NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business Examination 2020-21

MARK SCHEME

Notice to Markers

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A2. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Types of marks awarded:

There are three types of mark:

M = Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A = Answer
B = Correct answer independent of method

Please note that A marks cannot be scored if the M marks have not been earned (i.e. M0 followed by A1 is not possible) unless there is a special case allowing this which will be indicated in the mark scheme.

If a candidate obtains full marks or no marks in a section, it is only necessary to enter the score on the script. If there is a part score, then each mark scored must be shown as M1, A1 or B1 on the script.

Error Carried Forward (ECF):

If a candidate makes a numerical error in a calculation, and the answer is used in a subsequent part of the question, the marking procedure is as follows. If the candidate's working shows a method which is clearly correct, then the M marks can still be awarded (and 'ECF' should be written on the script) but no A marks are available. The only exception to this is when there is a follow through permitted. This is denoted by A1ft and B1ft – and allowing a follow through will usually be indicated in the mark scheme as well.

If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Misreads:

Where a candidate incorrectly copies information from a question this may be marked as a misread using the annotation MR. In this case a one mark penalty should be applied and all other marks (including A marks) may be awarded for error carried forward. Markers should use discretion to award fewer marks if the question becomes much simpler because of the misread. If an inappropriate result is reached as a result of the misread (eg cos Θ = 2 or a probability greater than 1) the final A mark(s) should not be awarded. Any misread cases which are unusual or difficult to mark should be highlighted in the Marker Report.

Where a candidate miscopies their own work this is an error NOT a misread.

Section A

Question A1

X lies at $(\frac{c}{5}, 0)$ **[M1]**

Y lies at $(0, \frac{c}{3})$ **[M1]**

$\frac{1}{2} \times \text{their } \frac{c}{5} \times \text{their } \frac{c}{3} = 120$ **[M1]**

$c = \pm 60$ (Both answers needed) **[A1]**

Question A2

0.6 seen **[B1]**

Their 0.6^7 **[M1]**

0.02799… (can be implied) **[A1]**

0.0280 to 3 significant figures. Do not accept 0.028. Allow follow through provided a more accurate answer is seen earlier. **[A1ft]**

Question A3

$9^2 - 4 \times a \times 3 > 0$ **[M1]**

Reaches $a < \cdots$ $(6\frac{3}{4})$ **[M1]**

Largest possible value is 6. **[A1]**

Question A4

${}^8C_3 \times m^5 \times 4^3$ **[M1]**

Accept equivalent binomial coefficients, allow presence of x but do not accept reversed powers.

Reaches $m^5 = \cdots$ $(\frac{1}{32})$ (There must now be no x term) **[M1]**

$m = \frac{1}{2}$ or equivalent **[A1]**

<u>Special case</u>: if the binomial coefficients are written the wrong way round e.g. ${}^3C_8$ instead of ${}^8C_3$, but the correct answer still emerges, then award M0 M1 A1.

Question A5

(a) $\log_n\left(\frac{\frac{9}{4}\times\frac{10}{3}}{\frac{3}{4}}\right)$ Correct use of power law and either addition or subtraction law. **[M1]**

[A1]

$= \log_n 10$

[B1]

(b) $(n =)10$

Question A6

Correct order of solving (arcsin, then divides by $\frac{5}{2}$) seen or implied anywhere **[M1]**

Finds any value of $\frac{5}{2}\theta$ which does not have to be in the range (probably an answer rounding to -75) **[M1]**

Realises search is from 0 to 900 degrees **[M1]**

$\theta =$ anything rounding to 102, 114, 246, 258 (degrees) **(A1)** any two correct; **(A2)** all correct. One mark is lost if there are any extra solutions in the range. Ignore solutions outside the range. **[A2]**

Question A7

Attempts to differentiate [sight of x, x^{-1} or equivalent, or ke^{4x} $(k \neq -4)$ is sufficient for this mark] $[112x + \frac{3}{x} - 16e^{4x}]$ **[M1*]**

Attempts to differentiate a second time [sight of their 112, x^{-2} or equivalent, or me^{4x} $(m \neq -4, -16)$ is sufficient for this mark] $[112 - \frac{3}{x^2} - 64e^{4x}]$ **[M1*]**

