NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business
Time-Controlled Assessment

2019-20

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 40 marks.**
SECTION B	**Answer THREE questions ONLY. This section carries 60 marks.**
The marks for each question are indicated in square brackets [ ].	
Guide Time: 2 hours **The Guide Time is how long you are expected to spend completing this Time-Controlled Assessment. You are allowed 24 hours in total to complete and submit.**	

- You **MUST** show **ALL** of your working. This is very important. You will score no marks if there is not enough working shown – even if your answer is correct.
- An approved calculator may be used in the assessment.
- All work must be completed independently. The penalty for collusion is a mark of zero.
- Due to the nature of the questions, there should be no need to use external sources of information to answer them. If you do use external sources of information you must ensure you reference these. Plagiarism is a form of academic misconduct and will be penalised.
- Work must be submitted by the deadline provided. Your Study Centre can be contacted only for guidance on submission of work and cannot comment on the contents of the assessment.
- Your work can be word-processed or handwritten. Once complete, any handwritten work will need to be clearly photographed/scanned and inserted into a single word-processed file for submission
- Work must be submitted in a single word-processed file using the standard NCUK cover page.

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

Point M $(2, -1)$ is the mid-point of the line AB. Point B lies at $(10, -6)$.

Find the equation of the line which passes through point A and is perpendicular to the line AB.

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. **[4]**

Question A2

The probability it rains on Wednesday is p. The probability it rains on Thursday is $2p$. The probability it rains on neither day is $\frac{5}{9}$.

Show that $p = \frac{1}{6}$. **[4]**

Question A3

(a) Divide $2x^3 - 7x^2 - 33x + 18$ by $(2x - 1)$. **[3]**

(b) Hence factorise $2x^3 - 7x^2 - 33x + 18$ completely. **[1]**

Question A4

The numbers $(9x + 2)$, $(12x - 8)$ and $(11x + 10)$ are three consecutive terms of an arithmetic progression.

Find the value of x and the common difference. **[3]**

Question A5

Solve the equation **[4]**

$$4^{3x} = 1240[4^{(x+3)}]$$

Give your answer to **3** significant figures.

In this question, 1 mark will be given for the correct use of significant figures.

Question A6

Solve the equation $10\tan\theta = -7 \qquad (0° \leq \theta \leq 360°)$ **[3]**

Question A7

A curve has gradient function $\frac{dy}{dx} = 6x^2 - 8x - 5$.

Find the equation of the curve, given that it passes through the point $(3, q)$ where q is a constant. **[3]**

Question A8

The mean of 8 readings is $4\frac{1}{2}$ and the standard deviation is $\frac{\sqrt{21}}{2}$.

Find the sum of the readings and show that the sum of the squares of the readings is 204. **[4]**

Question A9

Two events, A and B are such that $p(A) = 0.48$, $p(B) = 0.64$ and $p(A|B) = 0.5$.

Find $p(A \cap B)$ and $p(A \cup B)$. **[4]**

Question A10

If $y = e^{3kx}\ln x$, where k is a constant, find the <u>exact</u> value of $\frac{dy}{dx}$ when $x = \frac{1}{3k}$. **[4]**

Question A11

Find **[3]**

$$\int (4x-3)\cos\left(\frac{1}{2}x\right)dx.$$

Section B
Answer <u>THREE</u> questions ONLY. This section carries 60 marks.

Question B1

(a) The line with equation $y = 3x + 5$ and the line with equation $y = 1 - 5x$ intersect at point P. Find the coordinates of point P. **[3]**

(b) **i.** Factorise $9x^2 - 6x - 80$. **[2]**

ii. Use your answer to part i to solve the equation $9x^2 - 6x - 80 = 0$.

Give your answers in the form $\frac{a}{b}$ where a and b are integers. **[1]**

iii. Find the range of values satisfying $9x^2 - 6x - 80 < 0$. **[2]**

(c) When $2x^3 - 3x^2 + 4x + k$ is divided by $(x - 2)$, the remainder is the same as when $x^3 + x^2 - 3x + 1$ is divided by $(x + 3)$.

Use the Remainder Theorem to find the value of k. **[4]**

(d) Show that the expansion of $(2 - \frac{1}{2}x)^7$ **[4]**

is $128 - 224x + 168x^2 - 70x^3 + \frac{35}{2}x^4 - \frac{84}{32}x^5 + \frac{7}{32}x^6 - \frac{1}{128}x^7$.

