Field Guide to

Geometrical Optics

John E. Greivenkamp

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University of Arizona

SPIE Field Guides
Volume FG01

John E. Greivenkamp, Series Editor

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Welcome to the *SPIE Field Guides*! This volume is one of the first in a new series of publications written directly for the practicing engineer or scientist. Many textbooks and professional reference books cover optical principles and techniques in depth. The aim of the *SPIE Field Guides* is to distill this information, providing readers with a handy desk or briefcase reference that provides basic, essential information about optical principles, techniques, or phenomena, including definitions and descriptions, key equations, illustrations, application examples, design considerations, and additional resources. A significant effort will be made to provide a consistent notation and style between volumes in the series.

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John E. Greivenkamp, *Series Editor*
Optical Sciences Center
The University of Arizona

Field Guide to Geometrical Optics

The material in this *Field Guide to Geometrical Optics* derives from the treatment of geometrical optics that has evolved as part of the academic programs at the Optical Sciences Center at the University of Arizona. The development is both rigorous and complete, and it features a consistent notation and sign convention. This material is included in both our undergraduate and graduate programs. This volume covers Gaussian imagery, paraxial optics, first-order optical system design, system examples, illumination, chromatic effects and an introduction to aberrations. The appendices provide supplemental material on radiometry and photometry, the human eye, and several other topics.

Special acknowledgement must be given to Roland V. Shack and Robert R. Shannon. They first taught me this material "several" years ago, and they have continued to teach me throughout my career as we have become colleagues and friends. I simply cannot thank either of them enough.

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This *Field Guide* is dedicated to my wife, Kay, and my children, Jake and Katie. They keep my life in focus (and mostly aberration free).

John E. Greivenkamp
Optical Sciences Center
The University of Arizona

Table of Contents

Glossary x

Fundamentals of Geometrical Optics 1

Sign Conventions 1
Basic Concepts 2
Optical Path Length 3
Refraction and Reflection 4
Optical Spaces 5
Gaussian Optics 6
Refractive and Reflective Surfaces 7
Newtonian Equations 8
Gaussian Equations 9
Longitudinal Magnification 10
Nodal Points 11
Object-Image Zones 12
Gaussian Reduction 13
Thick and Thin Lenses 14
Vertex Distances 15
Thin Lens Imaging 16
Object-Image Conjugates 17
Afocal Systems 18
Paraxial Optics 19
Paraxial Raytrace 20
YNU Raytrace Worksheet 21
Cassegrain Objective Example 22
Stops and Pupils 24
Marginal and Chief Rays 25
Pupil Locations 26
Field of View 27
Lagrange Invariant 28
Numerical Aperture and F-Number 29
Ray Bundles 30
Vignetting 31
More Vignetting 32
Telecentricity 33
Double Telecentricity 34
Depth of Focus and Depth of Field 35
Hyperfocal Distance and Scheimpflug Condition 36

Table of Contents (cont.)

Optical Systems 37

- Parity and Plane Mirrors 37
- Systems of Plane Mirrors 38
- Prism Systems 39
- More Prism Systems 40
- Image Rotation and Erection Prisms 41
- Plane Parallel Plates 42
- Objectives 43
- Zoom Lenses 44
- Magnifiers 45
- Keplerian Telescope 46
- Galilean Telescope 47
- Field Lenses 48
- Eyepieces 49
- Relays 50
- Microscopes 51
- Microscope Terminology 52
- Viewfinders 53
- Single Lens Reflex and Triangulation 54
- Illumination Systems 55
- Diffuse Illumination 56
- Integrating Spheres and Bars 57
- Projection Condenser System 58
- Source Mirrors 59
- Overhead Projector 60
- Schlieren and Dark Field Systems 61

Chromatic Effects 62

- Dispersion 62
- Optical Glass 63
- Material Properties 64
- Dispersing Prisms 65
- Thin Prisms 66
- Thin Prism Dispersion and Achromatization 67
- Chromatic Aberration 68
- Achromatic Doublet 69

Table of Contents (cont.)

Monochromatic Aberrations 70
- Monochromatic Aberrations 70
- Rays and Wavefronts 71
- Spot Diagrams 72
- Wavefront Expansion 73
- Tilt and Defocus 74
- Spherical Aberration 75
- Spherical Aberration and Defocus 76
- Coma 77
- Astigmatism 78
- Field Curvature 79
- Distortion 80
- Combinations of Aberrations 81
- Conics and Aspherics 82
- Mirror-Based Telescopes 83

Appendices 84
- Radiometry 84
- Radiative Transfer 85
- Photometry 86
- Sources 87
- Airy Disk 88
- Diffraction and Aberrations 89
- Eye 90
- Retina and Schematic Eyes 91
- Ophthalmic Terminology 92
- More Ophthalmic Terminology 93
- Film and Detector Formats 94
- Photographic Systems 95
- Scanners 96
- Rainbows and Blue Skies 97
- Matrix Methods 98
- Common Matrices 99
- Trigonometric Identities 100
- Equation Summary 101

Bibliography 107
Index 111

Glossary

Unprimed variables and symbols are in object space.
Primed variables and symbols are in image space.

Frequently used variables and symbols:

a	Aperture radius
A, A'	Object and image areas
B'	Image plane blur criterion
BFD	Back focal distance
c	Speed of light
C	Curvature
CC	Center of curvature
d, d'	Front and rear principal plane shifts
D	Diopters
D	Diameter
D	Airy disk diameter
DOF	Depth of focus, geometrical
E, E_V	Irradiance and illuminance
EFL	Effective focal length
EP	Entrance pupil
ER	Eye relief
f, f_E	Focal length or effective focal length
f_F, f'_R	Front and rear focal lengths
$f/\#$	F-number
$f/\#_W$	Working F-number
δf	Longitudinal chromatic aberration
F, F′	Front and rear focal points
FFD	Front focal distance
FFOV	Full field of view
FOB	Fractional object
FOV	Field of view
h, h'	Object and image heights
H	Lagrange invariant
H	Normalized field height
H, H_V	Exposure
HFOV	Half field of view
I	Optical invariant
I, I_V	Intensity and luminous intensity
L	Object-to-image distance
L, L_V	Radiance and luminance

Glossary (cont.)

L_H	Hyperfocal distance
L_{NEAR}, L_{FAR}	Depth of field limits
LA	Longitudinal aberration
m	Transverse or lateral magnification
$\overline{m}$	Longitudinal magnification
m_V	Visual magnification (microscope)
M, M_V	Exitance and luminous exitance
MP	Magnifying power (magnifier or telescope)
MTF	Modulation transfer function
n	Index of refraction
N, N'	Front and rear nodal points
NA	Numerical aperture
OPL	Optical path length
OTL	Optical tube length
P	Partial dispersion ratio
P, P′	Front and rear principal points
PSF	Point spread function
Q	Energy
r_P	Pupil radius
R	Radius of curvature
s	Surface sag or a separation
s, s'	Object and image vertex distances
S	Seidel aberration coefficient
SR	Strehl ratio
t	Thickness
T	Temperature
TA	Transverse aberration
$\mathrm{TA_{CH}}$	Transverse axial chromatic aberration
TIR	Total internal reflection
Δt	Exposure time
u, $\bar{u}$	Paraxial angles; marginal and chief rays
U	Real marginal ray angle
V	Abbe number
V, V′	Surface vertices
W	Wavefront error
W_{IJK}	Wavefront aberration coefficient
WD	Working distance
x, y	Object coordinates
x', y'	Image coordinates

Glossary (cont.)

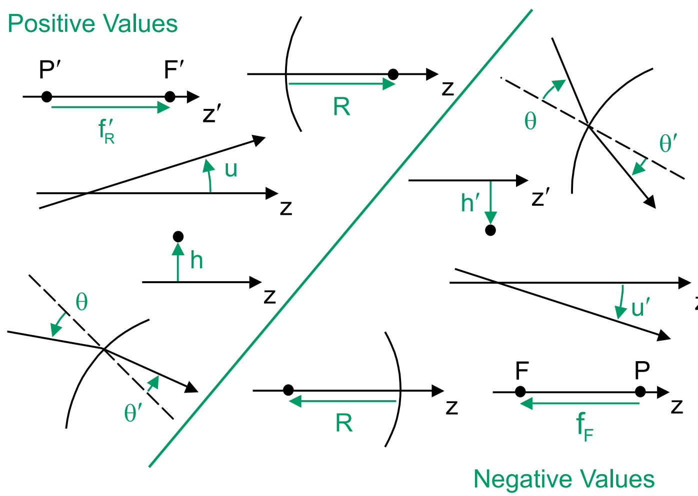
x_P, x_P	Normalized pupil coordinates
XP	Exit pupil
$y, \bar{y}$	Paraxial ray heights; marginal and chief rays
z	Optical axis
z, z'	Object and image distances
δz	Image plane shift
δz	Depth of focus, diffraction
$\Delta z, \Delta z'$	Object and image separations
α	Dihedral angle or prism angle
δ	Prism deviation
δ_{MIN}	Angle of minimum deviation
$\delta\phi$	Longitudinal chromatic aberration
Δ	Prism dispersion
ε	Prism secondary dispersion
$\varepsilon_X, \varepsilon_Y$	Transverse ray errors
ε_Z	Longitudinal ray error
θ	Angle of incidence, refraction or reflection
θ	Azimuth pupil coordinate
θ_C	Critical angle
$\theta_{1/2}$	Half field of view angle
κ	Conic constant
λ	Wavelength
ν	Abbe number
ρ	Reflectance
ρ	Normalized pupil radius
τ	Reduced thickness
ϕ	Optical power
Φ, Φ_V	Radiant and luminous power
$\omega, \bar{\omega}$	Optical angles; marginal and chief rays
Ω	Solid angle
Ж	Lagrange invariant

Sign Conventions

Throughout this **Field Guide**, a set of fully consistent **sign conventions** is utilized. This allows the signs of results and variables to be easily related to the diagram or to the physical system.

- The axis of symmetry of a rotationally symmetric optical system is the **optical axis** and is the z-axis.
- All distances are measured relative to a reference point, line, or plane in a Cartesian sense: **directed distances** above or to the right are positive; below or to the left are negative.
- All angles are measured relative to a reference line or plane in a Cartesian sense (using the right-hand rule): counterclockwise angles are positive; clockwise angles are negative.
- The **radius of curvature** of a surface is defined to be the directed distance from its vertex to its center of curvature.
- Light travels from left to right (from –z to +z) in a medium with a positive index of refraction.
- The signs of all indices of refraction following a reflection are reversed.

To aid in the use of these conventions, all directed distances and angles are identified by arrows with the tail of the arrow at the reference point, line, or plane.



Basic Concepts

Geometrical optics is the study of light without diffraction or interference. Any object is comprised of a collection of independently radiating point sources.

First-order optics is the study of perfect optical systems, or optical systems without aberrations. Analysis methods include **Gaussian optics** and **paraxial optics**. Results of these analyses include the imaging properties (image location and magnification) and the radiometric properties of the system.

Aberrations are the deviations from perfection of the optical system. These aberrations are inherent to the design of the optical system, even when perfectly manufactured. Additional aberrations can result from manufacturing errors.

Third-order optics (and higher-order optics) includes the effects of aberrations on the system performance. The image quality of the system is evaluated. The effects of diffraction are sometimes included in the analysis.

Index of refraction n:

$$n \equiv \frac{\text{Speed of Light in Vacuum}}{\text{Speed of Light in Medium}} = \frac{c}{v} \qquad v = \frac{c}{n}$$

$$c = 2.99792458 \times 10^8 \text{ m/s}$$

> Following a reflection, light propagates from right to left, and its velocity can be considered to be negative. Using velocity instead of speed in the definition of n, the index of refraction is now also negative.

Wavelength λ and **frequency** ν:

$$\lambda = \frac{v}{\nu} \qquad \text{in vacuum: } \lambda = \frac{c}{\nu}$$

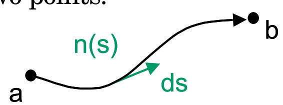
The **wavenumber** w is the number of wavelengths per cm.

$$w = \frac{1}{\lambda} \quad \text{units of cm}^{-1}$$

Optical Path Length

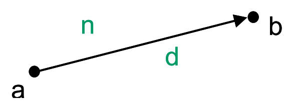
Optical path length OPL is proportional to the time required for light to travel between two points.

$$OPL = \int_a^b n(s)ds$$



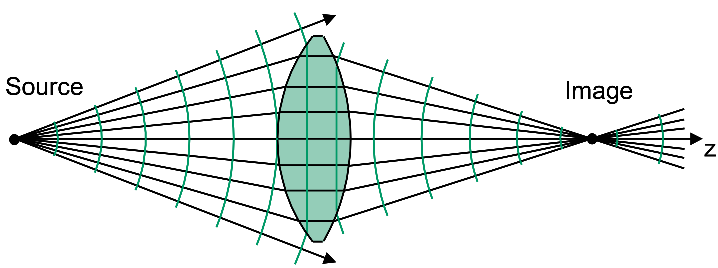
In a homogeneous medium:

$$OPL = nd$$



Wavefronts are surfaces of constant OPL from the source point.

Rays indicate the direction of energy propagation and are normal to the wavefront surfaces.



In a perfect optical system or a first-order optical system, all wavefronts are spherical or planar.

Fermat's principle: The path taken by a light ray in going from point a to point b through any set of media is the one that renders its OPL equal, in the first approximation, to other paths closely adjacent to the actual path.

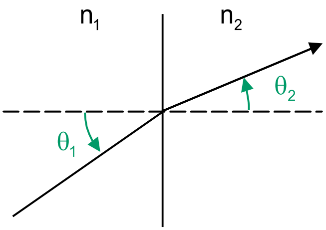
The OPL of the actual ray is either an extremum (a minimum or a maximum) with respect to the OPL of adjacent paths or equal to the OPL of adjacent paths.

In a medium of uniform index, light rays are straight lines.

In a first-order or paraxial imaging system, all of the light rays connecting a source point to its image have equal OPLs.

Refraction and Reflection

Snell's law of refraction:

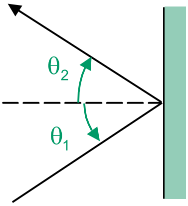


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

The incident ray, the refracted ray and the surface normal are coplanar.

When propagating through a series of parallel interfaces, the quantity $n \sin\theta$ is conserved.

Law of reflection:



$$\theta_1 = -\theta_2$$

The incident ray, the reflected ray and the surface normal are coplanar.

Reflection equals refraction with $n_2 = -n_1$.

Total internal reflection TIR occurs when the angle of incidence of a ray propagating from a higher index medium to a lower index medium exceeds the **critical angle**.

$$\sin\theta_C = \frac{n_2}{n_1}$$

At the critical angle, the angle of refraction θ_2 equals 90°

n_1	θ_C
1.3	50.3°
1.4	45.6°
1.5	41.8°
1.6	38.7°
1.7	36.0°
1.8	33.7°
1.9	31.8°
2.0	30.0°

Critical angles for $n_2 = 1.0$

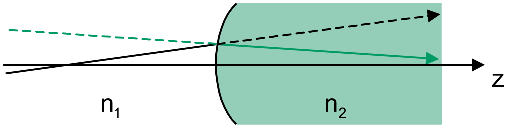
The **reflectance** ρ of an interface between n_1 and n_2 is given by the **Fresnel reflection coefficients**. At normal incidence with no absorption,

$$\rho = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$

Optical Spaces

Any optical surface creates two **optical spaces**: an **object space** and an **image space**. Each optical space extends from –∞ to +∞ and has an associated index of refraction. There are real and virtual segments of each optical space.

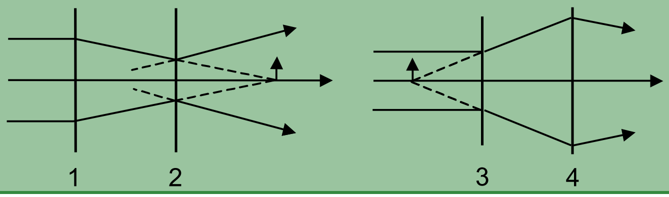
Rays can be traced from optical space to optical space. Within any optical space, a ray is straight and extends from –∞ to +∞ with real and virtual segments. Rays from adjoining spaces meet at the common optical surface.



A **real object** is to the left of the surface; a **virtual object** is to the right of the surface. A **real image** is to the right of the surface; a **virtual image** is to the left of the surface. In an optical space with a negative index (light propagates from right to left), left and right are reversed in these descriptions of real and virtual.

If a system has N optical surfaces, there are N + 1 optical spaces. A single object or image exists in each space. The real segment of an optical space is the volume between the surfaces defining entry and exit into that space. It is also common to combine multiple optical surfaces into a single element and only consider the object and image spaces of the element; the intermediate spaces within the element are ignored.

In a multi-element system, the use of real and virtual may become less obvious. For example, the real image formed by Surface 1 becomes virtual due to the presence of Surface 2, and this image serves as the virtual object for Surface 2. In a similar manner, the virtual image produced by Surface 3 can be considered to be a real object for Surface 4.



Gaussian Optics

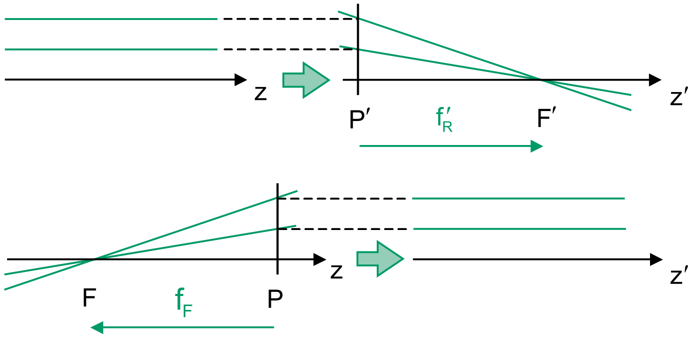
Gaussian optics treats imaging as a mapping from object space into image space. It is a special case of a **collinear transformation** applied to rotationally symmetric systems, and it maps points to points, lines to lines and planes to planes. The corresponding object and image elements are called **conjugate elements**.

- Planes perpendicular to the axis in one space are mapped to planes perpendicular to the axis in the other space.
- Lines parallel to the axis in one space map to conjugate lines in the other space that either intersect the axis at a common point (**focal system**), or are also parallel to the axis (**afocal system**).
- The **transverse magnification** or **lateral magnification** is the ratio of the image point height from the axis h' to the conjugate object point height h:

$$m \equiv \frac{h'}{h}$$

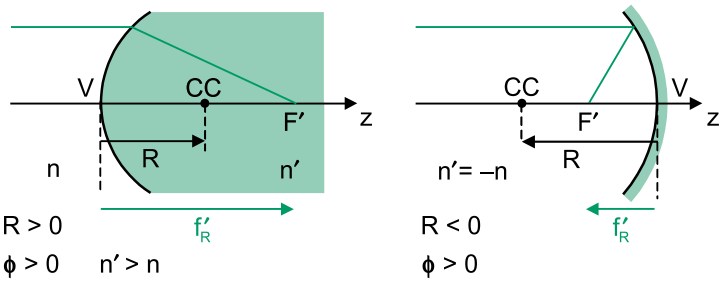
The **cardinal points and planes** completely describe the focal mapping. They are defined by specific magnifications:

F	**Front focal point/plane**	$m = \infty$
F′	**Rear focal point/plane**	$m = 0$
P	**Front principal plane**	$m = 1$
P′	**Rear principal plane**	$m = 1$



The **front and rear focal lengths** (f_F and f'_R) are defined as the directed distances from the front and rear principal planes to the respective focal points.

Refractive and Reflective Surfaces



The **radius of curvature** R of a surface is defined to be the distance from its vertex to its center of curvature CC.

The front and rear principal planes (P and P′) of an optical surface are coincident and located at the **surface vertex** V.

Power of an optical surface:

$$\phi = (n'-n)C = \frac{(n'-n)}{R}$$

Curvature:

$$C = \frac{1}{R}$$

The **effective** (or **equivalent**) **focal length** (EFL or f_E) is defined as

$$f = f_E \equiv \frac{1}{\phi}$$

The "effective" in EFL is actually unnecessary; this quantity is the **focal length** f. The front and rear focal lengths are related to the EFL:

$$f_F = -\frac{n}{\phi} = -nf_E \qquad f'_R = \frac{n'}{\phi} = n'f_E$$

$$f_E = -\frac{f_F}{n} = \frac{f'_R}{n'} \qquad \frac{f'_R}{f_F} = -\frac{n'}{n}$$

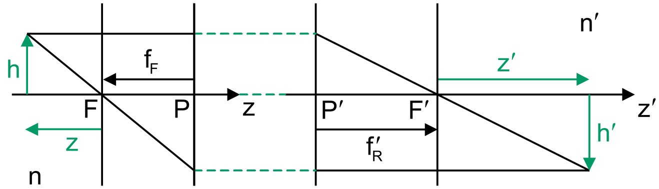
A **reflective surface** is a special case with $n' = -n$:

$$\phi = -2nC = -\frac{2n}{R}$$

$$f_F = f'_R = -\frac{n}{\phi} = -nf_E = \frac{R}{2} = \frac{1}{2C}$$

Newtonian Equations

For a focal imaging system, an object plane location is related to its conjugate image plane location through the transverse magnification associated with those planes. The **Newtonian equations** characterize this Gaussian mapping when the axial locations of the conjugate object and image planes are measured relative to the respective focal points. By definition, the front and rear focal lengths continue to be measured relative to the principal planes. The Newtonian equations result from the analysis of similar triangles.



$$z = -\frac{f_F}{m} \qquad z' = -mf'_R \qquad zz' = f_F f'_R$$

$$\frac{z}{n} = \frac{f_E}{m} \qquad \frac{z'}{n'} = -mf_E \qquad \left(\frac{z}{n}\right)\left(\frac{z'}{n'}\right) = -f_E^2$$

The front and rear focal points map to infinity ($m = \infty$ and 0). The two principal planes are conjugate to each other ($m = 1$).

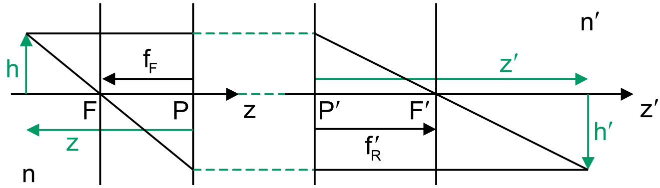
The cardinal points, and the associated focal lengths and power, completely specify the mapping from object space into image space for a focal system. Gaussian imagery aims to reduce any focal imaging system, regardless of the number of surfaces, to a single, unique set of cardinal points.

The EFL of a system is determined from its front or rear focal length in the same manner used for a single surface:

$$f_E = -\frac{f_F}{n} = \frac{f'_R}{n'} \qquad f = f_E \equiv \frac{1}{\phi}$$

Gaussian Equations

The **Gaussian equations** describe the focal mapping when the respective principal planes are the references for measuring the locations of the conjugate object and image planes.



$$z = -\frac{(1-m)}{m}f_F \qquad \frac{z}{n} = \frac{(1-m)}{m}f_E$$

$$z' = (1-m)f'_R \qquad \frac{z'}{n'} = (1-m)f_E$$

$$m = -\left(\frac{z'}{z}\right)\left(\frac{f_F}{f'_R}\right) \qquad m = \frac{z'/n'}{z/n}$$

$$\frac{f'_R}{z'} + \frac{f_F}{z} = 1 \qquad \frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E}$$

When the Newtonian and Gaussian equations are expressed in terms of the EFL or power (f_E or ϕ), all of the axial distances appear as a ratio of the physical distance to the index of refraction in the same optical space. This ratio is called a **reduced distance** and is usually denoted by a Greek letter, for example τ represents the reduced distance associated with the thickness t:

$$\tau = \frac{t}{n}$$

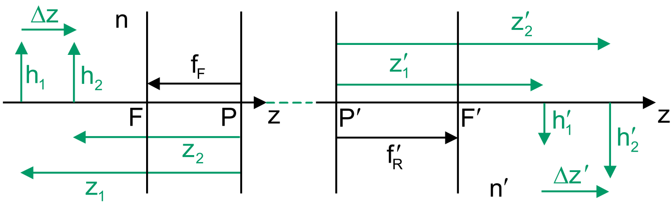
The EFL is the reduced focal length: it equals the reduced rear focal length or minus the reduced front focal length.

A ray angle multiplied by the refractive index of its optical space is called an **optical angle**:

$$\omega = nu$$

Longitudinal Magnification

The **longitudinal magnification** relates the distances between pairs of conjugate planes.



$$\Delta z = z_2 - z_1 \qquad \Delta z' = z'_2 - z'_1$$

$$m_1 = \frac{h'_1}{h_1} \qquad m_2 = \frac{h'_2}{h_2}$$

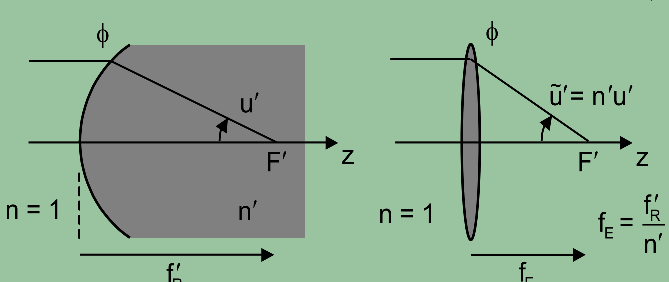
$$\frac{\Delta z'}{\Delta z} = -\left(\frac{f'_R}{f_F}\right)m_1 m_2 \qquad \frac{\Delta z'/n'}{\Delta z/n} = m_1 m_2$$

These equations are valid for widely separated planes. As the plane separation approaches zero, the local longitudinal magnification $\overline{m}$ is obtained.

$$\overline{m} = \left(\frac{n'}{n}\right)m^2 \qquad \frac{\Delta z'/n'}{\Delta z/n} = m^2$$

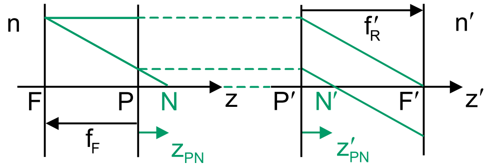
Since m varies with position, $\overline{m}$ is a function of z and z'.

The use of reduced distances and optical angles allows a system to be represented as an air-equivalent system with thin lenses. Consider the example of a refracting surface and its thin lens equivalent. Both have the same power ϕ.



Nodal Points

Two additional cardinal points are the front and rear **nodal points** (N and N′) that define the location of unit angular magnification for a focal system. A ray passing through one nodal point of a system is mapped to a ray passing through the other nodal point having the same angle with respect to the optical axis.



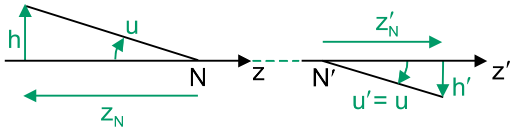
$$z_{PN} = z'_{PN} = f_F + f'_R$$

$$z_{PN} = z'_{PN} = (n' - n)f_E \qquad m_N = -\frac{f_F}{f'_R} = \frac{n}{n'}$$

Both nodal points of a single refractive or reflective surface are located at the center of curvature of the surface:

$$z_{PN} = z'_{PN} = R$$

The angular subtense of an image as seen from the rear nodal point equals the angular subtense of the object as seen from the front nodal point.



