2002 AMC 10B Problems

Contents

Problem 1

The ratio $\frac{2^{2001}\cdot 3^{2003}}{6^{2002}}$ is:

$\mathrm{(A)}\ \frac{1}{6} \qquad \mathrm{(B)}\ \frac{1}{3} \qquad \mathrm{(C)}\ \frac{1}{2} \qquad \mathrm{(D)}\ \frac{2}{3} \qquad \mathrm{(E)}\ \frac{3}{2}$

Solution

Problem 2

For the nonzero numbers a, b, and c, define

$$(a,b,c)=\frac{abc}{a+b+c}$$

Find $(2,4,6)$.

$\mathrm{(A)}\ 1 \qquad \mathrm{(B)}\ 2 \qquad \mathrm{(C)}\ 4 \qquad \mathrm{(D)}\ 6 \qquad \mathrm{(E)}\ 24$

Solution

Problem 3

The arithmetic mean of the nine numbers in the set $\{9, 99, 999, 9999, \ldots, 999999999\}$ is a 9-digit number M, all of whose digits are distinct. The number M does not contain the digit

$\mathrm{(A)}\ 0 \qquad \mathrm{(B)}\ 2 \qquad \mathrm{(C)}\ 4 \qquad \mathrm{(D)}\ 6 \qquad \mathrm{(E)}\ 8$

Solution

Problem 4

What is the value of

$$(3x-2)(4x+1)-(3x-2)4x+1$$

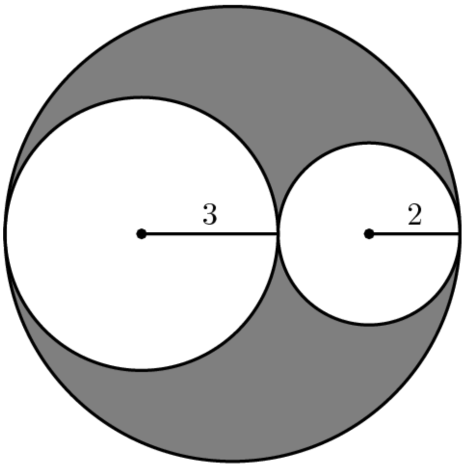
when $x=4$?

$\mathrm{(A)}\ 0 \qquad \mathrm{(B)}\ 1 \qquad \mathrm{(C)}\ 10 \qquad \mathrm{(D)}\ 11 \qquad \mathrm{(E)}\ 12$

Solution

Problem 5

Circles of radius 2 and 3 are externally tangent and are circumscribed by a third circle, as shown in the figure. Find the area of the shaded region.



$\mathrm{(A)}\ 3\pi \qquad \mathrm{(B)}\ 4\pi \qquad \mathrm{(C)}\ 6\pi \qquad \mathrm{(D)}\ 9\pi \qquad \mathrm{(E)}\ 12\pi$

Solution

Problem 6

For how many positive integers n is n^2-3n+2 a prime number?

$\mathrm{(A)}\ \text{none} \qquad \mathrm{(B)}\ \text{one} \qquad \mathrm{(C)}\ \text{two} \qquad \mathrm{(D)}\ \text{more than two, but finitely many} \qquad \mathrm{(E)}\ \text{infinitely many}$

Solution

Problem 7

Let n be a positive integer such that $\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{n}$ is an integer. Which of the following statements is **not** true?

$\mathrm{(A)}\ 2 \text{ divides } n \qquad \mathrm{(B)}\ 3 \text{ divides } n \qquad \mathrm{(C)}\ 6 \text{ divides } n \qquad \mathrm{(D)}\ 7 \text{ divides } n \qquad \mathrm{(E)}\ n>84$

Solution

Problem 8

Suppose July of year N has five Mondays. Which of the following must occur five times in the August of year N? (Note: Both months have 31 days.)

$\textbf{(A)}\ \text{Monday} \qquad \textbf{(B)}\ \text{Tuesday} \qquad \textbf{(C)}\ \text{Wednesday} \qquad \textbf{(D)}\ \text{Thursday} \qquad \textbf{(E)}\ \text{Friday}$

Solution

Problem 9

Using the letters A, M, O, S, and U, we can form five-letter "words". If these "words" are arranged in alphabetical order, then the "word" $USAMO$ occupies position

$\text{(A) } 112 \qquad \text{(B) } 113 \qquad \text{(C) } 114 \qquad \text{(D) } 115 \qquad \text{(E) } 116$

Solution

Problem 10

Suppose that a and b are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions a and b. Then the pair (a, b) is

$\text{(A) } (-2,1) \qquad \text{(B) } (-1,2) \qquad \text{(C) } (1,-2) \qquad \text{(D) } (2,-1) \qquad \text{(E) } (4,4)$

Solution

Problem 11

The product of three consecutive positive integers is 8 times their sum. What is the sum of the squares?

$\text{(A) } 50 \qquad \text{(B) } 77 \qquad \text{(C) } 110 \qquad \text{(D) } 149 \qquad \text{(E) } 194$

Solution

Problem 12

For which of the following values of k does the equation $\frac{x-1}{x-2} = \frac{x-k}{x-6}$ have no solution for x?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Solution

Problem 13

Find the value(s) of x such that $8xy - 12y + 2x - 3 = 0$ is true for all values of y.

