1. **(D)** Since Mike tipped \$2, which was $10\% = 1/10$ of his bill, his bill must have been $2 \cdot 10 = 20$ dollars. Similarly, Joe tipped \$2, which was $20\% = 1/5$ of his bill, so his bill must have been $2 \cdot 5 = 10$ dollars. The difference between their bills is therefore \$10.

2. **(C)** First we have
$$(1 \star 2) = \frac{1+2}{1-2} = -3.$$
Then
$$((1 \star 2) \star 3) = (-3 \star 3) = \frac{-3+3}{-3-3} = \frac{0}{-6} = 0.$$

3. **(B)** Since $2x + 7 = 3$ we have $x = -2$. Hence
$$-2 = bx - 10 = -2b - 10, \quad \text{so} \quad 2b = -8, \text{ and } b = -4.$$

4. **(B)** Let w be the width of the rectangle. Then the length is $2w$, and
$$x^2 = w^2 + (2w)^2 = 5w^2.$$
The area is consequently $w(2w) = 2w^2 = \frac{2}{5}x^2$.

5. **(A)** If Dave buys seven windows separately he will purchase six and receive one free, for a cost of \$600. If Doug buys eight windows separately, he will purchase seven and receive one free, for a total cost of \$700. The total cost to Dave and Doug purchasing separately will be \$1300. If they purchase fifteen windows together, they will need to purchase only 12 windows, for a cost of \$1200, and will receive 3 free. This will result in a savings of \$100.

6. **(B)** The sum of the 50 numbers is $20 \cdot 30 + 30 \cdot 20 = 1200$. Their average is $1200/50 = 24$.

7. **(B)** Because (rate)(time) = (distance), the distance Josh rode was $(4/5)(2) = 8/5$ of the distance that Mike rode. Let m be the number of miles that Mike had ridden when they met. Then the number of miles between their houses is
$$13 = m + \frac{8}{5}m = \frac{13}{5}m.$$
Thus $m = 5$.

8. **(C)** The symmetry of the figure implies that $\triangle ABH$, $\triangle BCE$, $\triangle CDF$, and $\triangle DAG$ are congruent right triangles. So
$$BH = CE = \sqrt{BC^2 - BE^2} = \sqrt{50 - 1} = 7,$$
and $EH = BH - BE = 7 - 1 = 6$. Hence the square $EFGH$ has area $6^2 = 36$.

OR

As in the first solution, $BH = 7$. Now note that $\triangle ABH$, $\triangle BCE$, $\triangle CDF$, and $\triangle DAG$ are congruent right triangles, so
$$\text{Area}(EFGH) = \text{Area}(ABCD) - 4\text{Area}(\triangle ABH) = 50 - 4\left(\tfrac{1}{2} \cdot 1 \cdot 7\right) = 36.$$

9. **(B)** There are three X's and two O's, and the tiles are selected without replacement, so the probability is

$$\frac{3}{5}\cdot\frac{2}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{1}=\frac{1}{10}.$$

OR

The three tiles marked X are equally likely to lie in any of $\binom{5}{3}=10$ positions, so the probability of this arrangement is $1/10$.

10. **(A)** The quadratic formula yields

$$x=\frac{-(a+8)\pm\sqrt{(a+8)^2-4\cdot 4\cdot 9}}{2\cdot 4}.$$

The equation has only one solution precisely when the value of the discriminant, $(a+8)^2-144$, is 0. This implies that $a=-20$ or $a=4$, and the sum is -16.

OR

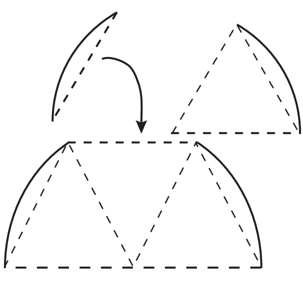
The equation has one solution if and only if the polynomial is the square of a binomial with linear term $\pm\sqrt{4x^2}=\pm 2x$ and constant term $\pm\sqrt{9}=\pm 3$. Because $(2x\pm 3)^2$ has a linear term $\pm 12x$, it follows that $a+8=\pm 12$. Thus a is either -20 or 4, and the sum of those values is -16.

11. **(B)** The unit cubes have a total of $6n^3$ faces, of which $6n^2$ are red. Therefore

$$\frac{1}{4}=\frac{6n^2}{6n^3}=\frac{1}{n},\quad \text{so}\quad n=4.$$

12. **(B)** The trefoil is constructed of four equilateral triangles and four circular segments, as shown. These can be combined to form four 60° circular sectors. Since the radius of the circle is 1, the area of the trefoil is

$$\frac{4}{6}\left(\pi\cdot 1^2\right)=\frac{2}{3}\pi.$$



13. **(E)** The condition is equivalent to

$$130n>n^2>2^4=16,\quad \text{so}\quad 130n>n^2\ \ \text{and}\ \ n^2>16.$$

This implies that $130>n>4$. So n can be any of the 125 integers strictly between 130 and 4.

