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## 2020 AMC 10B Problems

<div>2020 AMC 10B (Answer Key)</div> <div>Printable version:   AoPS Resources (<a href="http://www.artofproblemsolving.com/Forum/resources.php?c=182&amp;cid=43&amp;year=2020">http://www.artofproblemsolving.com/Forum/resources.php?c=182&amp;cid=43&amp;year=2020</a>) • PDF (<a href="http://www.artofproblemsolving.com/Forum/resources/files/usa/USA-AMC_10-2020-43">http://www.artofproblemsolving.com/Forum/resources/files/usa/USA-AMC_10-2020-43</a>)</div>
<div>Instructions</div> <div><div>1. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.</div><div>2. You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered if the year is before 2006, 1.5 points for each problem left unanswered if the year is after 2006, and 0 points for each incorrect answer.</div><div>3. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor and erasers (and calculators that are accepted for use on the SAT if before 2006. No problems on the test will <i>require</i> the use of a calculator).</div><div>4. Figures are not necessarily drawn to scale.</div><div>5. You will have <b>75 minutes</b> working time to complete the test.</div></div>
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### Problem 1

What is the value of

$$1 - (-2) - 3 - (-4) - 5 - (-6)?$$

- (A)  $-20$       (B)  $-3$       (C)  $3$       (D)  $5$       (E)  $21$

Solution

### Problem 2

Carl has 5 cubes each having side length 1, and Kate has 5 cubes each having side length 2. What is the total volume of these 10 cubes?

- (A) 24      (B) 25      (C) 28      (D) 40      (E) 45

Solution

Problem 3

The ratio of  $w$  to  $x$  is  $4 : 3$ , the ratio of  $y$  to  $z$  is  $3 : 2$ , and the ratio of  $z$  to  $x$  is  $1 : 6$ . What is the ratio of  $w$  to  $y$ ?

- (A)  $4 : 3$       (B)  $3 : 2$       (C)  $8 : 3$       (D)  $4 : 1$       (E)  $16 : 3$

Solution

Problem 4

The acute angles of a right triangle are  $a^\circ$  and  $b^\circ$ , where  $a > b$  and both  $a$  and  $b$  are prime numbers. What is the least possible value of  $b$ ?

- (A) 2      (B) 3      (C) 5      (D) 7      (E) 11

Solution

Problem 5

How many distinguishable arrangements are there of 1 brown tile, 1 purple tile, 2 green tiles, and 3 yellow tiles in a row from left to right? (Tiles of the same color are indistinguishable.)

- (A) 210      (B) 420      (C) 630      (D) 840      (E) 1050

Solution

Problem 6

Driving along a highway, Megan noticed that her odometer showed 15951 (miles). This number is a palindrome-it reads the same forward and backward. Then 2 hours later, the odometer displayed the next higher palindrome. What was her average speed, in miles per hour, during this 2-hour period?

- (A) 50      (B) 55      (C) 60      (D) 65      (E) 70

Solution

Problem 7

How many positive even multiples of 3 less than 2020 are perfect squares?

- (A) 7      (B) 8      (C) 9      (D) 10      (E) 12

Solution

Problem 8

Points  $P$  and  $Q$  lie in a plane with  $PQ = 8$ . How many locations for point  $R$  in this plane are there such that the triangle with vertices  $P, Q$ , and  $R$  is a right triangle with area 12 square units?

