NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics
Part 2 (Business) Examination

MARK SCHEME

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<u>Notice to Markers</u>

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (<u>http://www.ncuk.ac.uk</u>). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:
All <u>correct</u> answers should be rewarded regardless of the number of significant figures used, with the exception of question A7. For this question, 1 discretionary mark is available which will <u>only</u> be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

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Section A

Question A1

Mid-point lies at (-1, -6). **[B1]**

Finds gradient of PQ, $(-\frac{1}{3})$ **[M1]**

inverts and changes sign (3) **[M1]**

$y + 6 = 3(x + 1)$ or in any equivalent form. **[A1]**

Question A2

$\frac{11}{16} \times \frac{10}{15} \times \frac{9}{14}$ **(M1)** for any one fraction **[M1]**

Multiplies their fractions **[M1]**

$= \frac{33}{112}$ `or equivalent, or anything rounding to 0.295 **[A1]**

Question A3

Substitutes $x = -k$ into expression and forms a quadratic equation ($x^2 - 8k + 12 = 0$ or equivalent) **[M1*]**

Factorises or uses formula $[(k-6)(k-2) = 0$ or $k = \frac{8 \pm \sqrt{[(-8)^2 - 4 \times 1 \times 12]}}{2 \times 1}]$ **[M1]**

$k = 2$ or 6 (Both needed) **[A1]**

Question A4

$\frac{ar^5}{ar^2} = \frac{-3125}{200}$ (either way up) **[M1]**

Reaches $r^3 = \cdots$ or $\frac{1}{r^3} = \cdots$ $(-\frac{125}{8}$ or $-\frac{8}{125})$ **[M1]**

$r = -\frac{5}{2}$ or equivalent **[A1]**

Substitutes their value of r into either expression **[M1]**

$a = 32$ **[A1]**

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Question A5

$\frac{8m^6 \times 6m}{3m^3} = \frac{48m^7}{3m^3} = 16m^4 = 81$

Shows a correct knowledge anywhere how to handle indices when multiplying or dividing. **[M1]**

Shows a correct knowledge anywhere how to handle indices when using a power or a square root. **[M1]**

$m = \frac{3}{2}$ or equivalent **[A1]**

Question A6

Multiplies out and attempts to integrate (Presence of x^5, x^3 or $16x$ is sufficient for this mark) **[M1*]**

$\frac{x^5}{5} - \frac{8x^3}{3} + 16x$ (Correct answer – ignore + c) **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= \frac{123}{5}$ or equivalent **[A1]**

Please note: if the answer appears with no working, this scores 0.

Question A7

Uses correct version of cosine formula $(21^2 = 18^2 + 19^2 - 2\times18\times19\times \cos A)$ **[M1]**

Rearranges correctly to reach $\cos A = \cdots$ $(\frac{244}{684})$ **[M1]**

$A = 1.206036 \ldots$ (radians) **[A1+]**

$= 1.21$ (radians) to 3 significant figures. **[A1ft]**

+This mark can be implied if this line is not seen but 1.21 appears.
Allow follow through provided a more accurate answer is seen earlier.
Special case: if a candidate works in degrees and writes 69.1 (without having shown a more accurate answer) give M1 M1 A0 A1(ft).

Question A8

$\frac{p + 2p + (3p - 5) + (2p + 21)}{4}$ $(= 38)$ Adds expressions and divides by 4 **[M1]**

Sets equal to 38 and reaches $p = \cdots$ **[M1]**

$p = 17$ **[A1]**

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Question A9

Uses $p(B|A) = \frac{p(B \cap A)}{p(A)}$ **[M1]**

$p(A) = 0.8$ or equivalent **[A1]**

Uses $p(A \cup B) = p(A) + p(B) - p(A \cap B)$ **[M1]**

$p(B) = 0.44$ or equivalent **[A1]**

Question A10

For $p(\text{sack is below 30.7 grams}) = \frac{30.7 - 30}{0.4} (= 1.75)$ **[M1]**

Finds Φ(their 1.75) $(= 0.9599$ or $0.96)$ **[M1]**

Spots the symmetry or goes through process again to find p(sack is above or below 29.3 grams) **[M1]**

Uses any method to find required area **[M1]**

= anything rounding to 92% (must be in percentage form) **[A1]**

Question A11

Uses Product Rule in its correct form **[M1]**

$= 4^x \times \frac{1}{x} + 4^x \ln 4 \times \ln x$ **[A2]**

(A1) for one part correct; **(A2)** for all correct. Accept in any form, which does not have to be simplified.

