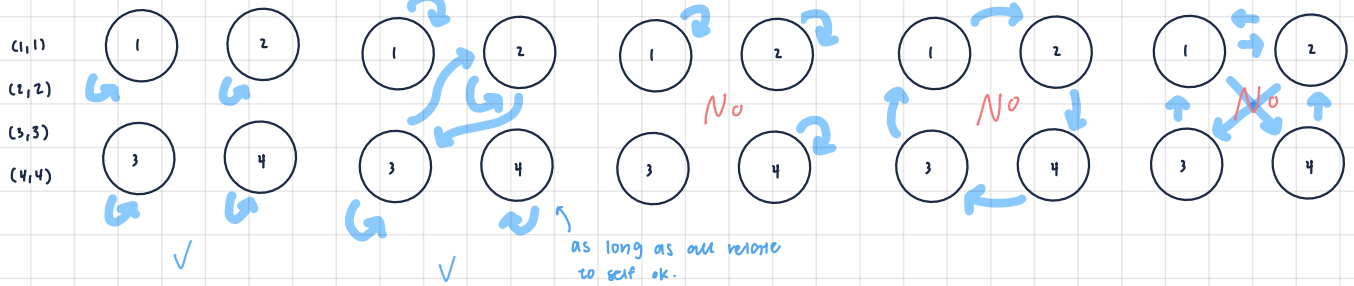# RELATIONS

SPECIAL PROPERTIES wk 2 - 1

↳ reflexive, symmetric, transitive, anti-symmetric

REFLEXIVE

a relation on set A is reflexive if and only if for every element of set A, that element relates to itself.
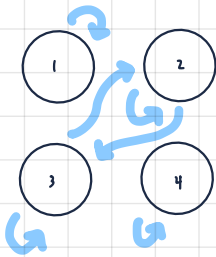
$\forall x \in A \quad x R x$ — means (X,X) is an element of relation R

↳ x relates to itself : (X,X) → gen. example : (1,2) = 1 R 2 = 1 relates to 2

(1,1)
(2,2)
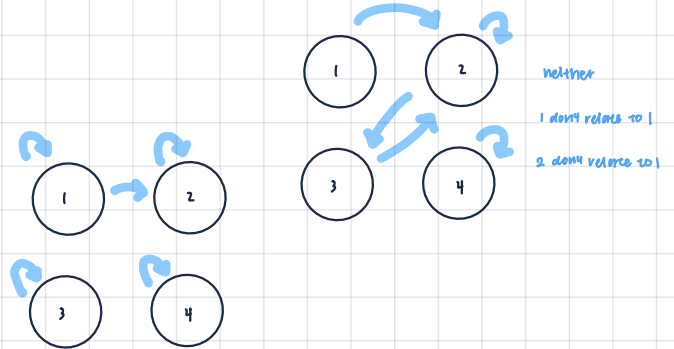(3,3)
(4,4)

No   No   No

✓   ✓

as long as all relate to self ok.

EVERY ELEMENT MUST RELATE TO ITSELF IN ONE STEP!

represent in matrix

diagonal = relate to self = reflexive

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

neither
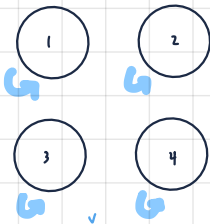1 don't relate to 1
2 don't relate to 1
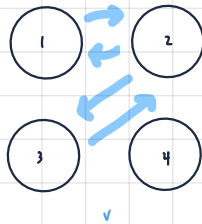
reflexive but not symmetric, 1 R 2 but no 2 R 1

SYMMETRIC

a relation R on set A is symmetric if and only if $\forall x, y \in A \ (x R y \rightarrow y R x)$
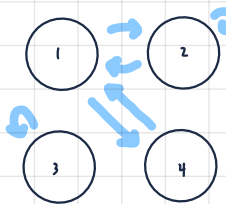
(if (a,b) is in the relation, then so is (b,a))

✓   ✓

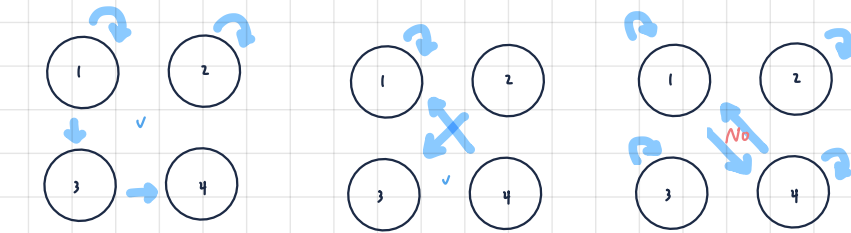reflexive + symmetric   symmetric

represent in matrix

$$\begin{matrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{matrix}$$

it is symmetric!

ANTI·SYMMETRIC

a relation R on set A is anti-symmetric if and only if $\forall x, y \in A \ ((xRy) \wedge (yRx) \rightarrow x = y)$

if (3,4) is in R, then (4,3) isn't → if (a,b) is in the relation, then (b,a) isn't, unless a=b

anti-symmetric   anti-symmetric   NOT ANTI-SYMMETRIC

w/ this example
prove it is anti-symmetric

"NEVER HAVE 2 DIFF ELEMENTS THAT RELATE TO EACH OTHER"
meaning an element can relate to itself

look at

(1,1)	(2,1)	(3,1)	(4,1)
(1,2)	(2,2)	(3,2)	(4,2)
(1,3)	(2,3)	(3,3)	(4,3)
(1,4)	(2,4)	(3,4)	(4,4)

→ plug into $\forall_{x,y} \in A \ ((xRy) \wedge (yRX)) \rightarrow x = y)$

let's do 1,2   1,3

$(1R2 \wedge 2R1) \rightarrow 1 = 2$ $(1R3 \wedge 3R1) \rightarrow 1 = 3$

$(F \wedge F) \rightarrow F$ $T \wedge F \rightarrow F$

$F \rightarrow F = T$ ✓ $F \rightarrow F$ ✓

we are trying to find what violates the formula, there is none in this example

↳ only way for it to be violated is

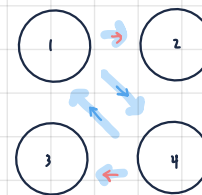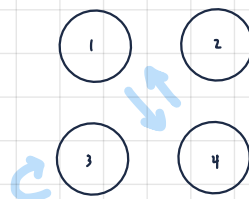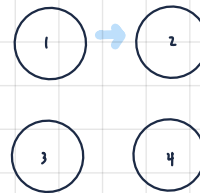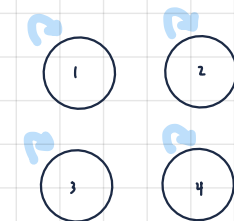

symmetric   ## NOT symmetric

anti-symmetric

not anti-symmetric

not symmetric
not anti-symmetric

[ANTI·SYMMETRIC ≠ NOT SYMMETRIC]