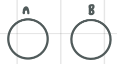
INDEPENDENCE + CONDITIONAL PROBABILITY
wk 2-1 cont. of ch.2

REMEMBER OF MUTUALLY EXCLUSIVE :

- like male vs female →
- $A \cap B = \emptyset$



} two things cannot happen at the same time

↳ another ex: cannot be a freshman + a senior at the same time, but they are still related

UNCONDITIONAL

independent : if occurance of one event does not affect the probability of the other event → one thing has nothing to do with another → 2 UNRELATED things

↳ $P(A|B) = A$, A is not affected by B

CONDITIONAL PROBABILITY (aka dependent) of event A given event B : is the probability of A when event B has occurred → $P(A|B)$, $P(A) \neq P(A|B)$

| = given

$A|B$ = A given B

→ ex: randomly select person

A = taller than 72 inches   B = NBA PLAYER

$P(A) < P(A|B)$

↑ if person is NBA , very likely to be > 72 in.

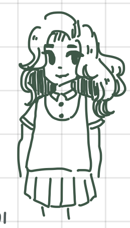
↳ FORMULA

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

→ rewrite w/ $P(A \cap B) = P(A)\,P(B|A) = P(B)\,P(A|B)$ ← MULTIPLICATION AKA PROB. OF INTERSECTION FORMULA

when A + B are INDEPENDENT then → $P(A|B) = P(A)$ + $P(B|A) = P(B)$ → $P(A \cap B) = P(A)P(B)$

↳ proves independence



DOING MATH TO DETERMINE INDEPENDENCE

↳ EXAMPLE 1 : Rolling a die once. (by default : 6 sided + balanced) $A = \{1,3,5\}$, $B = \{1,2,3,4\}$, $C = \{2,4,6\}$, $D = \{1,3\}$

↳ A + C are mutually exclusive (disjoint)

- can't happen simultaneously

↳ A + B are not disjoint

↳ B + C are not disjoint

↳ A + B are independent : how do we know? we prove w/ $P(A \cap B) = P(A)P(B)$ see if A + B r independent

↳ B + C are independent

↳ A + C are dependent

↑ they are mutually exclusive

$P(A \cap B) = P(\{1,3\}) = \frac{2}{6} = \frac{1}{3}$ ✓ INDEPENDENT

$P(A) = P(\{1,3,5\}) = \frac{3}{6}$

$P(B) = P(\{1,2,3,4\}) = \frac{4}{6}$

$\frac{3}{6} \cdot \frac{4}{6} = \frac{12}{36} = \frac{1}{3}$ ✓

or

use conditional probability formula

$P(A) = P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/3}{4/6} = \frac{1}{3} \cdot \frac{6}{4} = \frac{1}{2}$

$P(A) = \frac{1}{2} = P(A|B)$

unconditional + conditional probability are the same, so they are independent

★ THE TRICK !

- if A + B are mutually exclusive / disjoint → A + B are dependent
- if A + B are overlapped → could be dependent or independent
- A + B are independent → A + B are not mutually exclusive

if $A \to B$, then $\overline{B} \to \overline{A}$

↳ contrapositive

EXAMPLE 2 : Randomly selected a person from a big mall : A = { the person is athletic } , B = { the person is a male } , C = { the person is female }

↳ B + C are mutually exclusive

↳ A + B are not mutually exclusive

↳ A + C are not mutually exclusive

↳ B + C are dependent → we know if $P(B|C) = 0$ then $P(B) = 0$ but we know $P(B)$ is 0. The condition affects the probability

↳ A + C are ____?____

↳ A + B are ____?____

} → depends on stereotypes, can be debated

COMBINATORICS: COUNTING TECHNIQUES

PREMUTATION:

selecting r items from a set of n items where order matters + repeating is not allowed

↳ $P(n,r)$, $_nP_r$ = $\frac{n!}{(n-r)!}$

COMBINATION:

selecting r items from a set of n items where order does not matter + repeating is not allowed

↳ $C(n,r)$, $_nC_r$ = $\frac{n!}{r!(n-r)!}$

FOUNDEMENTAL COUNTING PRINCIPLE

aka multiplication principle but we don't call it that bc it's reserved 4 smth else

there are k steps to complete a task + n_1 ways to complete step 1, n_2 ways to complete step 2, ... n_k,

then there are a total of $\underset{\uparrow \text{ multiplied together}}{n_1 n_2 \ldots n_k}$ ways to complete the task.

EXAMPLE 1: How many ways are there to choose 7 people to form a committee (all members play the same role) from a class of 50 people?

order don't matter → combination

$_{50}C_7$ = 99884400

EXAMPLE 2: How many ways are there to choose 3 people to form a cabinet (president, vice president, treasurer) from a class of 50 people?

order matters → premutation

$_{50}P_3$ = 117600

EXAMPLE 3: How many car license plates are available if the rule is 3 letters followed by four numbers?

