SMOL REVIEW:

- we need the transition probability matrix $P^{(n)}$, got by doing P^n
  - ↳ + the distribution P_n where $P_n = P_0 P^{(n)}$ → will get n probabilities
- steady state distribution : FIND IT BY → raise the matrix to a big #
  - ↳ do the $P^T - I$ matrix

6.3 COUNTING PROCESS

THE BINOMIAL COUNTING PROCESS

USED TO MODEL ARRIVAL

REVIEW

$X \sim$ BINOMIAL($\overset{n}{10}$, $\overset{p}{0.4}$) → KEYWORDS: "trials", "# of successes" / $E(X) = np$ $SD(X) = \sqrt{np(1-p)}$ / X = # of successes over a fixed # of trials

$X \sim$ POISSON (λ) → KEYWORDS : "POISSON", HAS TIME INTERVALS / X = # OF RARE EVENTS PER TIME UNIT

$X \sim$ GEOMETRIC (P) → KEYWORDS. "UNTIL SUCCESS" " HOW MANY TIMES TILL FIRST SUCCESS?" / X = # of trials until 1st success

- BINOMIAL COUNTING PROCESS WILL BE STATED IN PROBLEM
  - OR 2/3 OF Δ, λ, or P WILL BE GIVEN

WHAT IT IS :

it's usually small
↑
frame/ time interval :

- ASSUME THAT BERNOULLI TRIALS OCCUR AT EQUAL TIME INTERVALS , EVERY Δ MINUTES , N TRIALS OCCUR DURING TIME $t = n\Delta$ → $n = t/\Delta$

X represents # of successes during time t, X has a Binomial ($n = \frac{t}{\Delta}$, P)

$E(X) = \frac{t}{\Delta} \cdot P$ = # of success during time t , $SD(X) = \sqrt{nP(1-P)}$ ← REMEMBER $n = t/\Delta$

↳ then $\frac{t}{\Delta} \cdot P = \lambda t$, therefore $\boxed{P = \lambda t}$

λ = frequency , # of successes per time unit

Δ = time frame

P = probability of success

t = total time

- typically will get 2/3 of λ, Δ, or P, find the third using the formulas
  - ↳ λ IS ALMOST ALWAYS GIVEN

MORE DEFINITIONS

NUMBER OF TRIALS BECOMES # OF FRAMES →

$\lambda = P/\Delta$ = arrival rate per time unit

$\Delta = P/t$ = time frame (total # of trials = number of Δ during time t)

$X(t/\Delta)$ = number of arrivals by time t / t/Δ total number of trials

$Y \sim$ Geometric(P) = number of frames until next arrival

$E(Y) = 1/P$, $Var(Y) = \frac{1-P}{P^2}$

$T = Y\Delta$ = INTERARRIVAL TIME BETWEEN ARRIVALS (TIME BETWEEN 2 ARRIVALS)

↳ $E(T) = 1/\lambda$, $Var(T) = \frac{1-P}{\lambda^2}$

QUESTIONS SOUND LIKE :

To use Binomial counting process to model number of arrivals, where Δ needs to be small so that no more than 1 arrival during a Δ

↑ time of arrival is important

EXAMPLES

EXAMPLE 6 :

Tasks are sent to a supercomputer at an average rate of 6 tasks per minute.

Their arrivals are modeled by a Binomial counting process with 2 second frames.

a) compute the probability of more than 2 tasks sent during 10 seconds.

λ = 6 / minute = $6/60$ / second = 0.1 / second

Δ = 2 seconds $P = \lambda\Delta = (0.1)2 = 0.2$

n = total time = $\frac{10}{2 \leftarrow \Delta \text{ (frame)}} = 5$

X = # of arrival during n = 5

$X \sim$ Binomial (n = 5, p = 0.2)

= $P(X > 2) = 1 - P(X \leq 2)$

= 1 − Binomcdf (5, 0.2, 2)

= 0.05792

b) Compute the probability of more than 20 tasks sent during 100 seconds. Do not use approximation.

λ = 0.1 / second $P = \lambda\Delta = (0.1)2 = 0.2$ $n = \frac{100}{2} = 50$

Δ = 2 seconds

$X \sim$ Binomial (50, .2)

$P(X > 20) = 1 - P(X \leq 20)$

= 1 - BINOMIALCDF(50, 0.2, 20)

≈ .000320664

EXAMPLE 7:

Jobs are sent to a printer at the average rate of 2 jobs per minute. Binomial counting process is used to model these jobs.

a) What frame length Δ gives the probability 0.1 of an arrival during any given frame?

$\lambda = 2$ /min $\quad \Delta = p/\lambda = 0.1/2 = 0.05$

$p = 0.1$

b) With this value of Δ, compute the expectation + standard deviation for the number of jobs sent to the printer during a 1-hour period.

\# of successes

means binomial

$E(X) = \frac{t}{\Delta} \cdot p$ $\qquad SD(X) = \sqrt{np(1-p)}$

this is n

$= \frac{60}{.05} \cdot 0.1 \qquad = \sqrt{1200(0.1)(0.9)}$

$= 1200(0.1) = 120 \qquad = 6\sqrt{3} = 10.39230485$

EXAMPLE 8:

On average, every 12 seconds a customer makes a call using a certain phone card. Calls are modeled by a Binomial Counting Process with a 2-second frame. Find the mean n variance for the time, in seconds, between 2 consecutive calls

\# of arrivals

WE DON'T KNOW WHAT DIST. T HAS

HAS GEOMETRIC DIST.

$T = Y\Delta$

BUT

$E(T) = 1/\lambda$, $Var(T) = \frac{1-p}{\lambda^2}$

FORMULAS GIVEN

COME FROM FORMULA

NEED TO FIND λ + p

$\lambda = 1/12$ / second $\qquad \Delta = 2$ seconds (given)

$p = \lambda\Delta = 1/12 \cdot 2 = 1/6$

T = time between 2 arrivals

$E(T) = 1/\lambda = \frac{1}{1/12} = 12$

$Var(T) = \frac{(1-p)}{\lambda^2} = \frac{(1-1/6)}{(1/12)^2} = \frac{5}{6} \cdot \frac{144}{1} = \frac{720}{6} = 120$

EXAMPLE 9:

Messages arrive at a transmission center according to a Binomial counting process with 30 frames per minute. The average arrival rate is 40 messages per hour. Compute the mean + SD of the number of messages arriving between 10 am - 10:30 am

\# of successes

$\lambda = 40/60 = 2/3$ /minute $\qquad p = \frac{2}{3} \cdot \frac{1}{30} = \frac{2}{90} = \frac{1}{45}$ $\qquad E(X) = np = 900 \cdot \frac{1}{45} = 20$

$\Delta = 1/30$ minute $\qquad T = 30$ min (10 - 10:30 AM) $\qquad X$ = # of arrivals between 10 - 10:30 $\qquad SD(X) = \sqrt{np(1-p)} = \sqrt{900(1/45)(44/45)} = \sqrt{\frac{176}{9}}$

$n = T/\Delta = \frac{30}{1/30} = 900 \qquad X \sim$ BINOMIAL($n = 900$, $p = 1/45$)