

Question A6



NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business Examination 2018-19

Question A3

Examination Session

Semester Two

Time Allowed2 Hours 40 minutes
(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A Answer ALL questions. This section carries 45 marks.

SECTION B Answer 4 questions ONLY. This section carries 80 marks.

The marks for each question are indicated in square brackets [].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Find a) $p(X=0)$

b) $p(Y)$

$$(S < x) \cap S = (S - x - S) \cap S = (S + x + S) \cap S$$

14

Section A

Answer ALL questions. This section carries 45 marks.

Question A1

Solve the simultaneous equations $8p - 3q = 24$

$$6p + 2q = 1$$

[4]

All working must be shown. Just giving the answers, even the correct ones, will score no marks if this working is not seen.

Question A2

When a coin is tossed, it can land Heads or Tails. The coin is biased (unfair) in that $p(\text{Heads}) = 0.68$.

The coin is tossed three times.

Find the probability that it lands Tails each time. Give your answer to 2 significant figures.

In this question, 1 mark will be given for the correct use of significant figures.

[4]

Question A3

a) Write the expression $x^2 - 10x + 19$ in the form $(x + a)^2 + b$ where a and b are integers.

[2]

b) Hence solve the equation $x^2 - 10x + 19 = 0$ giving your answers in surd form.

[2]

Question A4

In a convergent geometric series, the first term is 40 times larger than the common ratio and the sum to infinity is 24.

Find the first term and the common ratio.

[3]

Question A5

Solve the equation

$$\log_4(x^2 + 4x + 3) - \log_4(x^2 - x - 2) = 2 \quad (x > 2)$$

All working must be shown. Just giving the answer, even the correct one, will score no marks if this working is not seen.

[4]

Question A6

Solve the equation $\tan(\theta + 40^\circ) = \frac{3}{5}$ $(0 < \theta < 360^\circ)$ [3]

Question A7

Find

a) Point A lies at $(-1, 4)$ and point B lies at $(2, \frac{2}{x})$. Find the area of the region bounded by the line AB and the curve $y = \frac{2}{x}$. [4]

Question A8

During one week, the numbers of absent students at a college were as follows:

14, 18, 11, 20, 17.

Find the mean and show that the variance is 10. [3]

Question A9

The table below shows the sales of cars over a 4-week period.

Week	Number of cars sold	Three point moving average
1	$3a + 3$	
2	$5a - 2$	$a^2 - 14$
3	$2a + 5$	24
4	b	

Find the value of a and the value of b . [4]

Question A10

Two events, X and Y , are such that $p(X) = \frac{13}{20}$, $p(Y|X) = \frac{5}{13}$ and $p(X \cup Y) = \frac{19}{20}$.

Find a) $p(X \cap Y)$; [2]

b) $p(Y)$. [2]

Question A11

- a) A student invested £5000 over 3 years. The compound interest earned after this time was £624.32. Find the percentage rate of interest. [2]
- b) A car lost 18% of its value during one year and, at the end of the year, was worth £1968. Find its value at the start of the year. [2]

Question A12

A curve has equation $5x^2 - 4xy^2 + 2y^3 = 1$.

Find $\frac{dy}{dx}$ in terms of x and y . [4]

Section B begins on the next page.

Section B

Answer 4 questions ONLY. This section carries 80 marks.

Question B1

- a) Point A lies at $(-1, 4)$ and point B lies at $(-7, 6)$.

Find the equation of the line which passes through the point $(-3, 8)$ and is perpendicular to line AB.

[3]

- b) Divide $6x^3 - 23x^2 - 5x + 4$ by $(x - 4)$.

[3]

- c) i. Find the range of values which satisfy $x^2 - 4 \leq 0$.

[3]

- ii. Find the range of values which satisfy $x^2 - 4x > 0$.

[3]

- iii. Hence state the integers which satisfy both $x^2 - 4 \leq 0$ and $x^2 - 4x > 0$.

[1]

- d) The 6th term of an arithmetic series is 40. The first term is 5 times larger than the common difference.

- i. Find the first term and the common difference.

[2]

- ii. Find the sum of the first 25 terms.

[3]

- e) Find the coefficient of the term in x^3 in the expansion of $(4 - \frac{1}{4}x)^8$.

[2]

Section B continues on the next page.

Question B2

- a) The variables x and y are connected by the formula

$$y = 24 e^{kx} - 8$$

b) A car loses 18% of its value during one year and, at the end of the year, where k is a constant.

