NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business

Examination

2016-17

Examination Session	**Time Allowed**
Semester Two	2 Hours 40 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A Answer ALL questions. This section carries 45 marks.

SECTION B Answer 4 questions ONLY. This section carries 80 marks.

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Line l_1 has equation $3x - y - 5 = 0$ and line l_2 has equation $9x + 2y + 30 = 0$.

Find the coordinates of the point where the two lines intersect. **[4]**

Question A2

A bag contains 3 red beads and 4 blue beads. One bead is taken from the bag and not replaced. 5 green beads are then added to the bag and a second bead is drawn.

Find the probability that the first bead is blue and the second bead is green. **[3]**

Question A3

Solve the quadratic equation $x^2 + 10x + 23 = 0$ **by completing the square.**

Give your answers in the form $a + \sqrt{b}$ and $a - \sqrt{b}$ where a and b are integers. **[4]**

Question A4

In the expansion of $(k + 2x)^5$ the coefficient of the term in x^2 is 90 times larger than the coefficient of the term in x^3 in the expansion of $(k + x)^4$.

Find the two possible values of k. *All working must be shown.* **[4]**

Question A5

Solve the equation $\log_4 x + \log_4(x - 6) = 2 \quad (x > 6)$. *Show all working*. **[4]**

Question A6

Solve the equation $\cos 2\theta = 0.54$ for $0 \le \theta \le 2\pi$. **[4]**

Question A7

A curve has equation $y = x^3 + \ln x$.

Write down $\frac{dy}{dx}$ and hence find its value when $x = 0.7$

Give your answer to **3** significant figures. **[4]**

In this question 1 mark will be given for the correct use of significant figures.

Question A8

During one week the mean number of hours of sunshine per day over the first 6 days was 5.5 hours. After the 7th day the mean had gone up to 6.0 hours per day.

Find the number of hours of sunshine on the 7th day. **[3]**

Question A9

Two events A and B are such that $p(A) = 0.36$ and $p(A \cap B) = 0.234$

Events A and B are independent.

Find $p(B)$ and $p(A \cup B)$. **[3]**

Question A10

A machine produces nails. A sample of 16 nails is taken and the lengths recorded. A 95% confidence interval has width 2.45 mm.

Find the standard deviation of the lengths of the nails. **[3]**

Question A11

A student invested £1200 and, after 4 years, had gained £231 compound interest.

Find the percentage rate of interest. **[4]**

Question A12

Evaluate

$$\int_0^{\ln 3} 8x\, e^{2x} \; dx$$

Give your answer in the form $p \ln q - r$ where p, q and r are integers. **[5]**

All working must be shown.

Section B
Answer 4 questions ONLY. This section carries 80 marks.

Question B1

a) Solve the inequality $x^2 - 7x - 8 \leq 0$. **[4]**

b) The function $f(x)$ is defined as $f(x) = x^3 + 7x^2 - x - 7$.

i. Use the Factor Theorem to show that $(x+7)$ is a factor of $f(x)$. **[2]**

ii. Divide $f(x)$ by $(x+7)$. **[2]**

iii. Hence factorise $f(x)$ completely. **[1]**

c) The first term of an arithmetic series is 8 and the 31st term is 203.

i. Find the common difference. **[2]**

ii. Find the sum of the first 49 terms. **[3]**

d) A geometric series has common ratio $\frac{4}{5}$ and sum to infinity 12000.

i. Show that the first term is 2400. **[1]**

ii. Find which term in the series is the first to fall below 1. **[3]**

iii. Find the sum of the first 9 terms. **[2]**

Question B2

a) Two variables x and y are connected by the equation

$$y = A(2^{kx}) + 8$$

where A and k are constants.

When $x = 0,\ y = 12$.

i. Write down the value of A. **[1]**

When $x = 20, y = 72$.

ii. Find the value of k. **[3]**

iii. Write down an expression for $\frac{dy}{dx}$ and hence find its value when $x = 12$. **[3]**

b) Solve the equation $e^x + 4e^{0.5x} - 21 = 0$.

