2004 AMC 10B Problems

Contents

Problem 1

Each row of the Misty Moon Amphitheater has 33 seats. Rows 12 through 22 are reserved for a youth club. How many seats are reserved for this club?

$\mathrm{(A)}\ 297\qquad\mathrm{(B)}\ 330\qquad\mathrm{(C)}\ 363\qquad\mathrm{(D)}\ 396\qquad\mathrm{(E)}\ 726$

Solution

Problem 2

How many two-digit positive integers have at least one 7 as a digit?

$\mathrm{(A)}\ 10\qquad\mathrm{(B)}\ 18\qquad\mathrm{(C)}\ 19\qquad\mathrm{(D)}\ 20\qquad\mathrm{(E)}\ 30$

Solution

Problem 3

At each basketball practice last week, Jenny made twice as many free throws as she made at the previous practice. At her fifth practice she made 48 free throws. How many free throws did she make at the first practice?

$\mathrm{(A)}\ 3\qquad\mathrm{(B)}\ 6\qquad\mathrm{(C)}\ 9\qquad\mathrm{(D)}\ 12\qquad\mathrm{(E)}\ 15$

Solution

Problem 4

A standard six-sided die is rolled, and P is the product of the five numbers that are visible. What is the largest number that is certain to divide P?

$\mathrm{(A)}\ 6 \qquad \mathrm{(B)}\ 12 \qquad \mathrm{(C)}\ 24 \qquad \mathrm{(D)}\ 144 \qquad \mathrm{(E)}\ 720$

Solution

Problem 5

In the expression $c \cdot a^b - d$, the values of a, b, c, and d are 0, 1, 2, and 3, although not necessarily in that order. What is the maximum possible value of the result?

$\mathrm{(A)}\ 5 \qquad \mathrm{(B)}\ 6 \qquad \mathrm{(C)}\ 8 \qquad \mathrm{(D)}\ 9 \qquad \mathrm{(E)}\ 10$

Solution

Problem 6

Which of the following numbers is a perfect square?

$\mathrm{(A)}\ 98! \cdot 99! \qquad \mathrm{(B)}\ 98! \cdot 100! \qquad \mathrm{(C)}\ 99! \cdot 100! \qquad \mathrm{(D)}\ 99! \cdot 101! \qquad \mathrm{(E)}\ 100! \cdot 101!$

Solution

Problem 7

On a trip from the United States to Canada, Isabella took d U.S. dollars. At the border she exchanged them all, receiving 10 Canadian dollars for every 7 U.S. dollars. After spending 60 Canadian dollars, she had d Canadian dollars left. What is the sum of the digits of d?

$\mathrm{(A)}\ 5 \qquad \mathrm{(B)}\ 6 \qquad \mathrm{(C)}\ 7 \qquad \mathrm{(D)}\ 8 \qquad \mathrm{(E)}\ 9$

Solution

Problem 8

Minneapolis-St. Paul International Airport is 8 miles southwest of downtown St. Paul and 10 miles southeast of downtown Minneapolis. Which of the following is closest to the number of miles between downtown St. Paul and downtown Minneapolis?

$\mathrm{(A)}\ 13 \qquad \mathrm{(B)}\ 14 \qquad \mathrm{(C)}\ 15 \qquad \mathrm{(D)}\ 16 \qquad \mathrm{(E)}\ 17$

Solution

Problem 9

A square has sides of length 10, and a circle centered at one of its vertices has radius 10. What is the area of the union of the regions enclosed by the square and the circle?

$\mathrm{(A)}\ 200 + 25\pi \qquad \mathrm{(B)}\ 100 + 75\pi \qquad \mathrm{(C)}\ 75 + 100\pi \qquad \mathrm{(D)}\ 100 + 100\pi \qquad \mathrm{(E)}\ 100 + 125\pi$

Solution

Problem 10

A grocer makes a display of cans in which the top row has one can and each lower row has two more cans than the row above it. If the display contains 100 cans, how many rows does it contain?

