SYLLABUS DAY

★ EMAIL FOR ANY QUESTIONS! ★

JK NO SHE JUMPED RIGHT INTO LEC.

- will have 2 attempts for quizzes
- quiz 1 over syllabus

CHAPTER 2: PROBABILITY (PROBS THE HARDEST CHAPTER)

WK1 – 1

WHAT IS STATISTICS?

you learn 2 things

WHAT WE DO!

DESCRIPTIVE STAT

collect sample + study the samples →

SAMPLE PERCENTAGE

INFERENTIAL STAT

population as a whole usually what we're interested in

we need perimeters of what defines a population though.

→ WE INFER THE PERIMETERS

NEED PROBABILITY TO GO FROM SAMPLE → POPULATION!

NOTE. THE # OF EVENTS (SUBSETS). IS JUST A CONCEPT

↳ flip coin, there are probabilities for 4 subsets

$\{H\}, \{T\}, \{H,T\}, \emptyset$ — SIMPLE EVENT: $\{H\}, \{T\}$

$P(\{H\}) = .5$

$P(\{T\}) = .5$

$P(\{H,T\}) = 1$ → does not mean roll twice, means the probability of getting H or T

$P(\emptyset) = 0$

WHAT IS PROBABILITY? – A NUMBER THAT MEASURES CHANCE

HOW DO WE COMPUTE PROBABILITY: we use

BASIC PROBABILITY THEORY →

- SET THEORY

KEYWORDS:

PROBABILITY EXPERIMENT: an experiment where the outcome is uncertain

Ω SAMPLE SPACE: a collection of all possible outcomes

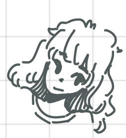
EVENT: any subset of the sample space

→ total # of events $= 2^n$, $n =$ # of possible outcomes

$\emptyset$ EMPTY EVENT: NO OUTCOMES

EXAMPLE: ROLL A DIE ONCE

- sample space $\Omega = \{1, 2, 3, 4, 5, 6\}$
- $2^6 = 64$ events → $\{1\} - \{6\}$, $\{1,2\} - \{1,6\}$, $\{2,1\}$ ...

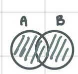


RANDOM PERSON FROM CLASS + MEASURE THERE HEIGHT

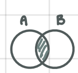
- cannot list all outcomes, there's too many

↳ there are ∞ possibilities since you could get super precise w/ the measurement.

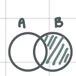
SET OPERATIONS: do stuff w/ sets to get new sets



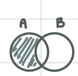
UNION: $A \cup B$, outcomes from A or B or both



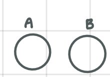
INTERSECTION. $A \cap B$, outcomes common in A + B



COMPLEMENT: $\bar{A}$ or A^c (A not) outcomes excluded from A



DIFFERENCE: $A \setminus B$ (A but not B) outcomes included in A but excluded from B



NO OVERLAP

DISJOINT: A + B are disjoint if $A \cap B = \emptyset$

same

MUTUALLY EXCLUSIVE / PAIRWISE DISJOINT: $A_1, A_2, + A_3$ are mutually exclusive if $A_i \cap A_j = \emptyset$ for any $i \neq j$

EXHAUSTIVE. $A_1, A_2, A_3, \ldots$ are exhaustive if $A_1 \cup A_2 \cup A_3 \ldots = \Omega$

DE MORGAN'S LAW: $\overline{A \cup B} = \bar{A} \cap \bar{B}$, $\overline{A \cap B} = \bar{A} \cup \bar{B}$

FORMULAS:

AXIOMS (we all agree)

- $P(\emptyset) = 0$ — probability of an $\emptyset$ is 0, roll a die will never not have an outcome
- $P(\Omega) = 1$ — SAMPLE SET WILL HAPPEN FOR SURE

↓

IF ALL EVENTS ARE EQUALLY LIKELY TO OCCUR, THEN

$$P(A) = \frac{\text{\# of outcomes contained in } A}{\text{\# of outcomes in } \Omega}$$

ROLL FAIR DIE ONCE, WHAT'S PROBABILITY OF $A = \{2, 4, 6\}$?

$$P(A) = \frac{3}{6} = 0.5$$

ADDITION FORMULA

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B) = P(A) + P(B)$ if A + B are disjoint
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$
- $P(A \cup B \cup C) = P(A) + P(B) + P(C)$ if A, B, C are mutually exclusive

$P(\bar{A}) = 1 - P(A)$ complement formula

$P(A \cap B) = P(A) \cdot P(B)$ if A + B are independent

independent: if occurance of one event does not affect the probability of the other event

CONDITIONAL PROBABILITY of event A given event B: is the probability of A when event B has occurred → $P(A \mid B)$

ex: randomly select person

A = taller than 72 inches B = NBA PLAYER

$P(A) < P(A \mid B)$

if person is NBA, very likely to be > 72 in.

EXAMPLE 2: SET OPERATIONS

DRAW A CARD FROM A STANDARD DECK (52 CARDS)

$\Omega = \{H_1, H_2 \ldots\}$ → 52

$A = \{\text{spade}\}$ → can list or descripe → 13

$B = \{\text{four}\}$ → 4

$A \cup B = 16$, 13 spades + 4 four − 1 spade four

$A \cap B = 1$

$\Omega \cup B = 52$ (big one)

$\Omega \cap B = 4$ (small one)

EXAMPLE 3:

equally likely!

DRAW A CARD FROM DECK, WHAT'S THE PROBABILITY THAT YOU GET A CARD THAT IS A FOUR?

$$P(A) = \frac{1}{52}$$

EXAMPLE 4:

equally likely!

ROLL A FAIR DIE TWICE. PROBABILITY THEIR SUM IS 6

1+5, 2+4, 3+3, 5+1, 4+2

$$P(A) = \frac{5}{36}$$

→ALL OUTCOMES (6 × 6, 36 OUTCOMES)

(1 1)	(1 2)	(1 3)	(1 4)	(1 5)	(1 6)
(2 1)	...				(2 6)
(3 1)	...				(3 6)
⋮					⋮
(6 1)	...				(6 6)

EXAMPLE 5:

AMOUNG DONORS AT A BLOOD CENTER, 1 in 2 gave O^+ type blood, 1 in 11 gave O^-, 1 in 4 gave A^+, + 1 in 20 gave A^-

what's the probability the 1st person who shows up tmr is ...

a) either A^- or O^-

$$P(A^- \cup O^-) = P(A^-) + P(O^-) = \frac{1}{20} + \frac{1}{11} =$$

b) neither A^+ nor O^+

$$P(\overline{A^+ \cup O^+}) = 1 - P(A^+ \cup O^+) = 1 - \left(\frac{1}{4} + \frac{1}{2}\right) = \frac{1}{4}$$