Give your answer in the form $y = \ln a$ where a is an integer. **[4]**

c)

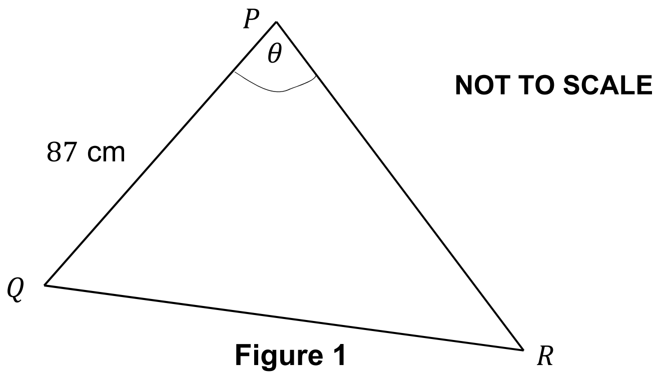

Figure 1

Figure 1 shows the acute-angled triangle PQR with $PQ = 87$ cm and angle $P = \theta$ where $\sin\theta = \frac{21}{29}$. The area of triangle PQR is 2205 cm^2.

i. Find the length of PR. **[2]**

ii. **Without** working out the value of θ, find $\cos\theta$. Give your answer in the form $\frac{m}{n}$ where m and n are integers. *All working must be shown*. **[2]**

iii. Hence find the length of QR. **[3]**

iv. Find the size of angle R. **[2]**

Question B3

a)

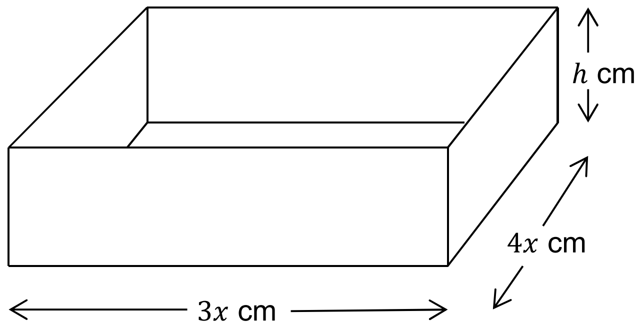

Figure 2

Figure 2 shows a cuboid which is $3x$ cm wide, $4x$ cm long and h cm high. **The cuboid has no top and no bottom** *i.e.* there are just the four walls. The total length of all the edges is 448 cm.

i. Express h in terms of x. **[2]**

ii. Show that the outside surface area, A, is given by $A = 1568x - 98x^2$. **[3]**

Each stage of your working must be clearly shown.

iii. Use $\frac{dA}{dx}$ to find the value of x which gives the maximum outside surface area. **[4]**

iv. Confirm that your outside surface area is a maximum. **[3]**

b)

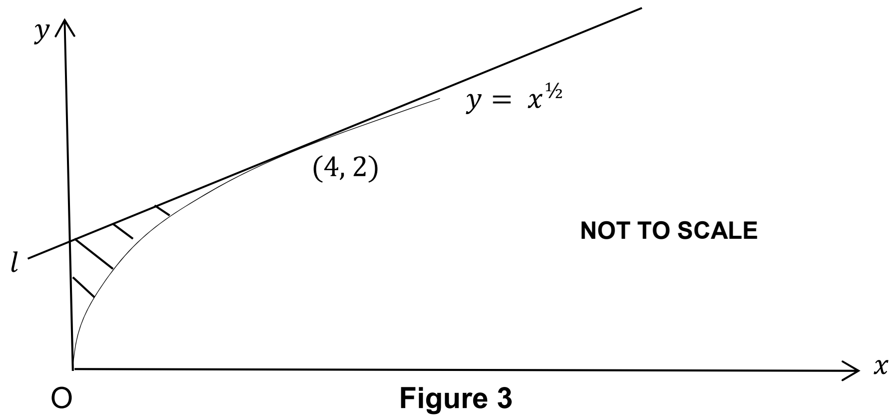

Figure 3

Figure 3 shows part of the curve $y = x^{½}$ and the tangent to the curve at the point $(4, 2)$ which is denoted by line l.

i. Show that the equation of line l is $y = \frac{1}{4}x + 1$. **[3]**

ii. Find the area, which is shaded on the diagram, that is bounded by the curve $y = x^{½}$, line l and the $y-$ axis. *Show all working*. **[5]**

Question B4

a) The midday temperature was recorded each day from 1 January until 31 March (90 days). The results are shown in the table below.