14. **(E)** The first and last digits must be both odd or both even for their average to be an integer. There are $5 \cdot 5 = 25$ odd-odd combinations for the first and last digits. There are $4 \cdot 5 = 20$ even-even combinations that do not use zero as the first digit. Hence the total is 45.

15. **(E)** Written as a product of primes, we have

$$3! \cdot 5! \cdot 7! = 2^8 \cdot 3^4 \cdot 5^2 \cdot 7.$$

A cube that is a factor has a prime factorization of the form $2^p \cdot 3^q \cdot 5^r \cdot 7^s$, where p, q, r, and s are all multiples of 3. There are 3 possible values for p, which are 0, 3, and 6. There are 2 possible values for q, which are 0 and 3. The only value for r and for s is 0. Hence there are $6 = 3 \cdot 2 \cdot 1 \cdot 1$ distinct cubes that divide $3! \cdot 5! \cdot 7!$. They are

$$\begin{aligned} 1 &= 2^0 3^0 5^0 7^0, \quad 8 = 2^3 3^0 5^0 7^0, \quad 27 = 2^0 3^3 5^0 7^0, \\ 64 &= 2^6 3^0 5^0 7^0, \quad 216 = 2^3 3^3 5^0 7^0, \quad \text{and} \quad 1728 = 2^6 3^3 5^0 7^0. \end{aligned}$$

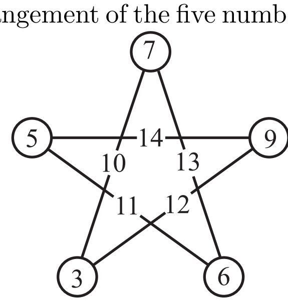
16. **(D)** Let $10a + b$ be the two-digit number. When $a + b$ is subtracted the result is $9a$. The only two-digit multiple of 9 that ends in 6 is $9 \cdot 4 = 36$, so $a = 4$. The ten numbers between 40 and 49, inclusive, have this property.

17. **(D)** Each number appears in two sums, so the sum of the sequence is

$$2(3 + 5 + 6 + 7 + 9) = 60.$$

The middle term of a five-term arithmetic sequence is the mean of its terms, so $60/5 = 12$ is the middle term.

The figure shows an arrangement of the five numbers that meets the requirement.



18. **(A)**

There are four possible outcomes,

$$\text{ABAA}, \quad \text{ABABA}, \quad \text{ABBAA}, \quad \text{and} \quad \text{BBAAA},$$

but they are not equally likely. This is because, in general, the probability of any specific four-game series is $(1/2)^4 = 1/16$, whereas the probability of any specific five-game series is $(1/2)^4 = 1/32$. Thus the first listed outcome is twice as likely as each of the other three. Let p be the probability of the occurrence $ABBAA$. Then the probability of $ABABA$ is also p, as is the probability of $BBAAA$, whereas the probability of $ABAA$ is $2p$. So

$$2p + p + p + p = 1, \quad \text{and} \quad p = \frac{1}{5}.$$

The only outcome in which team B wins the first game is $BBAAA$, so the probability of this occurring is $1/5$.

OR

To consider equally-likely cases, suppose that all five games are played, even if team A has won the series before the fifth game. Then the possible ways that team A can win the series, given that team B wins the second game, are

BBAAA, ABBAA, ABABA, ABAAB, and ABAAA.

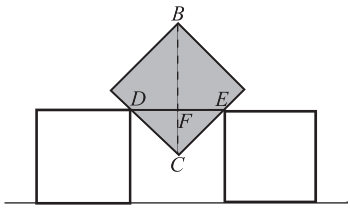
In only the first case does team B win the first game, so the probability of this occurring is $1/5$.

19. **(D)** Consider the rotated middle square shown in the figure. It will drop until length DE is 1 inch. Thus

$$FC = DF = FE = \frac{1}{2} \quad \text{and} \quad BC = \sqrt{2}.$$

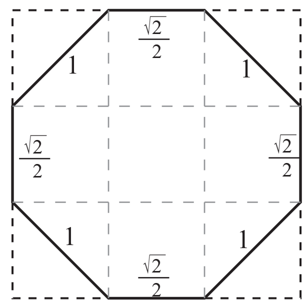
Hence $BF = \sqrt{2} - 1/2$. This is added to the 1 inch height of the supporting squares, so the overall height of point B above the line is

$$1 + BF = \sqrt{2} + \frac{1}{2} \text{ inches.}$$



20. **(A)** The octagon can be partitioned into five squares and four half squares, each with side length $\sqrt{2}/2$, so its area is

