2003 AMC 10A Problems

Contents

Problem 1

What is the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers?

$\mathrm{(A) \ } 0\qquad \mathrm{(B) \ } 1\qquad \mathrm{(C) \ } 2\qquad \mathrm{(D) \ } 2003\qquad \mathrm{(E) \ } 4006$

Solution

Problem 2

Members of the Rockham Soccer League buy socks and T-shirts. Socks cost $4 per pair and each T-shirt costs $5 more than a pair of socks. Each member needs one pair of socks and a shirt for home games and another pair of socks and a shirt for away games. If the total cost is $2366, how many members are in the League?

$\mathrm{(A) \ } 77\qquad \mathrm{(B) \ } 91\qquad \mathrm{(C) \ } 143\qquad \mathrm{(D) \ } 182\qquad \mathrm{(E) \ } 286$

Solution

Problem 3

A solid box is 15 cm by 10 cm by 8 cm. A new solid is formed by removing a cube 3 cm on a side from each corner of this box. What percent of the original volume is removed?

$\mathrm{(A) \ } 4.5\qquad \mathrm{(B) \ } 9\qquad \mathrm{(C) \ } 12\qquad \mathrm{(D) \ } 18\qquad \mathrm{(E) \ } 24$

Solution

Problem 4

It takes Mary 30 minutes to walk uphill 1 km from her home to school, but it takes her only 10 minutes to walk from school to her home along the same route. What is her average speed, in km/hr, for the round trip?

$\mathrm{(A) \ } 3\qquad \mathrm{(B) \ } 3.125\qquad \mathrm{(C) \ } 3.5\qquad \mathrm{(D) \ } 4\qquad \mathrm{(E) \ } 4.5$

Solution

Problem 5

Let d and e denote the solutions of $2x^{2}+3x-5=0$. What is the value of $(d-1)(e-1)$?

$\mathrm{(A) \ } -\frac{5}{2}\qquad \mathrm{(B) \ } 0\qquad \mathrm{(C) \ } 3\qquad \mathrm{(D) \ } 5\qquad \mathrm{(E) \ } 6$

Solution

Problem 6

Define $x\heartsuit y$ to be $|x-y|$ for all real numbers x and y. Which of the following statements is not true?

$\mathrm{(A) \ } x\heartsuit y=y\heartsuit x$ for all x and y

$\mathrm{(B) \ } 2(x\heartsuit y)=(2x)\heartsuit(2y)$ for all x and y

$\mathrm{(C) \ } x\heartsuit 0=x$ for all x

$\mathrm{(D) \ } x\heartsuit x=0$ for all x

$\mathrm{(E) \ } x\heartsuit y>0$ if $x\neq y$

Solution

Problem 7

How many non-congruent triangles with perimeter 7 have integer side lengths?

$\mathrm{(A) \ } 1\qquad \mathrm{(B) \ } 2\qquad \mathrm{(C) \ } 3\qquad \mathrm{(D) \ } 4\qquad \mathrm{(E) \ } 5$

Solution

Problem 8

What is the probability that a randomly drawn positive factor of 60 is less than 7?

$\mathrm{(A) \ } \frac{1}{10}\qquad \mathrm{(B) \ } \frac{1}{6}\qquad \mathrm{(C) \ } \frac{1}{4}\qquad \mathrm{(D) \ } \frac{1}{3}\qquad \mathrm{(E) \ } \frac{1}{2}$

Solution

Problem 9

Simplify

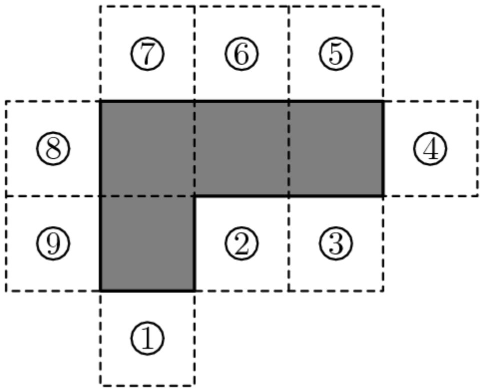
$$\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\sqrt{x}}}}.$$

$\mathrm{(A) \ } \sqrt{x}\qquad \mathrm{(B) \ } \sqrt[3]{x^{2}}\qquad \mathrm{(C) \ } \sqrt[27]{x^{2}}\qquad \mathrm{(D) \ } \sqrt[54]{x}\qquad \mathrm{(E) \ } \sqrt[81]{x^{80}}$

Solution

Problem 10

The polygon enclosed by the solid lines in the figure consists of 4 congruent squares joined edge-to-edge. One more congruent square is attached to an edge at one of the nine positions indicated. How many of the nine resulting polygons can be folded to form a cube with one face missing?



