### Deep Hash Embedding for Large-Vocab Categorical Feature Representations

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### **ABSTRACT**

Embedding learning for large-vocabulary categorical features (e.g. user/item IDs, and words) is crucial for deep learning, and especially neural models for recommendation systems and natural language understanding tasks. Typically, the model creates a huge embedding table that each row represents a dedicated embedding vector for every feature value. In practice, to handle new (i.e., out-of-vocab) feature values and reduce the storage cost, the hashing trick is often adopted, that randomly maps feature values to a smaller number of hashing buckets. Essentially, these embedding methods can be viewed as 1-layer wide neural networks with one-hot encodings.

In this paper, we propose an alternative embedding framework Deep Hash Embedding (DHE), with non-one-hot encodings and a deep neural network (embedding network) to compute embeddings on the fly without having to store them. DHE first encodes the feature value to a dense vector with multiple hashing functions and then applies a DNN to generate the embedding. DHE is collisionfree as the dense hashing encodings are unique identifiers for both in-vocab and out-of-vocab feature values. The encoding module is deterministic, non-learnable, and free of storage, while the embedding network is updated during the training time to memorize embedding information. Empirical results show that DHE outperforms state-of-the-art hashing-based embedding learning algorithms, and achieves comparable AUC against the standard one-hot encoding, with significantly smaller model sizes. Our work sheds light on design of DNN-based alternative embedding schemes for categorical features without using embedding table lookup.

### 1 INTRODUCTION

Machine learning is highly versatile to model various data types, including continuous features (e.g. item popularity), sparse features (e.g. text), and sequential features (e.g. speech signals). Among these, we focus on improving embedding learning for large-vocabulary categorical features. Specifically, we assume a categorical feature is defined by a vocabulary V, with the feature value is (exactly) one of the elements in V. For example, ID features are typically categorical features (e.g. each user has a unique ID). Another example is the 'device' feature, and 'iPhone 12' is a possible feature value.

Embedding learning has become the core technique for modeling categorical features, and various methods have been proposed and successfully applied in real-world applications, such as Matrix Factorization (MF) [32], word2vec [40], and node2vec [17]. The embedding learning technique greatly helps us understand the semantic meaning of feature values (e.g. words, movies, etc.). Embeddings have also become the cornerstone of deep models for

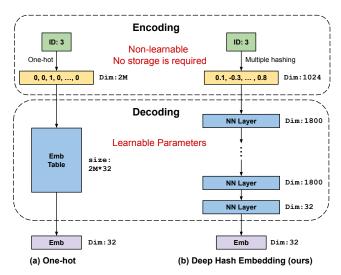


Figure 1: An illustration of one-hot based full embedding and Deep Hash Embedding (DHE) for generating 32-dim ID feature embeddings. The dimension numbers are from our experiments for providing a concrete example. The two models achieve similar AUC while DHE costs 1/4 of the full model size. DHE uses a dense hash encoding to obtain a unique identifier for each feature value, and applies a deep embedding network to generate the feature embedding. DHE doesn't perform any embedding lookup.

capturing more complex interactions among feature values (e.g. BERT [13], DeepFM [18]).

Despite the success of embedding learning in various domains like natural language processing [40] and knowledge graphs [5], there are several challenges when applying embedding learning to recommender systems (RecSys):

- Huge vocabulary size: Recommender systems usually need to
  handle categorical features with huge vocabulary sizes (e.g. billions of video IDs for online video sharing platforms). Moreover,
  in NLP tasks, it's common to construct sub-word vocabulary
  with methods like byte-pair encoding (BPE) [16] to reduce the
  vocabulary size (e.g. GPT-2 [44]). But it's generally infeasible to
  apply this approach to the categorical features in RecSys.
- Dynamic nature of input: Unlike vocabularies of 'words,' that
  are relatively static, the vocabulary in RecSys could be highly
  dynamic: new users and items enter the system on a daily basis,
  and stale items are gradually vanishing.

 Highly-skewed data distribution: The categorical features in RecSys usually follow highly skewed power-law distributions. The small number of training examples on infrequent feature values hurts the embedding quality [37] for the tail items significantly.

The one-hot encoding is widely adopted for embedding learning, that maps a feature value to a dedicated embedding vector in the embedding table. However, the size of the embedding table grows linearly with the vocabulary size, and could be gigantic for a large-vocab feature. In web-scale recommendation models, it is not surprising to have most of the parameters spent on the embedding table, while the neural network itself only accounts for a very small portion of parameters [11]. In practice, to better handle new (i.e., out-of-vocab / unseen) feature values and reduce the storage cost, the hashing trick [52] is often adopted, that randomly maps feature values to a smaller number of hashing buckets. Essentially, this embedding method can be viewed as a 1-layer wide neural network with one-hot encoding.

In this paper, we propose an alternative embedding framework Deep Hash Embedding (DHE), with *non-one-hot* encodings and a *deep* neural network (embedding network) to compute embeddings on the fly without having to store them. Specifically, we use multiple hashing to generate a unique, deterministic, dense, and real-valued vector as the encoding of the given feature value, and then the deep embedding network transforms the encoding to the final feature embeddings. The feature embeddings are used for training recommendation models (e.g. MF or deep recommendation models) end-to-end. We empirically find that DHE achieves better performance than hashing-based baselines, and results in similar AUC compared to one-hot full embeddings, with much smaller model sizes. As the computation bottleneck for DHE is in the deep embedding network, DHE significantly benefits from powerful hardware like GPUs and TPUs [29].

Our main contributions are listed as follows:

- We analyze various popular embedding methods, including hashing-based approaches for embedding categorical features.
   Unlike existing methods that heavily rely on one-hot encodings, we design a new encoding scheme based on *dense hash encoding*, which takes the first step to completely remove the huge embedding tables for large-vocab features.
- Inspired by recent successes of DNN, we propose to replace
  the commonly used embedding lookup (essentially a wide and
  shallow network) with *deep embedding networks* that transform
  hash encodings to feature embeddings. We address the trainability and expressiveness issues to enable the DNN for embedding
  generation.
- We propose Deep Hash Embedding (DHE) based on aforementioned dense hash encodings and deep embedding network.
   We further improve DHE to better generalize among feature values and to new values, via proposing side feature enhanced encodings.
- We conduct extensive experiments on two benchmark datasets for recommendation tasks with large-vocabulary categorical features. We compare with baselines and analyze the effect of various key components in DHE. The results suggest that DHE

Table 1: Notation.

Notation	Description
$\overline{V}$	set of feature values
$n \in \mathbb{N}$	vocabulary size
$d \in \mathbb{N}$	embedding dimension
$m \in \mathbb{N}$	hashed vocabulary size (usually $m < n$ )
$H:V\to [m]$	hash function mapping feature values to
	$\{1,2,\ldots,m\}$
$k \in \mathbb{N}$	number of hash functions, also the encoding
	length in DHE
$d_{ ext{NN}} \in \mathbb{N}$	the width of hidden layers in the embedding
	network
$h \in \mathbb{N}$	the number of hidden layers in the embedding
	network

outperforms various hashing-based baselines and is a promising alternative scheme to one-hot full embeddings.

