

## IFYMB004 MATHEMATICS (BUSINESS)

### APPENDIX C - FORMULA BOOKLET

2021/22

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## A - LINEAR EQUATIONS

### Straight Line

$y = mx + c$ , where  $m$  is the slope or gradient, and  $c$  is the  $y$ -intercept

Line through points  $(x_0, y_0)$  and  $(x_1, y_1)$

$$\frac{y - y_0}{x - x_0} = \frac{y_1 - y_0}{x_1 - x_0}$$

## C - QUADRATIC EQUATIONS, REMAINDER THEOREM, EQUATION OF A CIRCLE AND INEQUALITIES

### Quadratic Formula

If  $ax^2 + bx + c = 0$ , then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$   $a \neq 0$

### Equation of a circle

$$(x - a)^2 + (y - b)^2 = r^2, \text{ where } r \text{ is the radius and } (a, b) \text{ is the centre of the circle}$$

## D - INDICES, EXPONENTIAL AND LOGARITHMIC FUNCTIONS

$$\log_a x = \frac{\log_b x}{\log_b a} \quad e^{x \ln a} = a^x$$

## E - BINOMIAL EXPANSIONS, SEQUENCES AND SERIES

### Binomial Series

When  $n$  is a positive integer,

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n-1} a b^{n-1} + b^n$$

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

$$\binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

### Arithmetic Series

$a$  is the first term,  $d$  is the common difference

$n^{\text{th}}$  term:  $u_n = a + (n-1)d$ ,

Sum to  $n$  terms:  $S_n = \frac{n}{2}(2a + (n-1)d)$

### Geometric Series

$a$  is the first term,  $r$  is the common ratio

$n^{\text{th}}$  term:  $u_n = ar^{n-1}$

Sum to  $n$  terms:  $S_n = \frac{a(1-r^n)}{1-r}$

Sum to infinity:  $S_\infty = \frac{a}{1-r}$  provided  $|r| < 1$

## F - TRIGONOMETRIC FUNCTIONS

$$\sin^2 x + \cos^2 x = 1$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

## G & N - DIFFERENTIATION AND FURTHER DIFFERENTIATION

$f(x)$	$f'(x) = \frac{df(x)}{dx}$
$x^n$	$nx^{n-1}$
$e^{kx}$	$ke^{kx}$
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$

Chain Rule: If  $y$  is a function of  $u$  and  $u$  is a function of  $x$ , then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Product Rule: If  $y = uv$ , where  $u$  and  $v$  are functions of  $x$ , then

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Quotient Rule: If  $y = \frac{u}{v}$  where  $u$  and  $v$  are functions of  $x$ , then

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

## H &amp; O - INTEGRATION AND FURTHER INTEGRATION

$f(x)$	$\int f(x)dx$
$x^n, n \neq 1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln x $
$\frac{1}{ax+b}$	$\frac{1}{a} \ln ax+b $
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$-\ln \cos x $
$e^{kx}$	$\frac{1}{k}e^{kx}$

Integration by parts

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Applications of integration

$$\text{Area under a curve: } A = \int_a^b y dx$$

## I – INTRODUCTION TO STATISTICS

Measures of average and spread

### Ungrouped Data

Mean

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

Variance

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n x_i^2 - \bar{x}^2$$

Standard Deviation

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2 - \bar{x}^2}$$

### Grouped Data

$$\bar{x} = \frac{\sum_{i=1}^n x_i f_i}{\sum_{i=1}^n f_i}$$

$$\sigma^2 = \frac{\sum_{i=1}^n x_i^2 f_i}{\sum_{i=1}^n f_i} - \bar{x}^2$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^n x_i^2 f_i}{\sum_{i=1}^n f_i} - \bar{x}^2}$$

Change of origin/scale :

If  $X_i = k(x_i - a)$ ,

then  $\bar{X} = k(\bar{x} - a)$

$$\sigma_x = k\sigma_x. \quad \text{V2021 BS (iv.)}$$

## J - FURTHER PROBABILITY AND SET THEORY

$$\underline{P(A \cup B)} = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = \underline{P(A)P(B | A)} = P(B)P(A | B)$$