Question A12

$5x^2 - 4x - 21 = A(x-1)^2 + B(x+4)(x-1) + C(x+4)$ **[M1]**

$A = 3$; $B = 2$; $C = -4$ **(A1)** for each **[A3]**

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Section B

Question B1

a) i. Solves by any method and finds two critical values (0 and -9) **[M1]**

$x \geq -9$ and $x \leq 0$ (A1) for each or $-9 \leq x \leq 0$ (A1) for each end. **[A2]**

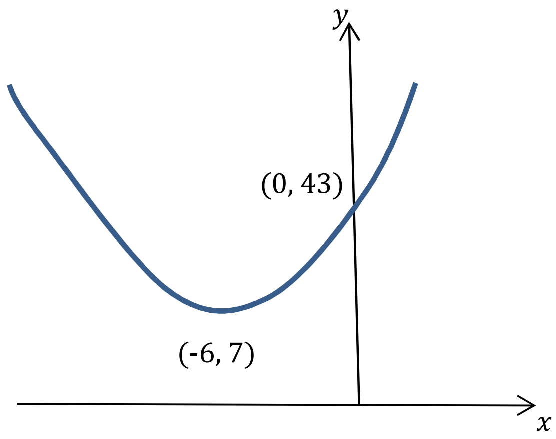
ii. Solves by any method and finds two critical values (3 and -3) **[M1]**

$x < -3$ and $x > 3$ (A1) for each **[A2]**

iii. 1, 2 and 3 **[B1]**

b) i. $(x+6)^2$ (B1) $+7$ (B1) **[B2]**

ii.



Correct shape above $x-$axis **[B1]**

(-6,7) and (0,43) shown **[B1]**

c) $(3x)^4 + {}^4C_1(3x)^3 \times(-\frac{1}{2}y) + {}^4C_2\,(3x)^2 \times(-\frac{1}{2}y)^2 + {}^4C_3\,(3x)\times(-\frac{1}{2}y)^3 + (-\frac{1}{2}y)^4$ **[B2]**

(B1) Any 2 unsimplified correct; (B2) all unsimplified correct
(Allow equivalent notation or ${}^xC_y$ for ${}^yC_x$)

$= 81x^4 - 54x^3y + \frac{27}{2}x^2y^2 - \frac{3}{2}xy^3 + \frac{1}{16}y^4$ **[B2]**

(B1) Any 2 correct; (B2) all correct (Allow coefficients in any form)

d) i. $(a = -119;\ d = 7)\ S_{87} = \frac{87}{2}[2 \times -119 + (87-1)\times 7]$ **[M1]**

Carries out correct calculation and reaches a value for S_{87} **[M1]**

$= 15834$ **[A1]**

ii. $(2k-1)-(k+4) = 33-(2k-1)$ or $\frac{(k+4)+33}{2} = 2k-1$ and attempts to solve **[M1]**

$k = 13$ **[A1]**

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Question B2

a) i. *Please note: (1) this is a 'show that' question and all working must be seen; (2) candidates who work backwards score no marks.*

$50 = 80e^{100k}$ and makes e^{100k} the subject $(= \frac{50}{80})$ **[M1*]**

Uses logs correctly and reaches $100k = \cdots$ $(\ln\left(\frac{50}{80}\right))$ **[M1*]**

$k = -0.0047(00036\ldots)$ (Both M marks scored and no errors seen) **[A1]**

ii. "Because the mass is decreasing" or "stone is eroding, becoming smaller, losing weight/mass" or any other valid explanation **[B1]**

iii. $M = 80e^{-0.0047 \times 330}$ **[M1]**

= anything rounding to 17 (grams) **[A1]**

iv. $\frac{dM}{dt} = 80 \times k \times e^{kt}$ (Please note: $80 \times kt \times e^{kt}$ is M0) **[M1]**

Substitutes $t = 100$ into their $\frac{dM}{dt}$ **[M1]**

= anything rounding to -0.235 (grams/year) **[A1]**

b) $\log_3[(x+2)(x-6)] = \log_3 9$ Uses log addition law correctly **[M1*]**

Writes RHS as a log **[M1*]**

Removes logs at the right time and forms a quadratic equation $(x^2 - 4x - 21 = 0)$ **[M1]**

