

## Homework Assignment #3

Posted on Sunday, 4/17/2016.  
Due 10PM, Monday, 4/25/2016.

1. (25 points) Exercise 30.2-5. Modify this exercise by making  $n$  a power of 5.
2. (15 points) Exercise 30.1-7. Generalize this exercise by replacing two sets  $A$  and  $B$  with three sets  $X$ ,  $Y$ , and  $Z$ .
3. (10 points) Exercise 30.1-3.
4. (15 points) Exercise 4.3-6. Modify this exercise by replacing 17 with the ceiling of  $\log n$  and replacing big  $O$  with big  $\Theta$ .
5. (15 points) Exercise 4.2-4. Modify this exercise by replacing  $3 \times 3$  with  $5 \times 5$ . Prove your answer.
6. (20 points) Assume you have an array  $X[1..n]$  of  $n$  elements. A majority element of  $X$  is defined to be an element occurring in more than  $n/2$  positions (e.g., if  $n = 6$  or  $n = 7$ , a majority element will occur in at least 4 positions). Assume that elements cannot be ordered or sorted, but can be compared for equality. (You might think of the elements as chips, and there is a tester that can be used to determine whether or not two chips are identical.)
  - a) (10 points) Design an algorithm to find a majority element in  $X$  or determine that no majority element exists. The time complexity of your algorithm should be  $O(n \log n)$ .
  - b) (10 points) Design an algorithm to find a majority element in  $X$  or determine that no majority element exists. The time complexity of your algorithm should be  $O(n)$ .

Prove the correctness and time complexity of each of your algorithms.