NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics
Part 2 (Business) Examination

MARK SCHEME

<u>Notice to Markers</u>

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:
All <u>correct</u> answers should be rewarded regardless of the number of significant figures used, with the exception of question A8. For this question, 1 discretionary mark is available which will <u>only</u> be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

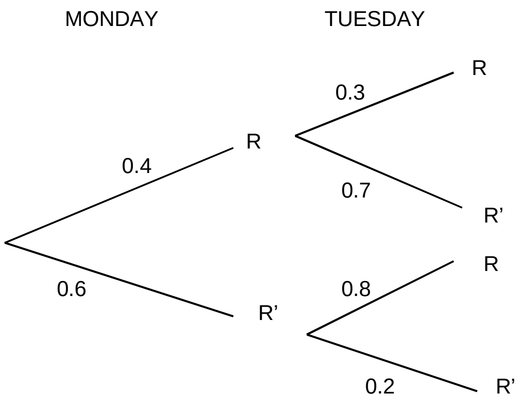
Gradient = $-\frac{4}{3}$ **[M1]**

$y - 7 = \text{their gradient}(x + 3)$ **[M1]**

$4x + 3y - 9 = 0$ **[A1]**

or writes $4x + 3y - k = 0$ **(M1)**; Substitutes $x = -3$ and $y = 7$ into their equation **(M1)**; $4x + 3y - 9 = 0$ **(A1)**

Question A2



R denotes the event 'rain'

(B1) Branches for Monday correct; **(B1)** Branches for Tuesday correct **[B2]**

$p(\text{one day only}) = \text{their } 0.4 \times \text{their } 0.7 + \text{their } 0.6 \times \text{their } 0.8$

(M1) for one correct part; **(M1)** for both parts correct and adding **[M2]**

$= 0.76$ or equivalent **[A1ft]**

Question A3

$$
\begin{array}{r|l}
 & x^2 \quad -x \quad -2 \\
\hline
x + 8 & x^3 + 7x^2 - 10x - 16 \\
 & \underline{x^3 + 8x^2} \\
 & \quad -x^2 - 10x \\
 & \quad \underline{-x^2 - 8x} \\
 & \qquad -2x - 16 \\
 & \qquad \underline{-2x - 16} \\
 & \qquad \underline{\ldots\ldots}
\end{array}
$$

Correct first division **[M1]**

Any correct subsequent division **[M1]**

Correct quotient **[A1]**

Question A4

$a = 1200$ **[B1]**

Their $a \times 0.2^6$ **[M1]**

$= 0.0768$ (Do not allow any rounding off here) **[A1]**

Question A5

Applies correct index addition and subtraction $(2^{x^2-13x+45})$ **[M1*]**

Writes RHS as an index (2^3) **[M1*]**

[If $x^2 - 13x + 45 = 3$ is seen, award both M1* marks]

Forms and solves a quadratic equation $(x^2 - 13x + 42 = 0)$ **[M1]**

$[(x-7)(x-6) = 0 \text{ or } x = \frac{13 \pm\sqrt{[(-13)^2 - 4 \times 1 \times 42]}}{2 \times 1}]$

$x = 6 \text{ or } 7$ (Both answers needed) **[A1]**

Question A6

Divides by x^3 or multiplies by x^{-3} $(x^2 - \frac{1}{x})$ **[M1*]**

Attempts to integrate (sight of $x^3 \text{ or} \ln x$ is sufficient for this mark) **[M1*]**

$\frac{x^3}{3} - \ln x$ (Correct answer) **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= \frac{26}{3} - \ln 3$ or any equivalent form, or anything rounding to 7.57 **[A1]**

Question A7

Correct use of cosine formula in any form $(BC^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos\theta)$ **[M1]**

Correct calculation reaching a value for BC. **[M1]**

$BC = 14$ (metres) **[A1]**

Question A8

Mean $= 8$ **[B1]**

Standard deviation $= \sqrt{[\frac{628}{6} - (\text{their mean})^2]}$ **[M1]**

$= 6.377042....$ **[A1+]**

$= 6.38$ to three significant figures **[A1ft]**

+This mark can be implied if this line is not seen but the 6.38 appears. Allow follow through provided a more accurate answer is seen earlier.