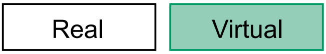
$$m \equiv \frac{h'}{h} = \frac{z'_N}{z_N}$$

If $n = n'$, $z_{PN} = z'_{PN} = 0$, and the nodal points are coincident with the respective principal planes. The magnification relationship now holds for the Gaussian object and image distances (z and z' are measured relative to P and P′):

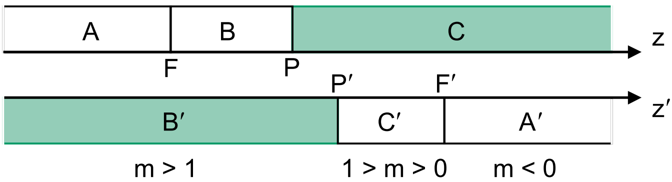
$$m \equiv \frac{h'}{h} = \frac{z'}{z} \qquad \text{when} \qquad n = n'$$

Object-Image Zones

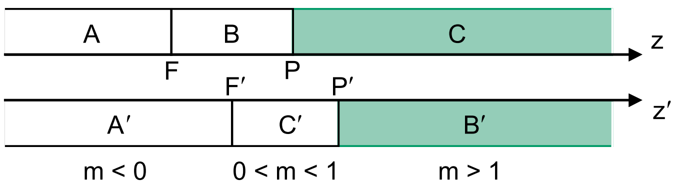
The **object-image zones** show the general image properties as a function of the object location relative to the cardinal points. An object in *Zone A* will map to an image in *Zone A′*, etc. All optical spaces extend from $-\infty$ to $+\infty$. A net reflective system (an odd number of reflections) inverts image space about P′.



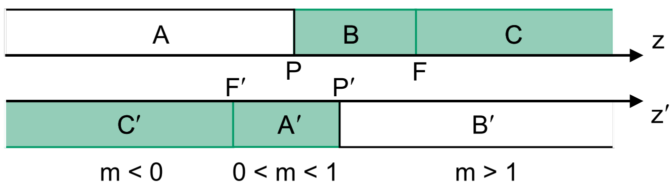
Positive Focal System $\phi > 0;\ n' > 0$



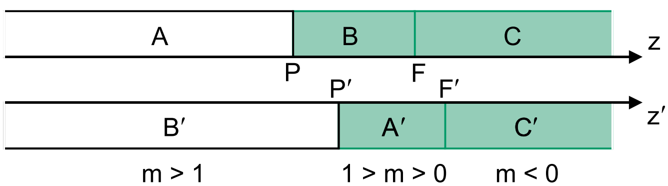
Positive Focal System – Reflective $\phi > 0;\ n' < 0$



Negative Focal System $\phi < 0;\ n' > 0$



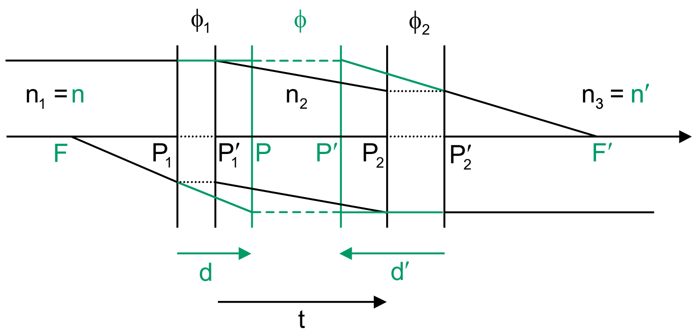
Negative Focal System – Reflective $\phi < 0;\ n' < 0$



Gaussian Reduction

Gaussian reduction is the process that combines multiple elements two at a time into a single equivalent focal system.

Two-component system:



The highlighted rays and quantities are associated with the equivalent reduced system.

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau \qquad \tau = \frac{t}{n_2}$$

$$\frac{d}{n} = \frac{\phi_2}{\phi}\tau \qquad \frac{d'}{n'} = -\frac{\phi_1}{\phi}\tau$$

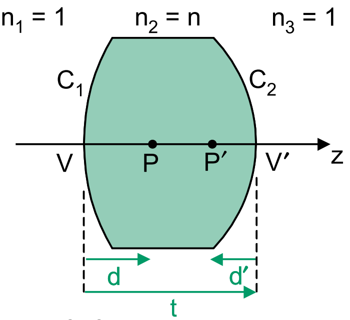
- P and P′ are the planes of unit system magnification.
- d is the shift in object space of the front system principal plane from the front principal plane of the first system.
- d' is the shift in image space of the rear system principal plane from the rear principal plane of the second system.
- t is the directed distance in the intermediate optical space from the rear principal plane of the first system to the front principal plane of the second system.
- Following reduction, the two original elements and the intermediate optical space n_2 are not needed.
- For multiple element systems, several reduction strategies are possible (two elements at a time):

$$1\ 2\ 3\ 4 \rightarrow (12)(34) \rightarrow (1234)$$

$$1\ 2\ 3\ 4 \rightarrow (12)\ 3\ 4 \rightarrow (123)\ 4 \rightarrow (1234)$$

Thick and Thin Lenses

Thick lens in air:



$$\tau = \frac{t}{n}$$

$$\phi_1 = (n-1)C_1$$

$$\phi_2 = -(n-1)C_2$$

$$\phi = (n-1)[C_1 - C_2 + (n-1)C_1C_2\tau]$$

$$d = \frac{\phi_2}{\phi}\tau \qquad d' = -\frac{\phi_1}{\phi}\tau$$

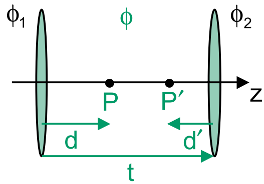
V and V′ are the surface vertices, and the nodal points are coincident with the principal planes.

Thin lens in air: $t \to 0$

$$\phi = (n-1)(C_1 - C_2) \qquad d = d' = 0$$

The principal planes and nodal points are located at the lens.

Two separated thin lenses in air:



$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t$$

$$d = \frac{\phi_2}{\phi}t \quad d' = -\frac{\phi_1}{\phi}t$$

The nodal points are coincident with the principal planes.

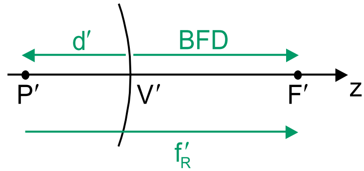
Optical power is sometimes measured in **diopters** (D), which have the units of m^{-1}.

$$\phi(\text{in D}) \equiv \frac{1}{f_E} \qquad (f_E \text{ in m})$$

When closely spaced elements are combined (t small), the system power is approximately the sum of the element powers.

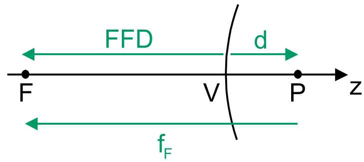
Vertex Distances

The **surface vertices** are the mechanical datums in a system and are often the reference locations for the cardinal points.



Back focal distance BFD:

$$BFD = f'_R + d'$$



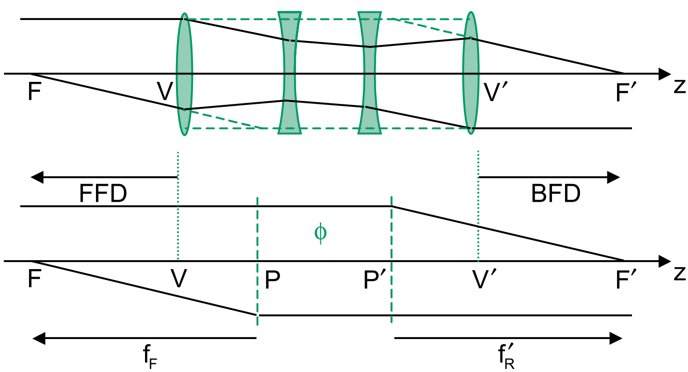
Front focal distance FFD:

$$FFD = f_F + d$$

Object and image vertex distances are determined using the Gaussian distances z, z':

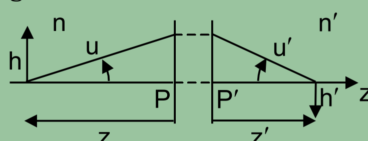
$$s = z + d \qquad\qquad s' = z' + d'$$

The utility of Gaussian optics and Gaussian reduction is that the imaging properties of any combination of optical elements can be represented by a system power or focal length, a pair of principal planes and a pair of focal points. In initial designs, the P – P′ separation is often ignored (i.e. a thin lens model).



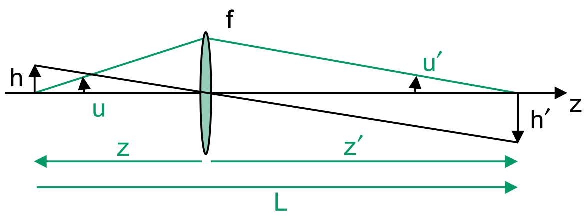
The Gaussian magnification may also be determined from the object and image ray angles:

$$m = \frac{(z'/n')}{(z/n)} = \frac{nu}{n'u'} = \frac{\omega}{\omega'}$$



Thin Lens Imaging

A **thin lens** is the most common element used in first-order layout. This idealized element has an optical power but no thickness and can be considered as a single refracting surface separating two spaces with the same index (usually air). The principal planes and nodal points are located at the lens.



$$f = f_E = f'_R = -f_F = \frac{1}{\phi} \qquad \frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

$$\frac{1}{m} = 1 + \frac{z}{f} \qquad m = 1 - \frac{z'}{f}$$

$$m = \frac{h'}{h} = \frac{z'}{z} = \frac{u}{u'}$$

The overall **object-to-image distance** for a thin lens in air is a function of the conjugate magnification.

$$L = z' - z = -\frac{(1-m)^2}{m} f_E$$

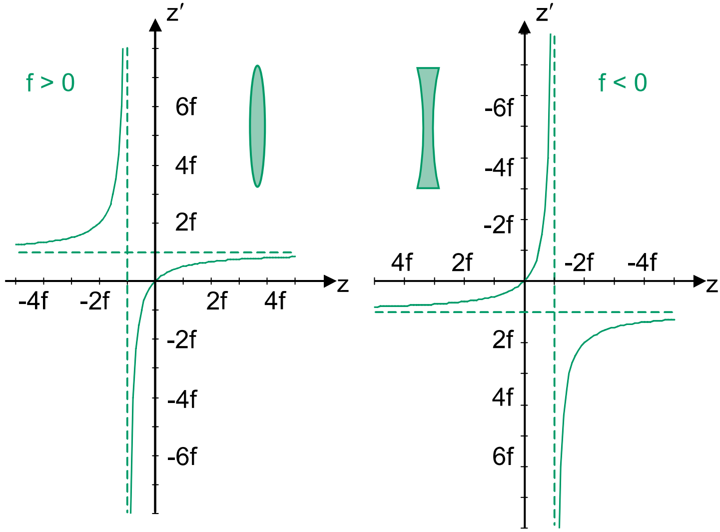
For each L, there are two possible magnifications and conjugates: the **reciprocal magnifications** m and $1/m$.

The minimum object-to-image distance with a real object and a real image occurs at **1:1 imaging**:

$$m = -1 \qquad L = 4f_E$$

Object-Image Conjugates

Distant objects (real or virtual) map to images located near the rear focal point. Objects near the front focal point map to distant images. The plots are for a thin lens in air, and the object and image distances are measured relative to the lens:



Real Objects: $z < 0$ Virtual Objects: $z > 0$
Real Images: $z' > 0$ Virtual Images: $z' < 0$

When the magnitude of the object distance z is more than a few times the magnitude of the system focal length, the image distance z' is approximately equal to the rear focal length. Here, $n = n' = 1$

$$|z| \gg |f|: \quad z' \approx f \qquad L = z' - z \approx f - z \qquad m = \frac{z'}{z} \approx \frac{f}{z}$$

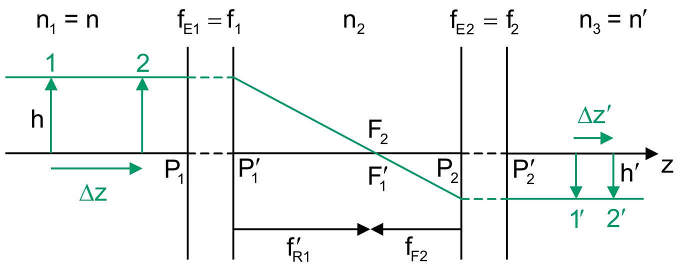
The fractional error in these approximations is about $|f|/|z|$, so they are very useful when the object distance more than 10–20 times the focal length. Most imaging problems can be solved with little or no computation.

There are similar approximations for distant images:

$$|z'| \gg |f|: \quad z \approx -f \qquad L = z' - z \approx z' + f \qquad m = \frac{z'}{z} \approx -\frac{z'}{f}$$

Afocal Systems

An **afocal system** is formed by the combination of two focal systems. The rear focal point of the first system is coincident with the front focal point of the second system. Rays parallel to the axis in object space are conjugate to rays parallel to the axis in image space. Common afocal systems are telescopes, binoculars and beam expanders.

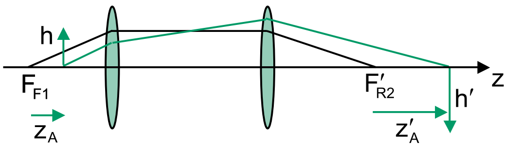


$$m = \frac{f_{F2}}{f'_{R1}} \qquad m = -\frac{f_{E2}}{f_{E1}} = -\frac{f_2}{f_1}$$

$$\overline{m} = \frac{n'}{n} m^2 \qquad \frac{\Delta z'/n'}{\Delta z/n} = m^2$$

The transverse and longitudinal magnifications are constant. Equispaced planes map into equispaced planes. The relative axial spacing changes by the longitudinal magnification $\overline{m}$.

Because the magnification is constant, the cardinal points are not defined for an afocal system, and the Gaussian and Newtonian equations cannot be used to determine conjugate planes. However, any pair of conjugate planes coupled with $\overline{m}$ can be used. A convenient pair is the front focal point of the first system F_{F1} and the rear focal point of the second system F'_{R2}.



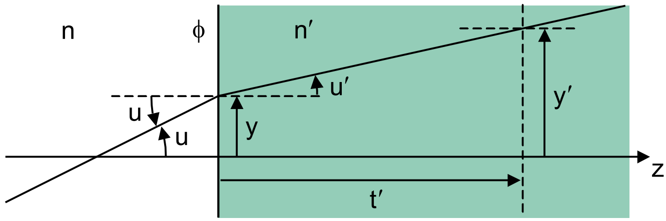
$$m = \frac{h'}{h}$$

$$z'_A = \overline{m} z_A$$

Paraxial Optics

Paraxial optics is a method of determining the first-order properties of an optical system that assumes all ray angles are small. A paraxial raytrace is linear with respect to ray angles and heights since all paraxial angles u are defined to be the tangent of the actual angle U. Rays in the vicinity of the optical axis are used, and the surface sag is ignored or negligible.

$$u = \sin U = \tan U$$



Refraction (or reflection) occurs at an interface between two optical spaces. The transfer distance t' allows the ray height y' to be determined at any plane within an optical space (including virtual segments).

$$\omega = nu \qquad \phi = (n' - n)C \qquad \tau' = \frac{t'}{n'}$$

Refraction or reflection: $n'u' = nu - y\phi$ $\qquad \omega' = \omega - y\phi$

Transfer: $y' = y + u't'$ $\qquad y' = y + \omega'\tau'$

This type of raytrace is also called an **YNU raytrace**. All rays propagate from object space to image space. A **reverse raytrace** allows the ray properties to be determined in the optical space upstream of a known ray segment. A ray can then be worked back to its origins in object space.

Refraction or reflection (reverse):

$$nu = n'u' + y\phi \qquad \omega = \omega' + y\phi$$

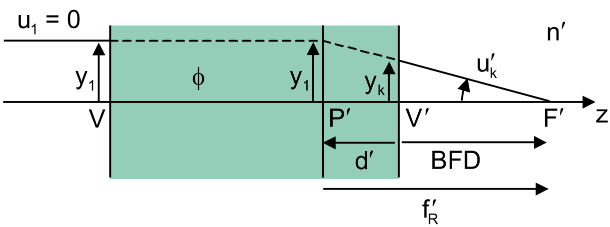
Transfer (reverse): $y = y' - u't'$ $\qquad y = y' - \omega'\tau'$

Paraxial Raytrace

The Gaussian properties of an optical system can be determined using a paraxial raytrace with particular rays.

Rear cardinal points:

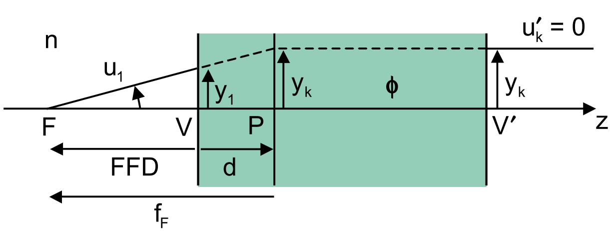
Trace a ray parallel to the axis in object space ($u = \omega = 0$). This ray must go through the rear focal point F′ of the system. The kth surface is the final surface in the system.



$$\phi = -\frac{n'u'_k}{y_1} = -\frac{\omega'_k}{y_1} \qquad f_E = \frac{1}{\phi} \qquad f'_R = \frac{n'}{\phi}$$

$$BFD = -\frac{n'y_k}{\omega'_k} = -\frac{y_k}{u'_k} \qquad d' = BFD - f'_R$$

Front cardinal points:



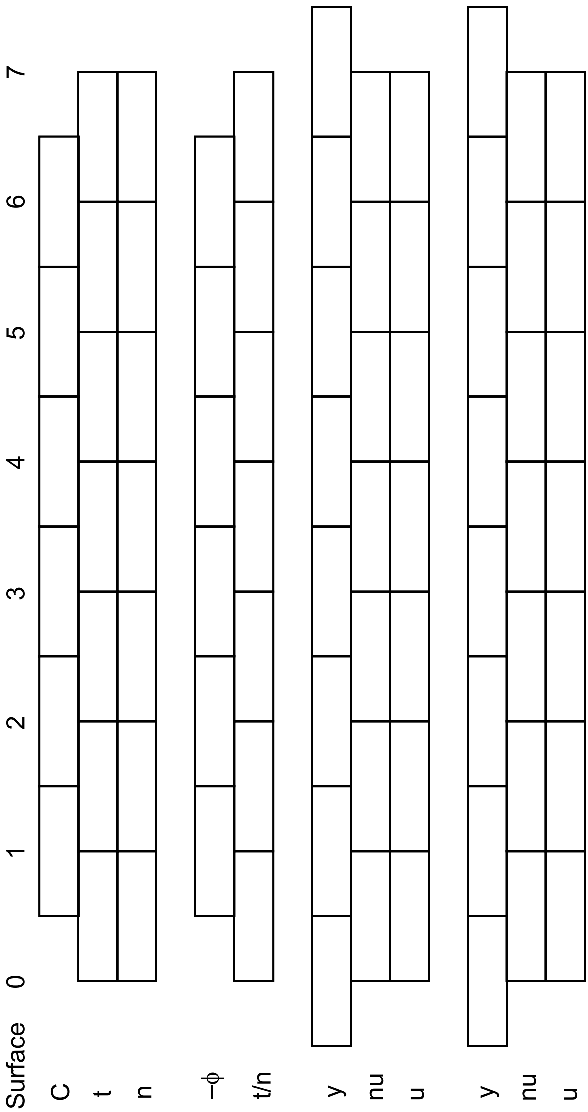
Trace a ray from the system front focal point F that emerges parallel to the axis in image space. The reverse raytrace equations are used to work from image space back to object space.

$$\phi = \frac{nu_1}{y_k} = \frac{\omega_1}{y_k} \qquad f_E = \frac{1}{\phi} \qquad f_F = -\frac{n}{\phi}$$

$$FFD = -\frac{ny_1}{\omega_1} = -\frac{y_1}{u_1} \qquad d = FFD - f_F$$

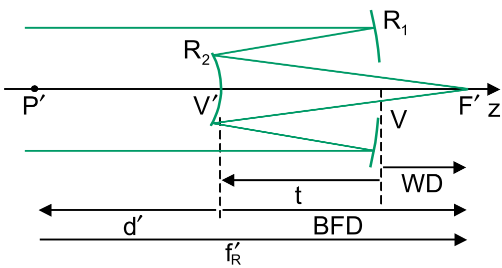
YNU Raytrace Worksheet

The **YNU raytrace worksheet** allows a systematic calculation method for tracing paraxial rays through an optical system. Its use is demonstrated in the following example.



Cassegrain Objective Example

Determine the rear cardinal points and power of the following **Cassegrain objective** example. Since the system is folded, the **working distance** WD is the distance from V to F′. The problem is solved by paraxial raytracing and also by Gaussian reduction.



$$R_1 = -200\ \text{mm}$$
$$t = -80\ \text{mm}$$
$$R_2 = -50\ \text{mm}$$
$$n_1 = n = 1$$
$$n_2 = -1$$
$$n_3 = n' = 1$$
$$C_1 = -0.005\ \text{mm}^{-1}$$
$$C_2 = -0.02\ \text{mm}^{-1}$$

Paraxial raytrace: $\omega' = \omega - y\phi \qquad y' = y + \omega'\tau'$

Surface	Object		V		V′		F′
C			-0.005		-0.02		
t		∞		-80		BFD	
n		1.0		-1.0		1.0	
$-\phi$			-0.01		0.04		
t/n		∞		80		100	
y	1.0		1.0		0.2		0.0
nu		0.0		-0.01		-0.002	
u		0.0		0.01		-0.002	

A ray launched at an arbitrary height of 1.0 with zero angle is traced until it crosses the axis at F′. The BFD can be directly solved on the raytrace sheet as the V′ to F′ distance. The arrows overlaying the worksheet indicate the raytrace procedure: the value of y is multiplied by $-\phi$ directly above and added to the previous nu to get nu in the next space. Similarly, the value of nu is multiplied by τ above and added to the previous y to obtain y at the next surface. ***continued….***

Cassegrain Objective Example

The analysis of the raytrace results:

$$\phi = -\frac{n'u_2'}{y_1} = -\frac{-0.002}{1.0} = 0.002\ \text{mm}^{-1} \qquad f_R' = f_E = \frac{1}{\phi} = 500\ \text{mm}$$

$$BFD = -\frac{y_2}{u_2'} = -\frac{0.2}{-0.002} = 100\ \text{mm}$$

$$d' = BFD - f_R' = BFD - f_E = -400\ \text{mm}$$

$$WD = BFD + t = 20\ \text{mm}$$

Gaussian reduction:

$$\phi_1 = (n_2 - n)C_1 = 0.01\ \text{mm}^{-1}$$

$$\phi_2 = (n' - n_2)C_2 = -0.04\ \text{mm}^{-1} \qquad \tau = \frac{t}{n_2} = 80\ \text{mm}$$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau = 0.002\ \text{mm}^{-1} \qquad f_R' = f_E = \frac{1}{\phi} = 500\ \text{mm}$$

$$d' = -n'\frac{\phi_1}{\phi}\tau = -400\ \text{mm}$$

$$BFD = f_R' + d' = f_E + d' = 100\ \text{mm}$$

$$WD = BFD + t = 20\ \text{mm}$$

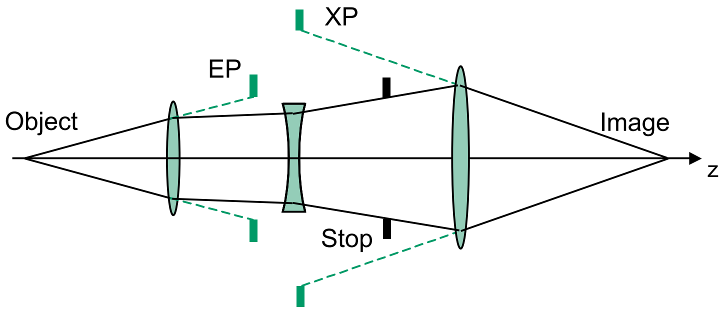
In a paraxial raytrace, t is the directed distance from the current surface to the next surface. As a result, real objects will usually have a positive distance to the first surface, as opposed to the typical negative Gaussian object distance z.

Surfaces are raytraced in **optical order**, not physical order. All planes of interest in an optical space must be analyzed before transferring to a reflective or refractive surface and entering the next optical space. Within an optical space, transfers move back or forth along the ray in that space without changing the ray angle. Real and virtual segments of the space can be accessed.

Stops and Pupils

The **aperture stop** is the aperture in the system that limits the bundle of light that propagates through the system from the axial object point. The stop can be one of the lens apertures or a separate aperture (**iris diaphragm**) placed in the system, however, the stop is always a physical surface.

The **entrance pupil** EP and the **exit pupil** XP are the images of the stop in object space and image space. The pupils define the cones of light entering and exiting the optical system from any object point.



There is a stop or pupil in each optical space. The EP is in the system object space, and the XP is in the system image space. Intermediate pupils are formed in other spaces.

There are two common methods to determine which aperture in a system serves as the system stop:

- Image each potential stop into object space. The pupil with the smallest angular size from the perspective of the axial object point corresponds to the stop. An analogous procedure can also be done in image space.
- Trace a ray through the system from the axial object point with an arbitrary initial angle. The aperture that is the stop will be proportionately closest to this ray. At each potential stop, form the ratio of the aperture radius a_k to the ray height at that surface $\tilde{y}_k$:

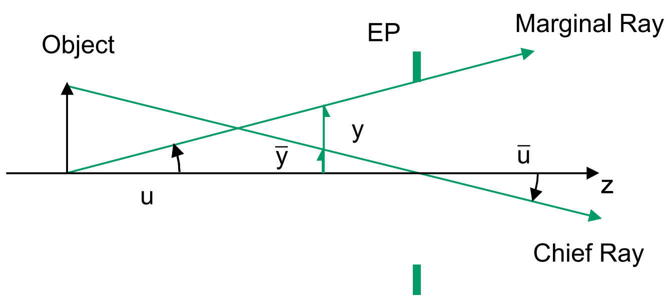
$$\text{Aperture Stop} \Rightarrow \text{Minimum}\left|\frac{a_k}{\tilde{y}_k}\right|$$

Marginal and Chief Rays

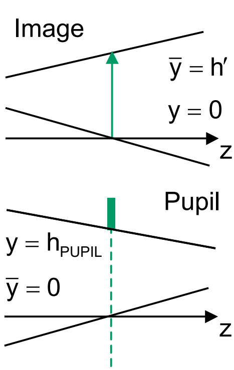
Rays confined to the y-z plane are called **meridional rays**. The marginal ray and the chief ray are two special meridional rays that together define the properties of the object, images and pupils.

The **marginal ray** starts at the axial object position, goes through the edge of the entrance pupil, and defines image locations and pupil sizes. It propagates to the edge of the stop and the edge of the exit pupil. The marginal ray height and ray angle are denoted by y and u.

The **chief ray** starts at the edge of the object, goes through the center of the entrance pupil, and defines image heights and pupil locations. It goes through the center of the stop and the center of the exit pupil. The chief ray height and ray angle are denoted by $\bar{y}$ and $\bar{u}$.



The heights of the marginal ray and the chief ray can be evaluated at any z in any optical space.



When the marginal ray crosses the axis, an image is located, and the size of the image is given by the chief ray height in that plane.

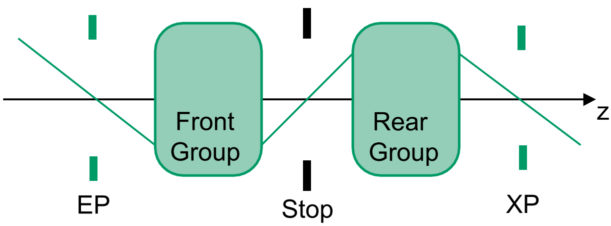
Whenever the chief ray crosses the axis, a pupil or the stop is located, and the pupil radius is given by the marginal ray height in that plane.

Intermediate images and pupils are often virtual.

Pupil Locations

The stop is a real object for the formation of both the entrance and exit pupil.

