THE NCUK INTERNATIONAL FOUNDATION YEAR
IFYMB002 Mathematics Business
Time-Controlled Assessment
2019-20

MARK SCHEME

<u>**Notice to Markers**</u>

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A5. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen. **Please note that there are many more M* marks in this Time-Controlled Assessment than in a test or examination**.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first n answers, in the order that they are written in the student's answer booklet (n being the number of questions required for the examination). Markers should not select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Misreads:

Where a candidate incorrectly copies information from a question this may be marked as a misread using the annotation MR. In this case a one mark penalty should be applied and all other marks (including A marks) may be awarded for error carried forward. Markers should use discretion to award fewer marks if the question becomes much simpler because of the misread. If an inappropriate result is reached as a result of the misread (e.g. cos Θ = 2 or a probability greater than 1) the final A mark(s) should not be awarded. Any misread cases which are unusual or difficult to mark should be highlighted in the Marker Report.

Where a candidate miscopies their own work this is an error NOT a misread.

Please note: this is an open book assessment so sufficient working must be seen in all questions.

Section A

Question A1

Finds coordinates of point A $(-6, 4)$ **[M1*]**

Finds gradient of AB $\left(-\frac{5}{8}\right)$, inverts and changes sign $(=\frac{8}{5})$ **[M1*]**

Correct form of equation $[\,y - \text{their } 4 = \text{their gradient}(x - \text{their} - 6)]$ **[M1]**

$8x - 5y + 68 = 0$ or equivalent but must be in this form. **[A1]**

Question A2

$(1-p)\ \times (1-2p)$ **[M1*]**

Sets equal to $\frac{5}{9}$ and forms a quadratic equation $[18p^2 - 27p + 4 = 0]$ **[M1*]**

Solves. This is dependent on the previous M mark. **[M1*]**

$p = \frac{1}{6}$. All M marks scored and no errors seen. If the $\frac{4}{3}$ is included and not discarded (placing in brackets is good enough to show non-inclusion) this mark is lost. **[A1]**

Question A3

(a)

$$
\begin{array}{r|l}
 & x^2 - 3x - 18 \\
\hline
2x - 1 & 2x^3 - 7x^2 - 33x + 18 \\
 & 2x^3 - x^2 \\
\hline
 & \quad -6x^2 - 33x \\
 & \quad -6x^2 + 3x \\
\hline
 & \qquad -36x + 18 \\
 & \qquad -36x + 18 \\
\hline
 & \qquad \ldots\ldots\ldots\ldots
\end{array}
$$

Correct first division† **[M1*]**

Any correct subsequent division† **[M1*]**

Correct quotient **[A1]**

†[Accept any other form of correct working, but quoting the answer with no working scores 0]

(b) $(2x-1)(x-6)(x+3)$ **[A1]**

Question A4

$(12x-8)-(9x+2)=(11x+10)-(12x-8)$ or $\frac{(11x\ +10)+(9x\ +2)}{2}=12x-8$ **[M1*]**

$x=7$ **[A1]**

Common difference $=11$ **[B1]**

Question A5

$4^{3x}\ \div\ 4^{(x\ +\ 3)}=1240$ and combines indices correctly $[4^{(2x\ -\ 3)}=1240]$ **[M1*]**

Uses logs correctly **[M1]**

$x=4.06903\ldots$ (can be implied) **[A1]**

$=4.07$ to 3 significant figures. Allow follow through provided a more accurate answer is seen earlier. **[A1ft]**

Question A6

Applies correct order (divides by 10, then takes arctan) **[M1*]**

Any correct value of θ which does not have to be in the right interval (probably an answer rounding to -35) **[M1]**

$\theta=$ anything rounding to $145, 325$ (degrees) [both answers needed]. This mark is lost if extra solutions in the range are given; ignore solutions outside the range. **[A1]**

Question A7

Attempts to integrate (sight of x^3, x^2 or x is sufficient for this mark); and there must be a constant in the answer. If this constant is missing, this mark and all subsequent marks, are lost. $[2x^3-4x^2-5x+c]$ **[M1*]**