Question A9

$\frac{5p + 3 + 21 + 32 + 7p - 2}{4} = 8p + 1$ **[M1]**

Rearranges and finds a value for p $(12p + 52 = 20p - 4) \ldots\ldots\ldots..p = \cdots)$ **[M1]**

$p = 7$ **[A1]**

Question A10

Finds $p(X < 10) = p(X \leq 9)$ $(= 0.5914)$ **[M1]**

Finds $p(X \leq 5)$ $(= 0.0553)$ and subtracts **[M1]**

= anything rounding to 0.536 **[A1]**

(Other valid methods score the marks. If the candidate works out the probabilities separately, award M1 for any two of $p(X = 6)$, $p(X = 7)$, $p(X = 8)$, $p(X = 9)$; another M1 for all four.)

Question A11

Uses Quotient Rule in its correct form **[M1*]**

$\frac{dy}{dx} = \frac{2x(x-2) - x^2 \times 1}{(x-2)^2}$ (No need for simplification) **[A1]**

Substitutes $x = 3$ into their $\frac{dy}{dx}$ (-3) **[M1]**

$y - 9 = \text{their gradient}(x - 3)$ **[M1]**

$y = -3x + 18$. **[A1]**

Question A12

Amount is £584 **[M1]**

$584 = 480 \times (1 + r)^5$ **[M1*]**

Rearranges correctly and reaches a value for r (≈ 1.04) **[M1]**

Rate = anything rounding to 4%. **[A1]**

Section B

Question B1

a) Gradient of $x + 4y = 1$ is $-\frac{1}{4}$ so $k = 4$ (Reaches value for k) **[M1*]**

Sets $(\text{their } k)x + (\text{their } k)^2 = 0$ and finds a value for x. **[M1*]**

Crosses $x-$ axis at $(-4, 0)$. **[A1]**

b) $x = y + 7$ $\quad or \quad$ $y = x - 7$ **[M1*]**

Substitutes into second equation and forms a quadratic equation.
$(3y^2 + 28y + 60 = 0)$ $\quad or \quad$ $(3x^2 - 14x + 11 = 0)$ **[M1*]**

Factorises or uses formula **[M1]**

$[(3y + 10)(y + 6) = 0$ $\quad or \quad$ $(3x - 11)(x - 1) = 0]$

$[y = \frac{-28 \pm\sqrt{28^2 - 4 \times 3 \times 60}}{2 \times 3}$ $\quad or \quad$ $x = \frac{14 \pm\sqrt{[(-14)^2 - 4 \times 3 \times 11]}}{2 \times 3}]$

Substitutes into either equation to find other unknown **[M1]**

$(1, -6)$ **(A1)** $\quad$ $(\frac{11}{3}, -\frac{10}{3})$ **(A1)** **[A2]**

(Allow equivalent fractions and decimals to at least two decimal places. Solutions do not need to be written as coordinates but it must be clear that they have been matched up correctly.)

c) i. Substitutes $x = \frac{1}{2}$ into expression and sets equal to 8 **[M1*]**

$a = 7$ **[A1]**

ii. Attempts to find discriminant and sets to greater than 0
$(b^2 - 4\times4\times9 > 0)$ **[M1]**

$(b > 12)$ so smallest value is 13. **[A1]**

d) i. $5719 = \frac{43}{2}[2\times7 + (43 - 1)\times d]$ **[M1]**

Rearranges correctly and finds a value for d. **[M1]**

$d = 6$ **[A1]**

ii. $(a = 4, r = 3)$ $\quad$ $S_n = \frac{4(3^n - 1)}{3 - 1} = 2000000$ **[M1]**

Reaches $3^n = \cdots$ (1000001) **[M1]**

Takes logs correctly **[M1]**

$(n = 12.575 \ldots)$ 13^{th} term **[A1]**

Question B2

a) *Please note: (1) this is a 'show that' question so all working must be seen; (2) candidates who work backwards score 0 marks.)*

i. $512 = 8e^{3k}$ and reaches $3^{3k} = \cdots \quad (64)$ **[M1*]**

Takes logs and reaches $3k = \cdots \ (\ln 64)$ **[M1*]**

$k = \quad \underline{\frac{1}{3}\ln 64 = \ln 64^{1/3}} \ = \ \ln 4$ (Both M marks scored and no errors) **[A1]**

