MATH 3Q03 - Numerical Methods

Sang Woo Park

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Course Outline

- Website: https://ms.mcmaster.ca/ bprotas/MATH3Q03/
- Textbook: *Numerical Mathematics*
- Supplemental references: *Approximation Theory and Approximation Practice*
- Five assignments

Contents

1 Introduction and Review

Definition 1.1 (Numerical analysis)**.** *Numerical analysis is a branch of applied mathematics, studying computational algorithms to solve problems in calculus and analysis (e.g., differentiation, integration, etc.). It focuses on the transforming continuous (infinite dimensional) problems to finite dimensional representations.*

Definition 1.2 (Scientific computing)**.** *Scientific computing focuses on efficient implementation of numerical algorithms on computers; modern computers are able to store data, compare data, and add, subtract, multiply, and divide. It aims to transform problems in calculus and analysis into problems in algebra.*

1.1 Errors in numerical computation

All numerical methods are *approximate*. In most practical problems, solutions need only possess a certain accuracy. Even when a solution is available analytically, its numerical value may only determine with a finite precision.

Numerical analysis seeks to understand and characterize the structure of errors so that approximations can be made arbitrarily accurate by using more computational time and memory. So we are equally interested in efficiency and complexity (number of operations required) of numerical methods (given the size of the problem and the desired accuracy of the solution).

Definition 1.3 (Types of error)**.** *There are four types of errors:*

1. Model error *from simplifying model by making assumptions.*
2. Measurement error *from determining parameters or data with imperfect accuracy.*
3. Truncation error *from approximating infinite sums by finite sums.*
4. Roundoff error *from representing real numbers with a finite number of bits (floating point numbers).*

Example 1.1.1 (Truncation error)**.** In theory, we would have to use infinitely many operation to compute e^x:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

However, we may approximate this sum by using a finite sum,

$$e^x \approx \sum_{n=0}^{k} \frac{x^n}{n!},$$

yielding a truncation error of $\varepsilon = \sum_{n=k+1}^{\infty} (x^n/n!)$. Note that $\varepsilon \to 0$ as $k \to \infty$.

Example 1.1.2 (Roundoff error)**.** Let a and b be adjacent floating point numbers. Real numbers $x \in (a, b)$ will be represented necessarily either by a or by b (rounding).

2 Interpolation

3 Approximation

4 Numerical Differentiation and Integration

5 Special Topics