NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business Examination

2020-21

Examination Session	**Time Allowed**
Semester Two	3 hours 40 minutes

INSTRUCTIONS TO STUDENTS

SECTION A Answer ALL questions. This section carries 45 marks.

SECTION B Answer 4 questions ONLY. This section carries 80 marks.

The marks for each question are indicated in square brackets [ ].

- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

The line with equation $5x + 3y - c = 0$ crosses the $x-$ axis at point X and the $y-$ axis at point Y.

The area of triangle OYX, where O is the origin, is 120 square units.

Find the values of c. **[4]**

Question A2

The probability that it rains on a given day is 0.4.

Find the probability that it does <u>not</u> rain for 7 consecutive days. **[4]**

Give your answer as a decimal to **3** significant figures.

In this question, 1 mark will be given for the correct use of significant figures.

Question A3

A quadratic equation is defined as $ax^2 + 9x + 3 = 0$ where a is an integer.

Find the largest possible value of a if the equation has two real distinct roots. **[3]**

Question A4

In the expansion of $(m + 4x)^8$ the coefficient of the x^3 term is 112.

Find the value of m. **[3]**

Question A5

(a) Write $2\log_n\left(\frac{3}{2}\right) + \log_n\left(\frac{10}{3}\right) - \log_n(\frac{3}{4})$ as a single logarithm. **[2]**

You are given the value of the expression in part (a) is equal to 1.

(b) State the value of n. **[1]**

Question A6

Solve $\sin(\frac{5}{2}\theta) = -0.966 \quad (0° \leq \theta \leq 360°)$ **[5]**

Question A7

If $y = 56x^2 + 3\ln x - 4e^{4x}$, find the <u>exact</u> value of $\frac{d^2y}{dx^2}$ when $x = \frac{1}{4}$. **[4]**

Question A8

Find **[3]**

$$\int (10x^2 - 3)^2 \ dx.$$

Question A9

Eight readings are shown below in ascending order (smallest to largest).

$$-1,\ 0,\ 2,\ x,\ 10,\ 15,\ 17,\ 23.$$

(a) Write down the mean in terms of x. **[1]**

You are given the median is 8.

(b) Find the standard deviation. **[3]**

Question A10

The probability a student has blue eyes is p.

Five students are selected at random.

Write down, in terms of p, the probability that exactly 3 or exactly 4 of the students have blue eyes.

Give your answer in the form $5p^3(p - a)(p - b)$ where a and b are integers. **[4]**

Question A11

A curve has equation $y = \ln(\tan x)$

Write down $\frac{dy}{dx}$ and hence find its value when $x = \frac{\pi}{4}$. **[3]**

Question A12

A curve has equation $y = \frac{1 - x}{x^2 + 8}$.

<u>Use the Quotient Rule</u> to find $\frac{dy}{dx}$ and hence find the coordinates of the stationary values. **[5]**

Section B
Answer <u>4</u> questions ONLY. This section carries 80 marks.

Question B1

(a) Solve the equations $3c - 5d = 5$ **[4]**

$$9c + 10d = 0$$

All working must be shown: just quoting the answers, even the correct ones, will score no marks if this working is not seen.

(b) The expression $x^2 - 7x + m$ is divided by $(x - k)$.

i. Write down the remainder in terms of m and k. **[1]**

You are given the remainder is -3 and $m = \frac{1}{2}k$.

ii. Find the possible values of k. **[3]**

(c) i. Factorise $63x^2 + 5x - 2$. **[2]**

ii. Hence, or otherwise, solve the inequality $63x^2 + 5x - 2 \geq 0$.

Give your critical values in the form $\frac{m}{n}$ where m and n are integers. **[2]**

(d) An arithmetic series has first term a and common difference d.

