2011 AMC 10A Problems

2011 AMC 10A (**Answer Key**)
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Instructions

1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer.
3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the SAT if before 2006. No problems on the test will *require* the use of a calculator).
4. Figures are not necessarily drawn to scale.
5. You will have **75 minutes** working time to complete the test.

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Problem 1

A cell phone plan costs $\$20$ each month, plus $5¢$ per text message sent, plus $10¢$ for each minute used over 30 hours. In January Michelle sent 100 text messages and talked for 30.5 hours. How much did she have to pay?

$\textbf{(A)}\ \$24.00 \qquad \textbf{(B)}\ \$24.50 \qquad \textbf{(C)}\ \$25.50 \qquad \textbf{(D)}\ \$28.00 \qquad \textbf{(E)}\ \$30.00$

Solution

Problem 2

A small bottle of shampoo can hold 35 milliliters of shampoo, whereas a large bottle can hold 500 milliliters of shampoo. Jasmine wants to buy the minimum number of small bottles necessary to completely fill a large bottle. How many bottles must she buy?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

Solution

Problem 3

Suppose $[a\ b]$ denotes the average of a and b, and $\{a\ b\ c\}$ denotes the average of a, b, and c. What is $\{\{1\ 1\ 0\}\ [0\ 1]\ 0\}$?

$\textbf{(A)}\ \frac{2}{9} \qquad \textbf{(B)}\ \frac{5}{18} \qquad \textbf{(C)}\ \frac{1}{3} \qquad \textbf{(D)}\ \frac{7}{18} \qquad \textbf{(E)}\ \frac{2}{3}$

Solution

Problem 4

Let X and Y be the following sums of arithmetic sequences:

$$\begin{eqnarray*} X &=& 10+12+14+\cdots+100,\\ Y &=& 12+14+16+\cdots+102. \end{eqnarray*}$$

What is the value of $Y - X$?

$\textbf{(A)}\ 92 \qquad \textbf{(B)}\ 98 \qquad \textbf{(C)}\ 100 \qquad \textbf{(D)}\ 102 \qquad \textbf{(E)}\ 112$

Solution

Problem 5

At an elementary school, the students in third grade, fourth grade, and fifth grade run an average of 12, 15, and 10 minutes per day, respectively. There are twice as many third graders as fourth graders, and twice as many fourth graders as fifth graders. What is the average number of minutes run per day by these students?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ \frac{37}{3} \qquad \textbf{(C)}\ \frac{88}{7} \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 14$

Solution

Problem 6

Set A has 20 elements, and set B has 15 elements. What is the smallest possible number of elements in $A \cup B$, the union of A and B?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 35 \qquad \textbf{(E)}\ 300$

Solution

Problem 7

Which of the following equations does NOT have a solution?

$\textbf{(A)}\ (x+7)^2=0$

$\textbf{(B)}\ |-3x|+5=0$

$\textbf{(C)}\ \sqrt{-x}-2=0$

$\textbf{(D)}\ \sqrt{x}-8=0$

$\textbf{(E)}\ |-3x|-4=0$

Solution

Problem 8

Last summer 30% of the birds living on Town Lake were geese, 25% were swans, 10% were herons, and 35% were ducks. What percent of the birds that were not swans were geese?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 60$

Solution

Problem 9

A rectangular region is bounded by the graphs of the equations $y=a, y=-b, x=-c,$ and $x=d$, where $a, b, c,$ and d are all positive numbers. Which of the following represents the area of this region?

$\textbf{(A)}\ ac+ad+bc+bd \qquad \textbf{(B)}\ ac-ad+bc-bd \qquad \textbf{(C)}\ ac+ad-bc-bd \qquad \textbf{(D)}\ -ac-ad+bc+bd \qquad \textbf{(E)}\ ac-ad-bc+bd$

Solution

Problem 10

A majority of the 30 students in Ms. Deameanor's class bought pencils at the school bookstore. Each of these students bought the same number of pencils, and this number was greater than 1. The cost of a pencil in cents was greater than the number of pencils each student bought, and the total cost of all the pencils was $\$17.71$. What was the cost of a pencil in cents?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 17 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 77$

Solution

Problem 11

Square $EFGH$ has one vertex on each side of square $ABCD$. Point E is on $\overline{AB}$ with $AE=7\cdot EB$. What is the ratio of the area of $EFGH$ to the area of $ABCD$?

$\textbf{(A)}\ \frac{49}{64} \qquad \textbf{(B)}\ \frac{25}{32} \qquad \textbf{(C)}\ \frac{7}{8} \qquad \textbf{(D)}\ \frac{5\sqrt{2}}{8} \qquad \textbf{(E)}\ \frac{\sqrt{14}}{4}$

Solution

Problem 12

The players on a basketball team made some three-point shots, some two-point shots, and some one-point free throws. They scored as many points with two-point shots as with three-point shots. Their number of successful free throws was one more than their number of successful two-point shots. The team's total score was 61 points. How many free throws did they make?

$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 17$

Solution

Problem 13

How many even integers are there between 200 and 700 whose digits are all different and come from the set {1,2,5,7,8,9}?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 72 \qquad \textbf{(D)}\ 120 \qquad \textbf{(E)}\ 200$

Solution

Problem 14

A pair of standard 6-sided fair dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the numerical value of the area of the circle is less than the numerical value of the circle's circumference?