Substitutes $x=3,\ y=q$ into their integrated expression and finds a value for c. **[M1]**

$y=\ 2x^3-4x^2-5x+q-3.$ **[A1]**

Question A8

Sum of readings $= 36$ **[B1]**

$\frac{\sqrt{21}}{2} = \sqrt{[\frac{\sum x^2}{8} - (4½)^2]}$ **[M1*]**

Rearranges correctly and finds a value for $\sum x^2$ **[M1*]**

$= 204$ (Both M marks scored and no errors seen) **[A1]**

Question A9

$p(A \cap B) = p(A|B) \times p(B)$ $(= 0.5 \times 0.64)$ **[M1]**

$= 0.32$ **[A1]**

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$ $(= 0.48 + 0.64 - \text{their } 0.32)$ **[M1]**

$= 0.8$ (Allow follow through provided 0 < their answer < 1). **[A1]**

Question A10

Uses Product Rule and obtains one correct part $[3ke^{3kx} \ln x + e^{3kx} \times x^{-1}]$ **[M1*]**

Correct answer **[A1]**

Substitutes $x = \frac{1}{3k}$ into their $\frac{dy}{dx}$ **[M1]**

$= 3ke \ln\left(\frac{1}{3k}\right) + 3ke$ or equivalent but it must be in exact form. **[A1]**

Question A11

Uses integration by parts in the right direction **[M1*]**

$= \underline{(4x - 3) \times 2\sin\left(\frac{1}{2}x\right)} - \int 4 \times 2\sin\left(\frac{1}{2}x\right) dx$ (A1) for underlined part **[A1]**

$= 2(4x - 3)\sin\left(\frac{1}{2}x\right) + 16\cos\left(\frac{1}{2}x\right) + c$ or equivalent **[A1]**

Section B

Question B1

(a) Finds one unknown, with working shown. **[M1*]**

Finds second unknown, with working shown. **[M1*]**

$(-\frac{1}{2}, \frac{7}{2})$ or $x = -\frac{1}{2}$, $y = \frac{7}{2}$ **[A1]**

(b) i. $(ax+b)(cx+d)$ where $ac = 9$ and $bd = \pm 80$ **[M1]**

$(3x+8)(3x-10)$ **[A1]**

ii. $x = -\frac{8}{3}$, $\frac{10}{3}$ or equivalent but must be in this form. Allow follow through on their part i. **[A1ft]**

iii. $x > -\frac{8}{3}$, $x < \frac{10}{3}$ **(A1+)** for each or $-\frac{8}{3} < x < \frac{10}{3}$ **(A1)** for each end. **[A2]**

+Please note: if this version of the answers is quoted, the two ranges can be separated by a space, a comma or the word 'and'. The final mark is lost if the word 'or' is seen.

(c) Substitutes $x = 2$ into first expression $(= 12 + k)$ **[M1*]**

Substitutes $x = -3$ into second expression $(= -8)$ **[M1*]**

Sets equal to each other and finds a value for k. **[M1]**

$k = -20$ **[A1]**

(d) $2^7 + {}^7C_6 \times 2^6 \times \left(-\frac{1}{2}x\right) + {}^7C_5 \times 2^5 \times (-\frac{1}{2}x)^2 + {}^7C_4 \times 2^4 \times (-\frac{1}{2}x)^3 + {}^7C_3 \times 2^3 \times (-\frac{1}{2}x)^4 + {}^7C_2 \times 2^2 \times (-\frac{1}{2}x)^5 + {}^7C_1 \times 2 \times (-\frac{1}{2}x)^6 + (-\frac{1}{2}x)^7$ **(M1*)** for any four correct unsimplified terms; **(M1*)** for remaining four correct unsimplified terms. (Do not accept reversed powers.) **[M2*]**

$128 - 224x + 168x^2 - 70x^3 + \frac{35}{2}x^4 - \frac{21}{8}x^5 + \frac{7}{32}x^6 - \frac{1}{128}x^7$. **(A1)** Fully correct working clearly leading to four correct terms; **(A1)** Fully correct working clearly leading to the remaining four correct terms. **[A2]**

(e) i. $r = 2$ **(B1)** $a = 6$ **(B1)** **[B2]**

ii. $\frac{\text{their } a[(\text{their } r)^{12} - 1]}{\text{their } r - 1}$ **[M1†]**

$= 24570$ Must be given in full with no rounding off **[A1†]**

†If 24570 appears with no working and either r or a is wrong, both marks are lost.