Factorises or uses formula $[(x-7)(x+3) = 0 \text{ or } x = \frac{4 \pm \sqrt{[(-4)^2 - 4 \times 1 \times -21]}}{2 \times 1}]$ **[M1]**

$x = 7$ If the -3 is included, this mark is lost. (Sight of -3 in brackets is good enough to indicate non-inclusion) **[A1]**

c) i. $\frac{1}{2} \times 16 \times x \times \sin 30 = 50$ **[M1]**

$x = 12\frac{1}{2}$ (metres) or equivalent **[A1]**

ii. $\cos\theta = \pm\sqrt{0.86}$ (The $\pm$ sign is not needed for this mark) **[M1]**

$\theta =$ anything rounding to 22, 158, 202, 338 (degrees)

(A1) any one correct; **(A2)** any two correct; **(A3)** all four correct. **[A3]**

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Question B3

a) i. $4h + 2(3x + 5x + 3x + 5x) = 168$ **[M1]**

$h = \frac{168 - 32x}{4}$ or $42 - 8x$ **[A1]**

ii. *Please note: this is a 'show that' question so all working must be seen.*

Uses $V = 3x \times 5x \times h$ **[M1*]**

$= 15x^2(\text{their } h)$ **[M1*]**

$630x^2 - 120x^3$ **[A1]**

iii. $\frac{dV}{dx} = 1260x - 360x^2$ Attempts to differentiate (sight of x or x^2 is sufficient for this mark) **[M1*]**

Correct answer **[A1]**

Sets their $\frac{dV}{dx}$ equal to 0 and finds a value for x. **[M1]**

$x = \frac{7}{2}$ or equivalent (Ignore any reference to 0) **[A1]**

iv. $\frac{d^2 V}{dx^2} = 1260 - 720x$ Attempts to differentiate a second time (sight of a constant term or the x is sufficient for this mark) **[M1*]**

Correct answer **[A1]**

This is negative (when $x = \frac{7}{2}$) so there is a maximum (or similar reason and conclusion) Allow follow through if their $\frac{d^2 V}{dx^2}$ is negative for their value of x. **[A1ft]**

or

takes a numerical value below $\frac{7}{2}$ but above 0 and shows $\frac{dV}{dx} > 0$ **(M1*)**

takes a numerical value above $\frac{7}{2}$ and shows $\frac{dV}{dx} < 0$ **(M1*)**

There is a maximum (at = $\frac{7}{2}$) or similar conclusion. **(A1ft)**

Allow follow through for their value of x and their value of $\frac{dV}{dx}$ provided a maximum occurs.

Part b) is on the next page.

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Question B3 – (continued)

b) i. *Please note: this is a 'show that' question so all working must be seen*.

$\frac{dy}{dx} = -\frac{1}{x^2} = -4$ when $x = \frac{1}{2}$ **[M1*]**

So gradient of normal is $(+)\frac{1}{4}$ which is not the gradient of $y = 4x$ (or similar words). (M mark scored and no errors seen.) **[A1]**

ii. Area of triangular part $= \frac{1}{2}$. **[B1]**

Rest of area = $\int_{1/2}^{a} \frac{1}{x}\ dx$ (Limits must be correct here) **[M1*]**

Substitutes limits into their integrated expression $(\ln x)$ and subtracts the right way round. This mark can be given as long as the integrand has undergone some change. **[M1]**

Sets their whole area equal to $\frac{1}{2} + \ln 5$ **[M1]**

Handles the logs correctly and reaches an equation with a single log on both sides. **[M1]**

$a = \frac{5}{2}$ or equivalent. **[A1]**

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Question B4

a) i.

Age	Frequency f	Mid-value x	$f \times x$	$f \times x^2$
$4 - 6$	12	5	60	300
$7 - 9$	10	8	80	640
$10 - 12$	7	11	77	847
$13 - 15$	6	14	84	1176
$16 - 20$	5	18	90	1620
	$\sum f = 40$		$\sum fx = 391$	$\sum fx^2 = 4583$

Finds $\sum fx$ **[M1]**

Divides their $\sum fx$ by 40 **[M1]**

= any answer rounding to 9.78 **[A1]**

Finds $\sum fx^2$ **[M1]**

Divides their $\sum fx^2$ by 40, subtracts (their mean)^2 and takes square root. **[M1]**

