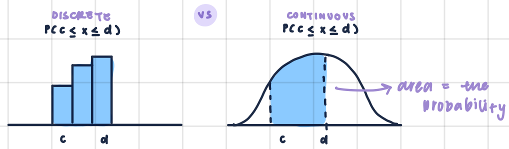
CH 4 CONTINUOUS DISTRIBUTION

WK 6-1

DIFFERENCE w/ DISCRETE

CONTINUOUS CASE IS ALWAYS

- always in intervals
- our cdf is an integral function instead of just adding



PDF : f(x)

a continuous random variable X on an interval $[a,b]$ has a probability density function $f(x)$ that satisfies the following two properties

1. $f(x)$ is non-negative for all values of x
2. $\int_a^b f(x)\,dx = 1$

$0 < P(x) < 1$, $\sum P(x) = 1$ } for discrete r.v. pdf

EXAMPLE 1: let X be a random variable w/ density function:

$f(x) = \frac{3}{14}(x + x^2)$ over $[0,2]$

a) verify that $f(x)$ is a density function

must fulfill 2 conditions = ① $f(x)$ must be non-negative

② the integral of $[0,2]$ (INTERVAL) $= 0$

$\int_0^2 f(x)\,dx = \frac{3}{14}\int_0^2 x + x^2\,dx = \frac{3}{14}\left[\frac{x^2}{2} + \frac{x^3}{3}\right]_0^2$

$\frac{3}{14}(2) + \frac{24}{42} = \frac{6}{14} + \frac{8}{14} = \frac{14}{14} = 1$ ✓

b) compute $P(X \le 1)$

$P(X \le 1) = \int_0^1 \frac{3}{14}(x+x^2)\,dx = \frac{3}{14}\left(\frac{x^2}{2} + \frac{x^3}{3}\right) = \frac{3}{14}\left(\frac{5}{6}\right) = \frac{15}{84} = \frac{5}{28}$

c) compute $F(x)$: cdf

$$F(x) \begin{cases} 0 & x < 0 \\ 3/14\left(\frac{x^2}{2} + \frac{x^3}{2}\right) & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

$x < 0$, $f(x) = \int_{-\infty}^x f(t)\,dt = 0$

$0 \le x \le 2$, $f(x) = \int_{-\infty}^0 f(t)\,dt + \int_0^x f(t)\,dt$

$= \int_{-\infty}^0 0\,dt + \int_0^x \frac{3}{14}(t+t^2)\,dt$

0 ; $\frac{3}{14}\left(\frac{x^2}{2} + \frac{x^3}{3}\right) = 1$

$x > 2$, $f(x) = \int_{-\infty}^x f(t)\,dt =$

$\int_{-\infty}^0 \overset{=0}{f(t)}\,dt + \int_0^2 \overset{=1}{f(t)}\,dt +$

$\int_2^x \overset{=0}{f(t)}\,dt = 1$

FOR ANY CONTINUOUS PROBABILITY:

1. $P(X = c) = 0$ (any single real #) because $\int_c^c dx = 0$
2. $P(X \le c) = P(X < c)$
3. $P(c \le X \le d) = P(c < X \le d) = P(c \le X < d) = P(c < X < d) = \int_c^d f(x)\,dx$

CDF : F(x)

1. $F(x) = P(X \le x)$
2. $0 \le F(x) \le 1$
3. F is non-decreasing
4. F is right-continuous
5. For any $a < b$, $P(a \le X \le b) = F(b) - F(a)$
6. $F'(x) = f(x)$

we can compute the probability if we know the cdf

derivative of cdf = pdf

$\rightarrow = P(X \le d) - P(X \le c)$

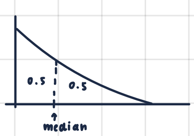
$= \int_{-\infty}^d f(x)\,dx - \int_{-\infty}^c f(x)\,dx = F(d) - F(c)$

$= \int_c^d f(x)\,dx$

EXAMPLE 2. MEDIAN A random variable X is assumed to have pdf

$f(x) = \frac{1}{x^2}$ over $(1, \infty)$

find median (50th percentile) of the distribution



$\int_1^M \frac{1}{x^2}\,dx = 0.5$

$\left[-\frac{1}{x}\right]_1^M = 1 - \frac{1}{M} = 0.5$, $M = 2$

MEAN, VARIANCE, SD

E: $\mu = E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

$E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx$

use short cut formula

V: $V(x) = E[(X-\mu)^2] = E[X^2] - \mu^2$

σ: $\sigma x = \sqrt{V(x)}$

EXAMPLE 3: EVσ find expected value, variance, n SD for $f(x) = 6x - 6x^2$ $[0,1]$

$E(x) = \int_0^1 x(6x - 6x^2)\,dx = \int_0^1 [6x^2 - 6x^3]\,dx = \left[2x^3 - \frac{3}{2}x^4\right]_0^1 = 2 - \frac{3}{2} = \frac{1}{2}$

$V(x) = E(x^2) - \left(\frac{1}{2}\right)^2 \rightarrow \frac{3}{10} - \frac{1}{4} = \frac{2}{40} = \frac{1}{20}$

$E(x^2) = \int_0^1 x^2(6x - 6x^2)\,dx = \int_0^1 [6x^3 - 6x^4]\,dx$

$= \frac{3}{2}x^4 - \frac{6}{5}x^5 \Big|_0^1 = \frac{3}{2} - \frac{6}{5} = \frac{3}{10}$

EXAMPLE 4: E, V, σ ∞

find median, expected value, variance, & SD of

$f(x) = 1/x^2$, $(1, \infty)$

$E(X) = \int_1^\infty x \frac{1}{x^2}\, dx = \int_1^\infty \frac{1}{x}\, dx$ (divergent)

so integral does not exist.

↳ this integral has a median, but no mean, variance, or SD bc the integral D.N.E

THEOREM

IF a & b are real constants, then to find the new random variable:

$$\left.\begin{aligned} E(aX + bY) &= aE(X) + bE(Y) \\ V(aX + bY) &= a^2 V(X) + b^2 V(Y) \\ SD(aX + bY) &= \sqrt{a^2 V(X) + b^2 V(Y)} \end{aligned}\right\} \text{assuming } x \text{ \& } y \text{ are independent}$$

4.2 FAMILIES OF CONTINUOUS DISTRIBUTIONS