2007 AMC 10A Problems

Contents

Problem 1

One ticket to a show costs $\$20$ at full price. Susan buys 4 tickets using a coupon that gives her a 25% discount. Pam buys 5 tickets using a coupon that gives her a 30% discount. How many more dollars does Pam pay than Susan?

$\mathrm{(A)}\ 2\qquad\mathrm{(B)}\ 5\qquad\mathrm{(C)}\ 10\qquad\mathrm{(D)}\ 15\qquad\mathrm{(E)}\ 20$

Solution

Problem 2

Define $a@b = ab - b^{2}$ and $a\#b = a + b - ab^{2}$. What is $\frac{6@2}{6\#2}$?

$\mathrm{(A)}\ -\frac{1}{2}\qquad\mathrm{(B)}\ -\frac{1}{4}\qquad\mathrm{(C)}\ \frac{1}{8}\qquad\mathrm{(D)}\ \frac{1}{4}\qquad\mathrm{(E)}\ \frac{1}{2}$

Solution

Problem 3

An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. It is filled with water to a height of 40 cm. A brick with a rectangular base that measures 40 cm by 20 cm and a height of 10 cm is placed in the aquarium. By how many centimeters does the water rise?

$\mathrm{(A)}\ 0.5\qquad\mathrm{(B)}\ 1\qquad\mathrm{(C)}\ 1.5\qquad\mathrm{(D)}\ 2\qquad\mathrm{(E)}\ 2.5$

Solution

Problem 4

The larger of two consecutive odd integers is three times the smaller. What is their sum?

$\mathrm{(A)}\ 4\qquad\mathrm{(B)}\ 8\qquad\mathrm{(C)}\ 12\qquad\mathrm{(D)}\ 16\qquad\mathrm{(E)}\ 20$

Solution

Problem 5

A school store sells 7 pencils and 8 notebooks for $\$4.15$. It also sells 5 pencils and 3 notebooks for $\$1.77$. How much do 16 pencils and 10 notebooks cost?

$\mathrm{(A)}\ \$1.76\qquad\mathrm{(B)}\ \$5.84\qquad\mathrm{(C)}\ \$6.00\qquad\mathrm{(D)}\ \$6.16\qquad\mathrm{(E)}\ \$6.32$

Solution

Problem 6

At Euclid High School, the number of students taking the AMC 10 was 60 in 2002, 66 in 2003, 70 in 2004, 76 in 2005, 78 in 2006, and is 85 in 2007. Between what two consecutive years was there the largest percentage increase?

$\text{(A)}\ 2002\ \text{and}\ 2003 \qquad \text{(B)}\ 2003\ \text{and}\ 2004 \qquad \text{(C)}\ 2004\ \text{and}\ 2005 \qquad \text{(D)}\ 2005\ \text{and}\ 2006 \qquad \text{(E)}\ 2006\ \text{and}\ 2007$

Solution

Problem 7

Last year Mr. Jon Q. Public received an inheritance. He paid 20% in federal taxes on the inheritance, and paid 10% of what he had left in state taxes. He paid a total of 10500 for both taxes. How many dollars was his inheritance?

$\text{(A)}\ 30000 \qquad \text{(B)}\ 32500 \qquad \text{(C)}\ 35000 \qquad \text{(D)}\ 37500 \qquad \text{(E)}\ 40000$

Solution

Problem 8

Triangles ABC and ADC are isosceles with $AB = BC$ and $AD = DC$. Point D is inside triangle ABC, angle ABC measures 40 degrees, and angle ADC measures 140 degrees. What is the degree measure of angle BAD?

$\text{(A)}\ 20 \qquad \text{(B)}\ 30 \qquad \text{(C)}\ 40 \qquad \text{(D)}\ 50 \qquad \text{(E)}\ 60$

Solution

Problem 9

Real numbers a and b satisfy the equations $3^{a} = 81^{b+2}$ and $125^{b} = 5^{a-3}$. What is ab?