- i. State the value of y when $x = 0$. [1]

Question A12
You are given $y = 40$ when $x = \frac{1}{3}$.

- ii. Find the value of k . Give your answer in the form $\ln b$ where b is an integer. [3]

- iii. Find $\frac{dy}{dx}$ and hence find its value when $x = -2$. [3]

- b) Simplify

$$\frac{1}{2} \log_a 36 + \frac{2}{3} \log_a 8 - 3 \log_a 2 - 2 \log_a 9.$$

Give your answer in the form $\log_a(\frac{1}{c})$ where c is an integer. [3]

All working must be shown.

Part c) is on the next page.

Question B2 – (continued)

- c) i. Given that $\cos 60^\circ = \frac{1}{2}$, prove that $\sin 60^\circ = \frac{\sqrt{3}}{2}$.

Each stage of your working must be clearly shown.

[2]

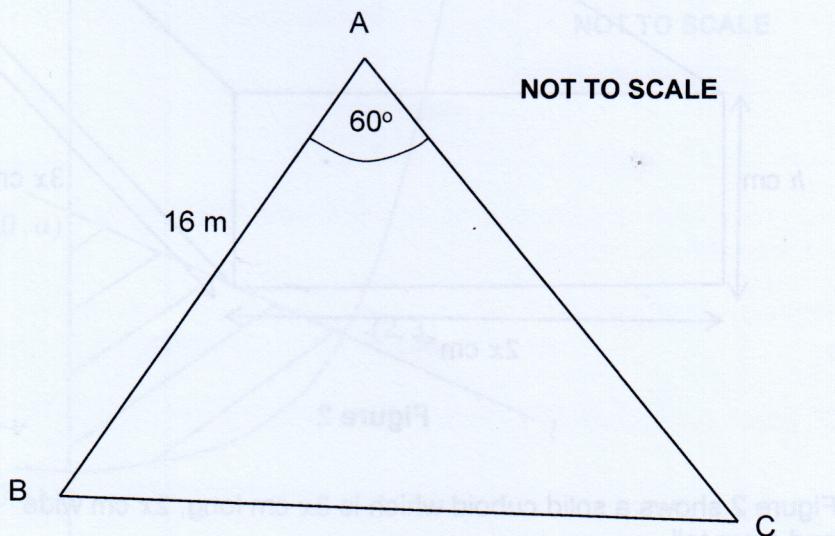


Figure 1

- Figure 1 shows acute-angled triangle ABC with $AB = 16 \text{ m}$ and angle $A = 60^\circ$. The area of triangle ABC is $80\sqrt{3} \text{ m}^2$.

- ii. Find the length of AC. [2]
- iii. Find the length of BC. Give your answer in the form $p\sqrt{q}$ where p and q are integers. [3]
- iv. Find $\sin C$. Give your answer in the form $\frac{n}{\sqrt{7}}$ where n is an integer. [2]
- v. Find the shortest distance from point B to AC. [1]

Section B continues on the next page.

Question B3

a)

