2004 AMC 10A Problems

Contents

Problem 1

You and five friends need to raise 1500 dollars in donations for a charity, dividing the fundraising equally. How many dollars will each of you need to raise?

$\mathrm{(A) \ } 250\qquad \mathrm{(B) \ } 300 \qquad \mathrm{(C) \ } 1500 \qquad \mathrm{(D) \ } 7500 \qquad \mathrm{(E) \ } 9000$

Solution

Problem 2

For any three real numbers a, b, and c, with $b\neq c$, the operation $\otimes$ is defined by: $\otimes(a,b,c)=\frac{a}{b-c}$. What is $\otimes(\otimes(1,2,3),\otimes(2,3,1),\otimes(3,1,2))$?

$\mathrm{(A) \ } -\frac{1}{2}\qquad \mathrm{(B) \ } -\frac{1}{4} \qquad \mathrm{(C) \ } 0 \qquad \mathrm{(D) \ } \frac{1}{4} \qquad \mathrm{(E) \ } \frac{1}{2}$

Solution

Problem 3

Alicia earns 20 dollars per hour, of which 1.45% is deducted to pay local taxes. How many cents per hour of Alicia's wages are used to pay local taxes?

$\mathrm{(A) \ } 0.0029\qquad \mathrm{(B) \ } 0.029 \qquad \mathrm{(C) \ } 0.29 \qquad \mathrm{(D) \ } 2.9 \qquad \mathrm{(E) \ } 29$

Solution

Problem 4

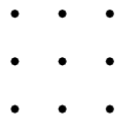
What is the value of x if $|x-1|=|x-2|$?

$(\mathrm{A})\ -\frac{1}{2} \qquad (\mathrm{B})\ \frac{1}{2} \qquad (\mathrm{C})\ 1 \qquad (\mathrm{D})\ \frac{3}{2} \qquad (\mathrm{E})\ 2$

Solution

Problem 5

A set of three points is randomly chosen from the grid shown. Each three point set has the same probability of being chosen. What is the probability that the points lie on the same straight line?



$(\mathrm{A})\ \frac{1}{21} \qquad (\mathrm{B})\ \frac{1}{14} \qquad (\mathrm{C})\ \frac{2}{21} \qquad (\mathrm{D})\ \frac{1}{7} \qquad (\mathrm{E})\ \frac{2}{7}$

Solution

Problem 6

Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and granddaughters have no daughters?

$(\mathrm{A})\ 22 \qquad (\mathrm{B})\ 23 \qquad (\mathrm{C})\ 24 \qquad (\mathrm{D})\ 25 \qquad (\mathrm{E})\ 26$

Solution

Problem 7

A grocer stacks oranges in a pyramid-like stack whose rectangular base is 5 oranges by 8 oranges. Each orange above the first level rests in a pocket formed by four oranges below. The stack is completed by a single row of oranges. How many oranges are in the stack?

$(\mathrm{A})\ 96 \qquad (\mathrm{B})\ 98 \qquad (\mathrm{C})\ 100 \qquad (\mathrm{D})\ 101 \qquad (\mathrm{E})\ 134$

Solution

Problem 8

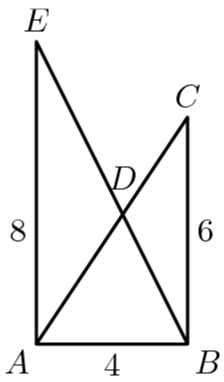
A game is played with tokens according to the following rule. In each round, the player with the most tokens gives one token to each of the other players and also places one token in the discard pile. The game ends when some player runs out of tokens. Players A, B, and C start with 15, 14, and 13 tokens, respectively. How many rounds will there be in the game?

$(\mathrm{A})\ 36 \qquad (\mathrm{B})\ 37 \qquad (\mathrm{C})\ 38 \qquad (\mathrm{D})\ 39 \qquad (\mathrm{E})\ 40$

Solution

Problem 9

In the figure, $\angle EAB$ and $\angle ABC$ are right angles. $AB=4, BC=6, AE=8$, and AC and BE intersect at D. What is the difference between the areas of $\triangle ADE$ and $\triangle BDC$?