$\mathrm{(A)}\ 2\qquad\mathrm{(B)}\ 3\qquad\mathrm{(C)}\ 4\qquad\mathrm{(D)}\ 5\qquad\mathrm{(E)}\ 6$

Solution

Problem 11

The sum of the two 5-digit numbers $AMC10$ and $AMC12$ is 123422. What is $A+M+C$?

$\mathrm{(A)}\ 10\qquad\mathrm{(B)}\ 11\qquad\mathrm{(C)}\ 12\qquad\mathrm{(D)}\ 13\qquad\mathrm{(E)}\ 14$

Solution

Problem 12

A point (x,y) is randomly picked from inside the rectangle with vertices $(0,0)$, $(4,0)$, $(4,1)$, and $(0,1)$. What is the probability that $x<y$?

$\mathrm{(A)}\ \frac{1}{8}\qquad\mathrm{(B)}\ \frac{1}{4}\qquad\mathrm{(C)}\ \frac{3}{8}\qquad\mathrm{(D)}\ \frac{1}{2}\qquad\mathrm{(E)}\ \frac{3}{4}$

Solution

Problem 13

The sum of three numbers is 20. The first is four times the sum of the other two. The second is seven times the third. What is the product of all three?

$\mathrm{(A)}\ 28\qquad\mathrm{(B)}\ 40\qquad\mathrm{(C)}\ 100\qquad\mathrm{(D)}\ 400\qquad\mathrm{(E)}\ 800$

Solution

Problem 14

Let n be the largest integer that is the product of exactly 3 distinct prime numbers d, e, and $10d+e$, where d and e are single digits. What is the sum of the digits of n?

$\mathrm{(A)}\ 12\qquad\mathrm{(B)}\ 15\qquad\mathrm{(C)}\ 18\qquad\mathrm{(D)}\ 21\qquad\mathrm{(E)}\ 24$

Solution

Problem 15

What is the probability that an integer in the set $\{1,2,3,...,100\}$ is divisible by 2 and not divisible by 3?

$\mathrm{(A)}\ \frac{1}{6}\qquad\mathrm{(B)}\ \frac{33}{100}\qquad\mathrm{(C)}\ \frac{17}{50}\qquad\mathrm{(D)}\ \frac{1}{2}\qquad\mathrm{(E)}\ \frac{18}{25}$

Solution

Problem 16

What is the units digit of 13^{2003}?

$\text{(A) } 1 \qquad \text{(B) } 3 \qquad \text{(C) } 7 \qquad \text{(D) } 8 \qquad \text{(E) } 9$

Solution

Problem 17

The number of inches in the perimeter of an equilateral triangle equals the number of square inches in the area of its circumscribed circle. What is the radius, in inches, of the circle?

$\text{(A) } \frac{3\sqrt{2}}{\pi} \qquad \text{(B) } \frac{3\sqrt{3}}{\pi} \qquad \text{(C) } \sqrt{3} \qquad \text{(D) } \frac{6}{\pi} \qquad \text{(E) } \sqrt{3}\pi$

Solution

Problem 18

What is the sum of the reciprocals of the roots of the equation

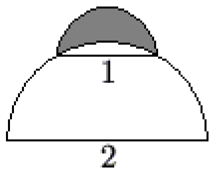
$\frac{2003}{2004}x + 1 + \frac{1}{x} = 0$?

$\text{(A) } -\frac{2004}{2003} \qquad \text{(B) } -1 \qquad \text{(C) } \frac{2003}{2004} \qquad \text{(D) } 1 \qquad \text{(E) } \frac{2004}{2003}$

Solution

Problem 19

A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a *prune*. Determine the area of this lune.



