Problem Set 1

Muthu Ram Kumar Avichi (1010188967)

Question 1 - Part a

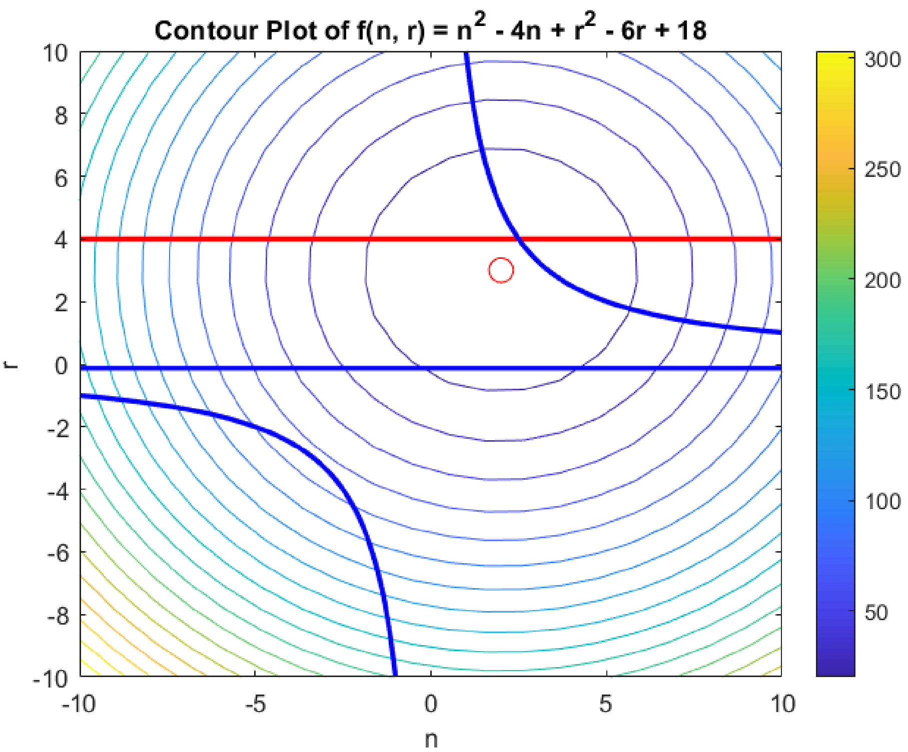
Matlab Code:

```
% Objective function
f = @(n, r) n.^2 - 4*n + r.^2 - 6*r + 18;
% Define curve limits.
max_n = 10;
min_n = -10;
max_r = 10;
min_r = -10;
% Meshgrid (n , r) and corresponding equations
nRange = linspace(min_n, max_n, 20);
rRange = linspace(min_r, max_r, 20);
[n, r] = meshgrid(nRange, rRange);
z = f(n, r);
% -------------------------------------------------------------------------
% -------------------------------------------------------------------------
% Defining the functions for calculating minima.
syms n2 r2;
fx = n2^2 - 4*n2 + r2^2 - 6*r2 + 18;
df_dn = diff(fx, n2);
df_dr = diff(fx, r2);
% Calculate the minimum of n and r by equation the first derivative to zero
mini_n = solve(df_dn == 0, n2);
mini_r = solve(df_dr == 0, r2);
% Verify if the variables solve the constraint equations
count = 0;
if mini_r > 4
   count = count+1;
end
if mini_n > 10/mini_r
   count = count + 1;
end
if count == 0
   % Since count is zero, both the constraints are inactive.
   disp("Both constraints are inactive")
   fminima = subs(fx, [n2, r2], [mini_n, mini_r]);

   disp("Minimum value of f");
   disp(fminima);
   disp("At n and r values:");
   disp(mini_n);
   disp(mini_r);
end
% -------------------------------------------------------------------------
% -------------------------------------------------------------------------
```

```
% Plot contours
figure;
contour(n, r, z, 20);
hold on;
% Plot the curves r - 4 = 0 and n = 10/r
fimplicit(@(n, r) r - 4, 'r-', 'LineWidth', 2);  % Curve r - 4 = 0
fimplicit(@(n, r) n - 10./r, 'b-', 'LineWidth', 2);  % Curve n = 10/r
plot(mini_n, mini_r, 'ro', 'MarkerSize', 10);
xlabel('n');
ylabel('r');
title('Contour Plot of f(n, r) = n^2 - 4n + r^2 - 6r + 18');
colorbar;
hold off;
```