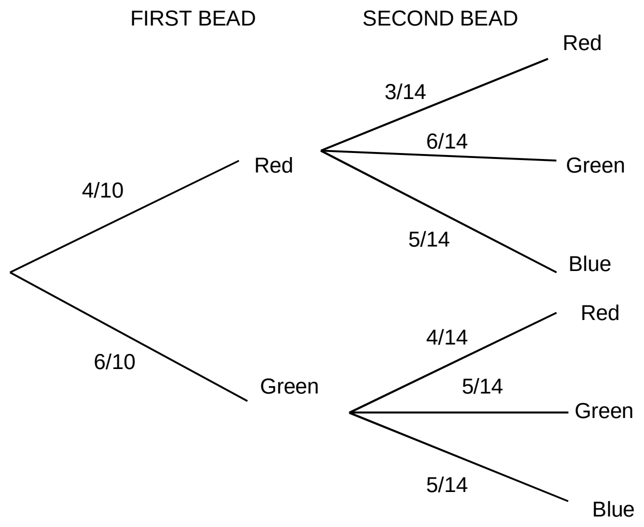
= anything rounding to 4.35 or 4.36 **[A1]**

ii. In the $7 - 9$ interval (Accept just $7 - 9$) **[B1]**

iii. Their mean $+ 5$ and their standard deviation unchanged **[B1ft]**

(anything rounding to 14.78 and 4.35 or 4.36)

b) i.



Allow equivalent fractions or decimals given to at least 3 decimal places where appropriate

(B1) Correct first pair or branches; **(B1ft)** for each correct second set of branches (allow follow through from their first branches) **[B3]**

Part b) is continued on the next page

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Question B4 – (continued)

ii. $\text{their } \frac{4}{10} \times \text{their } \frac{6}{14}$ and $\text{their} \frac{6}{10} \times \text{their } \frac{5}{14}$ **[M1]**

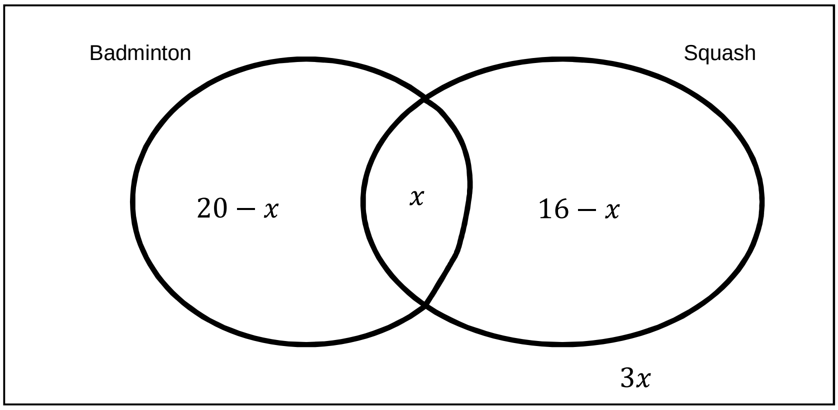
Adds their expressions **[M1]**

$= \frac{54}{140}$ or $\frac{27}{70}$ or equivalent or anything rounding to 0.386 **[A1ft]**

iii. $\dfrac{\text{their} \frac{4}{10} \times \text{ their} \frac{6}{14}}{\text{their answer to part ii}}$ **[M1]**

$= \frac{4}{9}$ or equivalent or anything rounding to 0.444 **[A1ft]**

c) E



Any two correct entries in terms of x **[B1]**

All entries correct with a rectangle but condone missing E **[B1]**

Forms an equation and finds a value for x. **[M1]**

$[(20 - x) + x + (16 - x) + 3x = 50 \ldots\ldots\ldots\ldots. x = 7]$

13 students play badminton only **[A1]**

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Question B5

a) i.

x	y	x^2	$x \times y$
2	9	4	8
5	10	25	50
7	11	49	77
10	12	100	120
11	13	121	143
$\sum x = 35$	$\sum y = 55$	$\sum x^2 = 299$	$\sum x \times y = 408$

Correct x^2 column **(B1)**; Correct $x \times y$ column **(B1)** **[B2]**

ii. $s_x^2 = \frac{\text{their} \sum x^2}{5} - 7^2 \quad (= 10.8)$ **[B1]**

$s_{xy} = \frac{\sum \text{their } xy}{5} - 7 \times 11 \quad (= 4.6)$ **[B1]**

$y - 11 = \frac{\text{their } s_{xy}}{\text{their } s_x^2}\ (x - 7)$ **[M1]**

$y - 11 = 0.43(x - 7)$ or $y = 0.43x + 8$ (or anything rounding to 0.43 and 8) **[A1]**

