# The MOSEK Python optimizer API manual Version 7.1 (Revision 32)

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# License agreement

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### Chapter 1

# Changes and new features in MOSEK

The section presents improvements and new features added to MOSEK in version 7.

#### 1.1 Platform support

In Table 1.1 the supported platform and compiler used to build MOSEK shown. Although RedHat is explicitly mentioned as the supported Linux distribution then MOSEK will work on most other variants of Linux. However, the license manager tools requires Linux Standard Base 3 or newer is installed.

#### 1.2 General changes

- The interior-point optimizer has been extended to semi-definite optimization problems. Hence, MOSEK can optimize over the positive semi-definite cone.
- The network detection has been completely redesigned. MOSEK no longer try detect partial networks. The problem must be a pure primal network for the network optimizer to be used.
- The parameter iparam.objective\_sense has been removed.
- The parameter iparam.intpnt\_num\_threads has been removed. Use the parameter iparam.num\_threads instead.
- MOSEK now automatically exploit multiple CPUs i.e. the parameter iparam.num\_threads is set to 0 be default. Note the amount memory that MOSEK uses grows with the number of threads employed.

Platform	OS version	C compiler
linux32x86	Redhat 5 or newer (LSB 3+)	Intel C 13.0 (gcc 4.3, glibc 2.3.4)
linux64x86	RedHat 5 or newer (LSB 3+)	Intel C 13.0 (gcc 4.3, glibc 2.3.4)
osx64x86	OSX 10.7 Lion or newer	Intel C 13.0 (llvm-gcc-4.2)
win32x86	Windows Vista, Server 2003 or newer	Intel C 13.0 (VS 2008)
win64x86	Windows Vista, Server 2003 or newer	Intel C 13.0 (VS 2008)

Interface	Supported versions
Java	Sun Java 1.6+
Microsoft.NET	2.1+
Python 2	2.6+
Python 3	3.1+

Table 1.1: Supported platforms

- The MBT file format has been replaced by a new task format. The new format supports semi-definite optimization.
- the HTML version of the documentation is no longer included in the downloads to save space. It is still available online.
- MOSEK is more restrictive about the allowed names on variables etc. This is in particular the case when writing LP files.
- MOSEK no longer tries to detect the cache sizes and is in general less sensitive to the hardware.
- The parameter is set iparam.auto\_update\_sol\_info is default off. In previous version it was by default on.
- The function relaxprimal has been deprecated and replaced by the function primalrepair.

#### 1.3 Optimizers

#### 1.3.1 Interior point optimizer

The factorization routines employed by the interior-point optimizer for linear and conic optimization problems has been completely rewritten. In particular the dense column detection and handling is improved. The factorization routine will also exploit vendor tuned BLAS routines.

#### 1.3.2 The simplex optimizers

• No major changes.

1.4. API CHANGES

#### 1.3.3 Mixed-integer optimizer

• A new mixed-integer for linear and conic problems has been introduced. It is from run-to-run determinitic and is parallelized. It is particular suitable for conic problems.

#### 1.4 API changes

- Added support for semidefinite optimization.
- Some clean up has been performed implying some functions have been renamed.

#### 1.5 Optimization toolbox for MATLAB

- A MOSEK equivalent of bintprog has been introduced.
- The functionality of the MOSEK version of linprog has been improved. It is now possible to employ the simplex optimizer in linprog.
- mosekopt now accepts a dense A matrix.
- An new method for specification of cones that is more efficient when the problem has many cones has introduced. The old method is still allowed but is deprecated.
- Support for semidefinite optimization problems has been added to the toolbox.

#### 1.6 License system

• Flexlm has been upgraded to version 11.11.

#### 1.7 Other changes

• The documentation has been improved.

#### 1.8 Interfaces

- Semi-definite optimization capabilities have been add to the optimizer APIs.
- A major clean up have occured in the optimizer APIs. This should have little effect for most users.
- A new object orientated interface called Fusion has been added. Fusion is available Java, MAT-LAB, .NET and Python.
- The AMPL command line tool has been updated to the latest version.

#### 1.9 Platform changes

- 32 bit MAC OSX on Intel x86 (osx32x86) is no longer supported.
- 32 and 64 bit Solaris on Intel x86 (solaris32x86,solaris64x86) is no longer supported.

#### 1.10 Summary of API changes

#### 1.10.1 Parameters

- dparam.callback\_freq removed.
- dparam.mio\_tol\_max\_cut\_frac\_rhs added.
- dparam.mio\_tol\_min\_cut\_frac\_rhs added.
- dparam.mio\_tol\_rel\_dual\_bound\_improvement added.
- dparam.presolve\_tol\_abs\_lindep added.
- dparam.presolve\_tol\_lin\_dep removed.
- dparam.presolve\_tol\_rel\_lindep added.
- iparam.bi\_clean\_optimizer Valid parameter values changed.
- iparam.cache\_size\_11 removed.
- iparam.cache\_size\_12 removed.
- iparam.check\_task\_data removed.
- iparam.cpu\_type removed.
- iparam.data\_check removed.
- iparam.intpnt\_basis Valid parameter values changed.
- iparam.intpnt\_num\_threads removed.
- iparam.intpnt\_order\_method Valid parameter values changed.
- $\bullet \ \ iparam.license\_allow\_overuse \ \mathrm{removed}.$
- iparam.license\_cache\_time removed.
- iparam.license\_check\_time removed.
- iparam.log\_expand added.
- iparam.log\_feasrepair removed.

- iparam.log\_feas\_repair added.
- iparam.lp\_write\_ignore\_incompatible\_items removed.
- iparam.mio\_cut\_cg added.
- iparam.mio\_cut\_cmir added.
- iparam.mio\_node\_optimizer Valid parameter values changed.
- iparam.mio\_probing\_level added.
- iparam.mio\_rins\_max\_nodes added.
- iparam.mio\_root\_optimizer Valid parameter values changed.
- iparam.mio\_use\_multithreaded\_optimizer added.
- iparam.num\_threads added.
- iparam.objective\_sense removed.
- iparam.optimizer Valid parameter values changed.
- iparam.presolve\_lindep\_abs\_work\_trh added.
- iparam.presolve\_lindep\_rel\_work\_trh added.
- iparam.presolve\_lindep\_work\_lim removed.
- iparam.presolve\_max\_num\_reductions added.
- iparam.read\_add\_anz removed.
- iparam.read\_add\_con removed.
- iparam.read\_add\_cone removed.
- iparam.read\_add\_qnz removed.
- iparam.read\_add\_var removed.
- iparam.read\_mps\_quoted\_names removed.
- iparam.read\_q\_mode removed.
- iparam.sim\_network\_detect removed.
- iparam.sim\_network\_detect\_hotstart removed.
- iparam.sim\_network\_detect\_method removed.
- iparam.sol\_quoted\_names removed.
- iparam.write\_mps\_obj\_sense removed.
- iparam.write\_mps\_quoted\_names removed.
- iparam.write\_mps\_strict removed.
- sparam.mio\_debug\_string added.

#### 1.10.2 Functions

- Env.axpy added.
- Env.dot added.
- Env.gemm added.
- Env.gemv added.
- Env.initenv removed.
- Env.licensecleanup added.
- Env.potrf added.
- Env.putcpudefaults removed.
- Env.putlicensedebug added.
- Env.putlicensedefaults removed.
- Env.putlicensepath added.
- Env.putlicensewait added.
- Env.syeig added.
- Env.syevd added.
- Env.syrk added.
- Task.append removed.
- Task.appendbarvars added.
- Task.appendcons changed.
- Task.appendsparsesymmat added.
- Task.appendvars changed.
- Task.checkdata removed.
- Task.core\_append removed.
- Task.core\_appendcones removed.
- Task.core\_removecones removed.
- Task.getaslicetrip removed.
- Task.getavec removed.
- Task.getavecnumnz removed.

- Task.getbarsj added.
- Task.getbarxj added.
- Task.getconname64 removed.
- Task.getdviolbarvar added.
- Task.getdviolcon added.
- Task.getdviolcones added.
- Task.getdviolvar added.
- Task.getintpntnumthreads removed.
- Task.getmemusagetask64 removed.
- Task.getname64 removed.
- Task.getnameapi64 removed.
- Task.getnameindex removed.
- Task.getnamelen64 removed.
- Task.getnumqobjnz removed.
- Task.getobjname64 removed.
- Task.getprosta added.
- Task.getpviolbarvar added.
- Task.getpviolcon added.
- Task.getpviolcones added.
- Task.getpviolvar added.
- Task.getqconk removed.
- Task.getskcslice added.
- Task.getskxslice added.
- Task.getslcslice added.
- Task.getslxslice added.
- Task.getsnxslice added.
- Task.getsolsta added.
- Task.getsolutioninfo added.

- Task.getsolutionstatus removed.
- Task.getsolutionstatuskeyslice removed.
- Task.getsucslice added.
- Task.getsuxslice added.
- Task.gettaskname64 removed.
- Task.getvarname64 removed.
- Task.getxcslice added.
- Task.getxxslice added.
- Task.getyslice added.
- Task.makesolutionstatusunknown removed.
- Task.netextraction removed.
- Task.netoptimize removed.
- Task.primalrepair added.
- Task.putacol added.
- Task.putaijlist removed.
- Task.putarow added.
- Task.putavec removed.
- Task.putaveclist64 removed.
- Task.putbaraij added.
- Task.putbarcj added.
- Task.putbarsj added.
- Task.putbarvarname added.
- Task.putbarxj added.
- Task.putconbound added.
- Task.putconboundlist added.
- Task.putconename added.
- Task.putconname added.
- Task.putmaxnumanz64 removed.

- Task.putmaxnumqnz64 removed.
- Task.putname removed.
- Task.putskcslice added.
- Task.putskxslice added.
- Task.putslcslice added.
- Task.putslxslice added.
- Task.putsnxslice added.
- Task.putsucslice added.
- Task.putsuxslice added.
- Task.putvarbound added.
- Task.putvarboundlist added.
- Task.putvarname added.
- Task.putxcslice added.
- Task.putxxslice added.
- Task.putyslice added.
- Task.readdata removed.
- Task.readtask added.
- Task.remove removed.
- Task.removecone removed.
- Task.removecones added.
- Task.removecons added.
- Task.removevars added.
- Task.toconic added.
- Task.undefsolution removed.
- Task.updatesolutioninfo added.
- Task.writetask added.

## Chapter 2

# About this manual

This manual covers the general functionality of MOSEK and the usage of the MOSEK Python API.

The MOSEK Python Application Programming Interface makes it possible to access the MOSEK optimizer from any Python application. The whole functionality of the native C API is available through a thin class-based interface using native Python types and exceptions. All methods in the interface are thin wrappers around functions in the native C API, keeping the overhead induced by the API to a minimum.

The API can be used in Python scripts as well as from the interactive Python command-line. The Python interface is particularly well-suited for fast prototyping of models and for debugging and displaying portions of a problem loaded from a file.

The Python interface consists of a mosek module that defines objects, functions and constants.

New users of the MOSEK Python API are encouraged to read:

- Chapter 4 on compiling and running the distributed examples.
- The relevant parts of Chapter 5, i.e. at least the general introduction and the linear optimization section.
- Chapter 9 for a set of guidelines about developing, testing, and debugging applications employing MOSEK.

This should introduce most of the data structures and functionality necessary to implement and solve an optimization problem.

Chapter 10 contains general material about the mathematical formulations of optimization problems compatible with MOSEK, as well as common tips and tricks for reformulating problems so that they can be solved by MOSEK.

Hence, Chapter 10 is useful when trying to find a good formulation of a specific model.

More advanced examples of modeling and model debugging are located in

• Chapter 14 which deals with analysis of infeasible problems,

 $\bullet$  Chapter 15 about the sensitivity analysis interface, and

Finally, the Python API reference material is located in

- Chapter A which lists all types and functions,
- Chapter B which lists all available parameters,
- Chapter C which lists all response codes, and
- $\bullet$  Chapter  ${\color{red} \mathbf{D}}$  which lists all symbolic constants.

# Chapter 3

# Getting support and help

## 3.1 MOSEK documentation

For an overview of the available MOSEK documentation please see  ${\tt mosek/7/docs/}$ 

in the distribution.

## 3.2 Bug reporting

If you think MOSEK is solving your problem incorrectly, please contact MOSEK support at

```
support@mosek.com
```

providing a detailed description of the problem. MOSEK support may ask for the task file which is produced as follows

```
task.writedata("data.task.gz")
task.optimize()
```

The task data will then be written to a binary file named data.task.gz which is useful when reproducing a problem.

## 3.3 Additional reading

In this manual it is assumed that the reader is familiar with mathematics and in particular mathematical optimization. Some introduction to linear programming is found in books such as "Linear programming" by Chvátal [1] or "Computer Solution of Linear Programs" by Nazareth [2]. For more theoretical aspects see e.g. "Nonlinear programming: Theory and algorithms" by Bazaraa, Shetty,

and Sherali [3]. Finally, the book "Model building in mathematical programming" by Williams [4] provides an excellent introduction to modeling issues in optimization.

Another useful resource is "Mathematical Programming Glossary" available at

 ${\rm http://glossary.computing.society.informs.org}$ 

# Chapter 4

# Testing installation and compiling examples

This chapter describes how to verify that the MOSEK Python API has been installed and works, and how to run the Python examples distributed with MOSEK.

To use the MOSEK Python API, a working MOSEK installation must be present — see the MOSEK Installation manual for instructions. Part of this installation are two versions of the Python interface; one for Python 2.5 and later, and one for the Python 3 series. Note that MOSEK can use one-dimensional arrays from the NumPy package from scipy.org. In case NumPy is not installed, the MOSEK/Python interface includes a minimal array implementation, mosek.array, that supports the basic array functionality for a limited number of native types.

A Python installer can be obtained from the official site:

http://www.python.org/

The NumPy package providing arrays and mathematical functionality can be obtained from:

http://numpy.scipy.org/

Note that the architecture of the Python binary and the MOSEK DLL must match: A 32 bit MOSEK can only be used from a 32 bit Python, and a 64 bit MOSEK can only be used from a 64 bit Python. The architecture of the Python binary can be checked using the following command:

```
python -c "print(__import__('platform').architecture())"
```

## 4.1 Microsoft Windows platform

MOSEK includes a binary Python that can be used interactively or for running scripts, but if you have a Python installed on your system you may prefer to use that instead. The MOSEK installer does *not* 

set up global paths to the included Python: To use it, either execute Python with full path, or set up the PATH environment variable to include the relevant bin directory.

The MOSEK/Python module structure is located under platform directory (mosek\7\tools\platform):

```
64 bit MOSEK, Python 2.5+ win64x86\python\2
32 bit MOSEK, Python 2.5+ win32x86\python\2
64 bit MOSEK, Python 3 win64x86\python\3
32 bit MOSEK, Python 3 win32x86\python\3
```

Examples using the MOSEK/Python interface are found in

 $mosek\7\tools\examples\python$ 

## 4.1.1 Running a Python example

```
To run one of the distributed examples, open a DOS box and type

C:
    cd "C:\Program Files\mosek\7\tools\examples\python"

then to execute example lo1 type
    python lo1.py
```

## 4.2 Implementation details

The MOSEK/Python module is implemented as a pure Python module using ctypes to call native DLLs. The CTypes module is included in the Python standard library from 2.5 on.

CTypes is considered an unsafe module, so it may be disabled under some circumstances, e.g. for web server scripts.

# Chapter 5

# Basic API tutorial

In this chapter the reader will learn how to build a simple application that uses MOSEK.

A number of examples is provided to demonstrate the functionality required for solving linear, conic, semidefinite and quadratic problems as well as mixed integer problems.

Please note that the section on linear optimization also describes most of the basic functionality needed to specify optimization problems. Hence, it is recommended to read Section 5.2 before reading about other optimization problems.

## 5.1 The basics

A typical program using the MOSEK Python interface can be described shortly:

- Create an environment object (Env).
- Set up some environment specific data and initialize the environment object.
- Create a task object (Task).
- Load a problem into the task object.
- Optimize the problem.
- Fetch the result.
- Dispose of the environment and task.

## 5.1.1 The environment and the task

The first MOSEK related step in any program that employs MOSEK is to create an environment object. The environment contains environment specific data such as information about the license file,

streams for environment messages etc. When this is done one or more task objects can be created. Each task is associated with a single environment and defines a complete optimization problem as well as task message streams and optimization parameters.

When done, tasks and environments may be disposed explicitly by calling the \_\_del\_\_ method. This is not strictly necessary, but it will free up allocated resources and checked-out licenses immediately instead of when the garbage collector runs.

In Python, the creation of an environment and a task would look something like this:

```
# Create an environment
env = mosek.Env()

# You may connect streams and other callbacks to env here.

# Create a task
task = env.Task()

# Load a problem into the task, optimize etc.
```

From Python 2.6 and later the with construction can be used to dispose objects automatically when they drop of out of the with-scope:

```
# Create an environment
with mosek.Env() as env:
    # You may connect streams and other callbacks to env here.

# Create a task
with env.Task() as task:
    # Load a problem into the task, optimize etc.
```

Please note that multiple tasks should, if possible, share the same environment.

