NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics
Part 2 (Business) Examination

MARK SCHEME

Notice to Markers

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A7. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Either writes equations as $3x - y = 5 \text{ and } 9x + 2y = -30$ and uses any method to find one unknown. **[M1]**

Uses any method to find second unknown **[M1]**

Coordinates are $(-\frac{4}{3}, -9)$ **(A1)** for each (Accept anything rounding to -1.33) **[A2]**

or accept $x = -\frac{4}{3}$ **(A1)** $y = -9$ **(A1)**

Or makes $y \text{ (or } x)$ the subject of both equations and sets equal to each other **(M1)**

$x = -\frac{4}{3}$ or equivalent, or any answer rounding to -1.33 (or $y = -9$) **(A1)**

Substitutes into either equation to find $y \text{ (or } x)$ **(M1)**

$y = -9 \text{ (or } x = -\frac{4}{3}$ or equivalent, or any answer rounding to -$1.33)$ **(A1)**

Question A2

Either $\frac{4}{7}$ or $\frac{5}{11}$ seen **[M1]**

Multiplies their probabilities **[M1]**

$= \frac{20}{77}$ or anything rounding to 0.26 **[A1]**

Question A3 *Please note: in this question, other methods score no marks.*

$(x+5)^2 - 25 + 23 \quad (\text{or } -2 \text{ for the } -25+23) = 0$ **[M1*]**

Reaches $(x \pm \text{their } 5)^2 = \pm\sqrt{\ldots}$ **[M1*]**

$x = -5 + \sqrt{2}$ **(A1)** or $-5 - \sqrt{2}$ **(A1)** or $x = -5 \pm \sqrt{2}$ **(A2)** **[A2]**

Question A4

$^5C_2 \times k^3 \times 2^2 \; (\times x^2)$ **(M1*)** $^4C_3 \times k \times (x^3)$ **(M1*)** **[M2*]**

[Allow ^xC_y for ^yC_x and presence of x]

Multiplies RHS by 90 or divides LHS by 90, sets equal to each other and reaches a value for k^2 $(= 9)$. [x must now not be present.] **[M1*]**

$k = \pm 3$ (Both needed) **[A1]**

Question A5

$\log_4[x(x-6)] = \log_4 16$ Uses log addition law correctly and writes RHS as a log **[M1*]**

Removes logs at the right time and forms a quadratic equation $(x^2 - 6x - 16 = 0)$ **[M1*]**

Factorises or uses formula $[(x-8)(x+2) = 0 \text{ or } x = \frac{6 \pm \sqrt{[(-6)^2 - 4 \times 1 \times -16]}}{2 \times 1}$ **[M1]**

$x = 8\ (\text{or} - 2)$ If the -2 is not discarded, this final mark is lost. Placing in brackets is good enough **[A1]**

Question A6

$2\theta =$ any answers rounding to $1,\ 5.28,\ 7.28,\ 11.57$ (Any two seen) **[M1]**

Divides by 2 at the right time **[M1]**

$\theta =$ anything rounding to 0.5, 2.6, 3.6, 5.8 (radians)

Any two correct **(A1)**; all correct **(A2)** **[A2]**

Question A7

$\frac{dy}{dx} = 3x^2 + \frac{1}{x}$ Attempts to differentiate. Sight of x^2 or reciprocal x is sufficient **[M1*]**

Substitutes $x = 0.7$ into their $\frac{dy}{dx}$. **[M1]**

= 2.89857….. (may be implied) **[A1]**

= 2.90 to three significant figures (Allow follow through provided a more accurate answer is seen earlier. **[A1ft]**

Question A8

Total for first 6 days = 6 × 5.5 (= 33 hours) **[M1]**

Total for 7 days = 6.0 × 7 (= 42 hours) **[M1]**

9 hours on the 7th day. **[A1]**

Question A9

$p(B) = 0.65$ **[B1]**

Uses $p(A \cup B) = p(A) + \text{their } p(B) - 0.234$ **[M1]**

= 0.776 **[A1]**

Question A10

Halves 2.45 **[M1]**

$\frac{1.96 \times s}{\sqrt{16}}$ **[M1]**

$s = 2.5$ (mm) **[A1]**

Question A11

$1200(1 + \frac{r}{100})^4$ **[M1]**

Adds principal and interest (£1431) and sets equal to their expression above **[M1]**

