2007 AMC 10B Problems

Contents

Problem 1

Isabella's house has 3 bedrooms. Each bedroom is 12 feet long, 10 feet wide, and 8 feet high. Isabella must paint the walls of all the bedrooms. Doorways and windows, which will not be painted, occupy 60 square feet in each bedroom. How many square feet of walls must be painted?

$\mathrm{(A)}\ 678 \qquad \mathrm{(B)}\ 768 \qquad \mathrm{(C)}\ 786 \qquad \mathrm{(D)}\ 867 \qquad \mathrm{(E)}\ 876$

Solution

Problem 2

Define the operation $\star$ by $a \star b = (a+b)b.$ What is $(3 \star 5) - (5 \star 3)?$

$\mathrm{(A)}\ -16 \qquad \mathrm{(B)}\ -8 \qquad \mathrm{(C)}\ 0 \qquad \mathrm{(D)}\ 8 \qquad \mathrm{(E)}\ 16$

Solution

Problem 3

A college student drove his compact car 120 miles home for the weekend and averaged 30 miles per gallon. On the return trip the student drove his parents' SUV and averaged only 20 miles per gallon. What was the average gas mileage, in miles per gallon, for the round trip?

$\mathrm{(A)}\ 22 \qquad \mathrm{(B)}\ 24 \qquad \mathrm{(C)}\ 25 \qquad \mathrm{(D)}\ 26 \qquad \mathrm{(E)}\ 28$

Solution

Problem 4

The point O is the center of the circle circumscribed about $\triangle ABC,$ with $\angle BOC = 120^\circ$ and $\angle AOB = 140^\circ.$ What is the degree measure of $\angle ABC?$

$\mathrm{(A)}\ 35 \qquad \mathrm{(B)}\ 40 \qquad \mathrm{(C)}\ 45 \qquad \mathrm{(D)}\ 50 \qquad \mathrm{(E)}\ 60$

Solution

Problem 5

In a certain land, all Arogs are Brafs, all Crups are Brafs, all Dramps are Arogs, and all Crups are Dramps. Which of the following statements is implied by these facts?

$\textbf{(A)}$ All Dramps are Brafs and are Crups.
$\textbf{(B)}$ All Brafs are Crups and are Dramps.
$\textbf{(C)}$ All Arogs are Crups and are Dramps.
$\textbf{(D)}$ All Crups are Arogs and are Brafs.
$\textbf{(E)}$ All Arogs are Dramps and some Arogs may not be Crups.

Solution

Problem 6

The 2007 AMC10 will be scored by awarding 6 points for each correct response, 0 points for each incorrect response, and 1.5 points for each problem left unanswered. After looking over the 25 problems, Sarah has decided to attempt the first 22 and leave only the last 3 unanswered. How many of the first 22 problems must she solve correctly in order to score at least 100 points?

$\textbf{(A)}\ 13 \qquad\textbf{(B)}\ 14 \qquad\textbf{(C)}\ 15 \qquad\textbf{(D)}\ 16 \qquad\textbf{(E)}\ 17$

Solution

Problem 7

All sides of the convex pentagon $ABCDE$ are of equal length, and $\angle A = \angle B = 90^\circ$. What is the degree measure of $\angle E?$

$\textbf{(A)}\ 90 \qquad\textbf{(B)}\ 108 \qquad\textbf{(C)}\ 120 \qquad\textbf{(D)}\ 144 \qquad\textbf{(E)}\ 150$

Solution

Problem 8

On the trip home from the meeting where this AMC10 was constructed, the Contest Chair noted that his airport parking receipt had digits of the form $bbcac,$ where $0 \le a < b < c \le 9,$ and b was the average of a and $c.$ How many different five-digit numbers satisfy all these properties?

