NCUK

# THE NCUK INTERNATIONAL FOUNDATION YEAR

## IFYMB002 Mathematics Business
## Time-Controlled Assessment

## 2019-20

## INSTRUCTIONS TO STUDENTS

| | |
|---|---|
| **SECTION A** | **Answer ALL questions. This section carries 40 marks.** |
| **SECTION B** | **Answer THREE questions ONLY. This section carries 60 marks.** |
| **The marks for each question are indicated in square brackets [ ].** | |
| **Guide Time: 2 hours**<br><br>**The Guide Time is how long you are expected to spend completing this Time-Controlled Assessment. You are allowed 24 hours in total to complete and submit.** | |

- You **MUST** show **ALL** of your working. This is very important. You will score no marks if there is not enough working shown – even if your answer is correct.
- An approved calculator may be used in the assessment.
- All work must be completed independently. The penalty for collusion is a mark of zero.
- Due to the nature of the questions, there should be no need to use external sources of information to answer them. If you do use external sources of information you must ensure you reference these. Plagiarism is a form of academic misconduct and will be penalised.
- Work must be submitted by the deadline provided. Your Study Centre can be contacted only for guidance on submission of work and cannot comment on the contents of the assessment.
- Your work can be word-processed or handwritten. Once complete, any handwritten work will need to be clearly photographed/scanned and inserted into a single word-processed file for submission
- Work must be submitted in a single word-processed file using the standard NCUK cover page.

# Section A
# Answer <u>ALL</u> questions. This section carries 40 marks.

**Question A1**

Point A lies at $(-4, 3)$ and point B lies at $(10, 7)$.

Line $l$ passes through point M which is the mid-point of AB, and is parallel to the line with equation $5x - 4y + 8 = 0$.

Find the equation of line $l$. **[ 3 ]**

**Question A2**

Three letters are selected from the word REPOSSESSIONS one after the other, with no replacement.

Find the probability that all three letters are 'S'. Give your answer to **3** significant figures. **[ 4 ]**

**In this question, 1 mark will be given for the correct use of significant figures.**

**Question A3**

When $x^3 + 2x^2 + 6x - 4$ is divided by $(x - k)$, the remainder is $(k^3 + 4)$.

<u>Use the Remainder Theorem</u> to find the values of $k$. **[ 4 ]**

**Question A4**

The coefficient of the $x^4$ term in the expansion of $(p + 3x)^6$ is 540.

Show that $p = \pm\frac{2}{3}$. **[ 3 ]**

**Question A5**

**(a)** Write $\frac{1}{2}\log_a 144 + 2\log_a 6 - \frac{3}{4}\log_a 16 - \log_a 6 \quad (a > 0)$ as a single logarithm **[ 2 ]**

**(b)** Hence find the value of $a$ given that

$$\frac{1}{2}\log_a 144 + 2\log_a 6 - \frac{3}{4}\log_a 16 - \log_a 6 = 2.$$ **[ 1 ]**

  

**Question A6**

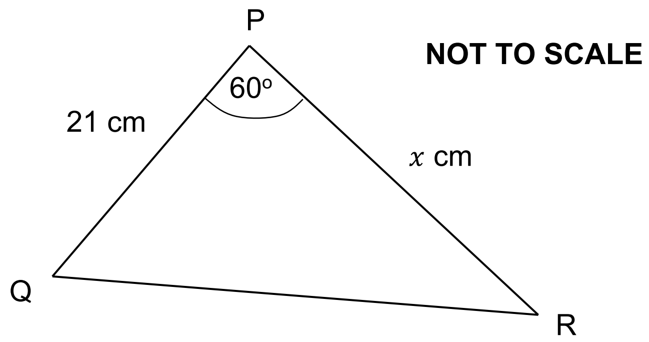


**Figure 1**

Figure 1 shows the acute angled triangle PQR where PQ $= 21$ cm, PR $= x$ cm and angle P $= 60°$. The area of triangle PQR is $126\sqrt{3}$ cm$^2$.

Find the value of $x$. **[ 3 ]**

**Question A7**

A curve has equation $y = c - 12x - 3x^2$ where $c$ is a constant.

Find the coordinates, in terms of $c$, of the stationary value on the curve. **[ 4 ]**

**Question A8**

Find **[ 3 ]**

$$\int \frac{8 - 4t + 2t^2}{t^3}\ dt.$$

**Question A9**

A discrete random variable, $X$, has probability distribution as given in the table below.

| $x$ | $-3$ | $-1$ | $0$ | $1$ | $2$ | $c$ |
|---|---|---|---|---|---|---|
| $p(X = x)$ | 0.28 | 0.3 | 0.06 | 0.07 | 0.15 | 0.14 |

**(a)** Show that $c = 5.5$ if $\mathrm{E}(X) = 0$. **[ 2 ]**

**(b)** Find $\mathrm{E}(X^2)$ and the standard deviation of $X$. **[ 3 ]**

**Question A10**

When £1000 is invested for 4 years, the compound interest is £188.