$\text{(A) } 2 \qquad \text{(B) } 4 \qquad \text{(C) } 5 \qquad \text{(D) } 8 \qquad \text{(E) } 9$

Solution

Problem 10

Coin A is flipped three times and coin B is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?

$\text{(A) } \frac{29}{128} \qquad \text{(B) } \frac{23}{128} \qquad \text{(C) } \frac{1}{4} \qquad \text{(D) } \frac{35}{128} \qquad \text{(E) } \frac{1}{2}$

Solution

Problem 11

A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

$\text{(A) } 10 \qquad \text{(B) } 25 \qquad \text{(C) } 36 \qquad \text{(D) } 50 \qquad \text{(E) } 60$

Solution

Problem 12

Henry's Hamburger Heaven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties, and any collection of condiments. How many different kinds of hamburgers can be ordered?

$\text{(A) } 24 \qquad \text{(B) } 256 \qquad \text{(C) } 768 \qquad \text{(D) } 40,320 \qquad \text{(E) } 120,960$

Solution

Problem 13

At a party, each man danced with exactly three women and each woman danced with exactly two men. Twelve men attended the party. How many women attended the party?

$\text{(A) } 8 \qquad \text{(B) } 12 \qquad \text{(C) } 16 \qquad \text{(D) } 18 \qquad \text{(E) } 24$

Solution

Problem 14

The average value of all the pennies, nickels, dimes, and quarters in Paula's purse is 20 cents. If she had one more quarter, the average would be 21 cents. How many dimes does she have in her purse?

$\text{(A) } 0 \qquad \text{(B) } 1 \qquad \text{(C) } 2 \qquad \text{(D) } 3 \qquad \text{(E) } 4$

Solution

Problem 15

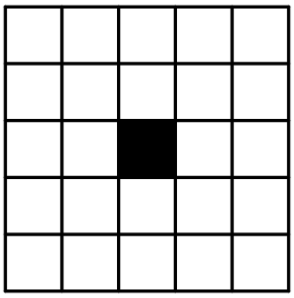
Given that $-4 \leq x \leq -2$ and $2 \leq y \leq 4$, what is the largest possible value of $\frac{x+y}{x}$?

$\mathrm{(A)}\ -1 \qquad \mathrm{(B)}\ -\frac{1}{2} \qquad \mathrm{(C)}\ 0 \qquad \mathrm{(D)}\ \frac{1}{2} \qquad \mathrm{(E)}\ 1$

Solution

Problem 16

The 5×5 grid shown contains a collection of squares with sizes from 1×1 to 5×5. How many of these squares contain the black center square?



$\mathrm{(A)}\ 12 \qquad \mathrm{(B)}\ 15 \qquad \mathrm{(C)}\ 17 \qquad \mathrm{(D)}\ 19 \qquad \mathrm{(E)}\ 20$

Solution

Problem 17

Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

$\mathrm{(A)}\ 250 \qquad \mathrm{(B)}\ 300 \qquad \mathrm{(C)}\ 350 \qquad \mathrm{(D)}\ 400 \qquad \mathrm{(E)}\ 500$