(e) The 7th and 8th terms of a geometric series are 384 and 768 respectively.

i. Find the common ratio and the first term. **[2]**

ii. Find the sum of the first 12 terms. Give your answer in full with no rounding off. **[2]**

Question B2

(a) Two variables, p and q, are connected by the formula

$$p = 512\left(8^{kq}\right) + q$$

where k is a constant.

i. State the value of p when $q = 0$. **[1]**

$p = 12$ when $q = 4$.

ii. Show that $k = -\frac{1}{2}$. **[3]**

iii. Find the value of p when $q = -\frac{4}{3}$. **[2]**

(b) Given $\log_x 2 + \log_x(x^2 - x - 12) = 2\log_a a \;\; (x > 4)$ where a is a positive constant, find the value of x. **[4]**

(c)

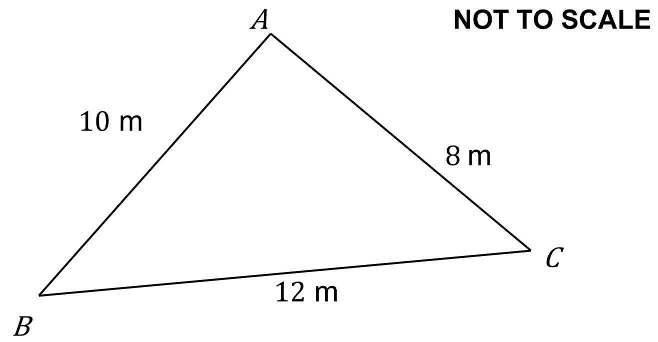

Figure 1

Figure 1 shows the acute angled triangle ABC with $AB = 10$ metres, $BC = 12$ metres and $AC = 8$ metres.

i. Find $\cos B$ giving your answer in the form $\frac{m}{n}$ where m and n are integers. **[3]**

ii. <u>Without</u> working out the size of angle B, find an expression for $\sin B$ giving your answer in surd form. **[2]**

iii. Show that $\sin C = \frac{5\sqrt{7}}{16}$. **[2]**

iv. Find the area of triangle ABC giving your answer in the form $r\sqrt{7}$ where r is an integer. **[2]**

v. Find the shortest distance from point A to the line BC. **[1]**

Question B3

(a)

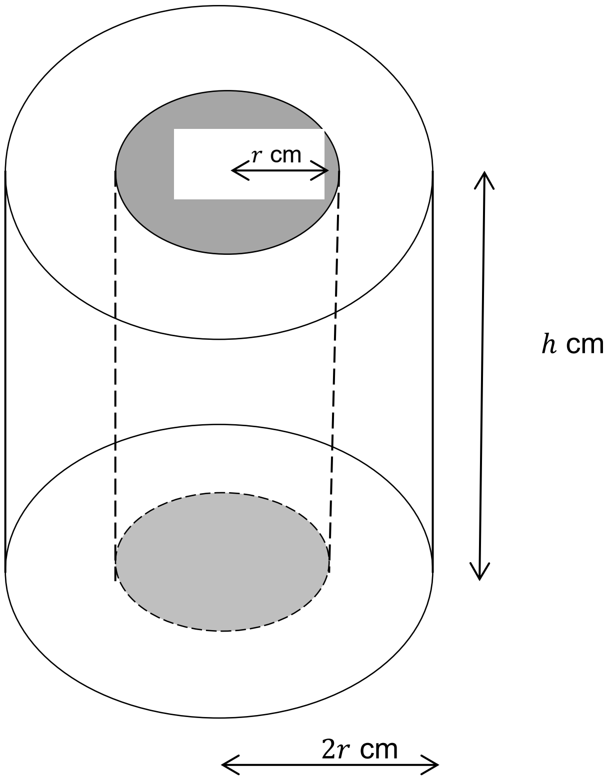

Figure 2

Figure 2 shows a solid cylinder of radius $2r$ cm and height h cm. A hole, of radius r cm, is drilled out through the centre. The volume of the remaining solid is 1296π cm^3.

i. Find h in terms of r. **[2]**

ii. Show that the <u>total</u> surface area, A, of the solid is given by **[3]**

$$A = \frac{2592\pi}{r} + 6\pi r^2$$

iii. <u>Use calculus</u> to find the value of r which makes the surface area a minimum. **[4]**

iv. Confirm that your value of r gives a minimum. **[3]**

Parts (b) and (c) are on the next page.

Question B3 – (continued)

(b) A curve has equation $y = e^{2x} - 7$.