The **pupil locations** can be found by tracing a paraxial ray starting at the center of the aperture stop. The ray is traced through the group of elements behind the stop and reverse traced through the group of elements in front of the stop. The intersections of this ray with the axis in object and image space determine the locations of the entrance and exit pupils.



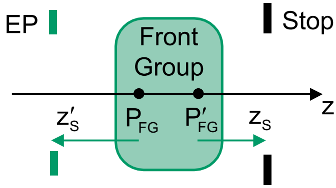
This ray becomes the chief ray when it is scaled to the object or image size. The marginal ray gives the pupil sizes.

The trial ray used to determine which aperture serves as the system stop can be scaled to the marginal ray.

$$y = \tilde{y}\left|\frac{a_k}{\tilde{y}_k}\right|_{\min} \qquad u = \tilde{u}\left|\frac{a_k}{\tilde{y}_k}\right|_{\min}$$

The pupil locations and sizes can also be found using Gaussian imagery. Imaging the stop through the rear group of elements to find the XP is straightforward, however for the EP, the stop is a real object to the right of the front group.

Light from the stop propagates from right to left to form the EP, and a negative index is assigned. The object and image distances are measured from the principal plane of the front group that is in the same optical space as the object (stop) or image (pupil).

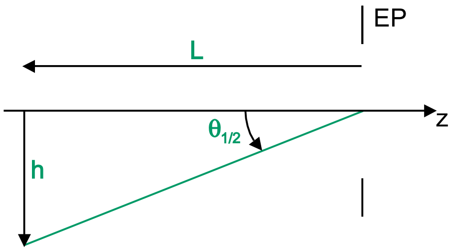


$$\frac{-1}{z'_S} = \frac{-1}{z_S} + \frac{1}{f_{FG}} \quad \text{(in air)}$$

Field of View

The **field of view** FOV of an optical system is often expressed as the maximum angular size of the object as seen from the entrance pupil. The maximum image height is also used. For finite conjugate systems, the maximum object height is useful.

Field of view FOV: the diameter of the object/image
Half field of view HFOV: the radius of the object/image



$$HFOV = \theta_{1/2} \text{ or } h$$

$$\tan(\theta_{1/2}) = \frac{h}{L}$$

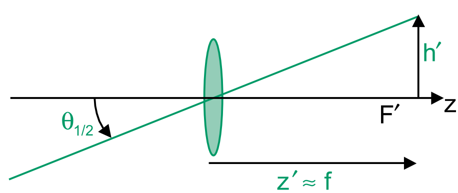
$$\bar{u} = \tan(\theta_{1/2}) = \frac{h}{L}$$

Full field of view FFOV is sometimes used for FOV to emphasize that this is a diameter measure. Since the EP is the reference position for the FOV, this defining ray becomes the chief ray of the system.

For distant objects, assuming a thin lens in air with the stop at the lens:

$$\bar{u} = \tan(\theta_{1/2}) \approx \frac{h'}{f}$$

$$h' \approx f\tan(\theta_{1/2}) = f\bar{u}$$



The system FOV can be determined by the maximum object size, the detector size, or by the field over which the optical system exhibits good performance. For rectangular image formats, horizontal, vertical and diagonal FOVs must be specified.

The **fractional object** FOB allows objects of different heights to be defined in terms of the HFOV. FOB 0 is an on-axis object, and FOB 1 is an object at the edge of the HFOV. Since objects are two dimensional, FOB 0.7 divides the circular FOV into two equal areas.

Lagrange Invariant

The linearity of paraxial optics provides a relationship between the heights and angles of any two rays propagating through the system. The **Lagrange invariant** (Ж or H) is formed with the paraxial marginal and chief rays:

$$Ж = H = n\bar{u}y - nu\bar{y} = \bar{\omega}y - \omega\bar{y}$$

This expression is invariant both on refraction and transfer, and it can be evaluated at any z in any optical space, and often allows for the completion of apparently partial information in an optical space by using the invariant formed in a different optical space. Many of the results obtained from raytrace derivations can also be simply obtained with the Lagrange invariant. The Lagrange invariant is particularly simple at images or objects ($y = 0$) and pupils ($\bar{y} = 0$):

Image: $Ж = -nu\bar{y} = -\omega\bar{y}$ Pupil: $Ж = n\bar{u}y = \bar{\omega}y$

If two rays other than the marginal and chief rays are used, the more general **optical invariant** I is formed.

Given two rays, a third ray can be formed as a linear combination of the two rays. The coefficients are the ratios of the pair-wise invariants of the values for the three rays at some initial z. The expressions are then valid at any z.

$$y_3 = Ay_1 + By_2 \qquad u_3 = Au_1 + Bu_2$$

$$A = I_{32}/I_{12} \qquad B = I_{13}/I_{12} \qquad I_{ij} = nu_iy_j - nu_jy_i$$

Changing the Lagrange invariant of a system scales the optical system. Doubling the invariant while maintaining the same object and image sizes and pupil diameters halves all of the axial distances (and the focal length).

The **throughput**, **etendue** or **AΩ product** in **radiometry** and **radiative transfer** are related to the square of the Lagrange invariant:

$$n^2A\Omega = \pi^2Ж^2$$

Numerical Aperture and F-Number

In an optical space of index n_k, the **numerical aperture** NA describes the axial cone of light in terms of the real marginal ray angle U_k:

$$NA \equiv n_k|\sin U_k| \approx n_k|u_k|$$

The **F-number** f/# describes the image-space cone of light for an object at infinity:

$$f/\# \equiv \frac{f_E}{D_{EP}} \qquad D_{EP} = \text{Diameter of the EP}$$

The NA and the f/# are related assuming a thin lens with the stop at the lens and infinite conjugates:

$$f/\# \approx \frac{1}{2NA} \qquad NA \text{ in image space}$$

While the f/# is an image-space, infinite-conjugate measure, this approximation allows f/#s to be defined for other optical spaces and conjugates. In particular, the **working F-number** $f/\#_W$ describes the image forming cone for finite conjugates:

$$f/\#_W \equiv \frac{1}{2NA} \approx (1-m)\, f/\# = (1-m)\frac{f_E}{D_{EP}} \qquad m = \text{Magnification}$$

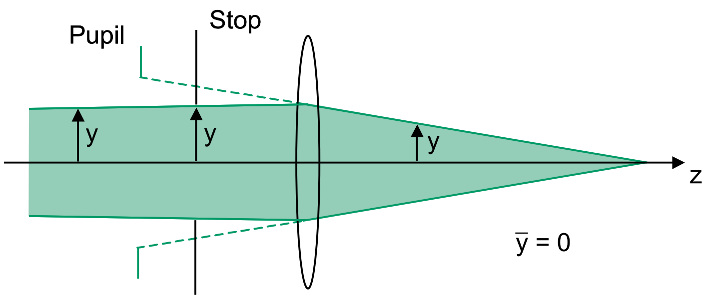
Fast optical systems have small numeric values for the f/#. Most lenses with adjustable stops have f/#s or **f-stops** labeled in increments of $\sqrt{2}$. The usual progression is f/1.4, f/2, f/2.8, f/4, f/5.6, f/8, f/11, f/16, f/22, etc, where each stop changes the area of the EP (and the light collection ability) by a factor of 2.

The Lagrange invariant relates the magnification between two pupils to the chief ray angles at the pupils.

$$Ж = n\bar{u}y_{PUPIL} = n'\bar{u}'y'_{PUPIL} \qquad m_{PUPIL} = \frac{y'_{PUPIL}}{y_{PUPIL}} = \frac{n\bar{u}}{n'\bar{u}'} = \frac{\bar{\omega}}{\bar{\omega}'}$$

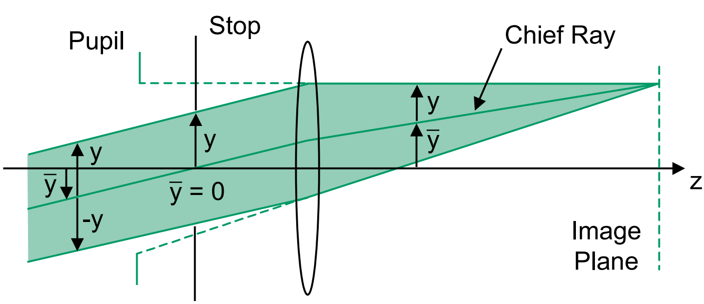
Ray Bundles

The **ray bundle** for an on-axis object is a rotationally symmetric spindle made up of sections of right circular cones. Each cone section is defined by the pupil and the object or image point in that optical space. The individual cone sections match up at the surfaces and elements.



At any z, the cross section of the bundle is circular, and the radius of the bundle is the marginal ray value.

For an off-axis object point, the ray bundle skews, and is comprised of sections of skew circular cones which are still defined by the pupil and object or image point in that optical space.



The cross section of the ray bundle at any z remains circular with a radius equal to the radius of the axial bundle. The off-axis bundle is centered about the chief ray height.

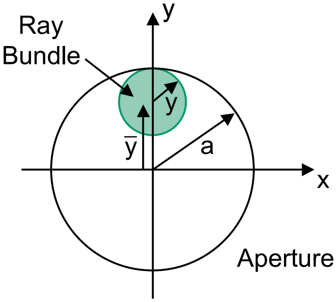
The maximum radial extent of the ray bundle at any z is

$$|y_{MAX}| = |y| + |\bar{y}|$$

Vignetting

While the stop alone defines the axial ray bundle, **vignetting** occurs when other apertures in the system, such as a lens clear aperture, block all or part of an off-axis ray bundle.

No vignetting occurs when all of the apertures pass the entire ray bundle from the object point. Each aperture radius a must equal or exceed the maximum height of the ray bundle at the aperture.



Unvignetted:

$$a \geq |y| + |\bar{y}|$$

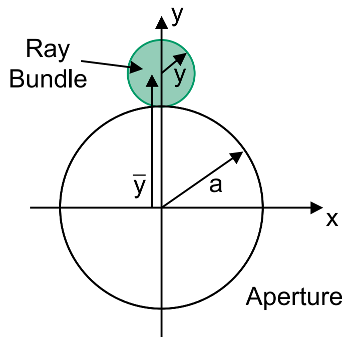
The maximum FOV supported by the system occurs when an aperture completely blocks the ray bundle from the object point.

Fully vignetted:

$$a \leq |\bar{y}| - |y|$$

and

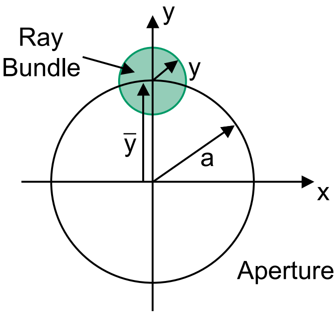
$$a \geq |y|$$



The second vignetting condition ensures that the aperture passes the marginal ray and is not the system stop. By definition, vignetting cannot occur at the aperture stop or at a pupil.

More Vignetting

A third vignetting condition is defined when an aperture passes about half of the ray bundle from an object point.



Half vignetted:

$$a = |\bar{y}|$$

and

$$a \geq |y|$$

The vignetting conditions are used in two different manners:

- For a given set of apertures, the FOV that the system will support with a prescribed amount of vignetting can be determined. A different chief ray defines each FOV.
- For a given FOV and vignetting condition, the required aperture diameters can be determined.

A system with vignetting will have an image that has full irradiance or brightness out to a radius corresponding to the unvignetted FOV limit. The irradiance will then begin to fall off, going to about half at the half-vignetted FOV, and decreasing to zero at the fully vignetted FOV. This fully vignetted FOV is the absolute maximum possible. This discussion ignores the obliquily factors of **radiative transfer**, such as the cosine fourth law.

The diameter of the aperture stop is very important design parameter for an optical system as it controls five separate performance aspects of the system:

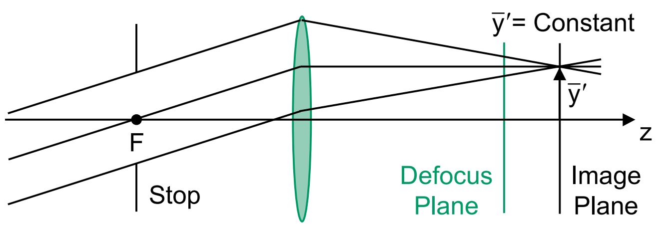
- The system FOV determined by vignetting.
- The radiometric or photometric speed of the system or its light collecting ability.
- The depth of focus and depth of field of the system.
- The amount of aberrations degrading image quality.
- The diffraction-based performance of the system.

While some of these aspects are interrelated, they all derive from different physical phenomena.

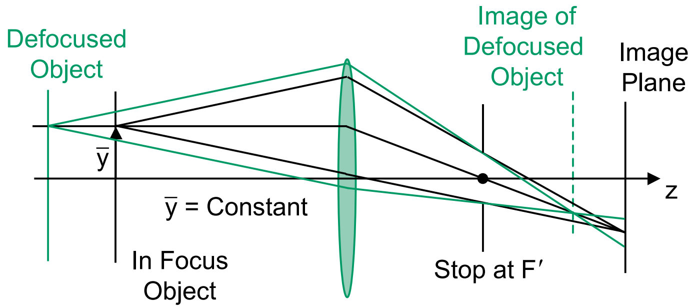
Telecentricity

In a **telecentric system**, the EP and/or the XP are located at infinity. **Telecentricity** in object or image space requires that the chief ray be parallel to the axis in that space. As a consequence, the apparent system magnification is constant even if the object or image plane is displaced from its nominal position. The image will be blurred, but of the correct size or magnification.

When the stop is located at the front focal plane of a focal system, the XP is at infinity, and the system is **image-space telecentric**. Defocus of the image plane or detector will not change the image height.



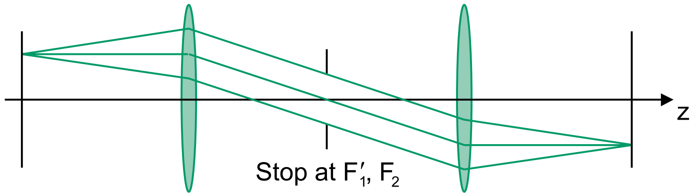
Placing the stop at the rear focal plane puts the EP at infinity and forms an **object-space telecentric** system. The blur from the defocused object is centered about the chief ray and the image height at the nominal image plane is constant.



Object-space telecentric systems are almost always used at finite conjugates. The maximum object size is limited to approximately the radius of the objective lens due to vignetting considerations.

Double Telecentricity

An afocal system is made **double telecentric** by placing the system stop at the common focal point. The chief ray is parallel to the axis in object space and image space, and both the EP and the XP are located at infinity. All double telecentric systems must be afocal.



Since the ray bundle is centered on the chief ray, this condition guarantees that height of the blur forming the image is independent of axial object shifts or image plane shifts.

Telecentricity is an important feature of many optical **metrology** systems as the apparent size of an inspected object does not change with focus, object position, or object thickness. Microscope objectives are often object-space telecentric to prevent "zooming" of out-of-focus planes when focusing through a thick, transparent specimen.

Defining the angular FOV relative to the EP or the XP is impossible if the system is telecentric in that particular optical space because the respective pupil is at infinity. The object height or image height can, however, be used.

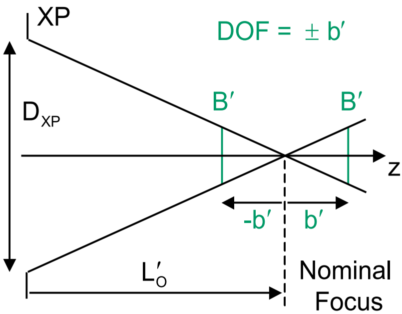
A second method for defining angular FOV is to measure the angular size of the object relative to the front nodal point N. This is useful because the angular sizes of the object and the image are equal when viewed from the respective nodal points. This definition of angular FOV fails for afocal systems which do not have nodal points. Double telecentric systems, being afocal, generally use the object height or the image height to define FOV.

The choice of using the EP or nodal point for angular FOV is of little consequence when the object is distant.

Depth of Focus and Depth of Field

There is often some allowable image blur that defines the performance requirement of an optical system. This maximum acceptable blur may result from the detector resolution or just the overall system resolution requirement. This blur requirement results in a first-order geometrical tolerance for the longitudinal position of the object or the image plane. No diffraction or aberrations are included.

The **depth of focus** DOF describes the amount the detector can be shifted from the nominal image position for an object before the resulting blur exceeds the blur diameter criterion B'.

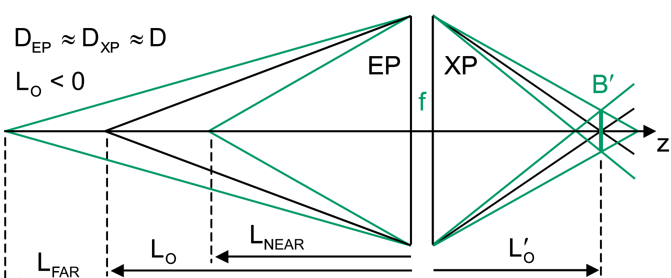


$$b' = \frac{B'L'_O}{D_{XP}} \approx \frac{B'z'}{D_{EP}}$$

$$DOF \approx \pm B' f/\#_W$$

$$DOF \approx \pm \frac{B'}{2NA}$$

There is also some range of object positions L_{FAR} to L_{NEAR}, the **depth of field**, that will appear in focus for a given detector or image plane position. The image plane blur criterion is met for these object positions. L_O is the object position corresponding to the image plane location L'_O. These results assume a thin lens with the stop at the lens.



$$L_{NEAR} \approx \frac{L_O f D}{fD - L_O B'} \qquad L_{FAR} \approx \frac{L_O f D}{fD + L_O B'}$$

Hyperfocal Distance and Scheimpflug Condition

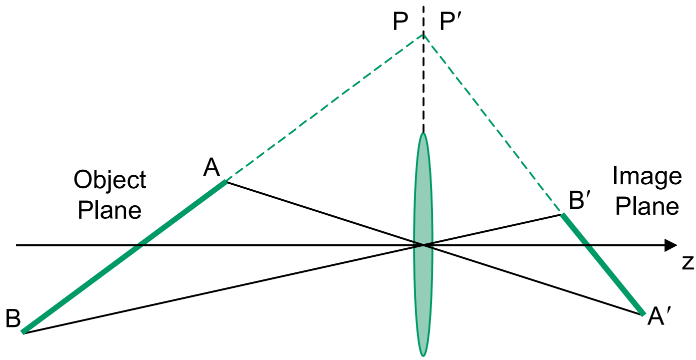
An important condition occurs when the far point of the depth of field L_{FAR} extends to infinity. The optical system is focused at the **hyperfocal distance** L_H, and all objects from L_{NEAR} to infinity meet the image plane blur criterion and are in focus.

$$L_H = -\frac{fD}{B'} \qquad L_{NEAR} \approx \frac{L_H}{2} \qquad L_{FAR} = \infty$$

The near focus limit is approximately half the hyperfocal object distance.

The depth of field and the hyperfocal distance help explain the practical operation of camera systems including the required focus precision, the needed number of focus zones, and how fixed-focus cameras work.

First-order optical systems image points to points, lines to lines and planes to planes. This condition holds even if the line or plane is not perpendicular to the optical axis. The **Scheimpflug condition** states that a tilted plane images to another tilted plane, and for a thin lens, the line of intersection lies in the plane of the lens.



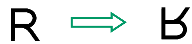
Even though the image is in focus, it will exhibit **keystone distortion** as the lateral magnification varies along the tilted object. This condition easily extends to a thick lens or system: the line of intersection is coincident in the front and rear principal planes of the system.

Parity and Plane Mirrors

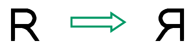
In addition to bending or folding the light path, reflection from a **plane mirror** introduces a **parity** change in the image.

Right Handed Parity (RH) Left Handed Parity (LH)

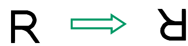
Invert – Image flip about a horizontal line.



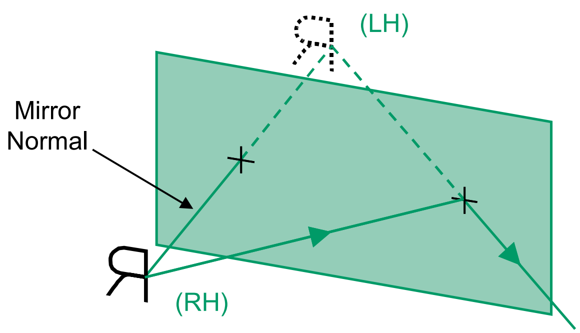
Revert – Image flip about a vertical line.



An inversion plus a reversion is equivalent to a 180° **image rotation**; no parity change.



An image seen by an even number of reflections maintains its parity. An odd number of reflections changes the parity. Parity is determined by looking back against the propagation direction towards the object or image in that optical space; let the light from the object or image come to you.

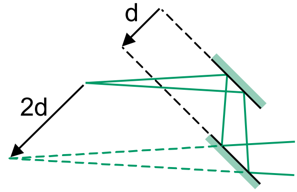


Each ray from an object obeys the law of reflection at a plane mirror surface, and a virtual image of the object is produced.

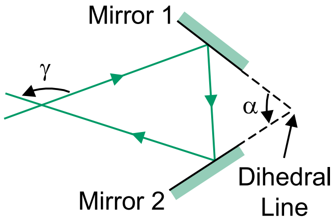
The rules of plane mirrors:
- The line connecting an object point and its image is perpendicular to the mirror and is bisected by the mirror.
- Any point on the mirror surface is equidistant from a given object point and its image point.
- The image parity is changed on reflection.

Systems of Plane Mirrors

The rules of plane mirrors are used sequentially at each mirror in a **system of plane mirrors**. Two **parallel plane mirrors** act as a **periscope** and displace the line of sight. There is no parity change, and all image rays are parallel to the corresponding object rays. The image is displaced by twice the perpendicular separation of the mirrors.



The **dihedral line** is the line of intersection of two **non-parallel plane mirrors**. In a plane perpendicular to the dihedral line (a **principal section**), the projected ray path is deviated by twice the angle between the mirrors (the **dihedral angle** α). This deviation is independent of the input angle.



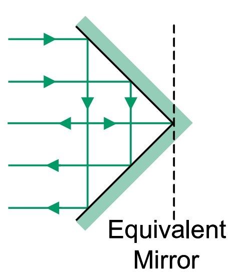
$$\gamma = 2\alpha$$

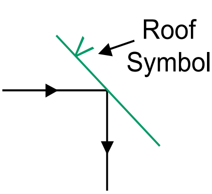
$\alpha < 90°$: The input and output rays cross.

$\alpha > 90°$: The input and output rays diverge.

The projection of the ray paths into a plane containing the dihedral line shows a simple reflection at the dihedral line.

When the dihedral angle is 90°, the input and output rays are anti-parallel. This **roof mirror** can replace any flat mirror to insert an additional reflection or parity change. An equivalent plane mirror is formed at the dihedral line. All rays through the roof mirror have the same optical path.





The dihedral line is often in the plane of the drawing, and the presence of a roof mirror is indicated by a "V" at the equivalent mirror or dihedral line.

Prism Systems

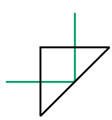
Prism systems can be considered systems of plane mirrors. If the angles of incidence allow, the reflection is due to TIR. Prisms fold the optical path and correct the image parity. Surfaces where TIR fails must have a reflective coating.

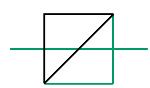
A **tunnel diagram** unfolds the optical path through the prism and shows the total length of the path through the prism. The prism is represented as a block of glass of the same thickness. The tunnel diagram aids in determining FOV, clear aperture, and vignetting. The addition of a roof mirror to the prism does not change the tunnel diagram.

Prisms are classified by the overall ray deviation angle and the number of reflections (# of R's).

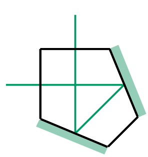
90° Deviation Prisms

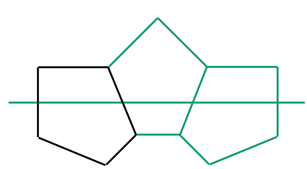
Right angle prism (1 R) – the deviation depends on the input angle and prism orientation.



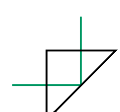


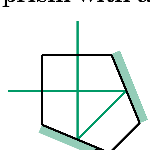
Pentaprism (2 R) – two coated surfaces at 45° produce a 90° deviation independent of the input angle. It is the standard optical metrology tool for defining a right angle.





Amici or **Roof prism** (2 R) – a right angle prism with a roof mirror.



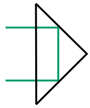
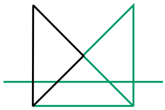


Reflex prism (3 R) – a pentaprism with an added roof mirror. Used in single lens reflex (SLR) camera viewfinders.

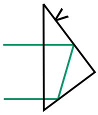
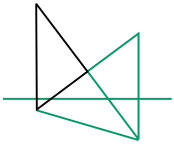
More Prism Systems

180° Deviation Prisms

Porro prism (2 R) – a right angle prism using the hypotenuse as the entrance face. It controls the deviation in one dimension.

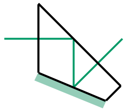
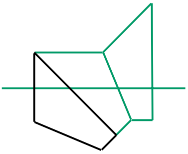



Corner cube (3 R) – three surfaces at 90°. The output ray of this retroreflector is anti-parallel to the input ray. The figures appear skewed due to the compound angles needed to represent a prism face and a roof edge when all three prism faces have equal angles with the optical axis.

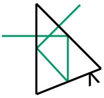
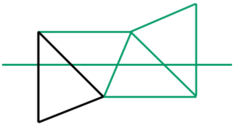



45° Deviation Prisms

45° prism (2 R) – half a pentaprism.

Schmidt prism (4 R) – a 3 R version without a roof also exists.

TIR often fails when prisms are used with fast f/# beams. In polarized light applications, TIR at the prism surfaces will change the polarization state of the light. In both these situations, silvered or coated prisms must be used.

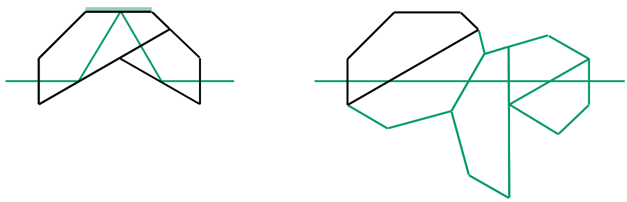
Prisms with entrance and exit faces normal to the optical axis can be used in converging or diverging light.

Image Rotation and Erection Prisms

Image Rotation Prisms – as the prism is rotated by θ about the optical axis, the image rotates by twice that amount (2θ).

Dove prism (1 R) – because of the tilted entrance and exit faces of the prism, it must be used in collimated light.

Reversion or K prism (3 R) – the upper face must be coated.



Pechan prism (5 R) – a small air gap provides a TIR surface inside the prism. This compact prism supports a wide FOV.

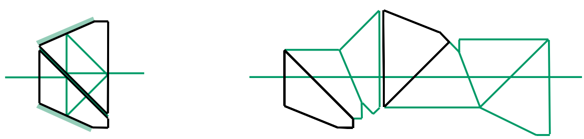
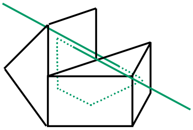
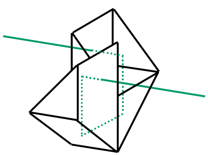


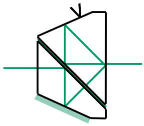
Image Erection Prisms – These prisms are inserted in an optical system to provide a fixed 180° image rotation.

Porro system (4 R) – two Porro prisms. This prism accounts for the displacement between the objective lenses and the eyepieces in binoculars.



Porro-Abbe system (4 R) – a variation of the Porro system where the sequence of reflections is changed.