Question B2

(a) **i.** $(p =)512$ **[B1]**

ii. Substitutes into formula and reaches $8^{4k} = \cdots (\frac{8}{512})$ **[M1*]**

Takes logs, or writes RHS as 8^{-2}, and solves. This mark is dependent on the previous M mark. **[M1*]**

$k = -\frac{1}{2}$ or equivalent (Both M marks scored and no errors seen) **[A1]**

iii. Substitutes $q = -\frac{4}{3}$ into formula. **[M1]**

$= 2046\frac{2}{3}$. Accept anything rounding to 2050 **[A1]**

(b) $\log_x(2x^2 - 2x - 24)$ on LHS Correctly applies collection of logs **[M1*]**

$\log_x(2x^2 - 2x - 24) = 2$ and reaches $2x^2 - 2x - 24 = x^2$
Removes logs correctly and at the right time **[M1*]**

Solves. This is dependent on both of the previous M marks. **[M1]**

$x = 6$. If $x = -4$ is quoted and not discarded (placing in brackets is sufficient to show non-inclusion) this mark is lost **[A1]**

(c) **i.** Uses cosine formula $[8^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \times \cos B]$ **[M1*]**

At least one line of correct intermediate working. **[M1*]**

$\cos B = \frac{3}{4}$ or equivalent but must be in this form, **[A1]**

ii. Uses a right-angled triangle or the identity $\cos^2\theta + \sin^2\theta \equiv 1$, or any other valid method. (Please note: If the candidate works backwards from part iii, this is M0) **[M1*]**

$\sin B = \frac{\sqrt{7}}{4}$ **[A1]**

iii. Uses sine formula $[\frac{\sin C}{10} = \frac{\text{their} \sin B}{8}]$ **[M1*]**

Reaches $\sin C = \frac{5\sqrt{7}}{16}$ having scored the M mark and no errors seen. **[A1]**

iv. $\frac{1}{2} \times 10 \times 12 \times \text{their} \sin B$ or $\frac{1}{2} \times 8 \times 12 \times \sin C$ **[M1*]**

$= 15\sqrt{7}$ (m^2) [must be in this form] **[A1]**

v. $\frac{5}{2}\sqrt{7}$ or anything rounding to 6.61 (m) **[B1]**

Question B3

(a) **i.** $1296\pi = \pi(2r)^2h - \pi r^2h = 3\pi r^2h$ **[M1]**

$h = \frac{432}{r^2}$ or equivalent **[A1]**

ii.

$$A = 2\pi(2r)h + 2\pi rh + 2[\pi(2r)^2 - \pi r^2]$$ **[M1*]**

Substitutes their h into their expression for surface area **[M1*]**

$A = \frac{2592\pi}{r} + 6\pi r^2$ Both M marks scored and no errors seen. **[A1]**

iii. Attempts to differentiate [sight of r^{-2} (or equivalent) or r is sufficient for this mark] $[-\frac{2592\pi}{r^2} + 12\pi r]$ **[M1*]**

Sets equal to 0 (this can be implied) **[M1]**

Reaches $r^3 = \cdots (\frac{2592}{12})$ **[M1]**

$r = 6$ and $\frac{dA}{dr}$ must be correct. **[A1]**

iv. Attempts to differentiate a second time (sight of r^{-3} (or equivalent) or the constant term is sufficient for this mark) **[M1*]**

$\frac{d^2A}{dr^2} = \frac{5184\pi}{r^3} + 12\pi$ **[A1]**

This is positive (when $x = 6$) so there is a minimum. Allow follow through on their $\frac{d^2A}{dr^2}$. **[A1ft]**

or takes a numerical value between 0 and 6 and shows $\frac{dA}{dr} < 0$ **(M1*)**

takes a numerical value above 6 and shows $\frac{dA}{dr} > 0$ **(M1*)**

thus there is a minimum (at $x = 6$). Allow follow through on their $\frac{dA}{dr}$. **(A1)**

Parts (b) and (c) are on the next page.