$\textbf{(A) } \frac{2}{3} \qquad \textbf{(B) } \frac{3}{2} \text{ or } -\frac{1}{4} \qquad \textbf{(C) } -\frac{2}{3} \text{ or } -\frac{1}{4} \qquad \textbf{(D) } \frac{3}{2} \qquad \textbf{(E) } -\frac{3}{2} \text{ or } -\frac{1}{4}$

Solution

Problem 14

The number $25^{64} \cdot 64^{25}$ is the square of a positive integer N. In decimal representation, the sum of the digits of N is

$\text{(A) } 7 \qquad \text{(B) } 14 \qquad \text{(C) } 21 \qquad \text{(D) } 28 \qquad \text{(E) } 35$

Solution

Problem 15

The positive integers A, B, $A-B$, and $A+B$ are all prime numbers. The sum of these four primes is

$\text{(A) even} \qquad \text{(B) divisible by } 3 \qquad \text{(C) divisible by } 5 \qquad \text{(D) divisible by } 7 \qquad \text{(E) prime}$

Solution

Problem 16

For how many integers n is $\frac{n}{20-n}$ the square of an integer?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 10$

Solution

Problem 17

A regular octagon $ABCDEFGH$ has sides of length two. Find the area of $\triangle ADG$.

$\textbf{(A) } 4+2\sqrt{2} \qquad \textbf{(B) } 6+\sqrt{2} \qquad \textbf{(C) } 4+3\sqrt{2} \qquad \textbf{(D) } 3+4\sqrt{2} \qquad \textbf{(E) } 8+\sqrt{2}$

Solution

Problem 18

Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

$\textbf{(A) } 8 \qquad \textbf{(B) } 9 \qquad \textbf{(C) } 10 \qquad \textbf{(D) } 12 \qquad \textbf{(E) } 16$

Solution

Problem 19

Suppose that $\{a_n\}$ is an arithmetic sequence with

$$a_1+a_2+\cdots+a_{100}=100 \text{ and } a_{101}+a_{102}+\cdots+a_{200}=200.$$

What is the value of $a_2 - a_1$?

$\mathrm{(A) \ } 0.0001 \qquad \mathrm{(B) \ } 0.001 \qquad \mathrm{(C) \ } 0.01 \qquad \mathrm{(D) \ } 0.1 \qquad \mathrm{(E) \ } 1$

Solution

Problem 20

Let $a, b,$ and c be real numbers such that $a-7b+8c = 4$ and $8a+4b-c = 7$. Then $a^2 - b^2 + c^2$ is

$\mathrm{(A) \ } 0 \qquad \mathrm{(B) \ } 1 \qquad \mathrm{(C) \ } 4 \qquad \mathrm{(D) \ } 7 \qquad \mathrm{(E) \ } 8$

Solution

Problem 21

Andy's lawn has twice as much area as Beth's lawn and three times as much as Carlos' lawn. Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as Andy's mower. If they all start to mow their lawns at the same time, who will finish first?

$\mathrm{(A) \ } \text{Andy} \qquad \mathrm{(B) \ } \text{Beth} \qquad \mathrm{(C) \ } \text{Carlos} \qquad \mathrm{(D) \ } \text{Andy and Carlos tie for first.} \qquad \mathrm{(E) \ } \text{All three tie.}$

Solution

Problem 22

Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90^{\circ}$. Let M and N be the midpoints of the legs OX and OY, respectively. Given $XN = 19$ and $YM = 22$, find XY.

$\mathrm{(A) \ } 24 \qquad \mathrm{(B) \ } 26 \qquad \mathrm{(C) \ } 28 \qquad \mathrm{(D) \ } 30 \qquad \mathrm{(E) \ } 32$

Solution

Problem 23

Let $\{a_k\}$ be a sequence of integers such that $a_1 = 1$ and $a_{m+n} = a_m + a_n + mn$, for all positive integers m and n. Then a_{12} is

$\mathrm{(A) \ } 45 \qquad \mathrm{(B) \ } 56 \qquad \mathrm{(C) \ } 67 \qquad \mathrm{(D) \ } 78 \qquad \mathrm{(E) \ } 89$

Solution

Problem 24

Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 feet and revolves at the constant rate of one revolution per minute. How many seconds does it take a rider to travel from the bottom of the wheel to a point 10 vertical feet above the bottom?

$\mathrm{(A) \ } 5 \qquad \mathrm{(B) \ } 6 \qquad \mathrm{(C) \ } 7.5 \qquad \mathrm{(D) \ } 10 \qquad \mathrm{(E) \ } 15$

Solution

Problem 25

When 15 is appended to a list of integers, the mean is increased by 2. When 1 is appended to the enlarged list, the mean of the enlarged list is decreased by 1. How many integers were in the original list?

$\mathrm{(A) \ } 4\qquad \mathrm{(B) \ } 5\qquad \mathrm{(C) \ } 6\qquad \mathrm{(D) \ } 7\qquad \mathrm{(E) \ } 8$

Solution

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