↓  
A and B are independent

$$P(A) = P(A|B) \Leftrightarrow A, B \text{ Independent}$$

$$P(B) = P(B|A)$$

## K - CORRELATION, LINEAR REGRESSION AND TIME SERIES

For a set of  $n$  points  $(x_i, y_i)$

Pearson's Product Moment Correlation Coefficient

$$r = \frac{s_{xy}}{s_x s_y}, \text{ where } -1 \leq r \leq 1$$

Regression line of  $y$  on  $x$

$$y - \bar{y} = \frac{s_{xy}}{s_{xx}}(x - \bar{x}) \quad \text{where } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

$$s_{xy} = \frac{\sum_{i=1}^n x_i y_i}{n} - \bar{x} \bar{y}, \quad s_x = \sqrt{\frac{\sum_{i=1}^n x_i^2}{n} - \bar{x}^2}, \quad s_y = \sqrt{\frac{\sum_{i=1}^n y_i^2}{n} - \bar{y}^2}$$

## L - PROBABILITY DISTRIBUTIONS

$X$  is a random variable taking values  $x_i$  with  $P(X = x_i) = p_i$  and  $\sum_{\text{all } x} p_i = 1$

Expected Value:  $\mu = E(X) = \sum x_i p_i$

Variance:  $\sigma^2 = \text{Var}(X) = E(X^2) - [E(X)]^2 = \sum x_i^2 p_i - \mu^2$

Binomial distribution  $X \sim B(n, p)$

Probability of  $x$  ( $0 \leq x \leq n$ ):  $P(X = x) = \binom{n}{x} p^x q^{n-x}$

where  $x = 0, 1, \dots, n$ ,  $0 < p < 1$ ,  $q = 1 - p$

Mean:  $\bar{x} = np$

Variance:  $\sigma_x^2 = npq$

**Normal Distribution****Standardised variable**

$$Z = \frac{X - \mu}{\sigma}$$

Confidence interval  $100(1 - \alpha)\%$ 

$$\left( \bar{x} - \frac{z\alpha}{2\sqrt{n}} \sigma, \bar{x} + \frac{z\alpha}{2\sqrt{n}} \sigma \right)$$

**Percentage Points of the Normal Distribution**

If  $Z$  has a normal distribution with mean = 0 and variance = 1, the table gives the values of  $z$  satisfying  $P(Z > z) = p$ .

$p$	$z$
0.3000	0.5244
0.2000	0.8416
0.1500	1.0364
0.1000	1.2816
0.0500	1.6449
0.0250	1.9600
0.0100	2.3263
0.0050	2.5758
0.0010	3.0902
0.0005	3.2905

## Binomial Cumulative Distribution Function

The table below show the value  $P(X \leq x)$ , where  $X$  has a binomial distribution with index  $n$  and parameter  $p$ .

		$p =$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
$n = 5$	$x = 0$	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313	
	1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875	
	2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000	
	3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125	
	4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688	
$n = 6$	$x = 0$	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156	
	1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094	
	2	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438	
	3	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563	
	4	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906	
	5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844	
$n = 7$	$x = 0$	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078	
	1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625	
	2	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266	
	3	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000	
	4	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734	
	5	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375	
	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922	
$n = 8$	$x = 0$	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039	
	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352	
	2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445	
	3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633	
	4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367	
	5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555	
	6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648	
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961

$p =$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	
$n = 9$	$x = 0$	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
	1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
	2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898
	3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
	4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
	5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
	6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980

$p =$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	
$n = 10$	$x = 0$	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
	2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
	3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
	4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
	5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
	6	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281
	7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453
	8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	

$p =$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	
$n = 12$	$x = 0$	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002
	1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032
	2	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193
	3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730
	4	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938
	5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872
	6	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.6128
	7	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062
	8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270
	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9972	0.9921	0.9807
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998