$\textbf{(A)}\ 12 \qquad\textbf{(B)}\ 16 \qquad\textbf{(C)}\ 18 \qquad\textbf{(D)}\ 20 \qquad\textbf{(E)}\ 25$

Solution

Problem 9

A cryptographic code is designed as follows. The first time a letter appears in a given message it is replaced by the letter that is 1 place to its right in the alphabet (assuming that the letter A is one place to the right of the letter Z). The second time this same letter appears in the given message, it is replaced by the letter that is $1+2$ places to the right, the third time it is replaced by the letter that is $1+2+3$ places to the right, and so on. For example, with this code the word "banana" becomes "cbodqg". What letter will replace the last letter s in the message

$$\text{"Lee's sis is a Mississippi miss, Chriss!"?}$$

$\textbf{(A)}\ g \qquad\textbf{(B)}\ h \qquad\textbf{(C)}\ o \qquad\textbf{(D)}\ s \qquad\textbf{(E)}\ t$

Solution

Problem 10

Two points B and C are in a plane. Let S be the set of all points A in the plane for which $\triangle ABC$ has area $1.$ Which of the following describes $S?$

$\textbf{(A)}\ \text{two parallel lines} \qquad\textbf{(B)}\ \text{a parabola} \qquad\textbf{(C)}\ \text{a circle} \qquad\textbf{(D)}\ \text{a line segment} \qquad\textbf{(E)}\ \text{two points}$

Solution

Problem 11

A circle passes through the three vertices of an isosceles triangle that has two sides of length 3 and a base of length 2. What is the area of this circle?

$\textbf{(A)}\ 2\pi \qquad \textbf{(B)}\ \frac{5}{2}\pi \qquad \textbf{(C)}\ \frac{81}{32}\pi \qquad \textbf{(D)}\ 3\pi \qquad \textbf{(E)}\ \frac{7}{2}\pi$

Solution

Problem 12

Tom's age is T years, which is also the sum of the ages of his three children. His age N years ago was twice the sum of their ages then. What is T/N?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Solution

Problem 13

Two circles of radius 2 are centered at $(2,0)$ and at $(0,2)$. What is the area of the intersection of the interiors of the two circles?

$\textbf{(A)}\ \pi-2 \qquad \textbf{(B)}\ \frac{\pi}{2} \qquad \textbf{(C)}\ \frac{\pi\sqrt{3}}{3} \qquad \textbf{(D)}\ 2(\pi-2) \qquad \textbf{(E)}\ \pi$

Solution

Problem 14

Some boys and girls are having a car wash to raise money for a class trip to China. Initially 40% of the group are girls. Shortly thereafter two girls leave and two boys arrive, and then 30% of the group are girls. How many girls were initially in the group?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 12$

Solution

Problem 15

The angles of quadrilateral $ABCD$ satisfy $\angle A = 2\angle B = 3\angle C = 4\angle D$. What is the degree measure of $\angle A$, rounded to the nearest whole number?

$\textbf{(A)}\ 125 \qquad \textbf{(B)}\ 144 \qquad \textbf{(C)}\ 153 \qquad \textbf{(D)}\ 173 \qquad \textbf{(E)}\ 180$

Solution

Problem 16

A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each of the juniors receive on the test?

$\textbf{(A)}\ 85 \qquad \textbf{(B)}\ 88 \qquad \textbf{(C)}\ 93 \qquad \textbf{(D)}\ 94 \qquad \textbf{(E)}\ 98$

Solution

Problem 17

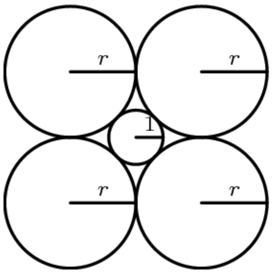
Point P is inside equilateral $\triangle ABC$. Points Q, R, and S are the feet of the perpendiculars from P to $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively. Given that $PQ=1$, $PR=2$, and $PS=3$, what is AB?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 3\sqrt{3} \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 4\sqrt{3} \qquad \textbf{(E)}\ 9$

Solution

Problem 18

A circle of radius 1 is surrounded by 4 circles of radius r as shown. What is r?

