produce the values of intensity during transit and occultation. Once this was achieved, the remaining code to find the temperature had already been written. It was able to output parameters about the system which have been written in detail in the table in the results section. The key parameters include the bond albedo, for which we obtained $(3.12 \pm 0.75)\ 10^{-3}$, and the Blackbody temperature, for which we obtained (2149.2 ± 281.76) K. The transit intensity was found from the mean of all the folded minima and its error being the individual error of each fold added in quadrature (see the appendix). This was especially significant when finding the secondary transit intensity; due to the very small secondary dip, its error was particularly large, approximately 20% which propagated through to following parameters, including albedo.

It was also attempted to plot albedo as a function of time as it was believed that any periodic motion would indicate dynamic cloud systems. This was done by keeping the plot unfolded and finding the albedo from each period individually. Unfortunately, the graph obtained showed no periodic motion, rather, random fluctuations. It is believed that the cause of this is that the noise of the signal shrouded the secondary transit. This would require a telescope with a greater resolution to observe due to the inability to fold the flux data.

This project could be extended by analysing spectral data from the system. During transit there may be dark lines in the spectrum due to molecules in the exoplanet's atmosphere scattering light from the star. By comparison with a table of spectral lines for each element, the atmospheric composition could be found. It would be expected that the presence of greenhouse gasses such as carbon dioxide and methane would increase the temperature past the blackbody prediction here. It could also confirm the presence of clouds if there are spectral lines matching that of water.

In addition, the introduction of limb-darkening effects into the model would also improve its similarity to the data. It can be seen that the model falls in intensity much quicker than the actual data. This is due to the fact that in reality, the exoplanet is obscuring a darker patch of its star than the model had predicted. This taken into account would cause a more gradual decline in intensity.

Finally, this can be extended by repetition with other Hot Jupiters so that direct comparisons can be made with Kepler 2b. This project was unable to reach this stage due to difficulty in finding exoplanets which produced a substantial drop in intensity for the secondary transit. Hence, a more accurate model could be utilised to recreate lightcurves which can then be analysed.

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Appendix, Error Formula

For the Intensity:

$$\delta I = I\sqrt{\frac{\sum_{i}^{n}\delta x_i^2}{n}}$$

For equation 2:

$$\delta r = r\sqrt{\left(\frac{\delta R}{R}\right)^2 + \left(\frac{\delta T}{2T}\right)^2}$$

For equation 3:

$$\delta a = a\sqrt{\left(\frac{\partial M}{3M}\right)^2 + \left(\frac{\delta G}{3G}\right)^2}$$

For equation 4:

$$\delta A_G = A_G\sqrt{\left(\frac{\delta I_0}{I_0}\right)^2 + \left(\frac{2\delta a}{a}\right)^2 + \left(\frac{2\delta r}{r}\right)^2}$$

For equation 7:

$$\delta A_B = A_B\sqrt{\left(\frac{\delta q}{q}\right)^2 + \left(\frac{\delta A_G}{A_G}\right)^2}$$

For equation 9:

$$\delta q = \left(\sum_{1}^{\pi} I(n)^2\right)^{1/2}$$

For equation: 10:

$$\delta L = \sqrt{\left(\frac{2\delta D}{D}\right)^2 + \left(\frac{2\delta D_v}{D_v}\right)^2 + \left(\frac{2\delta L_v}{L_v}\right)^2 + \left(\frac{\ln 10}{2.5}\delta m\right)^2}$$

For equation: 11:

$$\delta Temp = \frac{Temp}{4}\sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta A_B}{A_B}\right)^2 + \left(\frac{2\delta a}{a}\right)^2}$$

For equation 15:

$$\delta b = b * \sqrt{\left(\frac{\delta Transit\ time}{Transit\ time}\right)^2 + \left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta r}{r}\right)^2}$$