Substitutes $x = \frac{1}{4}$ into their $\frac{d^2y}{dx^2}$ **[M1]**

$= 64 - 64e$ or equivalent but must be in exact form. **[A1]**

Question A8

Multiplies out integrand $[100x^4 - 60x^2 + 9]$ **[M1]**

$= 20x^5 - 20x^3 + 9x + c$ **(A1)** for any two correct, **(A2)** for all correct and $+\,c$ **[A2]**

Question A9

(a) $\frac{66 + x}{8}$ or equivalent **[B1]**

(b) Finds x and the mean [$x = 6$ and mean $= 9$] **[M1]**

Standard deviation = $\sqrt{\left[\frac{\text{sum of squares of their readings (1184)}}{8} - \text{their mean } (9)^2\right]}$ **[M1]**

$\sqrt{67}$ or anything rounding to 8.19 **[A1]**

Question A10

$^5C_3 \times p^3(1-p)^2$ **(M1)** $^5C_4 \times p^4(1-p)$ and adds **(M1)** **[M2]**

Any correct unsimplified answer
$[e.g.\ 10p^3(1-2p+p^2) + 5p^4(1-p)$ or $10p^3 - 20p^4 + 10p^5 + 5p^4 - 5p^5]$ **[A1]**

$5p^3(p-1)(p-2)$ Answer in correct form **[A1]**

Question A11

$\frac{1}{\tan x}$ or $\frac{1}{\cos^2 x}$ (or equivalent) seen **[M1]**

$\frac{dy}{dx} = \frac{1}{\tan x \cos^2 x}$ or equivalent **[A1]**

$= 2$ [Allow follow through on their $\frac{dy}{dx}$] **[A1ft]**

Question A12

Uses Quotient Rule with correct denominator and one part of the numerator correct. **[M1*]**

$\frac{dy}{dx} = \frac{-(x^2 + 8) - 2x(1 - x)}{(x^2 + 8)^2}$ **[A1]**

Sets numerator equal to 0 and finds at least one value of x **[M1]**

Finds at least one value of y **[M1]**

$\left(-2, \frac{1}{4}\right)$; $\left(4, -\frac{1}{8}\right)$ or equivalent. Accept $x = -2,\ y = \frac{1}{4};\quad x = 4,\ y = -\frac{1}{8}$ **[A1]**

Section B

Question B1

(a) *Please note: all working must be seen.*

Solves to find either c or d **[M1*]**

Finds second unknown **[M1*]**

$c = \frac{2}{3}$ Accept anything rounding to 0.667 **[A1]**

$d = -\frac{3}{5}$ or equivalent **[A1]**

(b) i. $k^2 - 7k + m$ **[B1]**

ii. $k^2 - 7k + \frac{1}{2} = -3$ and forms a quadratic equation $[2k^2 - 13k + 6 = 0]$ **[M1]**

Solves. <u>This is dependent on the previous M mark</u>. **[M1]**

$k = \frac{1}{2}, 6$ (Both needed) **[A1]**

(c) i. $(ax + b)(cx + d)$ where $ac = 63,\ bd = \pm 2$ **[M1]**

$(7x - 1)(9x + 2)$ **[A1]**

ii. $x \le -\frac{2}{9}$ **(A1)** $x \ge \frac{1}{7}$ **(A1)** (Must be in the form $\frac{m}{n}$) **[A2]**

<u>Please note</u>: the two ranges can be separated by a space, a comma or the word 'or'. The final mark is lost if the word 'and' is seen.

