Electre

In: Multiple Attribute Decision Making

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The ELECTRE (Elimination et choix traduisant la réalité) method originated from Roy (1971) in the late 1960s. Since then Nijkamp and van Delft (1977) and Voogd (1983) have developed this method to its present state. The method dichotomizes preferred alternatives and nonpreferred ones by establishing outranking relationships. This method is most popular in Europe, especially among the French-speaking community.

6.1. Outranking Relationships

A greatly simplified car selection problem is employed to introduce the concept of an outranking relationship. A DM considers three cars based on two attributes: purchase price and gasoline efficiency. The three alternatives considered are A_1 = ($13,000, 18 MPG), A_2 = ($16,000, 21 MPG), and A_3 = ($18,000, 25 MPG). These alternatives nondominate each other because none of them excels the others in both attributes. Although we (as decision analysts) are not yet entitled to determine a preference between nondominated alternatives, a prior “likely preferred to” relationship might exist in the DM's mind. For instance, the DM is likely to prefer A, to A_2 because he or she does not want to pay an extra $3,000 for a gas mileage improvement of 3 MPG. Therefore, it is defined that A_1 outranks A_2. Furthermore, by the same token he or she may assess that A_2 outranks A_3. What then is the relationship between A_1 and A_3? Because these two alternatives are too separate, the DM may be hard put to assess a preference between them. Contrary to his or her previous judgments, the DM might assess that A_3 outranks A_1, which is quite acceptable, since human choice does not have to be logically transitive.

An outranking relationship can be concisely expressed by employing the binary relationship R. The notation (A_p R A_q) or ($A_p \rightarrow A_q$) (more useful in a directed graph) means that A_p outranks A_q. For example, the three assessments above are denoted as (A_1 R A_2), (A_2 R A_3), and (A_3 R A_1). Formally, an outranking relationship of (A_p R A_q) states that even though two alternatives A_p and A_q do not dominate each other, it is realistic to accept the risk of regarding A_p as almost surely better than A_q. Accordingly, the outranking relationship R is not required to be transitive. That is, (A_1 R A_2) and (A_2 R A_3) do not necessarily imply (A_1 R A_3). Such an outranking relationship is both ambiguous and practical. The outranking relationship in ELECTRE, however, is determined in an objective fashion by simultaneously employing concordance and discordance indexes.

6.2. Preferred Alternatives

Because the outranking relationship is not transitive, we may not eliminate A_q at once even when we have a relationship of (A_p R A_q) or ($A_p \rightarrow A_q$). We should consider all R relationships in a given problem and eliminate nonpreferred alternatives based on the overall dominance structure. For example, nine outranking relationships from eight alternatives are given: ($A_1 \rightarrow A_7$), ($A_2 \rightarrow A_3$), ($A_3 \rightarrow A_8$), ($A_4 \rightarrow A_2$), ($A_5 \rightarrow A_4$), ($A_5 \rightarrow A_7$), ($A_6 \rightarrow A_3$), ($A_7 \rightarrow A_4$), and ($A_8 \rightarrow A_6$). Figure 6.1 shows a digraph that represents the above nine outranking relationships all together. In this digraph each node represents an alternative. When a directed path begins in a node and comes back to this very node, we call the path a cycle. All nodes in a cycle are considered to have an equivalent preference. We then construct an acyclic digraph by combining the nodes in a cycle into a single node. We notice a cycle of $A_3 \rightarrow A_8 \rightarrow A_6 \rightarrow A_3$ in Figure 6.1.

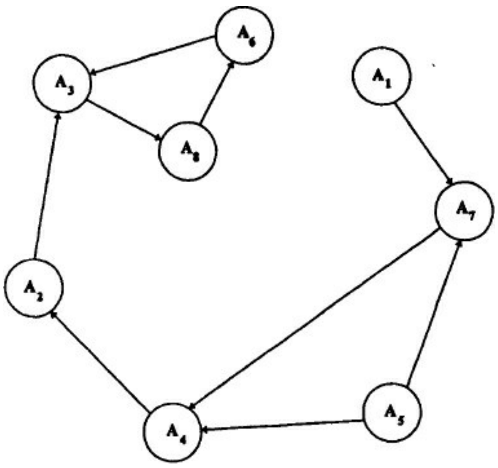

Figure 6.1. A Digraph for Eight Alternatives

The kernel (or core) of an acyclic digraph is a reduced set of nodes that is preferred to the set of nodes that do not belong to the kernel. In other words the kernel is a set of preferred alternatives defined by ELECTRE. The kernel K should satisfy the following two conditions:

1. Each node in K is not outranked by any other node in K.
2. Every node *not* in K is outranked by at least one node in K.

The digraph shown in Figure 6.1 is considered to illustrate the kernel identification process. First, we choose any nodes that have no entering arrows; these are A_1 and A_5. Then we add any alternatives that satisfy the above conditions. Only alternative A_2 can be added. Figure 6.2

shows the kernel of eight alternatives. The set of preferred alternatives defined by the kernel is K = {A_1, A_2, A_5}.

