WK 10-1
3/31/21

LET'S CONSTRUCT SOME RECURSIVE DEFINITIONS

I'M INTERESTED IN A SET $\{(a,b) \in \mathbb{N} \times \mathbb{N} \mid a+b \text{ is odd}\}$ $\mathbb{N} = \{0, 1, 2, \ldots\}$

- CONSTRUCT A RECURSIVE DEFINITION

DEFINITION OF S:

BASIS

$(0,1) \in S$

$(1,0) \in S$

RECURSIVE STEP

AFTER FINISHING PROOF WE SEE THAT WE DON'T NEED THIS IN THE DEF, IT'S ALREADY COVERED

if $(x,y) \in S$ then $\cancel{(x+1, y+1) \in S}$ + $(x+2, y) \in S$ + $(x, y+2) \in S$ $\cancel{+ (y,x) \in S}$ DON'T NEED THESE

~~if x!=y, then (x,y) != (y,x)~~

PROOF

CLAIM: everything in the set S is an element of $\{(a,b) \in \mathbb{N} \times \mathbb{N} \mid a+b \text{ is odd}\}$ ↳ RIGHT NOW WE ARE JUST PROVING EVERYTHING WE'RE GENERATING IS IN THE SET.

BASE CASE: $(0,1)$: $0+1 = 1$ = odd ✓

$(1,0)$: $1+0 = 1$ = odd ✓

INDUCTIVE CASE: ASSUME $x+y$ IS ODD

NEED TO SHOW:
- $(x+1)+(y+1)$ is odd
- $(x+2)+y$ is odd
- $x+(y+2)$ is odd

IF $x+y$ is odd, $x+y = 2k+1$ for some int k get all 3 to the $(x+y)$ form to explicitly use the definition of odd numbers

↳ NOTE all 3 of $(x+1)+(y+1)$, $(x+2)+y$, $x+(y+2)$ $= (x+y)+2$

$= 2k+1+2 = 2(k+1)+1$ ← odd by definition

NOW WE CONSTRUCT (a, b)

IMPROVED CLAIM →

CLAIM: everything in the set S is an element of $\{(a,b) \in \mathbb{N} \times \mathbb{N} \mid a+b \text{ is odd}\}$ is in S

GIVEN: (a,b) where $a+b$ is odd, $a, b \geq 0$

CASE 1: $a > b$

CASE 1.1: $a = 1$, use $(1,0) \in S$

CASE 1.2: $a > 1$, build (a,b) from $(a-2, b)$

↳ $(a-2) > 0$ since $a > 1$
- $a-2+b$ is odd since it is 2 less than an odd #

CASE 2: $b > a$ similar to $b > a$

CASE 2.1: $b = 1$, $(0,1) \in S$

CASE 2.2: $b > 1$, build (a,b) from $(a, b-2)$

WE INDUCTIVELY ASSUME SMALLER VALUES CAN BE CONSTRUCTED.

CASE 3: $a = b$

then $a+b$ is even, not possible

EXAMPLE:

$(11, 4)$ → HOW MIGHT WE BUILD THIS?

↳ WHAT IS THE ORDERED PAIR THAT CAME BEFORE $(11,4)$?

- $(9,4)$ ✓ works
- $(11,2)$ ✓ works

IF WE CAN GENERATE THESE, WE CAN GENERATE $(11,4)$

#1 TAKE AWAY:

JUST BECAUSE EVERY ELEMENT OF A SET HAS SOME PROPERTY DOES <u>NOT</u> MEAN EVERYTHING W/ THAT PROPERTY IS IN THE SET.

↳ every set of prime #'s are prime ≠ all prime #'s are in the set.

I WANT ORDERED PAIRS OF NATURAL NUMBERS IN WHICH :

THE ABSOLUTE DIFFERENCE OF THE COORDINATES IS AT MOST 2

↳ $|x-y| \le 2$

example: if we wanted $(2, 111)$

we'd get it like $(2, 109) \rightarrow (2, 107) \underbrace{\rightarrow \ldots \rightarrow}_{52 \text{ STEPS}} (2, 1) \rightarrow (0, 1)$

BASIS :

$\underbrace{(0,0), (0,1), (1,0), (2,0), (0,2)}_{5 \text{ CASES}}$ are in set S

SHOULD SEE 6 CASES

1. $(0,0) = 0$
2. $(0,1) = 1$
3. $(1,0) = 1$
4. $(2,0) = 2$
5. $(0,2) = 2$

RECURSIVE STEP :

if (x, y) is in S, then

$\underbrace{(x+1, y+1) \text{ is in S}}_{1 \text{ CASE}}$

6. SUPPOSE $|x-y| \le 2$

WHAT ABOUT $|(x+1) - (y+1)|$?

$$|(x+1) - (y+1)| = |(x-y) + (1-1)|$$
$$= |x-y|$$
$$\le 2$$

so $|(x+1) - (y+1)| \le 2$

RECURSIVE DEFINITION

GIVEN (a, b) WHERE $|a-b| \le 2$, $a, b \ge 0$, HOW TO CONSTRUCT (a, b)?

When can we use (+1/+1) rule?

if a, b are both > 0 , then we use the (+1/+1) rule on $(a-1, b-1)$

↳ $a-1$ + $b-1$ will both be at least 0 , since a, b are at least 1

so... $a-1, b-1$ will have absolute difference at most 2 .

WHAT IF a or b is 0? ARE WE GUARANTEED TO BE ABLE TO USE A BASE CASE?

IF ONE OF a or b is 0 , the other can be at most 2,

else absolute difference is more than 2, so the other is 0, 1, or 2

+ bases cover all these possibilities.