Question B3 – (continued)

(b) Attempts to differentiate [sight of ke^{2x} $(k \neq 1)$ is sufficient for this mark] $[2e^{2x}]$ **[M1*]**

Substitutes $x = \ln 3$ into their $\frac{dy}{dx}$, (18), inverts and changes sign $(-\frac{1}{18})$ **[M1]**

Finds y when $x = \ln 3$ (2) **[M1]**

$y - 2 = -\frac{1}{18}(x - \ln 3)$ or equivalent and $\frac{dy}{dx}$ must be correct. (Accept anything rounding to -0.0556 and 1.1, or anything rounding to 2.06 if equation given in $y = mx + c$ form.) **[A1]**

(c) Subtracts $4x + 24 \text{ from } 32x - 4x^2$ $(-4x^2 + 28x - 24)$ **[M1*]**

Attempts to integrate (sight of x^3, $x^2 \text{ or } x$ is sufficient for this mark) $[-\frac{4}{3}x^3 + 14x^2 - 24x]$. The limits do not need to be correct for this mark. **[M1*]**

Substitutes correct limits into their integrated expression and subtracts the right way round $[(-288 + 504 - 144) - \left(-\frac{4}{3} + 14 - 24\right)]$ **[M1*]**

$= 83\frac{1}{3}$ All M marks scored and the integration must be correct. **[A1]**

<u>or</u> attempts to integrate $32x - 4x^2$ $(16x^2 - \frac{4}{3}x^3)$ **(M1*)**

Substitutes correct limits into their integrated expression and subtracts the right way round
$[\left(288 - 14\frac{2}{3}\right) = 273\frac{1}{3}]$ **(M1*)**

Finds area under line (190) and subtracts from their $273\frac{1}{3}$ **(M1*)**

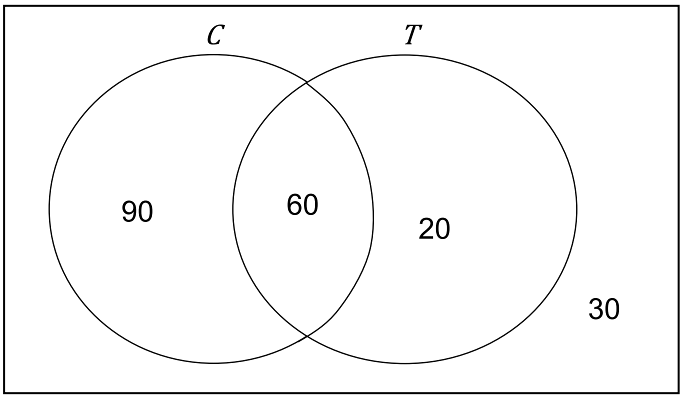
$= 83\frac{1}{3}$ All M marks scored and both areas must be correct. **(A1)**

Question B4

(a) **i.** Finds cumulative frequencies $[5, 30, 71, 98, 108, 114, 118, 120]$ **[B1]**

ii. $20 < t \leq 30$ (Allow $20 - 30$) **[B1ft]**

(b) **i.** E



(B1) for any two correct entries; **(B2)** all entries correct and enclosed in a rectangle. Condone missing E. **[B2]**

ii. $p(C \cup T)' = \frac{3}{20}$, $p(C' \cap T) = \frac{1}{10}$ and $p(C|T) = \frac{3}{4}$. **(B1)** for each **[B3]**
Accept equivalent fractions, decimals or percentages.