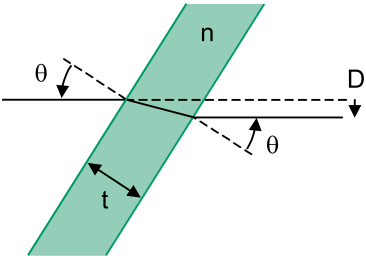




Pechan-roof prism (6 R) – a roof is added to a Pechan prism. This prism is used in compact binoculars and provides a straight-through line of sight.

Plane Parallel Plates

A ray passing through a **plane parallel plate** is displaced but not deviated; the input and output rays are parallel.



$$D = t\sin\theta\left[1 - \sqrt{\frac{1-\sin^2\theta}{n^2-\sin^2\theta}}\right]$$

$$D \approx t\theta\left(\frac{n-1}{n}\right)$$

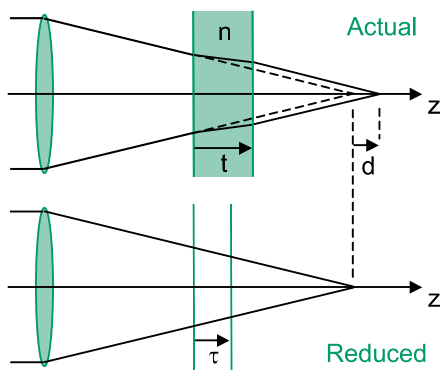
(in air)

An image formed through a plane parallel plate is longitudinally displaced, but its magnification is unchanged.

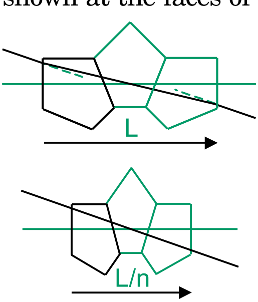
$$d \approx \left(\frac{n-1}{n}\right)t$$

$$d \approx \frac{t}{3} \quad \text{for} \quad n = 1.5$$

$$\tau = t - d = \frac{t}{n}$$

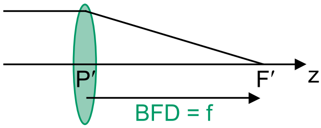


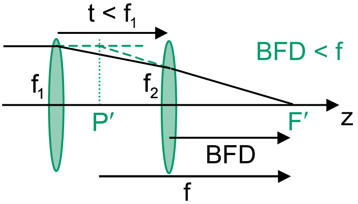
The **reduced thickness** τ gives the air-equivalent thickness of the glass plate. A **reduced diagram** shows the amount of air path needed to fit the plate in the system, and no refraction is shown at the faces of the plate. A **reduced tunnel diagram** shortens the length of a tunnel diagram by $1/n$ to show the air-equivalent length of the prism. Reduced diagrams can be placed directly onto system layout drawings to determine the required prism aperture sizes for a given FOV. Note that the OPL increases greatly when a prism or glass plate is inserted.



Objectives

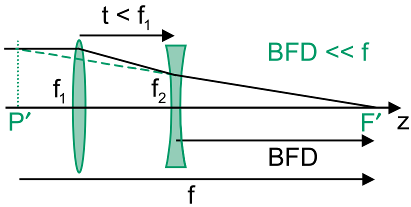
Objectives are lens element combinations used to image (usually) distant objects. To classify the objective, separated **groups** of lens elements are modeled as thin lenses. The **simple objective** is represented by a positive thin lens.

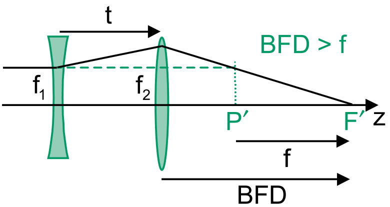




The **Petzval objective** consists of two separated positive groups of elements. The system rear principal plane is located between the groups.

The **telephoto objective** produces a system focal length longer than the overall system length (t + BFD). It consists of a positive group followed by a negative group.





The **reverse telephoto objective** or **retrofocus objective** consists of a negative group followed by a positive group. This configuration is used to produce a system with a BFD larger than the system focal length. While this configuration is used for many wide angle objectives, the term reverse telephoto specifically refers to the configuration, not the FOV.

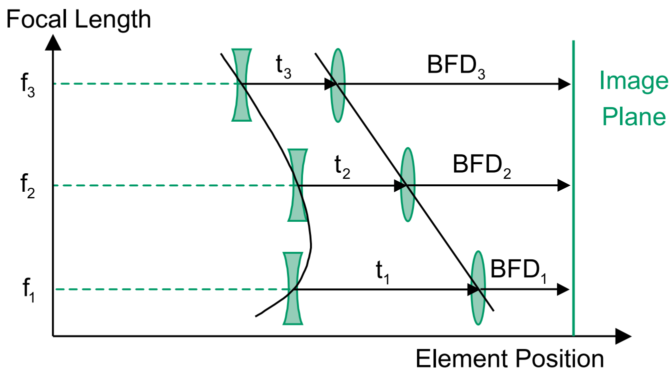
A **collimator** is a reversed objective. It creates a collimated beam from a source at the system front focal point, and the image of the source is projected to infinity. The degree of collimation is determined by the source size.

Zoom Lenses

A **zoom lens** is a variable focal length objective with a fixed image plane. The simplest example consists of two lens elements or groups (powers ϕ_1 and ϕ_2) where both the system focal length f and BFD vary with element spacing t.

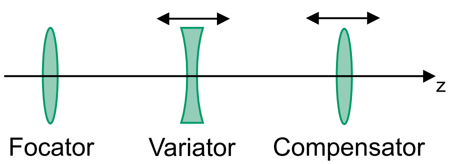
$$\phi = \frac{1}{f} = \phi_1 + \phi_2 - \phi_1\phi_2 t \qquad BFD = f + d' = f - \frac{\phi_1}{\phi} t$$

The pair of elements is moved relative to the fixed image plane to maintain focus as the focal length is varied. The element positions are shown for a **reverse telephoto zoom**. This configuration is attractive due to its large BFD.



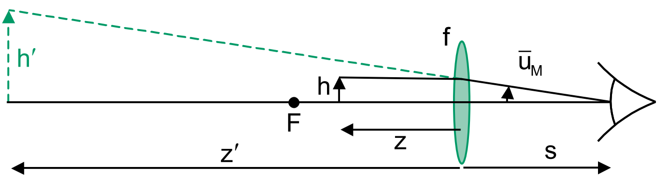
As the separation approaches the sum of the element focal lengths ($f_1 + f_2$), the system becomes afocal ($f \to \infty$). The zoom range of the analogous **telephoto zoom** is limited by its BFD as the rear element can run into the image plane when the element separation approaches f_1.

A mechanical cam provides the complicated lens motions required for these **mechanically compensated** zoom lenses. Zoom lenses often use multiple groups of moving elements. A common three group configuration uses a fixed front element and moving second and third groups.



Magnifiers

The largest image magnification possible with the unaided eye occurs when the object is placed at the **near point** of the eye, by convention, 250 mm or 10 in. from the eye. A **magnifier** is a single lens that provides an enlarged erect virtual image of a nearby object for visual observation.



The **magnifying power** MP is defined as (stop at the eye):

$$MP = \frac{\text{Angular size of the image (with lens)}}{\text{Angular size of the object at the near point}}$$

$$MP = \frac{\bar{u}_M}{\bar{u}_U} = \frac{h'/(z'-s)}{h/d_{NP}} \qquad d_{NP} = -250 \text{ mm}$$

$$MP = \frac{250 \text{ mm}(z'-f)}{f(z'-s)} \approx \frac{250 \text{ mm}}{f}$$

This approximation is the most common definition of the MP of a magnifier. It assumes that the lens is close to the eye and that the image is presented to a relaxed eye ($z' = \infty$).

The angular subtense θ of the image h' at the eye is

$$\theta = h\,MP/250 \text{ mm}$$

The resolution of the human eye is about 1 arc min. In order to resolve an object of size h, the required MP is then

$$MP \geq .075 \text{ mm}/h$$

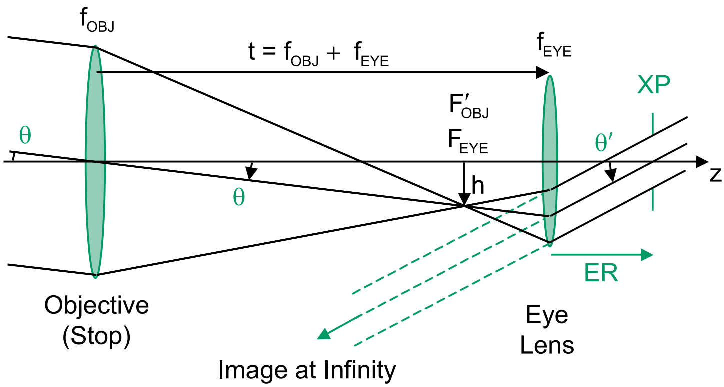
Magnifiers up to about 25X are practical; 10X is common.

Keplerian Telescope

Telescopes are afocal systems used for visual observation of distant objects. The image through the telescope subtends an angle θ' different from the angle subtended by the object θ. The **magnifying power** MP of a telescope is

$$MP = \frac{\theta'}{\theta} \qquad |MP| > 1 \quad \text{Telescope magnifies}$$
$$|MP| < 1 \quad \text{Telescope minifies}$$

A **Keplerian telescope** or **astronomical telescope** consists of two positive lenses separated by the sum of the focal lengths. The system stop is usually at or near the objective lens.



$$m = -\frac{f_{EYE}}{f_{OBJ}} \qquad MP = \frac{1}{m} = -\frac{f_{OBJ}}{f_{EYE}}$$

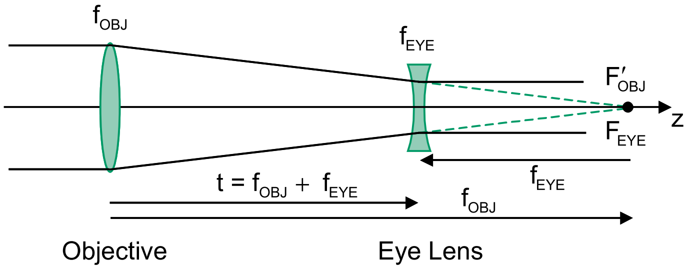
The image presented to the eye is inverted and reverted (rotated 180°), and the MP is negative. The eye should be placed at the real XP to couple the eye to the telescope. The XP position is the **eye relief** ER. The magnification of the afocal system relates the diameters of the EP and the XP.

$$ER = (1-m)f_{EYE} \qquad D_{XP} = |m|D_{EP} = \frac{D_{EP}}{|MP|}$$

The XP of a visual instrument is also known as the **eye circle** or the **Ramsden circle**.

Galilean Telescope

The **Galilean telescope** uses a positive lens and a negative lens to to obtain an erect image and a positive MP ($MP > 1$).



$$m = -\frac{f_{EYE}}{f_{OBJ}} \qquad MP = \frac{1}{m} = -\frac{f_{OBJ}}{f_{EYE}}$$

The XP is internal and not accessible to the eye. The FOV of the system is small. There is no intermediate image plane.

For a given $|MP|$, the Galilean telescope is shorter than the corresponding Keplerian telescope. Its FOV is also smaller.

A **reversed Galilean telescope** provides a minified erect image ($0 < MP < 1$). This configuration is used in door peepholes and many camera viewfinders. In these systems, the eye is often the system stop.

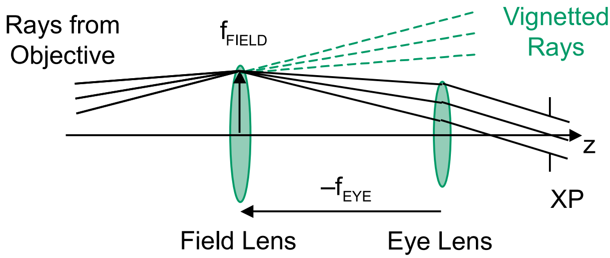
> The term telescope has come to mean any system used to view distant objects. Here, telescope specifically refers to an afocal system used with the eye. Large astronomical telescopes are actually objectives or cameras where an image array detector is placed at the system focal point.

Binoculars are a pair of parallel telescopes, one for each eye. The specification provided on telescopes and binoculars is of the form AXB (for example 7X35).

$$A = |MP| \qquad B = \text{Objective diameter in mm}$$

Field Lenses

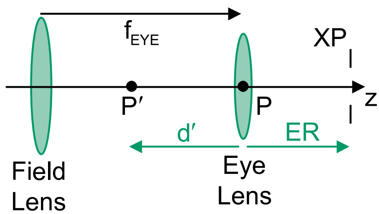
The FOV of the Keplerian telescope is limited by vignetting at the eye lens. A **field lens** placed at the intermediate image plane increases the FOV by bending the ray bundle into the aperture of the eye lens.



The combination of the field lens eye lens has the same focal length as the eye lens. The front principal plane of the combination remains at the eye lens, but the field lens shifts the rear principal plane to reduce the original eye relief by d'.

$$f = f_{EYE}$$

$$d' = -\frac{f_{EYE}^2}{f_{FIELD}}$$



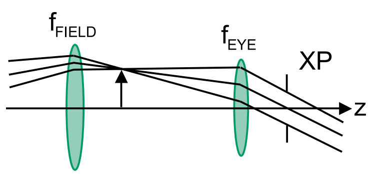
The field lens does not change the MP of the telescope or the size of the XP. Maintaining a usable ER limits the strength of the field lens and the FOV increase possible for a given eye lens diameter. Since the field lens is located at an image plane, dirt and imperfections on it become part of the image. In practice, the field lens is often displaced from the image plane to minimize these effects through defocus.

A Keplerian telescope can be considered to be the combination of an objective plus a magnifier. An **aerial image** (or an image formed in air) is formed at the common focal point by the objective. The eye lens magnifies this image and transfers it to infinity.

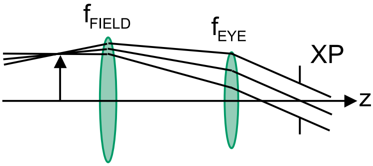
Eyepieces

An **eyepiece** or **ocular** is the combination of the field lens and the eye lens. A **simple eyepiece** does not have a field lens. A **compound eyepiece** has both an eye lens and a field lens.

A **field stop** can be placed at the intermediate image plane to restrict the system FOV. This aperture serves to limit the field to a well-corrected or non-vignetted region. **Reticles** and **graticles** provide alignment and measurement fiducial marks, and they are placed in the intermediate image plane to be superimposed on the image. Since both the reticle and the image are in focus, reticles must be clean and defect free.



Two special eyepiece configurations displace the field lens from the intermediate image plane. The intermediate image plane for a **Huygens eyepiece** falls between the two elements. The **Ramsden eyepiece** places the field lens behind the intermediate image. It is a good choice to use with reticles as the eyepiece does not change the magnification or size of the intermediate image. This eyepiece has about 50% more eye relief than the Huygens eyepiece. A **Kellner eyepiece** replaces the singlet eye lens of the Ramsden eyepiece with a doublet for color correction.

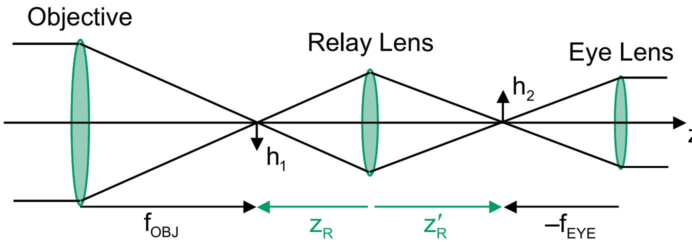


Hand-held instruments should have 15–20 mm of eye relief. Microscopes may have as little as 2–3 mm of eye relief. Other systems, such as riflescopes, should have a very long eye relief.

The XP should be made larger or smaller than the pupil of the eye so that vignetting does not occur with head or eye motion. The human eye pupil diameter varies from 2–8 mm, with a diameter of about 4 mm under ordinary lighting conditions. When overfilled, the eye becomes the system stop.

Relays

For terrestrial applications, the image orientation of a Keplerian telescope can be corrected using an image erection prism such as a Porro prism system or a Pechan-roof prism. A **relay lens** can also correct the image orientation.



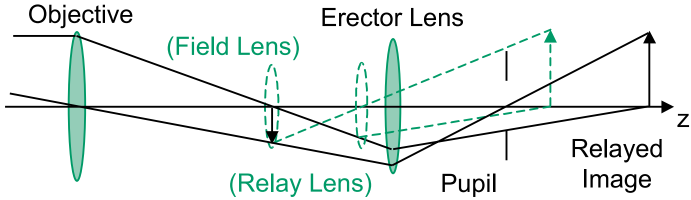
The net MP of the **relayed Keplerian telescope** is positive and equals the product of the magnification of the relay and the MP of the original Keplerian telescope.

$$m_R = \frac{z'_R}{z_R} \qquad\qquad MP = m_R MP_K = -\frac{z'_R f_{OBJ}}{z_R f_{EYE}}$$

Multiple relay lenses can be used to transfer the image over a long distance. Examples include periscopes, endoscopes and borescopes.

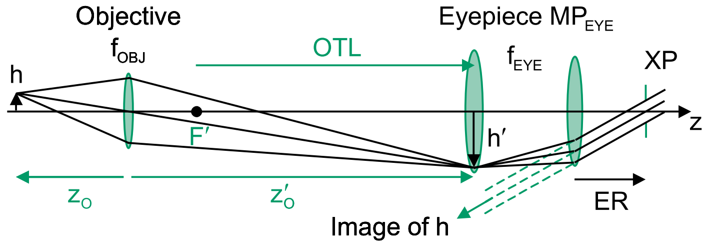
Field lenses can also be added at the intermediate images. A common arrangement is for each field lens to image the pupil into the following relay lens. All of the light collected by the objective is transferred down the optical system. The final field lens is part of the eyepiece.

The functions of a field lens and a relay lens can be combined into a single **erector lens**. This lens will require a diameter larger than the replaced field or relay lenses. The relayed image and pupil are shifted from their original positions.



Microscopes

A **microscope** is a sophisticated magnifier consisting of an objective plus an eyepiece.



The **visual magnification** is the product of the objective magnification and the eyepiece MP.

$$m_{OBJ} = \frac{z'_O}{z_O} \qquad MP_{EYE} = \frac{250 \text{ mm}}{f_{EYE}}$$

$$m_V = m_{OBJ} MP_{EYE} = \frac{z'_O}{z_O} \frac{250 \text{ mm}}{f_{EYE}}$$

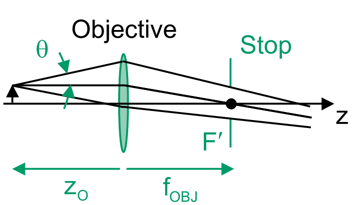
The **optical tube length** OTL of a microscope is defined as the distance from the rear focal point of the objective to the front focal point of the eyepiece (intermediate image). Standard values for the OTL are 160 mm and 215 mm. The OTL is a Newtonian image distance:

$$m_{OBJ} = -\frac{OTL}{f_{OBJ}} \qquad m_V = -\frac{OTL}{f_{OBJ}} \frac{250 \text{ mm}}{f_{EYE}}$$

The NA of a microscope objective is defined in object space by the half-angle of the accepted input ray bundle. Along with the objective magnification, the NA is inscribed on the objective barrel.

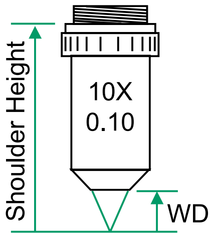
$$NA = n \sin\theta$$

Microscope objectives are often telecentric in object space. The stop is placed at the rear focal point of the objective so that the magnification does not change with object defocus.

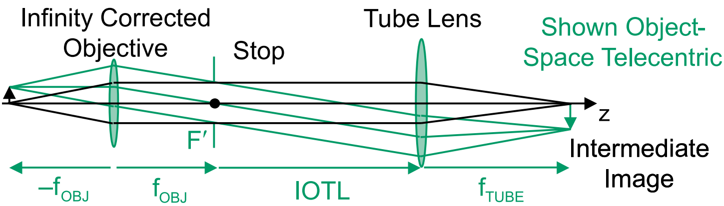


Microscope Terminology

- The **working distance** WD is the distance from the object to the first element of the objective; can be less than 1 mm for high-power objectives.
- The **mechanical tube length** is separation between the shoulder of the threaded mount of the objective and the end of the tube into which the eyepiece is inserted. Objectives and eyepieces must be used at their design conjugates and are not necessarily interchangeable between manufacturers.



- A set of **parfocal objectives** have different magnifications, but the same **shoulder height** and the same shoulder-to-intermediate image distance. As parfocal objectives are interchanged with a rotating turret, the image changes magnification but remains in focus.
- **Biological objectives** are aberration corrected assuming a cover glass between the object and the objective. The design of a **metallurgical objective** assumes no cover glass.
- Research-grade microscopes are usually designed using **infinity corrected objectives**. The object plane is the front focal plane of the objective, and a collimated beam results for each object point. There is no specific tube length, and an additional tube lens is used to produce the intermediate image presented to the eyepiece.



The magnification of the objective-tube lens combination is

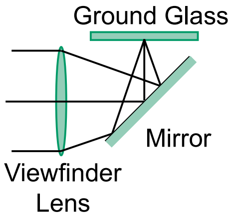
$$m_{OBJ} = -f_{TUBE}/f_{OBJ}$$

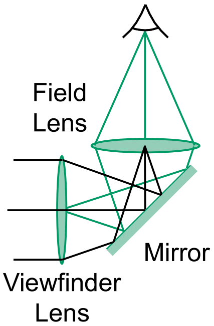
If the objective is object-space telecentric and f_{TUBE} equals the infinite optical tube length IOTL, the combination is afocal and double telecentric. This is a useful feature when using reticles in the eyepiece.

Viewfinders

Viewfinders allow for framing the scene in camera systems. The FOV of the viewfinder should match the FOV recorded by the camera.

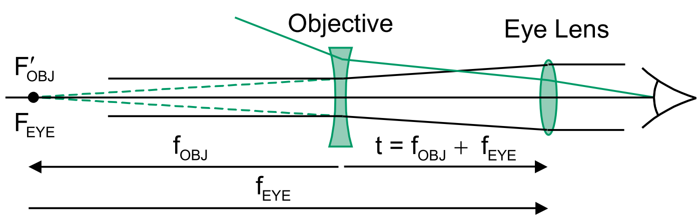
A **reflex viewfinder** is a waist-level viewfinder that uses an auxiliary objective on the camera. The dim image produced on a ground glass screen is erect but reverted.





A **brilliant reflex viewfinder** produces a much brighter image by replacing the ground glass with a field lens. The aperture of the viewfinder lens is imaged onto the eyes of the operator.

Reverse Galilean viewfinders ($MP < 1$) are common in point-and-shoot cameras, however the lack of an intermediate image plane prevents the use of a reticle for framing marks to define the FOV. The viewfinder stop is often at the eye.

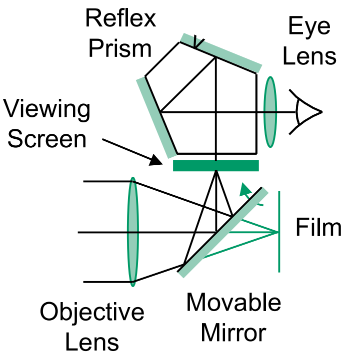


The **Van Albada viewfinder** adds framing marks by placing a partially reflecting coating on the negative lens of the reverse Galilean viewfinder. This resulting concave mirror images a framing mask or reticle (surrounding the positive eye lens) to the front focal plane of the eye lens. The framing marks, now imaged to infinity by the eye lens, are superimposed on the straight-through viewfinder image of the scene.

For near objects, **parallax** between the camera FOV and the viewfinder FOV is a problem with all of these viewfinders.

Single Lens Reflex and Triangulation

The **single lens reflex** SLR system solves the parallax problem by using the camera objective also for the viewfinder. The movable mirror directs the light path either through the viewfinder or to the film or detector. The ground glass is optically conjugate to the film, and the eye lens serves as a magnifier to view the image on this viewing screen. The **reflex prism** corrects the image parity and provides eye-level viewing. The ground glass viewing screen prevents vignetting by scattering light from the entire image into the eye lens. It can be replaced by a field lens, often a **Fresnel lens**, for light efficiency.



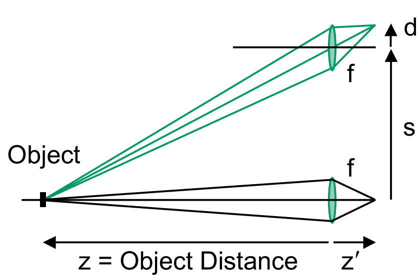
Because the viewfinder shares the objective lens, the SLR system is ideal for use with interchangeable camera lenses.

> Imaging (a real object and a real image) introduces a 180° image rotation. The optical magnification is negative, and the image is inverted and reverted.

The perspective difference or **parallax** between images produced by separated objectives can be used to **triangulate** the distance to an object. The object distance z is related to the relative image displacement d:

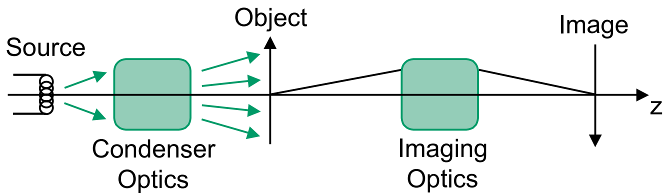
$$z = -\frac{sz'}{d} \approx -\frac{sf}{d}$$

Passive triangulation systems examine the two images produced by ambient scene light. **Active triangulation** sends a light beam out through one lens, and images the light reflected by the object with the other lens.



Illumination Systems

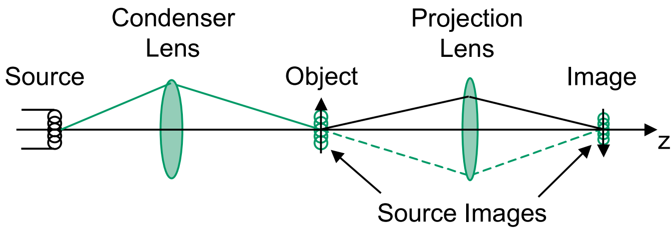
A **projector** is the general term for an imaging system that also provides the illumination for the object.



There are three basic classifications of illumination systems:

- **Diffuse illumination** – light with a large angular spread is incident on the object. This description would also include ambient or natural lighting conditions. There is no attempt to image the source into the imaging system. This type of system is simple and provides uniform illumination, but it is light inefficient.
- **Specular illumination** – the light source is imaged by the condenser optics into the EP of the imaging optics. Because of its good light efficiency, specular illumination is used for most optical systems designed with an integral light source.
- **Critical illumination** – the light source is imaged directly onto the object.

While very light efficient, critical illumination is rarely used. The source brightness distribution is superimposed directly on the object and therefore also appears as a brightness modulation of the image. A very uniform source is required; an example is a tungsten ribbon filament. The field of view of this type of system is typically small.

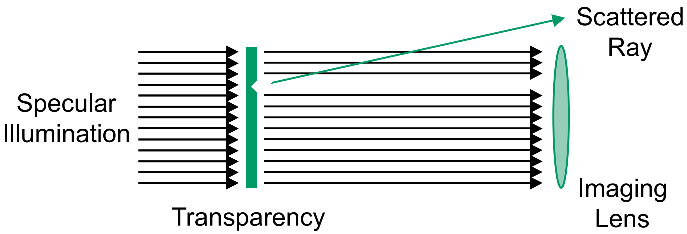


Diffuse Illumination

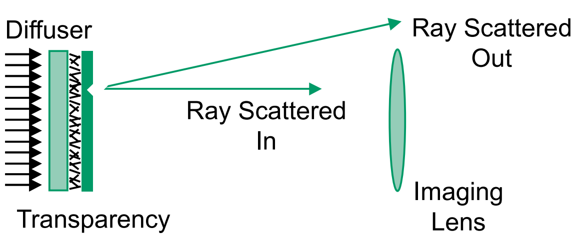
Diffuse illumination is usually achieved by the insertion of a **diffuser** into the system. Surface diffusers, such as ground glass, tend to be more efficient and less uniform than volume diffusers, such as opal glass or translucent plastic sheets.