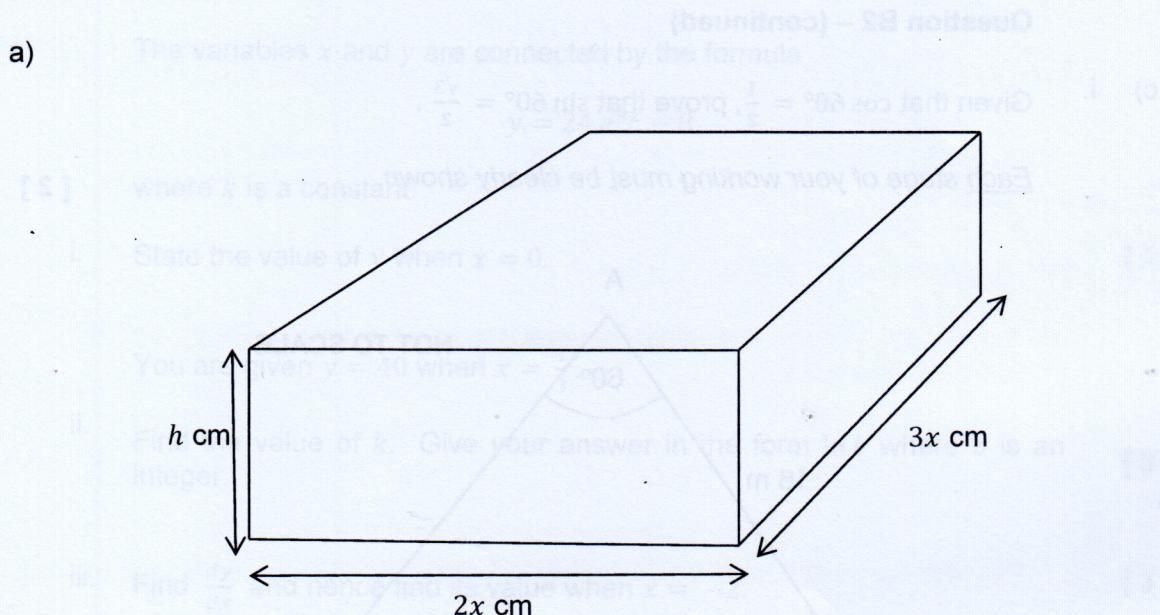
**Figure 2**

Figure 2 shows a solid cuboid which is $3x$ cm long, $2x$ cm wide and h cm tall.

The total surface area of the cuboid is 441 cm^2 .

- Find an expression for h in terms of x . [2]
- Show that the volume, V , of the cuboid is given by

$$V = \frac{9}{5}(147x - 4x^3)$$

Each stage of your working must be clearly shown. [3]

- Use $\frac{dV}{dx}$ to find the value of x which gives the maximum volume. [4]
- Confirm that your value of x gives a maximum. [3]

Part b) is on the next page.

Question B3 – (continued)

b)

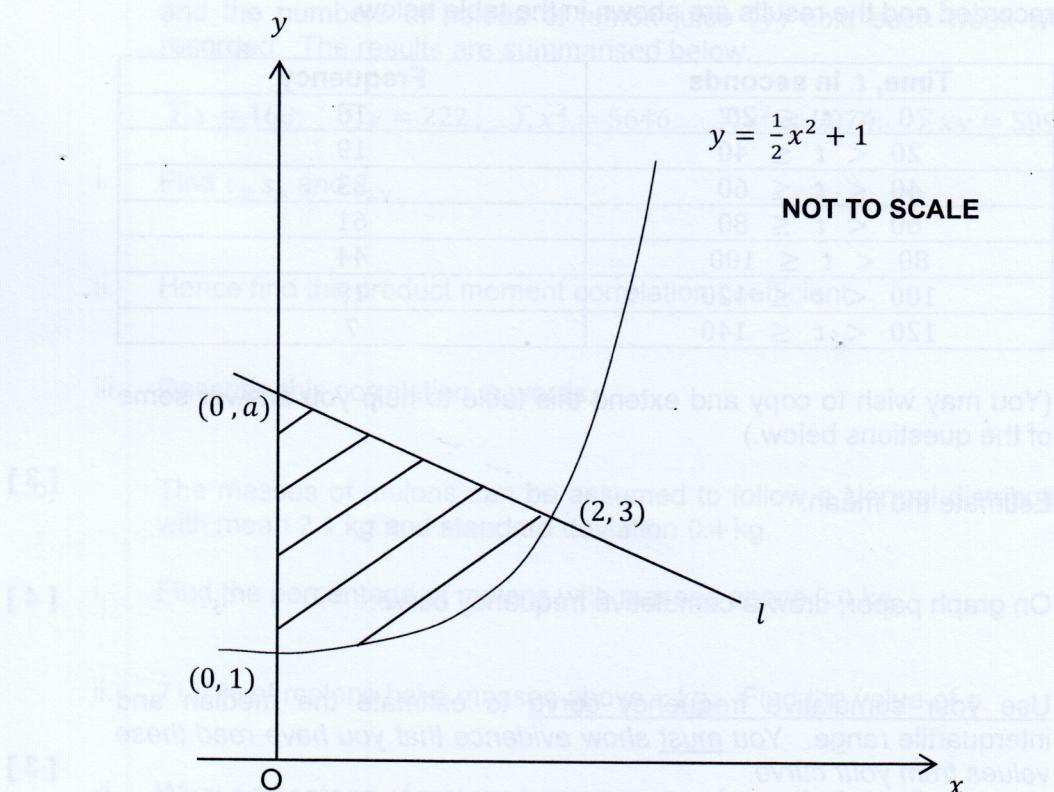
**Figure 3**

Figure 3 shows the curve $y = \frac{1}{2}x^2 + 1$ and line l which is a normal to the curve at the point $(2, 3)$.