$\text{(A)}\ -60 \qquad \text{(B)}\ -17 \qquad \text{(C)}\ 9 \qquad \text{(D)}\ 12 \qquad \text{(E)}\ 60$

Solution

Problem 10

The Dunbar family consists of a mother, a father, and some children. The average age of the members of the family is 20, the father is 48 years old, and the average age of the mother and children is 16. How many children are in the family?

$\text{(A)}\ 2 \qquad \text{(B)}\ 3 \qquad \text{(C)}\ 4 \qquad \text{(D)}\ 5 \qquad \text{(E)}\ 6$

Solution

Problem 11

The numbers from 1 to 8 are placed at the vertices of a cube in such a manner that the sum of the four numbers on each face is the same. What is this common sum?

$\text{(A)}\ 14 \qquad \text{(B)}\ 16 \qquad \text{(C)}\ 18 \qquad \text{(D)}\ 20 \qquad \text{(E)}\ 24$

Solution

Problem 12

Two tour guides are leading six tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the stipulation that each guide must take at least one tourist. How many different groupings of guides and tourists are possible?

$\text{(A)}\ 56 \qquad \text{(B)}\ 58 \qquad \text{(C)}\ 60 \qquad \text{(D)}\ 62 \qquad \text{(E)}\ 64$

Solution

Problem 13

Yan is somewhere between his home and the stadium. To get to the stadium he can walk directly to the stadium, or else he can walk home and then ride his bicycle to the stadium. He rides 7 times as fast as he walks, and both choices require the same amount of time. What is the ratio of Yan's distance from his home to his distance from the stadium?

$\text{(A)}\ \frac{2}{3} \qquad \text{(B)}\ \frac{3}{4} \qquad \text{(C)}\ \frac{4}{5} \qquad \text{(D)}\ \frac{5}{6} \qquad \text{(E)}\ \frac{7}{8}$

Solution

Problem 14

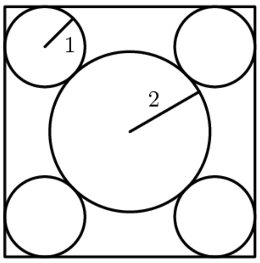
A triangle with side lengths in the ratio $3 : 4 : 5$ is inscribed in a circle with radius 3. What is the area of the triangle?

$\text{(A)}\ 8.64 \qquad \text{(B)}\ 12 \qquad \text{(C)}\ 5\pi \qquad \text{(D)}\ 17.28 \qquad \text{(E)}\ 18$

Solution

Problem 15

Four circles of radius 1 are each tangent to two sides of a square and externally tangent to a circle of radius 2, as shown. What is the area of the square?



$\text{(A)}\ 32 \qquad \text{(B)}\ 22 + 12\sqrt{2} \qquad \text{(C)}\ 16 + 16\sqrt{3} \qquad \text{(D)}\ 48 \qquad \text{(E)}\ 36 + 16\sqrt{2}$

Solution

Problem 16

Integers $a, b, c,$ and d, not necessarily distinct, are chosen independently and at random from 0 to 2007, inclusive. What is the probability that $ad - bc$ is even?

$\text{(A)}\ \frac{3}{8} \qquad \text{(B)}\ \frac{7}{16} \qquad \text{(C)}\ \frac{1}{2} \qquad \text{(D)}\ \frac{9}{16} \qquad \text{(E)}\ \frac{5}{8}$

Solution

Problem 17

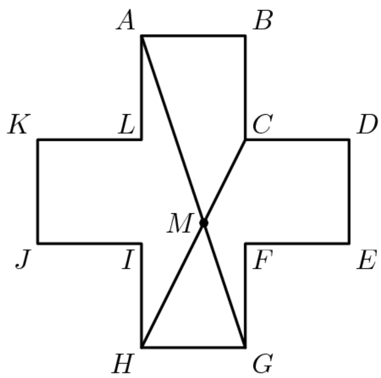
Suppose that m and n are positive integers such that $75m = n^3$. What is the minimum possible value of $m + n$?

$\text{(A)}\ 15 \qquad \text{(B)}\ 30 \qquad \text{(C)}\ 50 \qquad \text{(D)}\ 60 \qquad \text{(E)}\ 5700$

Solution

Problem 18

Consider the 12-sided polygon $ABCDEFGHIJKL$, as shown. Each of its sides has length 4, and each two consecutive sides form a right angle. Suppose that $\overline{AG}$ and $\overline{CH}$ meet at M. What is the area of quadrilateral $ABCM$?