Diffusers increase the apparent size of the source resulting in greater uniformity of illumination. This greater range of illumination angles also provides **scratch suppression** that will hide phase errors on the object, such as a scratch or defect in the substrate of the object transparency.

If specular or narrow angle illumination is used, this scratch will scatter the light out of the optical system, and the scratch will appear dark in the image.

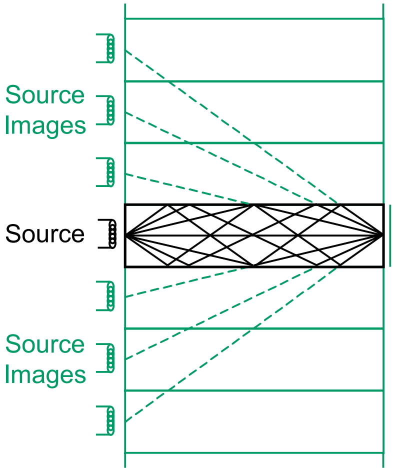


With diffuse illumination, many different input angles are present, and while some rays are scattered out of the system by the scratch, other rays will be scattered into the aperture of the imaging lens. The visibility of the scratch in the image is significantly decreased.

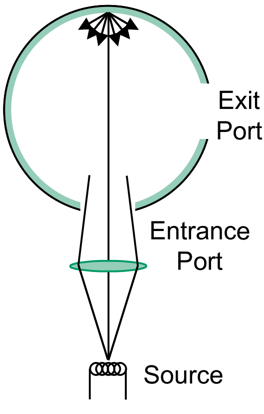


A scratch or defect in the transmission of the object is not hidden even by diffuse illumination. For example, a scratch in the emulsion of a transparency becomes part of the object and will be seen in the image.

Integrating Spheres and Bars



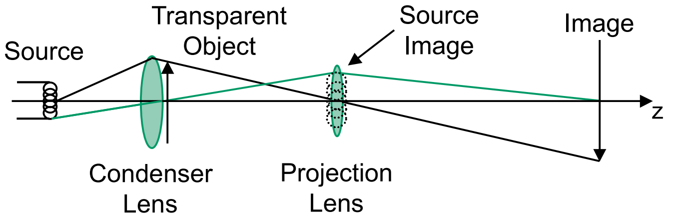
An **integrating bar** or **light pipe** provides diffuse light with a significant increase in efficiency over simple diffusers. The bar has a rectangular cross section with polished surfaces. The source is placed at one end of the bar, and TIR occurs at each face. The tunnel diagram shows that the transparency at the other end of the bar sees a rectangular array of source images. The effect is similar to a **kaleidoscope**. A greater range of illumination angles or diffuseness results. The bar geometry and the TIR critical angle limit the number of source images. With six polished faces, integrating bars are expensive. The source images produced by a **tapered integrating bar** (used to reduce the illuminated area) are located on a sphere. Hollow mirror tunnels can be used instead of solid glass.



The ultimate in diffuse illumination is provided by an **integrating sphere**. The inside of a hollow sphere is coated with a highly reflective diffuse white coating. Light directed into the entry port undergoes many random reflections before escaping through the exit port. The output light is extremely uniform with a brightness that is independent of viewing angle. The two ports are usually at 90° to prevent the direct viewing of the source and the first source reflection. Integrating spheres are also used in precision measurement radiometers by replacing the source with a detector.

Projection Condenser System

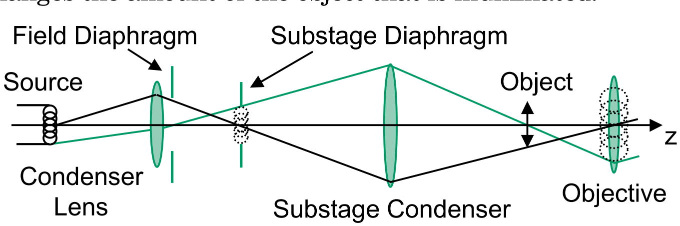
The most common example of specular illumination is the **projection condenser system**. A **condenser lens**, placed in close proximity to the transparent object, images the source into the pupil of the **projection lens**.



Each point on the object is illuminated by all parts of the source resulting in uniform illumination. The angular range of the illumination at the object is limited to the angular size of the source as seen from the object. The condenser lens serves as a field lens to bend source rays going through the edge of the object back into the projection lens. The condenser lens should be designed to be as fast as possible ($f/\#_W$ often faster than f/1 on the source side). The projection lens diameter must be larger than the size of the source image.

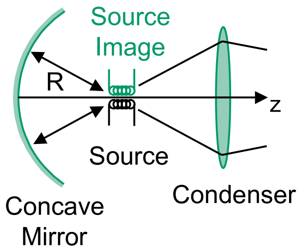
> The projection condenser system can be considered to be two coupled optical systems. The marginal ray of the condenser system becomes the chief ray of the imaging system, and the chief ray of the condenser system becomes the marginal ray of the imaging system.

Koehler illumination is a type of specular illumination often used in microscopes to provide control of the illumination. The **substage diaphragm** (at the source image) allows the overall light level to be varied, and the **field diaphragm** changes the amount of the object that is illuminated.

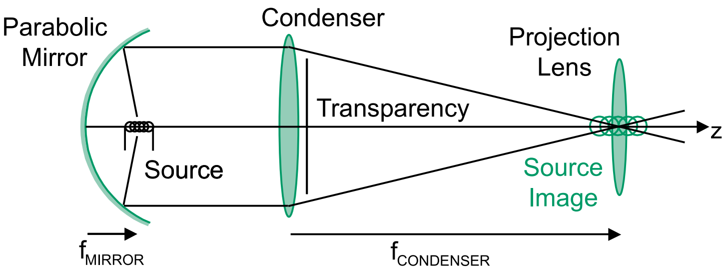


Source Mirrors

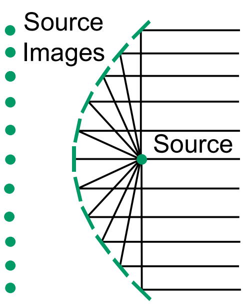
Placing a **concave mirror** behind the source can increase the light level in the projection system. The classic solution is to place the source at the center of curvature of the mirror. The source image is on top of or adjacent to the source. An improvement of less than a factor of two is obtained.



Dramatic increases in illumination level occur by placing the source at the focus of the concave mirror. The source image occurs at infinity. The solid angle of the mirror can be more than 2π sr, and the amount of light intercepted and reflected by the mirror can exceed the light directly collected by the condenser by a factor of ten or more. The designs of systems of this type almost ignore the forward light through the condenser. The mirror shape is usually parabolic.

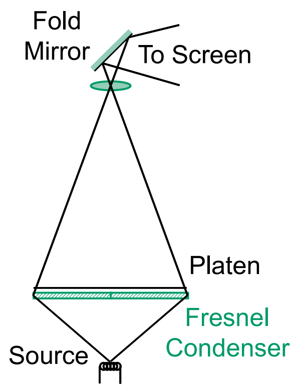


To provide a greater level of diffuseness, the surface of the parabola can be segmented into small flat mirrors. A virtual source is formed behind each facet. The details of the **faceted parabolic reflector** are complicated, but for design purposes it can be modeled as an extended source located at or near the concave mirror. The mirror aperture defines the extent of the extended source. The condenser lens images the collected sources into the aperture of the projection lens.

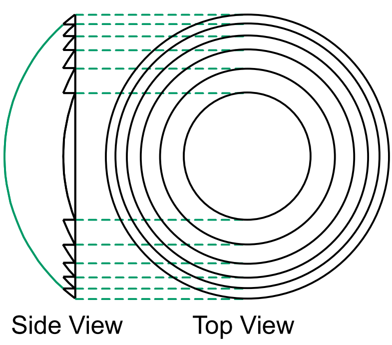


Overhead Projector

The **overhead projector** uses projection condenser illumination to project a large transparency onto a projection screen located behind the presenter. In addition to bending the light path, the fold mirror creates the proper image parity for the audience.



Because of the large size of the transparency, a conventional condenser lens is impractical and a **Fresnel lens** is used. The thick lens is collapsed into radial zones.



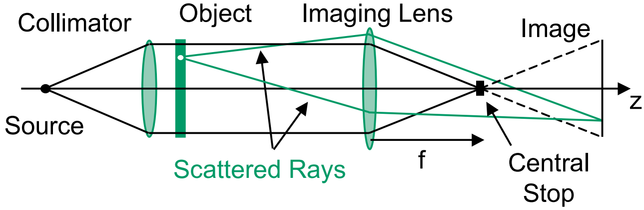
An image is produced by each zone, and these images add incoherently, so that the diffraction-based resolution is that of a single zone.

> To determine parity, the diffuse reflection from the **projection screen** introduces a parity change like any other reflection.

Heat management is a significant issue for most projectors. **Heat absorbing glass** or a **hot mirror** can be placed between the source and the condenser lens. In addition, a **concave cold mirror** behind the source allows the heat or infrared IR radiation to exit out the back of the system. A hot mirror reflects the IR light (the hot) and transmits the visible light. A cold mirror reflects the visible light (the cold) and transmits the IR light. A cooling fan is often required to supplement the heat management in the optical system.

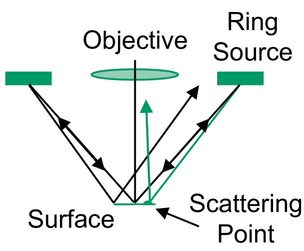
Schlieren and Dark Field Systems

Specular or narrow angle illumination can be used to identify features or defects on an object. In a **schlieren system**, light from a small source is collimated before passing through the object plane. An imaging lens forms an image of the source as well as the final image. The image of the source is blocked by an opaque disk or a knife edge. With no object present, the image appears black. When the object is inserted, any feature or imperfection on the object will scatter (or refract or diffract) some light past the obscuration. These localized areas on the object will appear bright in the image.



Some applications of the schlieren technique are aerodynamic flow visualization and inspecting glass for inhomogeneity and stria.

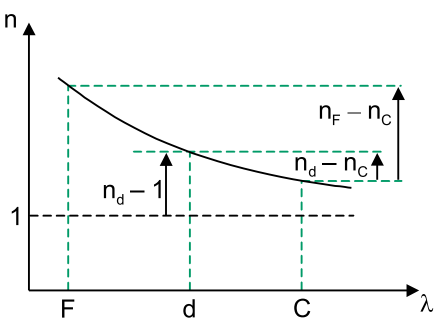
Dark field illumination is a variation of this technique using directional lighting. The light source is placed to the side of the objective lens, or in a ring around the lens. If the object is perfectly smooth (a mirror), a specular reflection within the FOV misses the objective, and the image is dark. Features or imperfections on the surface will scatter light into the objective and appear bright in the image. This technique is especially common in machine vision and reflection microscopy. Setups for transmission dark field measurements also exist.



With both techniques, the orientation of features, or the surface derivatives, can be measured using an oriented knife edge (schlieren) or by directional illumination (dark field).

Dispersion

Index of refraction is commonly measured and reported at the specific wavelengths of elemental spectral lines. Over the visible spectrum, the **dispersion** of the index of refraction for optical glass is about 0.5% (low dispersion) to 1.5% (high dispersion) of the mean value of the index.



F	(H)	486.1 nm
d	(He)	587.6 nm
C	(H)	656.3 nm
I	(Hg)	365.0 nm
h	(Hg)	404.7 nm
F′	(Cd)	480.0 nm
g	(Hg)	435.8 nm
e	(Hg)	546.1 nm
D	(Na)	589.3 nm
C′	(Cd)	643.8 nm
r	(He)	706.5 nm
t	(Hg)	1014.0 nm

For visible applications, the F, d and C lines are usually used.

Refractivity: $n_d - 1$ **Principal dispersion**: $n_F - n_C$

Abbe number (or reciprocal relative dispersion):

$$\nu = V = \frac{n_d - 1}{n_F - n_C} \qquad \frac{\text{Refractivity}}{\text{Principal dispersion}}$$

Typical values of the Abbe number for optical glass range from 25 to 65. Low ν-values indicate high dispersion.

Partial dispersion: $n_d - n_C$

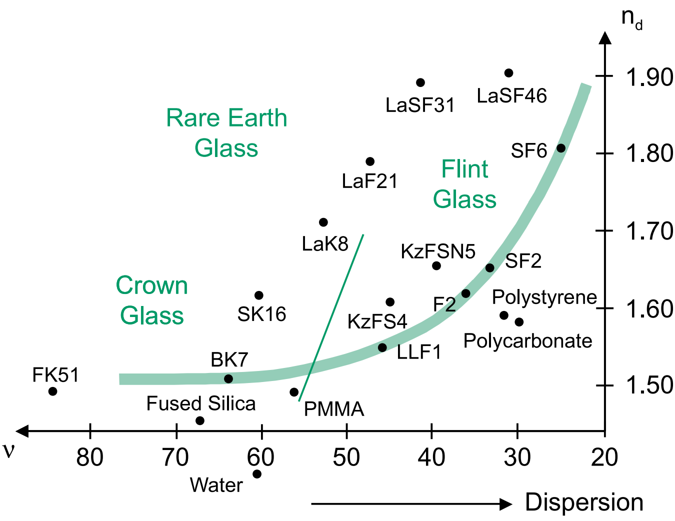
Relative partial dispersion ratio: $P = P_{d,C} = \dfrac{n_d - n_C}{n_F - n_C}$

The P-value gives the fraction of the total index change $n_F - n_C$ that occurs between the d and C wavelengths $n_d - n_C$. Due to the flattening of the dispersion curve, $P_{d,C} < 0.5$. P-values can also be defined for other sets of wavelengths:

$$P_{X,Y} = \frac{n_X - n_Y}{n_F - n_C}$$

Optical Glass

The **glass map** plots index of refraction versus Abbe number. By tradition, the Abbe number increases to the left, so that dispersion increases to the right. The **glass line** is the locus of ordinary optical glasses based on silicon dioxide.



The line at ν ~ 50 to 55 separates the glasses into the two primary classifications: crown glass (low dispersion) and flint glass (high dispersion).

The addition of lead oxide increases the dispersion and the index and moves the glass up the glass line. To increase the index without changing the dispersion, barium oxide is added. The **rare earth glasses** are lanthanum oxide based (instead of silicon dioxide) and provide high index and low dispersion.

Glasses away from the glass line are softer and more difficult to polish. Low index glasses are less dense and generally have better blue transmission.

Glasses are currently being reformulated to eliminate lead and arsenic. Lead is replaced with other elements, especially titanium. The new or environmentally safe glasses usually carry an N, S or E prefix (depending on the manufacturer).

Material Properties

The six-digit **glass code** specifies the index and the Abbe number:

$$abcdef \qquad n_d = 1.abc \quad \nu = de.f$$

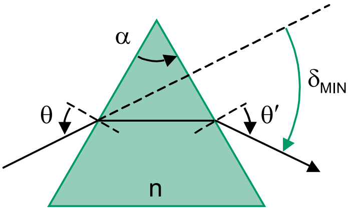
Material	Code	n_d	n_F	n_C	ν	P
N-FK51*	487845	1.48656	1.49056	1.48480	84.5	0.306
N-BK7	517642	1.51680	1.52238	1.51432	64.2	0.308
LLF1	548458	1.54814	1.55655	1.54457	45.8	0.298
N-KzFS4	613445	1.61336	1.62300	1.60922	44.5	0.301
N-F2	620364	1.62005	1.63208	1.61506	36.4	0.294
N-SK16	620603	1.62041	1.62756	1.61727	60.3	0.305
SF2	648339	1.64769	1.66123	1.64210	33.9	0.292
KzFSN5	654396	1.65412	1.66571	1.64920	39.6	0.298
N-LaK8	713538	1.71300	1.72222	1.70897	53.8	0.304
N-LaF21	788475	1.78800	1.79960	1.78301	47.5	0.301
N-SF6	805254	1.80518	1.82783	1.79608	25.4	0.287
N-LaSF31	881410	1.88067	1.89576	1.87429	41.0	0.297
N-LaSF46	901316	1.90138	1.92156	1.89307	31.6	0.292
Fused Silica	458678	1.45847	1.46313	1.45637	67.8	0.311
PMMA	492574	1.492	1.498	1.489	≈55	≈0.33
Polycarbonate	585299	1.585	1.600	1.580	≈30	≈0.25
Polystyrene	590311	1.590	1.604	1.585	≈31	≈0.26
Water	333560	1.333	1.337	1.331	≈60	≈0.33

* Schott Glass Technologies Inc. designation. Equivalent glasses can also be obtained from Ohara Corp. and Hoya Corp.

The properties of an individual sample, especially for the plastic materials and water, can vary from these catalog values. For precision systems, the measured indices of the actual glass should be used in final designs. The listed indices are measured relative to air ($n \approx 1.0003$), and the indices should be corrected for use in vacuum. In addition to index data at various wavelengths, the glass catalog lists other materials properties important for a design such as **coefficients of thermal expansion**, **temperature coefficients of refractive index**, **internal transmission** as a function of wavelength, several mechanical properties, and chemical resistance values (for example, **stain resistance**, **climatic resistance** and **acid resistance**).

Dispersing Prisms

At **minimum deviation**, the ray path through a **dispersing prism** is symmetric $\theta' = -\theta$. The ray is bent an equal amount at each surface. By sign convention, the deviation is negative for this prism orientation. The **angle of minimum deviation** is



For $\alpha = 60°$	
n	δ_{MIN}
1.3	–21.1°
1.4	–28.9°
1.5	–37.2°
1.6	–46.3°
1.7	–56.4°
1.8	–68.3°
2.0	–120°

$$\delta_{MIN} = \alpha - 2\sin^{-1}[n\sin(\alpha/2)]$$

The measurement of the index depends only on δ_{MIN} and the prism apex angle α:

$$n = \frac{\sin[(\alpha - \delta_{MIN})/2]}{\sin(\alpha/2)}$$

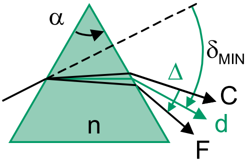
Prism spectrometers can obtain accuracies of one part in 10^6.

The details of the **prism dispersion** depend on the geometry used and the index dispersion curve. However, assuming the prism is used at or near δ_{MIN}, the average prism dispersion over a wavelength band (F to C) can be estimated:

$$\frac{d\delta}{d\lambda} = \frac{d\delta}{dn}\frac{dn}{d\lambda} \approx \frac{d\delta_{MIN}}{dn}\frac{\Delta n}{\Delta\lambda} \approx \frac{d\delta_{MIN}}{dn}\frac{(n_F - n_C)}{(\lambda_F - \lambda_C)}$$

where

$$\frac{d\delta_{MIN}}{dn} = \frac{-2\sin(\alpha/2)}{\cos[(\alpha - \delta_{MIN})/2]}$$



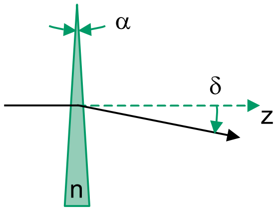
$\alpha = 60°$	BK7	F2
$\delta_{MIN}\,(n_d)$	–38.7°	–48.2°
$\Delta n/\Delta\lambda$	–.0474/µm	–.1002/µm
$d\delta/d\lambda$	4.18°/µm	10.2°/µm
Δ or $\delta_F - \delta_C$	–.79°	–1.92°

Blue light is deviated more than red light.

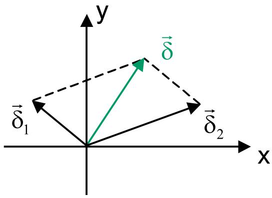
Thin Prisms

Thin prisms introduce small angular beam deviations and are useful as alignment devices. The beam deviation δ is approximately independent of the incident angle:

$$\delta \approx -(n-1)\alpha$$



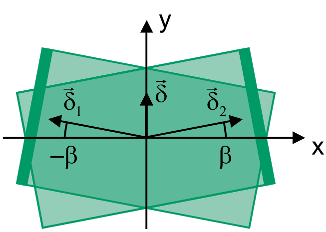
Thin prisms are used for optometric correction of strabismus (a misalignment of the axes of the eyes). The deviation is measured in **prism diopters**. A prism of 1 diopter deviates a beam by 1 cm at 1 m.



$$\vec{\delta} = \vec{\delta}_1 + \vec{\delta}_2$$

The beam deviation is parallel to a principal section of the prism and towards the thick end of the prism. The magnitude and direction of this deviation defines a vector perpendicular to the optical axis (in the x-y plane). The net deviation vector for a series of thin prisms is the sum of the component vectors.

A **Risley prism** consists of a pair of identical, but opposing, thin prisms. The prisms are counter-rotated by ±β to obtain a variable net deviation in a fixed direction (shown with the net deviation in the y-direction).



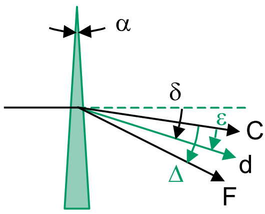
$$\delta_0 = |\vec{\delta}_1| = |\vec{\delta}_2|$$

$$\delta = 2\delta_0 \sin(\beta)$$

The Risley prism allows the fine angular alignment of an optical system by adjusting the prism orientations β.

Thin Prism Dispersion and Achromatization

The **dispersion of a thin prism** Δ measures the total angular spread from C to F light, and the **secondary dispersion** ε gives the spread from the C to d wavelengths. The results depend on the index n_d, Abbe number ν and partial dispersion ratio P of the glass.



Deviation: $\delta = -(n_d - 1)\alpha$

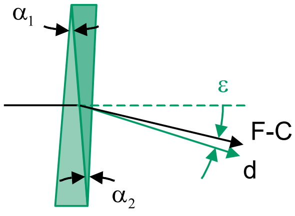
Dispersion: $\Delta = -(n_F - n_C)\alpha \qquad \Delta = \dfrac{\delta}{\nu}$

Secondary Dispersion: $\varepsilon = -(n_d - n_C)\alpha \qquad \varepsilon = P\Delta = P\dfrac{\delta}{\nu}$

An **achromatic thin prism** or **achromatic wedge** provides deviation without dispersion. Opposing prisms made from two different glasses (n_{d1}, ν_1, P_1 and n_{d2}, ν_2, P_2) are combined to force the dispersion between the F and C wavelengths to be zero. A deviation of δ is maintained for d light.

$$\frac{\alpha_1}{\delta} = \left(\frac{1}{\nu_2 - \nu_1}\right)\left(\frac{\nu_1}{n_{d1} - 1}\right)$$

$$\frac{\alpha_2}{\delta} = -\left(\frac{1}{\nu_2 - \nu_1}\right)\left(\frac{\nu_2}{n_{d2} - 1}\right)$$



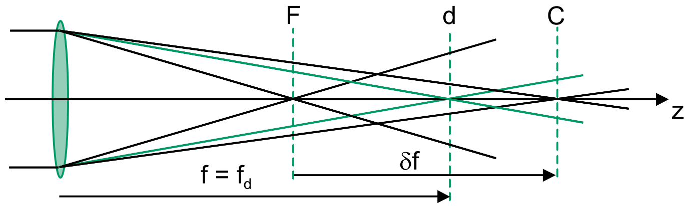
The high-dispersion prism is inverted to obtain an opposing deviation (as drawn, $\alpha_1 > 0$ and $\alpha_2 < 0$). While the F and C wavelengths are corrected, a residual secondary dispersion remains. For most glass pairs, d light will be bent more than the F and C wavelengths.

$$\frac{\varepsilon}{\delta} = \left(\frac{P_2 - P_1}{\nu_2 - \nu_1}\right) = \frac{\Delta P}{\Delta \nu}$$

A **direct vision prism** uses opposing prisms to provide dispersion without deviation of the d light.

Chromatic Aberration

Axial chromatic aberration or **axial color** is a variation of the system focal length with wavelength. This aberration derives from the dispersion of the glass.



$$\phi \equiv \frac{1}{f} = (n-1)(C_1 - C_2) \qquad \text{(Thin lens)}$$

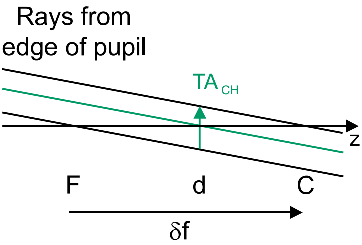
$$\delta f = f_C - f_F$$

$$\delta\phi = \phi_F - \phi_C$$

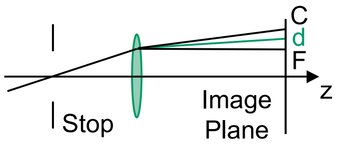
$$\frac{\delta f}{f} = \frac{\delta\phi}{\phi} = \frac{1}{\nu}$$

Since Abbe numbers are typically 30–70, the longitudinal chromatic aberration of a singlet is 1.5–3% of the focal length. The relative order of the foci is reversed for a negative lens.

Transverse axial chromatic aberration measures the image blur size due to axial chromatic aberration. It depends only on the glass and the pupil radius r_P (stop at the lens).



$$TA_{CH} = \frac{r_P}{\nu}$$



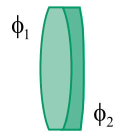
Lateral chromatic aberration or **lateral color** is caused by dispersion of the chief ray. The edge of the lens behaves like a prism. Off-axis image points will exhibit a radial color smear. The blur length increases linearly with image height. Each color has a different lateral magnification.

Achromatic Doublet

The thin lens **achromatic doublet** corrects longitudinal chromatic aberration by combining a positive element and a negative element. Two different glasses (ν_1, P_1 and ν_2, P_2) are used. The nominal powers and focal lengths are for d light.

$$\phi = \phi_1 + \phi_2 \qquad \phi_F = \phi_C$$

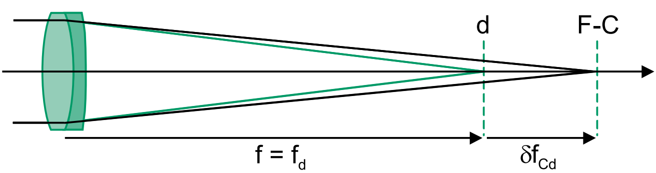
$$\frac{\phi_1}{\phi} = \frac{\nu_1}{\nu_1 - \nu_2} \qquad \frac{\phi_2}{\phi} = -\frac{\nu_2}{\nu_1 - \nu_2}$$



This result forces the same axial focus for F and C light, but d light can focus at a different location. This is the **secondary chromatic aberration** or **secondary color** of the doublet.

$$\delta\phi_{dC} = \phi_d - \phi_C \qquad \delta f_{Cd} = f_C - f_d$$

$$\frac{\delta\phi_{dC}}{\phi} = \frac{\delta f_{Cd}}{f} = \frac{P_2 - P_1}{\nu_2 - \nu_1} = \frac{\Delta P}{\Delta\nu}$$



On a plot of P versus ν, most glasses lie on a straight line.

$$\frac{\Delta P}{\Delta\nu} \approx 0.00045 \qquad \delta f_{Cd} \approx \frac{f}{2200}$$

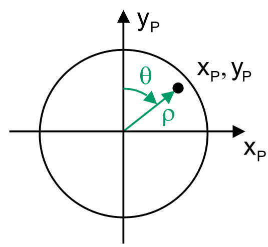
The use of the achromatic doublet reduces chromatic focal length variation by a factor of about 40 over the same focal length singlet. The d focus is inside the F-C focus.

