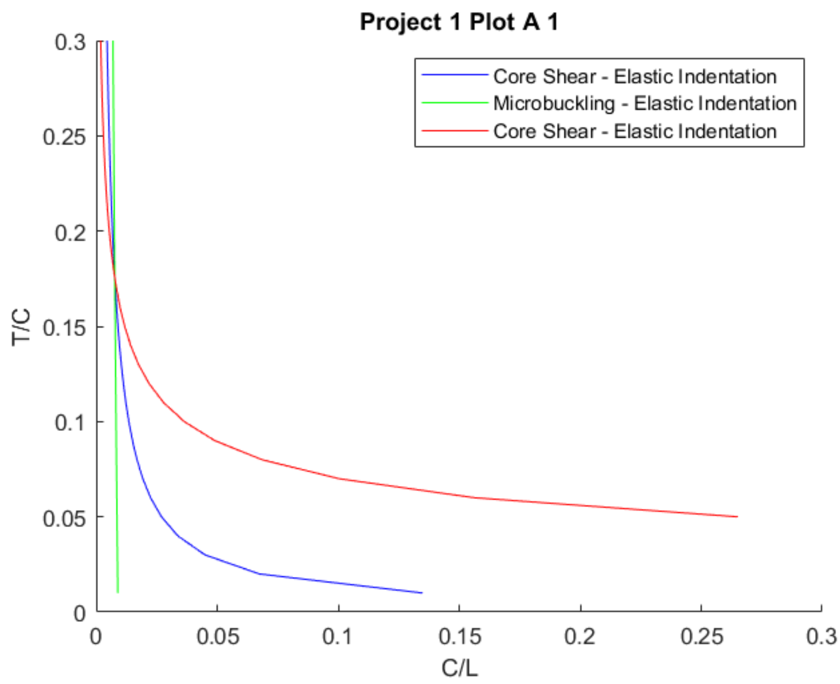
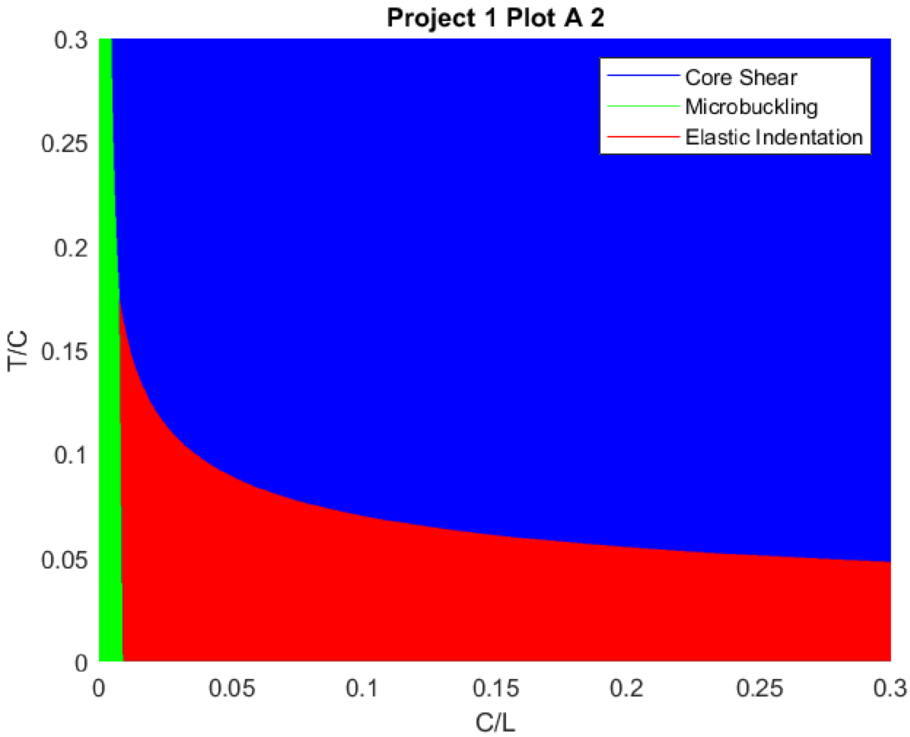
Project 1