(d) i. *Please note: this is a 'show that' question so all working must be seen.*

$a + 8d = 7a$ **[M1*]**

$d = \frac{3}{4}a$ (M mark scored and no errors seen) **[A1]**

ii. $5000 = \frac{40}{2}\left[2a + 39 \times \frac{3}{4}a\right]$ or $\frac{40}{2}[\,2 \times \frac{4}{3}d + 39 \times d]$ **[M1]**

$a = 8$ **(A1)** $d = 6$ **(A1)** **[A2]**

(e) $r = \frac{1}{2}p$ **[M1]**

Any correct unsimplified form $[e.g.\ \frac{1}{4}p\left(\frac{1}{2}p\right)^{n-1}]$ **[A1]**

$\frac{1}{2^{n+1}}p^n$ or $(\frac{1}{2})^{n+1}p^n$ (Accept in this form only) **[A1]**

Question B2

(a) **i.** Substitutes $q = -1$ into formula **[M1]**

$p = 34\frac{1}{4}$ or equivalent **[A1]**

ii. Substitutes $p = 8$ into formula and forms a quadratic equation in 2^q. **[M1]**
$[2^{2q} - 12(2^q) + 32 = 0]$

Solves. <u>This is dependent on the previous M mark</u>. $[2^q = 4,\ 8]$ **[M1]**

$q = 2,\ 3$ (both needed) **[A1]**

iii. p approaches 40 (or similar words) **[B1]**

(b) **i.** Any valid proof, with sufficient working, is fine e.g. 'If $m^0 = k$, $0 \log m = \log k,\ \text{so}\ \log k = 0,\ \text{thus}\ k = 1$' or '$1 = m^k \div m^k = m^{k-k} = m^0$ ', etc. **[M1*]**

ii. $\frac{24t^8}{2t^4 \times 4t} = \frac{24t^8}{8t^5}$ Correct handling of indices under the cube root **[M1]**

Correct handling of indices under multiplication or division **[M1]**

$t = \frac{1}{6}$ or equivalent, or anything rounding to 0.167 **[A1]**

(c) **i.** $\frac{1}{2} \times 65 \times 60 \times \sin A = 1800$ [Working backwards from part ii is M0] **[M1]**

$\sin A = \frac{12}{13}$ or equivalent but must be in this form. **[A1]**

ii. Uses a right-angled triangle, $\cos^2\theta + \sin^2\theta \equiv 1$ or any other valid method to give $\cos A = \frac{5}{13}$ **[M1*]**

iii. Uses cosine formula $[BC^2 = 65^2 + 60^2 - 2 \times 65 \times 60 \times \frac{5}{13}]$ **[M1]**

Calculates correctly, observing BODMAS. **[M1]**

$= 5\sqrt{193}$ (cm) (Accept this answer only) **[A1]**

Parts iv, v and vi are on the next page.

Question B2 – (continued)

iv. Uses sine formula [$\frac{\sin B}{60} = \frac{\text{their } \frac{12}{13}}{\text{their } BC}$] **[M1]**

Anything rounding to 52.9 (degrees) **[A1]**

v. *The answers to parts* v *and* vi *can be either way round*. *Sufficient working must be seen in both cases*.

$65 \sin A = 60$ (cm) **[M1*]**

vi. Area of triangle $ABC = \frac{1}{2} \times$ base $\times$ height, so $1800 = \frac{1}{2} \times 60 \times$ height

So height $= 60$ (cm) **[M1*]**

Question B3

(a) **i.** $10x + 16x + 2y = 408$ **[M1]**

$y = 204 - 13x$ or equivalent **[A1]**

ii. *Please note: this is a 'show that' question so all working must be seen*.

$A = 10xy - 2x \times 3x$ or $A = 10xy - 6x^2$ **[M1*]**

Substitutes their y into A $[10x(204 - 13x) - 6x^2]$ **[M1*]**

Reaches $A = 2040x - 136x^2$ having scored both M marks and no errors seen. **[A1]**

iii. Attempts to differentiate (sight of 2040 or x is sufficient for this mark) $[\frac{dA}{dx} = 2040 - 272x]$ **[M1*]**

Sets equal to 0 and finds a value for x **[M1]**

$x = \frac{15}{2}$ (m) or equivalent **[A1]**

iv. Attempts to differentiate a second time **[M1*]**

$\frac{d^2A}{dx^2} = -272$ **[A1]**

This is negative, so there is a maximum. [Allow follow through on their $\frac{d^2A}{dx^2}$ provided it gives a maximum] **[A1ft]**

<u>or</u> takes a numerical value between 0 and $\frac{15}{2}$ and shows $\frac{dA}{dx} > 0$ **(M1*)**

takes a numerical value above $\frac{15}{2}$ and shows $\frac{dA}{dx} < 0$ **(M1*)**

Thus there is a maximum when $x = \frac{15}{2}$ **(A1ft)** [Allow follow through on their $\frac{dA}{dx}$ provided it gives a maximum]

v. 7650 (m^2) [Allow follow through on their x] **[B1ft]**

Parts (b) and (c) are on the next page.