$p =$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	
$n = 15$	$x = 0$	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000
	1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005
	2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037
	3	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176
	4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592
	5	0.9999	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509
	6	1.0000	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036
	7	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000
	8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964
	9	1.0000	1.0000	1.0000	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491
	10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9937	0.9824
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9963
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$n = 20$	$p =$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
	$x = 0$	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000
	1	0.7358	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000
	2	0.9245	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002
	3	0.9841	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013
	4	0.9974	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059
	5	0.9997	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207
	6	1.0000	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577
	7	1.0000	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316
	8	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517
	9	1.0000	1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119
	10	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881
	11	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483
	12	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	0.8684
	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9935	0.9786	0.9423
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9793
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$p =$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
$x = 0$	0.2774	0.0718	0.0172	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000
1	0.6424	0.2712	0.0931	0.0274	0.0070	0.0016	0.0003	0.0001	0.0000	0.0000
2	0.8729	0.5371	0.2537	0.0982	0.0321	0.0090	0.0021	0.0004	0.0001	0.0000
3	0.9659	0.7636	0.4711	0.2340	0.0962	0.0332	0.0097	0.0024	0.0005	0.0001
4	0.9928	0.9020	0.6821	0.4207	0.2137	0.0905	0.0320	0.0095	0.0023	0.0005
5	0.9988	0.9666	0.8385	0.6167	0.3783	0.1935	0.0826	0.0294	0.0086	0.0020
6	0.9998	0.9905	0.9305	0.7800	0.5611	0.3407	0.1734	0.0736	0.0258	0.0073
7	1.0000	0.9977	0.9745	0.8909	0.7265	0.5118	0.3061	0.1536	0.0639	0.0216
8	1.0000	0.9995	0.9920	0.9532	0.8506	0.6769	0.4668	0.2735	0.1340	0.0539
9	1.0000	0.9999	0.9979	0.9827	0.9287	0.8106	0.6303	0.4246	0.2424	0.1148
10	1.0000	1.0000	0.9995	0.9944	0.9703	0.9022	0.7712	0.5858	0.3843	0.2122
11	1.0000	1.0000	0.9999	0.9985	0.9893	0.9558	0.8746	0.7323	0.5426	0.3450
12	1.0000	1.0000	1.0000	0.9996	0.9966	0.9825	0.9396	0.8462	0.6937	0.5000
13	1.0000	1.0000	1.0000	0.9999	0.9991	0.9940	0.9745	0.9222	0.8173	0.6550
14	1.0000	1.0000	1.0000	1.0000	0.9998	0.9982	0.9907	0.9656	0.9040	0.7878
15	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9971	0.9868	0.9560	0.8852
16	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9957	0.9826	0.9461
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9942	0.9784
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9927
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9980
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$p =$	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
$x = 0$	0.2146	0.0424	0.0076	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.5535	0.1837	0.0480	0.0105	0.0020	0.0003	0.0000	0.0000	0.0000	0.0000
2	0.8122	0.4114	0.1514	0.0442	0.0106	0.0021	0.0003	0.0000	0.0000	0.0000
3	0.9392	0.6474	0.3217	0.1227	0.0374	0.0093	0.0019	0.0003	0.0000	0.0000
4	0.9844	0.8245	0.5245	0.2552	0.0979	0.0302	0.0075	0.0015	0.0002	0.0000
5	0.9967	0.9268	0.7106	0.4275	0.2026	0.0766	0.0233	0.0057	0.0011	0.0002
6	0.9994	0.9742	0.8474	0.6070	0.3481	0.1595	0.0586	0.0172	0.0040	0.0007
7	0.9999	0.9922	0.9302	0.7608	0.5143	0.2814	0.1238	0.0435	0.0121	0.0026
8	1.0000	0.9980	0.9722	0.8713	0.6736	0.4315	0.2247	0.0940	0.0312	0.0081
9	1.0000	0.9995	0.9903	0.9389	0.8034	0.5888	0.3575	0.1763	0.0694	0.0214
10	1.0000	0.9999	0.9971	0.9744	0.8943	0.7304	0.5078	0.2915	0.1350	0.0494
11	1.0000	1.0000	0.9992	0.9905	0.9493	0.8407	0.6548	0.4311	0.2327	0.1002
12	1.0000	1.0000	0.9998	0.9969	0.9784	0.9155	0.7802	0.5785	0.3592	0.1808
13	1.0000	1.0000	1.0000	0.9991	0.9918	0.9599	0.8737	0.7145	0.5025	0.2923
14	1.0000	1.0000	1.0000	0.9998	0.9973	0.9831	0.9348	0.8246	0.6448	0.4278
15	1.0000	1.0000	1.0000	0.9999	0.9992	0.9936	0.9699	0.9029	0.7691	0.5722
16	1.0000	1.0000	1.0000	1.0000	0.9998	0.9979	0.9876	0.9519	0.8644	0.7077
17	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9955	0.9788	0.9286	0.8192
18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9986	0.9917	0.9666	0.8998
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9971	0.9862	0.9506
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9950	0.9786
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9984	0.9919
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9974
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 30

