Ch. 9 RELATIONS

WK1-1 (NO 2, MONDAY OFF)

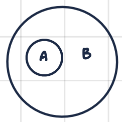
A **RELATION** from set A → set B is A × B — **CARTESIAN PRODUCT**

↳ ordered pairs w/ 1st coordinate from A + 2nd coordinate from B

* allows one value to map to multiple values!

• on its self: a relation on set A is a subset of A × A

refresher:



A = subset of B

ex: a relation from set A → set A is a relation on set A

ex: a relation from {1, 2, 3} to {4, 5, 6} is {(1,4), (2,5), (3,6)}

{(1,4), (2,6)}

{(1,4)}

{(1,4), (1,5), (1,6)}

↳ if (x, y) is an element of relation R, we say x relates to y

DEFINING A COMPOSITION

If R is a relation from X to Y + S is a relation from Y to Z: then the composition of S∘R is defined

General definition of a composition

+ (x, z) is an element of the composition iff (if + only if) there is some y such that (x, y) is an element of R + (y, z) is an element of S

R: {(1,4), (2,5), (3,4), (3,5), (3,6)} / {1, 2, 3} (x) → {4, 5, 6} (y)

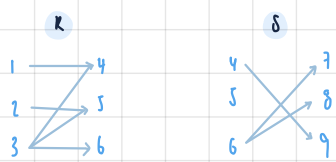
S: {(4,9), (6,7), (6,8)} / {4, 5, 6} (y) → {7, 8, 9} (z)

THESE 2 MUST MATCH FOR A COMPOSITION

is the composition S∘R defined?

yes, y's are the same

it's ok that 5 is not related to any item of the 2nd set *



THE COMPOSITION LOOKS LIKE:

→ S is left

S∘R

→ S is right bc S is the result of doing R

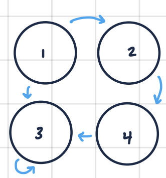
{(1,9), (3,7), (3,8), (3,9)}

DEFINED SIMILARLY TO COMPOSITION OF FUNCTIONS

COMPOSING A RELATION WITH ITSELF

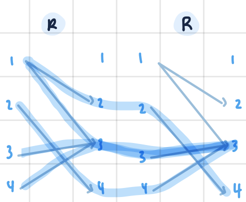
Let R be a relation on {1, 2, 3, 4}

R: {(1,2), (1,3), (2,4), (3,3), (4,3)} =



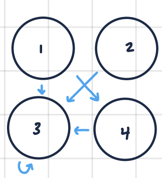
composition (R∘R, R^2):

1ST WAY TO FIND COMPOSITION



{(1,4), (1,3), (2,3), (3,3), (4,3)}

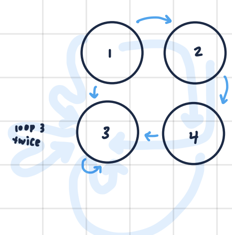
=



2ND WAY TO FIND COMPOSITION

FIND PATHS OF LENGTH 2:

→ this is a rule for compositions of a relation with itself!

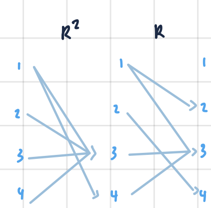


all beg + end of those paths are the (x, z) of the COMPOSITION!

CONCEPT OF CONNECTIVITY RELATION

• R ∪ R∘R → paths of length 1 + 2

• R^3 = R ∘ R^2 = R ∪ R^2 (no new edges added)



COMPOSITION R^3 =

{(1,3), (2,3), (3,3), (4,3)}

No new edges added = have found all paths

• R ∪ R^2 ∪ R^3 ... ∞ = R^* → CONNECTIVITY RELATION