



THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business Examination

2016-17

Examination Session
Semester Two

Time Allowed
2 Hours 40 minutes
(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A Answer ALL questions. This section carries 45 marks.

SECTION B Answer 4 questions ONLY. This section carries 80 marks.

The marks for each question are indicated in square brackets [].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

**DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE
INVIGILATOR**

Section A

Answer ALL questions. This section carries 45 marks.

Question A1

Point P lies at $(-7, -4)$ and point Q lies at $(5, -8)$.

Find the equation of the line which is perpendicular to the line PQ and passes through the mid-point of PQ . **[4]**

Question A2

A box holds 7 blue beads, 5 yellow beads and 4 green beads.

Three beads are taken from the box, one after the other, with no replacement.

Find the probability that none of the beads is yellow. **[3]**

Question A3

When $x^2 + 7x + 12$ is divided by $(x + k)$ the remainder is k .

Use the Remainder Theorem to find the two possible values of k . **[3]**

$$\frac{(x)^2 + 7x + 12}{(x + k)}$$

Question A4

The 3rd term of geometric progression is 200 and the 6th term is -3125 .

Find the common ratio and the first term. **[5]**

Question A5

Find the value of m if

$$\frac{(2m^2)^3 \times 6m}{\sqrt{(9m^6)}} = 81 \quad \text{[3]}$$

Question A6

Evaluate

$$\int_0^3 (x^2 - 4)^2 dx$$

[4]

All working must be shown. Just giving the answer, even if it is correct, will score no marks.

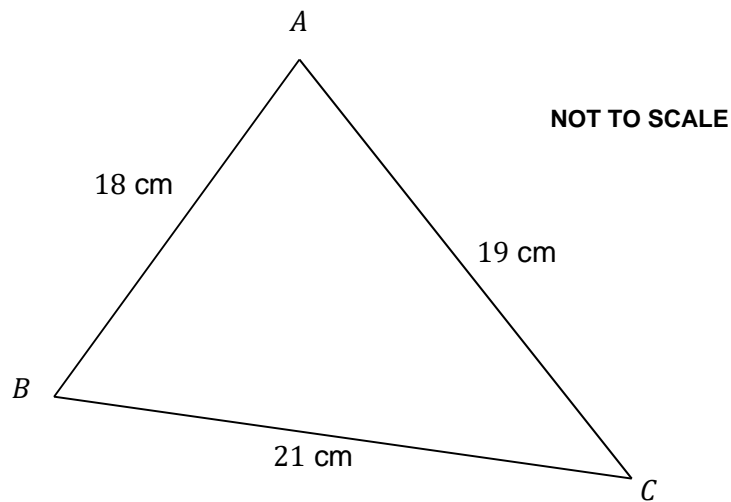
Question A7**Figure 1**

Figure 1 shows triangle ABC with $AB = 18$ cm, $AC = 19$ cm and $BC = 21$ cm.

Find the size of angle A . Give your answer in **radians** and to **3** significant figures.

In this question, 1 mark will be given for the correct use of significant figures.

[4]**Question A8**

The mean of the four numbers p , $2p$, $(3p - 5)$ and $(2p + 21)$ is 38.

Find the value of p .

[3]

Question A9

Two events, A and B , are such that

$$p(B|A) = 0.7, \quad p(A \cup B) = 0.68 \quad \text{and} \quad p(A \cap B) = 0.56.$$

Find $p(A)$ and $p(B)$.

[4]**Question A10**

Sacks of barley have a mean mass of 30 kg with standard deviation 0.4 kg.

The masses of these sacks can be assumed to follow a Normal distribution.

Find the percentage of sacks which have masses between 29.3 kg and 30.7 kg.

[5]**Question A11**

Differentiate $4^x \ln x$.

[3]**Question A12**

Write $\frac{5x^2 - 4x - 21}{(x + 4)(x - 1)^2}$ in the form $\frac{A}{x + 4} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2}$

where A , B and C are constants to be determined.

[4]

Section B
Answer 4 questions ONLY. This section carries 80 marks.

Question B1

- a) i. Solve the inequality $x^2 + 9x \leq 0$. **[3]**
- ii. Solve the inequality $x^2 - 9 > 0$. **[3]**
- iii. List the integers which satisfy **neither** $x^2 + 9x \leq 0$ **nor** $x^2 - 9 > 0$. **[1]**
- b) i. Write $x^2 + 12x + 43$ in the form $(x + a)^2 + b$ where a and b are integers. **[2]**
- ii. Sketch the graph of $y = x^2 + 12x + 43$ showing clearly the coordinates of any stationary values, and where the curve crosses the y - axis. **[2]**
- c) Expand $(3x - \frac{1}{2}y)^4$ writing each term in its simplest form. **[4]**
- d) i. Find the sum of the first 87 terms of the arithmetic series
 $-119, \quad -112, \quad -105, \dots$ **[3]**
- ii. The numbers $k + 4$, $2k - 1$ and 33 are three consecutive terms of an arithmetic progression.
 Find the value of k . **[2]**

Question B2

- a) The mass of a small stone on a beach is recorded and found to be 80 grams. Over the years the stone erodes (i.e. gradually wears away) due to the effect of the sea water. The mass of the stone, M , and the number of years since its mass was first recorded, t , are connected by the formula

$$M = 80e^{kt}$$

where k is a constant.

