WK 9-2
DAY IS 3/29

RECURSIVELY DEFINED SET

CH 5.3

EXAM 2
APRIL 16TH

- structural induction + recursive definitions (5.3)
- inductive review (5.1 + 5.2)
- solving certain types of recurrences (8.2)
- principle of inclusion - exclusion (8.5)
- PERMUTATION / combinatorics (6.3) ← review it
- trees + tree terminology (11.1)

BASIS:

$5 \in S$

↑ some elements that are in the set

RECURSIVE STEP:

generate more elements in the set aside from the existing ones

If $x \in S$, then $5x \in S$ ← nothing else in S

$5 \in S$ → so, $5 \cdot 5 \in S$ → {5, 25}

$25 \cdot 5 \in S$ → {5, 25, 125}

$125 \cdot 5 \in S$ → {5, 25, 125, 625}

$625 \cdot 5 \in S$ → {5, 25, 125, 625, 3125}

⋮

so on from here

what are some elements of S?

25, 125, 10, 35, 20

HOW do we put more elements in S?

1) take existing elements in S

2) plug it into the recursive definition

KEY IDEAS:

- JUST BC WE SHOW THAT EVERYTHING IN A SET HAS A PROPERTY
- THAT DOESN'T MEAN THAT EVERYTHING W/ THAT PROPERTY IS IN THE RECURSIVELY DEFINED SET

AND

((x-1)+1, (y-2)+2) = (x, y)

SO (x-1, y-2) WORKS FOR +1/+2 RULE

CLAIM 1: every element of S is a power of 5

" to show this, we'll first show every element in the basis power of 5. then, we'll show that given a power of 5, the recursive step will produce a power of 5. "

INDUCTION WAY

BASIS:

$5^1 = 5$

INDUCTIVE:

"ASSUME FOR STRUCTURAL INDUCTION: K IS A POWER OF 5 [CRITICAL ASSUMPTION] + $k \in S$ [GARBAGE ASSUMPTION]."

→ ASSUMPTION 1

ASSUMPTION 2 (we didn't even use it)

JUST ASSUMING $k \in S$ + NOT k IS A POWER OF 5 IS AN USELESS ASSUMPTION.

THE CRITICAL ASSUMPTION IS A MUST; BUT THE GARBAGE ASSUMPTION IS UNNECESSARY BUT WILL NOT BREAK THE PROOF.

ASSUMPTION 1

$5k = 5(k) = 5(5^n) = 5^{n+1}$ for some integer n

RESULT OF EXISTENTIAL INSTANTIATION

thus, $5k$ is also power of 5

RECURSIVE WAY

BASIS:

$(0,0) \in S$

RECURSIVE STEP:

if $(a,b) \in S$, then $(a+1, b+1) \in S$ and $(a+1, b+2) \in S$

CLAIM: for any $(x,y) \in S$, $x \le y$

CAN WE SHOW THIS USING STRUCTURAL INDUCTION? LET'S TRY:

BASIS:

$(0,0)$ $0 \le 0$ ✓

RECURSIVE:

ASSUME FOR STRUCTURAL INDUCTION $c \le d$

NEED TO SHOW $(c+1 \le d+1)$ + $(c+1 \le d+2)$

$c \le d$

$c+1 \le d+1$ add 1 to both sides, what we need

$c+1 \le d+1+1$ add 1 onto d-side can do this because the $\le$ is still true.

$c+1 \le d+2$ ✓ what we need

doesn't guarantee just because $4 < 10$, $(4, 10)$ is in the set

ARE ANY OF THESE IN S?

(0,0)	(2,3)	(6,6)	(4,10)	(6,5)	(monkey, monkey)
yes	yes	yes	no	no	no

CLAIM 2: $(x,y) \in S$ iff $x \le y \le 2x$; x, y integers ≥ 0

BASIS:

$(0,0) \in S$

RECURSIVE STEP:

if $(a,b) \in S$, then $(a+1, b+1) \in S$ + $(a+1, b+2) \in S$

NEW CLAIM:

$(x,y) \in S$ iff $x \le y \le 2x$; x, y integers ≥ 0

THIS IS A STRONGER CLAIM → AN IFF ↔ DOUBLE IMPLICATION

LET'S SHOW IF x, y ARE NON-NEGATIVE INTEGERS + $x \le y \le 2x$ then (x,y) is in S.

CASE 1: if $x = y = 0$, then by the basis, $(0,0) \in S$

CASE 2: if $x = y$, $x > 0$

we want to generate (x,y) using the +1/+1 rule

what pair generates (x,y) using that rule?

$(x-1, y-1)$ generates (x,y) using +1/+1

TO USE THIS, WE NEED TO GUARANTEE:

$x-1 \le y-1 \le 2(x-1)$ and x, y are non-negative integers.

CASE 3: IF $x < y$, $x > 0$

shows how you can build (6, 10) from (5,8) → (4,6) → (3,4) → (2,2) → (1,1) → (0,0)

BUILD (x,y) FROM $(x-1, y-2)$ USING +1, +2 RULE

$x-1 \le y-2$ because $x < y$

$y \le 2x$

$y-2 \le 2x-2$ sub 2 both sides

$y-2 \le 2(x-1)$ ✓ factor out 2

= valid pair

EXAMPLES THAT WORK

- say (6, 10) is in S

+1/+1 rule ∵ (5, 9) is in S since (5+1, 9+1) = (6, 10)

+1/+2 rule ∵ (5, 8) is in S since (5+1, 8+2) = (6, 10)

WE'RE TRYING TO FIND HOW EACH ELEMENT, GENERICALLY, OF S HAS BEEN BUILT:

see:

if (x,y) in S

$((x-1)+1, (y-1)+1) = (x,y)$

so $(x-1, y-1)$ works for +1/+1 rule