2013 AMC 10A Problems

Contents

Problem 1

A taxi ride costs \$1.50 plus \$0.25 per mile traveled. How much does a 5-mile taxi ride cost?

$\textbf{(A)}\ 2.25 \qquad \textbf{(B)}\ 2.50 \qquad \textbf{(C)}\ 2.75 \qquad \textbf{(D)}\ 3.00 \qquad \textbf{(E)}\ 3.75$

Solution

Problem 2

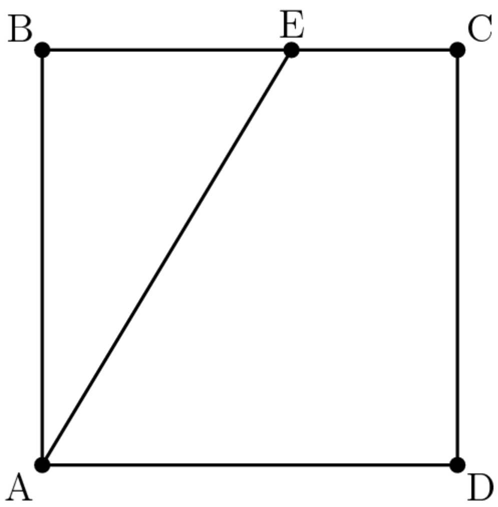
Alice is making a batch of cookies and needs $2\frac{1}{2}$ cups of sugar. Unfortunately, her measuring cup holds only $\frac{1}{4}$ cup of sugar. How many times must she fill that cup to get the correct amount of sugar?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 20$

Solution

Problem 3

Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE?



$\textbf{(A) }4\qquad\textbf{(B) }5\qquad\textbf{(C) }6\qquad\textbf{(D) }7\qquad\textbf{(E) }8$

Solution

Problem 4

A softball team played ten games, scoring 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 runs. They lost by one run in exactly five games. In each of their other games, they scored twice as many runs as their opponent. How many total runs did their opponents score?

$\textbf{(A) }35\qquad\textbf{(B) }40\qquad\textbf{(C) }45\qquad\textbf{(D) }50\qquad\textbf{(E) }55$

Solution

Problem 5

Tom, Dorothy, and Sammy went on a vacation and agreed to split the costs evenly. During their trip Tom paid $105, Dorothy paid $125, and Sammy paid $175. In order to share costs equally, Tom gave Sammy t dollars, and Dorothy gave Sammy d dollars. What is $t-d$?

$\textbf{(A) }15\qquad\textbf{(B) }20\qquad\textbf{(C) }25\qquad\textbf{(D) }30\qquad\textbf{(E) }35$

Solution

Problem 6

Joey and his five brothers are ages 3, 5, 7, 9, 11, and 13. One afternoon two of his brothers whose ages sum to 16 went to the movies, two brothers younger than 10 went to play baseball, and Joey and the 5-year-old stayed home. How old is Joey?

$\textbf{(A) }3\qquad\textbf{(B) }7\qquad\textbf{(C) }9\qquad\textbf{(D) }11\qquad\textbf{(E) }13$

Solution

Problem 7

A student must choose a program of four courses from a menu of courses consisting of English, Algebra, Geometry, History, Art, and Latin. This program must contain English and at least one mathematics course. In how many ways can this program be chosen?

$\textbf{(A) }6\qquad\textbf{(B) }8\qquad\textbf{(C) }9\qquad\textbf{(D) }12\qquad\textbf{(E) }16$

Solution

Problem 8

What is the value of $\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}$?

$\textbf{(A)}\ -1 \qquad\textbf{(B)}\ 1 \qquad\textbf{(C)}\ \frac{5}{3} \qquad\textbf{(D)}\ 2013 \qquad\textbf{(E)}\ 2^{4024}$

Solution

Problem 9

In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score?

$\textbf{(A)}\ 12 \qquad\textbf{(B)}\ 18 \qquad\textbf{(C)}\ 24 \qquad\textbf{(D)}\ 30 \qquad\textbf{(E)}\ 36$

Solution

Problem 10

A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

$\textbf{(A)}\ 15 \qquad\textbf{(B)}\ 30 \qquad\textbf{(C)}\ 40 \qquad\textbf{(D)}\ 60 \qquad\textbf{(E)}\ 70$

Solution

Problem 11

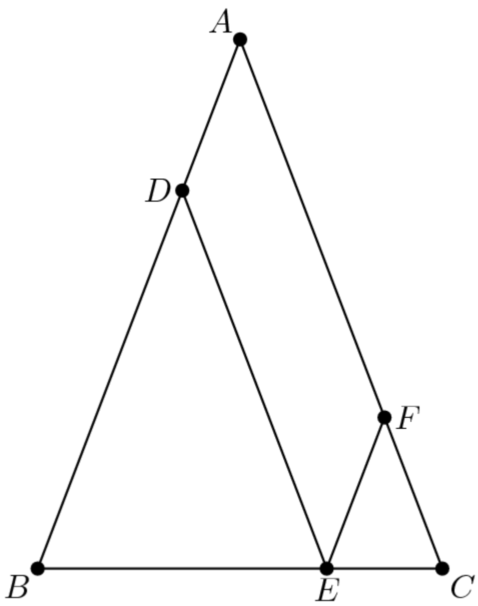
A student council must select a two-person welcoming committee and a three-person planning committee from among its members. There are exactly 10 ways to select a two-person team for the welcoming committee. It is possible for students to serve on both committees. In how many different ways can a three-person planning committee be selected?

$\textbf{(A)}\ 10 \qquad\textbf{(B)}\ 12 \qquad\textbf{(C)}\ 15 \qquad\textbf{(D)}\ 18 \qquad\textbf{(E)}\ 25$

Solution

Problem 12

In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points $D, E,$ and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?