Muthu Ram Kumar Avichi
Student No: 1010188967

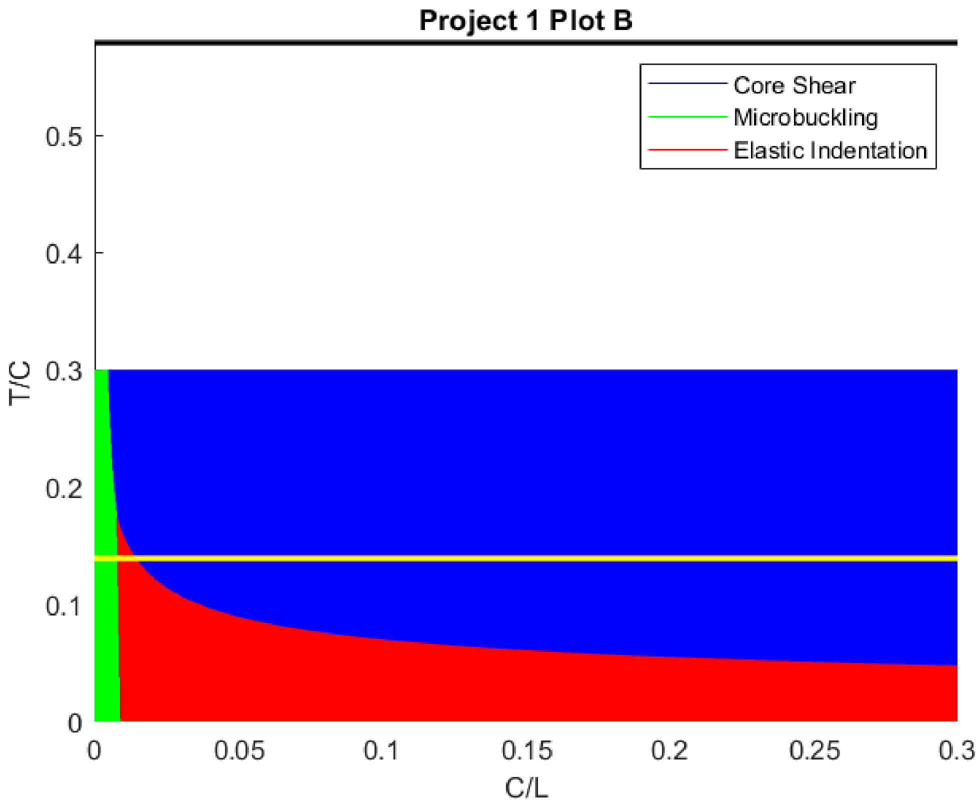
Output Plots



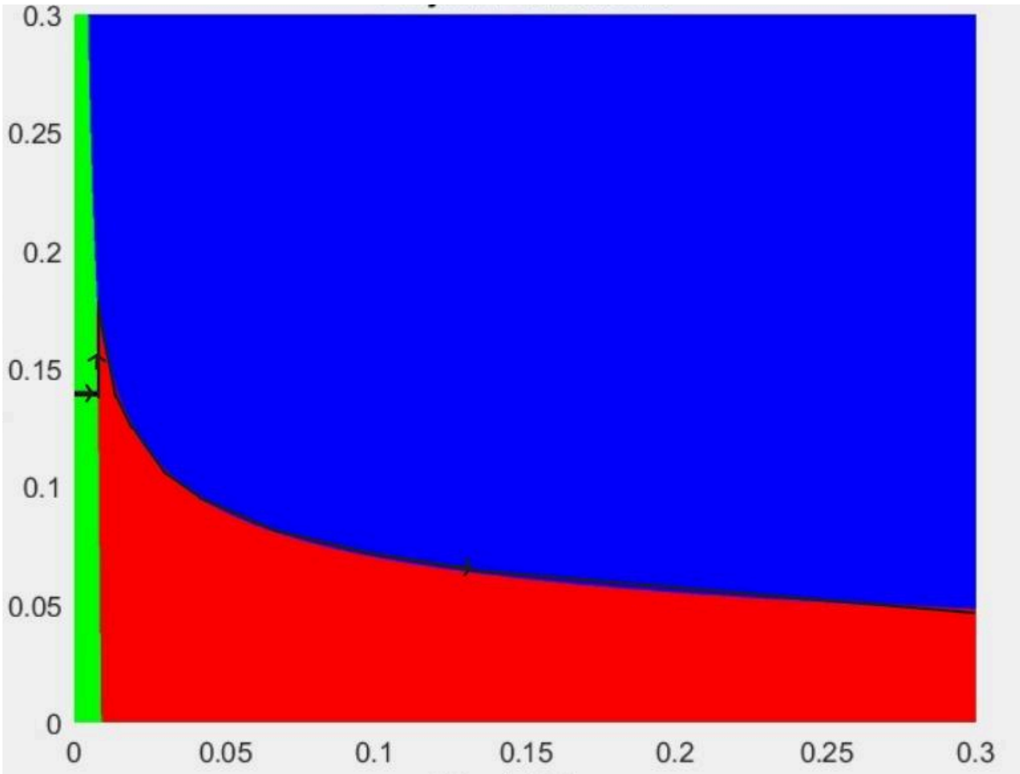
The values of the failure mechanism map is plotted for the pairs of failure conditions within the range of 0 to 0.3 for t/c and c/L pairs.



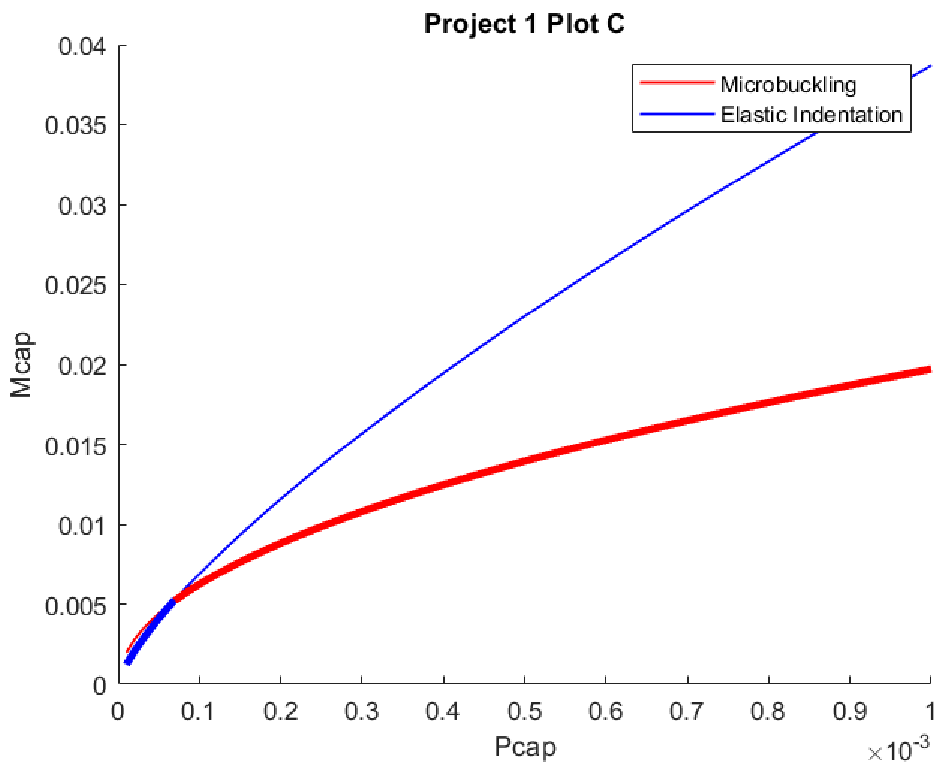
The contour plotting ensures that the imaginary parts of the curve are not represented on the failure mechanism map.



