NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business Examination

2017-18

MARK SCHEME

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A5. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Realises the need to solve simultaneous equations and uses any valid method to find one unknown. **[M1]**

Uses any valid method to find the second unknown. **[M1]**

$x = 4$ **[A1]**

$y = -3$ **[A1]**

Question A2

$\frac{16}{25} \times \frac{15}{24}$ **(M1)** $\frac{9}{25} \times \frac{8}{24}$ **(M1)** **[M2]**

Adds their probabilities **[M1]**

$\frac{13}{25}$ or equivalent **[A1]**

Question A3

Discriminant = $(-5)^2 - 4 \times 3 \times 3$ **[M1]**

$= -11$ **[A1]**

No real roots because the determinant is negative (allow follow through for their determinant). **[A1ft]**

Question A4

$^5C_2 \times p^3 \times 64p^2$ Allow $^2C_5$ and the presence of x for this mark. **[M1]**

Sets equal to 20 and reaches $p^5 = \cdots \left(\frac{20}{640}\right)$. There must now be no x present. **[M1]**

$p = \frac{1}{2}$ or equivalent. **[A1]**

Question A5

$\frac{1}{2} \times 3.7 \times x \times \sin 65 = 5$ **[M1]**

$x = 2.9821 \ldots$ (can be implied) **[M1]**

$= 3.0$ (cm) to two significant figures. Allow follow through. **[A1ft]**

Question A6

Recognises the 'hidden' quadratic equation **[M1*]**

Factorises or uses formula
$[(4^{0.5x}-1)(4^{0.5x}-16)=0$ or $4^{0.5x}=\frac{17 \pm \sqrt{[(-17)^2 - 4 \times 1 \times 16]}}{2 \times 1}]$ **[M1]**

$4^{0.5x}=1$ or 16 (can be implied) **[M1]**

$x=0$ or 4 **[A1]**

Question A7

Multiplies out the integrand $[\ \frac{1}{x^2}+\frac{4}{x}+4\ ]$ **[M1*]**

Attempts to integrate (sight of reciprocal x, $\ln x$ or $4x$ is sufficient for this mark) $[\ -\frac{1}{x}+4\ln x+4x\ ]$ **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$=\frac{9}{2}+\ln 16$ or equivalent, or anything rounding to 7.27 **[A1]**

Question A8

$p(A\cap B)=p(A)\times p(B)$ $(=0.44)$ or finds $p(A')$ $[=0.45]$ **[M1]**

Subtracts their 0.44 from 0.8 or draws a Venn diagram with at least two correct entries or multiplies their $p(A')$ by 0.8 **[M1]**

$p(A'\cap B)=0.36$ **[A1]**

Question A9

$3(p+15)=3p-1+2p+1+4p-3$ **[M1]**

Solves **[M1]**

$p=8$ **[A1]**

Question A10

$z = \frac{124-120}{5}$ $(= 0.8)$ **[M1]**

Finds $\phi(\text{their } 0.8)$ (0.7881) **[M1]**

Prob(length more than $124) = 1$ – their 0.7881 $(= 0.2119)$ **[M1]**

Realises the symmetry or repeats process for 116 **[M1]**

Anything rounding to 57.6% (Must be a percentage) **[A1]**

[Accept any other valid method]

Question A11

Under scheme X: Interest = $\frac{1200 \times 3 \times 3.2}{100} = (£115.20)$ **[M1*]**

Under scheme Y: Amount $1200\left(1 + \frac{3}{100}\right)^3$ $(= £1311.27)$ **[M1*]**

[Allow the longer step-by-step method]

Compound interest = their amount – 1200 $(= £111.27)$ **[M1]**

Scheme X better (Allow follow through) **[A1ft]**

by (anything rounding to) $£3.93$ (No follow through) **[A1]**

Question A12

Please note: substitution must be used in this question.

$du = 3x^2 dx$ or equivalent **[M1*]**

Writes integral in terms of u $[\,12\,u^5\ du\,]$ **[M1*]**

Integrates (sight of u^6 is sufficient) and turns expression back into terms in x **[M1]**

$2(x^3 - 6)^6 + c$ **[A1]**

Section B

Question B1

a) Finds gradient $(= -\frac{1}{2})$ **[M1]**

$y + 4 = -\frac{1}{2}(x - 6)$ or $y - 3 = -\frac{1}{2}(x + 8)$ or any equivalent form. **[A1]**

b) Multiplies out and forms a quadratic inequality with 0 on the RHS $[x^2 + x - 30 > 0]$ **[M1*]**

Factorises or uses formula $[(x + 6)(x - 5) = 0 \text{ or } x = \frac{-1 \pm \sqrt{[1^2 - 4 \times 1 \times -30]}}{2 \times 1}]$ **[M1]**

Finds two critical values (-6 and 5) **[M1]**

$x < -6$ **(A1)** $x > 5$ **(A1)** **[A2]**
Please note: the two ranges can be separated by a space, a comma or the word 'or'. The final mark is lost if the word 'and' is seen.