$\mathrm{(A)}\ 5 \qquad \mathrm{(B)}\ 8 \qquad \mathrm{(C)}\ 9 \qquad \mathrm{(D)}\ 10 \qquad \mathrm{(E)}\ 411$

Solution

Problem 11

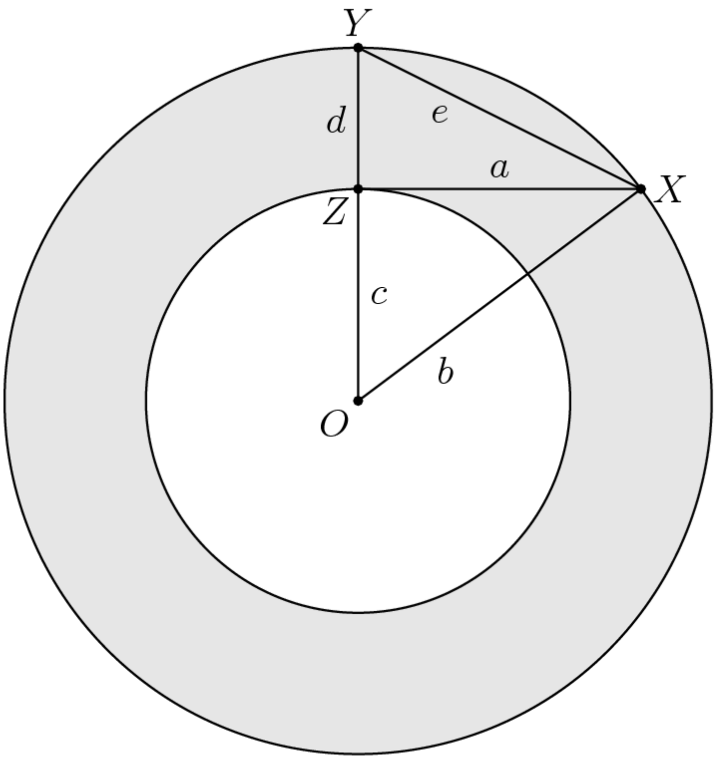
Two eight-sided dice each have faces numbered 1 through 8. When the dice are rolled, each face has an equal probability of appearing on the top. What is the probability that the product of the two top numbers is greater than their sum?

$\mathrm{(A)}\ \frac{1}{2} \qquad \mathrm{(B)}\ \frac{47}{64} \qquad \mathrm{(C)}\ \frac{3}{4} \qquad \mathrm{(D)}\ \frac{55}{64} \qquad \mathrm{(E)}\ \frac{7}{8}$

Solution

Problem 12

An annulus is the region between two concentric circles. The concentric circles in the figure have radii b and c, with $b > c$. Let OX be a radius of the larger circle, let XZ be tangent to the smaller circle at Z, and let OY be the radius of the larger circle that contains Z. Let $a = XZ$, $d = YZ$, and $e = XY$. What is the area of the annulus?



$\text{(A)}\ \pi a^2 \qquad \text{(B)}\ \pi b^2 \qquad \text{(C)}\ \pi c^2 \qquad \text{(D)}\ \pi d^2 \qquad \text{(E)}\ \pi e^2$

Solution

Problem 13

In the United States, coins have the following thicknesses: penny, 1.55 mm; nickel, 1.95 mm; dime, 1.35 mm; quarter, 1.75 mm. If a stack of these coins is exactly 14 mm high, how many coins are in the stack?

$\text{(A)}\ 7 \qquad \text{(B)}\ 8 \qquad \text{(C)}\ 9 \qquad \text{(D)}\ 10 \qquad \text{(E)}\ 11$

Solution

Problem 14

A bag initially contains red marbles and blue marbles only, with more blue than red. Red marbles are added to the bag until only $\frac{1}{3}$ of the marbles in the bag are blue. Then yellow marbles are added to the bag until only $\frac{1}{5}$ of the marbles in the bag are blue. Finally, the number of blue marbles in the bag is doubled. What fraction of the marbles now in the bag are blue?

$\text{(A)}\ \frac{1}{5} \qquad \text{(B)}\ \frac{1}{4} \qquad \text{(C)}\ \frac{1}{3} \qquad \text{(D)}\ \frac{2}{5} \qquad \text{(E)}\ \frac{1}{2}$

Solution

Problem 15

Patty has 20 coins consisting of nickels and dimes. If her nickels were dimes and her dimes were nickels, she would have 70 cents more. How much are her coins worth?

$\textbf{(A)}\ \$1.15 \qquad \textbf{(B)}\ \$1.20 \qquad \textbf{(C)}\ \$1.25 \qquad \textbf{(D)}\ \$1.30 \qquad \textbf{(E)}\ \$1.35$

Solution

Problem 16

Three circles of radius 1 are externally tangent to each other and internally tangent to a larger circle. What is the radius of the large circle?