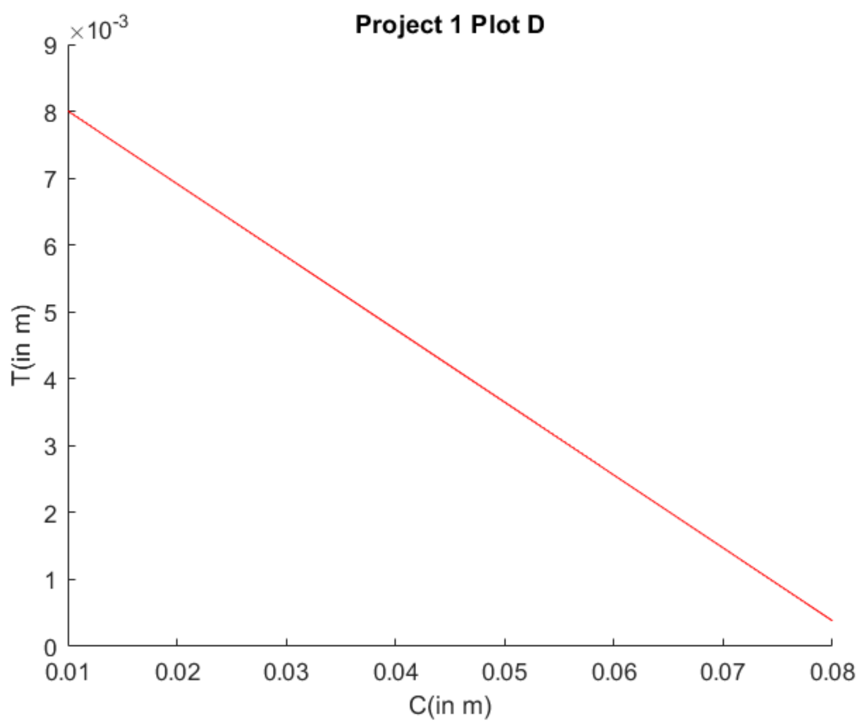
The yellow line



The black line with arrows represent the trajectory of optimal design on the mechanism map.



The plot with heavy line thickness represents the Minimum mass for load condition plot.



The T and C values for the given example beam geometry has to fall on this line. For a specific C value, the corresponding T value in the line follows optimal design criteria under minimum loading condition.

Explanations and Examples

Project - 1

Types of failure mechanisms & their respective load criteria

a. Microbuckling : $P_m = \frac{4btd\sigma_F}{L}$

b. core shear : $P_c = 2bd\tau_c$

c. elastic indentation : $P_i = bt\left(\frac{\pi^2 d E_f \sigma_c^2}{3L}\right)^{1/3}$

I. Finding the plot of intersection b/w microbuckling & core shear:

$$P_m = P_c$$

$$\frac{\cancel{4}^{2}\,\cancel{b}\cancel{d}\,t\,\sigma_F}{L} = \cancel{2}\,\cancel{b}\cancel{d}\,\tau_c$$

$$\frac{t}{L} \Rightarrow \boxed{\left(\frac{t}{c}\right)\left(\frac{c}{L}\right)_{cm} = \frac{\tau_c}{2\sigma_f}}$$

II. Plot of intersection b/w core shear and elastic indentation:

$$P_c = P_i$$

$$2\cancel{b}\,d\,\tau_c = \cancel{b}\,t\left(\frac{\pi^2 d\, E_f\,\sigma_c^2}{3L}\right)^{1/3}$$

$$\frac{8\,d^{\cancel{3}\,2}\,\tau_c^3}{t^3} = \frac{\pi^2 \cancel{d}\,E_f\,\sigma_c^2}{3L}$$

$$d = t+c$$

$$8\tau_c^3\left(\frac{(t+c)^2}{t^3}\right) = \frac{\pi^2 E_f\,\sigma_c^2}{3L}$$

We equate all the respective load conditions in pairs to find the failure criteria intersection equations in terms of (t/c) and (c/L).

$$\left(\frac{t}{L}+\frac{c}{L}\right)^2 = \frac{\pi^2 E_f \sigma_c^2}{24\tau_c^3} \times \left(\frac{t}{L}\right)^3$$

$$\left(\frac{t}{L}\right)^2 + \left(\frac{c}{L}\right)^2 + 2\left(\frac{t}{L}\right)\left(\frac{c}{L}\right) = \frac{\pi^2 E_f \sigma_c^2}{24\tau_c^3}\left(\frac{t}{c}\right)^3\left(\frac{c}{L}\right)^3$$

$$\left(\frac{t}{c}\right)^2 \times \cancel{\left(\frac{c}{L}\right)^2} + \cancel{\left(\frac{c}{L}\right)^2} + 2\left(\frac{t}{c}\right)\cancel{\left(\frac{c}{L}\right)^2} = \frac{\pi^2 E_f \sigma_c^2}{24\tau_c^3}\left(\frac{t}{c}\right)^3\left(\frac{c}{L}\right)^{\cancel{3}\,1}$$

$$\left(\frac{t}{c}+1\right)^2 = \frac{\pi^2 E_f \sigma_c^2}{24\tau_c^3}\left(\frac{t}{c}\right)^3\left(\frac{c}{L}\right)$$

$$\boxed{\left(\frac{c}{L}\right)_{ci} = \frac{\left(\frac{t}{c}+1\right)^2}{\left(\frac{\pi^2 E_f \sigma_c^2}{24\tau_c^3}\right)\left(\frac{t}{c}\right)^3}}$$

III. Plot of intersection b/w microbuckling & elastic indent.

$$P_m = P_i$$

$$\frac{4\cancel{b}\cancel{t}\, d\, \sigma_f}{L} = \cancel{b}\cancel{t}\left(\frac{\pi^2 d\, E_f \sigma_c^2}{3L}\right)^{1/3}$$

$$(4\sigma_f)^3\left(\frac{t}{L}+\frac{c}{L}\right)^{\cancel{3}\,2} = \left(\frac{\pi^2 E_f \sigma_c^2}{3}\right)\cancel{\left(\frac{t}{L}+\frac{c}{L}\right)}$$

Plot the curve by substituting t/c values in a range of 0 to 0.3, and calculating the respective c/l values. Plot this with c/l as x-axis and t/c as y-axis.