iii. Shows that $p(C) \times p(T) = \frac{3}{10}$ **[M1*]**

As $p(C \cap T) = \frac{3}{10}$ (this must be seen) then events C and T are independent (or similar argument) **[A1]**

iv. 'Events C and T can happen at the same time' or '$p(C \cap T) \neq 0$' or any other valid explanation. **[M1*]**

(c) **i.** 80% below x gives an approximate z value of 0.84 **[M1]**

$\left(0.84 = \frac{x - 420}{30}\right) =$ anything rounding to 445 (grams) **[A1]**

ii. Anything rounding to 0.137 **[B1]**

iii. 1 - $p(4 \text{ or less})$ $[1 - 0.6296]$ **[M1]**

Anything rounding to 0.370 **[A1]**

Part (d) is on the next page.

(d) $\mathrm{E}(X) = -2 \times 0.3 + (0 \times 0.2) + 1 \times 0.4 + 0.1 \times y \ (= 0.1y - 0.2)$ **[M1*]**

$\mathrm{E}(X^2) = 4 \times 0.3 + (0 \times 0.2) + 1 \times 0.4 + 0.1 \times y^2 \ (= 0.1y^2 + 1.6)$ **[M1*]**

Sets their $\mathrm{E}(X^2)$ equal to 13 times their $\mathrm{E}(X)$ and forms a quadratic equation
($y^2 - 13y + 42 = 0$ or equivalent) **[M1*]**

Solves. <u>This depends on the previous M mark</u>. **[M1]**

$y = 6 \text{ or } 7$ (Both needed) **[A1]**

Question B5

(a) *Please note: the Quotient Rule must be used*.

Uses Quotient Rule correctly with bottom line correct and one part of the top line correct **[M1*]**
$\left[\left(\frac{dy}{dx} = \right) \frac{(x+3) - (x-6)}{(x+3)^2} \right]$

$= \frac{9}{(x+3)^2}$ Must be in simplest form. **[A1]**

(b) i. $-6x + 12 + 4y \frac{dy}{dx} - 8 \frac{dy}{dx} = 0$ Correct implicit differentiation (sight of $4y \frac{dy}{dx} - 8 \frac{dy}{dx}$ is sufficient for this mark). **[M1*]**

Assembles $\frac{dy}{dx}$ terms on to one side and factorises. This mark is available only if there are at least two $\frac{dy}{dx}$ terms. **[M1]**

$\frac{dy}{dx} = \frac{3x - 6}{2y - 4}$ or equivalent **[A1]**

ii. Sets their $\frac{dy}{dx}$ equal to 0, finds a value for $x \ (= 2)$ and substitutes into original equation $(2y^2 - 8y = 0)$ **[M1*]**

$(2, 0)$, $(2, 4)$ Allow $x = 2$, $y = 0$; $x = 2, y = 4$ **[A1]**

(c) $du = (12x^2 - 8x - 16)\, dx$ or equivalent (may be seen in the integral) **[M1*]**

Writes integral in terms of u $\left[\frac{3}{4} \int u^{-2} \, du \right]$ **[M1*]**

Integrates $\left[\frac{3}{4} (-u^{-1} \right]$ and expresses answer in terms of x. **[M1*]**

$= -\frac{3}{4} \left(\frac{1}{4x^3 - 4x^2 - 16x + 3} \right) + c$ or equivalent **[A1]**

Question B5 – (continued)

(d) **i.** $1 = A(x-k) + B(x-2)$ or any other correct working **[M1*]**

$A = \frac{1}{2-k}$ **[M1*]**

$B = -\frac{1}{2-k}$ **[M1*]**

$p = 2 - k$ **[A1]**

ii. Uses previous result $(2-k)\int_0^1 \frac{1}{2-k}\left(\frac{1}{x-2} - \frac{1}{x-k}\right)\ dx$ **[M1*]**

Attempts to integrate (a log term is sufficient for this mark) $[\ln(x-2) - \ln(x-k)]$ **[M1*]**

Combines logs correctly at any stage, even after substituting in the limits $[\ln(\frac{x-2}{x-k})]$ **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round **[M1*]**

$= \ln\left[\frac{2(3-k)}{4-k}\right]$ **[A1]**