The 9th term is 7 times larger than the first term.

i. Show that $d = \frac{3}{4}a$. **[2]**

The sum of the first 40 terms is 5000.

ii. Find the value of d and the value of a. **[3]**

(e) A geometric series is defined as $\frac{1}{4}p + \frac{1}{8}p^2 + \frac{1}{16}p^3 + \ldots$

Write down the n^{th} term in terms of p in its simplest form. **[3]**

Question B2

(a) The variables p and q are connected by the formula

$$p = 2^{2q} - 12(2^q) + 40.$$

i. Find the value of p when $q = -1$. **[2]**

ii. Find the values of q when $p = 8$. **[3]**

iii. What happens to p when q becomes large and negative? **[1]**

(b) m (where $m \neq 0$) is a real number.

i. Prove that $m^0 = 1$. **[1]**

ii. Find the value of t if

$$\frac{30t^8 - 6t^8}{\sqrt[3]{8t^{12}} \times 4t} = \frac{1}{72}$$ **[3]**

Part (c) is on the next page.

Question B2 – (continued)

(c)

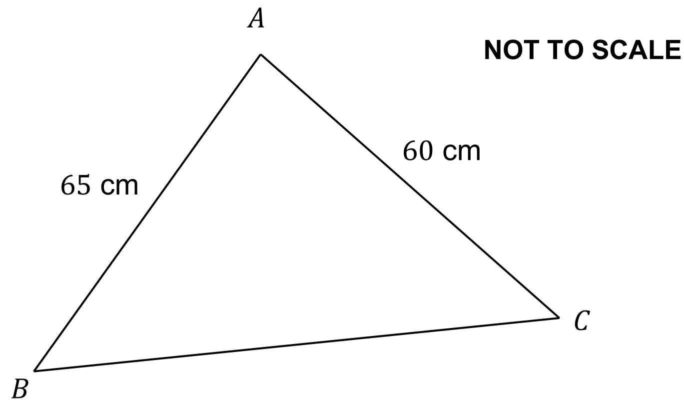

Figure 1

Figure 1 shows the acute-angled triangle ABC with $AB = 65$ cm and AC =60 cm. The area of triangle ABC is 1800 cm^2.

i. Find the value of $\sin A$, giving your answer in the form $\frac{c}{d}$ where c and d are integers. **[2]**

ii. Show that $\cos A = \frac{5}{13}$. **[1]**

iii. Find the length of BC. Give your answer in the form $5\sqrt{k}$ where k is an integer. **[3]**

iv. Find angle B. **[2]**

v. Find the shortest distance from B to AC. *Show your working*. **[1]**

vi. Find the shortest distance from B to AC using a different method. *Again, show your working*. **[1]**

Question B3

(a)

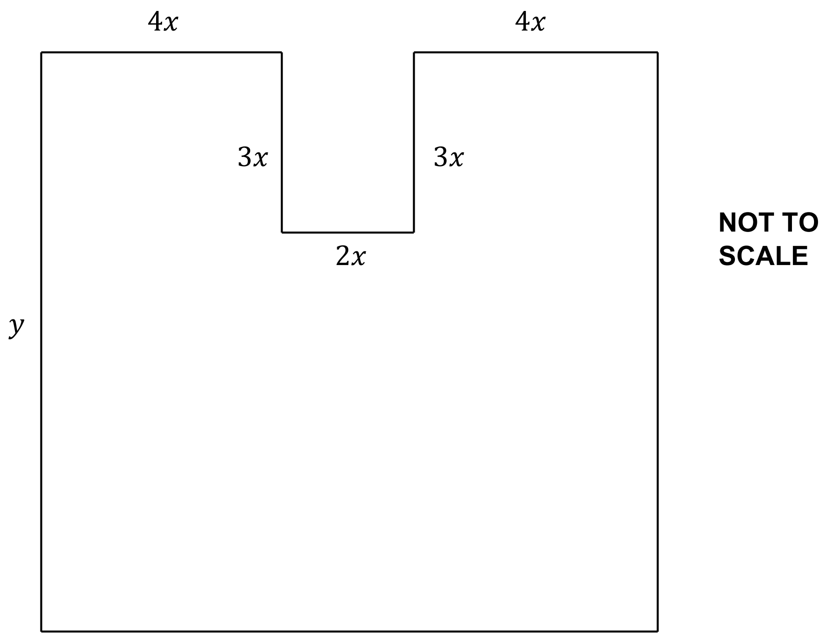

Figure 2

Figure 2 shows a rectangular field which has a smaller rectangle removed from one side. All measurements are in metres.