Please note: the Remainder Theorem must be used.

c) Substitutes $x = -2$ into first expression $(= 2k - 37)$ **[M1*]**

Substitutes $x = 3$ into second expression $(= 9k + 15)$ **[M1*]**

Adds 10 to first remainder or subtracts 10 from second, sets equal to each other and solves **[M1]**

$k = -6$ **[A1]**

d) $-60 + (77 - 1)d$ **(M1)** $= \frac{20}{2}[-120 + (20 - 1)d]$ **(M1)** **[M2]**

Solves **[M1]**

$d = 10$ **[A1]**

e) i. $(r = \frac{2}{5})$ Writes $375\,(\text{their } r)^{n-1} = 0.0001$ **[M1]**

Rearranges and uses logs correctly $[(n - 1)\log(\text{their } r) = \log\left(\frac{0.0001}{375}\right)]$ **[M1]**

Finds a value for $n - 1$ or n $[16.52 \ldots \text{ or } 17.52 \ldots]$ **[M1]**

18th term **[A1]**

Special case: if the candidate uses < in the first line and does not reverse the symbol when dividing by the log of their common ratio, then the A mark is normally lost.

ii. 625 (follow through for their common ratio) **[B1ft]**

Question B2

a) i. 50 **[B1]**

ii. *Please note: this is a 'show that' question so all working must be seen*.

Substitutes $N = 100,\ t = 7$ and rearranges to reach $e^{tk} = \cdots (\frac{100}{50})$ **[M1*]**

Takes logs and reaches $7k = \cdots [\ln\left(\frac{100}{50}\right)]$ **[M1*]**

$k \approx 0.1$ **[A1]**

iii. $\frac{dN}{dt} = 50 \times k \times e^{kt}$ $(\frac{dN}{dt} = 50 \times kt \times e^{kt}$ is M0) **[M1*]**

Substitutes $t = 12$ into their $\frac{dN}{dt}$ **[M1]**

= anything rounding to 16.6 (Allow anything rounding to 16.2 if a more accurate value for k is used) **[A1]**

b) $\log_2\left(\frac{x^2}{2x+6}\right) = \log_2 2$

Correct use of the logarithm power law or the subtraction law **[M1*]**

Adapts RHS, removes logs at the right time and forms a quadratic equation $[x^2 - 4x - 12 = 0]$ **[M1*]**

Factorises or uses formula $[(x-6)(x+2) = 0$ or $x = \frac{4 \pm \sqrt{[(-4)^2 - 4 \times 1 \times -12]}}{2 \times 1}]$ **[M1]**

$x = 6$ [If the -2 is also quoted, this mark is lost. Putting it in brackets is good enough to indicate its discard.] **[A1]**

c) i. Uses cosine formula $QR^2 = 12^2 + 16^2 - 2 \times 12 \times 16 \times \cos 60$ **[M1]**

Calculates correctly and in the right order **[M1]**

$QR = 4\sqrt{13}$ (cm) [must be in this form] **[A1]**

ii. Uses sine formula $[\frac{\sin R}{12} = \frac{\sin 60}{\text{their } QR}]$ **[M1]**

Anything rounding to 46.1 (degrees) **[A1]**

d) $2\theta =$ anything rounding to 52 or 232 **[M1]**

Extends range to 0 to 360 degrees for 2θ **[M1]**

Divides by 2 at the right time **[M1]**

$\theta = 26,\ 116$ (degrees) or anything rounding to these values. [Extra answers in the range lose the A mark. Ignore solutions outside the range] **[A1]**

Question B3

a) i. Attempts to differentiate (sight of x^3 or x^2 is sufficient) $(4x^3 + 12x^2)$ **[M1*]**

Substitutes $x = 1$ into their $\frac{dy}{dx}$ **[M1]**

Finds a gradient (16), changes sign and inverts $(-\frac{1}{16})$ **[M1]**

$y - 12 = -\frac{1}{16}(x - 1)$ or equivalent **[A1]**

ii. *Please note: this is a 'show that' question so all working must be seen*.