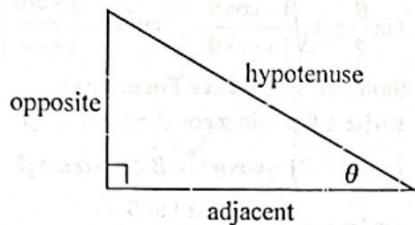
# Trig Cheat Sheet

## Definition of the Trig Functions

### Right triangle definition

For this definition we assume that

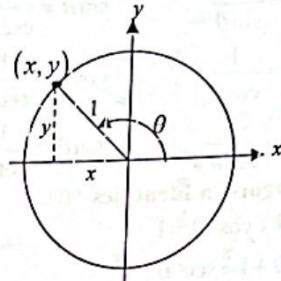
$$0 < \theta < \frac{\pi}{2} \text{ or } 0^\circ < \theta < 90^\circ.$$



$$\begin{aligned}\sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} \\ \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} \\ \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} & \cot \theta &= \frac{\text{adjacent}}{\text{opposite}}\end{aligned}$$

### Unit circle definition

For this definition  $\theta$  is any angle.



$$\begin{aligned}\sin \theta &= \frac{y}{1} = y & \csc \theta &= \frac{1}{y} \\ \cos \theta &= \frac{x}{1} = x & \sec \theta &= \frac{1}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$

## Facts and Properties

### Domain

The domain is all the values of  $\theta$  that can be plugged into the function.

$\sin \theta$ ,  $\theta$  can be any angle

$\cos \theta$ ,  $\theta$  can be any angle

$\tan \theta$ ,  $\theta \neq \left(n + \frac{1}{2}\right)\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$

$\csc \theta$ ,  $\theta \neq n\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$

$\sec \theta$ ,  $\theta \neq \left(n + \frac{1}{2}\right)\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$

$\cot \theta$ ,  $\theta \neq n\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$

### Range

The range is all possible values to get out of the function.

$-1 \leq \sin \theta \leq 1$   $\csc \theta \geq 1$  and  $\csc \theta \leq -1$

$-1 \leq \cos \theta \leq 1$   $\sec \theta \geq 1$  and  $\sec \theta \leq -1$

$-\infty < \tan \theta < \infty$   $-\infty < \cot \theta < \infty$

### Period

The period of a function is the number,  $T$ , such that  $f(\theta + T) = f(\theta)$ . So, if  $\omega$  is a fixed number and  $\theta$  is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

## Formulas and Identities

### Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Reciprocal Identities

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} & \sin \theta &= \frac{1}{\csc \theta} \\ \sec \theta &= \frac{1}{\cos \theta} & \cos \theta &= \frac{1}{\sec \theta} \\ \cot \theta &= \frac{1}{\tan \theta} & \tan \theta &= \frac{1}{\cot \theta} \end{aligned}$$

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Even/Odd Formulas

$$\begin{aligned} \sin(-\theta) &= -\sin \theta & \csc(-\theta) &= -\csc \theta \\ \cos(-\theta) &= \cos \theta & \sec(-\theta) &= \sec \theta \\ \tan(-\theta) &= -\tan \theta & \cot(-\theta) &= -\cot \theta \end{aligned}$$

### Periodic Formulas

If  $n$  is an integer.

$$\sin(\theta + 2\pi n) = \sin \theta \quad \csc(\theta + 2\pi n) = \csc \theta$$

$$\cos(\theta + 2\pi n) = \cos \theta \quad \sec(\theta + 2\pi n) = \sec \theta$$

$$\tan(\theta + \pi n) = \tan \theta \quad \cot(\theta + \pi n) = \cot \theta$$

### Double Angle Formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= 1 - 2 \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Degrees to Radians Formulas

If  $x$  is an angle in degrees and  $t$  is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \Rightarrow t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

### Half Angle Formulas (alternate form)

$$\begin{aligned} \sin \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{2}} & \sin^2 \theta &= \frac{1}{2}(1 - \cos(2\theta)) \\ \cos \frac{\theta}{2} &= \pm \sqrt{\frac{1 + \cos \theta}{2}} & \cos^2 \theta &= \frac{1}{2}(1 + \cos(2\theta)) \\ \tan \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} & \tan^2 \theta &= \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)} \end{aligned}$$

### Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

### Product to Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

### Sum to Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

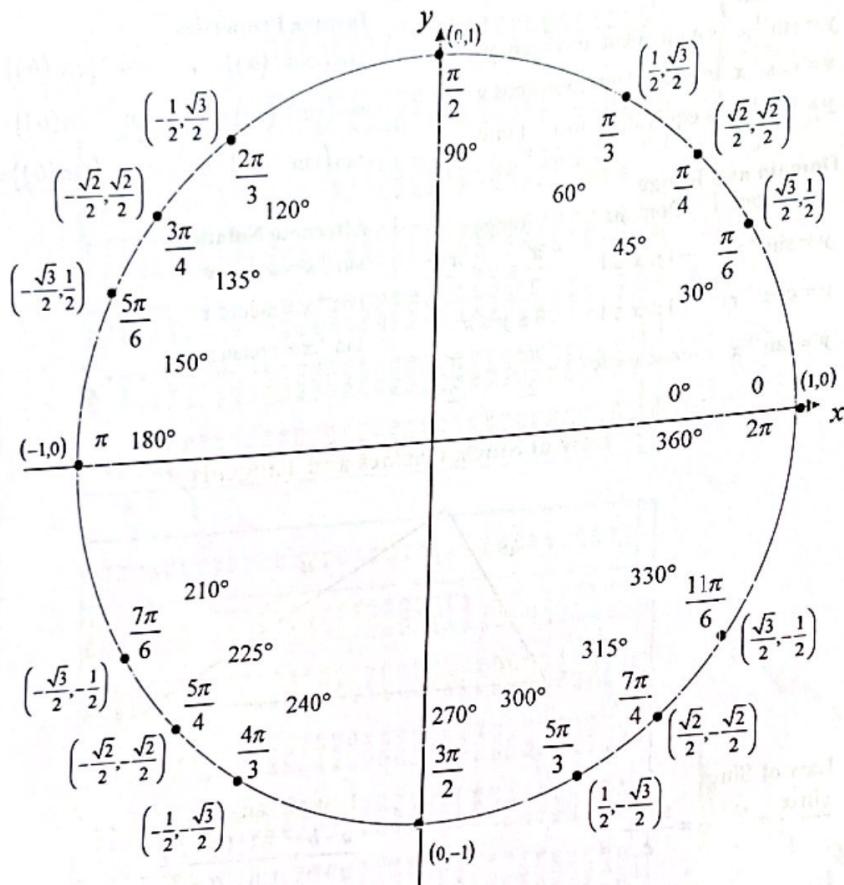
### Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta \quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

## Unit Circle



For any ordered pair on the unit circle  $(x, y)$  :  $\cos\theta = x$  and  $\sin\theta = y$

**Example**

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \quad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

## Inverse Trig Functions

### Definition

$y = \sin^{-1} x$  is equivalent to  $x = \sin y$

$y = \cos^{-1} x$  is equivalent to  $x = \cos y$

$y = \tan^{-1} x$  is equivalent to  $x = \tan y$

### Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

### Inverse Properties

$$\cos(\cos^{-1}(x)) = x \quad \cos^{-1}(\cos(\theta)) = \theta$$

$$\sin(\sin^{-1}(x)) = x \quad \sin^{-1}(\sin(\theta)) = \theta$$

$$\tan(\tan^{-1}(x)) = x \quad \tan^{-1}(\tan(\theta)) = \theta$$

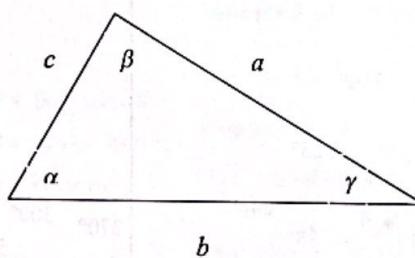
### Alternate Notation

$$\sin^{-1} x = \arcsin x$$

$$\cos^{-1} x = \arccos x$$

$$\tan^{-1} x = \arctan x$$

## Law of Sines, Cosines and Tangents



### Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

### Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

### Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos \frac{1}{2}(\alpha - \beta)}{\sin \frac{1}{2}\gamma}$$