Output:




```
>> q1a
Both constraints are inactive
Minimum value of f
5

At n and r values:
2

3
```

Explanation:

- To find the minima of the given function, the gradient of the function is evaluated and equated to zero.
- Both the given constraints are inactive for this condition since the minima lies at n = 2, r = 3 which falls within the constraint domain. When we substitute in the constraint equation, they satisfy the given condition and hence doesn't impact the solution.

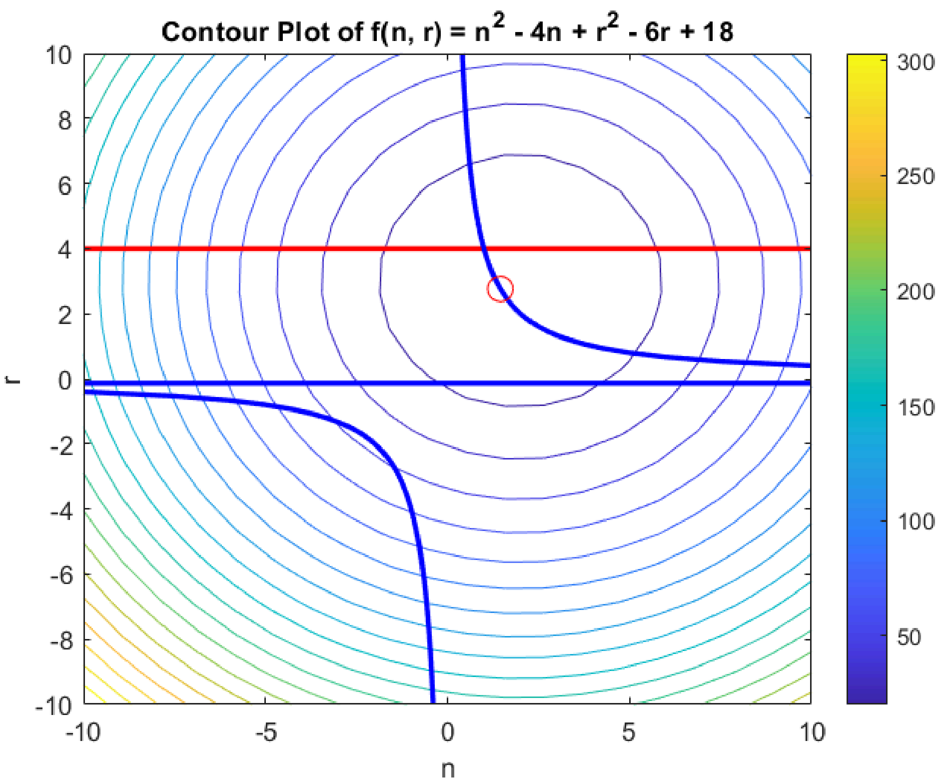
Question 1 - Part b

Matlab Code:

```
% Define the objective function
f = @(n, r) n.^2 - 4*n + r.^2 - 6*r + 18;
% Defining curve limits.
max_n = 10;
min_n = -10;
max_r = 10;
min_r = -10;
% Define the range for n and r variables
nRange = linspace(min_n, max_n, 20);
rRange = linspace(min_r, max_r, 20);
% Create a meshgrid for n and r
[n, r] = meshgrid(nRange, rRange);
% Evaluate the objective function at each point in the meshgrid
z = f(n, r);
% -----------------------------------------------------------------------
% ------------------------------------------------------------------------
% Defining the functions for performing differentiation operation
syms n2 r2 lambda;
fx = n2^2 - 4*n2 + r2^2 - 6*r2 + 18;
% Calculate the partial derivatives with respect to n and r
df_dn = diff(fx, n2);
df_dr = diff(fx, r2);
mini_n = solve(df_dn == 0, n2);
mini_r = solve(df_dr == 0, r2);
count = 0;
% Verify if the variables solve the constraint equations
if mini_r > 4
   count = count+1;
end
if mini_n > 4/mini_r
   count = count + 1;
end
% Calculations shown in notes.
calculated_n = 1.47;
calculated_r = 2.72;
if count > 0
   % Shown in calculation as well.
   disp("Constraint where r - 4/n <=0 are inactive")
```

```
    fminima = subs(fx, [n2, r2], [calculated_n, calculated_r]);
    disp("Minimum value of f");
    disp(fminima);
    disp("At n and r values:");
    disp(calculated_n);
    disp(calculated_r);
end
% -----------------------------------------------------------------------
% ------------------------------------------------------------------------
% Plot contours
figure;
contour(n, r, z, 20);            % 20 contour lines are plotted
hold on;
% Plotting the constraints
fimplicit(@(n, r) r - 4, 'r-', 'LineWidth', 2);  % Curve r - 4 = 0
fimplicit(@(n, r) n - 4./r, 'b-', 'LineWidth', 2);  % Curve n = 10/r