Substitutes $x = 0$ and $x = -3$ into their $\frac{dy}{dx}$ **[M1*]**

and shows both of them equal 0 **[A1]**

or sets their $\frac{dy}{dx}$ equal to 0 and factorises $[\,4x^2(x + 3) = 0]$ **(M1*)**

and states $x = 0,\ -3$ **(A1)**

iii. Attempts to find $\frac{d^2y}{dx^2}$ (sight of x^2 or x is sufficient) **[M1*]**

$12x^2 + 24x$ (correct answer) **[A1]**

= 0 and changes sign (when $x = 0$), so there is a point of inflexion (at $x = 0$) The "$x = 0$" has to be seen at least once. The candidate does not need to show that the sign changes, but it must be stated. **[A1ft]**

is positive (when $x = -3$) so there is a minimum (at $x = -3$)
The "$x = -3$" must be seen at least once. **[A1ft]**

[Allow follow through in both cases for their $\frac{d^2y}{dx^2}$ provided it gives the same outcome]

or Takes a numerical value below -3 and shows $\frac{dy}{dx} < 0$ **(M1*)**

Takes a numerical value between -3 and 0 and shows $\frac{dy}{dx} > 0$ **(M1*)**

Takes a numerical value above 0 and shows $\frac{dy}{dx} > 0$ **(M1*)**

Thus there is a point of inflexion when $x = 0$ and a minimum when $x = -3$. [Allow follow through for their $\frac{dy}{dx}$ provided it gives the same outcomes.] **(A1ft)**

Part b) is on the next page.

Question B3 - (continued)

b) i. Subtracts $\frac{4}{x^2}$ from $5 - x^2$ or subtracts areas after integrating **[M1]**

Attempts to integrate (sight of $5x$, x^3 or reciprocal x is sufficient for this mark) $[\, 5x - \frac{x^3}{3} + \frac{4}{x}\,]$ **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round $[\,\frac{28}{3} - \frac{26}{3}\,]$ **[M1]**

$= \frac{2}{3}$ **[A1]**

ii. Attempts to differentiate (sight of reciprocal x^3 is sufficient) $[-\frac{8}{x^3}]$ **[M1*]**

Substitutes $x = 2$ into their $\frac{dy}{dx}$ (-1) **[M1]**

$y - 1 = -(x - 2)$ or equivalent **[A1]**

iii. Attempts to differentiate (sight of $x -$ term is sufficient) $[-2x]$ **[M1*]**

Sets equal to the gradient of the normal in part ii and finds a value for x $(= -\frac{1}{2})$ **[M1]**

Coordinates are $(-\frac{1}{2}, \frac{19}{4})$ or equivalent. **[A1]**

Question B4 is on the next page.

Question B4

a) i.

Mid-value x	**Frequency f**	$x \times f$	**Cum. Freq.**
5	26	130	26
15	27	405	53
25	20	500	73
35	11	385	84
45	5	225	89
55	2	110	91
65	1	65	92

$$\sum(x \times f) = 1820$$

Works out $x \times f$ values **[M1]**

Divides their $\sum(x \times f)$ by 92 **[M1]**

= anything rounding to 19.8 **[A1]**

ii. Finds cumulative frequencies **[M1]**

Plots correct curve (a sketch is on page 15). 1 mark lost for each omitted/incorrect plot; 1 mark lost for each point missed by the curve by at least 1 mm (but allow ft for any incorrect plots); 1 mark lost if either axis not labelled correctly.
(Please note: a maximum total of 3 marks can be lost i.e. there are no negative scores. If the candidate plots the mid-values instead of the upper values in each interval, this will score A0.)

If graph paper is not used, award 1 mark out of the A3 if a reasonable curve is drawn. If a cumulative frequency polygon is drawn, award up to 2 marks out of the A3. **[A3]**

iii. Correctly reads off their median (should be around 17) **[B1]**

iv. Correctly reads off the number of e-mails for 35 seconds and the number of e-mails for 45 seconds **(B1)** [should be around 80 and 88 respectively] and subtracts **(B1)** [should be around 8] **[B2]**

v. Yes **(B1*)** plus reason **(B1)** such as: 'the mean and median are quite far apart'; there is 'bunching' at the lower end of the range; or any other valid comment which clearly indicates an understanding. ['the shape of the cumulative frequency curve is uneven' can be condoned] **[B2]**

*Only award if a reason, even a wrong one, follows.

Part b) is on the next page.