## 5.1.2 Example: Simple working example

The following simple example shows a working Python program which

- creates an environment and a task,
- reads a problem from a file,
- optimizes the problem, and
- writes the solution to a file.

```
simple.py ]

#
Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.

#
# File: simple.py
```

5.1. THE BASICS 23

```
# Purpose: Demonstrates a very simple example using MOSEK by
    # reading a problem file, solving the problem and
7
    # writing the solution to a file.
    import mosek
11
    import sys
12
13
    def streamprinter(msg):
14
15
         sys.stdout.write (msg)
         sys.stdout.flush ()
16
17
    if len(sys.argv) <= 1:</pre>
18
        print ("Missing argument, syntax is:")
19
20
        print (" simple inputfile [ solutionfile ]")
    else:
21
         # Create the mosek environment.
22
         env = mosek.Env ()
23
24
25
         # Create a task object linked with the environment env.
         # We create it with 0 variables and 0 constraints initially,
26
27
         # since we do not know the size of the problem.
         task = env.Task (0, 0)
28
         task.set_Stream (mosek.streamtype.log, streamprinter)
29
30
         # We assume that a problem file was given as the first command
31
         # line argument (received in 'argv')
32
         task.readdata (sys.argv[1])
33
        # Solve the problem
35
         task.optimize ()
36
37
         # Print a summary of the solution
38
         task.solutionsummary (mosek.streamtype.log)
40
         # If an output file was specified, write a solution
41
         if len(sys.argv) >= 3:
42
             # We define the output format to be OPF, and tell MOSEK to
43
44
             # leave out parameters and problem data from the output file.
             task.putintparam (mosek.iparam.write_data_format,
                                                                   mosek.dataformat.op)
45
             task.putintparam (mosek.iparam.opf_write_solutions, mosek.onoffkey.on)
             task.putintparam (mosek.iparam.opf_write_hints,
                                                                   mosek.onoffkey.off)
47
             task.putintparam (mosek.iparam.opf_write_parameters, mosek.onoffkey.off)
49
             task.putintparam (mosek.iparam.opf_write_problem,
                                                                   mosek.onoffkey.off)
50
             task.writedata (sys.argv[2])
```

### 5.1.2.1 Reading and writing problems

Use the Task.writedata function to write a problem to a file. By default, when not choosing any specific file format for the parameter iparam.write\_data\_format, MOSEK will determine the output file format by the extension of the file name:

```
Similarly, controlled by iparam.read_data_format, the function Task.readdata can read a problem from a file:

[simple.py]

simple.py]

task.readdata (sys.argv[1])
```

#### 5.1.2.2 Working with the problem data

An optimization problem consists of several components; objective, objective sense, constraints, variable bounds etc. Therefore, the interface provides a number of methods to operate on the task specific data, all of which are listed under the Task class-specification.

#### 5.1.2.3 Setting parameters

Apart from the problem data, the task contains a number of parameters defining the behavior of MOSEK. For example the <code>iparam.optimizer</code> parameter defines which optimizer to use. There are three kinds of parameters in MOSEK

- Integer parameters that can be set with Task.putintparam,
- Double parameters that can be set with Task.putdouparam, and
- string parameters that can be set with Task.putstrparam,

The values for integer parameters are either simple integer values or enum values.

A complete list of all parameters is found in Chapter B.

## 5.2 Linear optimization

The simplest optimization problem is a purely linear problem. A *linear optimization problem* is a problem of the following form:

Minimize or maximize the objective function

$$\sum_{j=0}^{n-1} c_j x_j + c^f \tag{5.1}$$

subject to the linear constraints

$$l_k^c \le \sum_{j=0}^{n-1} a_{kj} x_j \le u_k^c, \ k = 0, \dots, m-1,$$
 (5.2)

and the bounds

$$l_i^x \le x_j \le u_i^x, \ j = 0, \dots, n - 1,$$
 (5.3)

where we have used the problem elements:

m and n

which are the number of constraints and variables respectively,

x which is the variable vector of length n,

which is a coefficient vector of size n

$$c = \left[ \begin{array}{c} c_0 \\ c_{n-1} \end{array} \right],$$

 $c^f$  which is a constant,

A

c

which is a  $m \times n$  matrix of coefficients is given by

$$A = \begin{bmatrix} a_{0,0} & \cdots & a_{0,(n-1)} \\ & \cdots & \\ a_{(m-1),0} & \cdots & a_{(m-1),(n-1)} \end{bmatrix},$$

 $l^c$  and  $u^c$ 

which specify the lower and upper bounds on constraints respectively, and

 $l^x$  and  $u^x$ 

which specifies the lower and upper bounds on variables respectively.

Please note the unconventional notation using 0 as the first index rather than 1. Hence,  $x_0$  is the first element in variable vector x. This convention has been adapted from Python arrays which are indexed from 0.

## 5.2.1 Example: Linear optimization

The following is an example of a linear optimization problem:

having the bounds

$$\begin{array}{cccccc} 0 & \leq & x_0 & \leq & \infty, \\ 0 & \leq & x_1 & \leq & 10, \\ 0 & \leq & x_2 & \leq & \infty, \\ 0 & \leq & x_3 & \leq & \infty. \end{array}$$

## 5.2.1.1 Solving the problem

To solve the problem above we go through the following steps:

- Create an environment.
- Create an optimization task.
- Load a problem into the task object.
- Optimization.
- Extracting the solution.

Below we explain each of these steps. For the complete source code see section 5.2.1.2.

Create an environment.

Before setting up the optimization problem, a MOSEK environment must be created. All tasks in the program should share the same environment.

```
# Make mosek environment
with mosek.Env() as env:
```

Create an optimization task.

Next, an empty task object is created:

```
# Create a task object
with env.Task(0,0) as task:
# Attach a log stream printer to the task
task.set_Stream (mosek.streamtype.log, streamprinter)
```

We also connect a call-back function to the task log stream. Messages related to the task are passed to the call-back function. In this case the stream call-back function writes its messages to the standard output stream.

Load a problem into the task object.

Before any problem data can be set, variables and constraints must be added to the problem via calls to the functions Task.appendcons and Task.appendvars.

```
# Append 'numcon' empty constraints.

# The constraints will initially have no bounds.

task.appendcons(numcon)

# Append 'numvar' variables.

# The variables will initially be fixed at zero (x=0).

task.appendvars(numvar)
```

New variables can now be referenced from other functions with indexes in  $0, \ldots, \mathtt{numvar} - 1$  and new constraints can be referenced with indexes in  $0, \ldots, \mathtt{numcon} - 1$ . More variables / constraints can be appended later as needed, these will be assigned indexes from  $\mathtt{numvar}/\mathtt{numcon}$  and up.

Next step is to set the problem data. We loop over each variable index j = 0, ..., numvar - 1 calling functions to set problem data. We first set the objective coefficient  $c_j = c[j]$  by calling the function Task.putcj.

```
_____[lo1.py]______[task.putcj(j,c[j])
```

The bounds on variables are stored in the arrays

```
# Bound keys for variables
bkx = [mosek.boundkey.lo,
mosek.boundkey.ra,
mosek.boundkey.lo,
mosek.boundkey.lo]

# Bound values for variables
blx = [0.0, 0.0, 0.0, 0.0]
bux = [+inf, 10.0, +inf, +inf]
```

and are set with calls to Task.putvarbound.

```
# Set the bounds on variable j
# blx[j] <= x_j <= bux[j]
task.putvarbound(j,bkx[j],blx[j])
```

The Bound key stored in bkx specify the type of the bound according to Table 5.1. For instance bkx[0]=boundkey.lo means that  $x_0 \ge l_0^x$ . Finally, the numerical values of the bounds on variables are given by

$$l_i^x = blx[j]$$

and

$$u_j^x = \text{bux}[j].$$

Bound key	Type of bound	Lower bound	Upper bound
boundkey.fx	$\cdots = l_j$	Finite	Identical to the lower bound
boundkey.fr	Free	Minus infinity	Plus infinity
boundkey.lo	$l_j \leq \cdots$	Finite	Plus infinity
boundkey.ra	$l_j \leq \cdots \leq u_j$	Finite	Finite
boundkey.up	$\cdots \leq u_j$	Minus infinity	Finite

Table 5.1: Interpretation of the bound keys.

Recall that in our example the A matrix is given by

$$A = \left[ \begin{array}{rrrr} 3 & 1 & 2 & 0 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 0 & 3 \end{array} \right].$$

This matrix is stored in sparse format in the arrays:

```
62 asub = [ array([0, 1]),

63 array([0, 1, 2]),

64 array([0, 1]),

65 array([1, 2])]

66 aval = [ array([3.0, 2.0]),

67 array([1.0, 1.0, 2.0]),

68 array([2.0, 3.0]),

69 array([1.0, 3.0])]
```

The array aval[j] contains the non-zero values of column j and asub[j] contains the row index of these non-zeros.

Using the function Task.putacol we set column j of A

```
[lo1.py]
task.putacol(j, # Variable (column) index.
asub[j], # Row index of non-zeros in column j.
aval[j]) # Non-zero Values of column j.
```

Alternatively, the same A matrix can be set one row at a time; please see section 5.2.2 for an example.

Finally, the bounds on each constraint are set by looping over each constraint index  $i=0,\dots,\mathtt{numcon}-1$ 

```
# Set the bounds on constraints.

# blc[i] <= constraint_i <= buc[i]

for i in range(numcon):

task.putconbound(i,bkc[i],blc[i])

task.putconboundslice(0,numcon, bkc,blc,buc);
```

#### Optimization:

After the problem is set-up the task can be optimized by calling the function Task.optimize.

```
105 task.optimize()
```

Extracting the solution.

After optimizing the status of the solution is examined with a call to Task.getsolsta. If the solution status is reported as solsta.optimal or solsta.near\_optimal the solution is extracted in the lines below:

The Task.getxx function obtains the solution. MOSEK may compute several solutions depending on the optimizer employed. In this example the *basic solution* is requested by setting the first argument to soltype.bas.

#### 5.2.1.2 Source code for lo1

```
—[lo1.py]—
       Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
2
       File:
                lo1.py
       Purpose: Demonstrates how to solve small linear
                optimization problem using the MOSEK Python API.
    ##
    from __future__ import with_statement
11
    import sys
    import mosek
12
13
    # If numpy is installed, use that, otherwise use the
14
    # Mosek's array module.
15
16
        from numpy import array, zeros, ones
17
18
    except ImportError:
        from mosek.array import array, zeros, ones
19
20
    # Since the value of infinity is ignored, we define it solely
21
    # for symbolic purposes
22
    inf = 0.0
23
24
    # Define a stream printer to grab output from MOSEK
    def streamprinter(text):
26
         sys.stdout.write(text)
27
         sys.stdout.flush()
28
29
```

```
def main ():
      # Make mosek environment
31
       with mosek.Env() as env:
32
         # Create a task object
33
         with env. Task(0,0) as task:
34
           # Attach a log stream printer to the task
           task.set_Stream (mosek.streamtype.log, streamprinter)
36
37
38
           # Bound keys for constraints
           bkc = [mosek.boundkey.fx,
39
                  mosek.boundkey.lo,
40
                  mosek.boundkey.up]
41
42
           # Bound values for constraints
43
44
           blc = [30.0, 15.0, -inf]
           buc = [30.0, +inf, 25.0]
45
46
           # Bound keys for variables
           bkx = [mosek.boundkey.lo,
48
                  mosek.boundkey.ra,
49
                  mosek.boundkey.lo,
50
                  mosek.boundkey.lo]
51
52
           # Bound values for variables
53
           blx = [0.0, 0.0, 0.0, 0.0]
           bux = [+inf, 10.0, +inf, +inf]
55
56
           # Objective coefficients
57
           c = [3.0, 1.0, 5.0, 1.0]
58
           \mbox{\tt\#} Below is the sparse representation of the \mbox{\tt A}
60
           # matrix stored by column.
61
           asub = [ array([0, 1]),
62
                    array([0, 1, 2]),
63
                    array([0, 1]),
                    array([1, 2])]
65
66
           aval = [array([3.0, 2.0]),
                    array([1.0, 1.0, 2.0]),
67
                    array([2.0, 3.0]),
68
69
                    array([1.0, 3.0])]
70
           numvar = len(bkx)
71
           numcon = len(bkc)
72
73
           # Append 'numcon' empty constraints.
74
           # The constraints will initially have no bounds.
75
           task.appendcons(numcon)
77
           # Append 'numvar' variables.
78
           \mbox{\tt\#} The variables will initially be fixed at zero (x=0).
79
           task.appendvars(numvar)
80
81
           for j in range(numvar):
82
             # Set the linear term c_j in the objective.
83
84
             task.putcj(j,c[j])
85
             # Set the bounds on variable j
             # blx[j] \le x_j \le bux[j]
```

```
task.putvarbound(j,bkx[j],blx[j],bux[j])
89
             # Input column j of A
90
                                                 # Variable (column) index.
91
             task.putacol(j,
                           asub[i],
                                                 # Row index of non-zeros in column j.
92
                           aval[j])
                                                 # Non-zero Values of column j.
94
           # Set the bounds on constraints.
95
          # blc[i] <= constraint_i <= buc[i]</pre>
96
           for i in range(numcon):
97
               task.putconbound(i,bkc[i],blc[i],buc[i])
98
           task.putconboundslice(0,numcon, bkc,blc,buc);
99
100
           # Input the objective sense (minimize/maximize)
101
102
           task.putobjsense(mosek.objsense.maximize)
103
           # Solve the problem
104
           task.optimize()
105
106
           # Print a summary containing information
107
           # about the solution for debugging purposes
108
           task.solutionsummary(mosek.streamtype.msg)
109
110
           # Get status information about the solution
111
           solsta = task.getsolsta(mosek.soltype.bas)
112
113
           if (solsta == mosek.solsta.optimal or
114
115
               solsta == mosek.solsta.near_optimal):
             xx = zeros(numvar, float)
116
             task.getxx(mosek.soltype.bas, # Request the basic solution.
118
                         xx)
             print ("Optimal solution: ")
119
             for i in range(numvar):
120
                print ("x["+str(i)+"]="+str(xx[i]))
121
           elif (solsta == mosek.solsta.dual_infeas_cer or
122
                  solsta == mosek.solsta.prim_infeas_cer or
123
                  solsta == mosek.solsta.near_dual_infeas_cer or
124
125
                  solsta == mosek.solsta.near_prim_infeas_cer):
             print("Primal or dual infeasibility certificate found.\n")
126
127
           elif solsta == mosek.solsta.unknown:
             print("Unknown solution status")
128
           else:
129
             print("Other solution status")
130
131
132
     # call the main function
     try:
133
         main ()
134
     except mosek.Exception as e:
135
         print ("ERROR: %s" % str(e.errno))
136
137
         if e.msg is not None:
             print ("\t%s" % e.msg)
138
139
             sys.exit(1)
     except:
140
         import traceback
141
142
         traceback.print_exc()
         sys.exit(1)
143
144
     sys.exit(0)
```

## 5.2.2 Row-wise input

bkx = [mosek.boundkey.lo,

In the previous example the A matrix is set one column at a time. Alternatively the same matrix can be set one row at a time or the two methods can be mixed as in the example in section 5.10. The following example show how to set the A matrix by rows.

```
—[lo2.py]—
1
       Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
    #
      File:
               1o2.py
    #
       Purpose: Demonstrates how to solve small linear
                 optimization problem using the MOSEK Python API.
10
    import sys
11
    import mosek
12
    # If numpy is installed, use that, otherwise use the
13
    # Mosek's array module.
14
15
    try:
        from numpy import array,zeros,ones
16
    except ImportError:
17
        from mosek.array import array, zeros, ones
18
19
    # Since the actual value of Infinity is ignores, we define it solely
20
    # for symbolic purposes:
21
22
    inf = 0.0
23
    # Define a stream printer to grab output from MOSEK
24
    def streamprinter(text):
25
        sys.stdout.write(text)
26
27
        sys.stdout.flush()
28
    # We might write everything directly as a script, but it looks nicer
    # to create a function.
30
    def main ():
31
      # Make a MOSEK environment
      env = mosek.Env ()
33
      # Attach a printer to the environment
      env.set_Stream (mosek.streamtype.log, streamprinter)
35
      # Create a task
37
      task = env.Task(0,0)
38
39
      # Attach a printer to the task
      task.set_Stream (mosek.streamtype.log, streamprinter)
40
41
      # Bound keys for constraints
42
      bkc = [mosek.boundkey.fx,
43
             mosek.boundkey.lo,
44
             mosek.boundkey.up]
45
      # Bound values for constraints
      blc = [30.0, 15.0, -inf]
47
      buc = [30.0, +inf, 25.0]
      # Bound keys for variables
```

```
mosek.boundkey.ra,
              mosek.boundkey.lo,
52
              mosek.boundkey.lo]
53
       # Bound values for variables
       blx = [0.0, 0.0, 0.0, 0.0]
55
       bux = [+inf, 10.0, +inf, +inf]
       # Objective coefficients
57
       c = [3.0, 1.0, 5.0, 1.0]
59
60
       # We input the A matrix column-wise
61
       # asub contains row indexes
62
63
       asub = [array([0, 1, 2]),
                array([0, 1, 2, 3]),
64
65
                array([0, 3])]
         # acof contains coefficients
66
       aval = [ array([3.0, 1.0, 2.0]),
67
                array([2.0, 1.0, 3.0, 1.0]),
                array([2.0, 3.0])]
69
       numvar = len(bkx)
70
       numcon = len(bkc)
71
       # Append 'numcon' empty constraints.
72
       # The constraints will initially have no bounds.
73
       task.appendcons(numcon)
74
       #Append 'numvar' variables.
76
       # The variables will initially be fixed at zero (x=0).
       task.appendvars(numvar)
78
79
       for j in range(numvar):
         # Set the linear term c_j in the objective.
81
         task.putcj(j,c[j])
82
         # Set the bounds on variable j
83
         # blx[j] <= x_j <= bux[j]
84
         task.putbound(mosek.accmode.var,j,bkx[j],blx[j],bux[j])
86
87
       for i in range(numcon):
         task.putbound(mosek.accmode.con,i,bkc[i],blc[i],buc[i])
88
         # Input row i of A
89
90
         task.putarow(i,
                                              # Row index.
                      asub[i].
                                              # Column indexes of non-zeros in row i.
91
                      aval[i]);
                                              # Non-zero Values of row i.
92
93
       # Input the objective sense (minimize/maximize)
95
       task.putobjsense(mosek.objsense.maximize)
96
       # Optimize the task
98
       task.optimize()
100
       # Print a summary containing information
101
       # about the solution for debugging purposes
102
       task.solutionsummary(mosek.streamtype.msg)
103
105
       prosta = task.getprosta(mosek.soltype.bas)
       solsta = task.getsolsta(mosek.soltype.bas)
106
107
       # Output a solution
108
```

```
xx = zeros(numvar, float)
109
       task.getxx(mosek.soltype.bas,
110
111
112
       if solsta == mosek.solsta.optimal or solsta == mosek.solsta.near_optimal:
113
           print("Optimal solution: %s" % xx)
       elif solsta == mosek.solsta.dual_infeas_cer:
115
           print("Primal or dual infeasibility.\n")
116
117
       elif solsta == mosek.solsta.prim_infeas_cer:
           print("Primal or dual infeasibility.\n")
118
       elif solsta == mosek.solsta.near_dual_infeas_cer:
           print("Primal or dual infeasibility.\n")
120
       elif solsta == mosek.solsta.near_prim_infeas_cer:
121
           print("Primal or dual infeasibility.\n")
122
       elif mosek.solsta.unknown:
123
         print("Unknown solution status")
125
         print("Other solution status")
126
127
     # call the main function
128
129
         main ()
130
131
     except mosek.Exception as e:
         print ("ERROR: %s" % str(e.errno))
132
         if e.msg is not None:
133
             print ("\t%s" % e.msg)
134
             sys.exit(1)
135
136
     except:
         import traceback
137
138
         traceback.print_exc()
         sys.exit(1)
139
     sys.exit(0)
140
```

## 5.3 Conic quadratic optimization

Conic optimization is a generalization of linear optimization, allowing constraints of the type

$$x^t \in \mathcal{C}_t$$

where  $x^t$  is a subset of the problem variables and  $C_t$  is a convex cone. Actually, since the set  $\mathbb{R}^n$  of real numbers is also a convex cone, all variables can in fact be partitioned into subsets belonging to separate convex cones, simply stated  $x \in C$ .

