MATH 2C03 - Differntial Equations

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Course Outline

- Website: https://ms.mcmaster.ca/lovric/2C3.html
- Textbook: *Elementary Differential Equations with Boundary-Value Problems*
- Course pack must be bought.
- Assignments are online assignments.

Contents

1 Introduction

1.1 Differential equations

Definition 1.1. *Differential equation is an equation where the unknown object is a function or a set of functions, and which involve their derivaties.*

Definition 1.2. An ordinary differential equation (ODE) *is a differential equation involving one variable:* $f'(x)$

Example 1.1.1 (ODE)**.** The following differential equation represents exponential growth:

$$P'(t) = kP(t), k > 0$$

Definition 1.3. *If a function,* $f(x, y, z, \dots)$*, involves more than one variable, we must use partial derivatives, e.g.* $\partial f/\partial x, \partial^2 f/\partial x \partial y, \dots$*. This is called* a partial differential equation (PDE)*.*

Example 1.1.2 (PDE)**.** Let $c(x, t)$ represent the concentration at a location that is x units away from the source at time t. Then, we have

$$c_t(x, t) = Ac_{xx}(x, t)$$

Remark. Note that example 1.1.1 can be rearranged as follows:

$$\frac{P'(t)}{P(t)} = k$$

$P'(t)/P(t)$ represents the *relative rate of change*.

More often, k is not a constant. For example, we can incorporate seasonal variation to the exponential growth model.

Example 1.1.3.

$$\frac{P'(t)}{P(t)} = k \sin(at)$$

Definition 1.4. *If a model does not involve any chance effecs, it's* a deterministic model*. Otherwise, it's* a stochastic model*.*

Example 1.1.4 (Deterministic model)**.** Example 1.1.1 is a deterministic model.

Example 1.1.5 (Stochastic model)**.** Going back to example 1.1.1, we may define k as follows:

$$k = \begin{cases} 0.6 & 35\%\text{chance} \\ 0.5 & 65\%\text{chance} \end{cases}$$

This is a stochastic model.

1.2 Ordinary differential equations

All ordinary differential equations (ODEs) contain the following:

- An independent variable
- Unknown function
- Derivative of the function

In other words, all ordinary equations have the following form:

$$F(x, y, y', y'', \ldots, y^{(n)}) = 0.$$

Example 1.2.1. If $F(s,t) = s^2 + 2t - 4$, then

$$F(x, y) = x^2 + 2y - 4.$$

Note that this function is defined *implicitly.*

Example 1.2.2. If $F(s,t,u) = e^{su} - t^2$, then

$$F(x, y, y') = e^{xy'} = y^2.$$

This is a first order ODE.

Example 1.2.3. If $F(s,t,u,v,w) = w^2 - s + 4u$, then

$$F(x, y, y', y'', y''') = (y''')^2 - xy + 4y' = 0$$

This is a third order ODE.

Definition 1.5 (Order of an ODE)**.** *Order of an ODE is determined by the highest non-zero derivative. If the highest non-zero derivative is the n-th derivative, the ODE also has an order of n.*

Remark. We categorize differential equations because each category requires a different methods to find its solution.