The doublet design places **excess power** in the positive element that is cancelled by the negative element. Both elements contribute equal, but opposite, amounts of primary chromatic aberration. Large differences in Abbe number minimize the excess power and provide better performance.

Monochromatic Aberrations

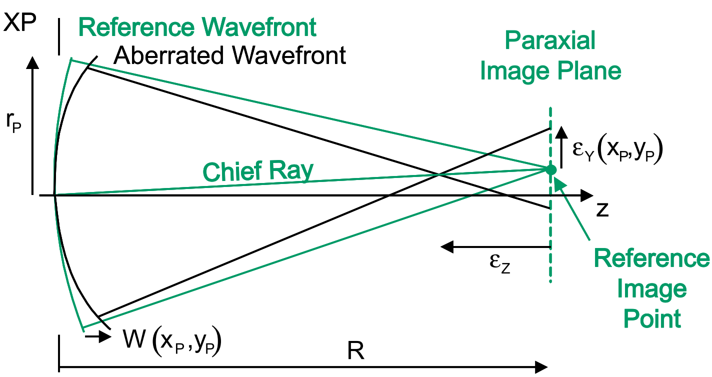
First-order or paraxial systems are ideal optical systems with perfect imagery. **Aberrations** describe the deviations of real systems from this perfection. Since the object is modeled as a collection of independently radiating point sources, light is propagated from a given object point to all points in the pupil of the system to analyze the aberrations.

The aberrations are a function of the **normalized pupil coordinates** x_P, y_P and the **normalized image height** H. Normalized polar pupil coordinates are also used. Note that by tradition, the azimuth angle θ is defined against the sign convention. The physical pupil radius is r_p .



$$x_p = \rho \sin\theta$$

$$y_p = \rho \cos\theta$$



A **reference image point** is defined by the intersection of the paraxial chief ray and the paraxial image plane. **Transverse ray errors** $\varepsilon_X, \varepsilon_Y$ and **longitudinal ray errors** ε_Z are measured relative to this reference image point. **Wavefront errors** W are measured in the XP relative to a **reference wavefront** or **reference sphere** centered on the reference image point. R is the radius of the reference sphere or the image distance. A positive wavefront error is shown.

Rays and Wavefronts

The wavefront error is the OPD difference between the actual wavefront and the reference wavefront. The wavefront error will change if the reference image point is moved.

$$W(x_P, y_P) = W_A(x_P, y_P) - W_R(x_P, y_P)$$

The rays are perpendicular to the wavefront. The **transverse ray errors** are related to the slope of the wavefront error:

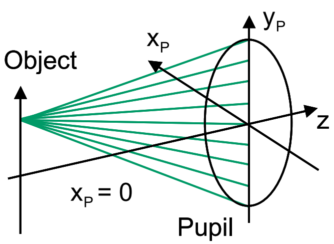
$$\varepsilon_Y(x_P, y_P) = -\frac{R}{r_P}\frac{\partial W(x_P, y_P)}{\partial y_P}$$

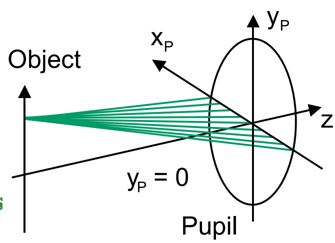
$$\frac{R}{r_P} = \frac{-1}{n'u'} \approx 2f/\#_W$$

$$\varepsilon_X(x_P, y_P) = -\frac{R}{r_P}\frac{\partial W(x_P, y_P)}{\partial x_P}$$

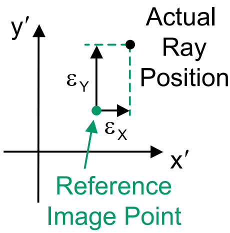
n' and u' are the image space index and marginal ray angle.

By rotational symmetry, only object points in the meridional plane need be considered. A **skew ray** leaves the meridional plane and intersects a general point in the pupil. Two special sets of rays are used for aberration analysis. **Tangential rays** or **meridional rays** intersect the pupil at $x_P = 0$.



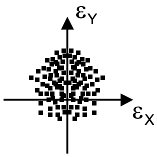


Sagittal rays or **transverse rays** intersect the pupil at $y_P = 0$.



Wave fans are plots of the wavefront error for these two sets of rays. **Ray fans** (or **ray intercept curves**) plot the transverse ray error. The **tangential ray fan** plots ε_Y versus y_P for $x_P = 0$. The **sagittal ray fan** plots ε_X versus x_P for $y_P = 0$.

Spot Diagrams



The **spot diagram** provides a geometrical estimate of the image blur produced by that system aberration. From a single object point, rays are traced through a uniform grid in the EP. Each ray corresponds to the same amount of energy. The spot diagram plots all of the ray intersections relative to the reference image point. The common grids are square, hexapolar and dithered.

The **spot centroid** relative to the reference image location is found by averaging the ray errors:

$$\bar{\varepsilon}_Y = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_{Yi} \qquad \bar{\varepsilon}_X = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_{Xi}$$

The spot size (max to min) is found by sorting through the transverse ray errors to find the total range in x and y.

A better measure of the spot size is the **root-mean-squared spot size** *RMS*. The ray errors are integrated or summed over the pupil:

$$RMS_Y = \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^1(\varepsilon_Y-\bar{\varepsilon}_Y)^2\rho\,d\rho\,d\theta\right]^{1/2} = \left[\frac{1}{N}\sum_{i=1}^{N}(\varepsilon_{Yi}-\bar{\varepsilon}_Y)^2\right]^{1/2}$$

$$RMS_X = \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^1(\varepsilon_X-\bar{\varepsilon}_X)^2\rho\,d\rho\,d\theta\right]^{1/2} = \left[\frac{1}{N}\sum_{i=1}^{N}(\varepsilon_{Xi}-\bar{\varepsilon}_X)^2\right]^{1/2}$$

A radial **RMS spot size** can also be determined:

$$RMS_R^2 = RMS_Y^2 + RMS_X^2$$

For a rotationally symmetric optical system, the spot diagram must be symmetric with respect to the meridional plane, and $\bar{\varepsilon}_X = 0$. In a similar fashion, the sagittal wave fan must be symmetric, and the sagittal ray fan is anti-symmetric. All of the aberration measures (including the wave fans, ray fans and spot diagrams) will vary with the image height H or FOV.

Wavefront Expansion

The **wavefront expansion** is a power series expansion for the wavefront aberrations inherent to a rotationally symmetric optical system. These aberrations are inherent to the design of the system. In order to satisfy the requirements of rotational symmetry, the expansion terms are H^2, ρ^2 and $H\rho\cos\theta$. The coefficient subscript encodes the powers of the corresponding polynomial term:

$$W_{IJK} \Rightarrow H^I \rho^J \cos^K\theta$$

W =	$W_{020}\rho^2$	**Defocus**
+	$W_{111}H\rho\cos\theta$	**Wavefront tilt**
	Third-Order Terms	
+	$W_{040}\rho^4$	**Spherical aberration (SA)**
+	$W_{131}H\rho^3\cos\theta$	**Coma**
+	$W_{222}H^2\rho^2\cos^2\theta$	**Astigmatism**
+	$W_{220}H^2\rho^2$	**Field curvature**
+	$W_{311}H^3\rho\cos\theta$	**Distortion**
	Fifth-Order Terms	
+	$W_{060}\rho^6$	**Fifth-order SA**
+	$W_{151}H\rho^5\cos\theta$	**Fifth-order linear coma**
+	$W_{422}H^4\rho^2\cos^2\theta$	**Fifth-order astigmatism**
+	$W_{420}H^4\rho^2$	**Fifth-order field curvature**
+	$W_{511}H^5\rho\cos\theta$	**Fifth-order distortion**
+	$W_{240}H^2\rho^4$	**Sagittal oblique SA**
+	$W_{242}H^2\rho^4\cos^2\theta$	**Tangential oblique SA**
+	$W_{331}H^3\rho^3\cos\theta$	**Cubic coma** } **Elliptical coma**
+	$W_{333}H^3\rho^3\cos^3\theta$	**Line coma** }
+	Higher order terms	

The wavefront terms are denoted by the order of their ray aberration, which is one less than the wavefront aberration order. Terms with no pupil dependence, **piston** (W_{000}) and **field-dependent phase** (W_{200}, W_{400}, *etc.*), are usually ignored.

Tilt and Defocus

Wavefront tilt describes a difference between the paraxial magnification and the actual magnification of the system.

$$W = W_{111} H \rho \cos\theta = W_{111} H y_P$$

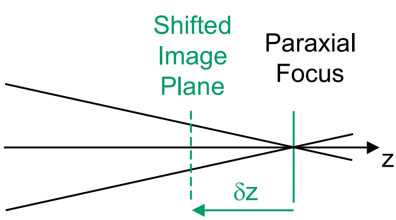
$$\varepsilon_Y = -\frac{R}{r_P} W_{111} H \qquad \varepsilon_X = 0$$

In a system with **defocus** W_{020}, the actual image plane is displaced from the paraxial image plane. More importantly, defocus allows the image plane or the reference image point to be shifted for aberration balance and better image quality. Recognizing that this shift is a user decision, the notation ΔW_{20} is used.

$$\Delta W = \Delta W_{20} \rho^2 = \Delta W_{20}(x_P^2 + y_P^2)$$

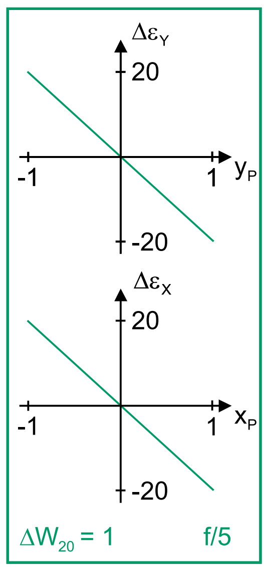
$$\Delta\varepsilon_Y = -2\frac{R}{r_P}\Delta W_{20} \rho \cos\theta = -2\frac{R}{r_P}\Delta W_{20} y_P$$

$$\Delta\varepsilon_X = -2\frac{R}{r_P}\Delta W_{20} \rho \sin\theta = -2\frac{R}{r_P}\Delta W_{20} x_P$$



$$\Delta W_{20} = \frac{\delta z}{8(f/\#)^2}$$

$$\delta z = 8(f/\#)^2 \Delta W_{20}$$



In a system that has a wavefront error W and transverse ray aberrations ε_Y, ε_X, an image plane shift changes the measured apparent aberration:

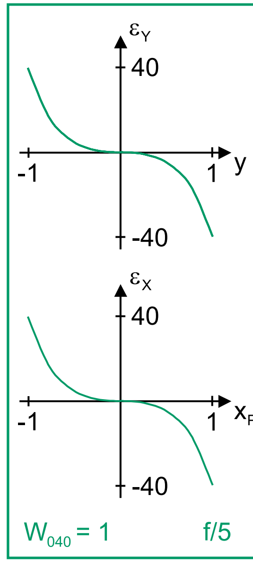
$$W = W + \Delta W$$

$$\varepsilon'_Y = \varepsilon_Y + \Delta\varepsilon_Y \qquad \varepsilon'_X = \varepsilon_X + \Delta\varepsilon_X$$

Moving the image plane changes the reference sphere, not the actual wavefront in the XP of the system.

Spherical Aberration

Spherical aberration causes the power or focal length of the system to vary with pupil radius.



$$W = W_{040}\rho^4 = W_{040}(x_P^2 + y_P^2)^2$$

$$\varepsilon_Y = -4\frac{R}{r_P}W_{040}\rho^3\cos\theta$$

$$\varepsilon_X = -4\frac{R}{r_P}W_{040}\rho^3\sin\theta$$

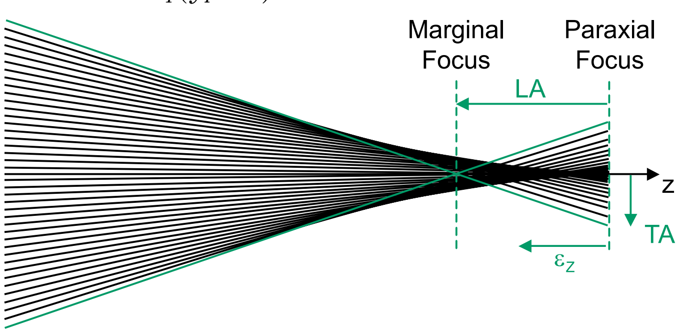
Ray fans:

$$\varepsilon_Y = -4\frac{R}{r_P}W_{040}y_P^3$$

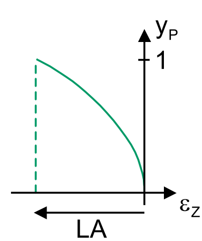
$$\varepsilon_X = -4\frac{R}{r_P}W_{040}x_P^3$$

The **transverse aberration** TA is the transverse ray error from the top of the pupil.

$$TA = \varepsilon_Y(y_P = 1)$$



The **longitudinal aberration** LA is the distance from paraxial focus to **marginal focus** (where the real marginal ray crosses the axis). The longitudinal ray errors ε_Z for SA are quadratic with y_P. The real marginal ray angle is U'.

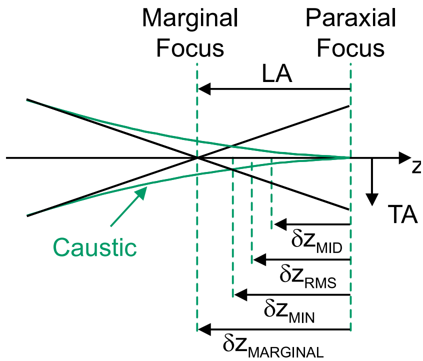


$$TA = -LA\tan U'$$

$$\varepsilon_Z \approx -16(f/\#)^2 W_{040}y_P^2$$

Spherical Aberration and Defocus

The image plane can be shifted from paraxial focus to obtain better image quality in the presence of SA. Focus criteria include mid focus, minimum RMS spot size, and **minimum circle** (where the marginal ray crosses the caustic).



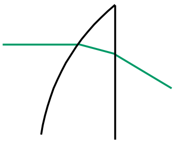
$$LA \approx -16(f/\#)^2 W_{040}$$

Mid focus:	$\Delta W_{20} = -W_{040}$	$\delta z = .5\ LA$
Min RMS:	$\Delta W_{20} = -1.33 W_{040}$	$\delta z = .67\ LA$
Min circle:	$\Delta W_{20} = -1.5 W_{040}$	$\delta z = .75\ LA$
Marginal focus:	$\Delta W_{20} = -2 W_{040}$	$\delta z = LA$

Mid focus corresponds to the **minimum wavefront variance** condition which is optimum for viewing isolated point sources such as stars. This condition is used for designing telescopes.

Spherochromatism is SA that varies with wavelength.

The power of a thin lens depends on the difference in surface curvatures. **Bending the lens** does not change its power, but its aberrations do change. The minimum SA occurs when the ray is bent the same at both surfaces. This is directly analogous to the angle of minimum deviation for prisms. For an object at infinity and n = 1.5, the correct lens shape is approximately convex-plano. At finite conjugates, a biconvex lens is used.



A positive thin lens has positive SA ($W_{040} > 0$), independent of lens bending. Bending can only change the magnitude of the SA. This is called **undercorrected SA** and is the situation shown in the figures.

Coma

Coma results when the magnification of the system varies with pupil position. An asymmetric blur is produced as the entire image blur is to one side of the paraxial image location. The image blur increases linearly with image height H.

$$W = W_{131} H \rho^3 \cos\theta$$

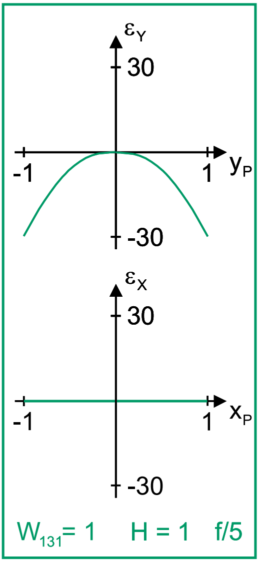
$$\varepsilon_Y = -\frac{R}{r_P} W_{131} H \rho^2 (2 + \cos 2\theta)$$

$$\varepsilon_X = -\frac{R}{r_P} W_{131} H \rho^2 \sin 2\theta$$

Ray fans:

$$\varepsilon_Y = -3\frac{R}{r_P} W_{131} H y_P^2$$

$$\varepsilon_X = 0$$

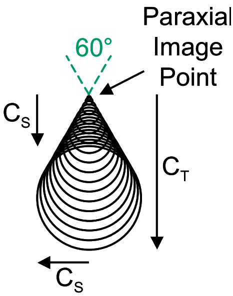


For a given object point, each annular zone in the pupil maps to a displaced circle of light in the image blur. The blur is contained in a 60 degree wedge, and about 55% of the light is contained in the first third of the pattern. Depending on the sign of the coma, the pattern can flare towards ($W_{131} > 0$) or away from ($W_{131} < 0$) the optical axis. H is assumed to represent a positive image height.

Tangential coma C_T and **sagittal coma** C_S are two other measures of coma:

$$C_T = -3\frac{R}{r_P} W_{131}$$

$$C_S = -\frac{R}{r_P} W_{131}$$



For a thin lens, coma varies with lens bending and the stop position. For any bending, there is a stop location that eliminates coma. This is the **natural stop position**.

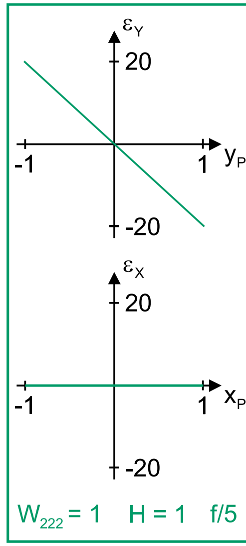
Astigmatism

In a system with **astigmatism**, the power of the optical system in horizontal and vertical meridians is different as a function of image height.

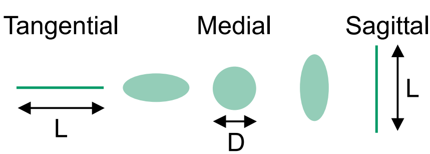
$$W = W_{222}H^2\rho^2\cos^2\theta = W_{222}H^2y_P^2$$

$$\varepsilon_Y = -2\frac{R}{r_P}W_{222}H^2y_P$$

$$\varepsilon_X = 0$$



With positive astigmatism, light from a vertical meridian is focused closer to the lens than light through the horizontal meridian. Each object point produces two perpendicular line images. These are the **tangential focus** and the **sagittal focus**. Sagittal focus is where the sagittal rays focus, and a line image in the meridional plane is formed. Tangential focus is where the tangential or meridional rays focus, and a line image is formed perpendicular to the meridional plane. Located between these two line foci is a circular focus called the **medial focus**.



$$L \approx 8(f/\#)W_{222}H^2$$

$$D = L/2 \approx 4(f/\#)W_{222}H^2$$

Each of these foci lies on a separate curved image plane. In the presence of astigmatism only:

Sagittal focus:	$\Delta W_{20} = 0$	$\delta z = 0$
Medial focus:	$\Delta W_{20} = -.5W_{222}H^2$	$\delta z \approx -4(f/\#)^2H^2W_{222}$
Tangential focus:	$\Delta W_{20} = -W_{222}H^2$	$\delta z \approx -8(f/\#)^2H^2W_{222}$

The field dependence of astigmatism is due to apparent foreshortening of the pupil at non-zero image heights. On axis, there is no astigmatism. This aberrational astigmatism is not caused by manufacturing errors.

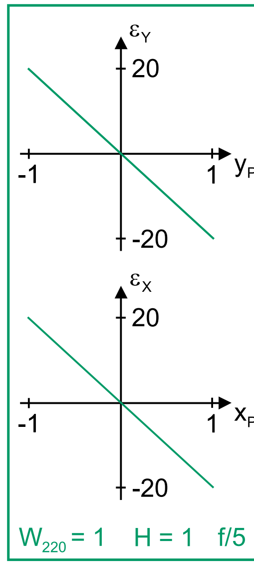
Field Curvature

Field curvature characterizes the natural tendency of optical systems to have curved image planes. A positive singlet has an inward bending image surface.

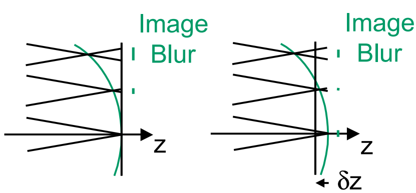
$$W = W_{220} H^2 \rho^2 = W_{220} H^2 (x_P^2 + y_P^2)$$

$$\varepsilon_Y = -2 \frac{R}{r_P} W_{220} H^2 y_P$$

$$\varepsilon_X = -2 \frac{R}{r_P} W_{220} H^2 x_P$$



A perfect image is formed on a curved surface, and the image blur at the paraxial image plane increases as H^2. A compromise flat image plane that reduces the average image blur occurs inside paraxial focus.



The field curvature is a bias curvature for the astigmatic image surfaces.

Sagittal surface: $\Delta W_{20} = -W_{220} H^2$

Medial surface: $\Delta W_{20} = -W_{220} H^2 - .5 W_{222} H^2$

Tangential surface: $\Delta W_{20} = -W_{220} H^2 - W_{222} H^2$

Petzval surface: $\Delta W_{20} = -W_{220} H^2 + .5 W_{222} H^2$

While not a good image surface, the **Petzval surface** represents the fundamental field curvature of the system. It depends only on the construction parameters of the system: surface curvatures and element indices of refraction.

These four image surfaces are equally spaced and occur in the same relative order: T-M-S-P or P-S-M-T. The best image quality occurs at medial focus. An **artificially flattened field** or medial surface can be obtained by balancing astigmatism and field curvature.

Distortion

Distortion occurs when image magnification varies with the image height H. Straight lines in the object are mapped to curved lines in the image. Points still map to points, so there is no image blur associated with distortion.

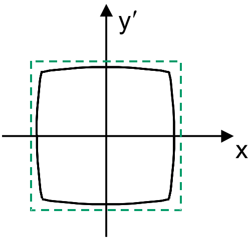
$$W = W_{311}H^3\rho\cos\theta = W_{311}H^3y_P$$

$$\varepsilon_Y = -\frac{R}{r_P}W_{311}H^3$$

$$\varepsilon_X = 0$$

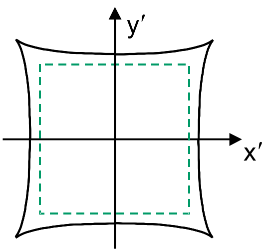
Distortion is a quadratic magnification error, and the image point position is displaced in a radial direction. The figures assume H represents a positive image height.

Barrel distortion results when the actual magnification becomes less than the paraxial magnification with increasing H. The corners of a square are pushed in towards the optical axis.



$$W_{311} > 0$$

$$\varepsilon_Y < 0 \text{ for } H > 0$$



Pincushion distortion results when the actual magnification becomes larger than the paraxial magnification with increasing H. The corners of a square are pulled away from the optical axis.

$$W_{311} < 0$$

$$\varepsilon_Y > 0 \text{ for } H > 0$$

The transverse ray fans for wavefront tilt and distortion both are constant with respect to y_P. These two aberration terms can be distinguished by their different field or H dependence: linear for wavefront tilt and cubic for distortion.

Combinations of Aberrations

A real system will be degraded by multiple aberrations, and the ray fans encode the aberration content in the dependence of the ray errors on x_P, y_P and H. A similar chart exists for wave fans.

Aberration	ε_Y vs. y_P	ε_X vs. x_P	H
Wavefront tilt W_{111}	constant	0	H
Distortion W_{311}	constant	0	H^3
Defocus ΔW_{20}	y_P	x_P	none
Field curvature W_{220}	y_P	x_P	H^2
Astigmatism W_{222}	y_P	0	H^2
Coma W_{131}	y_P^2	0	H
SA W_{040}	y_P^3	x_P^3	none

The slopes of the ray fans at the origins are especially important for deciphering the aberration content. Only defocus, field curvature and astigmatism produce a non-zero slope, but each has a different dependence on x_P and H. A positive slope of the $H = 0$ ray fan indicates that the image plane is inside paraxial focus, as a ray from the top of the pupil has not yet crossed the axis ($\varepsilon_Y > 0$ for $y_P > 0$). The image plane is outside paraxial focus for a negative slope. The magnitude of the slope is proportional to the separation.

Using normalized field and pupil coordinates gives the value of the wavefront aberration coefficients physical meaning. W_{IJK} is the amount of wavefront error associated with this aberration term at the edge of the pupil ($y_P = 1$) and the edge of the field ($H = 1$).

Aberration theory allows the **Seidel aberration coefficients** to be calculated from paraxial raytrace data. The Seidel coefficients are easily related to the wavefront aberrations:

$$S_I = 8W_{040} \qquad S_{II} = 2W_{131} \qquad S_{III} = 2W_{222}$$

$$S_{IV} = 4W_{220} - 2W_{222} \qquad S_V = 2W_{311}$$

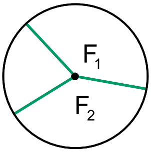
Conics and Aspherics

Because of the ease of fabrication and testing, most optical surfaces are flat or spherical. The introduction of **aspheric surfaces** provides more optimization variables for aberration correction. Rotational symmetry is maintained.

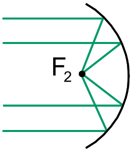
The first class of aspheric surfaces is generated by rotation of a **conic section** about the optical axis. Conics are defined by two foci. A source placed at one focus will image without aberration to the other focus. The sag of a conic is given by

$$s(r) = \frac{Cr^2}{1+(1-(1+\kappa)C^2r^2)^{1/2}}$$

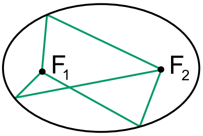
where C is the base curvature of the surface, r is the radial coordinate and κ is the conic constant. Conics are often used as reflecting surfaces.



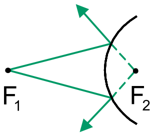
Circle: $\kappa = 0$
Both foci are at the center of curvature.



Parabola: $\kappa = -1$
One focus is at infinity, the other is at the focal point of the reflecting surface. Parabolas are used for imaging distant objects.



Ellipse: $-1 < \kappa < 0$
Both foci are real. Elliptical surfaces are used for relaying images.



Hyperbola: $\kappa < -1$
One focus is real, and the other is virtual. Hyperbolas are used as negative reflecting elements.

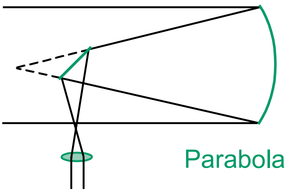
Other rotationally symmetric terms can be added to the conic to obtain a **generalized asphere**:

$$s(r) = \frac{Cr^2}{1+(1-(1+\kappa)C^2r^2)^{1/2}} + A_1r^2 + A_2r^4 + A_3r^6 + A_4r^8 + \ldots.$$

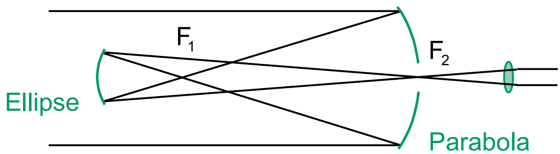
Mirror-Based Telescopes

The imaging properties of conic surfaces are used in the design of **mirror-based telescopes**.

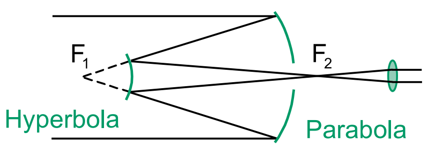
Newtonian telescope: a parabola with a fold flat. Analogous to a Keplerian refracting telescope.



Gregorian telescope: the parabola is followed by an ellipse to relay the intermediate image. As with a relayed Keplerian telescope, this design is good for terrestrial applications as it produces an erect image.



Cassegrain telescope: the parabola is combined with a hyperbolic secondary mirror to reduce the system length. The combination of the primary and secondary is the mirror equivalent of a telephoto objective.