- (A) 2      (B) 4      (C) 6      (D) 8      (E) 12

Solution

Problem 9

For how many ordered pairs of integers  $(x, y)$  satisfy the equation

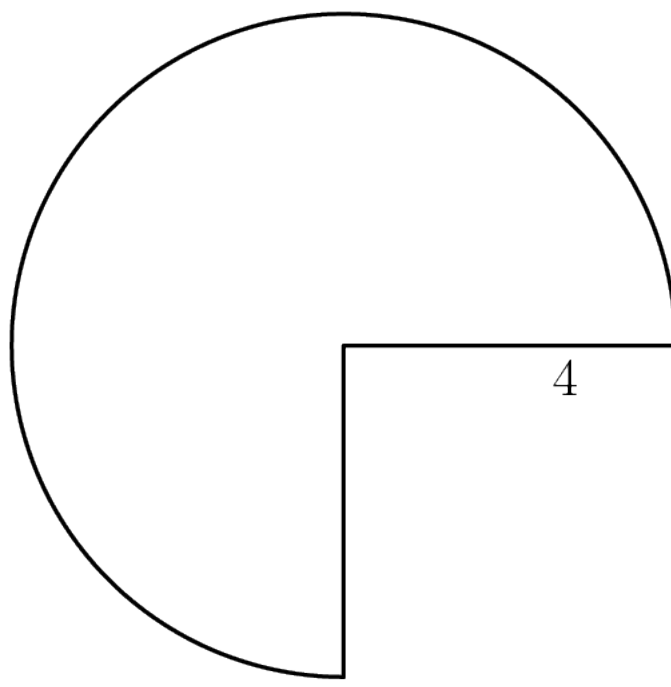
$$x^{2020} + y^2 = 2y?$$

- (A) 1      (B) 2      (C) 3      (D) 4      (E) infinitely many

Solution

Problem 10

A three-quarter sector of a circle of radius 4 inches along with its interior is the 2-D net that forms the lateral surface area of a right circular cone by taping together along the two radii shown. What is the volume of the cone in cubic inches?



- (A)  $3\pi\sqrt{5}$     (B)  $4\pi\sqrt{3}$     (C)  $3\pi\sqrt{7}$     (D)  $6\pi\sqrt{3}$     (E)  $6\pi\sqrt{7}$

Solution

## Problem 11

Ms. Carr asks her students to select 5 of the 10 books to read on her classroom reading list. Harold randomly selects 5 books from this list, and Betty does the same. What is the probability that there are exactly 2 books that they both select?

- (A)  $\frac{1}{8}$     (B)  $\frac{5}{36}$     (C)  $\frac{14}{45}$     (D)  $\frac{25}{63}$     (E)  $\frac{1}{2}$

Solution

## Problem 12

The decimal representation of

$$\frac{1}{20^{20}}$$

consists of a string of zeros after the decimal point, followed by a 9 and then several more digits. How many zeros are in that initial string of zeros after the decimal point?

- (A) 23    (B) 24    (C) 25    (D) 26    (E) 27

Solution

## Problem 13

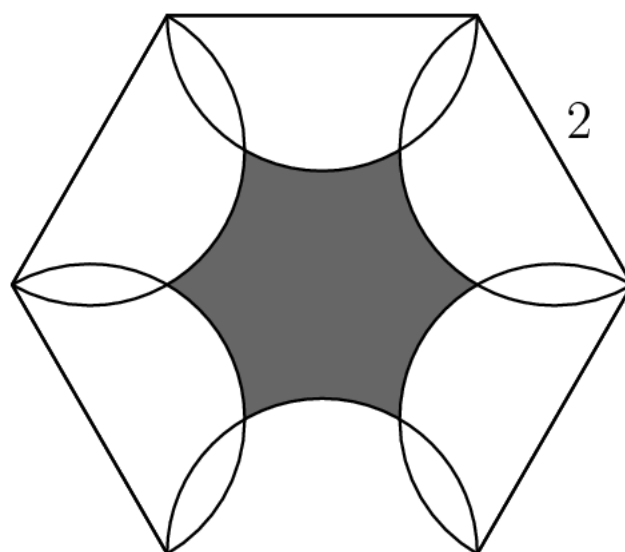
Andy the Ant lives on a coordinate plane and is currently at  $(-20, 20)$  facing east (that is, in the positive  $x$ -direction). Andy moves 1 unit and then turns  $90^\circ$  degrees left. From there, Andy moves 2 units (north) and then turns  $90^\circ$  degrees left. He then moves 3 units (west) and again turns  $90^\circ$  degrees left. Andy continues his progress, increasing his distance each time by 1 unit and always turning left. What is the location of the point at which Andy makes the 2020th left turn?

- (A)  $(-1030, -994)$     (B)  $(-1030, -990)$     (C)  $(-1026, -994)$     (D)  $(-1026, -990)$     (E)  $(-1022, -994)$

Solution

## Problem 14

As shown in the figure below, six semicircles lie in the interior of a regular hexagon with side length 2 so that the diameters of the semicircles coincide with the sides of the hexagon. What is the area of the shaded region — inside the hexagon but outside all of the semicircles?