plot(1.47, 2.72, 'ro', 'MarkerSize', 10);
xlabel('n');
ylabel('r');
title('Contour Plot of f(n, r) = n^2 - 4n + r^2 - 6r + 18');
colorbar;  % Display colorbar
hold off;
```

Output:



```
>> q1b
Constraint where r - 4/n <=0 are inactive
Minimum value of f
53593/10000

At n and r values:
    1.4700

    2.7200
```

Explanation:

- The calculated minima does not fall within the given constraint domain, hence the constraint r - 4/n <= 0 is active.
- The minima is calculated using Lagrange method and respective equations evaluated in Matlab. In this case the constraint function is simplified to 4 - 4/n <= 0 to increase ease of calculation.
- The value of the minima function is 6, at n = 1, r = 3.
- Hence, the constraint r <= 4 is inactive but the constraint r - 4/n <=0 is active.

Theoretical Calculations

Given function:

$$f(n,r) = n^2 - 4n + r^2 - 6r + 18$$

active constraint:

$$c(n,r) \; : \quad r - \frac{4}{n} \le 0$$

check if active.

substitute global minima to verify:

$$n = 2 \qquad r = 3$$

$$3 - \frac{4}{2} \le 0$$

$$1 \le 0$$

✗ this condition is not satisfied, hence

✓ the constraint is active.

inactive constraint:

$$c(n,r) \; : \quad r - 4 \le 0$$

$$3 - 4 \le 0$$

$$-1 \le 0$$

✓ this condition is ~~active~~ satisfied

✓ hence the condition is inactive.

for the lagrange solution, we substitute only the active constraint in the form of

$$L = f + \lambda \times c$$

$$L = n^2 - 4n + r^2 - 6r + 18 + \lambda\left(r - \frac{4}{n}\right)$$

now we find the partial derivates separately & equation to zero.

$$\frac{\partial L}{\partial n} = 0 \Rightarrow 2n - 4 + \lambda\left(\frac{4}{n^2}\right) = 0 \quad \text{--- } ①$$

$$\frac{\partial L}{\partial r} = 0 \Rightarrow 2r - 6 + \lambda = 0 \quad \text{--- } ②$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow r - \frac{4}{n} = 0 \quad \text{--- } ③$$

from ③

$$r = \frac{4}{n}$$

sub. in ②

$$2\left(\frac{4}{n}\right) - 6 + \lambda = 0$$

$$\lambda = 2\left(3 - \frac{4}{n}\right)$$

sub in ①

$$2(n-2) + 2\left(3 - \frac{4}{n}\right)\left(-\frac{4}{n^2}\right) = 0$$

$$n - 2 + \frac{12}{n^2} - \frac{16}{n^3} = 0$$

$$n^4 - 2n^3 + 12n - 16 = 0$$

by solving this equation approximately, we get

$$\boxed{n \approx 1.47}$$

$$r = \frac{4}{n}$$

$$\boxed{r = 2.72}$$

solving the Lagrangian, this is the solution that we get!