Homework Assignment #5

Posted on Sunday, 5/1/2016.
Due 10PM, Monday, 5/9/2016.

1. (20 points) You are making a plan to drive from Chicago to Miami. Your gas tank, when full, holds enough gas to travel m miles. You have a map that gives distances between gas stations along the route. Let $0 = d_1 < d_2 < \ldots < d_n$ be the locations of all the gas stations along the route where d_i is the distance from the 1^{st} gas station to the i-th gas station. You will drive from location 0 to location d_n. The distance between two neighboring gas stations is at most m miles. Your goal is to make as few gas stops as possible along the way.

   Give the most efficient algorithm you can to determine at which gas stations you should stop. Prove that your algorithm yields an optimal solution. Give the time complexity of your algorithm as a function of n. Prove the time complexity of your algorithm.

2. (20 points) Modify the fractional knapsack problem as follows. Instead of only one knapsack, there are two, each with a specified capacity. The two capacities may be equal or different. Each item can be taken fractionally but cannot be split between the two knapsacks. For instance, if 3/4 of an item is put into one knapsack, the remaining 1/4 will be discarded.

   Will any of the following greedy algorithms always produce an optimal solution? In each case, an order of the items is specified. The algorithm includes the items one at a time in that order as much as possible into the knapsack with the larger available capacity. When the two knapsacks have the same available capacity, the algorithm can use either.

   Prove the optimality of each algorithm below or disprove with a counter-example:

   (a) increasing order of weight
   (b) decreasing order of value
   (c) decreasing order of value to weight ratio

3. (20 points) Exercise 16.3-7. Modify this problem by changing ternary codewords to 6-ary codewords, i.e., codewords using the symbols 0, 1, 2, 3, 4, 5.

4. (20 points) Exercise 16.2-7.

5. (20 points) Problem 16-1.