Find the equation of the normal to the curve when $x = \ln 3$. **[4]**

(c)

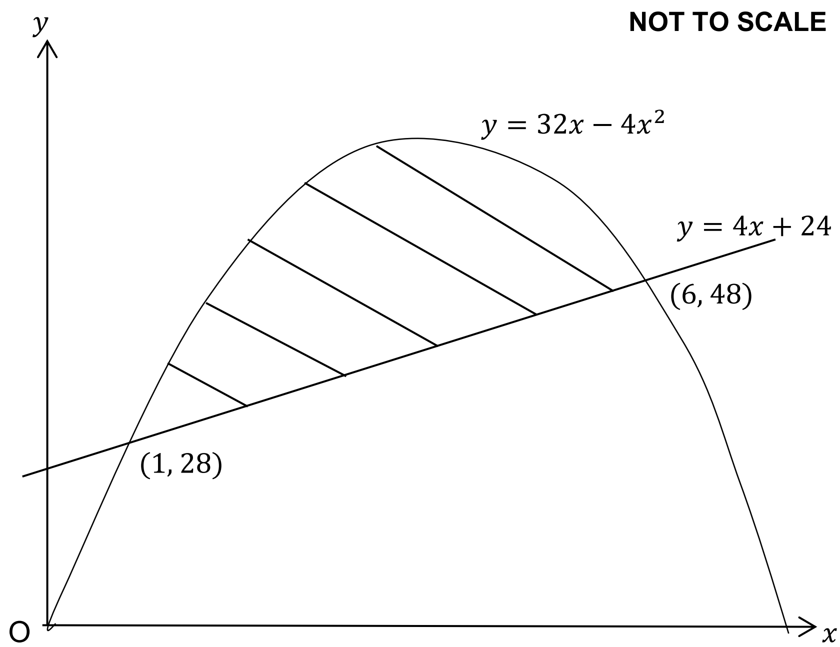

Figure 3

Figure 3 shows the curve $y = 32x - 4x^2$ and the line $y = 4x + 24$. The line intersects with the curve at the points $(1, 28)$ and $(6, 48)$.

The area, which is shaded on the diagram, is bounded by the curve $y = 32x - 4x^2$ and the line $y = 4x + 24$.

Show that this shaded area is $83\frac{1}{3}$ square units. **[4]**

Question B4

(a) A baker delivers bread to customers who are spread over a wide area. She keeps a record of delivery times on 120 occasions. The results are shown in the table below.

Time, t, in minutes	**Frequency**
$0 < t \leq 10$	5
$10 < t \leq 20$	25
$20 < t \leq 30$	41
$30 < t \leq 40$	27
$40 < t \leq 50$	10
$50 < t \leq 60$	6
$60 < t \leq 70$	4
$70 < t \leq 80$	2

i. Work out the cumulative frequencies. **[1]**

ii. In which interval does the median lie? **[1]**

(b) In a survey of 200 students, 150 said they like coffee, 80 said they like tea and 60 said they like both coffee and tea.

i. Show this information on a Venn diagram. **[2]**

One student is chosen at random.

C denotes the event 'the student likes coffee' and T denotes the event 'the student likes tea'.

ii. Work out $p(C \cup T)'$, $p(C' \cap T)$ and $p(C|T)$. **[3]**

iii. Show that events C and T are independent. **[2]**

iv. Explain why events C and T are not mutually exclusive. **[1]**

Parts (c) and (d) are on the next page.

Question B4 – (continued)

(c) The masses of coconuts can be assumed to follow a Normal distribution with mean 420 grams and standard deviation 30 grams. A coconut is described as 'large' if its mass is more than x grams. 20% of coconuts are described as 'large'.

i. Find the value of x. **[2]**

The coconuts are packed in boxes of 20. A box is chosen at random. Find the probability that the box contains

ii. exactly 2 large coconuts; **[1]**

iii. more than 4 large coconuts. **[2]**

(d) A discrete random variable, X, has probability distribution as given in the table below.

$\boldsymbol{x}$	-2	0	1	y
$\boldsymbol{p}(\boldsymbol{X}=\boldsymbol{x})$	0.3	0.2	0.4	0.1

You are given $\mathrm{E}(X^2) = 13\ \mathrm{E}(X)$.

Find the two possible values of y. **[5]**

Question B5

(a) Use the Quotient Rule to differentiate **[2]**

$y = \frac{x - 6}{x + 3}$

Give your answer in its simplest form.

(b) A curve has equation $-3x^2 + 12x + 2y^2 - 8y = 12$.

i. Find $\frac{dy}{dx}$ in terms of x and y. **[3]**

ii. Find the coordinates of the stationary values on the curve. **[2]**

(c) By using the substitution $u = 4x^3 - 4x^2 - 16x + 3$ find **[4]**

$$\int \frac{9x^2 - 6x - 12}{(4x^3 - 4x^2 - 16x + 3)^2} \; dx.$$

(d) **i.** Show that $\frac{1}{(x - 2)(x - k)}$ can be expressed in partial fractions in the **[4]**

form $\frac{1}{p}(\frac{1}{x - 2} - \frac{1}{x - k})$ where k and p are constants.

Find p in terms of k

ii. Hence find **[5]**

$$(2 - k)\int_3^4 \frac{1}{(x - 2)(x - k)} \; dx \qquad (k < 3).$$

Give your answer as a single logarithm in terms of k.

- This is the end of the Time-Controlled Assessment. -