



## THE NCUK INTERNATIONAL FOUNDATION YEAR

### IFYMB002 Mathematics Business Examination 2017-18

**Examination Session**  
Semester Two

**Time Allowed**  
2 Hours 40 minutes  
(including 10 minutes reading time)

### INSTRUCTIONS TO STUDENTS

**SECTION A** Answer ALL questions. This section carries 45 marks.

**SECTION B** Answer 4 questions ONLY. This section carries 80 marks.

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

**DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE  
INVIGILATOR**

## Section A

**Answer ALL questions. This section carries 45 marks.**

### Question A1

Line  $l_1$  has equation  $3x - 2y - 18 = 0$  and line  $l_2$  has equation  $2x + 5y + 7 = 0$ .

Find the coordinates of the point where lines  $l_1$  and  $l_2$  meet. **[ 4 ]**

### Question A2

A box holds 16 plain chocolates and 9 milk chocolates. Two chocolates are removed, one after the other and with no replacement.

Find the probability that the two chocolates removed are **either** both plain **or** both milk. **[ 4 ]**

### Question A3

A quadratic equation is defined as  $3x^2 - 5x + 3 = 0$ .

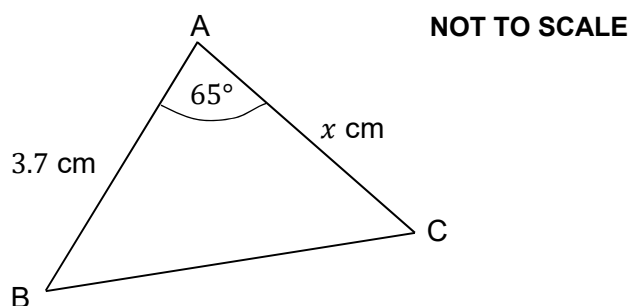
Find the discriminant and hence state the number of real roots, giving a reason for your answer. **[ 3 ]**

### Question A4

The coefficient of the term in  $x^2$  in the expansion of  $(p + 8px)^5$ , where  $p \neq 0$ , is 20.

Find the value of  $p$ . **[ 3 ]**

### Question A5



**Figure 1**

Figure 1 shows the acute-angled triangle ABC with  $AB = 3.7$  cm,  $AC = x$  cm and angle  $A = 65^\circ$ . The area of triangle ABC is  $5 \text{ cm}^2$ .

Find the value of  $x$ . Give your answer to **2** significant figures.

**In this question, 1 mark will be given for the correct use of significant figures.** **[ 3 ]**

**Question A6**

Solve the equation  $4^x - 17(4^{0.5x}) + 16 = 0$ .

**[ 4 ]****Question A7**

Evaluate

$$\int_1^2 \left(\frac{1}{x} + 2\right)^2 dx.$$

*All working must be shown. An answer, even the correct one, will receive no marks if this working is not seen.*

**[ 4 ]****Question A8**

Events  $A$  and  $B$  are such that  $p(A) = 0.55$  and  $p(B) = 0.8$ .

Events  $A$  and  $B$  are independent.

Find  $p(A' \cap B)$ .

**[ 3 ]****Question A9**

The table below shows the numbers of watches sold in a shop over three weeks.

Week	Number of watches sold	3-point moving average
1	$3p - 1$	
2	$2p + 1$	$p + 15$
3	$4p - 3$	

Find the value of  $p$ .

**[ 3 ]****Question A10**

A machine produces bolts which have a mean length of 120 mm and standard deviation 5 mm.

The lengths can be assumed to follow a Normal distribution.

Find the percentage of bolts whose lengths lie between 116 mm and 124 mm.

**[ 5 ]**

**Question A11**

A student invests £1200 for 3 years. There are two schemes, X and Y, under which she can invest the money.

Under scheme X, simple interest is paid at a rate of 3.2% per year.

Under scheme Y, compound interest is paid at a rate of 3% per year.

Which scheme gives more interest, and by how much?

*All working must be shown.*

**[ 5 ]**

**Question A12**

Use the substitution  $u = x^3 - 6$  to find

$$\int 36x^2(x^3 - 6)^5 \, dx.$$

*All working must be shown.*

**[ 4 ]**

**Section B**  
**Answer 4 questions ONLY. This section carries 80 marks.**

**Question B1**

- a) Find the equation of the line which passes through the points with coordinates  $(-8, 3)$  and  $(6, -4)$ . **[ 2 ]**
- b) Solve the inequality  $(x - 2)(x + 3) > 24$ . *Show all working.* **[ 5 ]**
- c) When  $5x^3 - kx + 3$  is divided by  $(x + 2)$ , the remainder is 10 less than the remainder when  $2x^2 + 3kx - 3$  is divided by  $(x - 3)$ .  
 Use the Remainder Theorem to find the value of  $k$ . **[ 4 ]**
- d) An arithmetic series has first term  $-60$ . The  $77^{\text{th}}$  term is equal to the sum of the first 20 terms.  
 Find the common difference. **[ 4 ]**
- e) A geometric series is defined as  $375, 150, 60, \dots$
- i. Which is the first term to fall below 0.0001? **[ 4 ]**
- ii. Find the sum to infinity. **[ 1 ]**

