Assignment 6

5.1 Computer Problems: 1

Q:

Make a table of the error of the three-point centered-difference formula for f (0), where f (x) = sin x − cos x, with h = 10−1,...,10−12, as in the table in Section 5.1.2. Draw a plot of the results. Does the minimum error correspond to the theoretical expectation?

A:

code:

```
format long
table = zeros(12,3);
for i = 1:12
    h = 10^(-i);
    table(i,1) = h;
    table(i,2) = ((sin(h)-cos(h))-(sin(-h)-cos(-h)))/(2*h);
    table(i,3) = cos(0)+sin(0)-table(i,2);
end
table
```

results:

```
table =

0.100000000000000   0.998334166468282   0.001665833531718
0.010000000000000   0.999983333416665   0.000016666583335
0.001000000000000   0.999999833333376   0.000000166666624
0.000100000000000   0.999999998332890   0.000000001667110
0.000010000000000   0.999999999984347   0.000000000015653
0.000001000000000   0.999999999973245   0.000000000026755
0.000000100000000   0.999999999473644   0.000000000526356
0.000000010000000   0.999999999473644   0.000000000526356
0.000000001000000   1.000000027229220  -0.000000027229220
0.000000000100000   1.000000082740371  -0.000000082740371
0.000000000010000   1.000000082740371  -0.000000082740371
0.000000000001000   1.000033389431110  -0.000033389431110
```

The minimum error is 0.000000000015653 when $h = 10^{-5}$. It corresponds to the theoretical expectation.

5.2 Computer Problems: 1(a, c)

Q:

Use the composite Trapezoid Rule with m = 16 and 32 panels to approximate the definite integral. Compare with the correct integral and report the two errors.

$$(a) \int_0^4 \frac{xdx}{\sqrt{x^2+9}}$$

$$(c) \int_0^1 xe^x dx$$

A:

(a)

code:

```
h = 1/4;
summ = 0;
for i=1:15
    summ = summ+i*h/((i*h)^2+9)^(0.5);
end
summ = 2*summ + 4/5;
0.5*h*summ
```

results:

```
ans =

    1.998638181470279
```

The correct integral is 2, the error is $1.36 * 10^{-3}$.

code:

```
h = 1/8;
summ = 0;
for i=1:31
    summ = summ+i*h/((i*h)^2+9)^(0.5);
end
summ = 2*summ + 4/5;
0.5*h*summ
```

results:

```
ans =

    1.999659678077911
```

The correct integral is 2, the error is $3.40 * 10^{-4}$.

(c)

code:

```
h = 1/16;
summ = 0;
for i=1:15
    summ = summ+i*h*exp(i*h);
end
summ = 2*summ + exp(1);
0.5*h*summ
```

results:

```
ans =

    1.001444027067708
```

The correct integral is 1, the error is $1.44 * 10^{-3}$.

code:

```
h = 1/32;
summ = 0;
for i=1:31
    summ = summ+i*h*exp(i*h);
end
summ = 2*summ + exp(1);
0.5*h*summ
```

results:

```
ans =

    1.000361038046700
```

The correct integral is 1, the error is $3.61 * 10^{-4}$.

5.5 Exercises: 3(a)

Q:

Approximate the integrals in Exercise 1, using n = 4 Gaussian Quadrature, and give the error.

$$(a) \int_{-1}^{1} (x^3 + 2x)$$

A:

$$\int_{-1}^{1} f(x)dx \approx c_1 f(x_1) + c_2 f(x_2) + c_3 f(x_3) + c_4 f(x_4)$$

$x_1 = -x_4$
$x_2 = -x_3$
$\because$ f(x) is an odd function
$\therefore f(x_1) = -f(x_4), f(x_2) = -f(x_3)$
$\because c_1 = c_4, c_2 = c_3$
$\therefore c_1 f(x_1) + c_2 f(x_2) + c_3 f(x_3) + c_4 f(x_4) = 0$
$\therefore \int_{-1}^{1} f(x)dx \approx 0$, the error is 0.