MOSEK can solve conic quadratic optimization problems of the form

minimize 
$$c^T x + c^f$$
  
subject to  $l^c \le Ax \le u^c$ ,  
 $l^x \le x \le u^x$ ,  
 $x \in \mathcal{C}$ , (5.4)

where the domain restriction,  $x \in \mathcal{C}$ , implies that all variables are partitioned into convex cones

$$x = (x^0, x^1, \dots, x^{p-1}), \text{ with } x^t \in \mathcal{C}_t \subseteq \mathbb{R}^{n_t}.$$

For convenience, the user only specify subsets of variables  $x^t$  belonging to cones  $C_t$  different from the set  $\mathbb{R}^{n_t}$  of real numbers. These cones can be a:

• Quadratic cone:

$$Q_n = \left\{ x \in \mathbb{R}^n : x_0 \ge \sqrt{\sum_{j=1}^{n-1} x_j^2} \right\}.$$

• Rotated quadratic cone:

$$Q_n^r = \left\{ x \in \mathbb{R}^n : 2x_0 x_1 \ge \sum_{j=2}^{n-1} x_j^2, \ x_0 \ge 0, \ x_1 \ge 0 \right\}.$$

From these definition it follows that

$$(x_4, x_0, x_2) \in \mathcal{Q}_3,$$

is equivalent to

$$x_4 \ge \sqrt{x_0^2 + x_2^2}.$$

Furthermore, each variable may belong to one cone at most. The constraint  $x_i - x_j = 0$  would however allow  $x_i$  and  $x_j$  to belong to different cones with same effect.

## 5.3.1 Example: Conic quadratic optimization

The problem

minimize 
$$x_3 + x_4 + x_5$$
  
subject to  $x_0 + x_1 + 2x_2 = 1$ ,  
 $x_0, x_1, x_2 \ge 0$ ,  $(5.5)$   
 $x_3 \ge \sqrt{x_0^2 + x_1^2}$ ,  
 $2x_4x_5 \ge x_2^2$ .

is an example of a conic quadratic optimization problem. The problem includes a set of linear constraints, a quadratic cone and a rotated quadratic cone.

#### 5.3.1.1 Source code

```
——[ cqo1.py ]—
    #
2
       Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
    # File:
              cqo1.py
      Purpose: Demonstrates how to solve small linear
                optimization problem using the MOSEK Python API.
    ##
    import sys
10
11
    import mosek
12
    # If numpy is installed, use that, otherwise use the
13
    # Mosek's array module.
14
15
        from numpy import array,zeros,ones
16
17
    except ImportError:
        from mosek.array import array, zeros, ones
18
19
    # Since the actual value of Infinity is ignores, we define it solely
20
    # for symbolic purposes:
21
    inf = 0.0
23
    # Define a stream printer to grab output from MOSEK
24
    def streamprinter(text):
25
        sys.stdout.write(text)
26
27
        sys.stdout.flush()
28
    # We might write everything directly as a script, but it looks nicer
    # to create a function.
30
    def main ():
31
      # Make a MOSEK environment
32
      env = mosek.Env ()
33
      # Attach a printer to the environment
      env.set_Stream (mosek.streamtype.log, streamprinter)
35
      # Create a task
37
      task = env.Task(0,0)
38
      # Attach a printer to the task
      task.set_Stream (mosek.streamtype.log, streamprinter)
40
      bkc = [ mosek.boundkey.fx ]
42
      blc = [ 1.0 ]
43
      buc = [ 1.0 ]
44
45
      c = [
                             0.0,
                                               0.0,
                                                                  0.0,
                                               1.0,
                             1.0.
47
      bkx = [ mosek.boundkey.lo,mosek.boundkey.lo,mosek.boundkey.lo,
48
              mosek.boundkey.fr,mosek.boundkey.fr,mosek.boundkey.fr ]
49
                            0.0,
                                              0.0,
50
51
                            -inf,
                                              -inf,
                                                                 -inf ]
      bux = \Gamma
                             inf.
                                               inf.
                                                                  inf.
52
                             inf,
                                               inf,
                                                                  inf ]
54
      asub = [array([0]), array([0]), array([0])]
```

```
aval = [ array([1.0]), array([1.0]), array([2.0]) ]
56
57
58
       numvar = len(bkx)
59
       numcon = len(bkc)
60
       NUMANZ = 4
       # Append 'numcon' empty constraints.
62
       # The constraints will initially have no bounds.
63
64
       task.appendcons(numcon)
65
       #Append 'numvar' variables.
66
       # The variables will initially be fixed at zero (x=0).
67
       task.appendvars(numvar)
69
70
       for j in range(numvar):
71
         # Set the linear term c_j in the objective.
         task.putcj(j,c[j])
72
         # Set the bounds on variable j
73
         # blx[j] <= x_j <= bux[j]
74
         task.putbound(mosek.accmode.var,j,bkx[j],blx[j],bux[j])
75
76
       for j in range(len(aval)):
77
78
         # Input column j of A
         task.putacol(j,
                                           # Variable (column) index.
79
                                           # Row index of non-zeros in column j.
80
                                           # Non-zero Values of column j.
                       aval[j])
81
       for i in range(numcon):
82
         task.putbound(mosek.accmode.con,i,bkc[i],blc[i],buc[i])
83
84
       # Input the cones
       task.appendcone(mosek.conetype.quad,
86
                        0.0,
                        [3,0,1])
88
       task.appendcone(mosek.conetype.rquad,
89
                        0.0,
                        [4, 5, 2]
91
92
       # Input the objective sense (minimize/maximize)
93
       task.putobjsense(mosek.objsense.minimize)
94
95
       # Optimize the task
96
       task.optimize()
       # Print a summary containing information
98
       # about the solution for debugging purposes
100
       task.solutionsummary(mosek.streamtype.msg)
       prosta = task.getprosta(mosek.soltype.itr)
101
       solsta = task.getsolsta(mosek.soltype.itr)
102
103
       # Output a solution
104
       xx = zeros(numvar, float)
105
       task.getxx(mosek.soltype.itr,
106
107
                              xx)
108
       if solsta == mosek.solsta.optimal or solsta == mosek.solsta.near_optimal:
109
110
          print("Optimal solution: %s" % xx)
       elif solsta == mosek.solsta.dual_infeas_cer:
111
112
           print("Primal or dual infeasibility.\n")
       elif solsta == mosek.solsta.prim_infeas_cer:
113
```

```
print("Primal or dual infeasibility.\n")
114
       elif solsta == mosek.solsta.near_dual_infeas_cer:
115
           print("Primal or dual infeasibility.\n")
116
117
       elif solsta == mosek.solsta.near_prim_infeas_cer:
           print("Primal or dual infeasibility.\n")
118
       elif mosek.solsta.unknown:
119
         print("Unknown solution status")
120
121
122
         print("Other solution status")
123
124
     # call the main function
125
126
     try:
         main ()
127
     except mosek.Exception as e:
128
         print ("ERROR: %s" % str(e.code))
129
         if msg is not None:
130
             print ("\t%s" % e.msg)
131
              sys.exit(1)
132
     except:
133
134
         import traceback
         traceback.print_exc()
135
136
         sys.exit(1)
     sys.exit(0)
137
```

#### 5.3.1.2 Source code comments

The only new function introduced in the example is **Task.appendcone**, which is called here:

The first argument selects the type of quadratic cone. Either conetype.quad for a quadratic cone or conetype.rquad for a rotated quadratic cone. The cone parameter 0.0 is currently not used by MOSEK — simply passing 0.0 will work.

The last argument is a list of indexes of the variables in the cone.

## 5.4 Semidefinite optimization

Semidefinite optimization is a generalization of conic quadratic optimization, allowing the use of matrix variables belonging to the convex cone of positive semidefinite matrices

$$\mathcal{S}_r^+ = \left\{ X \in \mathcal{S}_r : z^T X z \ge 0, \ \forall z \in \mathbb{R}^r \right\},\,$$

where  $S_r$  is the set of  $r \times r$  real-valued symmetric matrices.

MOSEK can solve semidefinite optimization problems of the form

minimize 
$$\sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle + c^f$$
 subject to  $l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle \leq u_i^c, \quad i = 0, \dots, m-1,$  
$$l_j^x \leq x_j \leq x_j \leq v_j^+, \quad j = 0, \dots, n-1,$$
 
$$x \in \mathcal{C}, \overline{X}_j \in \mathcal{S}_{r_j}^+, \quad j = 0, \dots, p-1$$

where the problem has p symmetric positive semidefinite variables  $\overline{X}_j \in \mathcal{S}_{r_j}^+$  of dimension  $r_j$  with symmetric coefficient matrices  $\overline{C}_j \in \mathcal{S}_{r_j}$  and  $\overline{A}_{i,j} \in \mathcal{S}_{r_j}$ . We use standard notation for the matrix inner product, i.e., for  $A, B \in \mathbb{R}^{m \times n}$  we have

$$\langle A, B \rangle := \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} A_{ij} B_{ij}.$$

Since all  $\overline{C}_j$ ,  $\overline{A}_{ij}$  are assummed to be symmetric, only their lower triangular parts are specified.

Some attention must be paid when formulating linear constraints involving semidefinite matrices. A common mistake is not to consider that, being all  $\overline{C}_j$ ,  $\overline{A}_{ij}$  symmetric, their off-diagonal entries are counted twice. Indeed in that case we can write

$$\langle \overline{A}, \overline{X} \rangle := \sum_{i=0}^{p-1} \overline{A}_{ii} \overline{X}_{ii} + 2 \sum_{i=0}^{p-1} \sum_{j=i+1}^{p-1} \overline{A}_{ij} \overline{X}_{ij},$$

and hence the contribution of each off-diagonal element to the linear constraint is double.

For instance, let's consider  $\overline{X} \in \mathcal{S}_3^+$  and a constraint of the form  $\overline{X}_{01} = 1$ . Introducing a symmetric matrix

$$A = \left[ \begin{array}{ccc} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right],$$

we write the constraint as

$$\langle A, \overline{X} \rangle = \overline{X}_{01} + \overline{X}_{10} = 2\overline{X}_{01} = 2.$$

Otherwise, we could use

$$A = \left[ \begin{array}{ccc} 0 & 0.5 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right],$$

and rewrite the constraint as

$$\langle A, \overline{X} \rangle = 0.5(\overline{X}_{10} + \overline{X}_{01}) = \overline{X}_{01} = 1.$$

## 5.4.1 Example: Semidefinite optimization

The problem

is a mixed semidefinite and conic quadratic programming problem with a 3-dimensional semidefinite variable

$$\overline{X} = \left[ \begin{array}{ccc} \overline{x}_{00} & \overline{x}_{10} & \overline{x}_{20} \\ \overline{x}_{10} & \overline{x}_{11} & \overline{x}_{21} \\ \overline{x}_{20} & \overline{x}_{21} & \overline{x}_{22} \end{array} \right] \in \mathcal{S}_3^+,$$

and a conic quadratic variable  $(x_0, x_1, x_2) \in \mathcal{Q}_3$ . The objective is to minimize

$$2(\overline{x}_{00} + \overline{x}_{10} + \overline{x}_{11} + \overline{x}_{21} + \overline{x}_{22}) + x_0,$$

subject to the two linear constraints

$$\overline{x}_{00} + \overline{x}_{11} + \overline{x}_{22} + x_0 = 1,$$

and

$$\overline{x}_{00} + \overline{x}_{11} + \overline{x}_{22} + 2(\overline{x}_{10} + \overline{x}_{20} + \overline{x}_{21}) + x_1 + x_2 = 1/2.$$

## 5.4.1.1 Source code

```
[sdo1.py]

##

2 # Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.

3 #

4 # File: sdo1.py

5 #

6 # Purpose: Demonstrates how to solve a small mixed semidefinite and conic quadratic optimization problem using the MOSEK Python API.

8 ##

9 import sys
```

```
import mosek
12
    # If numpy is installed, use that, otherwise use the
13
    # Mosek's array module.
14
15
        from numpy import array,zeros,ones
    except ImportError:
17
        from mosek.array import array, zeros, ones
18
19
    # Since the value of infinity is ignored, we define it solely
20
    # for symbolic purposes
22
    # Define a stream printer to grab output from MOSEK
24
25
    def streamprinter(text):
        sys.stdout.write(text)
        sys.stdout.flush()
27
    def main ():
29
      # Make mosek environment
30
      env = mosek.Env()
31
32
      # Create a task object and attach log stream printer
33
      task = env.Task(0,0)
34
      task.set_Stream(mosek.streamtype.log, streamprinter)
36
      # Bound keys for constraints
      bkc = [mosek.boundkey.fx,
38
             mosek.boundkey.fx]
39
      # Bound values for constraints
41
      blc = [1.0, 0.5]
      buc = [1.0, 0.5]
43
44
      \# Below is the sparse representation of the A
      # matrix stored by row.
46
      asub = [ array([0]),
               array([1, 2])]
48
      aval = [ array([1.0]),
49
50
                array([1.0, 1.0])]
51
       conesub = [0, 1, 2]
52
53
      barci = [0, 1, 1, 2, 2]
54
      barcj = [0, 0, 1, 1, 2]
55
      barcval = [2.0, 1.0, 2.0, 1.0, 2.0]
56
57
      barai = [array([0, 1, 2]),
58
                 array([0, 1, 2, 1, 2, 2])]
              = [array([0, 1, 2]),
60
      baraj
                  array([0, 0, 0, 1, 1, 2])]
61
      baraval = [array([1.0, 1.0, 1.0]),
62
                  array([1.0, 1.0, 1.0, 1.0, 1.0, 1.0])]
63
65
      numvar = 3
      numcon = len(bkc)
66
      BARVARDIM = [3]
67
```

```
# Append 'numvar' variables.
       # The variables will initially be fixed at zero (x=0).
70
       task.appendvars(numvar)
71
       # Append 'numcon' empty constraints.
73
       # The constraints will initially have no bounds.
       task.appendcons(numcon)
75
76
       # Append matrix variables of sizes in 'BARVARDIM'.
77
       # The variables will initially be fixed at zero.
78
       task.appendbarvars(BARVARDIM)
79
80
81
       # Set the linear term c_0 in the objective.
       task.putcj(0, 1.0)
82
83
       for j in range(numvar):
84
         # Set the bounds on variable j
85
         # blx[j] <= x_j <= bux[j]
         task.putvarbound(j, mosek.boundkey.fr, -inf, +inf)
87
88
       for i in range(numcon):
89
         # Set the bounds on constraints.
90
         # blc[i] <= constraint_i <= buc[i]</pre>
91
         task.putconbound(i, bkc[i], blc[i], buc[i])
92
93
         # Input row i of A
94
         task.putarow(i,
                                            # Constraint (row) index.
95
                       asub[i],
                                            # Column index of non-zeros in constraint j.
96
                       aval[i])
                                            # Non-zero values of row j.
97
       task.appendcone(mosek.conetype.quad,
99
                        0.0,
100
                        conesub)
101
102
       symc = \
         task.appendsparsesymmat(BARVARDIM[0],
104
105
                                   barcj,
106
                                   barcval)
107
108
       svma0 = 
109
         task.appendsparsesymmat(BARVARDIM[0],
110
                                   barai[0],
111
                                   baraj[0],
112
                                   baraval[0])
113
114
       syma1 = \
115
         task.appendsparsesymmat(BARVARDIM[0],
116
                                   barai[1],
117
                                   baraj[1],
118
                                   baraval[1])
119
120
       task.putbarcj(0, [symc], [1.0])
121
122
       task.putbaraij(0, 0, [syma0], [1.0])
123
       task.putbaraij(1, 0, [syma1], [1.0])
124
125
       # Input the objective sense (minimize/maximize)
126
```

```
127
       task.putobjsense(mosek.objsense.minimize)
128
       task.writedata("sdo1.task")
129
130
       # Solve the problem and print summary
131
       task.optimize()
132
       task.solutionsummary(mosek.streamtype.msg)
133
134
135
       # Get status information about the solution
       prosta = task.getprosta(mosek.soltype.itr)
136
       solsta = task.getsolsta(mosek.soltype.itr)
137
138
139
       if (solsta == mosek.solsta.optimal or
           solsta == mosek.solsta.near_optimal):
140
         xx = zeros(numvar, float)
141
142
         task.getxx(mosek.soltype.itr, xx)
143
         lenbarvar = BARVARDIM[0] * (BARVARDIM[0]+1) / 2
144
         barx = zeros(int(lenbarvar), float)
145
         task.getbarxj(mosek.soltype.itr, 0, barx)
146
147
         print("Optimal solution:\nx=%s\nbarx=%s" % (xx,barx))
148
149
       elif (solsta == mosek.solsta.dual_infeas_cer or
             solsta == mosek.solsta.prim_infeas_cer or
150
             solsta == mosek.solsta.near_dual_infeas_cer or
151
             solsta == mosek.solsta.near_prim_infeas_cer):
152
         print("Primal or dual infeasibility certificate found.\n")
153
154
       elif solsta == mosek.solsta.unknown:
         print("Unknown solution status")
155
156
         print("Other solution status")
157
158
     # call the main function
159
160
         main ()
161
     except mosek.Exception as e:
162
         print ("ERROR: %s" % str(e.errno))
163
         if e.msg is not None:
164
             print ("\t%s" % e.msg)
165
166
             sys.exit(1)
     except:
167
         import traceback
168
         traceback.print_exc()
169
         sys.exit(1)
170
     sys.exit(0)
```

#### 5.4.1.2 Source code comments

This example introduces several new functions. The first new function Task.appendbarvars is used to append the semidefinite variable:

```
task.appendbarvars(BARVARDIM)
```

Symmetric matrices are created using the function Task.appendsparsesymmat:

```
_[ sdo1.py ]-
        task.appendsparsesymmat(BARVARDIM[0],
104
                                   barci,
105
                                   barcj,
106
                                   barcval)
107
108
     svma0 = 
109
        task.appendsparsesymmat(BARVARDIM[0],
110
111
                                   barai[0].
                                   baraj[0],
112
113
                                   baraval[0])
114
     syma1 = \
115
        {\tt task.appendsparsesymmat(BARVARDIM[0],}
116
                                   barai[1],
117
                                   baraj[1],
118
                                    baraval[1])
119
```

The second argument specifies the dimension of the symmetric variable and the third argument gives the number of non-zeros in the lower triangular part of the matrix. The next three arguments specify the non-zeros in the lower-triangle in triplet format, and the last argument will be updated with a unique index of the created symmetric matrix.