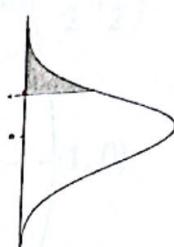
### Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(\alpha - \beta)}{\tan \frac{1}{2}(\alpha + \beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(\beta - \gamma)}{\tan \frac{1}{2}(\beta + \gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan \frac{1}{2}(\alpha - \gamma)}{\tan \frac{1}{2}(\alpha + \gamma)}$$

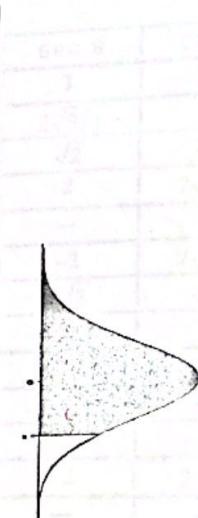
Table of Standard Normal Probabilities for Negative Z-scores



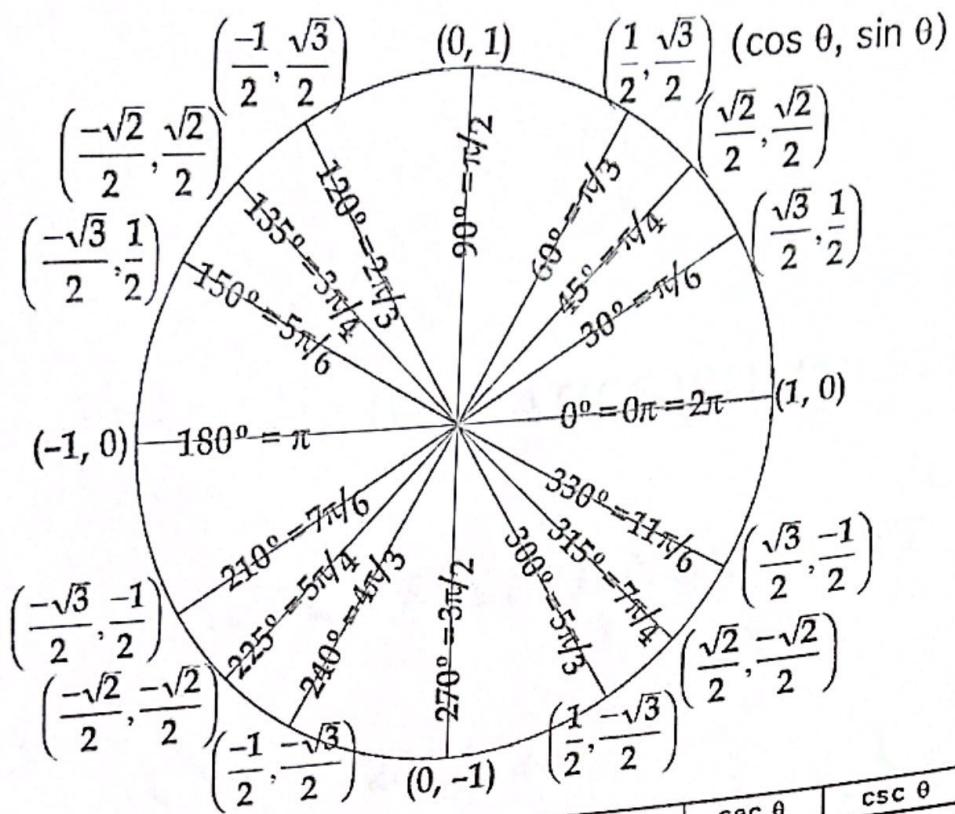
<i>z</i>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0012	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0053	0.0052	0.0051	0.0049
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0066	0.0064	0.0062
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0226	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0394	0.0387	0.0380	0.0370
-1.6	0.0548	0.0537	0.0527	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0653	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0754	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1818	0.1788	0.1762	0.1736	0.1711	0.1685	0.1653	0.1611	0.1560
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2338	0.2337	0.2296	0.2256	0.2217	0.2148	0.2104	0.2053
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3065	0.3030	0.3015	0.2981	0.2946	0.2912	0.2871	0.2835	0.2800	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.3974	0.3939	0.3897	0.3859	0.3818
-0.1	0.4602	0.4522	0.4483	0.4443	0.4364	0.4323	0.4286	0.4247	0.4204	0.4164
0.0	0.5000	0.4960	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641	0.4601