In section 2, we discuss various existing one-hot based embedding methods from the perspective of neural networks. Then we introduce DHE with dense hash encodings and deep embedding network, and an extension of using side features for better encodings, before we present our experimental results. Finally, we discuss related work, conclude our paper, and point out promising directions for future work.

### 2 PRELIMINARY: ONE-HOT BASED EMBEDDING LEARNING

Embedding learning has become a standard technique to handle categorical features like words, user/item IDs, categories, etc. The core idea is to map feature values into a continuous d-dimensional space. These learnable embeddings could be utilized by shallow models like word2vec [40] or MF [32], to directly measure the similarity between two feature values (e.g. large inner products between similar words' embeddings). Moreover, deep models like DeepFM [18] or BERT [13], can model more complex structures via considering interactions among the embeddings.

We define a general framework for describing various existing embedding methods as well as our proposed approach. The embedding function  $\mathcal{T}:V\to R^d$  maps a feature value (from feature vocabulary V with size |V|=n) to a d-dimensional embedding vector  $\mathbf{v}$ . Generally, the embedding function can be decomposed into two components:  $\mathcal{T}=F\circ E$ , where E is an encoding function to represent feature values in some spaces, and F is a decoding function to generate the embedding  $\mathbf{v}$ .

In this section, we introduce full and hashing-based embedding schemes with one-hot encodings. The notation is summarized in Table 1.

### 2.1 One-hot Full Embedding

This is the most straightforward and commonly used approach to embed categorical features, which assigns each feature value a unique d-dimensional embedding vector in an embedding table. Specifically, the encoding function E maps a feature value into a unique one-hot vector. In offline settings, this is easy to achieve even

Table 2: Comparison of embedding schemes. The model size of DHE is independent of n or m. DHE is based on dense hash encodings and deep neural networks. DHE can handle out-of-vocab values for online learning, and incorporate side features.

	One-Hot	The Hashing Trick [52]	Bloom Emb [47]	Hybrid Hashing [54]	Hash Emb [50]	Deep Hash Emb (DHE)
Model Size	O(nd)	O(md)	O(md)	O(md)	O(nk+md)	$O(kd_{NN}+(h-1)d_{NN}^2+dd_{NN})$
#Hash Functions	-	1	2~4	2	2	~1000
Encoding Vector	one-hot	one-hot	(multiple) one-hot	(multiple) one-hot	(multiple) one-hot	dense & real-valued
Decoding Function	1-layer NN	1-layer NN	1-layer NN	1-layer NN	1-layer NN	Deep NN
Handling OOV Values?	×	<b>~</b>	<b>V</b>	<b>~</b>	V	V
Emb Table Lookup?	~	<b>✓</b>	✓	<b>✓</b>	<b>✓</b>	×
Side Features for encoding?	×	×	×	×	×	<b>✓</b>

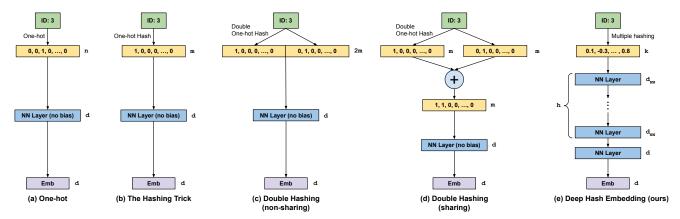


Figure 2: An illustration comparing various embedding schemes in the language of neural networks. Existing methods can be viewed as wide and shallow networks relying on one-hot encodings and embedding lookup, while our methods are deep and (relatively) narrow networks with dense hash encodings.

if the feature values are non-numeric types like string (e.g. feature values 'Japan' or 'India' for the categorical feature 'Country'), as we can scan and obtain a one-to-one mapping from feature values to  $\{1, 2, ..., n\}$ .

So we assume the feature values are already mapped to  $\{1, 2, \ldots, n\}$ , then the embedding approach creates an embedding table  $\mathbf{W}$  with a size of  $n \times d$ , and returns its i-th row  $\mathbf{W}_i$  for the feature value i. This is equivalent to the following: (i) we apply the encoding function E to encode feature value i with a one-hot encoding vector:  $E(i) = \mathbf{b} \in \{0, 1\}^n$  where  $b_i = 1$  and  $b_j = 0$  ( $j \neq i$ ); (ii) we then apply the decoding function F, a learnable linear transformation  $\mathbf{W} \in \mathbb{R}^{n \times d}$  to generate the embedding vector  $\mathbf{v}$ , that is,  $\mathbf{v} = F(\mathbf{b}) = \mathbf{W}^T \mathbf{b}$ . In short, the embedding lookup process can be viewed as a 1-layer neural network (without bias terms) based on the one-hot encoding.

### 2.2 One-hot Hash Embedding

Despite the simplicity and effectiveness of full embeddings, such a scheme has two major issues in large-scale or dynamic settings: (i) the size of the embedding table grows linearly with the vocabulary size, which could cause a huge memory consumption. For example, 100-dimensional embeddings for 1 billion video IDs alone costs near 400 GB of memory; (ii) in online learning settings where new values constantly arise, the full embedding scheme fails to handle unseen (out-of-vocab) feature values.

To address the above issues, various hashing-based methods have been proposed (e.g. [47, 50, 52]), and widely used in productionscale systems for handling large-vocab and out-of-vocab categorical features (e.g. *Youtube* [53] and *Twitter* [54]). Hashing has also been integrated into various libraries for feature processing (e.g. Tensor-Flow's feature column<sup>1</sup>, DeepCTR<sup>2</sup>).

The hashing trick [52] is a representative hashing method for reducing the dimension of the one-hot encoding for large vocabularies. The encoding function E still maps a feature value into a one-hot vector, but with a different (typically smaller) cardinality:  $E(s) = \mathbf{b} \in \{0,1\}^m$  where  $s \in V$ ,  $b_{H(s)}=1$  and  $b_j=0$  ( $j \neq H(s)$ ). The hash function H maps feature values (including unseen values) to  $\{1,2,\ldots,m\}$  where m is the hashed vocabulary size and usually smaller than n. The hash function H seeks to distribute hashing values as uniformly as possible to reduce collision, though it's inevitable when m < n. Similarly, the decoding function returns the H(s)-th row of the embedding table. In summary, the hashing trick uses hashing to map feature values into m-dim one-hot vectors, and then applies a 1-layer network to generate the embeddings.