Question B3 – (continued)

(b) Uses integration by parts in the right direction **[M1*]**

$= \underline{(12-8x)\times\frac{1}{2}e^{2x}} - \int -8\times\frac{1}{2}e^{2x}\ \ dx$ **(A1)** for underlined part **[A1]**

$= (6-4x)e^{2x} + 2e^{2x} + c$ **[A1]**

(c) Recognises the need to integrate

$\int_{-6}^{2}(36-\ x^2)dx,\quad \int_{2}^{4}\ \frac{128}{x^2}\ dx$ (At least one integral seen with correct limits) **[M1*]**

Attempts to integrate <u>at least one</u> of the expressions
$[36x - \frac{1}{3}x^3,\ \ -\frac{128}{x}]$ **[M1*]**

Substitutes limits into <u>both</u> integrated expressions and subtracts the right way round $[\left(72-\frac{8}{3}\right)-(-216+72),\quad -32-(-64)\quad = 213\frac{1}{3},\ 32]$ **[M1]**

Adds their areas **[M1]**

$= 245\frac{1}{3}$ or equivalent. Accept anything rounding to 245. **[A1]**

Question B4

(a) i. Continuous **(B1†)** with adequate reason **(B1)** such as 'data can take any value'. Be inclined to give benefit of the doubt if there is some uncertainty. **†**Award only if a reason follows, even if it is the wrong one. **[B2]**

ii. Finds cumulative frequencies (6, 16, 34, 59, 82, 98, 108, 112) **[M1]**

Plots correct curve (a sketch is on page 15). 1 mark lost for each omitted/incorrect plot; 1 mark lost for each point missed by the curve by at least 1 mm (but allow ft for any incorrect plots); 1 mark lost if either axis is not labelled correctly.
(Please note: a maximum total of 3 marks can be lost i.e. there are no negative scores. If the candidate plots the mid-values instead of the upper values in each interval, this will score A0.)

If graph paper is not used, award 1 mark out of the A3 if a reasonable curve is drawn. If a cumulative frequency polygon is drawn, award up to 2 marks out of the A3. **[A3]**

iii. *Please note: in part iii, marks are given for values taken from the candidate's curve, and there must be some evidence that this has been done.*

Reads off their median (around 39) **[B1ft]**

Reads off their LQ and UQ (around 27 and 51 respectively) **[B1ft]**

Subtracts their LQ from their UQ (around 24) **[B1ft]**

(b) i. $P(A \cap B) = P(A) \times P(B) = \frac{1}{5}$ **[M1]**

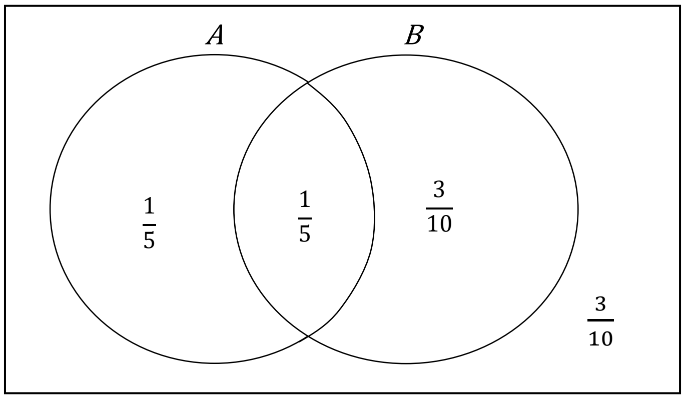
$P(A \cup B) = \frac{7}{10}$ **[A1]**

$P(A|B) = \frac{2}{5}$ **[B1]**

Part ii and part (c) are on the next page.