$$\left(\frac{t}{L}\right)^2+\left(\frac{c}{L}\right)^2+2\left(\frac{t}{L}\right)\left(\frac{c}{L}\right)=\frac{\pi^2 E_f \sigma_c^2}{192\sigma_f^3}$$

$$\left(\frac{t}{c}\right)^2\times\left(\frac{c}{L}\right)^2+\left(\frac{c}{L}\right)^2+2\left(\frac{t}{c}\right)\left(\frac{c}{L}\right)^2=\frac{\pi^2 E_f \sigma_c^2}{192\sigma_f^3}$$

$$\left(\frac{c}{L}\right)^2\left(\left(\frac{t}{c}\right)^2+1+2\left(\frac{t}{c}\right)\right)=\frac{\pi^2 E_f \sigma_c^2}{192\sigma_f^3}$$

$$\boxed{\left(\frac{c}{L}\right)_{mi}=\frac{\pi^2 E_f \sigma_c^2}{192\sigma_f^3}\times\frac{1}{\left(1+\frac{t}{c}\right)^2}}$$

IV. Optimal load performance calculation:

a. micro-buckling

$$\hat{P}_m=\frac{P_m}{bL\sigma_f}=\left(\frac{4\cancel{b}td\,\cancel{\sigma_f}}{L}\right)\times\frac{1}{\cancel{b}L\cancel{\sigma_f}}$$

$$=4\times\left(\frac{t}{L}\right)\times\left(\frac{t+c}{L}\right)$$

$$=4\times\left(\frac{t}{c}\times\frac{c}{L}\right)\times\left(\frac{t}{c}\times\frac{c}{L}+\frac{c}{L}\right)$$

$$\boxed{\hat{P}_m=4\times\frac{t}{c}\times\left(\frac{c}{L}\right)^2\times\left(\frac{t}{c}+1\right)}$$

b. core-shear

$$\hat{P}_c=\frac{P_c}{bL\sigma_f}=\frac{2\cancel{b}d\tau_c}{\cancel{b}L\sigma_f}$$

$$=\frac{2\tau_c}{\sigma_f}\times\left(\frac{t}{L}+\frac{c}{L}\right)$$

$$\boxed{\hat{P}_c=\frac{2\tau_c}{\sigma_f}\times\left(\frac{t}{c}+1\right)\times\frac{c}{L}}$$

Substitute P formulae for microbuckling, core-shear and elastic indentation to find the respective P cap equations in terms of t/c and c/l. Plot these as contours.

c. elastic Indentation.

$$\hat{P}_i = \frac{P_i}{bL\sigma_f} = \frac{\cancel{b}t}{\cancel{b}L\sigma_f}\left(\frac{\pi^2 d E_f \sigma_c^2}{3L}\right)^{\frac{1}{3}}$$

$$= \frac{1}{\sigma_f}\left(\frac{\pi^2 E_f \sigma_c^2}{3}\right)^{\frac{1}{3}} \times \left(\frac{t}{c}\times\frac{c}{L}\right)\left(\frac{t}{c}\times\frac{c}{L}+\frac{c}{L}\right)^{\frac{1}{3}}$$

$$\boxed{\hat{P}_i = \frac{1}{\sigma_f}\left(\frac{\pi^2 E_f \sigma_c^2}{3}\right)^{\frac{1}{3}} \times \left(\frac{t}{c}\times\frac{c}{L}\right)\times\left(\frac{c}{L}\right)^{\frac{1}{3}}\left(\frac{t}{c}+1\right)^{\frac{1}{3}}}$$

V Minimum mass function:

$$M = 2bLt\rho_f + bLc\rho_c$$

$$\hat{M} = \frac{M}{bL^2\rho_f} = \frac{2\cancel{b}\cancel{L}t\rho_f + \cancel{b}\cancel{L}c\rho_c}{\cancel{b}L^{\cancel{2}}\rho_f}$$

$$= 2\times\frac{t}{L} + \frac{c\rho_c}{L\rho_f}$$

$$= 2\times\frac{t}{c}\times\frac{c}{L} + \frac{c}{L}\times\frac{\rho_c}{\rho_f}$$

$$\boxed{\hat{M} = \frac{c}{L}\left(2\times\frac{t}{c}+\frac{\rho_c}{\rho_f}\right)}$$

$$x = \frac{c}{L} \qquad y = \frac{t}{c} \Rightarrow \boxed{\hat{M} = x\left(2y+\frac{\rho_c}{\rho_f}\right)}$$

$$\nabla_x \hat{M} = \frac{\partial}{\partial x}(\hat{M}) = \frac{\partial}{\partial x}\left(x\left(2y+\frac{\rho_c}{\rho_f}\right)\right) = 2y+\frac{\rho_c}{\rho_f}$$

To find M cap minima, we differentiate the M cap values separately w.r.t c/l (x) and t/c (y). To find the lagrange multiplier, we equate respective P cap for each failure mechanism differentiated w.r.t x and y.

$$\nabla_y \hat{M} = \frac{\partial}{\partial y}(\hat{M}) = \frac{\partial}{\partial y}\left(x\left(2y + \frac{P_c}{P_f}\right)\right) = 2x$$

(4a) <u>Micro buckling:</u>

$$\hat{P}_m = 4\underbrace{\left(\frac{t}{c}\right)}_{y}\underbrace{\left(\frac{c}{L}\right)^2}_{x}\left(\frac{t}{c}+1\right)$$

$$\hat{P}_m = 4yx^2(y+1)$$

$$\nabla_x \hat{P}_m = \frac{\partial}{\partial x}(4x^2 y(y+1)) = 8xy(y+1)$$

$$\nabla_y \hat{P}_m = \frac{\partial}{\partial y}(4x^2(y^2+y)) = 4x^2(2y+1)$$

$$\nabla_x \hat{M} = \lambda \nabla_x \hat{P}$$

$$2y + \frac{P_c}{P_f} = \lambda(8xy(y+1)) \quad \text{——— (1)}$$

$$\nabla_y \hat{M} = \lambda \nabla_y \hat{P}$$

$$\cancel{2x} = \lambda\left(\frac{\cancel{4x^2}(2y+1)}{2x}\right) \quad \text{——— (2)} \quad \boxed{x \neq 0}$$

$$(1) \div (2)$$

$$2y + \frac{P_c}{P_f} = \frac{\overset{4}{\cancel{8x}}y(y+1)}{\cancel{2x}(2y+1)}$$

$$\left(2y + \frac{P_c}{P_f}\right)(2y+1) = 4y(y+1)$$

$$\cancel{4y^2} + 2y\left(\frac{P_c}{P_f}+1\right) + \frac{P_c}{P_f} = \cancel{4y^2} + 4y$$

$$2y\left(\frac{P_c}{P_f} - 1\right) + \frac{P_c}{P_f} = 0$$

$$\boxed{y = \frac{\frac{P_c}{P_f}}{2\left(1 - \frac{P_c}{P_f}\right)}}$$

(b) Core shear:

$$\hat{P_c} = \frac{2\tau_c}{\sigma_f}\left(\frac{t}{c}+1\right)\left(\frac{c}{L}\right)$$