Although the hashing trick is able to arbitrarily reduce the cardinality of the original vocabulary V, it suffers from the embedding collision problem. Even in the ideal case (uniformly distributed), each embedding (in the embedding table) is shared by n/m feature values on average. This inevitably hurts the model performance, as the model cannot distinguish different feature values due to the same embedding representations. To alleviate this issue, multiple hash functions have been used to generate multiple one-hot encodings:  $E(s) = \mathbf{b} = [\mathbf{b}^{(1)}; \mathbf{b}^{(2)}; \dots; \mathbf{b}^{(k)}] \in \{0, 1\}^{m*k}$ 

 $<sup>{}^{-1}</sup> https://www.tensorflow.org/api\_docs/python/tf/feature\_column/categorical\_column\_with\_hash\_bucket$ 

<sup>&</sup>lt;sup>2</sup>https://github.com/shenweichen/DeepCTR

where  $b_{H^{(i)}(s)}^{(i)}$  =1 and  $b_j^{(i)}$  =0  $(j \neq H^{(i)}(s))$ . Here, k hash functions  $\{H^{(1)}, H^{(2)}, \dots, H^{(k)}\}$  are adopted to generate k one-hot encodings  $\{\mathbf{b}^{(1)}, \mathbf{b}^{(2)}, \dots, \mathbf{b}^{(k)}\}$ , and the concatenation is used as the encoding.

The core idea is that the concatenated encodings are less likely to be collided, thus reducing embedding collisions. We can lookup k embeddings in k embedding tables (respectively) and aggregate them into the final embedding. A common aggregation approach is 'add' [47, 50, 54], which can be simply expressed as  $\mathbf{v} = F(\mathbf{b}) = \mathbf{W}^T \mathbf{b} = \mathbf{W}^T [\mathbf{b}^{(1)}; \mathbf{b}^{(2)}; \dots; \mathbf{b}^{(k)}]$ . That is to say, multiple one-hot vectors are generated with different hash functions, and then the concatenation is fed into a 1-layer neural network without bias terms. However, creating k embedding tables costs more memory consumption, and thus it is common to just create and share a single embedding table [47, 50, 54]. Mathematically, it's equivalent to  $\mathbf{v} = \mathbf{W}^T \mathbf{b} = \mathbf{W}^T (\mathbf{b}^{(1)} + \mathbf{b}^{(2)} + \dots + \mathbf{b}^{(k)})$ . Note that existing methods didn't scale to large k and the most commonly used variant is double hashing (k=2) [47, 50, 54]. Figure 2 depicts the neural architectures of these embedding methods.

### 3 DEEP HASH EMBEDDINGS (DHE)

As introduced in the previous section, both full embeddings and hashing-based embeddings methods are essentially based on one-hot encodings and shallow networks. In this section, we propose Deep Hash Embeddings (DHE), an alternatives scheme for embedding learning in large-vocab or dynamic settings. DHE uses real-valued dense encodings and deep neural networks for generating embeddings without the need for any embedding lookup.

Following the general embedding framework  $\mathcal{T} = F \circ E$ , we propose several properties for designing good encodings, and then introduce our encoding function E and the decoding function F in DHE, followed by side-feature-enhanced encoding design for enabling generalization.

### 3.1 Encoding Design

What is a good encoding if we have no prior knowledge about feature values? This is the core question we seek to investigate in this section, and it also leads to our design of the encoding for DHE. We conclude the following properties for designing good encodings:

- Uniqueness: The encoding should be unique for each feature value. This is also the target of full embedding and multiple hashing methods. Otherwise, there are feature values that have to share the same encoding. The collided encodings make the subsequent decoding function impossible to distinguish different feature values, which typically hurts model performance.
- Equal Similarity: We think only having the uniqueness is not enough. An example is binary encoding, which uses the binary representation as the encoding of integers (e.g. IDs): e.g. H(9) = [1,0,0,1]. We can see that H(8) = [1,0,0,0] is more similar to H(9), compared with H(7) = [0,1,1,1]. We believe this introduces a wrong inductive bias (ID 8 and ID 9 are more similar), which may mislead the subsequent decoding function. The double hashing has a similar issue: the encodings of two feature values that collide in one hash function, are more similar than those of two values that have no collision in both hash

Table 3: Encoding comparison regarding the four properties: U: uniqueness; E-S: equal similarity; H-D: high-dimensionality; H-E: high entropy.

Encoding	Length	U	E-S	H-D	Н-Е
One-hot	n	~	<b>V</b>	~	×
One-hot Hash	m	×	~	~	×
<b>Double One-hot Hash</b>	2m	×	~	~	×
Binary	$\lceil \log n \rceil$	~	×	×	~
Identity	1	~	×	×	~
DHE (Dense Hash)	k	~	~	~	~

functions. As we don't know the semantic similarity among categorical features, we should make any two encodings be equally similar, and not introduce any inductive bias.

- High dimensionality: We hope the encodings are easy for the subsequent decoding function to distinguish different feature values. As high-dimensional spaces are often considered to be more separable (e.g. kernel methods), we believe the encoding dimension should be relatively high as well. For example, one-hot encoding has an extremely large dimensionality (n for full embedding and m for hash embedding). Another example is identity encoding which directly returns the ID number (assuming the feature values are integers): e.g. E(7) = [7]. Although this gives a unique encoding for each ID, it'd be extremely difficult for the following decoding function to generate embeddings based on the 1-dim encoding.
- **High Shannon Entropy**: The Shannon entropy [48] measures (in the unit of 'bits') the information carrying in a dimension. The high entropy requirement is to prevent redundant dimensions from the information theory perspective. For example, an encoding scheme may satisfy the above three properties, but, on some dimensions, the encoding values are the same for all the feature values. So we hope all dimensions are effectively used via maximizing the entropy on each dimension. For example, one-hot encodings have a very low entropy on every dimension, as the encoding on any dimension is 0 for most feature values. Therefore, one-hot encodings need extremely high dimensions (i.e., *n*) and is highly inefficient.

The formal definitions and analysis of the encoding properties can be found in Appendix, and we summarize the results in Table 3.

### 3.2 Dense Hash Encoding

After analyzing the properties of various encoding schemes, we found no existing scheme satisfies all the desired properties. Especially we found one-hot based encodings do not have the *High Entropy* property, while binary and identity encodings do not satisfy the *Equal Similarity* and *High dimensionality* properties. Inspired by this, we propose **Dense Hash Encoding**, which seeks to combine the advantage of the above encodings and satisfy all the properties.