Find the rate of interest as a percentage. **[ 3 ]**

**Question A11**

A curve has equation $y = \sin mx \cos mx$ where $m$ is a constant.

**(a)** Find $\frac{dy}{dx}$ **[ 2 ]**

**(b)** Find the equation of the tangent to the curve when $x = \frac{\pi}{3m}$. **[ 3 ]**

# Section B
# Answer <u>THREE</u> questions ONLY. This section carries 60 marks.

**Question B1**

**(a)** Solve the equations $5p + 2q = 4$

$$4p - 4q = -1$$ **[ 4 ]**

**(b)** **i.** Factorise $21x^2 + 2x - 8$. **[ 2 ]**

**ii.** Hence solve the equation $21x^2 + 2x - 8 = 0$. **[ 1 ]**

Give your answers in the form $\frac{a}{b}$ where $a$ and $b$ are integers.

**iii.** Find the range of values satisfying $21x^2 + 2x - 8 \geq 0$. **[ 2 ]**

**(c)** **i.** Divide $x^3 - x^2 - 20x$ by $(x - 5)$. **[ 2 ]**

**ii.** Hence, or otherwise, factorise $x^3 - x^2 - 20x$ completely. **[ 1 ]**

**(d)** In an arithmetic series, the 16th term is 120 less than the first term.

**i.** Find the common difference. **[ 2 ]**

The sum of the first $37$ terms is 0.

**ii.** Find the first term. **[ 3 ]**

**(e)** A geometric series has common ratio $\frac{2}{3}$ and the $9^{\text{th}}$ term is $256$.

**i.** Find the first term. Give your answer <u>in full</u> with no rounding off. **[ 2 ]**

**ii.** Find the sum to infinity. Give your answer <u>in full</u> with no rounding off. **[ 1 ]**

**Question B2**

**(a)** Two variables, $x$ and $y$, are connected by the formula

$$y = \frac{10}{e^{1.2x} + 4} \quad (x \geq 0)$$

**i.** Find the value of $y$ when $x = 0.1$ **[ 2 ]**

**ii.** Find the value of $x$ when $y = \frac{2}{3}$. **[ 3 ]**

**iii.** What happens to $y$ as $x$ becomes large? **[ 1 ]**

**(b)** Given

$$2\log_5 x + \log_5\left(1 + \frac{3}{x} - \frac{3}{x^2}\right) = 2\log_a a \quad (x > 1)$$

where $a$ is positive constant, find the value of $x$. **[ 4 ]**

**(c)** Solve the equation $\cos 3\theta = -0.891 \quad (-90° \leq \theta \leq 90°)$ **[ 5 ]**

**(d)**

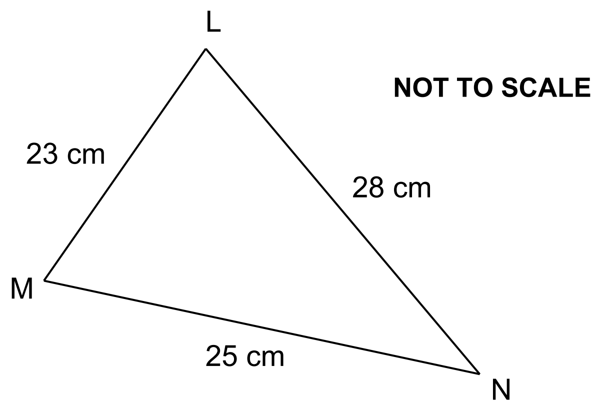


**Figure 2**

Figure 2 shows the acute angled triangle LMN where LM = 23 cm, MN = 25 cm and LN = 28 cm.

**i.** Find angle L. **[ 3 ]**

**ii.** Find angle M. **[ 2 ]**

**Question B3**

**(a)** A curve has equation $y = x^4 - 4x^3 + 17$.

**i.** Find $\frac{dy}{dx}$ **[ 2 ]**

There are two stationary values on the curve.

**ii.** Show that there is a stationary value at $x = 0$ and find the value of $x$ at the other stationary value. **[ 3 ]**

**iii.** Confirm there is a point of inflexion at $x = 0$ and find whether the other stationary value is a maximum or a minimum. **[ 4 ]**

**iv.** Sketch the curve showing clearly the coordinates of the stationary values and where the curve crosses the $y-$ axis. You do <u>not</u> have to show where the curve crosses the $x-$ axis. **[ 3 ]**

**(b) i.** Evaluate **[ 4 ]**

$$\int_{\ln 4}^{\ln 16} \left(e^{2x} - \frac{1}{2}\right) dx.$$

Give your answer in the form $a + \ln(\frac{1}{b})$ where $a$ and $b$ are integers.