$\text{(A) } \frac{1}{6}\pi - \frac{\sqrt{3}}{4} \qquad \text{(B) } \frac{\sqrt{3}}{4} - \frac{1}{12}\pi \qquad \text{(C) } \frac{\sqrt{3}}{4} - \frac{1}{24}\pi \qquad \text{(D) } \frac{\sqrt{3}}{4} + \frac{1}{24}\pi \qquad \text{(E) } \frac{\sqrt{3}}{4} + \frac{1}{12}\pi$

Solution

Problem 20

A base-10 three digit number n is selected at random. Which of the following is closest to the probability that the base-9 representation and the base-11 representation of n are both three-digit numerals?

$\text{(A) } 0.3 \qquad \text{(B) } 0.4 \qquad \text{(C) } 0.5 \qquad \text{(D) } 0.6 \qquad \text{(E) } 0.7$

Solution

Problem 21

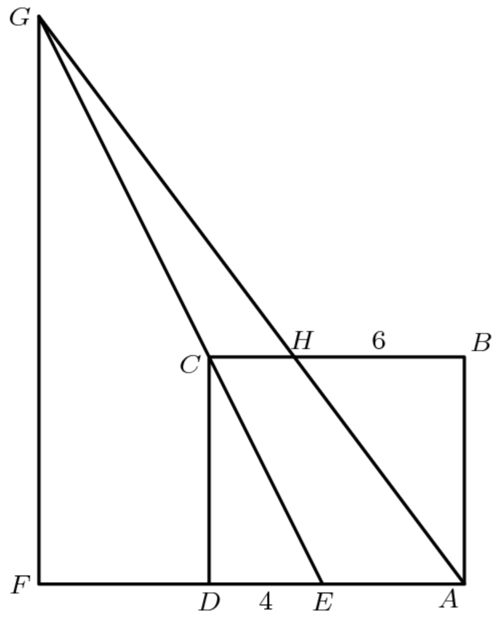
Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

$\text{(A) } 22 \qquad \text{(B) } 25 \qquad \text{(C) } 27 \qquad \text{(D) } 28 \qquad \text{(E) } 729$

Solution

Problem 22

In rectangle $ABCD$, we have $AB = 8$, $BC = 9$, H is on BC with $BH = 6$, E is on AD with $DE = 4$, line EC intersects line AH at G, and F is on line AD with $GF \perp AF$. Find the length of GF.

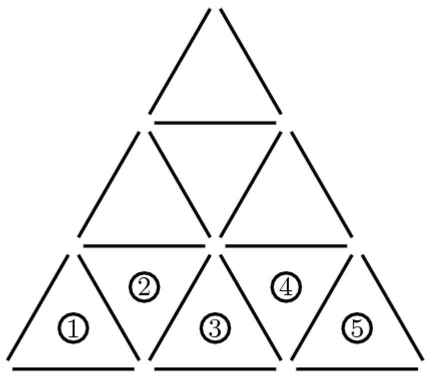


$(\mathrm{A})\ 16 \qquad (\mathrm{B})\ 20 \qquad (\mathrm{C})\ 24 \qquad (\mathrm{D})\ 28 \qquad (\mathrm{E})\ 30$

Solution

Problem 23

A large equilateral triangle is constructed by using toothpicks to create rows of small equilateral triangles. For example, in the figure we have 3 rows of small congruent equilateral triangles, with 5 small triangles in the base row. How many toothpicks would be needed to construct a large equilateral triangle if the base row of the triangle consists of 2003 small equilateral triangles?



$(\mathrm{A})\ 1,004,004 \qquad (\mathrm{B})\ 1,005,006 \qquad (\mathrm{C})\ 1,507,509 \qquad (\mathrm{D})\ 3,015,018 \qquad (\mathrm{E})\ 6,021,018$

Solution

Problem 24

Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

$(\mathrm{A})\ 8 \qquad (\mathrm{B})\ 9 \qquad (\mathrm{C})\ 10 \qquad (\mathrm{D})\ 11 \qquad (\mathrm{E})\ 12$

Solution

Problem 25

Let n be a 5-digit number, and let q and r be the quotient and the remainder, respectively, when n is divided by 100. For how many values of n is $q+r$ divisible by 11?

$(\mathrm{A})\ 8180 \qquad (\mathrm{B})\ 8181 \qquad (\mathrm{C})\ 8182 \qquad (\mathrm{D})\ 9000 \qquad (\mathrm{E})\ 9090$

Solution

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