Both stages must be seen

ii. Substitutes $t = \frac{7}{2}$ into expression **[M1]**

$= 1012$ **[A1]**

iii. $\frac{dx}{dt} = 8 \times k \times e^{kt}$ and substitutes $t = \frac{1}{2}$ into their $\frac{dx}{dt}$ **[M1]**

$= 16\ln 4$ or equivalent, or anything rounding to 22.2 **[A1]**

iv. $8e^{kt} > 0$ so $x > -12$ thus x will never reach -15 (or any similar and valid argument) **[B1]**

b) i. 27 **[B1]**

ii. 343 **[B1]**

iii. 81 **[B1]**

c) $\cos^2\theta = \frac{3}{4}$ **[M1]**

$\cos\theta = \pm\frac{\sqrt{3}}{2}$ (Mark not lost if $\pm$ omitted) **[M1]**

$\theta = \frac{\pi}{6}$, $\frac{5\pi}{6}$ **(A1)** for each **[A2]**

d) i. $\frac{\sin Q}{10.4} = \frac{\sin 48}{8.5}$ (Correct use of sine formula) **[M1]**

$Q =$ anything rounding to 65.4 (degrees) **[A1]**

ii. Area $= \frac{1}{2} \times 8.5 \times 10.4 \times \sin(180 - 48 - \text{their } Q)$ **[M1]**

= anything rounding to 40.6 (cm^2) **[A1ft]**

iii. Anything rounding to 7.73 (cm) **[B1]**

Question B3

a) i. Attempts to differentiate (x^3 or x^2 seen is sufficient for this mark) **[M1]**

$\frac{dy}{dx} = 12x^3 - 24x^2$ **[A1]**

ii. *Please note: this is a 'show that' question so sufficient working must be seen.*

Either sets their $\frac{dy}{dx}$ equal to 0 and attempts to solve an equation **[M1*]**

Confirms $x = 0$ and $x = 2$ (dependent on a correct equation which has been correctly factorised). **[A1]**

Or Substitutes $x = 0$ and $x = 2$ into their $\frac{dy}{dx}$ **(M1*)**

Confirms both $x = 0$ and $x = 2$ give $\frac{dy}{dx} = 0$ (dependent on a correct equation) **(A1)**

iii. Finds $\frac{d^2y}{dx^2}$ (sight of x^2 or x is sufficient for this mark) **[M1*]**

$\frac{d^2y}{dx^2} = 36x^2 - 48x$ **[A1]**

Shows $\frac{d^2y}{dx^2} = 0$ **and** states (but does not need to verify) there is a change of sign so giving a point of inflexion (conclusion needed) **[A1]**

Shows that $\frac{d^2y}{dx^2} > 0$ when $x = 2$ **[M1*]**

There is a minimum at $x = 2$ (conclusion needed) **[A1]**

or takes a numerical value of x below 0 and shows $\frac{dy}{dx} < 0$ **(M1*)**

takes a numerical value of x between 0 and 2 and shows $\frac{dy}{dx} < 0$ **(M1*)**