$y \quad\quad , x$

$$= \frac{2\tau_c}{\sigma_f}(y+1)(x)$$

$$\nabla_x \hat{P_c} = \frac{\partial}{\partial x}\left(\frac{2\tau_c}{\sigma_f}(y+1)(x)\right) = \frac{2\tau_c}{\sigma_f}(y+1)$$

$$\nabla_y \hat{P_c} = \frac{\partial}{\partial y}\left(\frac{2\tau_c}{\sigma_f}(y+1)(x)\right) = \frac{2\tau_c}{\sigma_f}(x)$$

$$\nabla_x \hat{M} = \lambda \nabla_x \hat{P_c}$$

$$2y + \frac{P_c}{P_f} = \lambda\left(\frac{2\tau_c}{\sigma_f}(y+1)\right) \quad \text{—— (1)}$$

$$\nabla_y \hat{M} = \lambda \nabla_y \hat{P_c}$$

$$\cancel{2x} = \frac{\cancel{2}\tau_c}{\sigma_f}(\cancel{x}) \quad \text{—— (2)}$$

$\boxed{x \neq 0}$

solving ① ÷ ②

$$2y + \frac{\rho_c}{\rho_f} = \cancel{x}\left(\frac{\frac{2\cancel{\tau_c}}{\cancel{\sigma_f}}(y+1)}{\frac{\cancel{\tau_c}}{\cancel{\sigma_f}}}\right)$$

$$\cancel{2y} + \frac{\rho_c}{\rho_f} = \cancel{x}(\cancel{2y} + 2)$$

$$\boxed{\frac{\rho_c}{\rho_f} = 2}$$

There is no optimal design trajectory for the failure region in core shear.

ⓒ Elastic indentation:

$$\hat{P}_i = \underbrace{\frac{1}{\sigma_f}\left(\frac{\pi^2 E_f \sigma_c^2}{3}\right)^{\frac{1}{3}}}_{K_i}\left(\underbrace{\frac{t}{c}}_{y} \times \underbrace{\frac{c}{L}}_{x}\right)\left(\frac{c}{L}\right)^{\frac{1}{3}}\left(\frac{t}{c}+1\right)^{\frac{1}{3}}$$

$$\hat{P}_i = K_i\,(yx)(x)^{1/3}(y+1)^{1/3}$$

$$\nabla_x \hat{P}_i = \frac{\partial}{\partial x}\left(K_i(yx)(x)^{1/3}(y+1)^{1/3}\right) = K_i(y)(y+1)^{1/3}\left(\frac{4}{3}x^{1/3}\right) \quad ①$$

$$\nabla_y \hat{P}_i = \frac{\partial}{\partial y}\left(K_i\left(x^{4/3}\right)\left(y\,(y+1)^{1/3}\right)\right)$$

$$= K_i x^{4/3} \times \left[y \times \frac{\partial}{\partial y}(y+1)^{1/3} + (y+1)^{1/3}\frac{\partial}{\partial y}(y)\right]$$

$$= K_i x^{4/3} \times \left[y \times \frac{1}{3} (y+1)^{-2/3} + (y+1)^{1/3} \right]$$

$$= K_i x^{4/3} (y+1)^{1/3} \left[\frac{y}{3y+3} + 1 \right]$$

$$= K_i x^{4/3} \left(\frac{\cancel{y}(4y+3)}{3y+3} \right) (y+1)^{1/3} \quad \text{—— (2)}$$

$$\nabla_x \hat{P}_i = \lambda \nabla_x \hat{M}$$

$$K_i \; y (y+1)^{1/3} \left(\frac{4}{3} x^{1/3} \right) = \lambda \left(2y + \frac{\rho_c}{\rho_f} \right) \quad \text{—— (1)}$$

$$\nabla_y \hat{P}_i = \lambda \nabla_y \hat{M}$$

$$K_i \frac{\cancel{x^{4/3}}}{x^{1/3}} \left(\frac{y(4y+3)}{3y+3} \right) = \lambda \left(2\cancel{y}\cancel{x} \right) \quad \text{—— (2)}$$

solving (1) ÷ (2)

$$\frac{\cancel{K_i} \; \cancel{y} \; \cancel{(y+1)^{1/3}} \left(\frac{4}{\cancel{3}} \cancel{x^{1/3}} \right) y}{\cancel{K_i} \; \cancel{x^{1/3}} \left(\frac{\cancel{y}(4y+3)}{\cancel{3}y+\cancel{3}1} \right) \cancel{(y+1)^{1/3}}} = \frac{\cancel{\lambda}\left(2y + \frac{\rho_c}{\rho_f} \right)}{\cancel{\lambda}(2)}$$

$$8y(y+1) = \left(2y + \frac{\rho_c}{\rho_f} \right)(4y+3)$$

$$\cancel{8y^2} + 8y = \cancel{8y^2} + y\left(6 + 4\frac{\rho_c}{\rho_f} \right) + \frac{3\rho_c}{\rho_f}$$

$$y\left(+2 - \frac{4\rho_c}{\rho_f} \right) = \frac{3\rho_c}{\rho_f}$$

Once we have all the equations, plot them on the same map to understand where the optimal trajectory lies. Based on the failure criteria and the line placement on the graph, we can plot the design trajectory for the minimum load criteria depending on whichever plot is at the minima.

