Problem Set 2

Muthu Ram Kumar Avichi (1010188967)

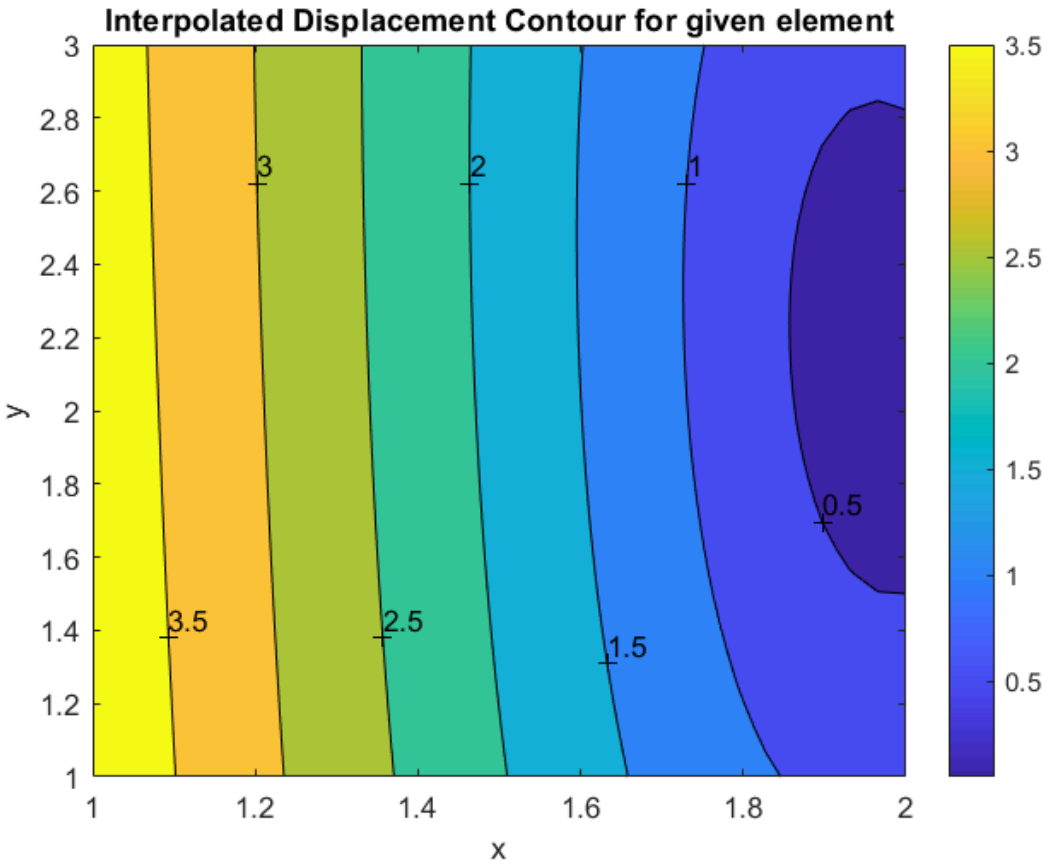
Question 2

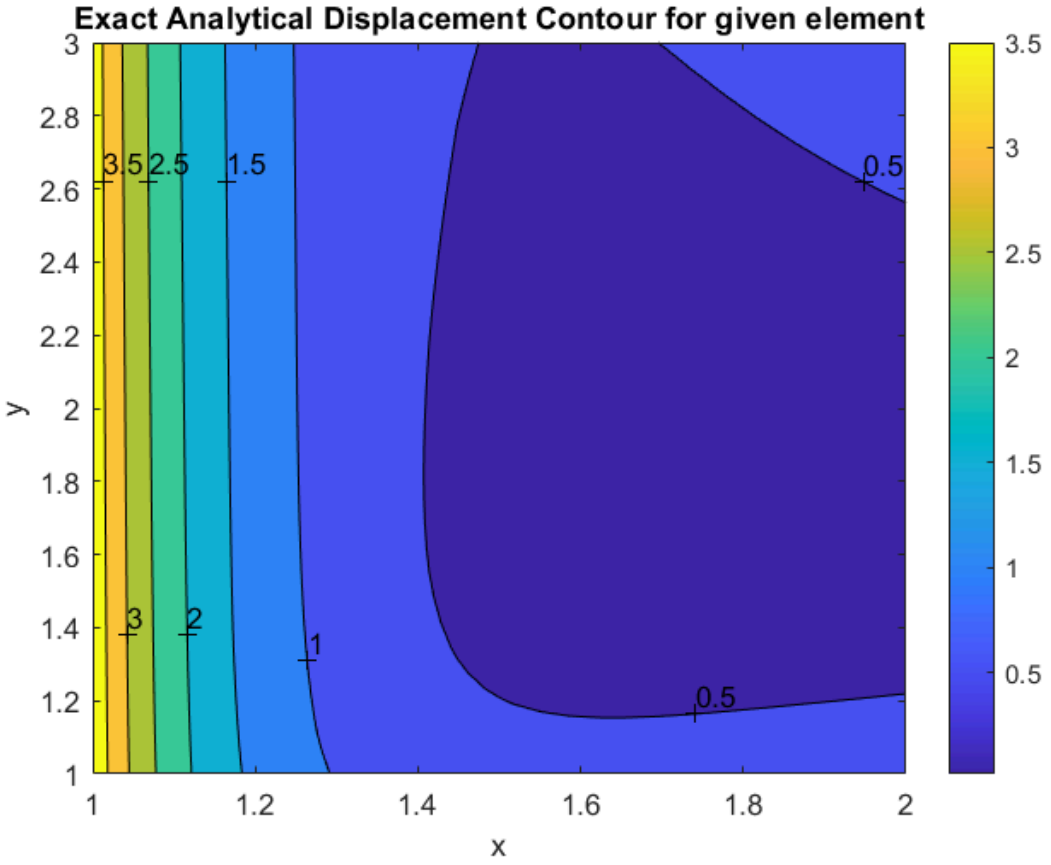
Matlab Code:

```
% ------------------------------------------------------------------------
% Matlab code for Problem Set 2 - Question 2
% ------------------------------------------------------------------------
% Coordinates of vertices of rectangle element declared.
x1 = 1;
y1 = 1;
x2 = 3;
y2 = 1;
x3 = 3;
y3 = 2;
x4 = 1;
y4 = 2;
% Variables declared to plot the rectangle element in the graph
lx = [x1 x2 x3 x4 x1];
ly = [y1 y2 y3 y4 y1];
% Displacements of the rectangle element declared.
syms xa ya
ux = (3/8)*xa - (2/3)*(xa/(ya^2));
uy = (4/(xa^3)) - (ya/8);
n = 4;                              % Number of nodes in the element.
dux = zeros(n, 1);
duy = zeros(n, 1);
for i = 1:4
   dux(i) = subs(ux, [xa, ya], [lx(i), ly(i)]);
   duy(i) = subs(uy, [xa, ya], [lx(i), ly(i)]);
end
% Preallocate C for efficiency
d = zeros(length(dux) + length(duy), 1);
% Interleave elements
d(1:2:end) = dux;  % Assign elements of A to odd indices of C
d(2:2:end) = duy;  % Assign elements of B to even indices of C
% Declare the variables to plot the contour.
jx = linspace(1, 3, 30);
jy = linspace(1, 2, 30);
[nx, ny] = meshgrid(jx, jy);
l = length(jx);
A = 2*1;
% Interpolated displacements variable initialised.
dx = zeros(l, l);
% Exact analytical displacements variable initialised.
adx = zeros(l, l);
% Nf matrix initialised from lecture equations
syms x y;
```

```
Nf = cell(4);
Nf{1} = (1/A)*(x - x2)*(y - y4);
Nf{2} = -(1/A)*(x - x1)*(y - y4);
Nf{3} = (1/A)*(x - x1)*(y - y1);
Nf{4} = -(1/A)*(x - x2)*(y - y1);
for i = 1:l
   for j = 1:l

       % Iteration to initialise Nf matrix -
       % substituted for each co-ordinate in the graph
       % Compute the interpolated linear displacement