Question B4 - (continued)

b) i. $\frac{17}{20} = \frac{2}{5} + \frac{3}{4} - p(K \cap M)$ **[M1]**

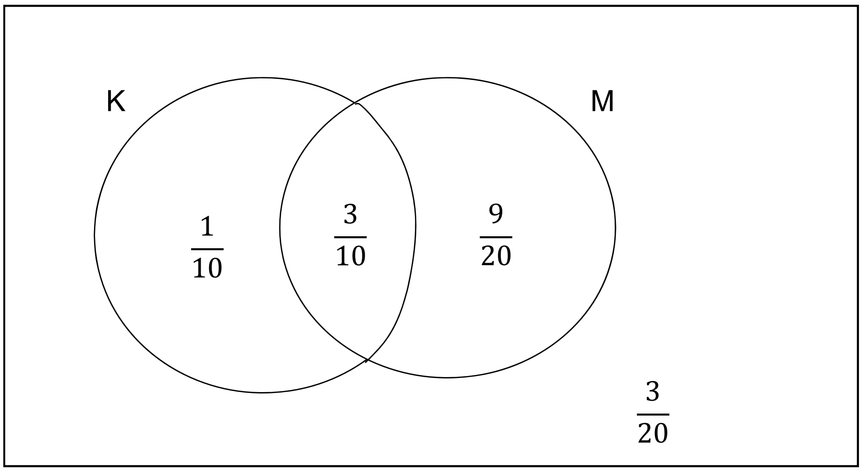
$p(K \cap M) = \frac{3}{10}$ **[A1]**

$p(M|K) = \frac{\text{their } p(K \cap M)}{p(K)} \left(= \frac{\frac{3}{10}}{\frac{2}{5}} \right)$ **[M1]**

$= \frac{3}{4}$ **[A1]**

[Allow equivalent fractions, decimals and percentages in answers]

ii. E



Any correct entry **(B1)**; any two correct entries **(B2)**; All entries correct and contained in rectangle **(B3)** [condone missing E] **[B3]**

iii. $\frac{11}{20}$ (Allow follow through) **[B1ft]**

Question B5 is on the next page.

Question B5

a) i.

x^2	y^2	xy
1	9	-3
25	1	-5
49	0	0
100	25	50
144	16	48

[B3]

(B1) for each correct column

ii. $\sum x = 35$; $\sum y = 5$; $\sum x^2 = 319$; $\sum y^2 = 51$; $\sum xy = 90$ **[M1*]**

$\bar{x} = 7$; $\bar{y} = 1$

$s_x = \sqrt{\left[\frac{319}{5} - 49\right]} \ (\approx 3.85)$ **[A1ft]**

$s_y = \sqrt{\left[\frac{51}{5} - 1\right]} \ (\approx 3.03)$ **[A1ft]**

$s_{xy} = \frac{90}{5} - 7 \times 1 \ (= 11)$ **[A1ft]**

These are follow through marks, so they can be given if the candidate's calculated values are used.
Special case: If the candidate obtains all 3 correct answers with no working, give 2 marks out of 4.

iii. Anything rounding to 0.94 [Allow follow through] **[A1ft]**

iv. (Very) strong positive **(B1)**. Other words can be used for 'strong' and can earn the mark as long as they convey similar meaning. **[B1ft]**

b) i. ${}^{20}C_{13} \times 0.7^{13} \times 0.3^{7}$ **or** uses tables with $p = 0.3$ and carries out prob(7 or less) – prob(6 or less) **[M1]**

= anything rounding to 0.164 **[A1]**

ii. Anything rounding to 0.107 **[B1]**

iii. Uses tables with $p = 0.3$ and finds probability that 6 or more do not have the disease [1 – prob(5 or less)] **[M1]**

Anything rounding to 0.584 **[A1]**

Part c) is on the next page.

Question B5 – (continued)

c). i. $p = 0.2$ **[B1]**

ii. $-3 \times 0.12 + -1 \times 0.08\ (+0 \times \text{their } p) + 1 \times 0.24 + 4 \times 0.3 + 7 \times 0.06$ **[M1]**

$= 1.42$ **[A1]**

iii. Finds $\mathrm{E}(X^2)$

$9 \times 0.12 + 1 \times 0.08\ (+0 \times \text{ their } p) + 1 \times 0.24 + 16 \times 0.3 + 49 \times 0.06$
$(= 9.14)$ **[M1]**

Finds their $\mathrm{E}(X^2)$ – their $(E(X))^2$ and takes square root **[M1]**

= anything rounding to 2.67 **[A1]**

Question B6 is on the next page.