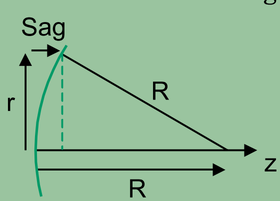


The Cassegrain design uses two conic surfaces to correct spherical aberration. The **Ritchey-Chretien telescope** is identical in layout, except that it uses two hyperbolic mirrors to correct coma as well as spherical aberration.

The **sag of a spherical surface** is often calculated using the parabolic approximation.

$$Sag = s(r) \approx \frac{r^2}{2R}$$

Valid for $Sag \ll r$



Radiometry

Radiometry characterizes the propagation of radiant energy through an optical system. Radiometry deals with the measurement of light of any wavelength; the basic unit is the **watt** W. The spectral characteristics of the optical system (source spectrum, transmission and detector responsivity) must be considered in radiometric calculations.

Radiometric terminology and units:

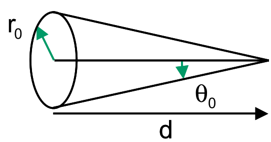
Energy	Q	Joules (J)	
Flux	Φ	W	Power
Intensity	I	W/sr	Power per unit solid angle
Irradiance	E	W/m^2	Power per unit area – incident
Exitance	M	W/m^2	Power per unit area – exiting
Radiance	L	W/m^{2}sr	Power per unit projected area per unit solid angle

In this simplified discussion, objects and images are assumed to be on-axis and perpendicular to the optical axis. With this assumption, the projected area equals the area.

The solid angle of a **right circular cone** is

$$\Omega = 2\pi(1-\cos\theta_0)$$

$$\Omega \approx \frac{\pi r_0^2}{d^2} \approx \pi\theta_0^2$$



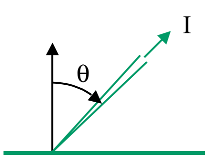
Exitance and irradiance are related by the **reflectance** of the surface ρ. Photographic research has shown that ρ = 18% for the average scene.

$$M = \rho E$$

The radiance of a **Lambertian source** (a perfectly diffuse surface) is constant. The intensity falls off with the apparent source size or the **projected area** (**Lambert's law**). The exitance of a Lambertian source is related to its radiance by π.

$$L = \text{constant} \qquad I = I_0\cos\theta$$

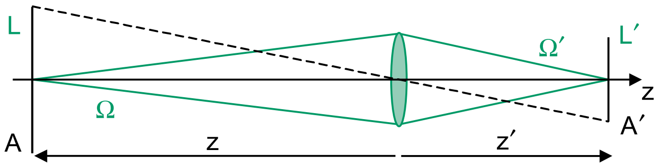
$$M = \pi L \qquad \pi L = \rho E$$



This relationship is π (instead of the expected 2π for a hemisphere) because of the falloff of the projected area with θ.

Radiative Transfer

Radiative transfer determines the amount of light from an object that reaches the image.



In air, the radiance and the **AΩ product** or **throughput** are conserved, and the flux collected by the lens Φ is transferred to the image area A'.

$$L = L' \qquad A\Omega = A'\Omega' \qquad m^2 = \frac{A'}{A} = \left(\frac{z'}{z}\right)^2$$

$$\Phi = LA\Omega = L'A'\Omega'$$

The image plane irradiance E' is

$$E' = \frac{\pi L}{4(1-m)^2(f/\#)^2} = \frac{\pi L}{4(f/\#_W)^2} = \pi L(NA)^2$$

This result is known as the **camera equation**. An on-axis Lambertian object and small angles are assumed. The object and image planes are perpendicular to the optical axis. Including obliquity factors associated with off-axis objects leads to the **cosine fourth law**. The image irradiance falls off as the $\cos^4$ of the field angle. Spectral dependence can also be added to these results.

Multiplying by the exposure time gives the **exposure** (J/m^2):

$$H = E'\Delta t$$

> The **mean solar constant** is 1368 W/m^2 outside the atmosphere of the earth, and the solar irradiance on the surface is about 1000 W/m^2.

In the general situation when the index is not unity, the **basic throughput** $n^2A\Omega$ and the **basic radiance** L /n^2 are invariant. Since throughput is based on areas, the basic throughput is proportional to the Lagrange invariant squared.

$$n^2A\Omega = \pi^2Ж^2$$

Photometry

Photometry is the subset of radiometry that deals with visual measurements, and luminous power is measured in **lumens** lm. All of the rules and results of radiometry and radiative transfer apply.

The lumen is a watt weighted to the visual **photopic response**. The peak response occurs at 555 nm, where the conversion is 683 lm/W. The dark adapted or **scotopic response** peaks at 507 nm with 1700 lm/W.

Photometric terminology and units:

Luminous power	Φ_V	lm
Luminous intensity	I_V	lm/sr
Illuminance	E_V	lm/m^2
Luminous exitance	M_V	lm/m^2
Luminance	L_V	lm/m^{2}sr
Exposure	H_V	lm s/m^2

Luminous Photopic Efficacy

λ (nm)	lm/W
400	0.3
420	2.7
440	15.7
460	41.0
480	95.0
500	221
520	485
540	652
560	680
580	594
600	425
620	260
640	120
660	41.7
680	11.6
700	2.8
720	0.7

Other common photometric units and conversions include:

I_V:	**candela** (cd)	= lm/sr
E_V:	**lux** (lx)	= lm/m^2
	foot-candle (fc)	= lm/ft^2
	1 fc	= 10.76 lx
L_V:	**foot-lambert** (fL)	= $\frac{1}{\pi}$cd/ft^2
	nit (nt)	= cd/m^2
	1 fL	= 3.426 nt
H_V:	**lux-second** (lx s)	= lm s/m^2

The unit **meter-candle-second** (mcs) is an obsolete unit of exposure equal to the lux-second.

Typical illuminance levels:

Sunny day:	10^5 lx	Moonlit night:	10^{-1} lx
Overcast day:	10^3 lx	Starry night:	10^{-3} lx
Interior:	10^2 lx	Desk lighting:	10^3 lx

Sources

Blackbody sources have a spectral radiance given by **Planck's equation**; T is the temperature and vacuum is assumed:

$$L_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{(e^{hc/\lambda kT}-1)}$$

$h = 6.626\times 10^{-34}$ J s
$c = 2.998\times 10^{8}$ m/s
$k = 1.381\times 10^{-23}$ JK^{-1}

or

$$L_\lambda = \frac{3.742\times 10^{-16}\ \mathrm{Wm^2}}{\pi\lambda^5}\frac{1}{(e^{0.01439\,\mathrm{mK}/\lambda T}-1)}$$

The units of L_λ are W/m^3 sr. **Thermal sources** must include a multiplicative **emittance** ε. If ε is constant, a graybody results, and non-gray bodies are characterized by ε(λ).

Wein's displacement law locates the peak wavelength of the blackbody distribution:

$$\lambda_{MAX}T = 2898\ \mu\mathrm{mK}$$

Sun:	6000K
Halogen Lamp:	3200K
Tungsten Lamp:	2800K
Room Temp:	300K

The total exitance for the blackbody source is given by the **Stefan-Boltzmann law**:

$$M = \pi L = \sigma T^4 \qquad \sigma = 5.6704\times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$$

Laser wavelengths:

HeNe	632.8 nm	Nd:YAG	1.064 µm
	543 nm	Doubled	532 nm
	1.15 µm	Tripled	354 nm
	1.52 µm	HeCd	442 nm
	3.39 µm	CO_2	10.6 µm
Ar ion	488 nm	F_2 excimer	157 nm
	515 nm	ArF excimer	193 nm
Kr ion	647 nm	KrF excimer	248 nm
Ruby	694 nm	Nitrogen	337 nm

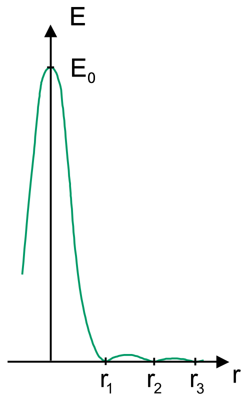
Some common wavelengths for **diode lasers** include (in nm): 635, 650, 670, 780, 808, 830, 850, 980, 1310 and 1550. The output wavelength can vary considerably. Examples of compound semiconductor materials used for diode lasers (and their corresponding wavelength ranges) are AlGaInP (630–680 nm), AlGaAs (780–880 nm) and InGaAsP (1150–1650 nm).

Airy Disk

Because of **diffraction** from the system stop, an aberration-free optical system does not image a point to a point. An **Airy disk** is produced having a bright central core surrounded by diffraction rings.

$$E = E_0\left[\frac{2J_1(\pi r/\lambda f/\#_W)}{\pi r/\lambda f/\#_W}\right]^2$$

where r is the radial coordinate, J_1 is a Bessel function, and $f/\#_W$ is the image space working $f/\#$.



	Radius r	Peak E	Energy in Ring (%)
Central maximum	0	$1.0\,E_0$	83.9
First zero r_1	$1.22\lambda f/\#_W$	0.0	
First ring	$1.64\lambda f/\#_W$	$0.017\,E_0$	7.1
Second zero r_2	$2.24\lambda f/\#_W$	0.0	
Second ring	$2.66\lambda f/\#_W$	$0.0041\,E_0$	2.8
Third zero r_3	$3.24\lambda f/\#_W$	0.0	
Third ring	$3.70\lambda f/\#_W$	$0.0016\,E_0$	1.5
Fourth zero r_4	$4.24\lambda f/\#_W$	0.0	

The **diameter of the Airy disk** (diameter to the first zero) is

$$D = 2.44\lambda f/\#_W$$

In visible light $\lambda \approx 0.5$ µm and $D \approx f/\#_W$ in µm

The **Rayleigh resolution criterion** states that two point objects can be resolved if the peak of one falls on the first zero of the other:

$$\text{Resolution} = 1.22\lambda f/\#_W$$

The **angular resolution** is found by dividing by the focal length (or image distance):

$$\text{Angular resolution} = \alpha = 1.22\lambda/D_{EP}$$

Diffraction and Aberrations

A system is said to be well corrected or **diffraction limited** if the total system wavefront aberration is less than $\lambda/4$. This requirement applied to the defocus coefficient ΔW_{20} leads to the **Rayleigh focus criterion** for diffraction-limited performance:

$$\delta z = \pm 8(f/\#)^2 \Delta W_{20} = \pm 2\lambda (f/\#)^2$$

In visible light $\lambda \approx 0.5\ \mu\text{m}$ and $\delta z \approx \pm(f/\#)^2$ in μm

The **diffraction-based point spread function** PSF equals the squared modulus of the Fourier transform of the pupil function. The result is scaled to the image plane coordinates (x', y'). The wavefront error W appears as a phase factor in the pupil of the system:

$$PSF(x', y') = \left| \Im\left\{ \text{cyl}\left(\frac{r_P}{D_{XP}}\right) e^{i2\pi W(x_P, y_P)/\lambda} \right\} \right|^2_{f_X = x'/\lambda f,\, f_Y = y'/\lambda f}$$

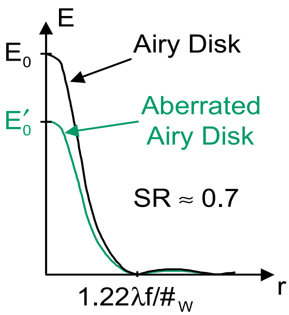
The cylinder function defines the pupil diameter. When $W = 0$, the diffraction-limited Airy disk results. The **modulation transfer function** MTF is the normalized Fourier transform of the PSF.

$$MTF(f_X, f_Y) = \frac{|\Im\{PSF(x', y')\}|}{|\Im\{PSF(x', y')\}|_{f_X = 0,\, f_Y = 0}}$$

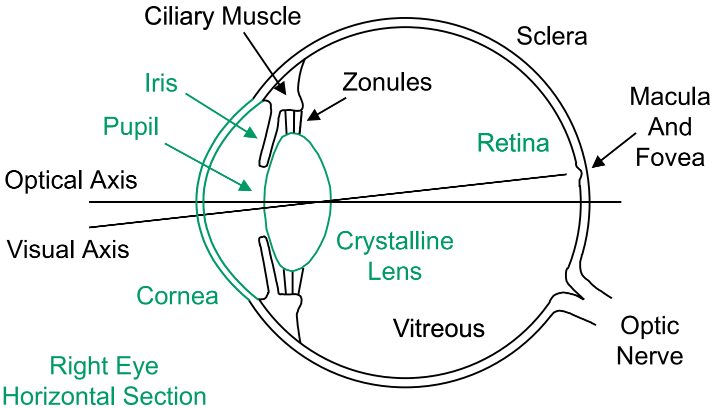
The **Strehl ratio** SR is a single-number measure of image quality for systems with small amounts of aberration.

$$SR = \frac{E_0'}{E_0}$$

The SR has a maximum value of one, and it measures the degradation of the Airy disk. Any reduction of the SR is directly proportional to the wavefront variance. The SR correlates well with image quality down to values of about 0.5.



Eye



The optical power of the **human eye** is about 60 D, of which the **cornea** provides 43 D. The base radius of curvature of the cornea is about 8 mm, and the overall length of the eye is about 25 mm. Since the **vitreous** ($n_V = 1.337$) fills the eye, the rear focal length differs from the focal length.

$$f \equiv \frac{1}{\phi} \approx 17 \text{ mm} \qquad f'_R = n_V f \approx 23 \text{ mm}$$

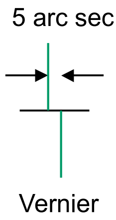
Anatomical variations between eyes can be as much as 25%. The **crystalline lens** is a gradient index element; it has a higher index at its center. The relaxed power of the lens is about 19 D, and the eye focuses at infinity. To view near objects, the **ciliary muscle** contracts, causing the lens power to increase. The lens bulges and its radii of curvature become steeper. The range of **accommodation** varies with age, but can be as much as 15 D. The **iris** is the stop of the eye. The **pupil** is the EP of the eye and has a typical diameter of about 4 mm, with a range of 2–8 mm.

The front and rear principal planes of the eye P and P′ are located about 1.6 mm and 1.9 mm, respectively, behind the vertex of the cornea. The system nodal points N and N′ are located near the anterior surface of the lens, 7.2 mm and 7.5 mm, respectively, from the corneal vertex. The visual axis of the eye is defined by the **macula** and is displaced about 5° nasally from the optical axis.

Retina and Schematic Eyes

The **retina** covers the interior of the globe of the eye. The **cones** provide color vision at daylight illumination levels. The highest cone density is at the **fovea** in the center of the **macula**. The macula is about 3 mm in diameter (11° FOV), and the fovea has a diameter of about 1.5 mm (5° FOV). The **rods** are more uniformly distributed over the retina and are used for dark-adapted vision.

The light **sensitivity** of the eye covers a dynamic range of 10^{10}–10^{14}. Most of this range comes from **dark adaptation** of the retina as the variation in the pupil area is only a factor of 16. For comparison, film and most electronic sensors have a dynamic range of only about 10^3–10^5.



Under bright illumination, the **resolution** of the eye is 1 arc min (1 mm at 3 m). This corresponds to about 100 lp/mm on the retina. The **vernier acuity** of the eye (the ability to line up two line segments) is about 5 arc sec (0.1 mm at 3 m).

Schematic eyes are simplified models of the eye. The simplest is the **reduced schematic eye**: a single refractive surface which approximates the paraxial properties of the eye (R = 5.65 mm, n = 1.333 and *length* = 22.6 mm).

A variety of more sophisticated eye modes have been developed; some model the aberration content of the eye. The following schematic eye provides a more complete model of the paraxial properties of the eye (Le Grand and El Hage). The crystalline lens is assumed to have a uniform index.

Surface	R (mm)	t (mm)	n	ϕ (D)
Anterior cornea	7.8	0.55	1.3771	48.35
Posterior cornea	6.5	3.05	1.3374	–6.11
Anterior lens	10.2	4.00	1.420	8.10
Posterior lens	–6.0	16.60	1.336	14.00

$\phi = 59.9$ D $\quad f = 16.9$ mm $\quad f'_R = 22.3$ mm

Ophthalmic Terminology

Emmotropia: Distant objects are imaged correctly onto the retina; normal vision.

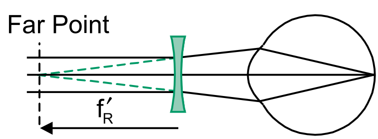
Myopia or **nearsighted**: the eye is too powerful for its axial length. Images of distant objects are in front of the retina; corrected with a negative spectacle lens.

Hyperopia or **farsighted**: the eye is too weak for its axial length. Images of distant objects are behind the retina; corrected with a positive spectacle lens. Accommodation can cause distant objects to be in focus.

Far point: the object distance that is in focus without accommodation. The far point is virtual with hyperopia.

Near point: the object distance that is in focus with maximum accommodation.

Spectacle lens: the rear focal point of the correcting lens should be placed at the far point of the relaxed eye. If the spectacle lens is placed at the front focal point of the eye, distant objects are brought into focus by shifting the rear focal point of the eye without changing the power or magnification of the eye.



Contact lens: applied to the surface of the cornea to change to the system power. The radius of curvature at the air interface is changed.

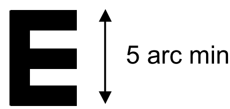
Presbyopia: the loss of accommodative response due to a stiffening of the crystalline lens with age. Occurs after age 40 and is compensated by additional positive spectacle power (as with bifocals or progressive lenses).

Visual astigmatism: a variation of the power of the eye with meridional cross section due to a non-rotationally symmetric cornea or lens. Linearly blurred images result. Because there is no field dependence, this effect is different from aberrational astigmatism W_{222}. Visual astigmatism is characterized by a wavefront aberration coefficient W_{022}.

Stiles-Crawford effect: the reduction in effectiveness of light rays entering the edge of the pupil due to the shape and orientation of the cones. The light efficiency as a function of pupil radius is approximately: 1 mm – 90%; 2 mm – 70%; 3 mm – 40% and 4 mm – 20 %.

More Ophthalmic Terminology

Snellen visual acuity VA: a single number measure of the resolution of the visual system based upon the ability of the subject to identify characters or symbols. The value 20/XX implies that the subject can identify a letter at 20 feet that a standard observer can just identify at XX feet. The 20/20 line of characters on the VA chart subtends 5 arc min. The letters on the 20/40 line subtend 10 arc min. Note that a 20/20 letter can be broken down into 5 segments of size 1 arc min. The human retina is capable of supporting a VA of better than 20/10. Metric VA is based upon distances in meters and reads as 6/6, etc.



Intra-ocular lens IOL: with age, the crystalline lens becomes opaque. The lens can be surgically removed and replaced with an artificial lens or IOL.

Refractive surgery techniques:

- **RK – Radial keratotomy**: A series of non-penetrating incisions are made in the periphery of the cornea to relax the cornea and change its shape.
- **PRK – Photorefractive keratectomy**: the outer layer (epithelium) of the cornea is removed to expose the body of the cornea (stroma). An excimer laser (193 nm) is used to ablate the stroma to change the corneal shape and power. The healing process must regrow the epithelium.
- **LASIK – Laser in situ keratomileusis**: a variation on PRK where a flap is shaved into the cornea to reveal the stroma and save the epithelium. The flap is replaced after ablation.
- **Phakic IOL**: a small addition lens surgically implanted in front of the natural lens to correct the power of the eye.

The resolution of the eye and diffraction combine to place practical limitations on the magnifying power MP of telescopes and the visual magnification m_V of microscopes.

$$|MP| \le 0.43\ D_{EP} \quad (D_{EP} \text{ in mm}) \qquad m_V \le 230\ NA$$

Visible light is assumed and the NA of the microscope objective is used. Powers in excess of these values only result in magnification of the just-resolved Airy disks. Extra magnification (or **empty magnification**) is often used so that the eye is not forced to work at the visual resolution limit.

Film and Detector Formats

Film format	Film width (mm)	Frame size (mm × mm)	Diagonal (mm)
120 (4:3)	61.5	60 × 45	75.0
220 (1:1)	61.5	60 × 60	84.9
220 (7:6)	61.5	70 × 60	92.2
220 (3:2)	61.5	90 × 60	108.2
126 (1:1)	35.0	28 × 28	40.0
110 (4:3)	16.0	17 × 13	21.4
135 (3:2)	35.0	36 × 24	43.3
Disk (4:3)		11 × 8	13.6
APS Classic (3:2)	24.0	25.0 × 16.7	30.1
APS HDTV (16:9)	24.0	30.2 × 16.7	34.5
APS Panoramic (3:1)	24.0	30.2 × 10.0	31.8

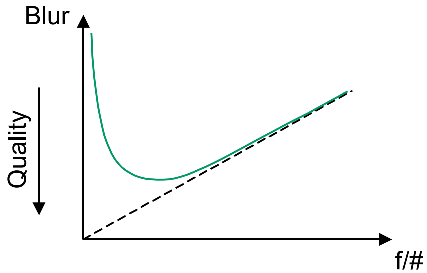
In photographic terms, a **standard lens** is one that produces an image perspective and FOV that somewhat matches human vision. A lens with a focal length equal to the diagonal of the format is usually considered standard. There is considerable variation in this definition as a standard lens for 35 mm camera (135 format) is historically 50–55 mm. Lenses that produce a larger FOV are called **wide angle lenses**, and lenses that produce a smaller FOV are **long focus lenses**.

Video format	Image size (mm × mm)	Diagonal (mm)
2/3 inch	8.8 × 6.6	11.0
1/2 inch	6.4 × 4.8	8.0
1/3 inch	4.8 × 3.6	6.0
1/4 inch	3.6 × 2.7	4.5

To match standard television format, **video sensors** or **focal plane arrays** are usually produced in a 4:3 format. This situation will likely change with the introduction of HDTV. Note that the format size (i.e. 2/3 inch) has little or nothing to do with the actual sensor size. These formats originated with vidicon or tube-type sensors and are the outer diameter of the glass tube required for the given active area. For the smaller formats, there is some variation in image size between manufacturers. A large variety of sensor formats exist for digital photography and scientific applications.

Photographic Systems

On a small-format photographic print, a blur diameter of 75 µm (0.003 in) is considered excellent image quality. Note that this corresponds to the resolution of the eye (1 arc min) at the standard near point of 250 mm. Blurs larger than about 200 µm are typically unacceptable. These blur sizes can be scaled by the enlargement ratio from the film to determine a blur requirement for the imaging lens.



A qualitative plot of **image blur** as a function of the f/# of an objective can be drawn. With large apertures, aberrations and depth of field errors are dominant, and the blur grows quickly with faster f/#s. When the system has a small aperture, diffraction dominates and there is a linear dependence of blur on the f/#. For many camera lenses, the minimum blur occurs at about f/5.6–8. Faster camera lenses are not produced because of the potential for reduced diffraction blur, but rather for their radiometric performance in low light level conditions or with fast shutter speeds. The best image quality is produced when the lens is stopped down several stops.

The ISO film speed specifies the required exposure:

$$H_V = E_V \Delta t = 0.8/ISO\ \# \qquad H_V \text{ is in lx s}$$

The **transmission** T and **optical density** D of film or a filter:

$$T = 10^{-D}$$

A white image is produced by equal amounts of the additive or **primary colors** red R, green G and blue B. Combinations two at a time produce the **complimentary** or **subtractive colors** cyan C, magenta M and yellow Y:

$$C = B + G \qquad M = B + R \qquad Y = G + R$$

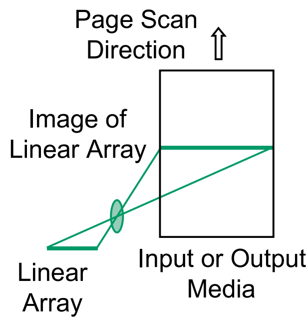
Cyan filters are also known as minus red, magenta are minus green and yellow are minus blue. White light W filtered by two subtractive filters produce a single primary color:

$$W - C - M = B \qquad W - C - Y = G \qquad W - M - Y = R$$

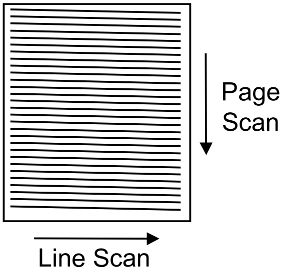
Scanners

There are three basic configurations for **scanners** based upon the source or detector configuration: area, line or spot. The **area scanner** uses a two-dimensional sensor. This is really just a camera.

A **linear array scanner** or **push broom scanner** uses a linear detector array or a linear array of sources such as LEDs. One line of the scene is imaged or recorded at a time. The scene is scanned by moving the two-dimensional output media or scene through the image of the linear array. Examples are thermal printers, high resolution film scanners, flatbed document scanners and earth resources satellites.



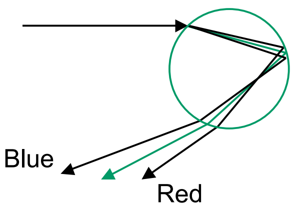
In a **flying spot scanner**, a point detector or source is scanned in a two-dimensional pattern over the scene or output surface. The two common options for the fast line scan in an optical flying spot scanner are a galvanometer mirror or a polygon scanner. The primary example is a laser printer where the page scan is accomplished by moving the photosensitive recording medium. Laser light shows use two galvanometer mirrors. CRTs are electron-based flying spot scanners.

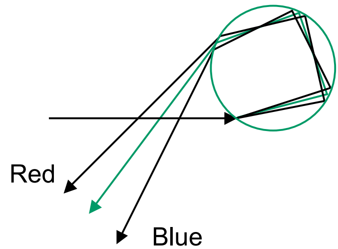


Two pertinent television definitions related to scanners:
Progressive scan: all of the TV lines are written in a single pass down the screen (HDTV and some scientific cameras).
Interlace scan: two fields are written per frame. Each field contains every other line in the image. In the U.S., the frame rate is 30 Hz, and the field rate is 60 Hz. Phosphor lag and the response of the eye combine the two fields into a single image without noticeable flicker.

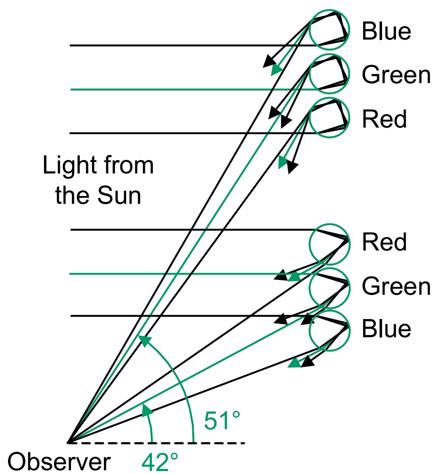
Rainbows and Blue Skies

Rainbows result from the combination of refraction, reflection and dispersion with a raindrop. The entering ray is refracted and dispersed twice. For the **primary rainbow**, there is single internal Fresnel reflection. There are two reflections for the **secondary rainbow**. In both cases, blue light is deviated more than red light.





In the primary rainbow, the droplets directing the red light to the observer are above those that direct the blue light. Because the angle of rotation is opposite, the colors of the secondary rainbow are reversed. The primary rainbow is at an angle of about 42°, and the secondary rainbow is at 51°. Each observer uses a different set of raindrops to view their individual rainbow.



Molecules in the atmosphere act as scattering centers for the incident sunlight. The primary scattering mechanism is **Rayleigh scattering** which has a $1/\lambda^4$ dependence. As a result, blue light is preferentially scattered, and the sky appears blue. The colors in **sunsets** occur for the same reason. The long path length through the atmosphere depletes the blue and green content of the direct sunlight at sunset, leaving reds and oranges.

Matrix Methods

Matrix methods are an alternate methodology of tracing paraxial rays where the ray height and ray angle at an input plane are propagated through the system using a series of matrix operations. The two fundamental operations are refraction and transfer.