$\mathrm{(A)}\ \frac{2+\sqrt{6}}{3} \qquad \mathrm{(B)}\ 2 \qquad \mathrm{(C)}\ \frac{2+3\sqrt{2}}{2} \qquad \mathrm{(D)}\ \frac{3+2\sqrt{3}}{3} \qquad \mathrm{(E)}\ \frac{3+\sqrt{3}}{2}$

Solution

Problem 17

The two digits in Jack's age are the same as the digits in Bill's age, but in reverse order. In five years Jack will be twice as old as Bill will be then. What is the difference in their current ages?

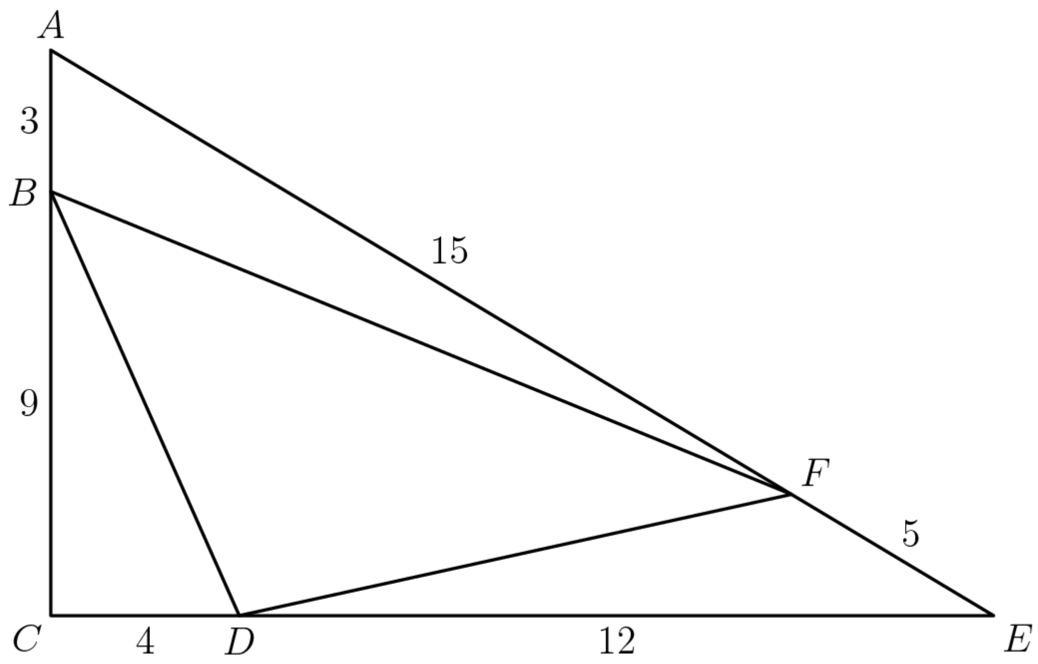
$\mathrm{(A)}\ 9 \qquad \mathrm{(B)}\ 18 \qquad \mathrm{(C)}\ 27 \qquad \mathrm{(D)}\ 36 \qquad \mathrm{(E)}\ 45$

Solution

Problem 18

In the right triangle $\triangle ACE$, we have $AC = 12$, $CE = 16$, and $EA = 20$. Points B, D, and F are located on AC, CE, and EA, respectively, so that $AB = 3$, $CD = 4$, and $EF = 5$. What is the ratio of the area of $\triangle DBF$ to that of $\triangle ACE$?

$\mathrm{(A)}\ \frac{1}{4} \qquad \mathrm{(B)}\ \frac{9}{25} \qquad \mathrm{(C)}\ \frac{3}{8} \qquad \mathrm{(D)}\ \frac{11}{25} \qquad \mathrm{(E)}\ \frac{7}{16}$



Solution

Problem 19

In the sequence $2001, 2002, 2003, \ldots$, each term after the third is found by subtracting the previous term from the sum of the two terms that precede that term. For example, the fourth term is $2001 + 2002 - 2003 = 2000$. What is the 2004^{th} term in this sequence?