**Question B2 is on the next page.**

**Question B2**

- a) i. Some fish were introduced into a pond and the number of fish,  $N$ , after  $t$  years from when the fish were first introduced is given by the formula

$$N = 50e^{kt}$$

where  $k$  is a constant.

How many fish were first introduced into the pond?

**[ 1 ]**

- ii. After 7 years, there were 100 fish.

Show that  $k$  is approximately 0.1

**[ 3 ]**

- iii. Find the value of  $\frac{dN}{dt}$  when  $t = 12$ .

**[ 3 ]**

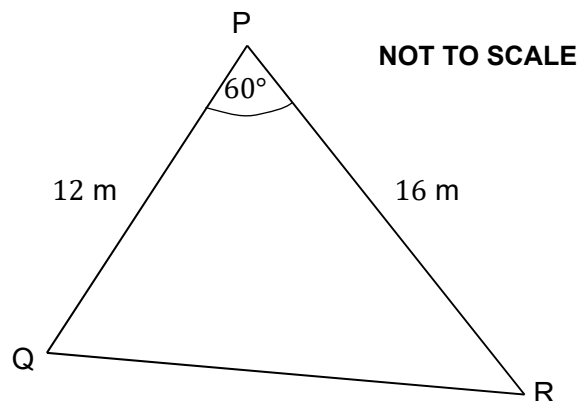
- b) Solve the equation

$$2 \log_2 x - \log_2(2x + 6) = 1 \quad (x > 0).$$

*All working must be shown. An answer, even the correct one, will receive no marks if this working is not seen.*

**[ 4 ]**

- c)



**Figure 2**

Figure 2 shows the acute-angled triangle PQR with  $PQ = 12$  m,  $PR = 16$  m and angle  $P = 60^\circ$ .

- i. Find QR. Give your answer in the form  $4\sqrt{n}$  where  $n$  is an integer.

**[ 3 ]**

- ii. Find angle R.

**[ 2 ]**

- d) Solve the equation  $\tan 2\theta = 1.28$  for  $0 \leq \theta \leq 180^\circ$ .

**[ 4 ]**

**Question B3**

- a) A curve has equation  $y = x^4 + 4x^3 + 7$ .
- Find the equation of the normal to the curve at the point  $(1, 12)$ . **[ 4 ]**
  - Show that there are stationary values at  $x = 0$  and  $x = -3$ . **[ 2 ]**
  - Confirm that there is a point of inflexion at  $x = 0$  and determine whether there is a maximum or a minimum at  $x = -3$ . **[ 4 ]**
- b)

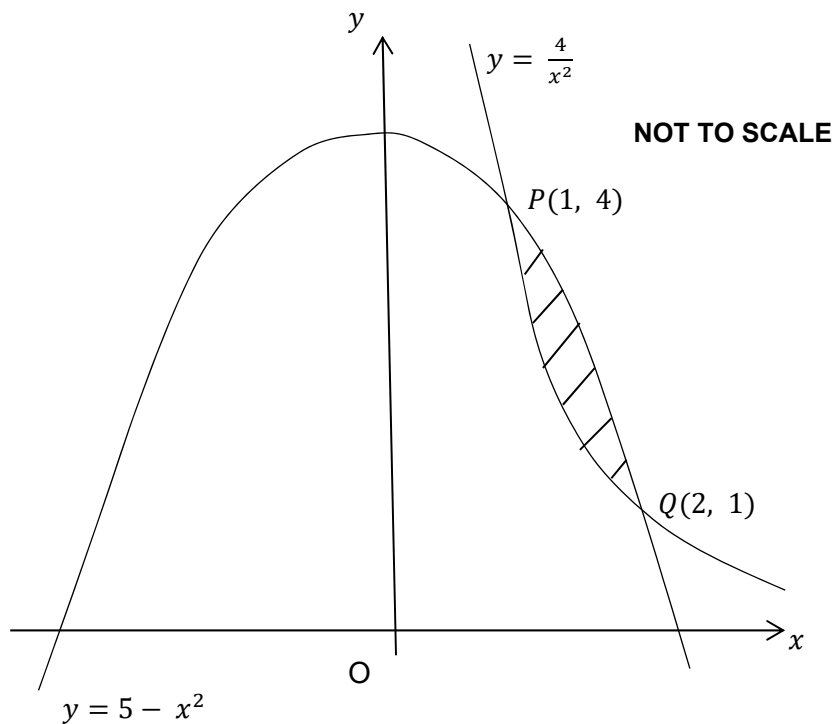
**Figure 3**

Figure 3 shows the curves  $y = 5 - x^2$  and  $y = \frac{4}{x^2}$ .