$$\boxed{y = \frac{3\frac{\rho_c}{\rho_f}}{2 - 4\frac{\rho_c}{\rho_f}}}$$

VI Minimum mass plots

~~Min~~ mass $\Rightarrow \hat{M} = \underbrace{\frac{c}{L}}_{x}\left(\underbrace{2\times\frac{t}{c}}_{y} + \underbrace{\frac{\rho_c}{\rho_f}}_{\bar{\rho}}\right)$

$$= x(2y + \bar{\rho}) \quad \text{——— } (1)$$

a. Micro buckling

$$\hat{P}_M = 4x^2 y(y+1)$$

$$x^2 = \frac{\hat{P}_M}{4y(y+1)} \Rightarrow x = \sqrt{\frac{\hat{P}_M}{4y(y+1)}} \quad \text{— } (2)$$

substituting (2) in (1)

$$\hat{M} = \sqrt{\frac{\hat{P}_M}{4y(y+1)}} \times (2y + \bar{\rho})$$

to find min. mass $\frac{d\hat{M}}{dy} = 0 \Rightarrow \frac{d}{dy}\left(\sqrt{\frac{\hat{P}_M}{4y(y+1)}} \times (2y+\bar{\rho})\right) = 0$

solving the differential, we get

Find c/l (x) value from each P cap failure mechanism and substitute the respective values in minimum mass equation to find the M cap equation in terms of P cap.

$$-\frac{\sqrt{P_M}\left(\left(2p-2\right)\alpha^{y}+\bar{p}\right)}{4\left(\alpha^{y}(\alpha^{y}+1)\right)^{3/2}}=0$$

$$(2p-2)y+\bar{p}=0$$

$$y=\frac{\bar{p}}{2(1-\bar{p})}$$

substituting y in $\hat{M}$

$$\hat{M}=\frac{\sqrt{\hat{P}_M}\left(\frac{\cancel{2}\bar{p}}{\cancel{2}(1-\bar{p})}+\bar{p}\right)}{2\sqrt{\frac{\bar{p}}{(1-\bar{p})2}\left(\frac{\bar{p}}{(1-\bar{p})2}+1\right)}}$$

$$=\frac{\sqrt{\hat{P}_M}\left(2\bar{p}-\bar{p}^2\right)/\cancel{(1-\bar{p})}}{\frac{\cancel{2}}{\cancel{2}}\sqrt{\frac{\bar{p}}{(1-\bar{p})}\left(\frac{2-\bar{p}}{1-\bar{p}}\right)}}$$

$$=\sqrt{\hat{P}_M}\,\frac{\bar{p}\,(2-\bar{p})}{\sqrt{\bar{p}\,(2-\bar{p})}}$$

$$\boxed{\hat{M}=\sqrt{\hat{P}_M}\,\sqrt{\bar{p}(2-\bar{p})}}$$

b. Core shear

$$\hat{P}_{cs}=\underbrace{\frac{2T_{cs}}{\sigma_f}}_{K}(y+1)x \qquad \hat{M}=x(2y+\bar{p})$$

$$\hat{P}_{cs}=K(y+1)x$$

$$\boxed{x=\frac{\hat{P}_c}{K(y+1)}}$$

$$\hat{M} = \frac{\hat{P_c}}{K(y+1)}(2y+\bar{P})$$

differentiating wrt y

$$\frac{d\hat{M}}{dy} = \frac{\hat{P}_c}{K}\left(\frac{2-\bar{P}}{(y+1)^2}\right) = 0$$

$$\Rightarrow y = \infty$$

This signifies that there is no plot for minimum mass in the core shear region.

c. Elastic Indentation

$$\hat{P}_i = K(y \times x)(x)^{\frac{1}{3}}(y+1)^{\frac{1}{3}}$$

$$\frac{1}{\sigma_f}\left(\frac{\pi^2 E_f \sigma_c^2}{3}\right)^{\frac{1}{3}}$$

$$\hat{M} = x(2y+\bar{P})$$

$$x^{\frac{4}{3}} = \frac{\hat{P}_i}{Ky(1+y)^{1/3}}$$

$$x = \left(\frac{\hat{P}_i}{Ky(1+y)^{1/3}}\right)^{3/4}$$

$$x = \left(\frac{\hat{P}_i}{K}\right)^{\frac{3}{4}} \times \frac{1}{y^{3/4}(1+y)^{1/4}}$$

substitute x in $\hat{M}$ equation

$$\hat{M} = \left(\frac{\hat{P_i}}{K}\right)^{\frac{3}{4}} \left(\frac{1}{y^{3/4}(1+y)^{1/4}}\right)(2y+\bar{P}) \quad \text{——— } (1)$$

differentiating wrt. y for $\hat{M}$

$$\frac{d\hat{M}}{dy} = \left(\frac{\hat{P_i}}{K}\right)^{\frac{3}{4}} \times \frac{d}{dy}\left(\frac{2y+\bar{P}}{y^{3/4}(1+y)^{1/4}}\right) = 0$$

$$\Rightarrow \left(\frac{\hat{P_i}}{K}\right)^{\frac{3}{4}} \times \frac{+(4\bar{P}-2)y - 3\bar{P}}{4y^{7/4}\cdot(y+1)^{5/4}} = 0$$

solving, $+(4\bar{P}-2)y - 3\bar{P} = 0$

$$\boxed{y = \frac{-3\bar{P}}{4\bar{P}-2}} \quad \text{——— } (2)$$

substitute (2) in (1)

$$\hat{M} = \left(\frac{\hat{P_i}}{K}\right)^{3/4} \frac{2y+\bar{P}}{y^{3/4}(1+y)^{1/4}}$$

num $\Rightarrow$ $2y+\bar{P} = 2\left(\frac{3\bar{P}}{2-4\bar{P}}\right) + \bar{P}$

$$= \frac{3\bar{P}}{1-2\bar{P}} + \bar{P}$$

$$= \frac{3\bar{P}+\bar{P}-2\bar{P}^2}{1-2\bar{P}} = \frac{4\bar{P}-2\bar{P}^2}{1-2\bar{P}}$$

$$= \frac{2\bar{P}(2-\bar{P})}{(1-2\bar{P})}$$

den $\Rightarrow$ $y^{3/4}(1+y)^{1/4} = \left(\frac{+3\bar{P}}{2-4\bar{P}}\right)^{3/4}\left(1+\frac{3\bar{P}}{2-4\bar{P}}\right)^{1/4}$