takes a numerical value of x above 2 and shows $\frac{dy}{dx} > 0$ **(M1*)**

Thus there is a point of inflexion at $x = 0$ **(A1)**

There is a minimum at $x = 2$ **(A1)**

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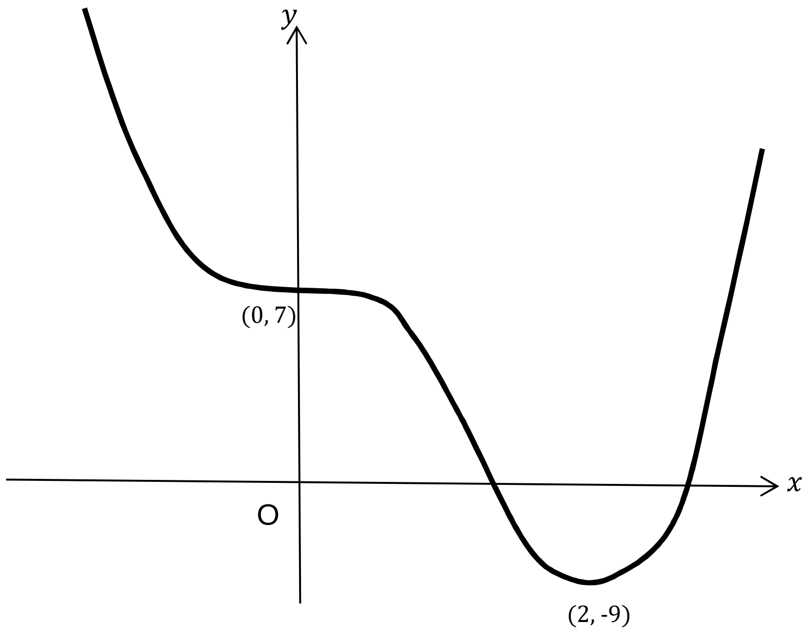
iv.



Correct shape (can be drawn anywhere) **[B1]**

$(0, 7)$ and $(2, \text{-}9)$ shown **[B1]**

b) i. *Please note: this is a 'show that' question so all working must be seen.*

Differentiates and finds a value for $\frac{dy}{dx}$. $\left(\frac{dy}{dx} = 2x - 6 = 2 \text{ when } x = 4\right)$ **[M1*]**

Inverts and changes sign $\left(= -\frac{1}{2}\right)$ **[M1*]**

The gradient of line l is (also) $-\frac{1}{2}$ so line l is a normal. **[A1]**
(Statement and conclusion needed)

ii. Area of triangular section = 1 **[B1]**

Attempts to integrate $x^2 - 6x + 9$ (a correct index for one of the x terms is sufficient for this mark) **[M1*]**

$= \frac{x^3}{3} - 3x^2 + 9x$ **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

Adds areas **[M1]**

$= \frac{4}{3}$ or equivalent, or anything rounding to 1.33 **[A1]**

Question B4

a) i.

Time, t, in seconds	Frequency (f)	Mid-value (x)	$f \times x$	Cumulative frequency
$3 \le t \le 4$	10	3.5	35	10
$4 < t \le 5$	32	4.5	144	42
$5 < t \le 6$	23	5.5	126.5	65
$6 < t \le 7$	8	6.5	52	73
$7 < t \le 8$	3	7.5	22.5	76
$8 < t \le 9$	2	8.5	17	78
$9 < t \le 10$	1	9.5	9.5	79
$10 < t \le 11$	1	10.5	10.5	80
			$\sum fx = 417$	

Finds $\sum fx$ $(= 417)$ **[M1]**

Divides their $\sum fx$ by 80 **[M1]**

= anything rounding to 5.21 **[A1]**

ii. Finds cumulative frequencies **[M1]**

Plots correct curve (a sketch is on page 15). 1 mark lost for each omitted/incorrect plot; 1 mark lost for each point missed by the curve by at least 1 mm (but allow ft for any incorrect plots); 1 mark lost if either axis not labelled correctly. **[A3]**
(<u>Please note</u>: a maximum total of 3 marks can be lost i.e. there are no negative scores. If the candidate plots the mid-values instead of the upper values in each interval, this will score A0.)

If graph paper is not used, award 1 mark out of the A3 if a reasonable curve is drawn. If a cumulative frequency polygon is drawn, award up to 2 marks out of the A3.

iii.
Reads off median from their graph (about 4.9 for a correct graph) **[B1ft]**

iv.
Yes **(B1+)** and a reason **(B1)** e.g. because there is bunching of data at lower end of the range; the median and mean are significantly different; spread is uneven; more thinly spread out at upper end of the range; or any description which shows an understanding. **[B2]**

[+This B1 can only be awarded if a reason (even a wrong one) follows]