Without loss of generality, we assume feature values are integers as we can map string values to integers with string hashing<sup>3</sup>. The proposed encoding function  $E: \mathbb{N} \to \mathbb{R}^k$  uses k hash functions to

 $<sup>^3</sup>$  There is basically no collision due to the large output space (2 $^{64}\approx 10^{19}$  values for 64-bit integers). An example is  $\it City Hash 64$ : https://github.com/google/cityhash

map a feature value to a k-dimensional dense and real-valued encodings. Specifically, we have  $E'(s) = [H^{(1)}(s), H^{(2)}(s), \ldots, H^{(k)}(s)]$  where  $H^{(i)}: \mathbb{N} \to \{1, 2, \ldots, m\}$ . Note that m in this case is not related to embedding table, and we just need to set it to a relatively large number ( $10^6$  in our experiments). Obviously, the integer-based E'(s) encoding is not suitable for the input to neural networks, as the input typically needs to be normalized for numeric stability. So we obtain the final encoding via transforming the encoding:  $E(s) = \operatorname{transform}(E'(s))$ . We consider to approximate one of the following commonly used distributions:

- **Uniform Distribution.** We use transform( $\mathbf{x}$ ) =  $(\mathbf{x} 1)/m * 2 1$  to convert the encoding E' to approximate uniform distribution U(-1, 1). As the hashing values are almost uniformly distributed among  $\{1, 2, ..., m\}$ , the approximation is reasonable with a large m.
- Gaussian Distribution. We first use the above transformation to obtain uniform distributed samples, and then apply the Box–Muller transform [6] to convert the uniformly distributed encodings (i.e., *U*(-1, 1)) to the standard normal distribution.

Empirically we found the two distributions work similarly well, and thus we choose the uniform distribution by default for simplicity.

Unlike existing hashing methods limited to a few hash functions, we choose a relatively large k for satisfying the high-dimensionality property (k=1024 in our experiments, though it's significantly smaller than n). We empirically found our method significantly benefits from larger k while existing hashing methods do not. Moreover, the proposed dense hash encodings also satisfy the other three properties. More details can be found in Appendix.

Note that the whole encoding process does not require any storage, as all the hashing computation can be done on the fly. This is also a nice property of using multiple hashing, as we obtain a more distinguishable higher-dimensional encoding without storage overhead. Computation-wise, as the calculation of each hashing is independent, the computation is very amenable for parallelization, highly suitable for hardwares like GPUs and TPUs. We can also accelerate the encoding process by storing O(k) parameters for the k hash functions, and it's still negligible compared with the whole model size. As an example, we use the universal hashing for integers [7] as the underlying hashing, and depict the encoding process in Algorithm 1. Other universal hashing (e.g. for strings) could also be adopted.

### 3.3 Deep Embedding Network

In DHE, the decoding function  $F:\mathbb{R}^k\to\mathbb{R}^d$  needs to transform a k-dim encoding vector to a d-dim embedding. Obviously, the encoding is not suitable for embedding lookup, as it's not one-hot. However, the mapping process is very similar to a highly non-linear feature transformation, which is often handled by neural networks. Therefore, we use powerful deep neural networks (DNN) to model such a complex transformation, because DNN is a universal function approximator [38] and has shown its expressive power in various complex tasks [13, 21].

However, the transformation task is highly challenging, even with DNNs. Essentially, the DNN needs to memorize the information (previously stored in the huge embedding table) in its weights.

### **Algorithm 1:** Dense Hash Encoding in DHE (on-the-fly).

```
Input: a feature value x \in \mathbb{N}, encoding length k, hash buckets m
Output: encod \in \mathbb{R}^k, a k-dim dense hash encoding for x
/* Using a fixed seed to generate the same hash functions at
   each encoding process. The generation can be skipped via
    storing O(k) parameters for hashing.
Set the Random Seed to 0.
for i \leftarrow 1 to k do
     /* a and b are randomly chosen integer with b \neq 0, p is a
        prime larger than \boldsymbol{m}
    a \leftarrow \text{RandomInteger()}
    b \leftarrow \text{RandomNonZeroInteger()}
     p \leftarrow \text{RandomPrimerLargerThan}(m)
     /* Applies universal hashing for integers
    encod[i] \leftarrow ((ax + b) \mod p) \mod m
/* Applies a transformation to normalize the encoding
encod \leftarrow transform(encod)
```

We hope the hierarchical structures and non-linear activations enable DNNs to express the embedding function more efficiently than the one-hot encodings (i.e., 1-layer wide NN). This is motivated by recent research that shows that deep networks can approximate functions with much fewer parameters compared with wide and shallow networks [36, 39].

Specifically, we use a feedforward network as the decoding function for DHE. We transform the k-dim encoding via h hidden layers with  $d_{\rm NN}$  nodes. Then, the outputs layer (with d nodes) transforms the last hidden layer to the d-dim feature value embedding. In practice,  $d_{\rm NN}$  is determined by the budget of memory consumption. We can see that the number of parameters in the DNN is  $O(k*d_{\rm NN}+(h-1)*d_{\rm NN}^2+d*d_{\rm NN})$ , which is independent of n or m. This is also the model size for DHE, as all the parameters in DHE are used for the DNN. A unique feature of DHE is that it does not use any embedding table lookup, while purely relies on hidden layers to compute embeddings on the fly.

However, we found that training the deep embedding network is quite challenging (in contrast, one-hot based shallow networks are much easier to train). We observed poor training and testing performance, presumably due to trainability and expressiveness issues. We found two techniques are important for improving the performance of DHE: batch normalization (BN) [27], and Mish activation [41]. The normalization techniques can generally stabilize and accelerate the training, and sometimes even improve the performance. We tried layer normalization (LN) [2] and batch normalization, and found BN is very helpful while LN is not. The expressiveness issue is relative unique to our task, as NNs are often considered to be highly expressive and easy to overfit. However, as the embedding generation task requires highly non-linear transformations from hash encodings to embeddings, we found the embedding network is underfitting, instead of overfitting in our case. We suspect the default ReLU activation is not expressive enough, as ReLU networks are piece-wise linear functions [1]. We tried various activation functions<sup>4</sup> and found the recently proposed Mish activation [41] consistently performs better than ReLU and others. We also found regularization methods like dropout are not beneficial, which again

 $<sup>^4</sup>$ we also tried tanh, elu [10], SIREN [49], and found them on par or inferior to ReLU.

verifies the bottleneck of our embedding network is underfitting. Please refer to the experiments for more analysis on the effect of these choices, and the impact of the depth h.

### 3.4 Side Features Enhanced Encodings for Generalization

An interesting extension for DHE utilizes side features for learning better encodings. This helps inject structure into our encodings, and enables better generalization among feature values, and to new values.

One significant challenge of embedding learning for categorical features is that we can only *memorize* the information for each feature value while we cannot *generalize* among feature values (e.g. generalize from ID 7 to ID 8) or to new values. This is due to the fact that the underlying feature representation does not imply any inherent similarity between different IDs. In contrast, if we train a convolutional neural network (CNN) for pixel-based image features, the model could generalize well to unseen images. Moreover, the skewed power-law distributions make the problem more severe, as it's hard to learn good embeddings for a large number of infrequent values due to the limited data. A typical method for achieving generalization is using side features which provide inherit similarities (e.g. bag-of-words features). However, these features are usually used as additional features for the recommendation model, and not used for improving the categorical feature representation learning.