Temperature, t, in ℃	Frequency
$-2 \le t \le 0$	6
$0 < t \le 2$	13
$2 < t \le 4$	24
$4 < t \le 6$	18
$6 < t \le 8$	17
$8 < t \le 12$	12

(You may wish to copy and extend this table to help you answer some of the following questions.)

i. Estimate the mean and standard deviation. **[6]**

ii. Explain why your answers in part i are estimates. **[1]**

iii. In which interval does the lower quartile lie? **[1]**

A day is selected at random.

iv. Find the probability that the midday temperature on that day was above 6℃. **[1]**

b) A student plays two games of squash. The probability that she wins the first game is $\frac{3}{5}$. If she wins the first game, the probability that she wins the second is $\frac{3}{4}$. If she does not win the first game, the probability that she wins the second is $\frac{1}{2}$.

i. Draw and label a tree diagram. **[2]**

ii. Find the probability that she wins <u>at least</u> one game. **[3]**

c) In a group of 100 people, 42 are female and 84 are right-handed. There are 35 right-handed females.

i. Draw a Venn diagram to show this information. **[3]**

A person is selected at random.

ii. If F denotes the event 'the person is female' and R denotes the event 'the person is right-handed', find $p(F' \cap R)$, $p(F' \cup R')$ and $p(F|R)$. **[3]**

Question B5

a) A car salesman keeps a record of the numbers of cars bought each week (x) and the number of cars sold each week (y). He keeps the record for six weeks. The results are shown in the table below.

Number of cars bought (x)	Number of cars sold (y)	x^2	y^2	xy
3	7			
8	1			
5	5			
2	3			
6	8			
3	6			
$\sum x = 27$	$\sum y = 30$			

i. Copy and complete the table. **[3]**

ii. Find s_x, s_y and s_{xy}. Hence find the Product Moment Correlation Coefficient. **[4]**

iii. Describe the correlation between the numbers of cars bought and the numbers of cars sold. **[1]**

b) A survey is carried out on the number of people who visited a café each day (N) and the number of loaves of bread that were used (L). The survey was carried out over several weeks. A student worked out the equation of the regression line of L on N as $L = -0.75N + 6$.

Give **two** reasons why this equation is unlikely to be correct. **[2]**

c) The masses of chocolate bars can be assumed to follow a Normal distribution with mean 320 grams and standard deviation 20. A chocolate bar is chosen at random.

i. Find the probability that its mass is below 306 grams. **[3]**

The chocolate bars are packed in boxes of 20.

ii. Find the probability that a box chosen at random will contain exactly 6 chocolate bars with mass below 306 grams. **[2]**

d) A discrete random variable, X, has probability distribution as given in the table below.

x	3	5	k	12
$p(X = x)$	0.1	0.25	0.45	0.2

i. $E(X) = 8.45$ Find the value of k. **[2]**

ii. Find the standard deviation. **[3]**

Question B6

a) A curve C has equation $-6x^2 + 6xy + 8y^2 = 0$.

i. Find $\frac{dy}{dx}$ in terms of x and y. *All working must be shown.* **[4]**

ii. Where there is a stationary point on curve C, x and y are connected by the equation $y = bx$. Write down the value of b. **[1]**

b) Use the Quotient Rule to find $\frac{dy}{dx}$ when $y = \frac{\sin x}{1 + \cos x}$.

Write your answer in its simplest form. **[3]**

c) Differentiate $e^{\cos x}$. **[2]**

d) i. Write $\frac{8}{4 - x^2}$ in the form $\frac{A}{2 + x} + \frac{B}{2 - x}$ where A and B are constants to be determined. **[3]**

ii. You are given

$$\int_{a}^{1.5} \frac{8}{4 - x^2}\ dx = 2\ \ln\left(\frac{7}{4}\right)$$

Find the value of a. **[7]**

This is the end of the examination.

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