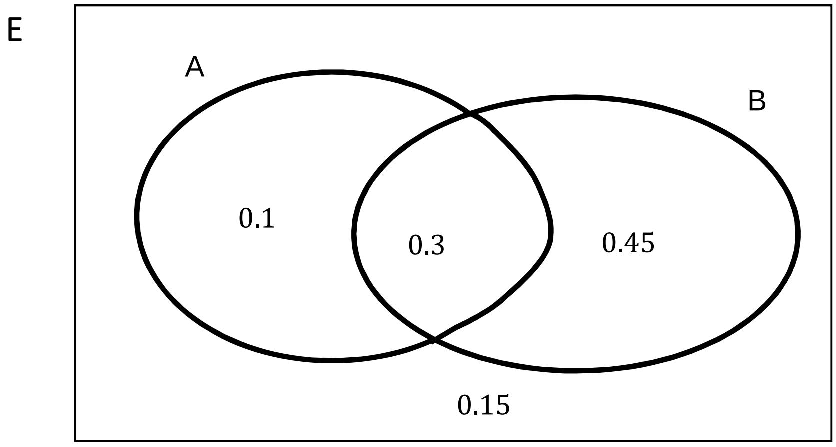
Parts b) and c) are on the next page.

Question B4 – (continued)

b) i. $p(A \cap B) = 0.3$ **[B1]**

$p(A \cup B) = 0.85$ **[B1]**

ii.



(B1) Any one entry correct; **(B2)** any two entries correct;
(B3) all entries correct with a rectangle (but condone missing E) **[B3]**

iii. $p(A' \cap B) = 0.45$ **[B1]**

$p(A \cup B') = 0.55$ **[B1]**

$p(A'|B) = 0.6$ **[B1]**

Accept any equivalent forms in all three cases.

c)

Yes **(B1+)** and a reason **(B1)** e.g. sets are disjoint/do not overlap; events cannot happen at the same time; $p(A \cap B) = 0$; one event must exclude the other; or any comment which shows a clear understanding. **[B2]**

[+This B1 can only be awarded if a reason (even a wrong one) follows]

Question B5

a) i. $s_x = \sqrt{[1284 \div 6 - 14^2]}$ (≈ 4.24) **[B1]**

$s_y = \sqrt{[3688 \div 6 - 24^2]}$ (≈ 6.22) **[B1]**

$s_{xy} = 2171 \div 6 - 14 \times 24$ (≈ 25.8) **[B1]**

$r =$ anything rounding to 0.98 **[B1]**

ii. $y - 24 = \frac{\text{their } s_{xy}}{\text{their } s_x^2}(x - 14)$ **[M1*]**

$y = 1.4x + 4$ (anything rounding to 1.4 and 4) **[A1]**

If the correct answer appears without working, give 1 mark out of 2.

iii. $r = 0.98$ **[B1ft]**

$y = 1.4x$ **(B1ft)** $+ 3.5$ **(B1ft)** **[B2ft]**

(Allow follow through on their answers in parts i and ii.)

b) i. $0.9332 = \Phi(1.5)$ **[M1]**

their $1.5 = \frac{25.3 - 25}{\sigma}$ **[M1]**

$\sigma = 0.2$ or equivalent **[A1]**

ii. $z = \frac{25-24.5}{\text{their } \sigma}$ or $\frac{24.5 - 25}{\text{their } \sigma}$ $(= 2.5)$ **[M1]**

Finds Φ(their 2.5) and subtracts from 1 $(= 0.0062)$ **[M1]**

0.62% (must be in percentage form) **[A1]**

c) i. $a^2 + 2a + 0.31 = 1$ and writes a three-term quadratic equation set equal to 0 $(a^2 + 2a - 0.69 = 0)$ **[M1]**

Solves by completing the square or using the formula **[M1]**

$[\ (a + 1\ )^2 - 1 - 0.69 = 0$ or $a = \frac{-2 \pm\sqrt{2^2 - 4 \times 1 \times -0.69}}{2 \times 1}\ ]$

$a = 0.3$ (If the -2.3 is included, this mark is lost.) **[A1]**

ii. $\mathrm{E}(X) = 1 \times 0.14 + 3 \times 0.17 + 5 \times \text{their } 2a + 7 \times \text{their } a^2$ **[M1]**