After one or more symmetric matrices have been created using Task.appendsparsesymmat, we can combine them to setup a objective matrix coefficient  $\bar{c}_j$  using Task.putbarcj, which forms a linear combination of one more symmetric matrices:

The second argument specify the semidefinite variable index j; in this example there is only a single variable, so the index is 0. The next three arguments give the number of matrices used in the linear combination, their indices (as returned by Task.appendsparsesymmat), and the weights for the individual matrices, respectively. In this example, we form the objective matrix coefficient directly from a single symmetric matrix.

Similary, a constraint matrix coefficient  $\overline{A}_{ij}$  is setup by the function Task.putbaraij:

```
task.putbaraij(0, 0, [syma0], [1.0])
task.putbaraij(1, 0, [syma1], [1.0])
```

where the second argument specifies the constraint number (the corresponding row of  $\overline{A}$ ), and the third argument specifies the semidefinite variable index (the corresponding column of  $\overline{A}$ ). The next three arguments specify a weighted combination of symmetric matrices used to form the constraint matrix coefficient.

After the problem is solved, we read the solution using Task.getbarxj:

\_\_\_\_[ sdo1.py ]-

task.getbarxj(mosek.soltype.itr, 0, barx)

The function returns the half-vectorization of  $\overline{x}_j$  (the lower triangular part stacked as a column vector), where the semidefinite variable index j is given in the second argument, and the third argument is a pointer to an array for storing the numerical values.

## 5.5 Quadratic optimization

MOSEK can solve quadratic and quadratically constrained convex problems. This class of problems can be formulated as follows:

minimize 
$$\frac{1}{2}x^{T}Q^{o}x + c^{T}x + c^{f}$$
subject to  $l_{k}^{c} \leq \frac{1}{2}x^{T}Q^{k}x + \sum_{j=0}^{n-1}a_{k,j}x_{j} \leq u_{k}^{c}, \quad k = 0, \dots, m-1,$ 

$$l_{j}^{x} \leq x_{j} \leq u_{j}^{x}, \quad j = 0, \dots, n-1.$$
(5.7)

Without loss of generality it is assumed that  $Q^o$  and  $Q^k$  are all symmetric because

$$x^T Q x = 0.5 x^T (Q + Q^T) x.$$

This implies that a non-symmetric Q can be replaced by the symmetric matrix  $\frac{1}{2}(Q+Q^T)$ .

The problem is required to be convex. More precisely, the matrix  $Q^o$  must be positive semi-definite and the kth constraint must be of the form

$$l_k^c \le \frac{1}{2} x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j$$
 (5.8)

with a negative semi-definite  $Q^k$  or of the form

$$\frac{1}{2}x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j \le u_k^c.$$
 (5.9)

with a positive semi-definite  $Q^k$ . This implies that quadratic equalities are *not* allowed. Specifying a non-convex problem will result in an error when the optimizer is called.

## 5.5.1 Example: Quadratic objective

The following is an example of a quadratic, linearly constrained problem:

This can be written equivalently as

$$\begin{array}{lll} \text{minimize} & 1/2x^TQ^ox + c^Tx \\ \text{subject to} & Ax & \geq & b \\ & x & \geq & 0, \end{array}$$

where

$$Q^o = \left[ \begin{array}{ccc} 2 & 0 & -1 \\ 0 & 0.2 & 0 \\ -1 & 0 & 2 \end{array} \right], c = \left[ \begin{array}{c} 0 \\ -1 \\ 0 \end{array} \right], A = \left[ \begin{array}{ccc} 1 & 1 & 1 \end{array} \right], \text{ and } b = 1.$$

Please note that MOSEK always assumes that there is a 1/2 in front of the  $x^TQx$  term in the objective. Therefore, the 1 in front of  $x_0^2$  becomes 2 in Q, i.e.  $Q_{0,0}^o = 2$ .

#### **5.5.1.1** Source code

```
——[ qo1.py ]————
        Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
    #
        File:
                 qo1.py
        Purpose: Demonstrate how to solve a quadratic
    #
                 optimization problem using the MOSEK Python API.
    ##
8
    import sys
10
    import os
11
12
    import mosek
13
    # If numpy is installed, use that, otherwise use the
15
    # Mosek's array module.
16
17
        from numpy import array,zeros,ones
18
19
    except ImportError:
        from mosek.array import array, zeros, ones
20
21
    # Since the actual value of Infinity is ignores, we define it solely
22
    # for symbolic purposes:
23
    inf = 0.0
24
25
    # Define a stream printer to grab output from MOSEK
    def streamprinter(text):
27
        sys.stdout.write(text)
28
        sys.stdout.flush()
29
30
```

```
# We might write everything directly as a script, but it looks nicer
    # to create a function.
32
    def main ():
      # Open MOSEK and create an environment and task
      # Make a MOSEK environment
35
      env = mosek.Env ()
      # Attach a printer to the environment
37
      env.set_Stream (mosek.streamtype.log, streamprinter)
38
39
      # Create a task
      task = env.Task()
40
      task.set_Stream (mosek.streamtype.log, streamprinter)
41
      # Set up and input bounds and linear coefficients
42
43
      bkc = [ mosek.boundkey.lo ]
      blc = [ 1.0 ]
44
      buc = [ inf ]
45
46
      bkx = [ mosek.boundkey.lo,
47
                mosek.boundkey.lo,
                mosek.boundkey.lo ]
49
      blx = [0.0, 0.0, 0.0]
50
      bux = [ inf, inf, inf ]
51
            = [ 0.0, -1.0, 0.0 ]
52
      asub = [ array([0]), array([0]), array([0]) ]
53
      aval = [array([1.0]), array([1.0]), array([1.0])]
54
      numvar = len(bkx)
56
      numcon = len(bkc)
      # Append 'numcon' empty constraints.
59
      # The constraints will initially have no bounds.
      task.appendcons(numcon)
61
      # Append 'numvar' variables.
      # The variables will initially be fixed at zero (x=0).
64
      task.appendvars(numvar)
66
      for j in range(numvar):
      # Set the linear term c_j in the objective.
68
        task.putcj(j,c[j])
69
70
        # Set the bounds on variable j
        # blx[j] <= x_j <= bux[j]
71
        task.putbound(mosek.accmode.var,j,bkx[j],blx[j],bux[j])
72
        # Input column j of A
73
74
        task.putacol( j,
                                          # Variable (column) index.
75
                      asub[j],
                                          # Row index of non-zeros in column j.
                      aval[j])
                                          # Non-zero Values of column j.
76
      for i in range(numcon):
77
        task.putbound(mosek.accmode.con,i,bkc[i],blc[i],buc[i])
78
80
      # Input the objective sense (minimize/maximize)
      task.putobjsense(mosek.objsense.maximize)
81
      # Set up and input quadratic objective
83
      qsubi = [ 0, 1, 2, 2 ]
85
      qsubj = [0, 1,
                          0, 2
      qval = [2.0, 0.2, -1.0, 2.0]
      task.putqobj(qsubi,qsubj,qval)
```

```
task.putobjsense(mosek.objsense.minimize)
90
91
92
       # Optimize
       task.optimize()
93
       # Print a summary containing information
       # about the solution for debugging purposes
95
       task.solutionsummary(mosek.streamtype.msg)
96
97
       prosta = task.getprosta(mosek.soltype.itr)
98
       solsta = task.getsolsta(mosek.soltype.itr)
100
       # Output a solution
101
       xx = zeros(numvar, float)
102
       task.getxx(mosek.soltype.itr,
103
105
       if solsta == mosek.solsta.optimal or solsta == mosek.solsta.near_optimal:
106
           print("Optimal solution: %s" % xx)
107
       elif solsta == mosek.solsta.dual_infeas_cer:
108
109
           print("Primal or dual infeasibility.\n")
       elif solsta == mosek.solsta.prim_infeas_cer:
110
           print("Primal or dual infeasibility.\n")
       elif solsta == mosek.solsta.near_dual_infeas_cer:
112
           print("Primal or dual infeasibility.\n")
113
       elif solsta == mosek.solsta.near_prim_infeas_cer:
114
           print("Primal or dual infeasibility.\n")
115
116
       elif mosek.solsta.unknown:
        print("Unknown solution status")
117
118
         print("Other solution status")
119
120
     # call the main function
121
122
     try:
         main()
     except mosek.Exception as e:
124
         print ("ERROR: %s" % str(e.errno))
125
         if e.msg is not None:
126
             import traceback
127
             traceback.print_exc()
128
             print ("\t%s" % e.msg)
129
         sys.exit(1)
130
131
     except:
         import traceback
132
         traceback.print_exc()
         sys.exit(1)
134
     print ("Finished OK")
135
     sys.exit(0)
136
```

## 5.5.1.2 Example code comments

Most of the functionality in this example has already been explained for the linear optimization example in Section 5.2 and it will not be repeated here.

This example introduces one new function, Task.putqobj, which is used to input the quadratic terms

of the objective function.

Since  $Q^o$  is symmetric only the lower triangular part of  $Q^o$  is inputted. The upper part of  $Q^o$  is computed by MOSEK using the relation

$$Q_{ij}^o = Q_{ji}^o.$$

Entries from the upper part may *not* appear in the input.

The lower triangular part of the matrix  $Q^o$  is specified using an unordered sparse triplet format (for details, see Section 5.13.3):

```
qsubi = [ 0, 1, 2, 2 ]
qsubj = [ 0, 1, 0, 2 ]
qval = [ 2.0, 0.2, -1.0, 2.0 ]
```

Please note that

- only non-zero elements are specified (any element not specified is 0 by definition),
- the order of the non-zero elements is insignificant, and
- only the lower triangular part should be specified.

Finally, the matrix  $Q^o$  is loaded into the task:

```
task.putqobj(qsubi,qsubj,qval) [qo1.py]
```

#### 5.5.2 Example: Quadratic constraints

In this section describes how to solve a problem with quadratic constraints. Please note that quadratic constraints are subject to the convexity requirement (5.8).

Consider the problem:

$$\begin{array}{ll} \text{minimize} & x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2 \\ \text{subject to} & 1 & \leq & x_1 + x_2 + x_3 - x_1^2 - x_2^2 - 0.1x_3^2 + 0.2x_1x_3, \\ & x \geq 0. \end{array}$$

This is equivalent to

$$\begin{array}{ll} \text{minimize} & 1/2x^TQ^ox + c^Tx \\ \text{subject to} & 1/2x^TQ^0x + Ax & \geq & b, \end{array}$$

where

$$Q^{o} = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 0.2 & 0 \\ -1 & 0 & 2 \end{bmatrix}, c = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, b = 1.$$

$$Q^{0} = \begin{bmatrix} -2 & 0 & 0.2 \\ 0 & -2 & 0 \\ 0.2 & 0 & -0.2 \end{bmatrix}.$$

#### 5.5.2.1 Source code

```
—[ qcqo1.py ]—
       Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
2
    # File:
              qcqo1.py
    # Purpose: Demonstrates how to solve small linear
7
                optimization problem using the MOSEK Python API.
8
    import sys
10
11
12
    import mosek
    # If numpy is installed, use that, otherwise use the
13
    # Mosek's array module.
14
15
        from numpy import array,zeros,ones
16
    except ImportError:
17
        from mosek.array import array, zeros, ones
18
19
    # Since the actual value of Infinity is ignores, we define it solely
20
    # for symbolic purposes:
21
22
    inf = 0.0
    # Define a stream printer to grab output from MOSEK
24
25
    def streamprinter(text):
        sys.stdout.write(text)
26
        sys.stdout.flush()
28
    # We might write everything directly as a script, but it looks nicer
    # to create a function.
30
    def main ():
31
32
      # Make a MOSEK environment
      env = mosek.Env ()
33
      # Attach a printer to the environment
34
      env.set_Stream (mosek.streamtype.log, streamprinter)
35
36
      # Create a task
37
      task = env.Task(0,0)
38
      # Attach a printer to the task
      task.set_Stream (mosek.streamtype.log, streamprinter)
40
42
      # Set up and input bounds and linear coefficients
```

```
bkc = [ mosek.boundkey.lo ]
       blc = [ 1.0 ]
buc = [ inf ]
45
46
47
       bkx = [ mosek.boundkey.lo,
48
49
                   mosek.boundkey.lo,
                  mosek.boundkey.lo ]
50
       blx = [ 0.0, 0.0, 0.0 ]
bux = [ inf, inf, inf ]
51
52
53
             = [ 0.0, -1.0, 0.0 ]
54
55
       asub = [ array([0]), array([0]), array([0]) ]
aval = [ array([1.0]), array([1.0]), array([1.0]) ]
56
57
58
59
       numvar = len(bkx)
       numcon = len(bkc)
60
       NUMANZ = 3
       # Append 'numcon' empty constraints.
62
       # The constraints will initially have no bounds.
63
       task.appendcons(numcon)
64
65
        #Append 'numvar' variables.
66
        # The variables will initially be fixed at zero (x=0).
67
        task.appendvars(numvar)
69
        #Optionally add a constant term to the objective.
70
       task.putcfix(0.0)
71
72
       for j in range(numvar):
         # Set the linear term c_j in the objective.
74
          task.putcj(j,c[j])
75
          # Set the bounds on variable j
76
          # blx[j] <= x_j <= bux[j]
77
          task.putbound(mosek.accmode.var,j,bkx[j],blx[j],bux[j])
          # Input column j of A
79
80
          task.putacol(j,
                                               # Variable (column) index.
                        asub[j],
                                               # Row index of non-zeros in column j.
81
                                               # Non-zero Values of column j.
                        aval[j])
82
83
       for i in range(numcon):
84
          task.putbound(mosek.accmode.con,i,bkc[i],blc[i],buc[i])
85
86
87
        # Set up and input quadratic objective
88
       qsubi = [ 0,  1,  2,  2  ]
qsubj = [ 0,  1,  0,  2  ]
qval = [ 2.0, 0.2, -1.0, 2.0 ]
89
91
92
       task.putqobj(qsubi,qsubj,qval)
93
94
95
       # The lower triangular part of the Q^0
       # matrix in the first constraint is specified.
96
        # This corresponds to adding the term
       \# - x0^2 - x1^2 - 0.1 x2^2 + 0.2 x0 x2
98
       qsubi = [ 0,
100
                         1,
       qsubj = [ 0, 1,
                                 2, 0 ]
101
```

```
qval = [-2.0, -2.0, -0.2, 0.2]
102
103
       # put Q^0 in constraint with index 0.
104
105
       task.putqconk (0, qsubi,qsubj, qval);
106
107
       # Input the objective sense (minimize/maximize)
108
       task.putobjsense(mosek.objsense.minimize)
109
110
       # Optimize the task
111
112
       task.optimize()
113
       # Print a summary containing information
114
       # about the solution for debugging purposes
115
       task.solutionsummary(mosek.streamtype.msg)
116
117
       prosta = task.getprosta(mosek.soltype.itr)
118
       solsta = task.getsolsta(mosek.soltype.itr)
119
120
       # Output a solution
121
122
       xx = zeros(numvar, float)
       task.getxx(mosek.soltype.itr,
123
124
125
       if solsta == mosek.solsta.optimal or solsta == mosek.solsta.near_optimal:
126
           print("Optimal solution: %s" % xx)
127
       elif solsta == mosek.solsta.dual_infeas_cer:
128
129
           print("Primal or dual infeasibility.\n")
       elif solsta == mosek.solsta.prim_infeas_cer:
130
131
           print("Primal or dual infeasibility.\n")
       elif solsta == mosek.solsta.near_dual_infeas_cer:
132
           print("Primal or dual infeasibility.\n")
133
       elif solsta == mosek.solsta.near_prim_infeas_cer:
134
           print("Primal or dual infeasibility.\n")
135
       elif mosek.solsta.unknown:
         print("Unknown solution status")
137
138
         print("Other solution status")
139
140
     # call the main function
141
142
     try:
         main ()
143
     except mosek.Exception as e:
144
         print ("ERROR: %s" % str(code))
145
146
         if msg is not None:
             print ("\t^{s}" % e.msg)
147
              sys.exit(1)
148
     except:
149
         import traceback
150
151
         traceback.print_exc()
         sys.exit(1)
152
     sys.exit(0)
```

The only new function introduced in this example is Task.putqconk, which is used to add quadratic terms to the constraints. While Task.putqconk add quadratic terms to a specific constraint, it is also possible to input all quadratic terms in all constraints in one chunk using the Task.putqcon function.