The perimeter of the field is 408 metres.

i. Express y in terms of x. **[2]**

ii. Show that the area of the field, A, is given by **[3]**

$$A = 2040x - 136x^2$$

iii. Use calculus to find the value of x which makes the area a maximum. **[3]**

iv. Use calculus to confirm your value of x gives a maximum. **[3]**

v. Find this maximum area. **[1]**

(b) Find **[3]**

$$\int (12 - 8x)e^{2x}\, dx.$$

Part (c) is on the next page.

(c) Question B3 – (continued)

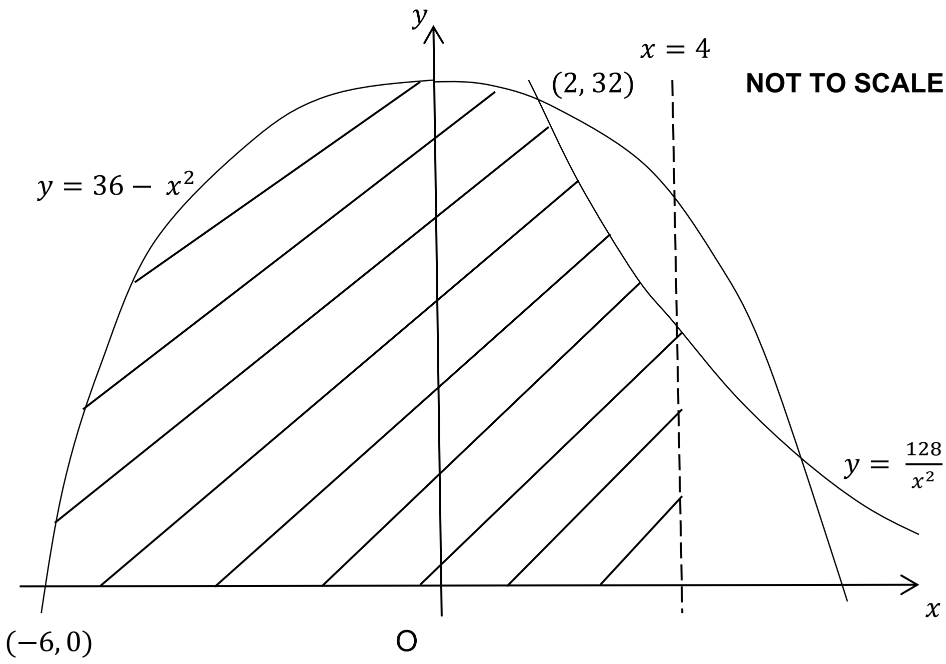

Figure 3

Figure 3 shows the curves $y = 36 - x^2$ and $y = \frac{128}{x^2}$ (for $x > 0$)

The curve $y = 36 - x^2$ crosses the $x -$ axis at $(-6, 0)$ and meets the curve $y = \frac{128}{x^2}$ at $(2, 32)$.

The line $x = 4$ is also shown.

Find the area, which is shaded on the diagram, that is bounded by both curves, the line $x = 4$ and the $x -$ axis. **[5]**

All working must be shown. Just giving the answer, even the correct one, will score no marks if this working is not seen.

Section B continues on the next page.

Question B4

(a) 112 students were asked to record how long it took them to compose their next text message. The results are shown in the table below.

Time (t) seconds	**Frequency**
$0 < t \leq 10$	6
$10 < t \leq 20$	10
$20 < t \leq 30$	18
$30 < t \leq 40$	25
$40 < t \leq 50$	23
$50 < t \leq 60$	16
$60 < t \leq 70$	10
$70 < t \leq 80$	4

(You may wish to copy this table and extend it to help you answer some of the questions below).

i. Is this an example of continuous or discrete data? Give a reason for your answer. **[2]**

ii. On graph paper, draw a cumulative frequency curve. **[4]**

iii. <u>Use your cumulative frequency curve</u> to estimate the median and interquartile range. **[3]**

You <u>must</u> show evidence that you have read these values from your graph.