After 100 years, the mass of the stone is 50 grams.

- i. Show that the value of k is approximately -0.0047 . *Each stage of your working must be clearly shown.* [3]
 - ii. Explain why k is negative. [1]
 - iii. Find the predicted mass of the stone when $t = 330$. [2]
 - iv. Find the value of $\frac{dM}{dt}$ when $t = 100$. [3]
- b) Solve the equation $\log_3(x + 2) + \log_3(x - 6) = 2$ ($x > 6$) [5]
- Show all of your working.*

c)

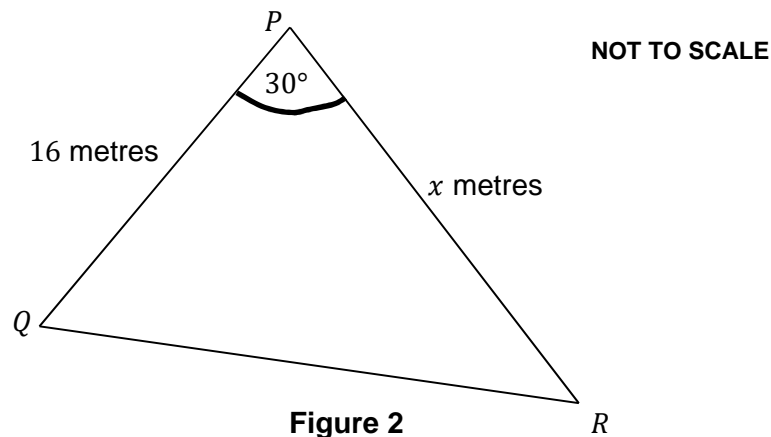


Figure 2 shows triangle PQR with $PQ = 16$ metres, $PR = x$ metres and angle $P = 30^\circ$. The area of triangle PQR is 50 m^2 .

- Find the value of x . [2]
- d) Solve the equation $\cos^2 \theta = 0.86$ for $0^\circ \leq \theta \leq 360^\circ$. [4]

Question B3

a)

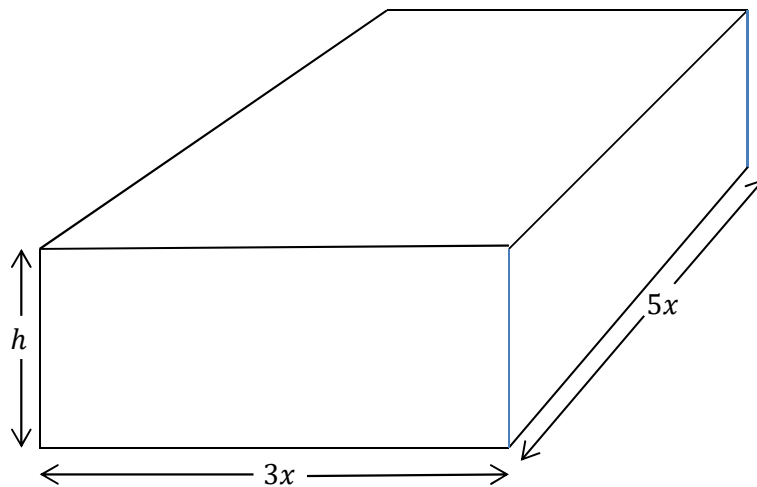
**Figure 3**

Figure 3 shows a cuboid which is $5x$ metres long, $3x$ metres wide and h metres high.

The total length of all the edges is 168 metres.

i. Find h in terms of x . [2]

ii. Show that the volume of the cuboid, V , is given by

$$V = 630x^2 - 120x^3.$$

Each stage of your working must be clearly shown. [3]

iii. Use $\frac{dV}{dx}$ to find the value of x which gives the maximum volume. [4]

iv. Confirm that your volume is a maximum. [3]

Part b) is on the next page.

Question B3 – (continued)

b)