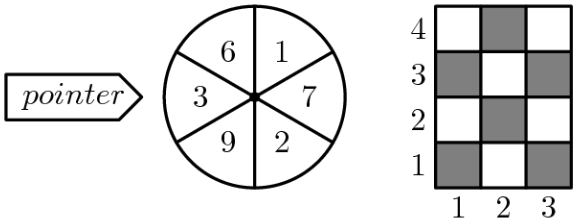


$\textbf{(A) } \sqrt{2} \qquad \textbf{(B) } 1+\sqrt{2} \qquad \textbf{(C) } \sqrt{6} \qquad \textbf{(D) } 3 \qquad \textbf{(E) } 2+\sqrt{2}$

Solution

Problem 19

The wheel shown is spun twice, and the randomly determined numbers opposite the pointer are recorded. The first number is divided by $4,$ and the second number is divided by $5.$ The first remainder designates a column, and the second remainder designates a row on the checkerboard shown. What is the probability that the pair of numbers designates a shaded square?



$\textbf{(A) } \frac{1}{3} \qquad \textbf{(B) } \frac{4}{9} \qquad \textbf{(C) } \frac{1}{2} \qquad \textbf{(D) } \frac{5}{9} \qquad \textbf{(E) } \frac{2}{3}$

Solution

Problem 20

A set of 25 square blocks is arranged into a 5×5 square. How many different combinations of 3 blocks can be selected from that set so that no two are in the same row or column?

$\textbf{(A) } 100 \qquad \textbf{(B) } 125 \qquad \textbf{(C) } 600 \qquad \textbf{(D) } 2300 \qquad \textbf{(E) } 3600$

Solution

Problem 21

Right $\triangle ABC$ has $AB = 3, BC = 4,$ and $AC = 5.$ Square $XYZW$ is inscribed in $\triangle ABC$ with X and Y on $\overline{AC}, W$ on $\overline{AB},$ and Z on $\overline{BC}.$ What is the side length of the square?

$\textbf{(A) } \frac{3}{2} \qquad \textbf{(B) } \frac{60}{37} \qquad \textbf{(C) } \frac{12}{7} \qquad \textbf{(D) } \frac{23}{13} \qquad \textbf{(E) } 2$

Solution

Problem 22

A player chooses one of the numbers 1 through $4.$ After the choice has been made, two regular four-sided (tetrahedral) dice are rolled, with the sides of the dice numbered 1 through $4.$ If the number chosen appears on the bottom of exactly one die after it has been rolled, then the player wins 1 dollar. If the number chosen appears on the bottom of both of the dice, then the player wins 2 dollars. If the number chosen does not appear on the bottom of either of the dice, the player loses 1 dollar. What is the expected return to the player, in dollars, for one roll of the dice?

$\textbf{(A) } -\frac{1}{8} \qquad \textbf{(B) } -\frac{1}{16} \qquad \textbf{(C) } 0 \qquad \textbf{(D) } \frac{1}{16} \qquad \textbf{(E) } \frac{1}{8}$

Solution

Problem 23

A pyramid with a square base is cut by a plane that is parallel to its base and 2 units from the base. The surface area of the smaller pyramid that is cut from the top is half the surface area of the original pyramid. What is the altitude of the original pyramid?

$\textbf{(A)}\ 2 \qquad\textbf{(B)}\ 2+\sqrt{2} \qquad\textbf{(C)}\ 1+2\sqrt{2} \qquad\textbf{(D)}\ 4 \qquad\textbf{(E)}\ 4+2\sqrt{2}$

Solution

Problem 24

Let n denote the smallest positive integer that is divisible by both 4 and $9,$ and whose base-10 representation consists of only 4's and 9's, with at least one of each. What are the last four digits of $n?$

$\textbf{(A)}\ 4444 \qquad\textbf{(B)}\ 4494 \qquad\textbf{(C)}\ 4944 \qquad\textbf{(D)}\ 9444 \qquad\textbf{(E)}\ 9944$

Solution

Problem 25

How many pairs of positive integers (a,b) are there such that a and b have no common factors greater than 1 and

$$\frac{a}{b} + \frac{14b}{9a}$$

is an integer?

$\textbf{(A)}\ 4 \qquad\textbf{(B)}\ 6 \qquad\textbf{(C)}\ 9 \qquad\textbf{(D)}\ 12 \qquad\textbf{(E)}\ \text{infinitely many}$

Solution

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