One-hot based full embeddings inherit the property of categorical features, and generate the embeddings independently (i.e., the embeddings for any two IDs are independent). Thus, one-hot based schemes can be viewed as **decentralized** architectures that are good at *memorization* and unable to achieve *generalization*. In contrast, the DHE scheme is a **centralized** solution: any weight change in the embedding network will affect the embeddings for all feature values. We believe the centralized structure provides a potential opportunity for generalization.

Unfortunately, the hash encodings in DHE do not break through the generalization limitation of categorical features, as the representation implies no meaningful similarity as well. However, as the decoding function of DHE is a neural network, we have a great flexibility to modify the encoding function of DHE, like incorporating side features. We propose side feature enhanced encodings for DHE, and hope this will enable the generalization among feature values, and to new values.

We take movie tag features (i.e., multivalent features where each feature value is a subset of the vocabulary) as an example to show how to enhance the encoding in DHE for movie embeddings. The enhanced encoding is obtained by directly concatenating the binary tag vector and the hash encodings. If the dimensionality of the tag vector is too high, we could use locality-sensitive hashing [12] to significantly reduce the cardinality while preserving the tag similarity. The enhanced encoding is then fed into the deep embedding network for generating the movie embedding. We think the hash encoding provides a unique identifier for *memorization* while the tag features enable the *generalization* ability.

We think the enhanced encoding may enable better generalization capability for the movie embedding, due to the generalizable encoding and centralized embedding network (i.e., all the movie embeddings need to go through the same network). Note that using side features for movie embedding generation is different from feeding side features into the recommendation model, mainly because it doesn't solve the generalization problem for the movie embedding. Thus the movie embedding may overfit, and we cannot generalize the movie embedding to new movies. However, the two approaches are compatible to be used at the same time.

### 4 EXPERIMENTS

We conduct extensive experiments to investigate the following research questions:

- RQ1: How does DHE compare against one-hot based full embedding and hash embedding methods?
- **RQ2**: What's the effect of various encoding schemes for DHE?
- **RQ3**: How does the number of hash functions *k* affect DHE and other hashing methods?
- RQ4: What's the influence of different embedding network architectures, depths, normalization and activation functions?
- **RQ5:** What's the effect of the side feature enhanced encoding?
- RQ6: What's the efficiency and GPU acceleration effect of DHE?

### 4.1 Experimental Setup

4.1.1 Dataset. We use two commonly used public benchmark datasets for evaluating recommendation performance:

- Movielens-20M is a widely used benchmark for evaluating collaborative filtering algorithms [20]. The dataset includes 20M user ratings on movies<sup>5</sup>.
- Amazon Books is the largest category in a series of datasets introduced in [42], comprising large corpora of product reviews crawled from *Amazon.com*. We used the latest 5-core version crawled in 2018<sup>6</sup>. The dataset is known for its high sparsity.

The dataset statistics are shown in Table 4. As in [22], we treat all ratings as observed feedback, and sort the feedback according to timestamps. For each user, we withhold their last two actions, and put them into validation set and test set respectively. All the rest are used for model training.

4.1.2 Backbone Recommendation Models. We adopt the Generalized Matrix Factorization (GMF) and Multi-layer Perceptron (MLP) from [23] as the backbone recommendation models to evaluate the performance of different embedding approaches. We use the two methods to represent both shallow and deep recommendation models. GMF is a shallow model that calculates a weighted sum on the element-wise product of user and item embeddings. With equal weights, GMF is reduced to the classic MF method. Conversely, MLP is a deep model that applies several fully-connected layers on the concatenation of user and item embeddings. Similar deep models have been adopted for recommendation and CTR prediction [9, 18]. The MLP we used has three hidden layer (with [256, 128, 64] nodes), and an output layer to generate the d-dim embedding.

<sup>&</sup>lt;sup>5</sup>https://grouplens.org/datasets/movielens/20m/

 $<sup>^6</sup>https://nijianmo.github.io/amazon/index.html\\$ 

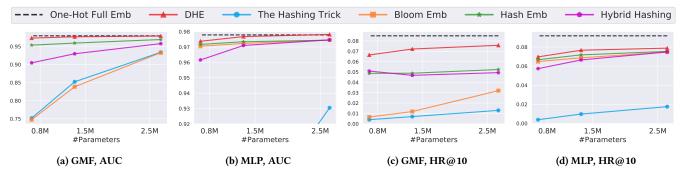


Figure 3: Performance with different model sizes on Movielens. The full embedding method costs about 5M parameters.

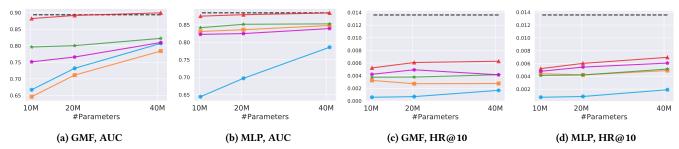


Figure 4: Performance with different model sizes on Amazon. The full embedding method costs about 80M parameters.

**Table 4: Dataset statistics** 

Dataset	#users	#items	total vocab size	#actions	sparsity
MovieLens	138K	27K	165K	20M	99.47%
Amazon	1.9M	0.7M	2.6M	27M	99.99%

4.1.3 Metrics. We use AUC and Hit Rate@10 (HR@10) to evaluate overall and top-K ranking, respectively. AUC [14] is a widely used metric in recommendation [22, 46], and especially in CTR prediction [18, 55]. The AUC measures the probability of ranking pairs of a positive item and negative items in the right order, and thus random guesses achieve an AUC of 0.5. HR@10 is commonly adopted in measuring top-K recommendation performance [22, 43], which counts the fraction of times that the test items are ranked as top-10 items. Note that since we only have one test item for each user, Hit Rate@10 is equivalent to Recall@10 and is proportional to Precision@10. We evaluate the metrics via ranking the full item set, as a recent paper shows that sampling on items will lead to biased estimations [33].

### 4.2 Baselines

The one-hot **Full Embedding** is a standard way to handle categorical features, which uses a dictionary to map each feature value to a unique one-hot vector. However, to adapt to online learning settings where new items constantly appear and stale items gradually vanish, or to reduce storage cost, hashing-based methods are often adopted. We use the follow hashing-based baselines:

- The Hashing Trick [52] A classical approach for handling large-vocab categorical features, which uses a single hash function to map feature value into a smaller vocab. The method often suffers from collision problems.
- Bloom Embedding [47] Inspired by bloom filter [4], Bloom Embedding generates a binary encoding with multiple hash functions. Then a linear layer is applied to the encoding to recover the embedding for the given feature value.
- Hash Embedding (HashEmb) [50] HashEmb uses multiple (typically two) hash functions and lookups the corresponding embeddings. Then a weighted sum of the embeddings is adopted, where the weights are learned and dedicated for each feature value.
- Hybrid Hashing [54] A recently proposed method uses onehot full embedding for frequent feature values, and uses double hashing for others.

