wk 4-1

★ what are the maximal, greatest, least, minimal elements?

one that all elements relate to → **Greatest:** 0 → any integer a com * c if c = 0. There will always be a solution

★ def. of being greatest / least means being the only maximal / minimal!

Maximal: 0

Least: 1 → relates to every non-negative integer, given non-negative an integer b, 1 | b, since 1 * b = b

(c might not be 1, we choose c = b)

Minimal: 1 because we have one least

1.5 EQUIVALENCE RELATIONS

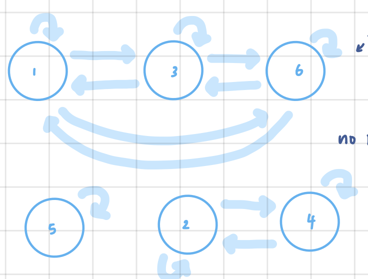
wk 4-1

→ DIVIDES UP ELEMENTS INTO **EQUIVALENCE CLASSES** → a set of elements that has the property that an element in the class is related to all elements in the class n no others

- → two elements are related iff they are in the same equivalence class
- → every element is exactly one equivalence class
- → symmetric

HOW WOULD AN EQUIVALENCE RELATION W/ EQUIVALENCE CLASSES {1, 3, 6}, {2, 4}, and {5} look like?

THIS IS ONE RELATION

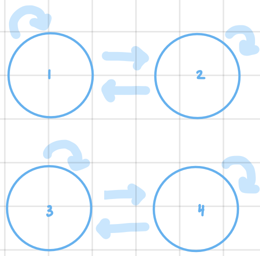


they're in same class, so they relate to each other

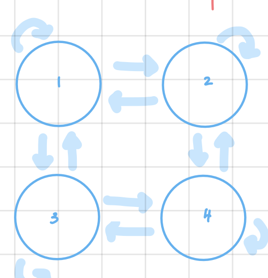
no body in different classes are related to each other

FOR AN EQUIVALENCE CLASS OF n ELEMENTS, THERE ARE n^2 ARROWS

EXAMPLES:



{1, 2}, {3, 4}



NOT EQUIVALENCE RELATION

LACK TRANSITIVITY

WRONG EQUIVALENCE CLASSES:

{1, 3} → element must be in exactly one class, element 2 + 4 are not in a class (violation:)

{1, 2}, {1, 3}, {2, 4}, {3, 4} → element 1 is in 2 classes, element cannot be in more than 1 class.

{1, 2, 3, 4} → 2 doesn't relate to 3, violation

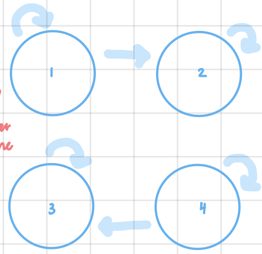
{1, 3}, {2, 4} → 3 can't relate to 4, but they do... violation

are 3 + 2 in the same class?

2R4 should be and 3R4 should be

2+3 SHOULD BE IN THE SAME CLASS

, **all equivalence relations are transitive**



only allowed to relate to other elements in the equivalence class

NOT EQUIVALENCE RELATION

1R2 so 1 + 2 must be in the same class

so 2 should relate to 2

(likewise w/ 3 + 4)

violates symmetry

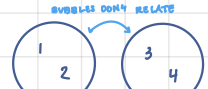
KNOWING EQUIVALENCE RELATIONS, WE SHOULD BE ABLE TO DRAW THE DIAGRAMS

THE EQUIVALENCE RELATION ON {1, 2, 3, 4} W/ JUST 1 equivalence class {1, 2, 3, 4}

communicates the whole relation w/o drawing

→ w/ 2 equivalence classes {1, 2}, {3, 4}

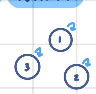
BUBBLES DON'T RELATE



WITHIN EACH BUBBLE, EVERYONE IS RELATED

AS A CONSEQUENCE OF THIS PROPERTY, all equivalence relations are **transitive + symmetric + reflexive**

special case →



} transitive n symmetric n reflexive

ANY RELATION WHICH IS REFLEXIVE, SYMMETRIC, N TRANSITIVE IS AN EQUIVALENCE RELATION

ANY RELATION WHICH IS REFLEXIVE, ANTI - SYMMETRIC, N TRANSITIVE IS A PARTIAL ORDER

LET'S LOOK AT SOME RELATIONS:

let R be a relation on the integers

$a R b$ iff $\left\lfloor \frac{a}{6} \right\rfloor = \left\lfloor \frac{b}{6} \right\rfloor$

floor

is this reflexive?

for any int. a, floor (a/6) = floor (a/6) ✓

is this symmetric?

if floor (a/6) = floor (b/6) then floor (b/6) = floor (a/6) ✓

is it transitive?

if floor (a/6) = floor (b/6) and floor (b/6) = floor (c/6) then floor (a/6) = floor (c/6) ✓

what are the equivalence classes of this relation?

$\ldots \underbrace{\{0,1,2,3,4,5\}}_{\text{floor}(x/6)=0}, \underbrace{\{6,7,8,9,10,11\}}_{=1}, \underbrace{\{12,13,14,15,16,17\}}_{=2} \ldots$

let R be a relation on the integers

$a R b$ iff $(a \bmod 6) = (b \bmod 6)$

$\{\ldots 1, 7, 13 \ldots\}$ congruent to 1 mod 6

$\{\ldots 2, 8, 14 \ldots\}$ 2 mod 6

$\{\ldots 3, 9, 15, \ldots\}$ 3 mod 6

$\vdots$

$a R b$ iff $f(a) = f(b)$ is an equivalence relation for any function $f()$

same reasons as before