Ch. 3 PT2 COMPUTE & INTERPRET EXPECTED VALUE, VARIENCE, & STANDARD DEVIATION OF A RANDOM VARIABLE

EXPECTED VALUE AKA MEAN

OF A RANDOM VARIABLE IS

(could be negative)

$\mu_X = E(X) = \sum_j x_j f(x_j)$ ↳ interpreted as the long run average

use function notation not probability for mean

VARIANCE always positive

$V(X) = \sigma^2 = E[(X-\mu)^2] = \sum (x_j - \mu)^2 f(x_j)$

will always be positive #

SHORT CUT WAY → $V(X) = E[X^2] - \mu^2$
WHEN CALCULATING MANUALLY
(we don't do this one we have calc.)

STANDARD DEVIATION

$SD(X) = \sigma = \sqrt{V(X)}$

- can think of this as the average distance to the mean in the long run
- has same measurement as x

larger SD = win alot, lose a lot

small SD = win not much, lose not much

WILL HAVE THE SAME UNIT!

EXAMPLE.

M	-1	4
P(M)	5/6	1/6

AMOUNT OF $ AFTER GAMBLING

$E(M)$: in the long run do you gain/lose $ & how much $(-1)(\frac{5}{6}) + 4(\frac{1}{6}) = -\frac{5}{6} + \frac{4}{6} = \frac{-1}{6}$

↗ by the end of playing like 1000 times, you lose 1/6

$Var(M)$: $(-1-(-\frac{1}{6}))^2 \cdot \frac{5}{6} + (4-(-\frac{1}{6}))^2 \cdot \frac{1}{6} = \frac{1625}{108}$

$SD(M)$: the risk high or low $\sqrt{\frac{1625}{108}} \approx 3.8789$

two tests could have the same expected value but different standard deviation

CREATING A NEW RANDOM VAR.

+ FINDING ITS EXPECTED VALUE & SD

FIND EXPECTED VALUE + SD FOR A NEW r.v. Y which is a function of the old r.v. X (x_i, $i = 1, 2, \ldots$)

we use: $E(aX) = aE(X)$

$V(aX) = a^2 V(X)$

$SD(aX) = |a| SD(X)$

$E(aX + bY) = aE(X) + bE(Y)$

$V(aX + bY) = a^2(V(X)) + b^2(V(Y))$ if x & y are independent

$SD(aX + bY) = \sqrt{a^2 V(X) + b^2 V(Y)}$

EXAMPLE:

Example 7 Shares of company A costs $10 per share and give a profit of X%. Independently of A, shares of company B cost $50 per share and give a profit of Y%. Deciding how to invest $1000, Mr. W chooses between 3 portfolios:

(a) 100 shares of A
(b) 50 shares of A and 10 shares of B
(c) 20 shares of B.

The distribution of X is given by probabilities
P(X = -3) = 0.3, P(X = 0) = 0.2, P(X = 3) = 0.5.
The distribution of Y is given by probabilities
P(Y = -3) = 0.4, P(Y = 3) = 0.6

Compute expectations and variances of the total profit in dollars generated by portfolios (a), (b), and (c) and compare.

FINDING E, V, SD ON CALCULATOR

L_1	L_2	L_3
1	1/6	$(1-3.5)^2$
2	1/6	$(2-3.5)^2$
3	1/6	$(3-3.5)^2$
4	1/6	$(4-3.5)^2$
5	1/6	$(5-3.5)^2$
6	1/6	$(6-3.5)^2$

① ENTER POSSIBLE VALUES & CORRESPONDING PROBABILITIES TO L_1 + L_2 (in STAT)

② EXPECTED VALUE: LIST → MATH → sum(L_1*L_2) → ENTER

③ VARIANCE: $(L_1 - \mu)^2$ STO L_3, LIST → MATH → SUM(L_2*L_3) → ENTER

μ = E(X)

$E(X) = 3.5$

$V(X) = 2.9166$

$SD \approx 1.7$

solution

x	-3	0	3
P(x)	0.3	0.2	0.5

y	-3	3
P(y)	0.4	0.6

$E(X) = (-3)(0.3) + (3)(0.5) = 0.6$

$V(X) = (-3-0.6)^2 * 0.3 + (0-0.6)^2 * 0.2 + (3-0.6)^2 * 0.5 = 6.84$

a) profit = $100 * 10 * X\% = 10X$ (100 shares, $10 each; if x ⊕ = profit, if x ⊖ = lose $)

$E(Profit) = E(10X) = 10E(X) = 10 * 0.6 = 6$

$V(Profit) = V(10X) = 10^2 V(X) = 100 \cdot 6.84 = 684\$^2$

higher variance higher risk

b) profit $= (50 * 10 * X\%) + (10 * 50 * Y\%) = 5X + 5Y$

$E(Profit) = E(5X + 5Y) = 5E(X) + 5E(Y) = 6$

$V(Profit) = V(5X + 5Y) = 25V(X) + 25V(Y) = 387\$^2$ (5^2 = 25)

lower variance lower risk

c) profit = $(10 * 50 * Y\%) = 5Y$ (10 shares, $50 each)

$E(Profit) = (-3)(.4) + (3)(0.6) = .6$

$V(Profit) = (-3-.6)^2 * .4 + (3-6)^2 * .6 = 8.64$

3.4 SPECIAL DISCRETE PROBABILITY DISTRIBUTIONS

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BERNOULLI DISTRIBUTION

BERNOULLI EXPERIMENT: an experiment that has two possible outcomes, one is called success w/ probability p + the other one is failure w/ probability 1-P

X is called a Bernoulli random variable if

X = 1 if success is observed, X = 0 if failure is observed

$P(1) = p$ $P(0) = 1 - p$

EXAMPLE: Roll a fair die once, success = if "six" is observed and failure = if any other number is observed. X has a Bernoulli distribution with $p = 1/6$

↳ this is Bernoulli based on how you define the outcomes

- success = 6, failure = any other number
  - $1/6$ ↑ (success), $5/6$ ↑ (failure)

- many experiments irl can be defined as Bernoulli

BINOMIAL DISTRIBUTION

since we defined Bernoulli, we can define binomial distribution as a summation of Bernoullis

1) there are a fixed number of bernoulli trials, n

2) trials are repeated under identical situations, and are independent

let X = number of success in n trials, X follows a binomial distribution + is denoted as $X \sim B(n, p)$. $X = 0, 1, 2, \ldots, n$

EXAMPLE: Roll a fair die 10 times in a row, for each trial, success = if "six" is observed and failure = if any other number is observed. X has a Bernoulli distribution of $p = 1/6$ and $n = 10$.

FORMULAS:

$$P(X) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, \ldots n$$

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$$

$E(X) = np$ $V(X) = np(1-p)$ $SD(X) = \sigma_x = \sqrt{np(1-p)} = \sqrt{npq}$

GEOMETRIC DISTRIBUTION

POISSON DISTRIBUTION