(b) i. Events A and B are such that $P(A) = \frac{2}{5}$ and $P(B) = \frac{1}{2}$.

Events A and B are independent.

Find $P(A \cup B)$ and $P(A|B)$. **[3]**

ii. Draw a Venn diagram to show this information. **[2]**

(c) A student plays two games of squash. The probability that he wins the first game is $\frac{7}{10}$. If he wins the first game, the probability that he wins the second is $\frac{3}{4}$. If he does not win the first game, the probability he wins the second is $\frac{2}{3}$.

i. Draw a fully labelled tree diagram. **[2]**

ii. The student wins the second game.

Find the probability he also won the first game. **[4]**

Question B5

(a) At the beginning of 2019 the value of a house was £187200.

The value rose by 4% during 2018.

i. Find the value at the beginning of 2018. **[2]**

During 2019 the value fell by 5%.

ii. Find the value of the house at the beginning of 2020. Give your answer in full with no rounding off. **[2]**

iii. Find the overall percentage change in the value of the house over the two years. Your answer must state clearly if this is an increase or a decrease. **[2]**

(b) A boy tries to hit a coconut by throwing balls at it. He reckons that the probability of hitting it with any one ball is 0.4.

He throws 10 balls, one after the other and hits the coconut x times.

i. Find the smallest possible value of x if the probability of this happening is more than 0.75. **[1]**

ii. What important assumption is being made? **[1]**

Part (c) is on the next page.

(c) A discrete random variable, X, has probability distribution as given in the table below.

$\boldsymbol{x}$	0	k	$3k$	$5k$	$7k$
$\boldsymbol{P(X) = x}$	p	0.15	0.25	0.2	0.3

i. State the value of p. **[1]**

ii. Find $\mathrm{E}(X)$ in terms of k. **[2]**

iii. Find $\mathrm{Var}(X)$ in terms of k. **[3]**

Another random variable, Y, is connected to X by the formula

$$Y = 3X + 2k.$$

iv. Write down, in terms of k, $\mathrm{E}(Y)$ and $\mathrm{Var}(Y)$. **[2]**

(d) The masses of oranges can be assumed to follow a Normal distribution with mean 108 grams and standard deviation 6 grams.

What percentage of oranges have a mass of 117 grams or more? **[2]**

(e) The masses of lemons can also be assumed to follow a Normal distribution with standard deviation 10 grams.

A sample of 49 lemons is taken and the mean mass is found to be m grams.

The lower limit of a 95% confidence interval is 94.2 grams.

Find m and the upper limit of the confidence interval. **[2]**

Section B continues on the next page.

Question B6

(a) Curve C has equation $-3x^2 + 2xy + y^2 + \frac{1}{3}y - 1 = 0$.

i. Find $\frac{dy}{dx}$ in terms of x and y. **[4]**

ii. Show that stationary values occur when $y = 3x$. **[1]**

iii. Find the coordinates of the stationary values on curve C. **[3]**

(b) Use the substitution $u = 1 + \cos\theta$ to find

$$\int \frac{3\sin\theta}{(1+\cos\theta)^3}\, d\theta.$$ **[4]**

(c) i. Write $\frac{5x + 8}{x^2 + 3x + 2}$ in the form $\frac{A}{x+2}$ and $\frac{B}{x+1}$ where A and B are constants to be determined. **[3]**

ii. Hence evaluate

$$\int_1^2 \frac{5x + 8}{x^2 + 3x + 2}\, dx.$$

Give your answer in the form $\ln k$ where k is an integer.

All working must be shown. Just giving the answer, even the correct one, will score no marks if this working is not seen. **[5]**

- This is the end of the examination. -