# 5.6 The solution summary

All computations inside MOSEK are performed using finite precision floating point numbers. This implies the reported solution isonly be an approximate optimal solution. Therefore after solving an optimization problem it is important to investigate how good an approximation the solution is. This can easily be done using the function Task.solutionsummary which reports how much the solution violate the primal and dual constraints and the primal and dual objective values. Recall for a convex optimization problem the optimality conditions are:

- The primal solution must satisfy all the primal constraints.
- The dual solution much satisfy all the dual constraints.
- The primal and dual objective values must be identical.

Thus the solution summary reports information that makes it possible to evaluate the quality of the solution obtained.

In case of a linear optimization problem the solution summary may look like

```
Basic solution summary

Problem status : PRIMAL_AND_DUAL_FEASIBLE

Solution status : OPTIMAL

Primal. obj: -4.6475314286e+002 Viol. con: 2e-014 var: 0e+000

Dual. obj: -4.6475316001e+002 Viol. con: 7e-009 var: 4e-016
```

The summary reports information for the basic solution. In this case we see:

- The problem status is primal and dual feasible which means the problem has an optimal solution. The problem status can be obtained using Task.getprosta.
- The solution status is optimal. The solution status can be obtained using Task.getsolsta.
- Next information about the primal solution is reported. The information consists of the objective value and violation measures for the primal solution. In this case violations for the constraints and variables are small meaning the solution is very close to being an exact feasible solution. The violation measure for the variables is the worst violation of the solution in any of the bounds on the variables.

The constraint and variable violations are computed with Task.getpviolcon and Task.getpviolvar.

- Similarly for the dual solution the violations are small and hence the dual solution is feasible. The
  constraint and variable violations are computed with Task.getdviolcon and Task.getdviolvar
  respectively.
- Finally, it can be seen that the primal and dual objective values are almost identical. Using Task.getprimalobj and Task.getdualobj the primal and dual objective values can be obtained.

To summarize in this case a primal and a dual solution with small feasiblity violations are available. Moreover, the primal and dual objective values are almost identical and hence it can be concluded that the reported solution is a good approximation to the optimal solution.

Now what happens if the problem does not have an optimal solution e.g. it is primal infeasible. In that case the solution summary may look like

```
Basic solution summary
Problem status : PRIMAL_INFEASIBLE
Solution status : PRIMAL_INFEASIBLE_CER
Dual. obj: 3.5894503823e+004 Viol. con: 0e+000 var: 2e-008
```

i.e. MOSEK reports that the solution is a certificate of primal infeasibility. Since the problem is primal infeasible it does not make sense to report any information about the primal solution. However, the dual solution should be a certificate of the primal infeasibility. If the problem is a minimization problem then the dual objective value should be positive and in the case of a maximization problem it should be negative. The quality of the certificate can be evaluated by comparing the dual objective value to the violations. Indeed if the objective value is large compared to the largest violation then the certificate highly accurate. Here is an example

```
Basic solution summary
Problem status : PRIMAL_INFEASIBLE
Solution status : PRIMAL_INFEASIBLE_CER
Dual. obj: 3.0056574100e-005 Viol. con: 9e-013 var: 2e-011
```

of a not so strong infeasibility certificate because the dual objective value is small compared to largest violation.

In the case a problem is dual infeasible then the solution summary may look like

```
Basic solution summary

Problem status : DUAL_INFEASIBLE

Solution status : DUAL_INFEASIBLE_CER

Primal. obj: -1.4500853392e+001 Viol. con: 0e+000 var: 0e+000
```

Observe when a solution is a certificate of dual infeasibility then the primal solution contains the certificate. Moreoever, given the problem is a minimization problem the objective value should negative and the objective should be large compared to the worst violation if the certificate is strong.

# 5.7 Integer optimization

An optimization problem where one or more of the variables are constrained to integer values is denoted an integer optimization problem.

#### 5.7.1 Example: Mixed integer linear optimization

In this section the example

maximize 
$$x_0 + 0.64x_1$$
  
subject to  $50x_0 + 31x_1 \le 250$ ,  
 $3x_0 - 2x_1 \ge -4$ ,  
 $x_0, x_1 \ge 0$  and integer (5.10)

is used to demonstrate how to solve a problem with integer variables.

#### **5.7.1.1** Source code

The example (5.10) is almost identical to a linear optimization problem except for some variables being integer constrained. Therefore, only the specification of the integer constraints requires something new compared to the linear optimization problem discussed previously. In MOSEK these constraints are specified using the function Task.putvartype as shown in the code:

The complete source for the example is listed below.

```
—[milo1.py]—
    ##
         Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
    #
         File:
    #
                  milo1.py
         Purpose: Demonstrates how to solve a small mixed
    #
                   integer linear optimization problem using the MOSEK Python API.
    ##
10
    import sys
11
    import mosek
12
    # If numpy is installed, use that, otherwise use the
    # Mosek's array module.
14
    try:
15
        from numpy import array,zeros,ones
16
    except ImportError:
17
        from mosek.array import array, zeros, ones
18
19
    # Since the actual value of Infinity is ignores, we define it solely
    # for symbolic purposes:
21
    inf = 0.0
22
23
    # Define a stream printer to grab output from MOSEK
24
    def streamprinter(text):
        sys.stdout.write(text)
26
        sys.stdout.flush()
28
    # We might write everything directly as a script, but it looks nicer
29
    # to create a function.
    def main ():
31
      # Make a MOSEK environment
32
      env = mosek.Env ()
33
      # Attach a printer to the environment
34
      env.set_Stream (mosek.streamtype.log, streamprinter)
35
36
      # Create a task
      task = env.Task(0.0)
38
      # Attach a printer to the task
39
40
      task.set_Stream (mosek.streamtype.log, streamprinter)
```

```
bkc = [ mosek.boundkey.up, mosek.boundkey.lo ]
      blc = [
                                      -4.0 ]
                           -inf,
43
      buc = [
                           250.0,
                                                inf ]
44
45
      bkx = [ mosek.boundkey.lo, mosek.boundkey.lo ]
46
47
      blx = [
                             0.0.
                                                 0.0 ]
      bux = [
                             inf,
                                                inf ]
48
49
      c = [
                             1.0,
                                                 0.64]
50
51
      asub = [ array([0, 1]),
                                     array([0,
                                                  1])
52
      aval = [ array([50.0, 3.0]), array([31.0, -2.0]) ]
53
      numvar = len(bkx)
55
56
      numcon = len(bkc)
57
      # Append 'numcon' empty constraints.
58
      # The constraints will initially have no bounds.
      task.appendcons(numcon)
60
61
      #Append 'numvar' variables.
62
      # The variables will initially be fixed at zero (x=0).
63
      task.appendvars(numvar)
64
65
      for j in range(numvar):
66
        \# Set the linear term c_{-j} in the objective.
67
        task.putcj(j,c[j])
68
        # Set the bounds on variable j
69
        # blx[j] <= x_j <= bux[j]
70
        task.putvarbound(j,bkx[j],blx[j],bux[j])
        # Input column j of A
72
        task.putacol(j,
                                          # Variable (column) index.
73
                      asub[j],
                                          # Row index of non-zeros in column j.
74
                      aval[j])
                                          # Non-zero Values of column j.
75
      task.putconboundlist(range(numcon),bkc,blc,buc)
77
78
      # Input the objective sense (minimize/maximize)
79
      task.putobjsense(mosek.objsense.maximize)
80
81
      # Define variables to be integers
82
      task.putvartypelist([ 0, 1 ],
83
                           [ mosek.variabletype.type_int,
84
85
                             mosek.variabletype.type_int ])
86
      # Optimize the task
87
      task.optimize()
89
      # Print a summary containing information
      # about the solution for debugging purposes
91
      task.solutionsummary(mosek.streamtype.msg)
92
      prosta = task.getprosta(mosek.soltype.itg)
94
      solsta = task.getsolsta(mosek.soltype.itg)
96
      # Output a solution
```

```
xx = zeros(numvar, float)
100
       task.getxx(mosek.soltype.itg,xx)
101
102
       if solsta in [ mosek.solsta.integer_optimal, mosek.solsta.near_integer_optimal ]:
103
           print("Optimal solution: %s" % xx)
104
       elif solsta == mosek.solsta.dual_infeas_cer:
           print("Primal or dual infeasibility.\n")
106
       elif solsta == mosek.solsta.prim_infeas_cer:
107
108
           print("Primal or dual infeasibility.\n")
       elif solsta == mosek.solsta.near_dual_infeas_cer:
109
110
           print("Primal or dual infeasibility.\n")
       elif solsta == mosek.solsta.near_prim_infeas_cer:
111
           print("Primal or dual infeasibility.\n")
112
       elif mosek.solsta.unknown:
113
         if prosta == mosek.prosta.prim_infeas_or_unbounded:
114
             print("Problem status Infeasible or unbounded.\n")
         elif prosta == mosek.prosta.prim_infeas:
116
             print("Problem status Infeasible.\n")
117
         elif prosta == mosek.prosta.unkown:
118
             print("Problem status unkown.\n")
119
120
         else:
             print("Other problem status.\n")
121
122
           print("Other solution status")
123
124
     # call the main function
125
     try:
126
127
         main ()
     except mosek.Exception as msg:
128
129
         #print "ERROR: %s" % str(code)
         if msg is not None:
130
             print ("\t%s" % msg)
131
             sys.exit(1)
132
     except:
133
         import traceback
134
         traceback.print_exc()
135
         sys.exit(1)
136
137
     sys.exit(0)
```

#### 5.7.1.2 Code comments

Please note that when Task.getsolutionslice is called, the integer solution is requested by using soltype.itg. No dual solution is defined for integer optimization problems.

#### 5.7.2 Specifying an initial solution

Integer optimization problems are generally hard to solve, but the solution time can often be reduced by providing an initial solution for the solver. Solution values can be set using Task.putsolution (for inputting a whole solution) or Task.putsolutioni (for inputting solution values related to a single variable or constraint).

It is not necessary to specify the whole solution. By setting the iparam.mio\_construct\_sol parameter

to <code>onoffkey.on</code> and inputting values for the integer variables only, will force MOSEK to compute the remaining continuous variable values.

If the specified integer solution is infeasible or incomplete, MOSEK will simply ignore it.

## 5.7.3 Example: Specifying an integer solution

Consider the problem

```
 \begin{array}{ll} \text{maximize} & 7x_0 + 10x_1 + x_2 + 5x_3 \\ \text{subject to} & x_0 + x_1 + x_2 + x_3 \leq 2.5 \\ & x_0, x_1, x_2 \text{ integer }, x_0, x_1, x_2, x_3 \geq 0 \end{array}
```

The following example demonstrates how to optimize the problem using a feasible starting solution generated by selecting the integer values as  $x_0 = 0, x_1 = 2, x_2 = 0$ .

```
_____[mioinitsol.py]____
    ##
2
    #
         Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
    #
         File:
                  mioinitsol.py
    #
         Purpose: Demonstrates how to solve a small mixed
                   integer linear optimization problem using the MOSEK Python API.
    ##
    import sys
10
11
    import mosek
12
13
    # If numpy is installed, use that, otherwise use the
14
    # Mosek's array module.
15
16
17
        from numpy import array,zeros,ones
    except ImportError:
18
        from mosek.array import array, zeros, ones
19
20
    # Since the actual value of Infinity is ignores, we define it solely
21
    # for symbolic purposes:
    inf = 0.0
23
25
    # Define a stream printer to grab output from MOSEK
26
27
    def streamprinter(text):
        sys.stdout.write(text)
28
        sys.stdout.flush()
29
30
31
    # We might write everything directly as a script, but it looks nicer
32
    # to create a function.
33
    def main ():
        # Make a MOSEK environment
35
        env = mosek.Env ()
36
        # Attach a printer to the environment
37
        env.set_Stream (mosek.streamtype.log, streamprinter)
```

```
39
        # Create a task
40
        task = env.Task(0,0)
41
        # Attach a printer to the task
42
        task.set_Stream (mosek.streamtype.log, streamprinter)
43
45
        bkc = [ mosek.boundkey.up ]
46
        blc = [ -inf,
47
        buc = [ 2.5
48
49
        bkx = [ mosek.boundkey.lo,
50
51
                 mosek.boundkey.lo,
                 mosek.boundkey.lo,
52
53
                 mosek.boundkey.lo ]
54
        blx = [0.0, 0.0, 0.0, 0.0]
55
        bux = [ inf, inf, inf, inf ]
57
        c = [7.0, 10.0, 1.0, 5.0]
58
59
        asub = [ 0, 0, 0, 0 ]
60
        acof = [ 1.0, 1.0, 1.0, 1.0]
61
62
        ptrb = [ 0, 1, 2, 3 ]
63
        ptre = [1, 2, 3, 4]
64
65
        numvar = len(bkx)
66
        numcon = len(bkc)
67
68
        # Input linear data
69
        task.inputdata(numcon,numvar,
70
71
                        c,0.0,
                       ptrb, ptre, asub, acof,
72
                        bkc, blc, buc,
                       bkx, blx, bux)
74
75
        # Input objective sense
76
        task.putobjsense(mosek.objsense.maximize)
77
78
        # Define variables to be integers
79
        task.putvartypelist([ 0, 1, 2 ],
80
                             [ mosek.variabletype.type_int,
81
82
                               mosek.variabletype.type_int,
83
                               mosek.variabletype.type_int])
84
        # Construct an initial feasible solution from the
        # values of the integer value specified
86
        task.putintparam(mosek.iparam.mio_construct_sol,
87
88
                         mosek.onoffkey.on);
89
        # Assign values 0,2,0 to integer variables. Important to
        # assign a value to all integer constrained variables.
91
        task.putxxslice(mosek.soltype.itg,0,3,[0.0, 2.0, 0.0])
92
93
        # Optimize
94
95
        task.optimize()
```

```
97
         # Did mosek construct a feasible initial solution ?
         if task.getintinf(mosek.iinfitem.mio_construct_solution) > 0:
98
             print("Objective value of constructed integer solution: %-24.12e" % task.getdouinf(mosek.dinfitem.mio_construct_solu
99
100
             print("Intial integer solution construction failed.");
101
102
         if task.solutiondef(mosek.soltype.itg):
103
104
105
             # Output a solution
             xx = zeros(numvar, float)
106
107
             task.getxx(mosek.soltype.itg, xx)
             print("Integer optimal solution")
108
             for j in range(0,numvar) :
109
                  print("\tx[%d] = %e" % (j,xx[j]))
110
111
112
             print("No integer solution is available.")
113
     # call the main function
114
115
     try:
         main ()
116
     except mosek.Exception as e:
117
         print ("ERROR: %s" % str(e.errno))
118
119
         if e.msg is not None:
             print ("\t%s" % e.msg)
120
         sys.exit(1)
121
122
     except:
         import traceback
123
124
         traceback.print_exc()
         svs.exit(1)
125
```

# 5.8 The solution summary for mixed integer problems

The solution summary for a mixed-integer problem may look like

```
Integer solution solution summary
Problem status : PRIMAL_FEASIBLE
Solution status : INTEGER_OPTIMAL
Primal. obj: 4.0593518000e+005    Viol. con: 4e-015    var: 3e-014    itg: 3e-014
```

The main diffrence compared to continous case covered previously is that no information about the dual solution is provided. Simply because there is no dual solution available for a mixed integer problem. In this case it can be seen that the solution is highly feasible because the violations are small. Moreoever, the solution is denoted integer optimal. Observe itg: 3e-014 implies that all the integer constrained variables are at most 3e-014 from being an exact integer.

# 5.9 Response handling

After solving an optimization problem with MOSEK an approriate action must be taken depending on the outcome. Usually, the expected outcome is an optimal solution, but there may be several situations where this is not the result. E.g., if the problem is infeasible or nearly so or if the solver ran out of memory or stalled while optimizing, the result may not be as expected.

This section discusses what should be considered when an optimization has ended unsuccessfully.

Before continuing, let us consider the four status codes available in MOSEK that is relevant for the error handing:

#### The termination code:

The termination provides information about why the optimizer terminated. For instance if a time limit has been specified (this is common for mixed integer problems), the termination code will tell if this termination limit was the cause of the termination. Note that reaching a prespecified time limit is not considered an exceptional case. It must be expected that this occurs occasionally.

Note that if we want to report, e.g., that the optimizer terminated due to a time limit or because it stalled but with a feasible solution, we have to consider *both* the termination code, *and* the solution status.