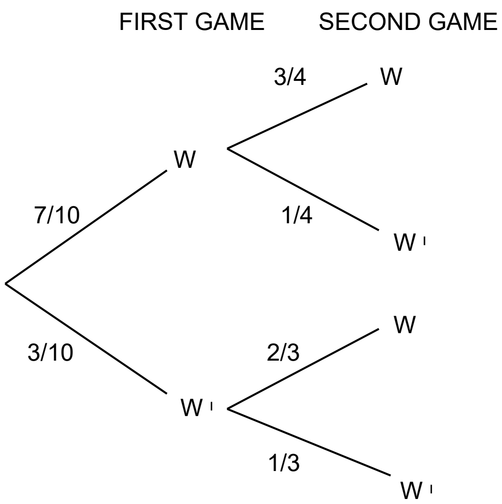
Question B4 – (continued)

ii. E



(B1) for any 2 correct entries; **(B2)** for all entries correct and diagram enclosed in a rectangle. Condone missing E. **[B2]**

(c) i.



W denotes 'wins game'

Set of branches under first game correct **[B1]**

Set of branches under second game correct **[B1]**

ii. Finds probability he wins, then wins; loses then wins $[\frac{7}{10}\times\frac{3}{4}, \quad \frac{3}{10}\times\frac{2}{3}]$ **[M1]**

Adds their probabilities $[\frac{29}{40}]$ **[M1]**

Divides their $\frac{7}{10}\times\frac{3}{4}$ by their $\frac{29}{40}$ **[M1]**

$= \frac{21}{29}$ or anything rounding to 0.724 **[A1]**

Question B5

(a) **i.** $\frac{187200 \times 100}{104}$ or equivalent **[M1]**

$= 180000$ (pounds) **[A1]**

ii. $187200 \times \frac{95}{100}$ or equivalent **[M1]**

$= 177840$ (pounds) [must be given in full] **[A1]**

iii. $\frac{\text{their } 177840 - \text{their } 180000}{\text{their } 180000} \times 100$ **[M1]**

$= 1.2\%$ decrease [Increase/decrease must be specified. -1.2% is A0. Allow follow through on their parts i and ii] **[A1ft]**

(b) **i.** 5 **[B1]**

ii. Independence (is assumed) [Allow similar words] **[B1]**

(c) **i.** 0.1 or equivalent **[B1]**

ii. $(0 \times 0.1) + k \times 0.15 + 3k \times 0.25 + 5k \times 0.2 + 7k \times 0.3$ **[M1]**

$= 4k$ **[A1]**

iii. $\mathrm{E}(X^2) = (0 \times 0.1) + k^2 \times 0.15 + 9k^2 \times 0.25 + 25k^2 \times 0.2$
$+49k^2 \times 0.3 \ (= 22.1k^2)$ **[M1]**

$\mathrm{Var}(X) = \text{their } \mathrm{E}(X^2) - \text{ their } [\mathrm{E}(X)]^2$ **[M1]**

$6.1k^2$ or equivalent **[A1]**

iv. $\mathrm{E}(Y) = 14k$ **(B1ft)** $\mathrm{Var}(Y) = 54.9k^2$ **(B1ft)** [Allow follow through] **[B2ft]**

(d) $z = \frac{117 - 108}{6} = 1.5$ **[M1]**

6.68% (must be a percentage) **[A1]**

(e) $m = 97$ (grams) **[B1]**

Upper limit = 99.8 (grams) [Allow follow through on their m] **[B1ft]**

Question B6

(a) i. $-6x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} + \frac{1}{3}\frac{dy}{dx} = 0$

Correct use of Product Rule (sight of $2y + 2x\frac{dy}{dx}$ is sufficient for this mark) **[M1*]**

Correct implicit differentiation (sight of $2x\frac{dy}{dx}$, $2y\frac{dy}{dx}$ or $\frac{1}{3}\frac{dy}{dx}$ is sufficient for this mark) **[M1*]**

Assembles $\frac{dy}{dx}$ terms on to one side and factorises. (This mark is available only if there are at least two $\frac{dy}{dx}$ terms) **[M1]**