$= 4.28$ **[A1]**

Question B6

a) i. $-4x + 8y^2 + 16xy\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$

Correct use of Product Rule **[M1*]**

Correct implicit differentiation (sight of $16xy\frac{dy}{dx}$ or $3y^2\frac{dy}{dx}$ is sufficient) **[M1*]**

Factorises and reaches an expression for $\frac{dy.}{dx}$ (This mark can only be given if there are at least two $\frac{dy}{dx}$ terms in the expression) **[M1]**

$\frac{dy}{dx} = \frac{4x - 8y^2}{16xy - 3y^2}$ **[A1]**

ii. $\frac{dy}{dx} = 0$ **[M1*]**

There is a stationary value (or turning point or similar words). **[A1]**

b) i.

$2 = A(1-x) + B(1+x)$ **[M1]**

$A = 1$ **(A1)** $B = 1$ **(A1)** **[A2]**

ii. Uses previous result **[M1]**

Attempts to integrate (sight of a log term is sufficient for this mark) **[M1]**

$\ln(1+x) - \ln(1-x)$ (Correct integrated expression) **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

Gathers expression up into a single logarithm $[\ln\left(\frac{1+a}{1-a}\right)]$ **[M1*]**

Sets equal to $\ln\left(\frac{2}{3}\right)$, removes logs and obtains a value for a **[M1]**

$a = -\frac{1}{5}$ or equivalent **[A1]**

(If the negative sign is missed in the third line, M1 M1 A0 M1 M1 M1 A0 is possible, giving a maximum of 5 out of 7.)

Part c) is on the next page.

Question B6 – (continued)

c) $\frac{dy}{dx} = 1$ or $du = dx$ **[M1*]**

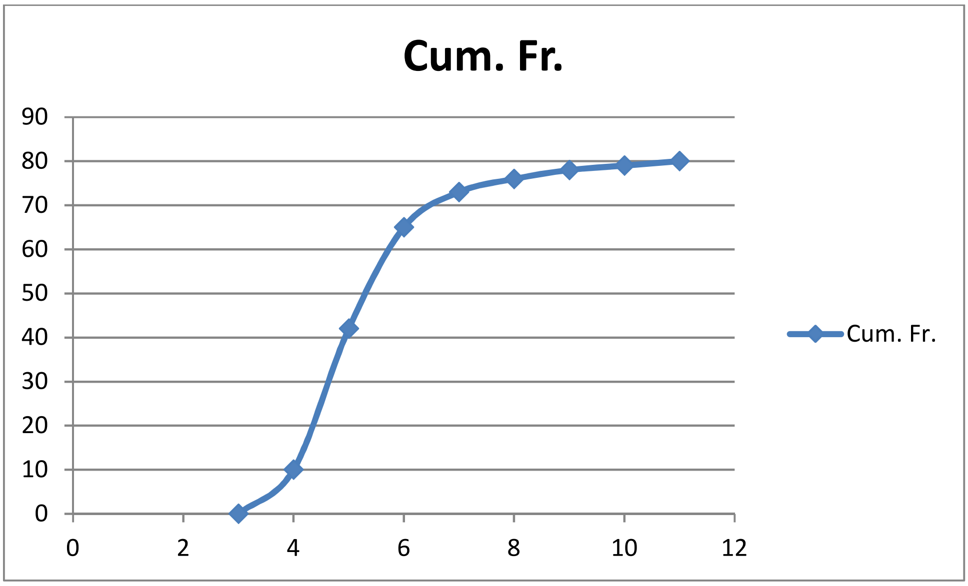
Writes integral in terms of u $\left(\int \frac{u-3}{u}\, du\right)$ **[M1*]**

Writes integrand as $1 - \frac{3}{u}$ and attempts to integrate (u or $\ln u$ seen) **[M1]**

$= (x+3) - 3\ln(x+3) + c$ (Correct answer which must be in x, and $+ c$) **[A1]**

Please note: the question has asked for integration by substitution, so if the candidate has used another method, this scores 0.

Sketch of the cumulative frequency curve for question B4



Time, t (seconds)

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