The following pseudo code demonstrates a best practice way of dealing with the status codes.

```
if ( the solution status is as expected )
{
  The normal case:
    Do whatever that was planned. Note the response code is ignored because the solution has the expected status.
    Of course we may check the response anyway if we like.
}
else
{
    Exceptional case:
    Based on solution status, response and termination codes take appropriate action.
}
```

In the following example the pseudo code has implemented. The idea of the example is to read an optimization problem from a file, e.g., an MPS file and optimize it. Based on status codes an appropriate action is taken, which in this case is to print a suitable message.

```
-[response.py]-
1
    # Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
    # File:
                  response.py
    # Purpose:
                  This examples demonstrates proper response handling.
6
    import mosek
10
    import sys
11
    def streamprinter(text):
12
        sys.stdout.write(text)
13
        sys.stdout.flush()
14
15
    def main(args):
16
      if len(args) < 1:</pre>
17
        print ("No input file specified")
18
```

```
19
        return
20
      else:
        print ("Inputfile: %s" % args[0])
21
22
      with mosek.Env() as env:
23
        with env.Task(0,0) as task:
          task.set_Stream (mosek.streamtype.log, streamprinter)
25
26
          task.readdata(args[0])
27
          e = None
28
29
          trmcode = None
30
          try:
            trmcode = task.optimize()
31
          except mosek.MosekException as err:
32
33
34
          solsta = task.getsolsta(mosek.soltype.itr)
35
36
          if solsta in [ mosek.solsta.optimal,
37
                           mosek.solsta.near_optimal ]:
38
            print ("An optimal basic solution is located.")
39
            task.solutionsummary(mosek.streamtype.log)
40
41
          elif solsta in [ mosek.solsta.dual_infeas_cer,
                            mosek.solsta.near_dual_infeas_cer ]:
42
            print ("Dual infeasibility certificate found.")
43
44
          elif solsta in [ mosek.solsta.prim_infeas_cerl,
                            mosek.solsta.near_prim_infeas_cer ]:
45
            printf("Primal infeasibility certificate found.\n");
46
          elif solsta == mosek.solsta.sta_unknown:
47
            # The solutions status is unknown. The termination code
            # indicating why the optimizer terminated prematurely.
49
            print ("The solution status is unknown.")
50
51
            if trmcode is not None:
              print ("Termination code: %s" % str(trmcode))
52
               #print mosek.getcodedesc(trmcode)
            elif e is not None:
54
              print ("Error:")
55
56
              print (e)
57
58
             print ("An unexpected solution status is obtained.")
59
    if __name__ == '__main__':
60
      import sys
61
      main(sys.argv[1:])
```

# 5.10 Problem modification and reoptimization

Often one might want to solve not just a single optimization problem, but a sequence of problem, each differing only slightly from the previous one. This section demonstrates how to modify and re-optimize an existing problem. The example we study is a simple production planning model.

## 5.10.1 Example: Production planning

A company manufactures three types of products. Suppose the stages of manufacturing can be split into three parts, namely Assembly, Polishing and Packing. In the table below we show the time required for each stage as well as the profit associated with each product.

Product no.	Assembly (minutes)	Polishing (minutes)	Packing (minutes)	Profit (\$)
0	2	3	2	1.50
1	4	2	3	2.50
2	3	3	2	3.00

With the current resources available, the company has 100,000 minutes of assembly time, 50,000 minutes of polishing time and 60,000 minutes of packing time available per year.

Now the question is how many items of each product the company should produce each year in order to maximize profit?

Denoting the number of items of each type by  $x_0, x_1$  and  $x_2$ , this problem can be formulated as the linear optimization problem:

and

$$x_0, x_1, x_2 \ge 0.$$

The following code loads this problem into the optimization task.

```
-[ production.py ]-
    # Create a MOSEK environment
35
    env = mosek.Env ()
36
37
    # Create a task
38
    task = env.Task(0,0)
39
    # Attach a printer to the task
40
    task.set_Stream (mosek.streamtype.log, streamprinter)
42
    # Bound keys for constraints
43
44
    bkc = [mosek.boundkey.up,
                  mosek.boundkey.up,
45
                  mosek.boundkey.up]
    # Bound values for constraints
47
    blc = array ([-inf, -inf, -inf])
48
    buc = array ([100000.0, 50000.0, 60000.0])
49
50
    # Bound keys for variables
51
    bkx = [mosek.boundkey.lo,
52
                   mosek.boundkey.lo,
53
                   mosek.boundkey.lo]
    # Bound values for variables
55
```

```
blx = array ([ 0.0, 0.0, 0.0])
     bux = array ([+inf, +inf, +inf])
57
     # Objective coefficients
     csub = array([ 0, 1,
                                2])
60
     cval = array([ 1.5, 2.5, 3.0 ])
62
     # We input the A matrix column-wise
63
     # asub contains row indexes
     asub = array([ 0, 1, 2,
65
                    0, 1, 2,
                    0, 1, 2])
67
     # acof contains coefficients
     acof = array([ 2.0, 3.0, 2.0,
69
                    4.0, 2.0, 3.0,
                    3.0, 3.0, 2.0])
     # aptrb and aptre contains the offsets into asub and acof where
72
     # columns start and end respectively
     aptrb = array([ 0, 3, 6 ])
74
     aptre = array([ 3, 6, 9 ])
75
     numvar = len(bkx)
77
     numcon = len(bkc)
79
     # Append the constraints
80
     task.appendcons(numcon)
81
82
     # Append the variables.
     task.appendvars(numvar)
84
     # Input objective
86
     task.putcfix(0.0)
     task.putclist(csub,cval)
     # Put constraint bounds
     task.putconboundslice(0, numcon, bkc, blc, buc)
91
     # Put variable bounds
93
     task.putvarboundslice(0, numvar,bkx, blx, bux)
94
     # Input A non-zeros by columns
96
     for j in range(numvar):
         ptrb,ptre = aptrb[j],aptre[j]
98
         task.putacol(j,
99
                      asub[ptrb:ptre],
100
                      acof[ptrb:ptre])
101
102
     # Input the objective sense (minimize/maximize)
103
     task.putobjsense(mosek.objsense.maximize)
104
105
106
     # Optimize the task
107
     task.optimize()
108
     # Output a solution
110
     xx = zeros(numvar, float)
111
112
     {\tt task.getsolutionslice(mosek.soltype.bas,}
                           mosek.solitem.xx,
113
```

```
114 0,numvar,

115 xx)

116 print ("xx =", [i for i in xx])
```

## 5.10.2 Changing the A matrix

Suppose we want to change the time required for assembly of product 0 to 3 minutes. This corresponds to setting  $a_{0,0} = 3$ , which is done by calling the function Task.putaij as shown below.

The problem now has the form:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2$$
  
subject to  $3x_0 + 4x_1 + 3x_2 \le 100000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 50000$ ,  
 $2x_0 + 3x_1 + 2x_2 \le 60000$ , (5.11)

and

$$x_0, x_1, x_2 \ge 0.$$

After changing the A matrix we can find the new optimal solution by calling Task.optimize again.

## 5.10.3 Appending variables

We now want to add a new product with the following data:

Product no.	Assembly (minutes)	Polishing (minutes)	Packing (minutes)	Profit (\$)
3	4	0	1	1.00

This corresponds to creating a new variable  $x_3$ , appending a new column to the A matrix and setting a new value in the objective. We do this in the following code.

```
-[production.py]-
     # Append a new varaible x_3 to the problem */
122
     task.appendvars(1)
123
124
     # Set bounds on new varaible
125
     task.putbound(mosek.accmode.var,
126
                    task.getnumvar()-1,
127
                    mosek.boundkey.lo,
128
129
130
                    +inf)
131
```

```
# Change objective
132
     task.putcj(task.getnumvar()-1,1.0)
133
134
     # Put new values in the A matrix
     acolsub = array([0, 2])
136
     acolval =
                 array([4.0, 1.0])
137
138
     task.putacol(task.getnumvar()-1, # column index
139
140
                   acolsub,
                   acolval)
141
```

After this operation the problem looks this way:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2 + 1.0x_3$$
  
subject to  $3x_0 + 4x_1 + 3x_2 + 4x_3 \le 100000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 50000$ ,  
 $2x_0 + 3x_1 + 2x_2 + 1x_3 \le 60000$ , (5.12)

and

$$x_0, x_1, x_2, x_3 \ge 0.$$

## 5.10.4 Reoptimization

When

Task.optimize is called MOSEK will store the optimal solution internally. After a task has been modified and

Task.optimize is called again the solution will automatically be used to reduce solution time of the new problem, if possible.

In this case an optimal solution to problem (5.11) was found and then added a column was added to get (5.12). The simplex optimizer is well suited for exploiting an existing primal or dual feasible solution. Hence, the subsequent code instructs MOSEK to choose the simplex optimizer freely when optimizing.

```
# Change optimizer to simplex free and reoptimize
task.putintparam(mosek.iparam.optimizer,mosek.optimizertype.free_simplex)
task.optimize()
```

#### 5.10.5 Appending constraints

Now suppose we want to add a new stage to the production called "Quality control" for which 30000 minutes are available. The time requirement for this stage is shown below:

Product no.	Quality control (minutes)
0	1
1	2
2	1
3	1

This corresponds to adding the constraint

$$x_0 + 2x_1 + x_2 + x_3 \le 30000$$

to the problem which is done in the following code:

```
-[production.py]-
    # Append a new constraint
    task.appendcons(1)
146
    # Set bounds on new constraint
148
    task.putconbound( task.getnumcon()-1, mosek.boundkey.up,-inf, 30000)
149
150
    # Put new values in the A matrix
151
    arowsub = array([0, 1, 2, 3])
153
     arowval = array([1.0, 2.0, 1.0, 1.0])
154
155
     task.putarow(task.getnumcon()-1, # row index
156
157
                  arowsub,
                  arowval)
158
```

# 5.11 Solution analysis

## 5.11.1 Retrieving solution quality information with the API

Information about the solution quality may be retrieved in the API with the help of the following functions:

- Task.getsolutioninfo: Obtains information about objective values and the solution violations of the constraints.
- Task.analyzesolution: Print additional information about the solution, e.g basis condition number and optionally a list of violated constraints.
- Task.getpviolcon, Task.getpviolvar, Task.getpviolbarvar, Task.getpviolcones, Task.getdviolcon, Task.getdviolvar, Task.getdviolbarvar, Task.getdviolcones. Obtains violation of the individual constraints.

# 5.12 Efficiency considerations

Although MOSEK is implemented to handle memory efficiently, the user may have valuable knowledge about a problem, which could be used to improve the performance of MOSEK This section discusses some tricks and general advice that hopefully make MOSEK process your problem faster.

#### Avoiding memory fragmentation:

MOSEK stores the optimization problem in internal data structures in the memory. Initially MOSEK will allocate structures of a certain size, and as more items are added to the problem the structures are reallocated. For large problems the same structures may be reallocated many times causing memory fragmentation. One way to avoid this is to give MOSEK an estimated size of your problem using the functions:

- Task.putmaxnumvar. Estimate for the number of variables.
- Task.putmaxnumcon. Estimate for the number of constraints.
- Task.putmaxnumcone. Estimate for the number of cones.
- Task.putmaxnumbarvar. Estimate for the number of semidefinite matrix variables.
- $\bullet$  Task.putmaxnumanz. Estimate for the number of non-zeros in A.
- Task.putmaxnumqnz. Estimate for the number of non-zeros in the quadratic terms.

None of these functions change the problem, they only give hints to the eventual dimension of the problem. If the problem ends up growing larger than this, the estimates are automatically increased.

## Do not mix put- and get- functions:

For instance, the functions Task.putacol and Task.getacol. MOSEK will queue put- commands internally until a get- function is called. If every put- function call is followed by a get-function call, the queue will have to be flushed often, decreasing efficiency.

In general get- commands should not be called often during problem setup.

Use the LIFO principle when removing constraints and variables:

MOSEK can more efficiently remove constraints and variables with a high index than a small index.

An alternative to removing a constraint or a variable is to fix it at 0, and set all relevant coefficients to 0. Generally this will not have any impact on the optimization speed.

Add more constraints and variables than you need (now):

The cost of adding one constraint or one variable is about the same as adding many of them. Therefore, it may be worthwhile to add many variables instead of one. Initially fix the unused variable at zero, and then later unfix them as needed. Similarly, you can add multiple free constraints and then use them as needed.

Use one environment (env) only:

If possible share the environment (env) between several tasks. For most applications you need to create only a single env.

Do not remove basic variables:

When doing re-optimizations, instead of removing a basic variable it may be more efficient to fix the variable at zero and then remove it when the problem is re-optimized and it has left the basis. This makes it easier for MOSEK to restart the simplex optimizer.

#### 5.12.1 API overhead

The Python interface is a thin wrapper around a native MOSEK library. The layer between the Python application and the native MOSEK library is made as thin as possible to minimize the overhead from function calls.

The methods in mosek. Env and mosek. Task are all written in C and resides in the module pymosek. Each method converts the call parameter data structures (i.e. creates a complete copy of the data), calls a MOSEK function and converts the returned values back into Python structures.

The following rules will often improve the performance of the MOSEK/Python API:

Reuse Env and Task whenever possible

There may be some overhead involved in creating and deleting task and environment objects, so if possible reuse these.

Make sure to delete task and environment when not in use anymore

Using the with-construction (available in python 2.6 and later) will allow automatic deletion of the environment and task. If this is not an option, use Env.\_del\_\_() and Task.\_\_del\_\_() to destroy the objects. Failing to do this may cause memory leaks in some cases.

Avoid input loops

Whenever possible imput data in large chunks or vectors instead of using loops. For small put- and get- methods there is a significant overhead, so for example inputting one row of the A-matrix at the time may be much slower than inputting the whole matrix.

For example, a loop with Task.putarow may be replaced with one Task.putarowlist, or a loop of Task.putqobjij may be replaced with Task.putqobj.

# 5.13 Conventions employed in the API

#### 5.13.1 Naming conventions for arguments

In the definition of the MOSEK Python API a consistent naming convention has been used. This implies that whenever for example numcon is an argument in a function definition it indicates the number of constraints.

In Table 5.2 the variable names used to specify the problem parameters are listed. The relation between the variable names and the problem parameters is as follows:

Python name	Python type	Dimension	Related problem parameter
numcon	int		m
numvar	int		n
numcone	int		t
numqonz	int		$q_{ij}^o$
qosubi	<pre>int[]</pre>	numqonz	$egin{array}{l} q_{ij}^o \ q_{ij}^o \end{array}$
qosubj	int[]	numqonz	$q_{ij}^{o}$
qoval			·
С	float[]	numvar	$c_{j}$
cfix	float		$c^f$
numqcnz	int		$q_{ij}^k$
qcsubk	int[]	qcnz	$egin{array}{l} q_{ij}^k \ q_{ij}^k \ q_{ij}^k \end{array}$
qcsubi	int[]	qcnz	$q_{ij}^{k}$
qcsubj	int[]	qcnz	$q_{ij}^{ec{k}}$
aptrb	int[]	numvar	$a_{ij}$
aptre	<pre>int[]</pre>	numvar	$a_{ij}$
asub	int[]	aptre[numvar-1]	$a_{ij}$
aval	float[]	aptre[numvar-1]	$a_{ij}$
blc	float[]	numcon	$l_k^c$
buc	float[]	numcon	$u_k^c$
blx	float[]	numvar	$l_k^x$
bux	float[]	numvar	$u_k^x$

Table 5.2: Naming convensions used in the MOSEK Python API.

Symbolic constant	Lower bound	Upper bound
boundkey.fx	finite	identical to the lower bound
boundkey.fr	minus infinity	plus infinity
boundkey.lo	finite	plus infinity
boundkey.ra	finite	finite
boundkey.up	minus infinity	finite

Table 5.3: Interpretation of the bound keys.

• The quadratic terms in the objective:

$$q^o_{\texttt{qosubi[t]},\texttt{qosubj[t]}} = \texttt{qoval[t]}, \ t = 0, \dots, \texttt{numqonz} - 1. \tag{5.13}$$

• The linear terms in the objective:

$$c_i = \mathbf{c}[\mathbf{j}], \ j = 0, \dots, \text{numvar} - 1 \tag{5.14}$$

• The fixed term in the objective:

$$c^f = \mathtt{cfix}.$$

• The quadratic terms in the constraints:

$$q_{\texttt{qcsubi[t]},\texttt{qcsubj[t]}}^{\texttt{qcsubk[t]}} = \texttt{qcval[t]}, \ t = 0, \dots, \texttt{numqcnz} - 1. \tag{5.15}$$

• The linear terms in the constraints:

$$\begin{array}{ll} a_{\texttt{asub}[\texttt{t}],\texttt{j}} = \texttt{aval}[\texttt{t}], & t = \texttt{ptrb}[\texttt{j}], \dots, \texttt{ptre}[\texttt{j}] - 1, \\ & j = 0, \dots, \texttt{numvar} - 1. \end{array} \tag{5.16}$$

• The bounds on the constraints are specified using the variables bkc, blc, and buc. The components of the integer array bkc specify the bound type according to Table 5.3. For instance bkc[2]=boundkey.lo means that  $-\infty < l_2^c$  and  $u_2^c = \infty$ . Finally, the numerical values of the bounds are given by

$$l_k^c = \mathtt{blc}[\mathtt{k}], \ k = 0, \ldots, \mathtt{numcon} - 1$$

and

$$u_k^c = \text{buc}[k], k = 0, \dots, \text{numcon} - 1.$$

• The bounds on the variables are specified using the variables bkx, blx, and bux. The components in the integer array bkx specify the bound type according to Table 5.3. The numerical values for the lower bounds on the variables are given by

$$l_i^x = \mathtt{blx}[\mathtt{j}], \ j = 0, \dots, \mathtt{numvar} - 1.$$

The numerical values for the upper bounds on the variables are given by

$$u_i^x = \text{bux}[j], j = 0, \dots, \text{numvar} - 1.$$

#### 5.13.1.1 Bounds

A bound on a variable or on a constraint in MOSEK consists of a bound key, as defined in Table 5.3, a lower bound value and an upper bound value. Even if a variable or constraint is bounded only from below, e.g.  $x \ge 0$ , both bounds are inputted or extracted; the value inputted as upper bound for  $(x \ge 0)$  is ignored.

#### 5.13.2 Vector formats

Three different vector formats are used in the MOSEK API:

Full vector:

This is simply an array where the first element corresponds to the first item, the second element to the second item etc. For example to get the linear coefficients of the objective in task, one would write

```
c = zeros(numvar,float)
task.getc(c)
```

where number of variables in the problem.

Vector slice:

A vector slice is a range of values. For example, to get the bounds associated constraint 3 through 10 (both inclusive) one would write

Please note that items in MOSEK are numbered from 0, so that the index of the first item is 0, and the index of the n 'th item is n-1.