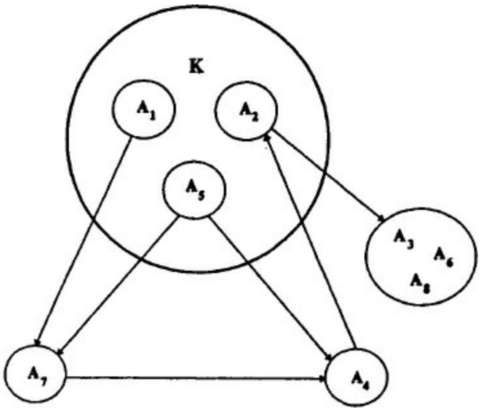

Figure 6.2. The Kernel of Figure 6.1

6.3. ELECTRE

The ELECTRE method formulates concordance and discordance indexes in order to obtain outranking relationships, then renders a set of preferred alternatives by forming a kernel. Concordance and discordance indexes can be viewed as measurements of satisfaction and dissatisfaction that a DM feels on choosing one alternative over the other. The method is illustrated by the following case.

Case 6.1. Budget Reduction Decision for Athletic Programs. This case was described in detail in Chapter 1, and its decision matrix was presented in Table 1.1. The "miscellaneous impact" attribute utilized a five-point scale ranging from *very low* (5) to *very high* (1) with *very low* being the best.

Step 1. Normalization. Attributes X_2 and X_3 are of benefit criterion (the more, the better) but attribute X_1 is of cost. Therefore, the values of X_1 (30, 29, 12) are inverted (1/30, 1/29, 1/12) in order to transform this attribute to a benefit one. Since each attribute has different measurement scales, a normalization is necessary to make their values comparable. The comparable ratings by vector normalization are given below:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} X_1 & X_2 & X_3 \end{array} \\ \begin{bmatrix} 0.3466 & 0.9126 & 0.4243 \\ 0.3587 & 0.3914 & 0.5657 \\ 0.8667 & 0.1179 & 0.7071 \end{bmatrix} \end{array}$$

where element r_{21} = 0.3587 was obtained from

$$(1/29)/\sqrt{(1/30)^2+(1/29)^2+(1/12)^2},$$

and r_{33} = 0.7071 was from $5/\sqrt{3^2+4^2+5^2}$.

Step 2. Weighted Normalization. The set of attribute weights for the Athletic Department problem were developed using the personal assessments of the department leaders. The number of people directly affected by dropping each sport was given a weight of 20%. The most important category, Athletic Department money saved, was weighted at 70%. The third attribute, miscellaneous, was weighted at 10%. These weights (w_1, w_2, w_3) = (0.2, 0.7, 0.1) are multiplied with each column of the normalized rating matrix. The weighted normalized ratings v_{ij} are computed as

$$\begin{array}{c c} & \begin{array}{ccc} X_1 & X_2 & X_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 0.0693 & 0.6388 & 0.0424 \\ 0.0717 & 0.2740 & 0.0566 \\ 0.1733 & 0.0825 & 0.0707 \end{bmatrix} \end{array}$$

Step 3. Concordance and Discordance Sets. For each pair of alternatives A_p and A_q (p, q = 1, 2, …, m and $p \neq q$), the set of attributes is divided into two distinct subsets. The concordance set, which is composed of all attributes for which alternative A_p is preferred to alternative A_q, can be written as

$$C(p, q) = \{j \mid v_{pj} \geq v_{qj}\}$$

where v_{pj} is the weighted normalized rating of alternative A_p with respect to the jth attribute. In other words, $C(p, q)$ is the collection of attributes where A_p is better than or equal to A_q.

The complement of $C(p, q)$, which is called the discordance set, contains all attributes for which A_p is worse than A_q. This can be written as

$$D(p, q) = \{j \mid v_{pj} < v_{qj}\}.$$

Note that $C(p, q)$ is not equal to $D(q, p)$ when tied ratings exist.

The concordance and discordance sets for the Athletic Department problem are obtained as

$C(1, 2) = \{2\}$ $D(1, 2) = \{1, 3\}$

$C(\mathrm{I}, 3)= \{2\}$ $D(\mathrm{I}, 3) = \{1, 3\}$

$C(2, 1) = \{1.3\}$ $D(2, 1) = \{2\}$

$C(2, 3) = \{2\}$ $D(2, 3)=\{1, 3\}$

$C(3, 1) = \{1, 3\}$ $D(3. 1) = \{2\}$

$C(3, 2) = \{1, 3\}$ $D(3, 2) = \{2\}$.

Step 4. Concordance and Discordance Indexes. The relative power of each concordance set is measured by means of the concordance index. The concordance index C_{pq} represents the degree of confidence in the pairwise judgments of ($A_p \rightarrow A_q$). The concordance index of $C(p, q)$ is defined as

$$C_{pq} = \sum_{j^*} w_{j^*}$$

where j^* are attributes contained in the concordance set $C(p, q)$.