$\frac{dy}{dx} = \frac{6x - 2y}{2x + 2y + \frac{1}{3}}$ or equivalent **[A1]**

ii. Makes top line of their $\frac{dy}{dx}$ equal to 0 and reaches $y = 3x$ [Allow this mark if their top line is wrong but still gives $y = 3x$] **[M1*]**

iii. Substitutes $y = 3x$ or $x = \frac{1}{3}y$ into the original equation and forms a quadratic equation in x or y $[12x^2 + x - 1 = 0,\ \ 4y^2 + y - 3 = 0]$ **[M1]**

Solves. <u>This is dependent on the previous M mark</u>. **[M1]**

Coordinates are $(-\frac{1}{3}, -1)$ and $(\frac{1}{4}, \frac{3}{4})$
[Allow $x = -\frac{1}{3}$, $y = -1$; $x = \frac{1}{4}$, $y = \frac{3}{4}$] **[A1]**

(b) $du = -\sin\theta\ d\theta$ or equivalent **[M1]**

Writes integral in terms of u [$\int -3u^{-3}\ du$] **[M1*]**

Integrates [$\frac{3}{2}u^{-2}\ (+c)$] and expresses answer in terms of θ. **[M1*]**

$= \frac{3}{2}(1 + \cos\theta)^{-2} + c$ or equivalent **[A1]**

Part (c) is on the next page.

Question B6 – (continued)

(c) **i.** $5x + 8 = A(x + 1) + B(x + 2)$ **[M1]**

$A = 2$ **[A1]**

$B = 3$ **[A1]**

ii. Uses previous part [$\int_1^2 \left(\frac{2}{x+2} + \frac{3}{x+1}\right) dx$] **[M1*]**

Attempts to integrate (a log term is sufficient for this mark) $[\,2\ln(x+2) + 3\ln(x+1)]$ **[M1*]**

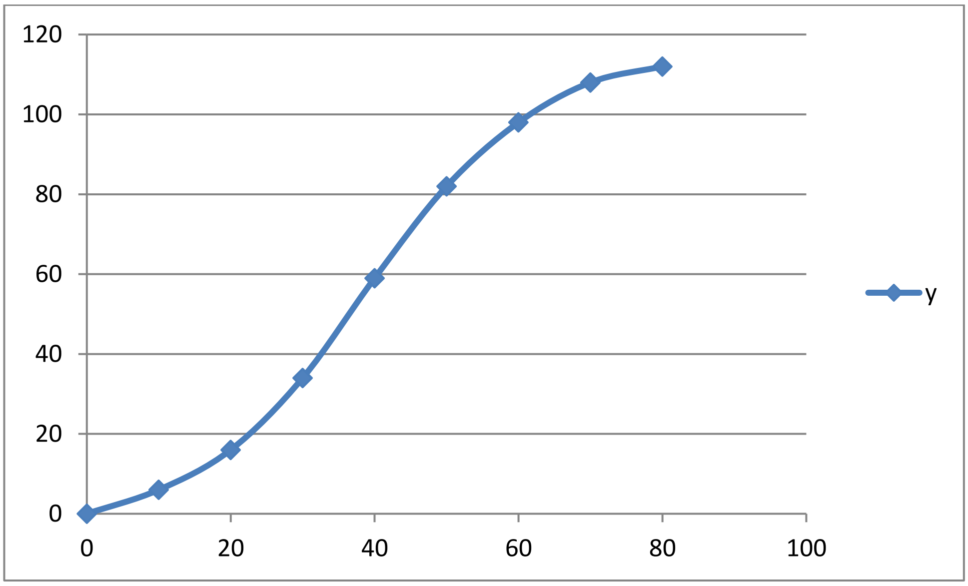
Uses the power law or combines logs correctly at any stage, even after substituting in the limits $\{\ln[(x+2)^2(x+1)^3]\}$ **[M1]**

Substitutes limits into their integrated expression and subtracts the right way round **[M1]**

$= \ln 6$ (Accept only in this form) **[A1]**

Cumulative frequency curve for B4 (a) ii

Cumulative frequency



Time (t) in seconds