$\textbf{(A) } 48 \qquad \textbf{(B) } 52 \qquad \textbf{(C) } 56 \qquad \textbf{(D) } 60 \qquad \textbf{(E) } 72$

Solution

Problem 13

How many three-digit numbers are not divisible by 5, have digits that sum to less than 20, and have the first digit equal to the third digit?

$\textbf{(A) } 52 \qquad \textbf{(B) } 60 \qquad \textbf{(C) } 66 \qquad \textbf{(D) } 68 \qquad \textbf{(E) } 70$

Solution

Problem 14

A solid cube of side length 1 is removed from each corner of a solid cube of side length 3. How many edges does the remaining solid have?

$\textbf{(A) } 36 \qquad \textbf{(B) } 60 \qquad \textbf{(C) } 72 \qquad \textbf{(D) } 84 \qquad \textbf{(E) } 108$

Solution

Problem 15

Two sides of a triangle have lengths 10 and 15. The length of the altitude to the third side is the average of the lengths of the altitudes to the two given sides. How long is the third side?

$\textbf{(A) } 6 \qquad \textbf{(B) } 8 \qquad \textbf{(C) } 9 \qquad \textbf{(D) } 12 \qquad \textbf{(E) } 18$

Solution

Problem 16

A triangle with vertices $(6, 5)$, $(8, -3)$, and $(9, 1)$ is reflected about the line $x = 8$ to create a second triangle. What is the area of the union of the two triangles?

$\textbf{(A) } 9 \qquad \textbf{(B) } \frac{28}{3} \qquad \textbf{(C) } 10 \qquad \textbf{(D) } \frac{31}{3} \qquad \textbf{(E) } \frac{32}{3}$

Solution

Problem 17

Daphne is visited periodically by her three best friends: Alice, Beatrix, and Claire. Alice visits every third day, Beatrix visits every fourth day, and Claire visits every fifth day. All three friends visited Daphne yesterday. How many days of the next 365-day period will exactly two friends visit her?

$\textbf{(A)}\ 48\qquad\textbf{(B)}\ 54\qquad\textbf{(C)}\ 60\qquad\textbf{(D)}\ 66\qquad\textbf{(E)}\ 72$

Solution

Problem 18

Let points $A = (0,0)$, $B = (1,2)$, $C = (3,3)$, and $D = (4,0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $\left(\frac{p}{q}, \frac{r}{s}\right)$, where these fractions are in lowest terms. What is $p + q + r + s$?

$\textbf{(A)}\ 54\qquad\textbf{(B)}\ 58\qquad\textbf{(C)}\ 62\qquad\textbf{(D)}\ 70\qquad\textbf{(E)}\ 75$

Solution

Problem 19

In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b-representation of 2013 end in the digit 3?

$\textbf{(A)}\ 6\qquad\textbf{(B)}\ 9\qquad\textbf{(C)}\ 13\qquad\textbf{(D)}\ 16\qquad\textbf{(E)}\ 18$

Solution

Problem 20

A unit square is rotated 45° about its center. What is the area of the region swept out by the interior of the square?

$\textbf{(A)}\ 1 - \frac{\sqrt2}{2} + \frac{\pi}{4}\qquad\textbf{(B)}\ \frac{1}{2} + \frac{\pi}{4}\qquad\textbf{(C)}\ 2 - \sqrt2 + \frac{\pi}{4}$
$\textbf{(D)}\ \frac{\sqrt2}{2} + \frac{\pi}{4}\qquad\textbf{(E)}\ 1 + \frac{\sqrt2}{4} + \frac{\pi}{8}$

Solution

Problem 21

A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^th pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

$\textbf{(A)}\ 720\qquad\textbf{(B)}\ 1296\qquad\textbf{(C)}\ 1728\qquad\textbf{(D)}\ 1925\qquad\textbf{(E)}\ 3850$

Solution

Problem 22

Six spheres of radius 1 are positioned so that their centers are at the vertices of a regular hexagon of side length 2. The six spheres are internally tangent to a larger sphere whose center is the center of the hexagon. An eighth sphere is externally tangent to the six smaller spheres and internally tangent to the larger sphere. What is the radius of this eighth sphere?

$\textbf{(A)}\ \sqrt{2}\qquad\textbf{(B)}\ \frac{3}{2}\qquad\textbf{(C)}\ \frac{5}{3}\qquad\textbf{(D)}\ \sqrt{3}\qquad\textbf{(E)}\ 2$

Solution

Problem 23

In $\triangle ABC, AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

$\textbf{(A)}\ 11\qquad\textbf{(B)}\ 28\qquad\textbf{(C)}\ 33\qquad\textbf{(D)}\ 61\qquad\textbf{(E)}\ 72$

Solution

Problem 24

Central High School is competing against Northern High School in a backgammon match. Each school has three players, and the contest rules require that each player play two games against each of the other school's players. The match takes place in six rounds, with three games played simultaneously in each round. In how many different ways can the match be scheduled?

$\textbf{(A)}\ 540\qquad\textbf{(B)}\ 600\qquad\textbf{(C)}\ 720\qquad\textbf{(D)}\ 810\qquad\textbf{(E)}\ 900$

Solution

Problem 25

All 20 diagonals are drawn in a regular octagon. At how many distinct points in the interior of the octagon (not on the boundary) do two or more diagonals intersect?

$\textbf{(A)}\ 49\qquad\textbf{(B)}\ 65\qquad\textbf{(C)}\ 70\qquad\textbf{(D)}\ 96\qquad\textbf{(E)}\ 128$

Solution

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