Sparse vector:

A sparse vector is given as an array of indexes and an array of values. For example, to input a set of bounds associated with constraints number 1, 6, 3, and 9, one might write

```
bound_index = [
                          1,
                                      6,
                                                   3,
bound_key = [boundkey.fr,boundkey.lo,boundkey.up,boundkey.fx]
lower_bound = [
                    0.0,
                                                 0.0,
                                                             5.0]
                                  -10.0,
upper_bound = [
                    0.0,
                                    0.0,
                                                 6.0,
                                                             5.0]
task.putboundlist(accmode.con, bound_index,
                  bound_key,lower_bound,upper_bound)
```

Note that the list of indexes need not be ordered.

#### 5.13.3 Matrix formats

The coefficient matrices in a problem are inputted and extracted in a sparse format, either as complete or a partial matrices. Basically there are two different formats for this.

#### 5.13.3.1 Unordered triplets

In unordered triplet format each entry is defined as a row index, a column index and a coefficient. For example, to input the A matrix coefficients for  $a_{1,2}=1.1$ ,  $a_{3,3}=4.3$ , and  $a_{5,4}=0.2$ , one would write as follows:

```
subi = array([ 1, 3, 5 ])
subj = array([ 2, 3, 4 ])
cof = array([ 1.1, 4.3, 0.2 ])
task.putaijlist(subi,subj,cof)
```

Please note that in some cases (like Task.putaijlist) only the specified indexes remain modified — all other are unchanged. In other cases (such as Task.putqconk) the triplet format is used to modify all entries — entries that are not specified are set to 0.

#### 5.13.3.2 Row or column ordered sparse matrix

In a sparse matrix format only the non-zero entries of the matrix are stored. MOSEK uses a sparse packed matrix format ordered either by rows or columns. In the column-wise format the position of the non-zeros are given as a list of row indexes. In the row-wise format the position of the non-zeros are given as a list of column indexes. Values of the non-zero entries are given in column or row order.

A sparse matrix in column ordered format consists of:

asub:

List of row indexes.

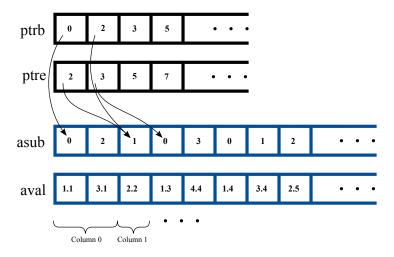


Figure 5.1: The matrix A (5.17) represented in column ordered packed sparse matrix format.

#### aval:

List of non-zero entries of A ordered by columns.

## ptrb:

Where ptrb[j] is the position of the first value/index in aval / asub for column j.

#### ptre:

Where ptre[j] is the position of the last value/index plus one in aval / asub for column j.

The values of a matrix A with numcol columns are assigned so that for

$$j = 0, \dots, numcol - 1.$$

We define

$$a_{\mathtt{asub}[k],j} = \mathtt{aval}[k], k = \mathtt{ptrb}[j], \dots, \mathtt{ptre}[j] - 1.$$

As an example consider the matrix

$$A = \begin{bmatrix} 1.1 & 1.3 & 1.4 \\ & 2.2 & & 2.5 \\ 3.1 & & 3.4 & \\ & & 4.4 & & \end{bmatrix}. \tag{5.17}$$

which can be represented in the column ordered sparse matrix format as

```
\begin{array}{lll} {\tt ptrb} & = & [0,2,3,5,7], \\ {\tt ptre} & = & [2,3,5,7,8], \\ {\tt asub} & = & [0,2,1,0,3,0,2,1], \\ {\tt aval} & = & [1.1,3.1,2.2,1.3,4.4,1.4,3.4,2.5]. \end{array}
```

Fig. 5.1 illustrates how the matrix A (5.17) is represented in column ordered sparse matrix format.

#### 5.13.3.3 Row ordered sparse matrix

The matrix A (5.17) can also be represented in the row ordered sparse matrix format as:

```
\begin{array}{lll} \mathtt{ptrb} &=& [0,3,5,7],\\ \mathtt{ptre} &=& [3,5,7,8],\\ \mathtt{asub} &=& [0,2,3,1,4,0,3,2],\\ \mathtt{aval} &=& [1.1,1.3,1.4,2.2,2.5,3.1,3.4,4.4]. \end{array}
```

### 5.13.4 Array objects

The MOSEK Python API provides a simple array object in the module mosekarr. This includes a one-dimensional dense array which can be of type Float, Int or Object, and a few operators and functions to create and modify array objects.

Arrays can be constructed in several ways:

```
# Create an array of integers
a0 = array([1,2,3],int)
# Create an array of floats
a1 = array([1,2,3],float)
# Create an integer array of ones
a2 = ones(10)
# Create an float array of ones
a3 = ones(10,float)
# Create a range of integers 5,6,...,9
a4 = range(5,10)
# Create and array of objects
a5 = array(['a string', 'b string', 10, 2.2])
```

A limited set of operations on arrays are available - these should work more or less like the equivalent Numeric operations:

```
a = ones(10,float)
b = 1.0 * arange(10)

# element-wise multiplication, addition and subtraction
c0 = a * b
c1 = a + b
c2 = a - b
```

```
# multiplly each element by 2.1
c4 = a * 2.1
# add 2 to each element
c5 = a + 2
```

If more advanced array operations is needed, it is necessary to install the Python Numeric package.

# 5.13.5 Typical problems using the Python API

Since all all type-information in Python is implicit, type-checking is performed only when required, and in certain cases it is necessary to explicitly write type information.

The MOSEK API currently *only* supports its own array object (mosek.array.array) and Pythonnumpy arrays. Other array or list compatible objects will are accepted but are converted.

Typically type errors occur in two situations:

- An array argument did not have the right type and could not be converted.
- An array was expected, but the argument was not an array and not a list-compatible object.

Furthermore, please note that mosek.array module only supports a limited set of array types: int32, int64, float64 and bool. The numerical types support normal simple mathematical operation (addition, subtraction, multiplication etc.)

# 5.14 The license system

By default a license token is checked out when Task.optimize is first called and is returned when the MOSEK environment is deleted. Calling Task.optimize from different threads using the same MOSEK environment only consumes one license token.

To change the license systems behavior to returning the license token after each call to Task.optimize set the parameter iparam.cache\_license to onoffkey.off. Please note that there is a small overhead associated with setting this parameter, since checking out a license token from the license server can take a small amount of time.

Additionally license checkout and checkin can be controlled manually with the functions Env.checkinlicense and Env.checkoutlicense.

## 5.14.1 Waiting for a free license

By default an error will be returned if no license token is available. By setting the parameter iparam.license\_waitMOSEK can be instructed to wait until a license token is available.

# Chapter 6

# Nonlinear API tutorial

This chapter provides information about how to solve general convex nonlinear optimization problems using MOSEK. By general nonlinear problems it is meant problems that cannot be formulated as a conic quadratic optimization or a convex quadratically constrained optimization problem.

In general it is recommended not to use nonlinear optimizer unless needed. The reasons are

- MOSEK has no way of checking whether the formulated problem is convex and if this assumption
  is not satisfied the optimizer will not work.
- The nonlinear optimizer requires 1st and 2nd order derivative information which is hard to provide correctly i.e. it is nontrivial to program the code that computes the derivative information.
- The specification of nonlinear problems requires C function callbacks. Such C function callbacks cannot be dump to disk and that makes it hard to report issues to MOSEK support.
- The algorithm employed for nonlinear optimization problems is not as good as the one employed for conic problems i.e. conic problems has special that can be exploited to make the optimizer faster and more robust.

This leads to following advices in decreasing order of importance.

- Consider reformulating the problem to a conic quadratic optimization problem if at all possible.
   In particular many problems involving polynomial terms can easily be reformulated to conic quadratic form.
- Consider reformulating the problem to a separable optimization problem because that simplifies
  the issue with verifying convexity and computing 1st and 2nd order derivatives significantly. In
  most cases problems on separable form also solves faster because of the simpler structure of the
  functions. In Section 6.1 some utility code that makes it easy to solve separable problems is
  discussed.
- Finally, if the problem cannot be reformulated to separable form then use a modelling language like AMPL or GAMS. The reason is the modeling language will do all the computing of function

values and derivatives. This eliminates an important source of errors. Therefore, it is strongly recommended to use a modelling language at the protype stage.

# 6.1 Separable convex (SCopt) interface

The MOSEK Python API provides a way to add simple non-linear functions composed from a limited set of non-linear terms. Non-linear terms can be mixed with quadratic terms in objective and constraints.

We consider a normal linear problem with additional non-linear terms z:

minimize 
$$z_0(x) + c^T x$$
 subject to 
$$l_i^c \leq z_i(x) + a_i^T x \leq u_i^c, \ i = 1 \dots m$$
 
$$z \in \mathbb{R}^n$$
 
$$x \in \mathbb{R}^n$$
 
$$z \cdot \mathbb{P}^n \to \mathbb{P}^{(m+1)}$$

Using the separable non-linear interface it is possible to add non-linear functions of the form

$$z_i(x) = \sum_{k=1}^{K_i} w_k^i(x_{p_{ik}}), \ w_k^i : \mathbb{R} \to \mathbb{R}$$

In other words, each non-linear function  $z_i$  is a sum of separable functions  $w_k^i$  of one variable each. A limited set of functions are supported; each  $w_k^i$  can be one of the separable functions:

Separable function	Operator name	
fx ln(x)	scopr.ent	Entropy function
$\int fe^{gx+h}$	scopr.exp	Exponential function
f ln(gx+h)	scopr.log	Logarithm
$f(x+h)^g$	scopr.pow	Power function

where f, g and h are constants.

This formulation does not guarantee convexity. For MOSEK to be able to solve the problem, following requirements must be met:

- If the objective is minimized, the sum of non-linear terms must be convex, otherwise it must be concave.
- Any constraint bounded below must be concave, and any constraint bounded above must be convex.
- Each separable term must be twice differentiable within the bounds of the variable it is applied to.

If these are not satisfied MOSEK may not be able to solve the problem or produce a meaningful status report. For details see section 6.1.3.

# 6.1.1 Adding separable terms

Separable terms — both objective and constraint terms — are added in one chunk and replaces any previously added non-linear terms. Each individual term can be describes by a set of values:

- opr, an indicator of which of the basic functions is applied,
- *i*, the constraint index for terms in constraints,
- j, the index of the variable the functions is applied to,
- f, the constant f in the basic function,
- $\bullet$  g, the constant g in the basic function, and
- h, the constant h in the basic function.

For example:

Term	opr	j	f	g	h
$0.1x_1\ln(x_1)$	scopr.ent	1	0.1	0.0	0.0
$e^{x_2+1.1}$	scopr.exp	2	1.0	1.0	1.1
$2.1x_1^{1.75}$	scopr.pow	1	2.1	1.75	0.0
$\sqrt{x_1}$	scopr.pow	1	1.0	0.5	0.0
$ln(x_2 + 1.2)$	scopr.log	2	1.0	1.0	1.2

The separable terms of the objective can now be defined by a set of arrays

```
mosek.scopr array opro # which method
int array oprjo # variable index
float array oprfo # f constant
float array oprgo # g constant
float array oprho # h constant
```

and the separable constraint terms can be defined the same way, only using an additional array indicating which constraint each term belongs in

```
mosek.scopr array oprc # which method
int array opric # constraint index
int array oprjc # variable index
float array oprfc # f constant
float array oprgc # g constant
float array oprhc # h constant
```

We can now input the separable terms using the Task.putSCeval function:

```
task.putSCeval(opro, oprjo, oprfo, oprgo, oprho, oprc, opric, oprjc, oprfc, oprgc, oprhc);
```

If we wish to input no objective terms, all opr\*o arguments may be None, and similarly, if we have no constraint terms, we may let all opr\*c be None.

This will replace all existing non-linear separable terms. To remove all non-linear separable terms, we can call Task.clearSCeval.

## 6.1.2 Example: Simple separable problem

We consider the convex separable problem

with 4 separable terms, two in the objective and two in the constraints.

The separable terms of the objective can be described by the arrays

```
–[demb781.py]–
    opro = [ mosek.scopr.exp, mosek.scopr.exp ]
    oprjo = [ 2, 3 ]
71
    oprfo = [ 1.0, 1.0 ]
72
    oprgo = [ 1.0, 1.0 ]
73
    oprho = [ 0.0, 0.0 ]
74
   and constraint terms by
                                                —[demb781.py]-
    oprc = [ mosek.scopr.exp, mosek.scopr.exp ]
77
    opric = [ 0, 0 ]
    oprjc = [ 4, 5 ]
79
    oprfc = [ 1.0, 1.0 ]
    oprgc = [ 1.0, 1.0 ]
81
    oprhc = [ 0.0, 0.0 ]
```

#### 6.1.2.1 Source code: demb781

```
—[demb781.py]—
    #
      Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
2
               demb781.py
    #
      File:
    #
      Purpose: Demonstrates how to solve a simple non-liner separable problem
       using the SCopt interface for Python. Then problem is this:
         Minimize e^x2 + e^x3
   #
         Such that e^x4 + e^x5
                                                      <= 1
q
    #
                       x0 + x1 - x2
                                                       = 0
10
                      - x0 - x1
                                                      = 0e+00
11
    #
                    0.5 x0
                                       - x4
                                                       = 1.3862944
    #
12
                                                 - x5 = 0
    #
13
   #
                   x0 ... x5 are unrestricted
14
    #
15
   ##
16
   from __future__ import with_statement
```

```
import sys
19
20
21
    import mosek
22
    def streamprinter(text):
         sys.stdout.write(text)
24
        sys.stdout.flush()
25
26
    def main ():
27
      with mosek.Env() as env:
28
         env.set_Stream (mosek.streamtype.log, streamprinter)
29
30
         with env. Task(0,0) as task:
          task.set_Stream (mosek.streamtype.log, streamprinter)
31
32
33
          numvar = 6
          numcon = 5
34
          bkc = [ mosek.boundkey.up,
36
                   mosek.boundkey.fx,
37
                   mosek.boundkey.fx,
38
                   mosek.boundkey.fx,
39
40
                   mosek.boundkey.fx ]
          blc = [ 0.0, 0.0, 0.0, 1.3862944, 0.0 ]
41
          buc = [ 1.0, 0.0, 0.0, 1.3862944, 0.0 ]
42
43
          bkx = [ mosek.boundkey.fr ] * numvar
44
          blx = [ 0.0 ] * numvar
45
          bux = [ 0.0 ] * numvar
46
          aptrb = [ 0, 0, 3, 6, 8 ]
48
          aptre = [ 0, 3, 6, 8, 10 ]
49
          asubi = [0, 1, 2, 3, 4]
50
          asubj = [ 0, 1, 2,
51
                     0, 1, 3,
                     0, 4,
53
54
                     1, 5]
          aval = [1.0, 1.0, -1.0,
55
                     -1.0, -1.0, -1.0,
56
57
                      0.5, -1.0,
                      1.0, -1.0]
58
59
          task.appendvars(numvar)
60
          task.appendcons(numcon)
61
62
          task.putobjsense(mosek.objsense.minimize)
63
          task.putvarboundslice(0, numvar, bkx, blx, bux)
65
           task.putconboundslice(0, numcon, bkc, blc, buc)
66
67
          task.putarowlist(asubi, aptrb, aptre, asubj, aval )
68
          opro = [ mosek.scopr.exp, mosek.scopr.exp ]
70
          oprjo = [ 2, 3 ]
71
72
          oprfo = [ 1.0, 1.0 ]
          oprgo = [ 1.0, 1.0 ]
73
74
          oprho = [ 0.0, 0.0 ]
75
```

```
76
           oprc = [ mosek.scopr.exp, mosek.scopr.exp ]
77
           opric = [ 0, 0 ]
78
           oprjc = [ 4, 5 ]
79
           oprfc = [ 1.0, 1.0 ]
80
           oprgc = [ 1.0, 1.0 ]
           oprhc = [ 0.0, 0.0 ]
82
83
84
           task.putSCeval(opro, oprjo, oprfo, oprgo, oprho,
                           oprc, opric, oprjc, oprfc, oprgc, oprhc)
85
           task.optimize()
87
88
           res = [ 0.0 ] * numvar
89
           task.getsolutionslice(
90
             mosek.soltype.itr,
             mosek.solitem.xx,
92
             0, numvar,
93
             res)
94
95
           print ( "Solution is: %s" % res )
96
97
    main()
```

# 6.1.3 Ensuring convexity and differentiability

Some simple rules can be set up to ensure that the problem satisfies MOSEK's convexity and differentiability requirements. First of all, for any variable  $x_i$  used in a separable term, the variable bounds must define a range within which the function is twice differentiable.

We can define these bounds as follows:

Separable function	Operator name	Safe $x$ bounds
fx ln(x)	scopr.ent	0 < x.
$fe^{gx+h}$	scopr.exp	$-\infty < x < \infty$ .
f ln(gx + h)	scopr.log	If $g > 0$ : $-h/g < x$ .
		If $g < 0$ : $x < -h/g$ .
$f(x+h)^g$	scopr.pow	If $g > 0$ and integer: $-\infty < x < \infty$ .
		If $g < 0$ and integer: either $-h < x$ or $x < -h$ .
		Otherwise: $-h < x$ .

To ensure convexity, we require that each  $z_i(x)$  is either a sum of convex terms or a sum of concave terms. The following table lists convexity for the relevant ranges for f > 0 — changing the sign of f switches concavity/convexity.

Separable function	Operator name	
fx ln(x)	scopr.ent	Convex within safe bounds.
$\int e^{gx+h}$	scopr.exp	Convex for all $x$ .
f ln(gx + h)	scopr.log	Concave within safe bounds.
$f(x+h)^g$	scopr.pow	If $g$ is even integer: convex within safe bounds.
		If g is odd integer: concave $(-\infty, -h)$ , convex $(-h, \infty)$ .
		If $0 < g < 1$ : concave within safe bounds.
		Otherwise: convex within safe bounds.