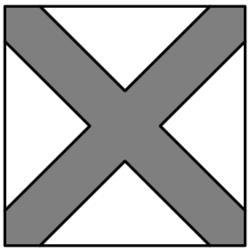


$\text{(A)}\ \frac{44}{3} \qquad \text{(B)}\ 16 \qquad \text{(C)}\ \frac{88}{5} \qquad \text{(D)}\ 20 \qquad \text{(E)}\ \frac{62}{3}$

Solution

Problem 19

A paint brush is swept along both diagonals of a square to produce the symmetric painted area, as shown. Half the area of the square is painted. What is the ratio of the side length of the square to the brush width?



$\text{(A) } 2\sqrt{2}+1 \qquad \text{(B) } 3\sqrt{2} \qquad \text{(C) } 2\sqrt{2}+2 \qquad \text{(D) } 3\sqrt{2}+1 \qquad \text{(E) } 3\sqrt{2}+2$

Solution

Problem 20

Suppose that the number a satisfies the equation $4 = a + a^{-1}$. What is the value of $a^4 + a^{-4}$?

$\text{(A) } 164 \qquad \text{(B) } 172 \qquad \text{(C) } 192 \qquad \text{(D) } 194 \qquad \text{(E) } 212$

Solution

Problem 21

A sphere is inscribed in a cube that has a surface area of 24 square meters. A second cube is then inscribed within the sphere. What is the surface area in square meters of the inner cube?

$\text{(A) } 3 \qquad \text{(B) } 6 \qquad \text{(C) } 8 \qquad \text{(D) } 9 \qquad \text{(E) } 12$

Solution

Problem 22

A finite sequence of three-digit integers has the property that the tens and units digits of each term are, respectively, the hundreds and tens digits of the next term, and the tens and units digits of the last term are, respectively, the hundreds and tens digits of the first term. For example, such a sequence might begin with the terms 247, 475, and 756 and end with the term 824. Let S be the sum of all the terms in the sequence. What is the largest prime factor that always divides S?

$\text{(A) } 3 \qquad \text{(B) } 7 \qquad \text{(C) } 13 \qquad \text{(D) } 37 \qquad \text{(E) } 43$

Solution

Problem 23

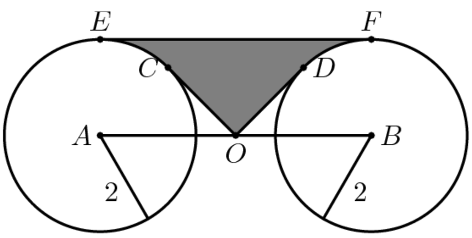
How many ordered pairs (m,n) of positive integers, with $m \geq n$, have the property that their squares differ by 96?

$\text{(A) } 3 \qquad \text{(B) } 4 \qquad \text{(C) } 6 \qquad \text{(D) } 9 \qquad \text{(E) } 12$

Solution

Problem 24

Circles centered at A and B each have radius 2, as shown. Point O is the midpoint of $\overline{AB}$, and $OA = 2\sqrt{2}$. Segments OC and OD are tangent to the circles centered at A and B, respectively, and EF is a common tangent. What is the area of the shaded region $ECODF$?



$\text{(A) } \frac{8\sqrt{2}}{3} \qquad \text{(B) } 8\sqrt{2}-4-\pi \qquad \text{(C) } 4\sqrt{2} \qquad \text{(D) } 4\sqrt{2}+\frac{\pi}{8} \qquad \text{(E) } 8\sqrt{2}-2-\frac{\pi}{2}$

Solution

Problem 25

For each positive integer n, let $S(n)$ denote the sum of the digits of n. For how many values of n is $n + S(n) + S(S(n)) = 2007?$

$\text{(A) } 1 \qquad \text{(B) } 2 \qquad \text{(C) } 3 \qquad \text{(D) } 4 \qquad \text{(E) } 5$

Solution

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