Takes logs correctly or takes 4th root. **[M1]**

$r =$ anything rounding to 4.5% **[A1]**

Question A12

Uses integration by parts in the right direction **[M1*]**

$8x \times \frac{1}{2}e^{2x}$ **(A1)** for first part - $\int_0^{\ln 3} 8 \times \frac{1}{2}e^{2x}\ dx$ **[A1]**

$= 8x \times \frac{1}{2}e^{2x} - 2e^{2x}$ **(A1)** for second part **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= 36 \ln 3 - 16$ (Must be in this form) **[A1]**

Section B

Question B1

a) Factorises or uses formula

$[(x-8)(x+1)(\leq 0)$ or $x = \frac{7 \pm \sqrt{[(-7)^2 - 4 \times 1 \times -8]}}{2 \times 1}$ **[M1]**

Finds two critical values (-1 and 8) **[M1]**

$x \geq -1$ **(A1)** $x \leq 8$ **(A1)** or $-1 \leq x \leq 8$ **(A1)** for each end **[A2]**

b) i. Substitutes $x = -7$ into expression **[M1*]**

Shows this equals 0. (There must be some evidence of this: just saying $f(x) = 0$ with no working is not enough) **[A1]**

ii.

$$\begin{array}{r|l} & x^2 - 1 \\ \hline x+7 & x^3 + 7x^2 - x - 7 \\ & \underline{x^3 + 7x^2} \\ & \quad -x - 7 \\ & \quad \underline{-x - 7} \\ & \quad \underline{\ldots\ldots\ldots} \end{array}$$

Any correct division **[M1]**

Correct quotient **[A1]**

iii. $(x+7)(x+1)(x-1)$ **[A1]**

c) i. $8 + (31-1)d = 203$ **[M1]**

$d = 6\frac{1}{2}$ or equivalent **[A1]**

ii. $S_{49} = \frac{49}{2}[2 \times 8 + (49-1) \times \text{their } d\,]$ **[M1]**

Calculates in correct order **[M1]**

= 8036 **[A1]**

d) i. $\frac{a}{1-\frac{4}{5}} = 12000$ and then writes $a = 2400$ **[M1*]**

ii. $2400 \times (\frac{4}{5})^{n-1} = 1$ **[M1]**

Takes logs correctly and reaches a value for n $(35.879 \ldots)$ **[M1]**

36th term (Candidates who use an inequality sign and forget to reverse it when dividing by $\log \frac{4}{5}$ score 2 marks out of 3). **[A1]**

iii. $S_9 = \frac{2400[1 - (\frac{4}{5})^9]}{1 - \frac{4}{5}}$ **[M1]**

Anything rounding to 10400 **[A1]**

Question B2

a) i. $A = 4$ **[B1]**

ii. $72 = \text{their } A\left(2^{20k}\right) + 8$ and reaches $20k = \cdots \ (\frac{64}{\text{their A}})$ **[M1]**

Takes logs correctly $(20k = (\frac{\log(64 \div \text{their } A)}{\log 2})$ **[M1]**

$k = \frac{1}{5}$ or equivalent **[A1]**

iii. $\frac{dy}{dx} = \text{their } A \times \text{their } k \times 2^{\text{their } k \times x} \ln 2$ **[M1]**

Substitutes $x = 12$ into their $\frac{dy}{dx}$ **[M1]**

= anything rounding to 2.93 (Follow through on their A and k) **[A1ft]**

b) Recognises the 'hidden' quadratic equation **[M1*]**

Factorises or uses formula **[M1]**

$[(e^{0.5x} + 7)(e^{0.5x} - 3) = 0$ or $0.5x = \frac{-4 \pm \sqrt{(4^2 - 4 \times 1 \times -21)}}{2 \times 1}$