We compare our **Deep Hashing Embedding (DHE)** against the above baselines. DHE uses a large number of hash functions (k=1024 in the experiments) to generate a unique identifier for each feature value, followed by a deep embedding network to generate the final embedding. DHE also differs in that it doesn't use any one-hot encoding and embedding table lookup.

Generally, the hashing-based methods are inferior to full embeddings in model performance (but with much smaller model sizes). We seek to investigate the performance gap through the comparisons.

### 4.3 Implementation Details

We implement all the methods using TensorFlow. The embedding dimension d for user and item embeddings is set to 32 for the

best Full Emb performance, searched among {8, 16, 32, 64}. For the recommendation model training, we use the Adam optimizer [31] with a learning rate of 0.001. We apply the embeddings schemes on both user and item embeddings. For HashEmb [50], we use dedicated weights (without collision) for each feature value for better performance. For Hybrid Hashing [54], we use dedicated embeddings for the top 10% of the most frequent feature values, and apply double hashing for the others. By default, we use k=2hash functions for hashing-based baselines (except for the hashing trick [52] which uses a single hash function), which is suggested by the authors [47, 50, 54]. The given model size budget decides the hashed vocabulary size m for hashing-based methods (e.g. a half of the full model size means m=n/2). For DHE, we use k=1024hash functions to generate the hash encoding vector, followed by a 5-layer feedforward neural network with Batch Normalization [27] and Mish activation function [41]. The width  $d_{NN}$  of the network is determined by the given model size. The m in DHE is set to  $10^6$ . To reduce the variance, all the results are the average of the outcomes from 5 experiments. The model training is accelerated with NVIDIA V-100 GPUs.

### 4.4 Performance Comparison (RQ1)

We plot the performance with 1/2, 1/4, and 1/8 of the full model size in Figure 3 and Figure 4 for the two datasets, respectively. We interpret the results via the following comparisons:

- DHE vs. hashing methods: We can see that DHE significantly outperforms hashing-based baselines in all cases. This is attributed to its unique hash encoding, which is free of collision and easy for the embedding network to distinguish. HashEmb [50] is the runner-up method, presumably due to the dedicated learned weights for better embedding aggregations. The Hash Trick [52] performs inferior to other methods, especially when the model size is small. This shows that the hash collisions severely hurt the performance.
- DHE vs. one-hot full emb: For the AUC metric, we observed that DHE effectively approximates Full Embedding's performance. In most cases, DHE achieves similar performance with only 1/4 of the full model size. This verifies the effectiveness of DHE's hash encoding and deep embedding network, and shows that it's possible to remove one-hot encodings and embedding tables without AUC loss. For the HR@10 metric, we found that there is a gap between Full Embedding and all hashing-based methods, especially on the Amazon dataset. We suspect hashing-based methods are not fine-grained enough to capture high-frequency signals and push the desired item to very top positions.

### 4.5 Comparison of Encoding Schemes (RQ2)

We investigate the effect of various encodings (not based on one-hot) that are suitable for DHE. We evaluate DHE with encodings mentioned in Section 3.1: identity encoding (1-dim, normalized into [0,1]), binary encoding, random Fourier feature encoding [51], and our proposed hashing encodings with the uniform or Gaussian distribution, and the results are shown in Table 5. We can see that our proposed hashing-based encodings are the best performers, where the Gaussian distribution variant has slightly better HR@10, and the uniform distribution variant has slightly better AUC. The

binary encoding is the runner-up method, and we think the inferior performance is due to its wrong inductive bias (some IDs have more similar encodings) and the relatively low dimensionality (i.e.,  $\lceil \log(n) \rceil$ ). The results also suggest that the random Fourier features [51] are not suitable for our case due to the difference between our problem and signal processing problems where the latter has an meaningful underlying 1-dim signal. This verifies the effectiveness of the hashing encodings which satisfy the four properties we proposed.

Table 5: Performance of DHE with different dense encoding schemes on Movielens. The memory budget is 1/4 of the full model size.

	Al	IJС	HR@10	
Encoding	GMF	MLP	GMF	MLP
Identity	94.99	95.01	5.35	5.36
Binary	97.38	97.36	6.62	6.81
Random Fourier Feature [51]	94.98	94.99	5.20	5.20
Hashing-Gaussian (ours)	97.62	97.66	7.29	7.69
Hashing-Uniform (ours, default)	97.66	97.68	7.27	7.64

## 4.6 Scalability Regarding the Number of Hash Functions (RQ3)

Both our method DHE and multiple hashing based methods utilize multiple hash functions to reduce collision. However, as existing hashing methods are limited to a few hash functions (typically 2), we investigate the scalability of DHE and the hashing baselines, in terms of the number of hash functions. Table 6 shows the performance with different k, the number of hash functions. Note that the encoding length of DHE is k, the same as the number of hash functions, while the encoding length for one-hot hashing based methods is m\*k.

With a small k (e.g.  $k \le 16$ ), the performance of DHE is inferior to the baselines, mainly because of the short encoding length of DHE (i.e., k). However, when  $k \ge 32$ , DHE is able to match the performance of alternative methods. When k further increases to more than 100, we can still observe performance gains of DHE, while the one-hot hashing baselines don't benefit from more hash functions. We suspect the reason of the poor utilization of multiple hashing is that each embedding will be shared k times more than single hashing (if sharing embedding tables), and this leads to more collisions. If creating k embedding tables (i.e., not sharing), given the same memory budget, the size for each table will be k times smaller, which again causes the collision issue. However, DHE is free of the collision and embedding sharing problems, and thus can scale to a large k.

# 4.7 Normalization and Activation Functions (RQ4)

Training the deep embedding network is much harder than training embedding lookup based shallow methods. We found there is a trainability issue as well as a unique expressiveness issue. We seek to improve the trainability via using normalization techniques, and we found the Batch Normalization (BN) [27] greatly stabilizes

Table 6: Performance with different number of hash functions. The results are the HR@10 (%) of the MLP recommendation model on Movielens. All the models uses 1/4 of the full model size. "-" means the setting exceeds the memory budget.

#hash functions (k)	2	4	8	16	32	128	512	1024	2048
Bloom Emb [47]	6.83	6.94	7.07	7.25	7.12	6.98	6.85	7.13	7.02
Hybrid Hashing [54]	6.86	6.92	7.13	7.19	7.10	7.03	7.02	7.10	7.02
Hash Emb [54]	7.23	7.34	-	-	-	-	-	-	-
DHE	3.86	4.67	6.25	6.85	7.11	7.26	7.52	7.51	7.52

and accelerates the training. For the expressiveness issue, we tried various activation functions for replacing ReLU, as ReLU networks are piece-wise linear functions [1] which may not be suitable for the complex transformation in our task. We found the recently proposed Mish [41] activation consistently outperforms alternatives including ReLU.