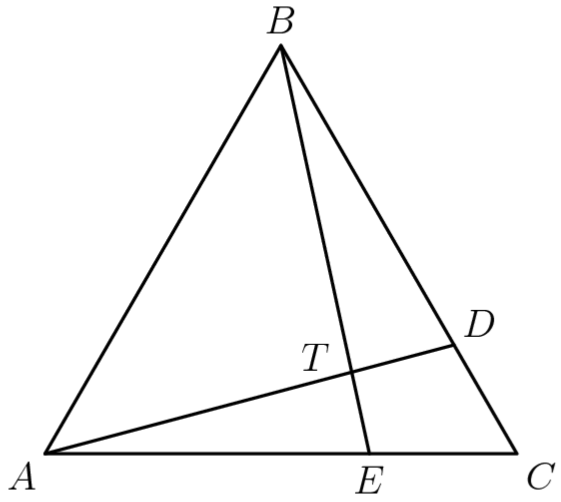
$\mathrm{(A)}\ -2004 \qquad \mathrm{(B)}\ -2 \qquad \mathrm{(C)}\ 0 \qquad \mathrm{(D)}\ 4003 \qquad \mathrm{(E)}\ 6007$

Solution

Problem 20

In $\triangle ABC$ points D and E lie on BC and AC, respectively. If AD and BE intersect at T so that $AT/DT=3$ and $BT/ET=4$, what is CD/BD?

$(A)\ \frac{1}{8} \qquad (B)\ \frac{2}{9} \qquad (C)\ \frac{3}{10} \qquad (D)\ \frac{4}{11} \qquad (E)\ \frac{5}{12}$



Solution

Problem 21

Let $1, 4, \ldots$ and $9, 16, \ldots$ be two arithmetic progressions. The set S is the union of the first 2004 terms of each sequence. How many distinct numbers are in S?

$(A)\ 3722 \qquad (B)\ 3732 \qquad (C)\ 3914 \qquad (D)\ 3924 \qquad (E)\ 4007$

Solution

Problem 22

A triangle with sides of $5, 12,$ and 13 has both an inscribed and a circumscribed circle. What is the distance between the centers of those circles?

$(A)\ \frac{3\sqrt{5}}{2} \qquad (B)\ \frac{7}{2} \qquad (C)\ \sqrt{15} \qquad (D)\ \frac{\sqrt{65}}{2} \qquad (E)\ \frac{9}{2}$

Solution

Problem 23

Each face of a cube is painted either red or blue, each with probability $1/2$. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

$(A)\ \frac{1}{4} \qquad (B)\ \frac{5}{16} \qquad (C)\ \frac{3}{8} \qquad (D)\ \frac{7}{16} \qquad (E)\ \frac{1}{2}$

Solution

Problem 24

In $\triangle ABC$ we have $AB = 7$, $AC = 8$, and $BC = 9$. Point D is on the circumscribed circle of the triangle so that AD bisects $\angle BAC$. What is the value of $\frac{AD}{CD}$?

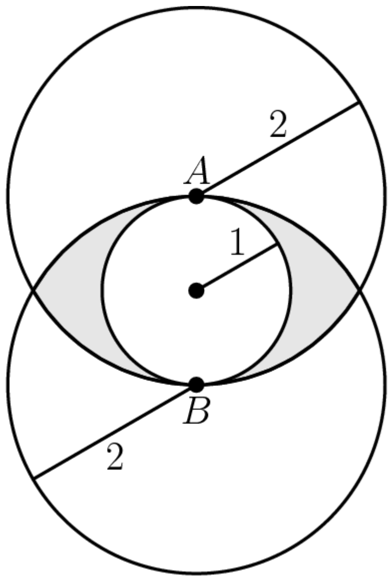
$(A)\ \frac{9}{8} \qquad (B)\ \frac{5}{3} \qquad (C)\ 2 \qquad (D)\ \frac{17}{7} \qquad (E)\ \frac{5}{2}$

Solution

Problem 25

A circle of radius 1 is internally tangent to two circles of radius 2 at points A and B, where AB is a diameter of the smaller circle. What is the area of the region, shaded in the picture, that is outside the smaller circle and inside each of the two larger circles?

$\mathrm{(A)}\ \frac{5}{3}\pi - 3\sqrt{2} \qquad \mathrm{(B)}\ \frac{5}{3}\pi - 2\sqrt{3} \qquad \mathrm{(C)}\ \frac{8}{3}\pi - 3\sqrt{3} \qquad \mathrm{(D)}\ \frac{8}{3}\pi - 3\sqrt{2} \qquad \mathrm{(E)}\ \frac{8}{3}\pi - 2\sqrt{3}$



Solution

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