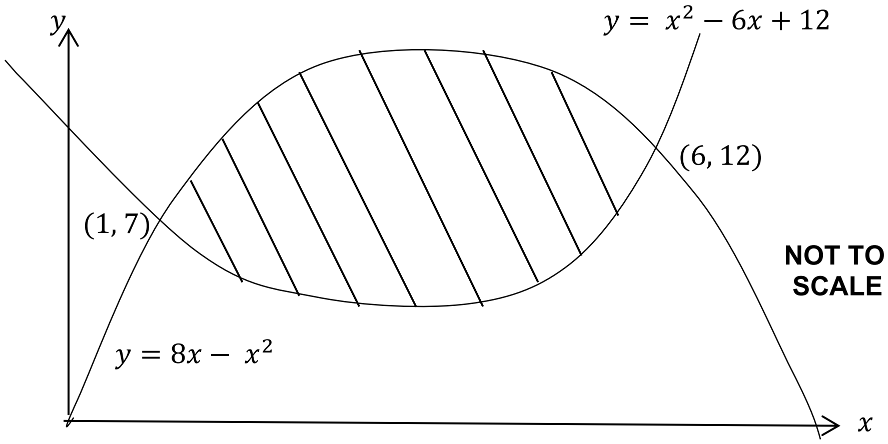


**Figure 3**

Figure 3 shows the curves $y = x^2 - 6x + 12$ and $y = 8x - x^2$. They intersect at the points $(1, 7)$ and $(6, 12)$.

**ii.** The shaded area on the diagram is bounded by both curves.

Show that this area is $41\frac{2}{3}$. **[ 4 ]**

**Question B4**

**(a)** A shop sells packets of crisps (potato chips). The masses of the packets can be assumed to follow a Normal distribution with mean 200 grams and standard deviation 8 grams.

**i.** What percentage of packets have masses between 204 grams and 206 grams? **[ 4 ]**

**ii.** 91% of the packets have masses below $x$ grams.

Find the value of $x$. **[ 2 ]**

**(b)** In a particular community, 15% of people are left-handed.

A sample of 30 people is chosen. Find the probability that:

**i.** less than 6 people are left-handed. **[ 1 ]**

**ii.** more than 27 people are right-handed. **[ 2 ]**

**(c)** $Y$ is a discrete random variable. $\mathrm{E}(Y) = 1.2$ and $\mathrm{Var}(Y) = 6$.

Another discrete random variable, $Z$, is connected to $Y$ by the formula $Z = 5Y + 4$.

Write down $\mathrm{E}(Z)$, $\mathrm{Var}(Z)$ and $\mathrm{E}(Z^2)$. **[ 3 ]**

**(d)** Events $A$ and $B$ are such that $p(A) = 4x$, $p(B) = 8x$ and $p(A \cap B) = 2x$ where $0 < x < 1$.

**i.** Write down, in terms of $x$, $p(A \cup B)$. **[ 1 ]**

**ii.** Write down $p(A|B)$. **[ 1 ]**

**iii.** Find the value of $x$ for which events $A$ and $B$ are independent. **[ 2 ]**

**(e)** Events $X$ and $Y$ are such that $p(X) = \frac{7}{10}$, $p(Y) = \frac{17}{40}$ and $p(X \cup Y) = \frac{33}{40}$.

**i.** Draw a Venn diagram. **[ 2 ]**

**ii.** Find $p(X' \cap Y)$ and $p(Y|X)$. **[ 2 ]**

  

**Question B5**

**(a)** A curve has equation $3x^2 - 12x + 2y^2 - 4y = -6$.

**i.** Find $\frac{dy}{dx}$ in terms of $x$ and $y$. **[ 3 ]**

**ii.** Find the coordinates of the stationary values on the curve. **[ 4 ]**

**(b) i.** A curve has equation $y = \frac{2k}{(x + k)(x + 2k)}$ where $k$ is a constant.

Write $\frac{2k}{(x + k)(x + 2k)}$ in partial fractions in the form $\frac{A}{x + k} + \frac{B}{x + 2k}$ where $A$ and $B$ are to be determined. **[ 3 ]**

**ii.** Hence evaluate

$$\int_0^{2k} \frac{2k}{(x+k)(x+2k)}\ dx.$$

Give your answer as a single logarithm. **[ 5 ]**

**(c)** Show that

$$\int_0^{\frac{\pi}{3}} 2x \cos x\ dx = \frac{\pi}{\sqrt{3}} - 1$$ **[ 5 ]**

**- This is the end of the Time-Controlled Assessment. -**