- (A)  $6\sqrt{3} - 3\pi$     (B)  $\frac{9\sqrt{3}}{2} - 2\pi$     (C)  $\frac{3\sqrt{3}}{2} - \frac{\pi}{3}$     (D)  $3\sqrt{3} - \pi$     (E)  $\frac{9\sqrt{3}}{2} - \pi$

Solution

Problem 15

Steve wrote the digits 1, 2, 3, 4, and 5 in order repeatedly from left to right, forming a list of 10, 000 digits, beginning 123451234512 . . . . He then erased every third digit from his list (that is, the 3rd, 6th, 9th, . . . digits from the left), then erased every fourth digit from the resulting list (that is, the 4th, 8th, 12th, . . . digits from the left in what remained), and then erased every fifth digit from what remained at that point. What is the sum of the three digits that were then in the positions 2019, 2020, 2021?

- (A) 7      (B) 9      (C) 10      (D) 11      (E) 12

Solution

Problem 16

Bela and Jenn play the following game on the closed interval  $[0, n]$  of the real number line, where  $n$  is a fixed integer greater than 4. They take turns playing, with Bela going first. At his first turn, Bela chooses any real number in the interval  $[0, n]$ . Thereafter, the player whose turn it is chooses a real number that is more than one unit away from all numbers previously chosen by either player. A player unable to choose such a number loses. Using optimal strategy, which player will win the game?

- (A) Bela will always win.      (B) Jenn will always win.      (C) Bela will win if and only if  $n$  is odd.  
(D) Jenn will win if and only if  $n$  is odd.      (E) Jenn will win if and only if  $n > 8$ .

Solution

Problem 17

There are 10 people standing equally spaced around a circle. Each person knows exactly 3 of the other 9 people: the 2 people standing next to her or him, as well as the person directly across the circle. How many ways are there for the 10 people to split up into 5 pairs so that the members of each pair know each other?

- (A) 11      (B) 12      (C) 13      (D) 14      (E) 15

Solution

Problem 18

An urn contains one red ball and one blue ball. A box of extra red and blue balls lie nearby. George performs the following operation four times: he draws a ball from the urn at random and then takes a ball of the same color from the box and returns those two matching balls to the urn. After the four iterations the urn contains six balls. What is the probability that the urn contains three balls of each color?

- (A)  $\frac{1}{6}$       (B)  $\frac{1}{5}$       (C)  $\frac{1}{4}$       (D)  $\frac{1}{3}$       (E)  $\frac{1}{2}$

Solution

Problem 19

In a certain card game, a player is dealt a hand of 10 cards from a deck of 52 distinct cards. The number of distinct (unordered) hands that can be dealt to the player can be written as  $158A00A4AA0$ . What is the digit  $A$ ?

- (A) 2      (B) 3      (C) 4      (D) 6      (E) 7

Solution

Problem 20

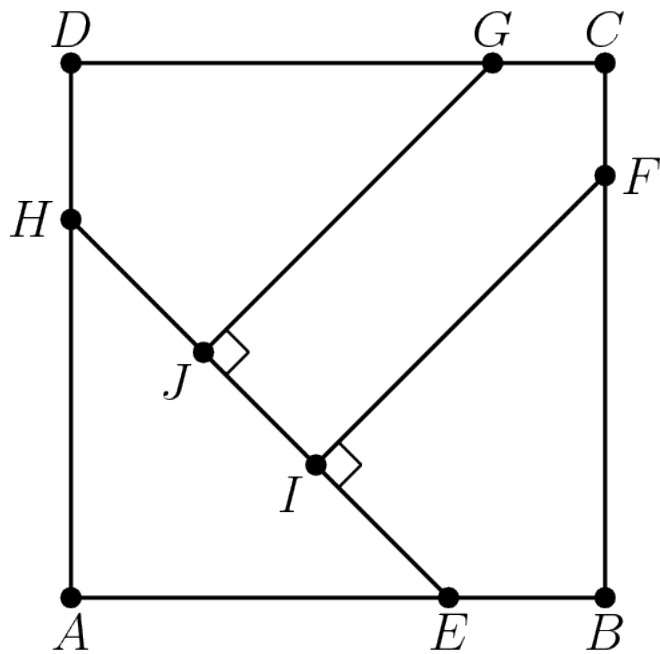
Let  $B$  be a right rectangular prism (box) with edges lengths 1, 3, and 4, together with its interior. For real  $r \geq 0$ , let  $S(r)$  be the set of points in 3-dimensional space that lie within a distance  $r$  of some point  $B$ . The volume of  $S(r)$  can be expressed as  $ar^3 + br^2 + cr + d$ , where  $a, b, c$ , and  $d$  are positive real numbers. What is  $\frac{bc}{ad}$ ?