       Nfc = zeros(4);
       for ck = 1:4
           Nfc(ck) = subs(Nf{ck}, [x, y], [jx(i), jy(j)]);
       end
       N = [Nfc(1) 0 Nfc(2) 0 Nfc(3) 0 Nfc(4) 0; 0 Nfc(1) 0 Nfc(2) 0 Nfc(3) 0
Nfc(4)];

       u{i, j} = N*d;

       dx(i,j) = sqrt( (u{i,j}(1))^2 + (u{i,j}(2))^2 );
       % Compute the exact analytical displacement
       adxi = subs(ux, [xa, ya], [jx(i), jy(j)]);
       adyi = subs(uy, [xa, ya], [jx(i), jy(j)]);
       adx(i,j) = sqrt( (adxi)^2 + (adyi)^2 );

   end
end
% ------------------------------------------------------------------------
% ------------------------------------------------------------------------
% Contours for interpolated displacement
figure(1);
contourf(ny, nx, dx);
clabel(contourf(ny, nx, dx));
colorbar;
hold on;
xlabel('x');
ylabel('y');
title('Interpolated Displacement Contour for given element');
% ------------------------------------------------------------------------
% Contours for exact displacement
figure(2);
contourf(ny, nx, adx);
clabel(contourf(ny, nx, adx));
colorbar;
hold on;
xlabel('x');
ylabel('y');
title('Exact Analytical Displacement Contour for given element');
```

Output:


Interpolated Displacement Contour for given element
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
y
1
1.2
1.4
1.6
1.8
2
x
3.5
3
2.5
2
1.5
1
0.5
3.5
3
2.5
2
1.5
1
0.5



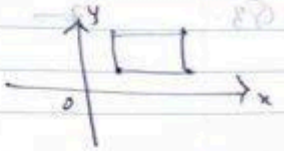
Theoretical Calculations

PS-2.

linear interpolation.

Q2

x_1	y_1	1	1
x_2	y_2	3	1
x_3	y_3	3	2
x_4	y_4	1	2



displacement $u = \frac{3x}{8} - \frac{2x}{3y^2}$ $\quad v = \frac{4}{x^3} - \frac{y}{8}$

$$\sqrt{u^2+v^2}$$

$$u = N(x,y)\,d^e$$

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}$$

where

$$N_1 = \frac{1}{A}(x-x_2)(y-y_4)$$

$$N_2 = \frac{-1}{A}(x-x_1)(y-y_4)$$

$$N_3 = \frac{1}{A}(x-x_1)(y-y_1)$$

$$N_4 = \frac{-1}{A}(x-x_2)(y-y_1)$$