- Find the equation of line l . Give your answer in the form $y = mx + c$. [3]
- Line l crosses the y – axis at the point $(0, a)$.
Write down the value of a . [1]
- Find the area, which is shaded on the diagram, that is bounded by the curve $y = \frac{1}{2}x^2 + 1$, line l and the y – axis. [4]

All working must be shown.

Section B continues on the next page.

Section B continues on the next page.

Question B4

- a) The times that 200 people spend filling their cars up with petrol are recorded and the results are shown in the table below.

Time, t , in seconds	Frequency
$0 < t \leq 20$	18
$20 < t \leq 40$	19
$40 < t \leq 60$	33
$60 < t \leq 80$	61
$80 < t \leq 100$	44
$100 < t \leq 120$	18
$120 < t \leq 140$	7

(You may wish to copy and extend this table to help you answer some of the questions below.)

- i. Estimate the mean. [3]
- ii. On graph paper, draw a cumulative frequency curve. [4]
- iii. Use your cumulative frequency curve to estimate the median and interquartile range. You must show evidence that you have read these values from your curve. [3]
- iv. Use your graph to estimate how many people take longer than 110 seconds to fill their cars with petrol. [2]
- b) Two events, A and B , have probabilities as follows:
- $$p(A \cap B) = 0.56; \quad p(A \cap B') = 0.24; \quad p(A \cup B)' = 0.06$$
- i. Show this information on a Venn diagram. [3]
- ii. Find $p(A)$, $p(A \cup B')$ and $p(A|B)$. [3]
- iii. Are events A and B independent? Give a reason for your answer. [2]

Section B continues on the next page.

Part b) is on the next page.

Question B5

- a) Over a period of six weeks, the numbers of bottles of orange juice (x) and the numbers of bottles of lemon juice (y) sold each week were recorded. The results are summarised below.

$$\Sigma x = 168; \quad \Sigma y = 222; \quad \Sigma x^2 = 5646; \quad \Sigma y^2 = 9074; \quad \Sigma xy = 5997.$$

- i. Find s_x , s_y and s_{xy} . [3]

- ii. Hence find the product moment correlation coefficient. [1]

- iii. Describe this correlation in words. [1]

- b) The masses of melons can be assumed to follow a Normal distribution with mean 2.4 kg and standard deviation 0.4 kg.

- i. Find the percentage of melons with masses above 3.0 kg. [3]

- ii. 71.9 % of melons have masses above x kg. Find the value of x . [3]

- iii. What percentage of melons have masses of exactly 2.6 kg? [1]

- c) In a large colony of bees, it is reckoned that 15% of them are drones.

30 bees are chosen at random. Find the probability that

- i. 3 or less bees are drones; [1]

- ii. more than 6 bees are drones; [2]

- iii. exactly 4 bees or exactly 5 bees are drones. [3]

- d) Two discrete random variables, Y and Z , are connected by the formula $Z = 4Y + 7$.

You are given $E(Z) = 24$ and $\text{Var}(Z) = 36$.

- i. Find $E(Y)$ [1]

- ii. Find the standard deviation of Y . [1]

Section B continues on the next page.

Question B6

- a) A curve C has equation

$$y = \frac{x^2 - 12}{x - 4}$$

- i. Use the Quotient Rule to find $\frac{dy}{dx}$ and hence find the coordinates of the stationary values on curve C .

[5]

Another curve K has equation $y = \sin^3 x$.

- ii. Find $\frac{dy}{dx}$ and hence find its value when $x = \frac{\pi}{3}$.

[3]

- b) By using the substitution $u = 1 + 3x$, find

$$\int \frac{9x}{1 + 3x} dx.$$

All working must be shown.

[4]

- c) i. Express $\frac{2(x - 2)}{x^2 - 1}$ in the form $\frac{A}{x + 1} + \frac{B}{x - 1}$ where A and B are

constants to be determined.

[3]

- ii. Hence evaluate

$$\int_2^5 \frac{2(x - 2)}{x^2 - 1} dx.$$

Give your answer in the form $\ln k$ where k is an integer.

All working must be shown. Just giving the answer, even the correct one, will score no marks if this working is not seen.

[5]

This is the end of the examination.

Section B continues on the next page.