- (A) 6      (B) 19      (C) 24      (D) 26      (E) 38

Solution

Problem 21

In square  $ABCD$ , points  $E$  and  $H$  lie on  $\overline{AB}$  and  $\overline{DA}$ , respectively, so that  $AE = AH$ . Points  $F$  and  $G$  lie on  $\overline{BC}$  and  $\overline{CD}$ , respectively, and points  $I$  and  $J$  lie on  $\overline{EH}$  so that  $\overline{FI} \perp \overline{EH}$  and  $\overline{GJ} \perp \overline{EH}$ . See the figure below. Triangle  $AEH$ , quadrilateral  $BFIE$ , quadrilateral  $DHJG$ , and pentagon  $FCGJI$  each has area 1. What is  $FI^2$ ?



- (A)  $\frac{7}{3}$       (B)  $8 - 4\sqrt{2}$       (C)  $1 + \sqrt{2}$       (D)  $\frac{7}{4}\sqrt{2}$       (E)  $2\sqrt{2}$

Solution

### Problem 22

What is the remainder when  $2^{202} + 202$  is divided by  $2^{101} + 2^{51} + 1$ ?

- (A) 100      (B) 101      (C) 200      (D) 201      (E) 202

Solution

### Problem 23

Square  $ABCD$  in the coordinate plane has vertices at the points  $A(1, 1)$ ,  $B(-1, 1)$ ,  $C(-1, -1)$ , and  $D(1, -1)$ . Consider the following four transformations:

- $L$ , a rotation of  $90^\circ$  counterclockwise around the origin;
- $R$ , a rotation of  $90^\circ$  clockwise around the origin;
- $H$ , a reflection across the  $x$ -axis; and
- $V$ , a reflection across the  $y$ -axis.

Each of these transformations maps the squares onto itself, but the positions of the labeled vertices will change. For example, applying  $R$  and then  $V$  would send the vertex  $A$  at  $(1, 1)$  to  $(-1, -1)$  and would send the vertex  $B$  at  $(-1, 1)$  to itself. How many sequences of 20 transformations chosen from  $\{L, R, H, V\}$  will send all of the labeled vertices back to their original positions? (For example,  $R, R, V, H$  is one sequence of 4 transformations that will send the vertices back to their original positions.)

- (A)  $2^{37}$       (B)  $3 \cdot 2^{36}$       (C)  $2^{38}$       (D)  $3 \cdot 2^{37}$       (E)  $2^{39}$

Solution

### Problem 24

How many positive integers  $n$  satisfy

$$\frac{n + 1000}{70} = \lfloor \sqrt{n} \rfloor?$$

(Recall that  $\lfloor x \rfloor$  is the greatest integer not exceeding  $x$ .)

- (A) 2      (B) 4      (C) 6      (D) 30      (E) 32

Solution

### Problem 25

Let  $D(n)$  denote the number of ways of writing the positive integer  $n$  as a product

$$n = f_1 \cdot f_2 \cdots f_k,$$

where  $k \geq 1$ , the  $f_i$  are integers strictly greater than 1, and the order in which the factors are listed matters (that is, two representations that differ only in the order of the factors are counted as distinct). For example, the number 6 can be written as 6,  $2 \cdot 3$ , and  $3 \cdot 2$ , so  $D(6) = 3$ . What is  $D(96)$ ?

- (A) 112
- (B) 128
- (C) 144
- (D) 172
- (E) 184

Solution

See also

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