= $(26^3)(10^4)$

wk 2-2

SELECTION

W/ REPLACEMENT

HOW MOST STATISTIC FORMULAS ARE DERIVED

- rolling a die always replaced
  - ↳ models a perfect world

w/R
① suppose we are drawing cards w/ replacement

the 1st, 2nd, 3rd card's prob. of being a heart are all 1/4

③ probability of the 3rd card is heart is also 1/4!

based on assumption we don't look at any cards we've drawn.

IF PROB. OF W/ REPLACEMENT + W/O ARE THE SAME, THEN WHAT'S THE DIFFERENCE?

- the conditional probability is different!

w/o replacement $P(H_2|H_1) \neq P(H_2)$ ← dependent

w/ replacement $P(H_2|H_1) = P(H_2)$ ← independent

★ HOW THE SAMPLE IS COLLECTED IS IMPORTANT

should always be random + unbiased — don't ask same person twice

mostly samples are done w/o replacement

SELECTION

W/O REPLACEMENT:

HOW MOST APPLICATIONS IN REAL LIFE ARE DONE

- two cards are dealt in manner of playing bridge (not revealed till end)
  - ↳ once a card is drawn, it can't be drawn again

w/o R

ex: ① probability of 1st card is heart? 1/4 ← 13/52

② probability of 2nd card is heart? 1/4 surprise!

$P(H_2) = P(H_2 \cap H_1) + P(H_2 \cap \bar{H_1})$

2nd card being a ♥ — in set theory

these two are mutually exclusive because 1st one can't be ♥ + $\bar{♥}$

$H_2 = (H_2 \cap H_1) \cup (H_2 \cap \bar{H_1})$

$H_2 \cap H_1$ $\quad$ $H_2 \cap \bar{H_1}$

we will use 2 probability formulas! → $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A|B)\,P(B)$

$P(H_2) = P(H_1)\,P(H_2|H_1) + P(\bar{H_1})\,P(H_2|\bar{H_1})$

$= \frac{13}{52} \cdot \frac{12}{51} + \frac{39}{52} \cdot \frac{13}{51}$

$= \frac{1}{4}$

$P(H_1 \cap H_2)$ will be different w/ + w/o replacement

$\uparrow$ always creates dependence

HW RESTRICTION

6 different letters case sensitive

= after "A" or "a" is chosen only 25 letters left (50 choices)

6 different symbols, after "A" is picked there are 51 choices left

$\underbrace{\text{symbols}}_{\text{restricted to alphabet}}$

$P(H_1 \cap H_2)$ w/ replacement: $P(H_2 | H_1) P(H_1) = \frac{1}{16}$

$P(H_2 | H_1) = P(H_1) = \frac{13}{52} = \frac{1}{4}$

$P(H_1 \cap H_2)$ w/o replacement: $P(H_2 | H_1) P(H_1) = \frac{13}{52}\left(\frac{12}{51}\right) \rightarrow \frac{1}{4}\left(\frac{4}{17}\right) = \frac{4}{68} = \frac{1}{17}$

$P(H_1) = \frac{13}{52}$

$P(H_2 | H_1) = \frac{12}{51}$

EXAMPLE (like HW #6 2.14): → different in that after you check the 1st passcode you know if it works

THERE ARE 1000 TICKETS IN A BOX W/ ONLY ONE BEING THE WINNING TICKET. A PERSON DRAWS (ALL W/O REPLACEMENT) what's the probability of winning? W/O REPLACEMENT → assume all raffle tickets are revealed at once

a) 1 ticket

$P(\text{win}) = \frac{1}{1000}$

b) 10 tickets

$P(\text{win}) = \frac{10}{1000}$ ↪

$P(\text{win}) = P(\text{1st ticket is win ticket}) +$

$P(\text{2nd ticket is win ticket}) \ldots +$

$P(\text{10th ticket is win ticket})$

(consider we see result of all 10 at once)

↳ $P(\text{win}) = \frac{1}{1000} \cdot 10 = \frac{10}{1000}$

c) 100 tickets

$P(\text{win}) = \frac{100}{1000}$

(based on how b was done.)

d) 1000 tickets

$P(\text{win}) = 1$

Example 7 (computing probability when independence is assumed). Exercise 2.5 on page 33. A computer program is tested by 3 independent tests. When there is an error, these tests will discover it with probabilities 0.2, 0.3, and 0.5 respectively. What is the probability that it will be found by at least one test if there is an error? (The wording in textbook is: Suppose that the program contains an error, what is the probability that it will be found by at least one test?)

$P(T_1 \cup T_2 \cup T_3)$ USING THE COMPLEMENT + INDEPENDENCE!

use → $P(\bar{T_1} \cap \bar{T_2} \cap \bar{T_3})$

$P(T_1 \cup T_2 \cup T_3) = 1 - P(\bar{T_1} \cap \bar{T_2} \cap \bar{T_3})$

$= 1 - ((1-.2) \times (1-.3) \times (1-.5))$

$= 1 - ((.8)(.7)(.5))$

$= 1 - .28$

$= .72$