Note that the probabilities given in this table represent the area to the LEFT of the z-score.  
The area to the RIGHT of a z-score = 1 - the area to the LEFT of the z-score

Table of Standard Normal Probabilities for Positive Z-scores



<i>z</i>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5020	0.5050	0.5120	0.5190	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5198	0.5143	0.5178	0.5217	0.5257	0.5296	0.5336	0.5373	0.5411	0.5450
0.2	0.5393	0.5282	0.5371	0.5510	0.5587	0.5626	0.5664	0.5703	0.5741	0.5783
0.3	0.5679	0.6217	0.6255	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	0.6557
0.4	0.6354	0.6591	0.6628	0.6664	0.6706	0.6744	0.6782	0.6819	0.6854	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	0.7579	0.7610
0.7	0.7580	0.7642	0.7673	0.7710	0.7749	0.7787	0.7823	0.7852	0.7880	0.7906
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8463	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8663	0.8685	0.8708	0.8729	0.8749	0.8770	0.8790	0.8820	0.8850
1.2	0.8849	0.8869	0.8888	0.8907	0.8923	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9040	0.9066	0.9084	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	0.9191
1.4	0.9192	0.9222	0.9256	0.9286	0.9311	0.9332	0.9352	0.9370	0.9382	0.9394
1.5	0.9352	0.9385	0.9417	0.9443	0.9474	0.9494	0.9525	0.9545	0.9564	0.9583
1.6	0.9432	0.9463	0.9493	0.9514	0.9543	0.9571	0.9598	0.9626	0.9653	0.9683
1.7	0.9554	0.9584	0.9613	0.9643	0.9671	0.9701	0.9730	0.9759	0.9788	0.9817
1.8	0.9654	0.9684	0.9713	0.9743	0.9772	0.9802	0.9831	0.9860	0.9889	0.9918
1.9	0.9713	0.9749	0.9788	0.9828	0.9858	0.9888	0.9917	0.9947	0.9976	0.9995
2.0	0.9772	0.9818	0.9858	0.9898	0.9937	0.9977	0.9997	0.9999	0.9999	0.9999
2.1	0.9821	0.9856	0.9891	0.9921	0.9951	0.9981	0.9997	0.9999	0.9999	0.9999
2.2	0.9861	0.9894	0.9925	0.9955	0.9985	0.9997	0.9999	0.9999	0.9999	0.9999
2.3	0.9893	0.9926	0.9957	0.9987	0.9997	0.9999	0.9999	0.9999	0.9999	0.9999
2.4	0.9918	0.9948	0.9977	0.9997	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.5	0.9938	0.9968	0.9997	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.6	0.9953	0.9983	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.7	0.9963	0.9987	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.8	0.9974	0.9997	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.9	0.9981	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.0	0.9983	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.1	0.9986	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.2	0.9989	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.3	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997



	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\cot \theta$	$\sec \theta$	$\csc \theta$
$0^\circ = 0\pi = 2\pi$	0	1	0	---	1	--
$30^\circ = \pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}/3$	$\sqrt{3}$	$2\sqrt{3}/3$	2
$45^\circ = \pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$60^\circ = \pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\sqrt{3}/3$	2	$2\sqrt{3}/3$
$90^\circ = \pi/2$	1	0	--	0	--	1
$120^\circ = 2\pi/3$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\sqrt{3}/3$	-2	$2\sqrt{3}/3$
$135^\circ = 3\pi/4$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
$150^\circ = 5\pi/6$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\sqrt{3}/3$	$-\sqrt{3}$	$-2\sqrt{3}/3$	2
$180^\circ = \pi$	0	-1	0	--	-1	--
$210^\circ = 7\pi/6$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\sqrt{3}/3$	$\sqrt{3}$	$-2\sqrt{3}/3$	-2
$225^\circ = 5\pi/4$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
$240^\circ = 4\pi/3$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\sqrt{3}/3$	-2	$-2\sqrt{3}/3$
$270^\circ = 3\pi/2$	-1	0	--	0	--	-1
$300^\circ = 5\pi/3$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\sqrt{3}/3$	2	$-2\sqrt{3}/3$
$315^\circ = 7\pi/4$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
$330^\circ = 11\pi/6$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\sqrt{3}/3$	$-\sqrt{3}$	$2\sqrt{3}/3$	-2

-- undefined