$e^{0.5x} = (-7) \text{or } 3$ (can be implied: ignore -7 at this stage) **[A1]**

$x = \ln 9$ (This mark is lost if there is any attempt to use the -7). **[A1]**

c) i. $2205 = \frac{1}{2} \times 87 \times PR \times \frac{21}{29}$ **[M1]**

$PR = 70$ (cm) **[A1]**

ii. Uses $\cos^2\theta + \sin^2\theta = 1$ or a right-angled triangle, or any other valid method. **[M1*]**

$\cos\theta = \frac{20}{29}$ M mark scored and no errors seen. **[A1]**

iii. Uses correct version of the cosine formula **[M1]**

Calculates in correct order **[M1]**

$QR = \sqrt{4069}$ or anything rounding to 63.8 (cm) **[A1]**

iv. Uses sine formula in its correct form $\frac{\sin R}{87} = \frac{21/29}{\text{their } QR}$ **[M1]**

Angle R = anything rounding to 81 (degrees) or 1.4^c (ft on their QR) **[A1ft]**

Question B3

a) i. $448 = 4 \times 3x + 4 \times 4x + 4 \times h$ **[M1]**

$h = \frac{448 - 28x}{4}$ or $112 - 7x$ or equivalent **[A1]**

ii. *Please note: this is a 'show that' question so all working must be seen*.

$A = 2 \times 3x \times h + 2 \times 4x \times h$ **[M1*]**

Substitutes their h into their formula **[M1*]**

$= 14x(112 - 7x) = 1568x - 98x^2$. M marks scored and no errors seen. **[A1]**

iii. $\frac{dA}{dx} = 1568 - 196x$ Attempts to differentiate. Sight of 1568 or $x-$ term is sufficient. **[M1*]**

Correct answer **[A1]**

Sets equal to 0 and finds a value for x **[M1]**

$x = 8$ (cm) **[A1]**

iv. $\frac{d^2A}{dx^2} = -196$ Attempts to differentiate again. Presence of a constant term is sufficient. **[M1*]**

Correct answer **[A1]**

This is negative so there is a maximum (at $x = 8$) (reason and conclusion). Allow follow through on their x and $\frac{d^2A}{dx^2}$ provided their $\frac{d^2A}{dx^2}$ is negative. **[A1ft]**

or takes a numerical value of x between 0 and 8 and shows $\frac{dA}{dx} > 0$ **(M1*)**

takes a numerical value of x above 8 and shows $\frac{dA}{dx} < 0$ **(M1*)**

Thus there is a maximum (at $x = 8$) (or similar conclusion) **(A1ft)** (Allow follow through on their x and $\frac{dA}{dx}$)

Part b) is on the next page.

Question B3 – (continued)

b)

Please note this is a 'show that' question so all working must be seen.

i.

$\frac{dy}{dx} = \frac{1}{2}\, x^{-½}$ **[M1*]**

Substitutes $x = 4$ into their $\frac{dy}{dx}$ $(= \frac{1}{4})$ **[M1*]**

Correct working leading to $y = \frac{1}{4}x + 1$ (Both M marks scored and no errors seen). **[A1]**

ii. Area = $\int_0^4 \left(\frac{1}{4}x + 1 - \ x^{½} \right)\, dx$ Subtracts equations the right way round **[M1]**

Attempts to integrate. (Sight of $x, x^2 \text{ or } x^{3/2}$ is sufficient for this mark) **[M1*]**

$\frac{1}{8}x^2 + x - \frac{2}{3}x^{3/2}$ Correct answer **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round **[M1]**

$= \frac{2}{3}$ or equivalent, or anything rounding to 0.667 **[A1]**

(Allocate marks accordingly for any other valid method e.g. integrating both equations first and then subtracting the areas; or integrating along the $y -$ axis)

Question B4

a)