Solution

Problem 18

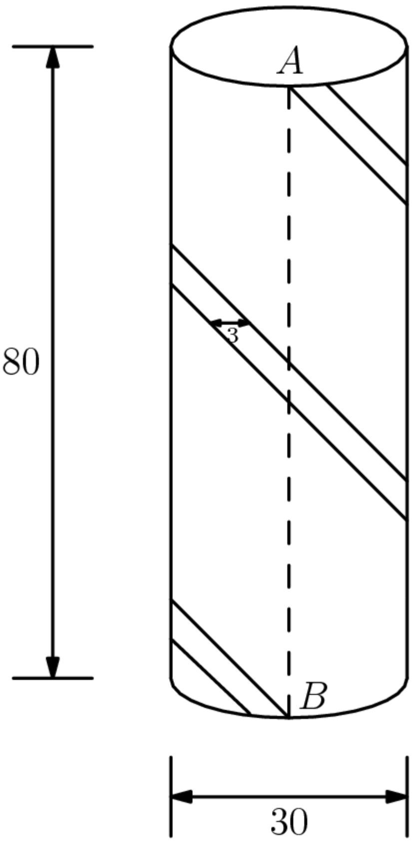
A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

$\mathrm{(A)}\ 1 \qquad \mathrm{(B)}\ 4 \qquad \mathrm{(C)}\ 36 \qquad \mathrm{(D)}\ 49 \qquad \mathrm{(E)}\ 81$

Solution

Problem 19

A white cylindrical silo has a diameter of 30 feet and a height of 80 feet. A red stripe with a horizontal width of 3 feet is painted on the silo, as shown, making two complete revolutions around it. What is the area of the stripe in square feet?

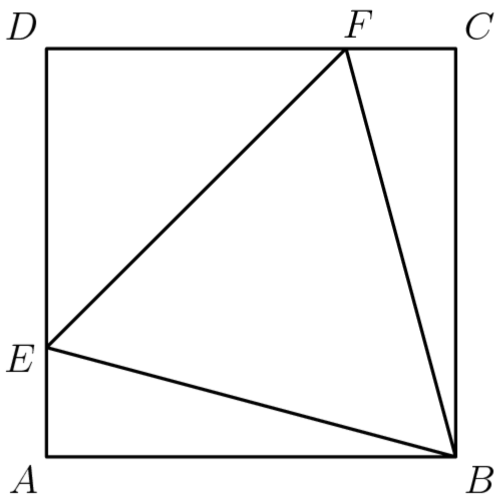


$\text{(A)}\ 120 \qquad \text{(B)}\ 180 \qquad \text{(C)}\ 240 \qquad \text{(D)}\ 360 \qquad \text{(E)}\ 480$

Solution

Problem 20

Points E and F are located on square $ABCD$ so that $\triangle BEF$ is equilateral. What is the ratio of the area of $\triangle DEF$ to that of $\triangle ABE$?

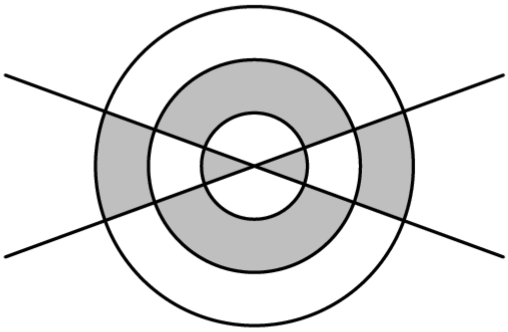


$\text{(A)}\ \frac{4}{3} \qquad \text{(B)}\ \frac{3}{2} \qquad \text{(C)}\ \sqrt{3} \qquad \text{(D)}\ 2 \qquad \text{(E)}\ 1+\sqrt{3}$

Solution

Problem 21

Two distinct lines pass through the center of three concentric circles of radii 3, 2, and 1. The area of the shaded region in the diagram is $\frac{8}{13}$ of the area of the unshaded region. What is the radian measure of the acute angle formed by the two lines? (Note: π radians is 180 degrees.)