Table 7 shows the results of with and without BN and Mish. We omit the results of Layer Normalization [2], and other activation functions, as we didn't observe performance improvement. We can see that both BN and Mish are critical for enabling deep networks for embedding generation, and improving DHE's performance. Note that for fair comparison, we only use BN and Mish for the embedding network in DHE, while use the same recommendation model (e.g. the MLP model) for all embedding methods.

Table 7: The effect of activations functions and normalization. (on Movielens with 1/4 of the full model size)

	AUG	HR@10 (%)		
<b>Activation functions</b>	GMF	MLP	GMF	MLP
Without Batch Normalization				
ReLU	97.33	97.47	6.66	7.01
Mish [41]	97.43	97.50	6.75	7.11
With Batch Normalization				
ReLU	97.54	97.59	7.17	7.47
Mish [41] (default)	97.66	97.67	7.29	7.60

### 4.8 The Effect of Depth (RQ4)

The embedding network in DHE takes a hash encoding vector and applies a deep neural network to generate the d-dim output embedding. Specifically, the embedding network consists of several hidden layers with  $d_{\rm NN}$  nodes, followed by an output layer with d nodes. We investigate whether deeper embedding networks are more effective against wide & shallow networks, via varying the number of hidden layers while keeping the same number of parameters. Figure 5 shows the results on Movielens. We observed that embedding networks with around five hidden layers are significantly better than wider and shallower networks. This is consistent with our motivation and theoretical results in [36], that deep networks are more parameter-efficient than shallow networks. However, we didn't see further improvement with more hidden layers, presumably because each layer's width is too narrow or due to trainability issues on deep networks.

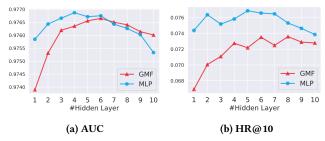


Figure 5: Performance with different depths on Movielens. All the datapoints are with the same #params (1/4 of the full model size). The network depth is #Hidden Layer plus one, where the last layer is the output layer for generating the embedding.

### 4.9 Neural Architectures (RQ4)

The default neural architecture for DHE is equal-width MLP, where each hidden layer has  $d_{NN}$  nodes. We also explore various architectures including Pyramid MLP (the width of a hidden layer is twice that of the previous layer), Inverted Pyramid MLP (opposite to Pyramid MLP), DenseNet [25]-like MLP (concatenate all previous layers' output as the input at each layer), and equal-width MLP with residual connections [21]. We adjust the width to make sure all the variants have the same number of parameters. The performance results are shown in Table 8. We can see that the simple equalwidth MLP performs the best, and adding residual connections also slightly hurts the performance. We suspect that the low-level representations are not useful in our case, so that the attempts (as in computer vision) utilizing low-level features (like DenseNet [25] or ResNet [21]) didn't achieve better performance. The (inverted) Pyramid MLPs also perform worse than the equal-width MLP, perhaps more tuning on the width multiplier (we used 2 and 0.5) is needed. The results also show it's challenging to design architectures for the embedding generalization tasks, as we didn't found useful prior to guide our designs.

Table 8: Performance with different neural architectures for the embedding network in DHE. All variants use 5 hidden layers with BN and Mish, and have the same number of parameter. (on Movielens with 1/4 of the full model size).

	AUG	C (%)	HR@10 (%)	
Emb Network	GMF	MLP	GMF	MLP
Pyramid MLP	97.20	97.49	6.39	7.14
Inverted Pyramid MLP	97.50	97.58	7.18	7.36
DenseNet-like MLP	97.53	97.50	6.89	7.18
Residual Equal-Width MLP	97.61	97.60	7.14	7.43
Equal-Width MLP (default)	97.66	97.68	7.27	7.64

### 4.10 Side Feature Enhanced Encodings (RQ5)

The encoding in DHE can be enhanced by side features. We use the 20 movie *Genres* (e.g. 'Comedy', 'Romance', etc.) in the Movielens dataset, as the side feature. Each movie has zero, one, or multiple genres, and we represent the feature with a 20-dim binary vector. The side features can be used in the encoding function of DHE,

and/or directly plugged into the MLP recommendation model (i.e., the MLP takes user, item, and genres vectors as the input).

The results are shown in Table 9. We can see that using side features only in the encoding and only in the MLP have similar performance. This shows DHE's item embeddings effectively capture the *Genres* information, and verifies the generalization ability of item embeddings generated by DHE with enhanced encodings. However, we didn't see further improvement of using the feature in both encoding and MLP. For other embedding methods, adding the feature to the MLP is helpful. However, unlike DHE, they fully rely on IDs and are unable to generate generalizable item embeddings.

Table 9: The effect of side feature enhanced encoding (on Movielens with 1/4 of the full model size).

	N	MLP	MLP (w	vith genres)
Item Embedding	AUC	HR@10	AUC	HR@10
DHE Encoding				
hash encoding (ID) only	97.67	7.64	97.72	7.62
Genres only	79.17	2.35	79.16	2.40
hash encoding (ID) +Genres	97.71	7.69	97.73	7.56
Hash Emb	97.34	7.12	97.42	7.17
Hybrid Hashing	97.22	6.87	97.31	6.84

### 4.11 Efficiency (RQ6)

One potential drawback of DHE is computation efficiency, as the neural network module in DHE requires a lots of computing resources. However, this is a common problem in all deep learning models, and we hope the efficiency issue could be alleviated by powerful computation hardware (like GPUs and TPUs, optimized for neural networks) that are improving very fast recently. We show the efficiency results in Table 10. With GPUs, DHE is about 9x slower than full embedding, and 4.5x slower than hash embeddings. However, we can see that DHE significantly benefits from GPU acceleration, while full embeddings don't. This is because the embedding lookup process in full embeddings is hard to accelerate by GPUs. The result conveys a promising message that more powerful computation hardware in the future could further accelerate DHE, and gradually close the efficiency gap. Moreover, DHE could also potentially benefit from NN acceleration methods, like quantization [19], pruning [15], distillation [24], binarization [26], etc.

Table 10: Time (in seconds) of embedding generation for 1M queries with a batch size of 100. Hash Emb and DHE uses 1/4 of the full model size.

	CPU	GPU	GPU Acceleration
Full Emb	3.4	3.4	-1%
Hash Emb	8.4	6.1	-26%
DHE	76.1	27.2	-64%

### 5 RELATED WORK

Embedding learning has been widely adopted, and representative examples include word2vec [40] and Matrix Factorization (MF) [32].

Other than 'shallow models,' embeddings are also the key component of deep models, like word embeddings for BERT [13]. There are various work on improving the performance or efficiency of embedding learning, via regularization [3, 8], dimensionality search [28], factorization [35], etc. These methods are built on top of the standard one-hot encoding, which don't change the underlying encoding representations.