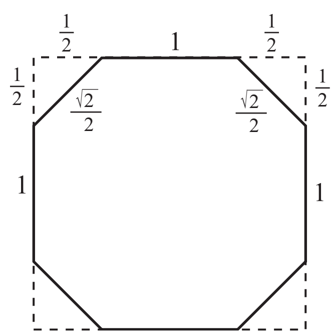
$$\left(5+4\cdot\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)^2=\frac{7}{2}.$$



OR

The octagon can be obtained by removing four isosceles right triangles with legs of length $1/2$ from a square with sides of length 2. Thus its area is

$$2^2-4\cdot\frac{1}{2}\left(\frac{1}{2}\right)^2=\frac{7}{2}.$$



21. **(B)** Because

$$1+2+\cdots+n=\frac{n(n+1)}{2},$$

$1+2+\cdots+n$ divides the positive integer $6n$ if and only if

$$\frac{6n}{n(n+1)/2}=\frac{12}{n+1}\text{ is an integer.}$$

There are five such positive values of n, namely, 1, 2, 3, 5, and 11.

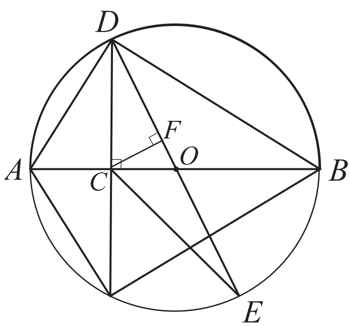
22. **(D)** The sets S and T consist, respectively, of the positive multiples of 4 that do not exceed $2005 \cdot 4 = 8020$ and the positive multiples of 6 that do not exceed $2005 \cdot 6 = 12{,}030$. Thus $S \bigcap T$, the set of numbers that are common to S and to T, consists of the positive multiples of 12 that do not exceed 8020. Let $\lfloor x \rfloor$ represent the largest integer that is less than or equal to x. Then the number of elements in the set $S \bigcap T$ is

$$\left\lfloor \frac{8020}{12} \right\rfloor = \left\lfloor 668 + \frac{1}{3} \right\rfloor = 668.$$

23. **(C)** Let O be the center of the circle. Each of $\triangle DCE$ and $\triangle ABD$ has a diameter of the circle as a side. Thus the ratio of their areas is the ratio of the two altitudes to the diameters. These altitudes are $\overline{DC}$ and the altitude from C to $\overline{DO}$ in $\triangle DCE$. Let F be the foot of this second altitude. Since $\triangle CFO$ is similar to $\triangle DCO$,

$$\frac{CF}{DC} = \frac{CO}{DO} = \frac{AO - AC}{DO} = \frac{\frac{1}{2}AB - \frac{1}{3}AB}{\frac{1}{2}AB} = \frac{1}{3},$$

which is the desired ratio.



OR

Because $AC = AB/3$ and $AO = AB/2$, we have $CO = AB/6$. Triangles DCO and DAB have a common altitude to $\overline{AB}$ so the area of $\triangle DCO$ is $\frac{1}{6}$ the area of $\triangle ADB$. Triangles DCO and ECO have equal areas since they have a common base $\overline{CO}$ and their altitudes are equal. Thus the ratio of the area of $\triangle DCE$ to the area of $\triangle ABD$ is $1/3$.

24. **(B)** The conditions imply that both n and $n+48$ are squares of primes. So for each successful value of n we have primes p and q with $p^2 = n+48$ and $q^2 = n$, and

$$48 = p^2 - q^2 = (p+q)(p-q).$$

The pairs of factors of 48 are

$$48 \text{ and } 1, \quad 24 \text{ and } 2, \quad 16 \text{ and } 3, \quad 12 \text{ and } 4, \quad \text{and} \quad 8 \text{ and } 6.$$

These give pairs (p,q), respectively, of

$$\left(\frac{49}{2}, \frac{47}{2}\right), \quad (13,11), \quad \left(\frac{19}{2}, \frac{13}{2}\right), \ (8,4), \quad \text{and} \quad (7,1).$$

Only $(p,q) = (13,11)$ gives prime values for p and for q, with $n = 11^2 = 121$ and $n+48 = 13^2 = 169$.

25. **(D)** We have

$$\frac{\text{Area}(ADE)}{\text{Area}(ABE)} = \frac{AD}{AB} = \frac{19}{25} \quad \text{and} \quad \frac{\text{Area}(ABE)}{\text{Area}(ABC)} = \frac{AE}{AC} = \frac{14}{42} = \frac{1}{3},$$

so

$$\frac{\text{Area}(ABC)}{\text{Area}(ADE)} = \frac{25}{19} \cdot \frac{3}{1} = \frac{75}{19},$$

and

$$\frac{\text{Area}(BCED)}{\text{Area}(ADE)} = \frac{\text{Area}(ABC) - \text{Area}(ADE)}{\text{Area}(ADE)} = \frac{75}{19} - 1 = \frac{56}{19}.$$

Thus

$$\frac{\text{Area}(ADE)}{\text{Area}(BCED)} = \frac{19}{56}.$$