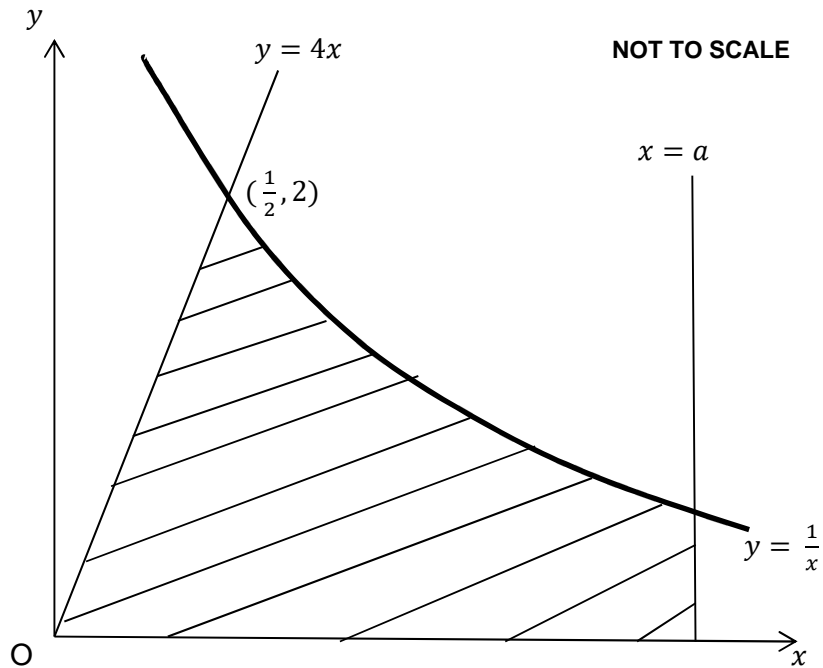


Figure 4

Figure 4 shows the curve $y = \frac{1}{x}$ and the line $y = 4x$. The line $x = a$ is also shown.

The curve $y = \frac{1}{x}$ and the line $y = 4x$ cross at $\left(\frac{1}{2}, 2\right)$.

- i. Show that the line $y = 4x$ is **not** a normal to the curve $y = \frac{1}{x}$ at the point $\left(\frac{1}{2}, 2\right)$. **[2]**

- ii. The area that is bounded by the x – axis, the lines $y = 4x$ and $x = a$, and the curve $y = \frac{1}{x}$ (which is shaded on the diagram) is equal to $\frac{1}{2} + \ln 5$.

Find the value of a . **[6]**

Question B4

- a) The ages of 40 people are shown in the table below.

Age (in years)	Frequency
4 – 6	12
7 – 9	10
10 – 12	7
13 – 15	6
16 – 20	5

(You may wish to copy and extend this table to help you answer some of the following questions.)

- i. Estimate the mean and standard deviation. [6]
 - ii. In which interval does the median lie? [1]
 - iii. Write down the mean and standard deviation of the ages of these people after exactly 5 years. [1]
- b) A bag holds 4 red beads and 6 green beads. A student takes out one bead and does **not** replace it. After the bead is taken, 5 blue beads are added to the bag. The student then takes a second bead.
- i. Draw, and label, a tree diagram. [3]
 - ii. Find the probability that the second bead is green. [3]
 - iii. Given that the second bead is in fact green, find the probability that the first bead was red. [2]
- c) In a group of 50 students, 20 play badminton and 16 play squash.
 x students play both games and $3x$ students play neither game.
 Draw a Venn Diagram and work out how many students play badminton only. [4]

Question B5

- a) Five pairs of readings were recorded during some trials and the results are shown below.

x	y	x^2	$x \times y$
2	9		
5	10		
7	11		
10	12		
11	13		
$\sum x = 35$	$\sum y = 55$		

- i. Copy and complete the table. [2]
- ii. Find s_x^2 and s_{xy} .

Hence find the equation of the regression line of y on x . [4]

- b) When a student drives through a large city, she passes 15 sets of traffic lights. The probability that she is stopped at any of these lights is 0.4.

If X represents the number of times that she is stopped, find the probability of

- i. $X < 7$ [1]
- ii. $X \geq 9$ [2]
- iii. $X = 5$ [2]

- c) At the beginning of 2016 the value of a house was £322000. This was a 15% increase on the value at the beginning of 2015.

- i. Find the value of the house at the beginning of 2015. [2]

At the beginning of 2017 the value of the house had fallen by 15% of its value at the beginning of 2016.

- ii. Find the value at the beginning of 2017. [2]
- iii. Find the overall percentage change in the value of the house over the two years from the beginning of 2015 to the beginning of 2017. [2]

- d) i. A discrete random variable, X , has $E(X) = 9.6$ and $\text{Var}(X) = 7.84$.

Find $E(X^2)$. [1]

- ii. Find the standard deviation of X . [1]

- iii. Find $\text{Var}(5X)$. [1]

Question B6

a) A curve C has equation $-6x^2 - 24x + y^2 - 8y = 12$.

i. Show that

$$\frac{dy}{dx} = \frac{6x + 12}{y - 4} \quad [4]$$

ii. Find the value of x which gives a stationary value. [1]

iii. Find the coordinates of the two points on curve C where stationary values occur. [3]

b) By writing $\sec x$ as $\frac{1}{\cos x}$ show that the differentiation of $\sec x$ is $\sec x \tan x$. [3]

c) i. Use the substitution $u = x^3 - 1$ to evaluate

$$\int_1^2 x^2 e^{x^3 - 1} dx \quad [4]$$

All working must be shown. Just giving the answer, even if it is correct, will score no marks.

ii. Use integration by parts to find

$$\int x^2 e^{2x} dx \quad [5]$$

This is the end of the examination.

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