STATS 3U03

Sang Woo Park

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Course Outline

- Textbook: Inroduction to stochastic processes
- Requirement: 5 assignments, 2 tests, and 1 final
- Test 1: Friday, February 10th
- Test 2: Friday, March 17th

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1 Introduction

1.1 Review

Definition 1.1 (Independent random variables)**.** *X and Y are independent iff for any $a, b \in \mathbb{R}$, $P(X \leq a, Y \leq b) = P(X \leq a)P(Y \leq b)$*

1.2 Stochastic processes

Definition 1.2 (Stochastic process)**.** *Let T be a subset of $[0, +\infty]$. For each $t \in T$, let X_t be a random variable. Then, the collection of $\{X_t : t \in T\}$ is called a stochastic process. Simply put, a stochastic process is just a family of random variables.*

Example 1.2.1. Let $T = \{0\}$. Then, $\{X_0\}$ is a stochastic process.

Example 1.2.2. Let $T = \{1, 2, 3, \dots, m\}$ be a set of finite natural numbers. Then, $\{X_1, X_2, X_3, \dots, m\}$ is a stochastic process.

Example 1.2.3. Let $T = \{0, 1, 2, \dots\}$ be a set of all non-negative integers. Then, $\{X_1, X_2, X_3, \dots\}$ is a stochastic process.

Example 1.2.4. Let $T = [0, +\infty)$ be a set of all non-negative real numbers. Then, $\{X_t : t \geq 0\}$ is a stochastic process.

Definition 1.3 (Time index)**.** *Let T be time index. If $T = \{0, 1, 2, \dots\}$, then the time is discrete. If $T = [0, \infty)$, then time is continuous.*

Definition 1.4 (State Space)**.** *State space, S, is the space space where the random variable takes the values.*

Given a sample space, Ω, and time index T, we can define $X_t(w) \in S$, where $w \in \Omega$ and $t \in T$ to describe a stochastic process.

We can further categorize a stochastic process by considering the following two cases: countable and uncountable state space. Time index can also be categorized as follows: discrete and continuous time. Note that each stochastic process must belong to one of the four categories.

Remark. *Every stochastic process can be described by the following three factors:*

1. *Time index*
2. *State space*
3. *Dependence relation*

Example 1.2.5. Let $S = \{0, 1\}$ and $T = \{0, 1, 2, \dots\}$. Given,

$$X_n = \begin{cases} 1 & \text{with probability of } 1/2 \\ 0 & \text{with probability of } 1/2 \end{cases}$$

$\{X_0, X_1, X_2, \dots\}$ is a stochastic process and is often noted as Bernoulli trials.