Temperature, t, in ℃	Frequency f	Mid-value x	$f \times x$	$f \times x^2$
$-2 \leq t \leq 0$	6	-1	-6	6
$0 < t \leq 2$	13	1	13	13
$2 < t \leq 4$	24	3	72	216
$4 < t \leq 6$	18	5	90	450
$6 < t \leq 8$	17	7	119	833
$8 < t \leq 12$	12	10	120	1200

i. $\sum f \times x \, (= 408)$ [Finds the $f \times x$ values and adds] **[M1]**

Divides their $\sum f \times x$ by 90 **[M1]**

Mean = anything rounding to 4.53 (degrees) **[A1]**

$\sum f \times x^2 (= 2718)$ [Finds the $f \times x^2$ values and adds] **[M1]**

Divides their $\sum f \times x^2$ by 90, subtracts square of their mean and takes square root. **[M1]**

Standard deviation = anything rounding to 3.1 **[A1]**

ii. Mid-values have been used. **[B1]**

iii. $2 < t \leq 4$ interval (Accept 2 – 4) **[B1]**

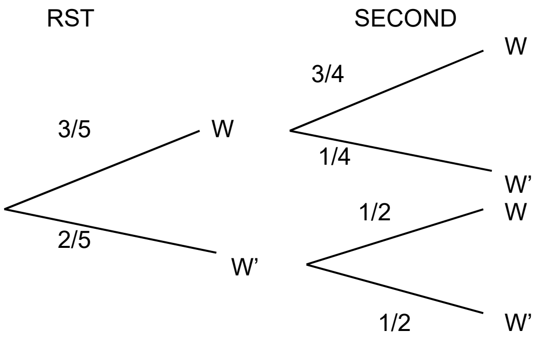
iv. $\frac{29}{90}$ or anything rounding to 0.322 **[B1]**

Parts b) and c) are on the next page

Question B4 – (continued)

b)

i. W denotes win; W' denotes no win



[B2]

(B1) for correct first set of branches; **(B1)** for correct second set.

ii. Finds probability she loses both $\left(\frac{2}{5} \times \frac{1}{2}\right)$ **[M1]**

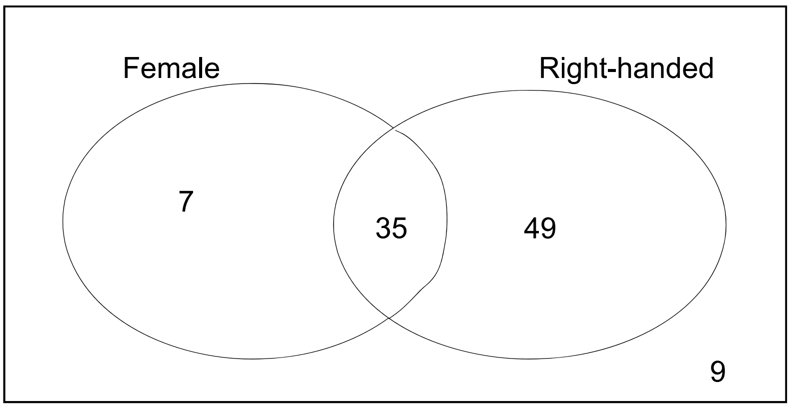
Subtracts from 1 **[M1]**

or finds $p(WW)$, $p(WW')$ and $p(W'W)$ **(M1)**

Adds answers **(M1)**

$= \frac{4}{5}$ or equivalent **[A1]**

c) i. E



(B1) any one correct entry **(B2)** any 2 correct entries; **(B3)** all correct
One mark lost if rectangle not drawn, but condone missing E. **[B3]**

ii. $p(F' \cap R) = \frac{49}{100}$ $p(F' \cup R') = \frac{65}{100}$ $p(F|R) = \frac{35}{84}$

(B1) for each. Allow decimal equivalents or percentages. **[B3]**

Question B5

a)