Once we have all equations, plot on a Mcap (y) vs Pcap (x) curve to find the minimum mass plot.

$$= \left(\frac{3\bar{\rho}}{2-4\bar{\rho}}\right)^{3/4} \left(\frac{2-4\bar{\rho}+3\bar{\rho}}{2-4\bar{\rho}}\right)^{1/4}$$

$$= \left(\frac{3\bar{\rho}}{2-4\bar{\rho}}\right)^{3/4} \left(\frac{2-\bar{\rho}}{2-4\bar{\rho}}\right)^{1/4}$$

$$= \frac{(3\bar{\rho})^{3/4}(2-\bar{\rho})^{1/4}}{2-4\bar{\rho}}$$

$$\hat{M} = \left(\frac{\hat{P}_i}{K}\right)^{\frac{3}{4}} \frac{\left(2\bar{\rho}\left(\frac{2-\bar{\rho}}{\cancel{1-2\bar{\rho}}}\right)\right)}{\left(\frac{(3\bar{\rho})^{3/4}(2-\bar{\rho})^{1/4}}{\cancel{2-4\bar{\rho}}}\right)}$$

$$\boxed{\hat{M} = \left(\frac{\hat{P}_i}{K}\right)^{3/4} \times 4\bar{\rho}\left(\frac{2-\bar{\rho}}{3\bar{\rho}}\right)^{3/4}}$$

$$K = \left(\frac{\pi^2 E_f \sigma_c^2}{3}\right)^{1/3} \times \frac{1}{\sigma_f}$$

$$= \left(\hat{P}_i\right)^{3/4}\left(\frac{3}{\pi^2 E_f \sigma_c^2}\right)^{1/4} \times \left(\frac{\sigma_f}{\cancel{\sigma_f}}\right)^{3/4} \times 4\bar{\rho} \times \left(\frac{2-\bar{\rho}}{3\bar{\rho}}\right)^{3/4}$$

$$= \left(\frac{\hat{P}_i \sigma_f}{\cancel{\sigma_f}}\right)^{3/4} \left(\frac{\cancel{3}(2-\bar{\rho})^3}{\cancel{27}_{9}\,\bar{\rho}^3 \times (\pi^2 E_f \sigma_c^2)}\right)^{1/4} \times 4\bar{\rho}$$

$$\boxed{\hat{M} = \left(\frac{\hat{P}_i \sigma_f}{\cancel{\sigma_f}}\right)^{3/4} \left(\frac{\bar{\rho}(2-\bar{\rho})^3}{9(\pi^2 E_f \sigma_c^2)}\right)^{1/4} \times 4}$$

Substituting $\bar{E} = \frac{E_f}{\sigma_f}$ & $\bar{\sigma_c} = \frac{\sigma_c}{\sigma_f}$

$$\hat{M} = (\hat{P}_i)^{3/4}\left(\frac{\bar{\rho}(2-\bar{\rho})^3}{9\pi^2 \bar{E}\,\bar{\sigma}^2}\right)^{1/4} \times 4$$

In this curve, there will be two equations. We need to plot the set of curves with lowest mass Mcap values. Hence, we find the point of intersection and plot the equations piecewise on the Mcap vs Pcap curve.

VII Example Beam calculations.

- given are P, b & L values.
- we substitute P in $\hat{P} = \frac{P}{bL\sigma_f}$ equation to find $\hat{P}$ values.
- then verifying with the plot, the values calculated for $\hat{P}$ is $\sim 0.13 \times 10^{-4}$
- This value of $\hat{P}$ corresponds to the microbuckling region.
- Substituting the $\hat{P}$ in the corresponding microbuckling $\hat{M}$ minima equation gives an $\hat{M}$ value.

$$\hat{M} = \frac{2 \times t}{L} + \frac{\rho_c}{\rho_f} \times \frac{c}{L}$$

- from previous calculations, substituting in $\hat{M}$ equation

$$\cancel{\hat{M}} \times \quad \boxed{t = \left(\hat{M} - \frac{\rho_c}{\rho_f L} \times c\right)\frac{L}{2}}$$

- Now, we plot c from a range of 0 to 0.08 to get corresponding t values in that range.