The discordance index, on the other hand, measures the power of $D(p, q)$. The discordance index of $D(p, q)$, which represents the degree of disagreement in ($A_p \rightarrow A_q$), can be defined as

$$D_{pq} = (\sum_{j^*} |v_{pj^*} - v_{qj^*}|)/(\sum_{j} |v_{pj} - v_{qj}|)$$

where j^* are attributes that are contained in the discordance set $D(p, q)$.

For the given problem, a concordance index C_{21} is obtained as follows: Since $C(2, 1) = \{1, 3\}$, $C_{21} = \Sigma w^*_j = w_1 + w_3 = 0.2 + 0.1 = 0.3$. A discordance index D21 is calculated as follows: Since $D(2, 1) = \{2\}$, the formula for D_{21} takes the form:

$$D_{21} = (\sum |v_{22} - v_{12}|)/(\sum_{j=1}^{3} |v_{2j} - v_{1j}|) = 0.9565.$$

The complete list of concordance and discordance indexes is as follows:

$C_{12} = 0.7$ $D_{12} = 0.0435$

$C_{13} = 0.7$ $D_{13} = 0.1921$

$C_{21} = 0.3$ $D_{21} = 0.9565$

$C_{23} = 0.7$ $D_{23} = 0.3766$

$C_{31} = 0.3$ $D_{31} = 0.8079$

$C_{32} = 0.3$ $D_{32} = 0.6234$.

Step 5. Outranking Relationships. The dominance relationship of alternative A_p over alternative A_q becomes stronger with a higher concordance index C_{pq} and a lower discordance index D_{pq}. The method defines that A_p outranks A_q when $C_{pq} \geq \bar{\boldsymbol{C}}$ and $D_{pq} < \bar{\boldsymbol{D}}$, where $\bar{\boldsymbol{C}}$ and $\bar{\boldsymbol{D}}$ are the averages of C_{pq} and D_{pq}, respectively.

For the given problem, $\bar{C}$ = (0.7 + 0.7 +… + 0.3)/6 = 0.5 and $\bar{D}$ = (0.0435 + 0.1921 + … + 0.6234)/6 = 0.5.

TABLE 6.1
Determination of Outranking Relationship

C_{pq}	*Is* $(C_{pq} \geq \bar{C})$?	D_{pq}	*Is* $(D_{pq} < \bar{D})$?	*Is* $(A_p \rightarrow A_q)$?
C_{12}	Yes	D_{12}	Yes	Yes
C_{13}	Yes	D_{13}	Yes	Yes
C_{21}	No	D_{21}	No	No
C_{23}	Yes	D_{23}	Yes	Yes
C_{31}	No	D_{31}	No	No
C_{32}	No	D_{32}	No	No

Table 6.1 illustrates the determination of outranking relationships. Three outranking relationships are obtained: ($A_1 \rightarrow A_2$), ($A_1 \rightarrow A_3$), and ($A_2 \rightarrow A_3$). Only alternative A_1 remains in the kernel, which makes A_1 the optimal choice. The Athletic Department did in fact eliminate the ski program (i.e., A_1), for which it received harsh criticism. However, by ELECTRE analysis we determined that dropping the ski program was indeed the best choice and that criticism of the Athletic Department was unfounded.

6.4. Complementary Analysis

A weakness of ELECTRE might lie in its use of the critical threshold values $\bar{C}$ and $\bar{D}$. These values are rather arbitrary, although their impact upon the ultimate result may be significant. We also notice that ELECTRE does not indicate a preference among nodes (alternatives) in the kernel K. The net outranking relationship is introduced to address these problems. First, complementary ELECTRE defines the net concordance index C_p, which measures the degree to which the dominance of alternative A_p over competing alternatives exceeds the dominance of competing alternatives over A_p. Similarly, the net discordance index D_p measures the relative weakness of alternative A_p with respect to other alternatives. These net indexes are mathematically denoted as

$$C_p = \sum_{\substack{k=1 \\ k \neq p}}^{m} C_{pk} - \sum_{\substack{k=1 \\ k \neq p}}^{m} C_{kp}$$

$$D_p = \sum_{\substack{k=1 \\ k \neq p}}^{m} D_{pk} - \sum_{\substack{k=1 \\ k \neq p}}^{m} D_{kp}.$$

Obviously, an alternative A_p has a greater preference with a higher C_p and a lower D_p. Hence the final selection should satisfy the condition that its net concordance index should be at a maximum and its net discordance index at a minimum. If both these conditions are not satisfied, the alternative that scores the highest average rank can be selected as the final solution.

For the Athletic Department problem, the net concordance and discordance indexes are

$C_1 = 0.8$ $D_1 = -1.53$

$C_2 = 0.0$ $D_2 = 0.67$

$C_3 = -0.8$ $D_3 = 0.86$

where, for example, they are calculated by

$C_1 = (C_{12} + C_{13}) - (C_{21} + C_{31}) = 0.8$

$D_3 = (D_{31} + D_{32}) - (D_{13} + D_{23}) = 0.86.$

The preference ranks based on the net concordance index and the net discordance index in this case are identical: [A_1, A_2, A_3] where A_1 is the first rank and A_3 is the last.

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