The hashing trick [52] is a classic method enabling handling large-vocab features and out-of-vocab feature values with one-hot encodings. As only a single hash function is adopted, the collision issue becomes severe when the number of hashing buckets is small. To alleviate this, various improved methods [47, 50, 54] are proposed based on the idea of using multiple hash functions to generate multiple one-hot encodings. Our method also adopts hash functions for generating the encoding. However, our method doesn't rely on one-hot encodings. Also, our approach is able to scale to use a large number of hash functions, while existing methods are limited to use a few (typically two) hash functions.

There is an orthogonal line of work using similarity-preserving hashing for embedding learning. For example, HashRec [30] learns preference-preserving binary representation for efficient retrieval, where a low hamming distance between the embeddings of a user and an item indicates the user may prefer the item. Some other methods utilize locality-sensitive hashing [12] to reduce feature dimensions while maintaining their similarities in the original feature spaces [34, 45]. The main difference is that the hashing we used are designed for reducing collision, while the hashing used in these methods seeks to preserve some kind of similarity.

Very recently, a new line of work on coordinate-based MLPs has been explored in computer vision and computer graphics for modeling high-frequency signals in low-dimensional domains [49, 51], including audio signals (1-dim), images (2-dim), and 3D shapes (3-dim), etc. The ID features (e.g. {1,2,...,n}) in our case could also be viewed as a 1-dimensional signal, and we seek to recover the embedding at each time-division. However, as the 1-dim signal in our case has no meaningful implication (e.g. adjacent IDs have similar embeddings), these approaches are not suitable for our problem. We also empirically verified this with the random Fourier features [51] and sinusoidal representation networks (SIREN) [49], proposed in this line of research.

### 6 CONCLUSIONS AND FUTURE WORK

In this work, we revisited the widely adopted one-hot based embedding methods, and proposed an alternative embedding framework (DHE), based on dense hash encodings and deep neural networks. DHE does not lookup embeddings, and instead computes embeddings on the fly through its hashing functions and an embedding network. This avoids creating and maintaining huge embedding tables for training and serving. With multiple hash functions, the dense hash encoding generates a unique high-dimensional vector for each feature value, and can be computed on-the-fly without keeping any parameters for the encodings. Empirical results show that DHE outperforms hashing-based methods and achieves comparable AUC performance against full embeddings, with much smaller model sizes. As a DNN-based embedding framework, DHE could

benefit significantly from future deep learning advancement in modeling and hardware, that will further improve DHE's performance and efficiency.

In the future, we plan to investigate several directions for extending and improving DHE: (i) handling multivalent features like bag-of-words; (ii) jointly modeling multiple features with DHE; (iii) hybrid approaches using both embedding tables and neural networks, for balancing efficiency and performance.

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### **Appendix**

### A Analysis on Encoding Properties

We formally define and analyze the encoding properties. For demonstration, we use a setting similar to what we used in the experiments:  $n=10^6$ ,  $m=10^6$ , and k=1024.

### A.1 Uniqueness

*Definition .1 (Uniqueness in encoding).* An encoding functions E is a unique encoding if  $P(E(x)) = P(E(y)) < \epsilon, \forall x, y \in V$ , where  $\epsilon$  is a near-zero constant.

Obviously the *identity encoding*, *one-hot encoding*, and *binary encoding* satisfy the uniqueness property.

For hashing methods, the probability of having collision is  $1-e^{-\frac{n(n-1)}{2m}}$  where m is the total number of hashing buckets ( $m^2$  buckets for double hashing), according to [7]. The probability is 1.0, and 0.39 for *one-hot hashing* and *double one-hot hashing*, respectively. For DHE, the number of possible hashing buckets is  $m^k=10^{6144}$ , and the collision rate is extremely small. Thus we can safely assume there is no collision.

### A.2 Equal Similarity

Definition .2 (Equal similarity in encoding). An encoding functions E is a equally similar encoding if  $\mathbb{E}[\text{Euclidean\_distance}(E(x) - E(y))] = c, <math>\forall x, y \in V$ , where c is a non-zero constant.

Obviously the *identity encoding*, and *binary encoding* doesn't satisfy the property.

For one-hot based hashing methods, the expected Euclidean distance is  $k * \frac{m-1}{m}$ . For DHE, the expectation is:

$$\mathbb{E}[(E(x) - E(y))^{2}] = \mathbb{E}[E^{2}(x) - 2E(x)E(y) + E^{2}(y)]$$

$$= \mathbb{E}[E^{2}(x)] - 2\mathbb{E}[E(x)]\mathbb{E}[E(y)] + \mathbb{E}[E^{2}(y)]$$

$$= \frac{m(2m+1)(m+1)}{3} - \frac{(m+1)^{2}}{2}$$

### A.3 High Dimensionality

This is a subjective property, and we generally think larger than 100-dim can be considered as high-dimensional spaces. Following this, the 1-dim *identity encoding* and the  $\lceil \log n \rceil$ -dim *binary encoding* doesn't satisfy the property.

### A.4 High Shannon Entropy

Definition .3 (High Shannon Entropy). An encoding functions E has the high entropy property if for any dimension i, the entropy  $H(E(x)_i) = H^*$ ,  $(x \in V)$ , where  $H^* = \log o$  is the max entropy for o outcomes (e.g.  $H^* = 1$  for binary outcome).

As the zeros and ones are uniformly distributed at each dimension in *binary encoding*, the entropy equals to  $H^* = 1$ . Similarly, the entropy of *identity encoding* also reaches the maximal entropy  $H = -\sum_{i=1}^{n} \frac{1}{n} \log \frac{1}{n} = \log n = H^*$ .

 $H=-\sum_{i=1}^n \frac{1}{n} \log \frac{1}{n} = \log n = H^*.$  For one-hot full embedding, at each dimension, the probability is  $\frac{1}{n}$  for having 1, and  $\frac{n-1}{n}$  for having 0. So the entropy  $H=-\frac{1}{n}\log \frac{1}{n}-\frac{n-1}{n}\log \frac{n-1}{n}$ , which quickly converges to zero with a large n. The entropy is significantly less than  $H^*=1$ 

For one-hot hashing, the probability of having ones is  $\frac{n}{m} \cdot \frac{1}{n} = \frac{1}{m}$ , and for zeros it's  $\frac{m-1}{m}$ . Therefore the entropy  $H = -\frac{1}{m} \log \frac{1}{m}$ 

Deep Hash Embedding for Large-Vocab Categorical Feature Representations

 $\frac{m-1}{m}\log\frac{m-1}{m},$  which is near zero due to the large m. Double one-hot hashing has a similar conclusion.

For DHE, at each dimension, the encodings are uniformly distributed among  $[m] = \{1, 2, ..., m\}$ . Therefor the entropy  $H = \{1, 2, ..., m\}$ 

 $-\sum_{i=1}^m \frac{1}{m}\log \frac{1}{m} = \log m = H^*,$  which reaches the maximal entropy.