Number of cars bought (x)	Number of cars sold (y)	x^2	y^2	xy
3	7	9	49	21
8	1	64	1	8
5	5	25	25	25
2	3	4	9	6
6	8	36	64	48
3	6	9	36	18
$\sum x = 27$	$\sum y = 30$	$\sum x^2 = 147$	$\sum y^2 = 184$	$\sum xy = 126$

i. **(B1)** for each correct column completed. **[B3]**

ii. s_x = anything rounding to 2.06 **[B1]**

s_y = anything rounding to 2.38 **[B1]**

$s_{xy} = -1.5$ **[B1]**

Product Moment Correlation Coefficient = anything rounding to -0.3 (but do not allow this mark if there is no sign of the earlier answers.) **[B1]**

iii. (Very) weak negative or other similar description. **[B1]**

b) Negative gradient probably unlikely; one would expect the number of loaves consumed to rise along with the number of people using the café. The 6 is also probably unlikely: one would expect this to be somewhere near zero because if no people use the café, no bread will be used. **[B1]** **[B1]**

c) i. $z = \frac{320 - 306}{20}$ or $\frac{306 - 320}{20}$ $(= \pm 0.7)$ **[M1]**

Φ(their 0.7) (= 0.758) and subtracts from 1 **[M1]**

Probability = 0.242 or equivalent **[A1]**

ii. $^{20}C_6 \times \text{their } 0.242^6 \times (1 - \text{their } 0.242)^{14}$ (Allow $^6C_{20}$ for this mark) **[M1]**

= anything rounding to 0.16 (Allow follow through from part i) **[A1ft]**

d) i. $3 \times 0.1 + 5 \times 0.25 + k \times 0.45 + 12 \times 0.2 = 8.45$ **[M1]**

$k = 10$ **[A1]**

ii. $\mathrm{E}(X^2) = 3^2 \times 0.1 + 5^2 \times 0.25 + \text{their } k^2 \times 0.45 + 12^2 \times 0.2$ $(= 80.95)$ **[M1]**

$\mathrm{Var(X)} = \text{their } E(X^2) - 8.45^2$ and then takes square root **[M1]**

= anything rounding to 3.09 **[A1]**

Question B6

a)

i. $-12x + 6x\frac{dy}{dx} + 6y + 16y\frac{dy}{dx} = 0$ Correct use of Product Rule **[M1*]**

Use of implicit differentiation (sight of $6x\frac{dy}{dx}$ or $16y\frac{dy}{dx}$ is sufficient) **[M1*]**

Factorises and reaches $\frac{dy}{dx} = \cdots$ (There must be two or more $\frac{dy}{dx}$ terms for this mark to be awarded) **[M1]**

$\frac{dy}{dx} = \frac{12x - 6y}{6x + 16y}$ or equivalent **[A1]**

ii. $b = 2$ **[B1]**

b) Uses correct version of the Quotient Rule **[M1*]**

$\frac{dy}{dx} = \frac{(1 + \cos x)\cos x - \sin x\,(-\sin x)}{(1 + \cos x)^2}$ Correct unsimplified **[A1]**

$= \frac{2}{(1 + \cos x)^2}$ **[A1]**

c) Sight of $-\sin x$ or $e^{\cos x}$ **[M1]**

$-\sin x\ e^{\cos x}$ **[A1]**

d) i. $8 = A(2 - x) + B(2 + x)$ **[M1]**

$A = 2$ **[A1]**

$B = 2$ **[A1]**

ii. Uses previous answer and attempts to integrate (presence of a log term is sufficient for this mark) **[M1*]**

$2\ln(2 + x) - 2\ln(2 - x)$ (Allow ft on their A and/or B) **[A1ft]**

$2\ln(\frac{2 + x}{2 - x})$ Combines logs (can be seen at any stage) **[M1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$2\ln[\,7 \div \frac{2 + a}{2 - a}]$ Obtains correct expression in terms of a. **[A1]**

Sets equal to $2\ln\left(\frac{7}{4}\right)$, removes logs and attempts to solve equation **[M1]**

$a = \frac{6}{5}$ or equivalent **[A1]**

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