(If the correct equation appears with no working, this 1 out of 4.)

b) i. 0.6098 or anything rounding to 0.61 **[B1]**

ii. Finds $p(X \le 8)$ and subtracts from 1 **[M1]**

$= 0.095$ **[A1]**

iii. Either ${}^{15}C_5 \times 0.4^5 \times 0.6^{10}$
or finds $p(X \le 5) - p(X \le 4) \quad (0.4032 - 0.2173)$ **[M1]**

= anything rounding to 0.186 **[A1]**

c) i. $322000 \div 1.15$ or any other equivalent valid method **[M1]**

$= £280000$ **[A1]**

ii. 322000×0.85 or any other equivalent valid method **[M1]**

$= £273700$ (Accept $£274000$) **[A1]**

iii. $\frac{\text{their answer to part i} - \text{their answer to part ii}}{\text{their answer to part i}} \times 100$ **[M1]**

$= 2\frac{1}{4}\%$ or equivalent **[A1ft]**

d) i. 100 **[B1]**
ii. 2.8 **[B1]**
iii. 196 **[B1]**

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Question B6

a) *Please note: this is a 'show that' question so all working must be seen.*

i. $-12x - 24 + 2y\frac{dy}{dx} - 8\frac{dy}{dx} = 0$ Correct attempt at implicit differentiation (sight of $2y\frac{dy}{dx}$ or $8\frac{dy}{dx}$ is sufficient) **[M1*]**

Correct answer **[A1]**

Factorises and makes $\frac{dy}{dx}$ the subject ($\frac{12x + 24}{2y - 8}$) **[M1*]**

$\frac{dy}{dx} = \frac{6x + 12}{y - 4}$ (Both M marks scored and no errors seen) **[A1]**

ii. $x = -2$ **[B1]**

iii. Substitutes their value of x into original expression and obtains a quadratic equation in y ($y^2 - 8y + 12 = 0$ or equivalent) **[M1*]**

Factorises or uses formula **[M1]**

$[(y-2)(y-6) = 0$ or $y = \frac{8 \pm \sqrt{[(-8)^2 - 4 \times 1 \times 12]}}{2 \times 1}]$

Coordinates are (-2, 2) and (-2,6) **[A1]**

b) *Please note: this is a 'show that' question so all working must be seen.*

$\frac{dy}{dx} = \frac{(0) - (-\sin x)}{\cos^2 x}$ Use of Quotient Rule in its correct form **[M1*]**

$= \frac{\sin x}{\cos x} \times \frac{1}{\cos x}$ (can be either way round, but this step must be seen) **[M1*]**

$= \sec x \tan x$ (or other way round) Both M marks scored and no errors seen. **[A1]**

or uses Chain Rule

$u = \cos x$ and $\frac{du}{dx} = -\sin x$; $y = \frac{1}{u}$ and $\frac{dy}{du} = -u^{-2}$ **(M1*)**

$\frac{dy}{dx} = -\frac{1}{\cos^2 x} \times -\sin x = \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$ (can be either way round, but this step must be seen **(M1*)**

$= \sec x \tan x$ (or other way round) Both M marks scored and no errors seen. **(A1)**

Part c) is on the next page

Question B6 – (continued)

c) i. $\frac{du}{dx} = 3x^2$ or $du = 3x^2\ dx$ **[M1*]**

Writes integral in terms of u ($\int_0^7 \frac{1}{3} \times e^u\ du$) (The limits do not need to have been changed for this mark) **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. If the limits have not been changed, the expression must be turned back into terms in x to score this mark **[M1]**

$= \frac{1}{3}(e^7 - 1)$ or equivalent, or anything rounding to 365 **[A1]**

(If the answer appears with no working, this scores 0.)

ii. Uses integration by parts in the right direction **[M1*]**

$x^2 \times \frac{1}{2}e^{2x}$ **(A1)** - $\int 2x \times \frac{1}{2}e^{2x}\ dx$ **[A1]**

Uses integration by parts again **[M1*]**

$2x \times \frac{1}{4}e^{2x}$ **(A1)** - $\int 2 \times \frac{1}{4}e^{2x}\ dx$ **[A1]**

$= \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + c$ **[A1]**

(All correct and $+\ c$. The expression does not have to be simplified)

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