Refraction: $\mathbf{R} = \begin{pmatrix} 1 & 0 \\ -\phi & 1 \end{pmatrix}$ **Transfer**: $\mathbf{T} = \begin{pmatrix} 1 & t/n \\ 0 & 1 \end{pmatrix}$

Successive application of these operands leads to the output ray:

$$\begin{pmatrix} y' \\ \omega' \end{pmatrix} = \mathbf{T}_k\mathbf{R}_k \ldots \mathbf{T}_3\mathbf{R}_2\mathbf{T}_2\mathbf{R}_1\mathbf{T}_1 \begin{pmatrix} y \\ \omega \end{pmatrix}$$

The matrix operations must be performed in optical order as is done is a paraxial raytrace. Each refraction operation propagates the ray into the next optical space. All of the individual operations can be combined into a single **system matrix** that connects the two planes. This composite matrix allows the internal details of the raytrace to be hidden, and the entire propagation takes place with a single operation.

$$\mathbf{M}_S = \mathbf{T}_k\mathbf{R}_k \ldots \mathbf{T}_3\mathbf{R}_2\mathbf{T}_2\mathbf{R}_1\mathbf{T}_1 \qquad \begin{pmatrix} y' \\ \omega' \end{pmatrix} = \mathbf{M}_S \begin{pmatrix} y \\ \omega \end{pmatrix}$$

Matrix methods allow two rays to be propagated at once by defining a ray matrix, shown here with the marginal and chief rays.

$$\mathbf{L} = \begin{pmatrix} y & \bar{y} \\ \omega & \bar{\omega} \end{pmatrix} \qquad \mathbf{L}' = \mathbf{M}_S\mathbf{L}$$

The **determinant of the ray matrix** is the Lagrange invariant or the optical invariant if two other rays are used.

$$|\mathbf{L}| = \begin{vmatrix} y & \bar{y} \\ \omega & \bar{\omega} \end{vmatrix} = \bar{\omega}y - \omega\bar{y} = n\bar{u}y - nu\bar{y} = \text{Ж}$$

The system matrix connecting any plane in object space to any plane in image space must have $-\phi$ as the "C" element.

$$\mathbf{M}_S = \begin{pmatrix} A & B \\ -\phi & D \end{pmatrix}$$

Common Matrices

The **conjugate matrix** connects an object plane to its conjugate image plane through the magnification m. The **afocal system matrix** between conjugate planes is found by setting $\phi = 0$:

$$\mathbf{M}_C = \begin{pmatrix} m & 0 \\ -\phi & 1/m \end{pmatrix} \qquad \mathbf{M}_A = \begin{pmatrix} m & 0 \\ 0 & 1/m \end{pmatrix}$$

Focal plane to focal plane matrix:

$$\mathbf{M}_F = \begin{pmatrix} 0 & 1/\phi \\ -\phi & 0 \end{pmatrix}$$

Nodal plane to nodal plane matrix:

$$\mathbf{M}_N = \begin{pmatrix} n/n' & 0 \\ -\phi & n'/n \end{pmatrix}$$

Thin lens matrix:

$$\mathbf{M}_{THIN} = \begin{pmatrix} 1 & 0 \\ -\phi & 1 \end{pmatrix}$$

Thick lens matrix (ϕ_1 and ϕ_2 are the powers of the two surfaces, and τ is the reduced thickness of the lens):

$$\mathbf{M}_{THICK} = \begin{pmatrix} 1-\phi_1\tau & \tau \\ -\phi & 1-\phi_2\tau \end{pmatrix}$$

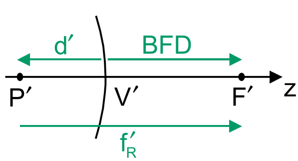
The system **vertex matrix** is the product of the component matrices interspersed with the appropriate transfer matrices. Given the elements of the vertex matrix, the cardinal points of the system can be determined:

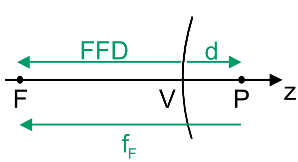
$$\mathbf{M}_V = \begin{pmatrix} A_V & B_V \\ C_V & D_V \end{pmatrix} \qquad \phi \equiv \frac{1}{f} = -C_V$$

$$f'_R = -\frac{n'}{C_V} \qquad f_F = \frac{n}{C_V}$$

$$\frac{d}{n} = \frac{D_V - 1}{C_V} \qquad \frac{d'}{n'} = \frac{1 - A_V}{C_V}$$

$$\frac{FFD}{n} = \frac{D_V}{C_V} \qquad \frac{BFD}{n'} = -\frac{A_V}{C_V}$$





Trigonometric Identities

$$\sin(-\alpha) = -\sin\alpha \qquad \cos(-\alpha) = \cos\alpha$$

$$\sin\alpha = +\cos(\alpha - 90°) = -\sin(\alpha - 180°) = -\cos(\alpha - 270°)$$

$$\cos\alpha = -\sin(\alpha - 90°) = -\cos(\alpha - 180°) = +\sin(\alpha - 270°)$$

$$\tan\alpha = -\cot(\alpha - 90°) = +\tan(\alpha - 180°) = -\cot(\alpha - 270°)$$

$$\sin^2\alpha + \cos^2\alpha = 1 \qquad 1 + \tan^2\alpha = \sec^2\alpha$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1 = \cos^2\alpha - \sin^2\alpha$$

$$\sin^2\alpha = \tfrac{1}{2}(1 - \cos 2\alpha) \qquad \cos^2\alpha = \tfrac{1}{2}(1 + \cos 2\alpha)$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$

$$\cos\alpha\cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$

$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha + \beta) + \tfrac{1}{2}\sin(\alpha - \beta)$$

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$

$$\sin\alpha - \sin\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$

$$\cos\alpha - \cos\beta = -2\sin\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$

$$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i} \qquad \cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$

Equation Summary

General equations (index, refraction, mirrors, etc.):

$$OPL = nd \qquad \rho = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \sin\theta_C = \frac{n_2}{n_1}$$

$$\tau = \frac{t}{n} \qquad \omega = nu$$

$$\gamma = 2\alpha \qquad d \approx \left(\frac{n-1}{n}\right)t = t - \tau$$

Power and focal length:

$$\phi = (n' - n)C = \frac{(n' - n)}{R} \qquad f_E \equiv \frac{1}{\phi} = -\frac{f_F}{n} = \frac{f'_R}{n'}$$

Newtonian equations (z, z' measured from F, F′):

$$\frac{z}{n} = \frac{f_E}{m} \qquad \frac{z'}{n'} = -mf_E \qquad \left(\frac{z}{n}\right)\left(\frac{z'}{n'}\right) = -f_E^2$$

Gaussian equations and imaging (z, z' measured from P, P′):

$$\frac{z}{n} = \frac{(1-m)}{m}f_E \qquad \frac{z'}{n'} = (1-m)f_E \qquad m = \frac{z'/n'}{z/n} = \frac{\omega}{\omega'}$$

$$\frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E} \qquad \frac{\Delta z'/n'}{\Delta z/n} = m_1 m_2 \qquad \overline{m} = \left(\frac{n'}{n}\right)m^2$$

$$z_{PN} = z'_{PN} = f_F + f'_R \qquad m_N = -\frac{f_F}{f'_R} = \frac{n}{n'}$$

Gaussian reduction:

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau \qquad \frac{d}{n} = \frac{\phi_2}{\phi}\tau \qquad \frac{d'}{n'} = -\frac{\phi_1}{\phi}\tau$$

$$BFD = f'_R + d' \qquad FFD = f_F + d$$

Equation Summary

Thin lens:

$$\phi = (n-1)(C_1 - C_2) \qquad L = z' - z = -\frac{(1-m)^2}{m} f_E$$

Afocal systems:

$$m = -\frac{f_{E2}}{f_{E1}} = -\frac{f_2}{f_1} \qquad \overline{m} = \frac{n'}{n} m^2 \qquad \frac{\Delta z'/n'}{\Delta z/n} = m^2$$

Paraxial raytrace:

$$n'u' = nu - y\phi \qquad \omega' = \omega - y\phi$$

$$y' = y + u't' \qquad y' = y + \omega'\tau'$$

FOV, stops and pupils:

$$\bar{u} = \tan(\theta_{1/2}) \qquad Ж = H = n\bar{u}y - nu\bar{y}$$

$$NA \equiv n_k |\sin U_k| \approx n_k |u_k| \qquad f/\# \equiv \frac{f_E}{D_{EP}} \approx \frac{1}{2NA}$$

$$f/\#_W \approx (1-m) f/\# \qquad m_{PUPIL} = \frac{\bar{\omega}}{\bar{\omega}'}$$

Vignetting:

Un:	Half:	Fully:
$a \geq \lvert y \rvert + \lvert \bar{y} \rvert$	$a = \lvert \bar{y} \rvert$	$a \leq \lvert \bar{y} \rvert - \lvert y \rvert$
	$a \geq \lvert y \rvert$	$a \geq \lvert y \rvert$

Depth of focus and hyperfocal distance:

$$DOF \approx \pm B' f/\#_W \approx \pm \frac{B'}{2NA}$$

$$L_H = -\frac{fD}{B'} \qquad L_{NEAR} \approx -\frac{L_H}{2}$$

Equation Summary

Magnifiers, telescopes and microscopes:

$$MP = \frac{250\,\text{mm}}{f}$$

$$MP = \frac{1}{m} = -\frac{f_{OBJ}}{f_{EYE}}$$

$$m_V = m_{OBJ} MP_{EYE}$$

Dispersion:

$$\nu = V = \frac{n_d - 1}{n_F - n_C} \qquad P = P_{d,C} = \frac{n_d - n_C}{n_F - n_C}$$

$$n = \frac{\sin[(\alpha - \delta_{MIN})/2]}{\sin(\alpha/2)}$$

Thin prisms:

$$\delta \approx -(n-1)\alpha \qquad \Delta = \frac{\delta}{\nu} \qquad \varepsilon = P\Delta = P\frac{\delta}{\nu}$$

$$\frac{\alpha_1}{\delta} = \left(\frac{1}{\nu_2 - \nu_1}\right)\left(\frac{\nu_1}{n_{d1} - 1}\right) \qquad \frac{\alpha_2}{\delta} = -\left(\frac{1}{\nu_2 - \nu_1}\right)\left(\frac{\nu_2}{n_{d2} - 1}\right)$$

$$\frac{\varepsilon}{\delta} = \left(\frac{P_2 - P_1}{\nu_2 - \nu_1}\right) = \frac{\Delta P}{\Delta \nu}$$

Chromatic aberration and achromats:

$$\frac{\delta f}{f} = \frac{\delta \phi}{\phi} = \frac{1}{\nu} \qquad TA_{CH} = \frac{r_P}{\nu}$$

$$\frac{\phi_1}{\phi} = \frac{\nu_1}{\nu_1 - \nu_2} \qquad \frac{\phi_2}{\phi} = -\frac{\nu_2}{\nu_1 - \nu_2}$$

$$\frac{\delta \phi_{dC}}{\phi} = \frac{\delta f_{Cd}}{f} = \frac{\Delta P}{\Delta \nu}$$

Equation Summary

Surface sag:

$$s(r) \approx \frac{r^2}{2R} \qquad s(r) = \frac{Cr^2}{1+(1-(1+\kappa)C^2r^2)^{1/2}}$$

Radiometry and radiative transfer:

$$\Omega = 2\pi(1-\cos\theta_0) \qquad \Omega \approx \frac{\pi r_0^2}{d^2} \approx \pi\theta_0^2$$

$$L = \frac{M}{\pi} = \frac{\rho E}{\pi} \qquad \Phi = LA\Omega$$

$$E' = \frac{\pi L}{4(1-m)^2(f/\#)^2} = \frac{\pi L}{4(f/\#_W)^2} = \pi L(NA)^2$$

$$H = E'\Delta T$$

Diffraction limited systems:

$$D = 2.44\lambda f/\#_W \qquad D \approx f/\#_W \text{ in } \mu\text{m} \qquad (\lambda = 0.5\ \mu\text{m})$$

$$\delta z = \pm 2\lambda(f/\#)^2 \qquad \delta z \approx \pm(f/\#)^2 \text{ in } \mu\text{m} \qquad (\lambda = 0.5\ \mu\text{m})$$

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Index

1:1 imaging, 16
180° deviation prisms, 40
45° deviation prisms, 40
45° prism, 40
90° deviation prisms, 39

AΩ product, 28, 85
Abbe number, 62–64, 67–69
aberration theory, 81
accommodation, 90, 92
achromatic doublet, 69
achromatic thin prism, 67
achromatic wedge, 67
achromatization, 67
acid resistance, 64
active triangulation, 54
aerial image, 48
afocal system, 6, 18, 34, 44, 46, 47, 52, 99, 102
Airy disk, 88, 89, 93
Amici prism, 39
angle of minimum deviation, 76
angular resolution, 88
aperture stop, 24, 26, 31, 32
area scanner, 96
artificially flattened field, 79
aspherics, 82
astigmatism, 73, 78, 79, 81, 92
astronomical telescope, 46–48, 50, 83
axial chromatic aberration, 68
axial color, 68

back focal distance (BFD), 15, 20, 22, 43, 44, 99, 101
barrel distortion, 80
basic radiance, 85
bending a lens, 76
binoculars, 18, 41, 47
biological objectives, 52
BK7, 64, 65
blackbody sources, 87
brilliant reflex viewfinder, 53

camera equation, 85
cardinal points and planes, 6, 8, 11, 12, 15, 18, 20, 22, 99
Cassegrain objective, 22, 23
Cassegrain telescope, 83
chief ray, 25–30, 32–34, 58, 68, 69, 98
ciliary muscle, 90
circle (geometric), 82
climatic resistance, 64
coefficients of thermal expansion, 64
cold mirror, 60
collinear transformation 6
collimator, 43
coma, 73, 77, 81, 83
complimentary colors, 95
compound eyepiece, 49
concave mirror,53, 59, 60
condenser, 55, 58–60
cones (of the eye), 91, 92
conics, 82, 83
conjugates, 6, 8–10, 17, 18, 27, 29, 33, 52, 54, 76, 99
conjugate matrix 99
contact lens 92
cornea, 90, 92, 93
corner cube, 40
cosine fourth law, 32, 85
critical angle, 4, 57
critical illumination, 55
crystalline lens, 90-93
cubic coma, 73

Index

dark adaptation, 91
dark field illumination, 61
dark field system, 61
defocus 33, 48, 51, 73, 74, 76, 81, 89
depth of field, 32, 35, 36, 95
depth of focus (DOF), 32, 35, 102
determinant of the ray matrix, 98
diffraction, 2, 35, 88, 89, 93, 95
diffraction limited system 89, 104
diffuse illumination, 55–57
diffuser, 56, 57
dihedral angle, 38
dihedral line, 38
diode lasers, 87
diopters, 14, 66
direct vision prism, 67
directed distances, 1, 6
dispersing prism, 65
dispersion, 62, 63, 65, 67, 68, 97, 103
dispersion of a thin prism, 67
distortion, 36, 73, 80, 81
double telecentricity, 34, 52
dove prism, 41

effective (or equivalent) focal length (EFL) 7–9, 15, 17, 28, 43, 44, 46, 48, 68, 69, 75, 90, 94, 101
ellipse, 82, 83
elliptical coma, 73
emittance, 87
emmotropia, 92
empty magnification, 93
energy, 84, 88
entrance pupil (EP) 24–27, 29, 33, 34, 46, 55, 72, 90
erector lens, 50
etendue, 28
excess power, 69
exit pupil (XP), 24–26, 33–35, 46–49, 70, 74, 89
exitance, 84, 86, 87
exposure, 85, 86, 95
eye, 45–49, 53, 54, 66, 90-93, 95, 96
eye circle, 46
eye relief (ER), 46, 48, 49
eyepiece, 49

F2, 64, 65
faceted parabolic reflector, 59
far point, 92
farsighted, 92
Fermat's principle, 3
field curvature, 73, 79, 81
field diaphragm, 58
field lens, 48–50, 53, 54, 58
field of view (FOV), 27, 31, 32, 39, 41, 43, 47, 48, 49, 53, 55, 61, 72, 91, 94, 102
field stop, 49
field-dependent phase, 73
fifth-order astigmatism, 73
fifth-order distortion, 73
fifth-order field curvature, 73
fifth-order linear coma, 73
fifth-order spherical aberration, 73
film and detector formats 94
first-order optics, 2
flux, 84, 85
flying spot scanner, 96
F-number ($f/\#$), 29, 102

Index

focal length, 7–9, 15, 17, 28, 43, 44, 46, 48, 68, 69, 75, 90, 94, 101
focal plane arrays, 94
focal plane to focal plane matrix, 99
focal system, 6, 8, 11–13
fractional object (FOB), 27
frequency, 2
Fresnel lens, 54, 60
Fresnel reflection coefficients, 4
front focal length, 6–9
front cardinal points, 20
front focal distance (FFD), 15, 20, 99, 101
front focal plane, 6, 33, 52, 53
front focal point, 6, 17, 18, 20, 43, 51, 92
front principal plane, 6, 13, 48
full field of view (FFOV), 27
fused silica, 64

Galilean telescope, 47
Gaussian equations, 9, 101
Gaussian optics, 2, 6, 15
Gaussian reduction, 13, 15, 22, 23, 101
generalized asphere, 82
glass code, 64
glass map, 63
graticles, 49
Gregorian telescope, 83

half field of view (HFOV), 27
heat absorbing glass, 60
heat management, 60
hot mirror, 60
human eye, 45–49, 53, 54, 66, 90-93, 95, 96
Huygens eyepiece, 49
hyperbola, 82
hyperfocal distance, 36, 102
hyperopia, 92

illuminance, 86
illumination systems, 55
image blur, 35, 68, 72, 77, 79, 80, 95
image erection prisms, 41
image rotation, 37, 41, 54
image rotation prisms, 41
image space 5, 6, 8, 12, 13, 18–20, 24, 26, 29, 33, 34, 71, 88, 98
image-space telecentric, 33
index of refraction, 1, 2, 5, 9, 62, 63
infinity corrected objectives, 52
integrating bar, 57
integrating sphere, 57
intensity, 84
interlace scan, 96
internal transmission, 64
intra-ocular lens, 93
invert, 37
iris, 24, 90
irradiance, 32, 84, 85

K prism, 41
kaleidoscope, 57
Kellner eyepiece, 49
Keplerian telescope, 46–48, 50, 83
keystone distortion, 36
Koehler illumination, 58

Index

Lagrange invariant, 28, 29, 85, 98, 102
Lambertian source, 84
laser in situ keratomileusis, 93
laser wavelengths, 87
LASIK, 93
lateral chromatic aberration, 68
lateral color, 68
lateral magnification, 6, 36, 68
law of reflection, 4, 37
light pipe, 57
line coma, 73
linear array scanner, 96
long focus lenses, 94
longitudinal aberration, 75
longitudinal magnification, 10, 18
longitudinal ray errors, 70, 75
lumens, 86
luminance, 86
luminous exitance, 86
luminous intensity, 86
luminous photopic efficacy, 86
luminous power, 86

macula, 90, 91
magnifier, 45, 48, 51, 54, 103
magnifying power (MP), 45-48, 50, 51, 53, 93, 103
marginal focus, 75, 76
marginal ray, 25, 26, 29, 30, 31, 58, 71, 75, 76
matrices, 98, 99
matrix methods, 98
mean solar constant, 85
mechanical tube length, 52
mechanically compensated zoom, 44
medial focus, 78, 79
medial surface, 79
meridional rays, 25, 71, 78
metallurgical objective 52
metrology, 34, 39
microscope, 34, 49, 51, 52, 58, 93, 103
minimum circle, 76
minimum deviation, 65, 76
minimum wavefront variance, 76
mirror-based telescopes, 83
modulation transfer function (MTF), 89
myopia, 92

natural stop position, 77
near point, 45, 92, 95
nearsighted, 92
Newtonian equations, 8, 18, 101
Newtonian telescope, 83
nodal plane to nodal plane matrix, 99
nodal points, 11, 14, 16, 34, 90
normalized image height, 70
normalized pupil coordinates, 70
numerical aperture (NA), 29, 51, 93, 102

object space, 5, 6, 8, 13, 18–20, 24, 33, 34, 51, 52, 98
object-image conjugates, 17
object-image zones, 12

Index

objectives, 22, 23, 33, 34, 43, 44, 46–48, 50–54, 61, 93, 95
object-space telecentric, 33, 34, 52
object-to-image distance, 16
ocular, 49
optical angle, 9, 10
optical axis, 11, 19, 36, 40, 41, 66, 77, 80, 82, 84, 85, 90
optical density, 95
optical invariant, 28, 98
optical order, 23, 98
optical path length (OPL), 3
optical spaces, 5, 9, 12, 13, 19, 23–30, 34, 37, 98
optical tube length (OTL), 51, 52
overhead projector, 60

parabola, 59, 82, 83
parallax, 53, 54
parallel plane mirrors, 38
paraxial optics, 2, 19, 28
paraxial raytrace, 19, 20, 22, 23, 81, 98, 102
parfocal objectives, 52
parity, 37–39, 54, 60
partial dispersion, 62, 67
passive triangulation system, 54
Pechan prism, 41
Pechan-roof prism, 41, 50
Pentaprism, 39, 40
periscope, 38, 50
Petzval objective, 43
Petzval surface, 79
phakic IOL, 93
photometric units, 86
photometry, 86
photopic respons,e 86
photorefractive keratectomy (PRK), 93
pincushion distortion, 80
piston, 73
Planck's equation, 87
plane mirror, 37–39
plane parallel plate, 42
point spread function (PSF), 89
polycarbonate, 64
polystyrene, 64
Porro prism, 40, 41, 50
Porro system, 41
Porro-Abbe system, 41
power of an optical surface, 7
presbyopia, 92
primary colors, 95
primary rainbow, 97
principal dispersion, 62
principal section, 38, 66
prism diopters, 66
prism dispersion, 65, 67
prism systems, 39–41
progressive scan, 96
projected area, (Lambert's law) 84
projection condenser system, 58
projection lens, 58, 59
projection screen, 60
projector, 55, 60
pupil (of the eye), 90–92
pupil locations, 25, 26
push broom scanner, 96

radial keratotomy (RK), 93
radiance, 84, 85, 87
radiative transfer, 28, 32, 85, 86, 104

Index

radiometry, 84–86, 104
radius of curvature, 1, 7, 90, 92
rainbow, 97
Ramsden circle, 46
Ramsden eyepiece, 49
rare earth glasses 63
ray bundle, 30–32
ray fans, 71, 72, 75, 77, 80, 81
ray intercept curves, 71, 72, 75, 77, 80, 81
Rayleigh criterion, 88, 89
Rayleigh scattering, 97
real image, 5
real object, 5
rear cardinal points, 20, 22
rear focal length, 6–9, 17, 90, 92
rear focal point/plane, 6, 8, 17, 18, 20, 33, 51, 92
rear principal plane, 6, 36, 43, 48, 90
reciprocal magnifications, 16
reduced diagram, 42
reduced distance, 9, 10
reduced schematic eye, 91
reduced thickness, 42, 99
reduced tunnel diagram, 42
reference image point, 70–72, 74
reference sphere, 70, 74
reference wavefront 70, 71
reflectance, 4, 84
reflex prism, 39, 54
reflex viewfinder, 53
refraction matrix, 98
refractive surgery techniques, 93
refractivity, 62
relative partial dispersion ratio, 62
relay lens, 50
relayed Keplerian telescope, 50, 83
resolution of the eye, 91, 93, 95
reticle ,49, 52, 53
retina, 91–93
retrofocus objective, 43
reverse Galilean viewfinder, 53
reverse raytrace, 19, 20
reverse telephoto objective, 43
reverse telephoto zoom, 44
reversed Galilean telescope, 47
reversion prism, 41
revert, 37
right angle prism, 39, 40
right circular cone, 30, 84
Risley prism, 66
Ritchey-Chretien telescope, 83
RMS spot size 72, 76
rods, 91
roof mirror, 38, 39
roof prism, 39
root-mean-squared spot size, 72, 76

sag of a spherical surface, 83, 104
sagittal coma, 77
sagittal focus, 78
sagittal oblique spherical aberration, 73
sagittal ray fan, 71, 72

Index

sagittal rays or transverse rays, 71, 78
sagittal surface, 79
scanners, 96
Scheimpflug condition, 36
schematic eyes, 91
schlieren system, 61
Schmidt prism, 40
scotopic response, 86
scratch suppression, 56
secondary chromatic aberration, 69
secondary color, 69
secondary dispersion, 67
secondary rainbow, 97
Seidel aberration coefficients, 81
sensitivity of the eye, 91
shoulder height, 52
sign conventions, 1
simple eyepiece, 49
simple objective, 43
single lens reflex system, 39, 54
skew ray, 71
SLR system, 39, 54
Snell's law of refraction, 4
Snellen visual acuity, 93
source mirror, 59
spectacle lens, 92
specular illumination, 55, 58
spherical aberration (SA), 73, 75, 76, 83
spherochromatism, 76
spot diagram, 72
stain resistance, 64
standard lens, 94
Stefan-Boltzmann law, 87
Stiles-Crawford effect, 92
Strehl ratio, 89
substage diaphragm, 58
subtractive colors, 95
sunsets, 97
surface vertices, 7, 14, 15
system matrix, 98, 99
system of plane mirrors, 38

tangential coma, 77
tangential focus, 78
tangential oblique spherical aberration, 73
tangential rays or meridional rays, 25, 71, 78
tangential surface, 79
tapered integrating bar, 57
telecentricity, 33, 34
telephoto objective, 43, 83
telephoto zoom, 44
telescopes, 46–50, 83
temperature coefficients of refractive index, 64
thermal sources, 87
thick lens, 14, 36, 60
thick lens matrix, 99
thin lens, 10, 14–17, 27, 29, 35, 36, 43, 68, 69, 76, 77, 102
thin lens matrix, 99
thin prisms, 66, 103
third-order optics, 2
throughput, 28, 85
total internal reflection (TIR), 4
transfer matrix, 98
transmission, 56, 61, 63, 64, 84, 95
transverse aberration, 75
transverse axial chromatic aberration, 68, 103

Index

transverse magnification, 6, 8
transverse ray errors, 70–72
trigonometric identities, 100
tunnel diagram, 39, 42, 57
two separated thin lenses, 14
two-component system, 13

undercorrected (SA), 76

Van Albada viewfinder, 53
vernier acuity, 91
vertex distances, 15
vertex matrix, 99
video sensors, 94
viewfinders, 39, 47, 53
vignetting, 31–33, 39, 48, 49, 54, 102
virtual image, 5
virtual object, 5
visual astigmatism, 92
visual magnification 51, 93
vitreous, 90

wave fans, 71, 72, 81
wavefront errors, 70
wavefront expansion, 73
wavefront tilt, 73, 74, 80, 81
wavefronts, 3, 71
wavelength, 2
wavenumber, 2
Wein's displacement law, 87
wide angle lenses, 94
working distance (WD), 22, 23, 52
working f-number, 29

YNU raytrace, 19, 21
YNU raytrace worksheet, 21

zoom lens, 44

John E. Greivenkamp is a Professor at the Optical Sciences Center of the University of Arizona where he has taught courses in optical engineering since 1991. After receiving a Ph.D. from the Optical Sciences Center in 1980, he was employed by Eastman Kodak. He is a fellow of SPIE–The International Society for Optical Engineering, of the Optical Society of America, and he has served a member of the National Research Council Committee on Optical Science and Engineering (COSE).

Professor Greivenkamp's research interests include interferometry and optical testing, optical fabrication, ophthalmic optics, optical measurement systems, optical systems design, and the optics of electronic imaging systems.

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