Question B6

a) $10x + 8y^3 + 24xy^2\frac{dy}{dx} - 12y\frac{dy}{dx} = 0$

Correct use of Product Rule ($\pm 8y^3 \pm 24xy^2\frac{dy}{dx}$ is sufficient evidence) **[M1*]**

Correct use of implicit differentiation ($y^2\frac{dy}{dx}$ or $y\frac{dy}{dx}$ is sufficient) **[M1*]**

Collects $\frac{dy}{dx}$ terms on to one side and factorises (there must be at least two $\frac{dy}{dx}$ terms for this mark to be awarded) **[M1]**

$\frac{dy}{dx} = \frac{-10x - 8y^3}{24xy^2 - 12y}$ or equivalent **[A1]**

b) Uses Quotient Rule $[\frac{dy}{dx} = \frac{2x(x-1) - (x^2+3)}{(x-1)^2}]$ **[M1*]**

Sets top line equal to 0 and forms a quadratic equation $[x^2 - 2x - 3 = 0]$ **[M1]**

Factorises or uses formula $[(x-3)(x+1) = 0 \text{ or } x = \frac{2 \pm \sqrt{[(-2)^2 - 4 \times 1 \times -3]}}{2 \times 1}]$ **[M1]**

Finds two values of x and substitutes into original equation to find $y-$ values **[M1]**

$(3,\ 6),\quad (-1, -2)$ **[A1]**

Special case: if the Product Rule is used, the M1* mark is lost and the candidate must arrive at the correct quadratic equation for the next M mark. The following M1, M1 and A1 marks are then possible. [Maximum 4 out of 5].

c) $\cos x$ or $\frac{1}{\sin x}$ seen **[M1]**

$\frac{\cos x}{\sin x}$ or $\cot x$ **[A1]**

d) i. $x + 5 = A(x+2) + B(x-1)$ **[M1]**

$A = 2$ **(A1)** $B = -1$ **(A1)** **[A2]**

ii. Uses previous result $[\int \left(\frac{2}{x-1} - \frac{1}{x+2}\right) dx]$ **[M1*]**

Attempts to integrate (sight of a logarithm is sufficient) **[M1*]**

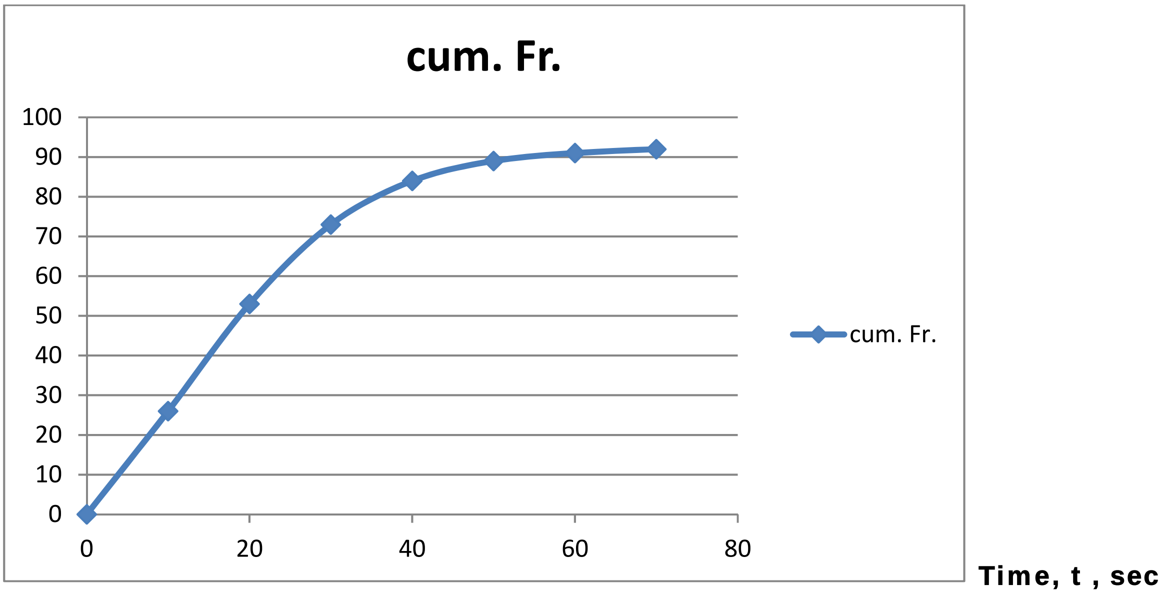
$\ln\left(\frac{C(x-1)^2}{x+2}\right)$ or $\ln\left(\frac{(x-1)^2}{x+2}\right) + c$ **[A1]**

e) Uses integration by parts in the right direction **[M1*]**

$6x(-\cos x)$ **(A1)** $- \int 6(-\cos x)\ dx$ (A1) for first part only **[A1]**

$-6x\cos x + 6\sin x + c$ **[A1]**

Sketch for Question B4 a) ii.



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