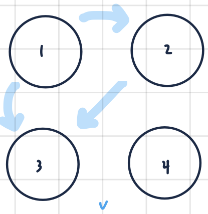
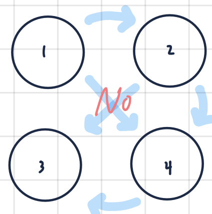
WK 2-2: TRANSITIVE

TRANSITIVE

a relation R on set A is transitive iff $\forall x,y,z \in A\ ((xRy \wedge yRz) \to xRz)$



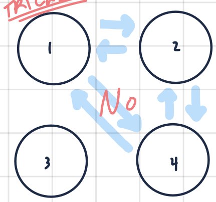
$(1R2 \wedge 2R3) \to 1R3$



No

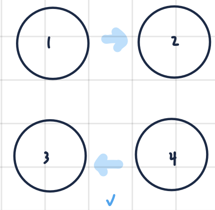
$1R2 \wedge 2R3 \to 1R3$
FALSE

TRICKERY



No

★ $1R2 \wedge 2R1 \to 1R1$
FALSE



there are no values of x, y, z we can plug in to make the implication
x never relate to y + y relates to z

PROVING / DISPROVING A RELATION IS REFLEXIVE/SYMMETRIC/ANTI-SYMMETRIC/TRANSITIVE

QUICK REVIEW:

- reflexivity $\forall x \in A\ xRx$
- symmetry $\forall x,y \in A\ (xRy \to yRx)$
- anti-symmetry $\forall x,y \in A\ ((xRy \wedge yRx) \to x=y)$
- transitive $\forall x,y,z \in A\ ((xRy \wedge yRz) \to xRz)$

★ they're all universally quantified → prove for all, use generic elements
disprove → show one counter-example

Let R be a relation on integers, xRy iff $\underbrace{2x+y \text{ is a multiple of } 3}_{\text{DEFINITION OF R}}$

P/DP REFLEXIVE:

CLAIM: for any element a in A, a R a →

$2a + a$ = multiple of 3

→ we plug in a for both var. in 2x+y bc def. of transitivity is xRx

- let a be a generic element of A

$2a + a = 3a$

a is a multiple of 3 (since a is an integer, 3a is 3 times an integer, so it is a multiple of 3 by def. of multiples)

so a R a by def of R

✓ reflexive

P/DP SYMMETRY:

CLAIM: 4R1 but not 1R4 (plug in as xRy to def. 2x+y)

$2*4 + 1 = 8+1 = 9$ ✓ true
$2*1 + 4 = 2+4 = 6$ ✓ true
} both are mult. of 3

CONTRADICTION! DON'T WORK

CLAIM: R is symmetric

let a and b be generic integers

we need to prove an implication where

$2a+b$ is a multiple of 3 then $2b+a$ is a multiple of 3

★ DIRECT PROOF: we need to prove bRa assuming aRb

assume aRb show bRa

$2a+b$ = a multiple of 3 (by def. of R)

$2a+b = 3X$ (for some integer X, by def. of multiples)

$4a+2b = 6X$ (mult. 2 by both sides)

$2b+a = 6X-3a$ (sub. 3a from both sides)

$2b+a = 3(2X-a)$ (factor out 3)

2X-a is an integer, since X + a are integers, so 2b+a is a mult. of 3

∴ bRa : conclude R is symmetric

P/DP ANTI-SYMMETRY: NEED TO PROVE $\forall x, y \in A\ ((xRy \wedge yRx) \rightarrow x=y)$ is false w/ a counter-example

- we need $(xRy \wedge yRx)$ = true + $x=y$ = false
- we can use $(1,4)$ + $(4,1)$

$2*1 + 4 = 6$, which is a mult. of 3, so $1R4$

$2*4 + 1 = 9$, which is a mult. of 3, so $4R1$

$1R4 \wedge 4R1 \rightarrow 1=4$ is false since $1 \neq 4$

∴ R is not anti-symmetric

* cannot say a relation is anti-symmetric if it is not symmetric!

P/DP TRANSITIVITY: $\forall x, y, z \in A\ ((xRy \wedge yRz) \rightarrow xRz)$

we think it is transitive

let a, b, c be generic integers → need to prove aRc

→ we can only falsify it if aRb, bRc but a is not related to c

$2a + b = 3x$ (for some int x)

$2b + c = 3y$ (for some int y)

$2a + 3b + c = 3x + 3y$ (add ^ those 2 eqs.)

$2a + c = 3x + 3y - 3b$ (minus 3)

$2a + c = 3(x + y - b)$

$2a + c$ = a multiple of 3