Matlab Code

```
% Matlab code for Aerospace Structures Project - 1
% ---------------------------------------------------------------------------
% Part A - Failure Mechanism Map
% Define tbyc values from 0.01 to 0.30 with a 0.01 increment
tbyc = 0.01:0.01:0.30;
% Initialize arrays for failure mode intersections
mccbyl = zeros(1, 30);
cicbyl = zeros(1, 30);
micbyl = zeros(1, 30);
% Define face sheet properties
rhof = 1600 + (30 * 1);
Ef = (40 + 88) * (10^9);
sigmaf = (200 + (100 * 9)) * (10^6);
% Define foam core properties
rhoc = 20 + (5 * 67);
sigmac = (0.5 + 6.5 * ((67 / 100)^(1.5))) * (10^6);
tauc = (0.5 + 4.5 * ((67 / 100)^(1.5))) * (10^6);
% Calculate constants to use in the equations
cnum = pi^2 * Ef * sigmac^2;
cden = 192 * sigmaf^3;
cden2 = 24 * tauc^3;
K = cnum / cden;
K2 = cnum / cden2;
for i = 1:30
% Microbuckling and Core Shear
mccbyl(i) = tauc / (2 * sigmaf * tbyc(i));
% Microbuckling and Indentation
tbycc = (tbyc(i) + 1)^2;
micbyl(i) = sqrt(K / tbycc);
% Core Shear and Indentation
cicbyl(i) = tbycc / (K2 * tbyc(i)^3);
end
% Create a meshgrid
num_values = 1000;
newtbyc = linspace(0, 0.30, num_values);
newcbyl = linspace(0, 0.30, num_values);
[ny, nx] = meshgrid(newtbyc, newcbyl);
% Calculate Pmcap, Pccap, and Picap
Pmcap = 4 .* ny .* nx .* (ny + 1) .* nx;
Pccap = 2 .* (tauc / sigmaf) .* (ny + 1) .* nx;
Ki = ((pi^2 .* Ef .* sigmac^2 / 3).^(1/3)) ./ sigmaf;
Picap = Ki .* ny .* nx .* (ny + 1).^(1/3) .* nx.^(1/3);
% Find the minimum values and their corresponding indices
min_values = min(min(Pmcap, Pccap), Picap);
index_matrix = zeros(size(min_values));
for i = 1:num_values
for j = 1:num_values
if min_values(i, j) == Pmcap(i, j)
index_matrix(i, j) = 1;
elseif min_values(i, j) == Pccap(i, j)
index_matrix(i, j) = 2;
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else
index_matrix(i, j) = 3;
end
end
end
% -----------------------------------------------------------------------
% ------------------------------------------------------------------------
% Part b - Optimal Design Trajectory
%Failure due to microbuckling
rhobar = rhoc/rhof;
ymb = rhobar / (2 * (1 - rhobar));
%Failure due to elastic indentation - the line plotted is out of bounds.
yei = ( 3 * rhobar ) / (2 - 4 * rhobar);
% Point of intersection between microbuckling optimal design curve and
% microbuckling - indentation failure plot pair.
tbycc = (ymb + 1)^2;
intersection_x1 = sqrt(K / tbycc);
% -----------------------------------------------------------------------
% ------------------------------------------------------------------------
% PART C - Minimal Mass vs Load
Pcap = 0.00001:0.00001:0.001;
Ebar = Ef/sigmaf;
sigmabar = sigmac/sigmaf;
den = 9*(pi^2)*(sigmabar^2)*(Ebar);
rhoe = rhobar*((2 - rhobar)^3);
Kei = 4 * ((rhoe/den)^(1/4));
Mcapei = Kei.*(Pcap.^(3/4));
Mcapmb = sqrt(Pcap.*(rhobar*(2 - rhobar)));
% Find the point of intersection
intersection_x2 = Pcap(abs(Mcapei - Mcapmb) < 0.0001); % Adjust the tolerance
as needed
Pcap1 = Pcap(Pcap <= intersection_x2); % x-values for the first segment
Pcap2 = Pcap(Pcap >= intersection_x2); % x-values for the second segment
% corresponding y-values
Mcapei1 = Kei.*(Pcap1.^(3/4));
Mcapmb1 = sqrt(Pcap2.*(rhobar*(2 - rhobar)));
% -----------------------------------------------------------------------
% ------------------------------------------------------------------------
% Part D - Example Beam Optimal Design
Pgiven = 75 * 10^3;
Lgiven = 2.5;
bgiven = 200 * 10^-3;
Pf = Pgiven / (bgiven * Lgiven * sigmaf);
% The value shows that for the load condition, minimum mass condition falls
% in the microbuckling region. (~0.13 * 10^-3)
Mf = sqrt(Pf*(rhobar*(2 - rhobar)));
cf = 0.01:0.01:0.08;
tf = ( Lgiven * Mf - rhobar .* cf ) / 2;
% -----------------------------------------------------------------------
% ------------------------------------------------------------------------
% Plot A 1 - Failure Region Plot
figure(1);
hold on;
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```
plot(mccbyl, tbyc, 'b');
plot(micbyl, tbyc, 'g');
plot(cicbyl(5:end), tbyc(5:end), 'r');
% Additional plot settings
xlabel('C/L');
ylabel('T/C');
title('Project 1 Plot A 1');
legend('Core Shear - Elastic Indentation','Microbuckling - Elastic
Indentation', 'Core Shear - Elastic Indentation');
% -------------------------------------------------------------------------
% --------------------------------------------------------------------------
% Plot A 2 - Contour Plot
figure(2);
hold on;
plot(mccbyl, tbyc, 'b');
plot(micbyl, tbyc, 'g');
plot(cicbyl(5:end), tbyc(5:end), 'r');
custom_colormap = [0 1 0; 0 0 1; 1 0 0];
colormap(custom_colormap); % Apply the custom colormap
contourf(nx, ny, index_matrix, 'LineStyle', 'none');
% Additional plot settings
xlabel('C/L');
ylabel('T/C');
title('Project 1 Plot A 2');
legend('Core Shear','Microbuckling', 'Elastic Indentation');
% -------------------------------------------------------------------------
% --------------------------------------------------------------------------
% Plot B - Optimal Design Plots
figure(3);
hold on;
plot(mccbyl, tbyc, 'b');
plot(micbyl, tbyc, 'g');
plot(cicbyl(5:end), tbyc(5:end), 'r');
custom_colormap = [0 1 0; 0 0 1; 1 0 0];
colormap(custom_colormap); % Apply the custom colormap
contourf(nx, ny, index_matrix, 'LineStyle', 'none');
% Plot the first part of optimal design curve
% plot([0, intersection_x1], [ymb, ymb], 'black', 'LineWidth', 2);
% Plot for both the lagrange multiplier calculated design trajectories
plot([0, 0.3], [ymb, ymb], 'yellow', 'LineWidth', 2);
plot([0, 0.3], [yei, yei], 'black', 'LineWidth', 2);
% Additional plot settings
xlabel('C/L');
ylabel('T/C');
title('Project 1 Plot B');
legend('Core Shear','Microbuckling', 'Elastic Indentation');
% -------------------------------------------------------------------------
% --------------------------------------------------------------------------
% Plot C - Mass Minima Plots
figure(4);
hold on;
plot(Pcap, Mcapmb, 'r', 'LineWidth',1);
plot(Pcap, Mcapei, 'b','LineWidth',1);
```

```
% Plot the curve segments
plot(Pcap1, Mcapei1, 'blue', 'LineWidth', 3); % Plot the first segment of
curve 1 in blue
hold on; % Keep the current figure active
plot(Pcap2, Mcapmb1, 'red', 'LineWidth', 3); % Plot the second segment of
curve 2 in red
% Additional plot settings
xlabel('Pcap');
ylabel('Mcap');
title('Project 1 Plot C');
legend('Microbuckling', 'Elastic Indentation');
% -------------------------------------------------------------------------
% --------------------------------------------------------------------------
% Part D - Example Beam Geometry Plot
figure(5);
hold on;
plot(cf, tf, 'r');
% Additional plot settings
xlabel('C(in m)');
ylabel('T(in m)');
title('Project 1 Plot D');
hold off;
```