Ch.9 INFERENTIAL STATS

SYMBOLS

	PARAMETRIC (POPULATION)	STATISTIC (SAMPLE)
MEAN	μ, μ_1, μ_2	$\bar{x}, \bar{x}_1, \bar{x}_2$
VARIANCE	$\sigma^2, \sigma_1^2, \sigma_2^2$	s^2, s_1^2, s_2^2
SD	$\sigma, \sigma_1, \sigma_2$	s, s_1, s_2
PROPORTION	p, p_1, p_2	$\hat{p}, \hat{p}_1, \hat{p}_2$
SIZE	(none)	n, n_1, n_2

9.1 POINT-ESTIMATE

- a single number. use a number computed from a sample to estimate the parameter

THE TWO METHODS

METHOD OF MOMENT

FIND MOMENT OF POPULATION → $M_1 = E(X)$, $M_2 = E(X^2)$ (POP. MEAN, VARIANCE)

MOMENT OF SAMPLE → $m_1 = \frac{1}{n}\sum X_i = \bar{x}$ mean

$m_2 = \frac{1}{n}\sum X_i^2$ — ALL INDEPENDENT N IDENTICAL DISTRIBUTION

SET THEM EQUAL.

GET EQUATION + SOLVE

METHOD OF MAXIMUM LIKELYHOOD

FIND JOINT PROBABILITY MASS FUNCTION (PMF)

OR JOINT PROBABILITY DENSITY FUNCTION (PDF)

FOR $x = (x_1, x_2, x_3, \dots x_n)$

MAXIMIZE IT BY TAKING DERIVATIVES

SET DERIVATIVE = 0

SIDENOTE 4. VARIANCE:

$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$ in probability chapter 8 ← UNBIASED } referred to as sample variance

$m_2' = \frac{\sum (x_i - \bar{x})^2}{n}$ in THIS chapter ← BIASED } referred to as sample central moment

LOG RECAP

$\ln e^{\square} = \square$

$\ln x^n y^m = \ln x^n + \ln y^m$
$= n \ln x + m \ln y$

$\frac{d}{d\triangle} \ln \triangle = \frac{1}{\triangle}$

CALC RECAP

$\frac{d}{dx} \ln f(x) = 0 \rightarrow x = \text{CRITICAL \#}$

EXAMPLE FOR DISCRETE VARIABLE

X IS A D.V.

X	3	7
P(X)	θ	$1-\theta$

$(x_1, x_2, \dots, x_8) = (7, 3, 3, 7, 3, 7, 7, 7)$ → THESE ARE RANDOM DATA SAMPLE } USE THEM TO:

θ = parameter

GOAL: ESTIMATE θ

· M.O. MOMENT

$E(X) = \sum X_i P(X_i)$

1) 1ST POP. MOMENT $(M_1) = E(X) = 3\theta + 7(1-\theta) = 7 - 4\theta$

2) 1ST SAMPLE MOMENT $= \frac{1}{n}\sum X_i = \bar{x}$

$\bar{x} = \frac{7+3+3+7+3+7+7+7}{8} = 5.5$

SET = TO $\bar{x}$

$7 - 4\theta = \bar{x} \rightarrow \hat{\theta} = \frac{7 - \bar{x}}{4}$ } GENERAL FORMULA

HAT BECAUSE ITS AN ESTIMATE

$= 0.375$

M.O. MAX. LIKELIHOOD

each are independent + identical

$f(x_1, x_2, x_3, \dots, x_8) = f(7, 3, 3, 7, 3, 7, 7, 7)$

JOINT PMF $= (1-\theta)^5 \theta^3$ (7's, 3's)

1) let $f = \theta^3 (1-\theta)^5$

2) take derivative → $\ln f = 3 \ln \theta + 5 \ln (1-\theta)$

$\frac{d}{d\theta} \ln f = \frac{3}{\theta} - \frac{5}{1-\theta}$

3) set = 0

$\frac{3}{\theta} - \frac{5}{1-\theta} = 0$, $\theta = \frac{3}{8}$

$\hat{\theta} = \frac{3}{8} = 0.375$

→ **NOT GUARANTEED BOTH METHOD SAME ANSWER.**

EXAMPLE FOR CONTINUOUS VARIABLE

X HAS A NORMAL DISTRIBUTION W/ UNKNOWN μ + σ

$X \sim$ normal $\quad f(X) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(X-\mu)^2}{2\sigma^2}}$

$X_1, X_2, X_3, \ldots, X_n \sim f(X)$ ← TREAT X_i as individual fixed numbers.

↳ USE THEM TO FIND MEAN + σ

JOINT PDF

WE WANT $f(X_1, X_2, \ldots X_n) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(X_1-\mu)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(X_2-\mu)^2}{2\sigma^2}} \cdot \ldots \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(X_n-\mu)^2}{2\sigma^2}}$

JOINT PDF $= \quad f = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^n e^{-\frac{1}{2\sigma^2}\sum (X_i-\mu)^2}$

take derivative of ln f

M.O. LIKELIHOOD

$$\ln f = -n \ln \sqrt{2\pi} - n \ln \sigma - \frac{1}{2\sigma^2} \sum (X_i - \mu)^2$$

μ + σ are parameters, we find them

we take partial derivative

$$\frac{d}{d\mu} \ln f = \frac{1}{2\sigma^2} \sum 2(X_i - \mu) = \frac{1}{\sigma^2} (\sum X_i - n\mu) = 0$$

$$\mu = \frac{\sum X_i}{n} = \bar{X}, \quad \mu = \bar{X}$$

$$\ln f = -n \ln \sqrt{2\pi} - n \ln \sigma - \frac{1}{2\sigma^2} \sum (X_i - \mu)^2$$

$$\frac{d}{d\sigma} \ln(f) = 0 - \frac{n}{\sigma} + \frac{1}{\sigma^3} \sum (X_i - \mu)^2 = 0$$

$$\sigma^2 = \frac{\sum (X_i - \bar{X})^2}{n}, \quad \hat{\sigma}^2 = \frac{\sum (X_i - \bar{X})^2}{n}$$

M.O. MOMENT

first pop. moment $= E(X) = \mu$

first sample moment $= \frac{\sum X_i}{n} = \bar{X}$

set =

$$\mu = \bar{X}, \quad \hat{\mu} = \bar{X}$$

second pop. central moment $= E(X-\mu)^2 = \sigma^2$

second sample central moment $= \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2$

set =

$$\sigma^2 = \frac{1}{n} \sum (X_i - \bar{x})^2, \quad \hat{\sigma}^2 = \frac{1}{n} \sum (X_i - \bar{X})^2$$