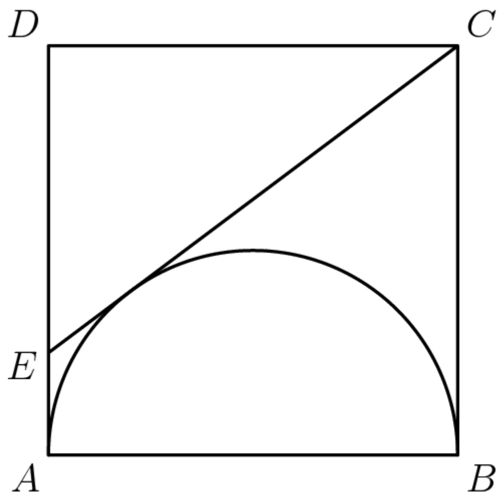


$\mathrm{(A) \ } \frac{\pi}{8} \qquad \mathrm{(B) \ } \frac{\pi}{7} \qquad \mathrm{(C) \ } \frac{\pi}{6} \qquad \mathrm{(D) \ } \frac{\pi}{5} \qquad \mathrm{(E) \ } \frac{\pi}{4}$

Solution

Problem 22

Square $ABCD$ has side length 2. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E. What is the length of $\overline{CE}$?

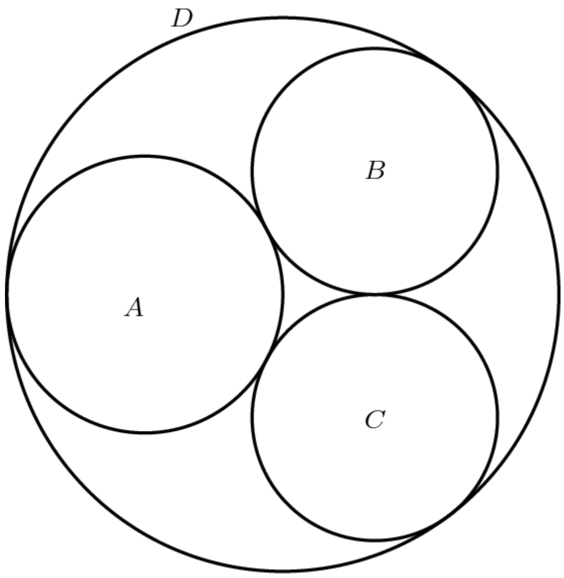


$\mathrm{(A) \ } \frac{2+\sqrt{5}}{2} \qquad \mathrm{(B) \ } \sqrt{5} \qquad \mathrm{(C) \ } \sqrt{6} \qquad \mathrm{(D) \ } \frac{5}{2} \qquad \mathrm{(E) \ } 5-\sqrt{5}$

Solution

Problem 23

Circles $A, B,$ and C are externally tangent to each other and internally tangent to circle D. Circles B and C are congruent. Circle A has radius 1 and passes through the center of D. What is the radius of circle B?



$\mathrm{(A)}\ \frac{2}{3} \qquad \mathrm{(B)}\ \frac{\sqrt{3}}{2} \qquad \mathrm{(C)}\ \frac{7}{8} \qquad \mathrm{(D)}\ \frac{8}{9} \qquad \mathrm{(E)}\ \frac{1+\sqrt{3}}{3}$

Solution

Problem 24

Let $a_1, a_2, \cdots$, be a sequence with the following properties.

(i) $a_1 = 1$, and

(ii) $a_{2n} = n \cdot a_n$ for any positive integer n.

What is the value of $a_{2^{100}}$?

$\mathrm{(A)}\ 1 \qquad \mathrm{(B)}\ 2^{99} \qquad \mathrm{(C)}\ 2^{100} \qquad \mathrm{(D)}\ 2^{4950} \qquad \mathrm{(E)}\ 2^{9999}$

Solution

Problem 25

Three pairwise-tangent spheres of radius 1 rest on a horizontal plane. A sphere of radius 2 rests on them. What is the distance from the plane to the top of the larger sphere?

$\mathrm{(A)}\ 3+\frac{\sqrt{30}}{2} \qquad \mathrm{(B)}\ 3+\frac{\sqrt{69}}{3} \qquad \mathrm{(C)}\ 3+\frac{\sqrt{123}}{4} \qquad \mathrm{(D)}\ \frac{52}{9} \qquad \mathrm{(E)}\ 3+2\sqrt{2}$

Solution

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