$\textbf{(A)}\ \frac{1}{36} \qquad \textbf{(B)}\ \frac{1}{12} \qquad \textbf{(C)}\ \frac{1}{6} \qquad \textbf{(D)}\ \frac{1}{4} \qquad \textbf{(E)}\ \frac{5}{18}$

Solution

Problem 15

Roy bought a new battery-gasoline hybrid car. On a trip the car ran exclusively on its battery for the first 40 miles, then ran exclusively on gasoline for the rest of the trip, using gasoline at a rate of 0.02 gallons per mile. On the whole trip he averaged 55 miles per gallon. How long was the trip in miles?

$\textbf{(A)}\ 140 \qquad \textbf{(B)}\ 240 \qquad \textbf{(C)}\ 440 \qquad \textbf{(D)}\ 640 \qquad \textbf{(E)}\ 840$

Solution

Problem 16

Which of the following is equal to $\sqrt{9-6\sqrt{2}}+\sqrt{9+6\sqrt{2}}$?

$\textbf{(A)}\ 3\sqrt{2} \qquad \textbf{(B)}\ 2\sqrt{6} \qquad \textbf{(C)}\ \frac{7\sqrt{2}}{2} \qquad \textbf{(D)}\ 3\sqrt{3} \qquad \textbf{(E)}\ 6$

Solution

Problem 17

In the eight-term sequence A, B, C, D, E, F, G, H, the value of C is 5 and the sum of any three consecutive terms is 30. What is $A+H$?

$\textbf{(A)}\ 17 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 43$

Solution

Problem 18

Circles $A, B,$ and C each have radius 1. Circles A and B share one point of tangency. Circle C has a point of tangency with the midpoint of $\overline{AB}$. What is the area inside Circle C but outside circle A and circle B ?

$\textbf{(A)}\ 3-\frac{\pi}{2} \qquad \textbf{(B)}\ \frac{\pi}{2} \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ \frac{3\pi}{4} \qquad \textbf{(E)}\ 1+\frac{\pi}{2}$

Solution

Problem 19

In 1991 the population of a town was a perfect square. Ten years later, after an increase of 150 people, the population was 9 more than a perfect square. Now, in 2011, with an increase of another 150 people, the population is once again a perfect square. Which of the following is closest to the percent growth of the town's population during this twenty-year period?

$\textbf{(A)}\ 42 \qquad \textbf{(B)}\ 47 \qquad \textbf{(C)}\ 52 \qquad \textbf{(D)}\ 57 \qquad \textbf{(E)}\ 62$

Solution

Problem 20

Two points on the circumference of a circle of radius r are selected independently and at random. From each point a chord of length r is drawn in a clockwise direction. What is the probability that the two chords intersect?

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{5} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{2}$

Solution

Problem 21

Two counterfeit coins of equal weight are mixed with 8 identical genuine coins. The weight of each of the counterfeit coins is different from the weight of each of the genuine coins. A pair of coins is selected at random without replacement from the 10 coins. A second pair is selected at random without replacement from the remaining 8 coins. The combined weight of the first pair is equal to the combined weight of the second pair. What is the probability that all 4 selected coins are genuine?

$\textbf{(A)}\ \frac{7}{11} \qquad \textbf{(B)}\ \frac{9}{13} \qquad \textbf{(C)}\ \frac{11}{15} \qquad \textbf{(D)}\ \frac{15}{19} \qquad \textbf{(E)}\ \frac{15}{16}$

Solution

Problem 22

Each vertex of convex pentagon $ABCDE$ is to be assigned a color. There are 6 colors to choose from, and the ends of each diagonal must have different colors. How many different colorings are possible?

$\textbf{(A)}\ 2520 \qquad \textbf{(B)}\ 2880 \qquad \textbf{(C)}\ 3120 \qquad \textbf{(D)}\ 3250 \qquad \textbf{(E)}\ 3750$

Solution

Problem 23

Seven students count from 1 to 1000 as follows:

•Alice says all the numbers, except she skips the middle number in each consecutive group of three numbers. That is, Alice says 1, 3, 4, 6, 7, 9, . . ., 997, 999, 1000.

•Barbara says all of the numbers that Alice doesn't say, except she also skips the middle number in each consecutive group of three numbers.

•Candice says all of the numbers that neither Alice nor Barbara says, except she also skips the middle number in each consecutive group of three numbers.

•Debbie, Eliza, and Fatima say all of the numbers that none of the students with the first names beginning before theirs in the alphabet say, except each also skips the middle number in each of her consecutive groups of three numbers.

•Finally, George says the only number that no one else says.

What number does George say?

$\textbf{(A)}\ 37 \qquad \textbf{(B)}\ 242 \qquad \textbf{(C)}\ 365 \qquad \textbf{(D)}\ 728 \qquad \textbf{(E)}\ 998$

Solution

Problem 24

Two distinct regular tetrahedra have all their vertices among the vertices of the same unit cube. What is the volume of the region formed by the intersection of the tetrahedra?

$\textbf{(A)}\ \frac{1}{12} \qquad \textbf{(B)}\ \frac{\sqrt{2}}{12} \qquad \textbf{(C)}\ \frac{\sqrt{3}}{12} \qquad \textbf{(D)}\ \frac{1}{6} \qquad \textbf{(E)}\ \frac{\sqrt{2}}{6}$

Solution

Problem 25

Let R be a square region and $n \ge 4$ an integer. A point X in the interior of R is called $n\text{-}ray$ partitional if there are n rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitional but not 60-ray partitional?

$\textbf{(A)}\ 1500 \qquad \textbf{(B)}\ 1560 \qquad \textbf{(C)}\ 2320 \qquad \textbf{(D)}\ 2480 \qquad \textbf{(E)}\ 2500$

Solution

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