The curves intersect at  $P(1, 4)$  and  $Q(2, 1)$ .

- Find the area, which is shaded on the diagram, that is bounded by both of the curves. **[ 4 ]**
- Find the equation of the tangent to  $y = \frac{4}{x^2}$  at point Q. **[ 3 ]**

The tangent at point Q is parallel to the normal at point R on the curve  $y = 5 - x^2$ .

- Find the coordinates of point R. **[ 3 ]**

**Question B4**

- a) The times taken for 92 e-mails to be delivered were recorded and the results are shown in the table below.

<b>Time, <math>t</math>, seconds</b>	<b>Frequency</b>
$0 < t \leq 10$	26
$10 < t \leq 20$	27
$20 < t \leq 30$	20
$30 < t \leq 40$	11
$40 < t \leq 50$	5
$50 < t \leq 60$	2
$60 < t \leq 70$	1

(You may wish to copy and extend this table to help you answer some of the questions below.)

- i. Estimate the mean. **[ 3 ]**
  - ii. Draw a cumulative frequency curve. **[ 4 ]**
  - iii. Use your cumulative frequency curve to estimate the median. **[ 1 ]**
  - iv. Use your cumulative frequency curve to estimate how many e-mails took between 35 and 45 seconds. **[ 2 ]**
  - v. Does this distribution appear to be skewed? Give a reason. **[ 2 ]**
- b) Events  $K$  and  $M$  are such that  $p(K) = \frac{2}{5}$ ,  $p(M) = \frac{3}{4}$  and  $p(K \cup M) = \frac{17}{20}$ .
- i. Find  $p(K \cap M)$  and  $p(M|K)$ . **[ 4 ]**
  - ii. Draw a Venn diagram to show this information. **[ 3 ]**
  - iii. Find  $p(K \cup M')$ . **[ 1 ]**



**Question B5**

- a) Five pairs of readings were recorded during some trials and the results are shown in the table below.

$x$	$y$	$x^2$	$y^2$	$xy$
1	-3			
5	-1			
7	0			
10	5			
12	4			

- i. Copy and complete the table. [ 3 ]
  - ii. Find  $s_x$ ,  $s_y$  and  $s_{xy}$ . *All working must be clearly shown.* [ 4 ]
  - iii. Hence find the product moment correlation coefficient. [ 1 ]
  - iv. Describe the correlation between  $x$  and  $y$ . [ 1 ]
- b) In a large forest 70% of the trees are affected by a disease.

A sample of 20 trees is taken.

Find the probability that:

- i. exactly 13 trees have the disease. [ 2 ]
  - ii. 17 or more trees have the disease. [ 1 ]
  - iii. less than 15 trees have the disease. [ 2 ]
- c) A discrete random variable,  $X$ , has probability distribution as given in the table below.

$x$	-3	-1	0	1	4	7
$p(X = x)$	0.12	0.08	$p$	0.24	0.3	0.06

- i. State the value of  $p$ . [ 1 ]
- ii. Find  $E(X)$ . [ 2 ]
- iii. Find the standard deviation of  $X$ . [ 3 ]

**Question B6**

- a) The equation of a curve is given by  $5x^2 + 8xy^3 - 6y^2 = 1$ .

Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . **[ 4 ]**

- b) A curve has equation

$$y = \frac{x^2 + 3}{x - 1}$$

Use the Quotient Rule to find  $\frac{dy}{dx}$  and hence find the coordinates of the stationary values. **[ 5 ]**

- c) Differentiate  $\ln(\sin x)$ . **[ 2 ]**

- d) i. Express  $\frac{x + 5}{(x - 1)(x + 2)}$  in the form  $\frac{A}{x - 1} + \frac{B}{x + 2}$  where  $A$  and  $B$  are constants to be determined. **[ 3 ]**

- ii. Hence find

$$\int \frac{x + 5}{(x - 1)(x + 2)} dx.$$

Give your answer as a single logarithm. **[ 3 ]**

- e) Use integration by parts to find

$$\int 6x \sin x \, dx. \quad \text{[ 3 ]}$$

**This is the end of the examination.**

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