# 6.1.4 SCopt Reference

Functions used to manipulate separable terms:

```
(opro, oprjo, oprfo, oprgo, oprho, oprc, opric, oprjc, oprfc, oprgc, oprhc)
    Replace all current non-linear separable terms with a new set.
     opro
         List of function indicators defining the objective terms; see scopr.
     oprjo
         List of variable indexes for the objective terms.
     oprfo
         List of f values for the objective terms
     oprgo
         List of g values for the objective terms
     oprho
         List of h values for the objective terms
     oprc
         List of function indicators defining the constraint terms; see scopr.
     opric
         List of variable indexes for the constraint terms.
     oprjc
         List of constraint indexes for the constraint terms.
         List of f values for the constraint terms
     oprgc
         List of g values for the constraint terms
     oprhc
         List of h values for the constraint terms
()
```

Remove all non-linear separable terms from the task.

Constants used to define :

Entropy function,  $fx\ln(x)$ 

Exponential function,  $fe^{gx+h}$ 

Logarithm, f ln(gx + h)

Power function,  $f(x+h)^g$ 

# Chapter 7

# Advanced API tutorial

This chapter provides information about additional problem classes and functionality provided in the Python API.

# 7.1 The progress call-back

Some of the API function calls, notably Task.optimize, may take a long time to complete. Therefore, during the optimization a call-back function is called frequently, to provide information on the progress of the call. From the call-back function it is possible

- to obtain information on the solution process,
- to report of the optimizer's progress, and
- to ask MOSEK to terminate, if desired.

# 7.1.1 Source code example

The following source code example documents how the progress call-back function can be used.

```
[ callback.py ]

1 ##
2 # Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.

3 #
4 # File: callback.py
5 #
6 # Purpose: To demonstrate how to use the progress
7 # callback.
8 #
9 # Use this script as follows:
10 # callback.py psim 25fv47.mps
11 # callback.py dsim 25fv47.mps
12 # callback.py intpnt 25fv47.mps
```

```
The first argument tells which optimizer to use
    #
14
                    i.e. psim is primal simplex, dsim is dual simplex
15
16
                   and intpnt is interior-point.
17
    from __future__ import with_statement
19
    import sys
20
21
    import mosek
22
    from mosek import *
24
25
    def makeUserCallback(maxtime):
26
27
        def userCallback(caller,
28
                          douinf,
                          intinf,
29
                          lintinf):
30
            opttime = 0.0
31
32
            if caller == callbackcode.begin_intpnt:
33
                print ("Starting interior-point optimizer")
34
35
            elif caller == callbackcode.intpnt:
                      = intinf[iinfitem.intpnt_iter
36
                itrn
                         = douinf[dinfitem.intpnt_primal_obj]
                         = douinf[dinfitem.intpnt_dual_obj ]
38
                dobj
                        = douinf[dinfitem.intpnt_time
                stime
39
40
                opttime = douinf[dinfitem.optimizer_time
41
                print ("Iterations: %-3d" % itrn)
                print (" Elapsed time: %6.2f(%.2f) " % (opttime,stime))
43
                print (" Primal obj.: %-18.6e Dual obj.: %-18.6e" % (pobj,dobj))
44
45
            elif caller == callbackcode.end_intpnt:
                print ("Interior-point optimizer finished.")
46
            elif caller == callbackcode.begin_primal_simplex:
                print ("Primal simplex optimizer started.")
48
49
            elif caller == callbackcode.update_primal_simplex:
                       = intinf[iinfitem.sim_primal_iter ]
50
                itrn
                         = douinf[dinfitem.sim_obj
51
52
                stime = douinf[dinfitem.sim_time
                                                             ٦
                opttime = douinf[dinfitem.optimizer_time
                                                            ٦
53
54
                print ("Iterations: %-3d" % itrn)
55
                print (" Elapsed time: %6.2f(%.2f)" % (opttime,stime))
56
                print (" Obj.: %-18.6e" % pobj )
57
            elif caller == callbackcode.end_primal_simplex:
58
                print ("Primal simplex optimizer finished.")
            elif caller == callbackcode.begin_dual_simplex:
60
                print ("Dual simplex optimizer started.")
61
            elif caller == callbackcode.update_dual_simplex:
62
                itrn
                        = intinf[iinfitem.sim_dual_iter
63
                         = douinf[dinfitem.sim_obj
64
                stime = douinf[dinfitem.sim_time
                                                             1
65
                opttime = douinf[dinfitem.optimizer_time
67
                print ("Iterations: %-3d" % itrn)
                print (" Elapsed time: %6.2f(%.2f)" % (opttime,stime))
68
                print (" Obj.: %-18.6e" % pobj)
69
            elif caller == callbackcode.end_dual_simplex:
```

```
71
                 print ("Dual simplex optimizer finished.")
             elif caller == callbackcode.begin_bi:
72
                 print ("Basis identification started.")
73
             elif caller == callbackcode.end_bi:
74
                 print ("Basis identification finished.")
75
             else:
77
                 pass
78
             if opttime >= maxtime:
79
                  # mosek is spending too much time. Terminate it.
80
81
                  return 1
82
83
             return 0
         return userCallback
84
85
86
     def msgPrinter(msg):
         sys.stdout.write(msg)
87
         sys.stdout.flush()
88
89
     def main(args):
90
91
       if len(args) < 3:</pre>
92
93
           print ("Too few input arguments. Syntax:")
           print ("\tcallback.py psim inputfile")
94
           print ("\tcallback.py dsim inputfile")
95
           print ("\tcallback.py intpnt inputfile")
96
97
98
       with mosek.Env() as env:
99
100
           with mosek. Task(env) as task:
               filename = args[2]
101
               task.readdata(filename)
102
103
               task.set_Stream(streamtype.log, msgPrinter)
104
                   args[1] == 'psim':
106
107
                    task.putintparam(iparam.optimizer,optimizertype.primal_simplex)
               elif args[1] == "dsim":
108
                    {\tt task.putintparam(iparam.optimizer,optimizertype.dual\_simplex)}
109
110
               elif args[1] == "intpnt":
                    task.putintparam(iparam.optimizer,optimizertype.intpnt)
111
112
               # Turn all MOSEK logging off (note that errors and other messages
113
               # are still sent through the log stream)
114
               task.putintparam(iparam.log, 0)
116
               usercallback = makeUserCallback(maxtime = 3600)
117
               task.set_Progress(usercallback)
118
119
               task.optimize()
120
121
               task.solutionsummary(streamtype.msg)
122
123
     if __name__ == '__main__':
124
125
         main(sys.argv)
```

# 7.2 Solving linear systems involving the basis matrix

A linear optimization problem always has an optimal solution which is also a basic solution. In an optimal basic solution there are exactly m basic variables where m is the number of rows in the constraint matrix A. Define

$$B \in \mathbb{R}^{m \times m}$$

as a matrix consisting of the columns of A corresponding to the basic variables.

The basis matrix B is always non-singular, i.e.

$$det(B) \neq 0$$

or equivalently that  $B^{-1}$  exists. This implies that the linear systems

$$B\bar{x} = w \tag{7.1}$$

and

$$B^T \bar{x} = w \tag{7.2}$$

each has a unique solution for all w .

MOSEK provides functions for solving the linear systems (7.1) and (7.2) for an arbitrary  $w_{-i}$ .

### 7.2.1 Identifying the basis

To use the solutions to (7.1) and (7.2) it is important to know how the basis matrix B is constructed. Internally MOSEK employs the linear optimization problem

maximize 
$$c^T x$$
  
subject to  $Ax - x^c = 0$ ,  $l^x \le x \le u^x$ ,  $l^c \le x^c \le u^c$ . (7.3)

where

$$x^c \in \mathbb{R}^m$$
 and  $x \in \mathbb{R}^n$ .

The basis matrix is constructed of m columns taken from

$$[A -I].$$

If variable  $x_j$  is a basis variable, then the j 'th column of A denoted  $a_{:,j}$  will appear in B. Similarly, if  $x_i^c$  is a basis variable, then the i 'th column of -I will appear in the basis. The ordering of the

basis variables and therefore the ordering of the columns of B is arbitrary. The ordering of the basis variables may be retrieved by calling the function

task.initbasissolve(basis)

where basis is an array of variable indexes.

This function initializes data structures for later use and returns the indexes of the basic variables in the array basis. The interpretation of the basis is as follows. If

then the *i*'th basis variable is  $x_i^c$ . Moreover, the *i* 'th column in B will be the *i*'th column of -I. On the other hand if

$$basis[i] \ge numcon$$
,

then the i 'th basis variable is variable

$$x_{\mathtt{basis}[i]-\mathtt{numcon}}$$

and the i 'th column of B is the column

$$A_{:,(\mathtt{basis}[i]-\mathtt{numcon})}$$

For instance if basis[0] = 4 and numcon = 5, then since basis[0] < numcon, the first basis variable is  $x_4^c$ . Therefore, the first column of B is the fourth column of -I. Similarly, if basis[1] = 7, then the second variable in the basis is  $x_{basis[1]-numcon} = x_2$ . Hence, the second column of B is identical to  $a_{:,2}$ .

# 7.2.2 An example

Consider the linear optimization problem:

minimize 
$$x_0 + x_1$$
  
subject to  $x_0 + 2x_1 \le 2$ ,  
 $x_0 + x_1 \le 6$ ,  
 $x_0, x_1 \ge 0$ . (7.4)

Suppose a call to Task.initbasissolve returns an array basis so that

basis[0] = 1, basis[1] = 2.

Then the basis variables are  $x_1^c$  and  $x_0$  and the corresponding basis matrix B is

$$\left[\begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array}\right].$$

Please note the ordering of the columns in B .

The following program demonstrates the use of Task.solvewithbasis.

```
____[solvebasis.py]____
    #
       Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
3
5
    #
      File
               : solvebasis.py
       Purpose : To demonstrate the usage of
                   MSK_solvewithbasis on the problem:
9
    #
    #
                   maximize x0 + x1
10
11
    #
                   st.
                           x0 + 2.0 x1 \le 2
12
                           x0 + x1 <= 6
13
    #
                           x0 >= 0, x1>= 0
    #
14
15
                    The problem has the slack variables
16
                    xc0, xc1 on the constraints
17
    #
                    and the variabels x0 and x1.
    #
19
    #
                    maximize x0 + x1
    #
21
                      x0 + 2.0 x1 -xc1
22
                      x0 + x1 -xc2 = 6
23
                      x0 >= 0, x1>= 0,
24
                       xc1 \le 0 , xc2 \le 0
    #
26
27
28
   import mosek
29
    def streamprinter(text):
30
       sys.stdout.write(text)
31
32
        sys.stdout.flush()
33
34
    def main():
        numcon = 2
35
        numvar = 2
36
37
        # Since the value infinity is never used, we define
38
        # 'infinity' symbolic purposes only
39
        infinity = 0
40
41
        c = [1.0, 1.0]
42
        ptrb = [0, 2]
43
        ptre = [2, 3]
        asub = [0, 1,
45
               0, 1]
46
        aval = [1.0, 1.0,
47
               2.0, 1.0]
48
        bkc = [mosek.boundkey.up,
                mosek.boundkey.up]
50
        blc = [-infinity,
52
                -infinity]
53
        buc = [2.0,
```

```
6.0]
55
56
         bkx = [mosek.boundkey.lo,
57
                  mosek.boundkey.lo]
58
         blx = [0.0,
59
                  0.0]
61
62
         bux = [+infinity,
63
                 +infinity]
         w1 = [2.0, 6.0]
64
         w2 = [1.0, 0.0]
65
         try:
66
67
           with mosek.Env() as env:
              with env.Task(0,0) as task:
68
                task.set_Stream (mosek.streamtype.log, streamprinter)
69
               task.inputdata(numcon, numvar,
71
                                С,
72
                                0.0,
                                ptrb,
73
                                ptre,
74
75
                                asub,
                                aval,
76
77
                                bkc,
                                blc.
78
                                buc,
79
80
                                bkx,
                                blx,
81
82
                                bux)
               task.putobjsense(mosek.objsense.maximize)
83
               r = task.optimize()
               if r != mosek.rescode.ok:
85
                  print ("Mosek warning:",r)
86
               basis = [0] * numcon
88
                task.initbasissolve(basis)
90
                #List basis variables corresponding to columns of B
91
               varsub = [0,1]
92
93
94
               for i in range(numcon):
                  if basis[varsub[i]] < numcon:</pre>
95
                    print ("Basis variable no %d is xc%d" % (i,basis[i]))
96
                  else:
97
                    print ("Basis variable no %d is x%d" % (i,basis[i] - numcon))
98
99
                \# solve Bx = w1
100
                # varsub contains index of non-zeros in b.
101
                # On return b contains the solution x and
102
               \mbox{\tt\#} varsub the index of the non-zeros in x.
103
104
               nz = 2
105
106
               nz = task.solvewithbasis(0, nz, varsub, w1)
               print ("nz = %s" % nz)
107
               print ("Solution to Bx = w1:")
108
109
               for i in range(nz):
110
111
                  if basis[varsub[i]] < numcon:</pre>
                    print ("xc %s = %s" % (basis[varsub[i]],w1[varsub[i]]))
112
```

```
113
                  else:
                    print ("x%s = %s" % (basis[varsub[i]] - numcon, w1[varsub[i]]))
114
115
                # Solve B^Tx = w2
116
                nz = 1
117
                varsub[0] = 0
118
119
                nz = task.solvewithbasis(1, nz, varsub, w2)
120
121
                print ("Solution to B^Tx = w2:")
122
123
                for i in range(nz):
124
125
                  if basis[varsub[i]] < numcon:</pre>
                    print ("xc %s = %s" % (basis[varsub[i]], w2[varsub[i]]))
126
127
                    print ("x %s = %s" % (basis[varsub[i]] - numcon, w2[varsub[i]]) )
128
         except Exception as e:
129
           print (e)
130
131
     if __name__ == '__main__':
132
133
       main()
```

In the example above the linear system is solved using the optimal basis for (7.4) and the original right-hand side of the problem. Thus the solution to the linear system is the optimal solution to the problem. When running the example program the following output is produced.

```
basis[0] = 1
Basis variable no 0 is xc1.
basis[1] = 2
Basis variable no 1 is x0.

Solution to Bx = b:

x0 = 2.000000e+00
xc1 = -4.000000e+00

Solution to B^Tx = c:

x1 = -1.000000e+00
x0 = 1.000000e+00
```

Please note that the ordering of the basis variables is

$$\left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right]$$

and thus the basis is given by:

$$B = \left[ \begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array} \right]$$

It can be verified that

$$\left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right] = \left[\begin{array}{c} -4 \\ 2 \end{array}\right]$$

is a solution to

$$\left[\begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array}\right] \left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right] = \left[\begin{array}{c} 2 \\ 6 \end{array}\right].$$

# 7.2.3 Solving arbitrary linear systems

MOSEK can be used to solve an arbitrary (rectangular) linear system

$$Ax = b$$

using the Task.solvewithbasis function without optimizing the problem as in the previous example. This is done by setting up an A matrix in the task, setting all variables to basic and calling the Task.solvewithbasis function with the b vector as input. The solution is returned by the function.

Below we demonstrate how to solve the linear system

$$\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$
 (7.5)

with b = (1, -2) and b = (7, 0).

```
---[ solvelinear.py ]-
    from __future__ import with_statement
       Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
       File
               : solvelinear.py
       Purpose : To demonstrate the usage of MSK_solvewithbasis
                   when solving the linear system:
       1.0 x1
                            = b1
10
       -1.0 	 x0 + 1.0 	 x1 = b2
       with two different right hand sides
13
       b = (1.0, -2.0)
15
16
    #
17
       and
18
19
    # b = (7.0, 0.0)
20
21
    import mosek
22
23
    def put_a( task,
24
               aval.
25
               ptrb,
27
28
               ptre,
               numvar,
                basis):
```

```
# Since the value infinity is never used, we define
31
         # 'infinity' symbolic purposes only
32
         infinity = 0
33
         skx = [mosek.stakey.bas] * numvar
35
         skc = [mosek.stakey.fix] * numvar
37
38
         task.appendvars(numvar)
         task.appendcons(numvar)
39
40
41
         for i in range(len(asub)):
42
43
           task.putacol(i,asub[i],aval[i])
44
         for i in range(numvar):
45
           task.putconbound(i,mosek.boundkey.fx,0.0,0.0)
46
47
         for i in range(numvar):
          task.putvarbound(i,
49
                             mosek.boundkey.fr,
50
                             -infinity,
51
                             infinity)
52
53
         # Define a basic solution by specifying
54
         # status keys for variables & constraints.
55
56
         for i in range(numvar):
57
           task.putsolutioni (mosek.accmode.var,
58
59
                               mosek.soltype.bas,
                               skx[i],
61
                               0.0,
62
                               0.0,
63
                               0.0,
64
                               0.0)
66
67
         for i in range(numvar):
           task.putsolutioni (mosek.accmode.con,
68
69
70
                               mosek.soltype.bas,
                               skc[i],
71
72
                               0.0,
                               0.0,
73
74
                               0.0,
75
                               0.0)
76
77
         task.initbasissolve(basis)
78
79
80
    def main():
81
        numcon = 2
82
        numvar = 2
83
         aval = [ [ -1.0 ],
85
                  [ 1.0, 1.0 ] ]
86
         asub = [ [ 1 ],
87
                  [0, 1]
88
```

```
ptrb = [ 0,1 ]
90
         ptre = [ 1,3 ]
91
92
         #int[]
                       bsub = new int[numvar];
93
         #double[]
                       b
                          = new double[numvar];
         #int[]
                      basis = new int[numvar];
95
96
97
         with mosek.Env() as env:
98
           with mosek. Task(env) as task:
99
             # Directs the log task stream to the user specified
100
101
             # method task_msg_obj.streamCB
             task.set_Stream(mosek.streamtype.log,
102
103
                              lambda msg : sys.stdout.write(msg))
             # Put A matrix and factor A.
             # Call this function only once for a given task.
105
106
             basis = [0] * numvar
107
             b = [0.0, -2.0]
108
             bsub = [0,
                            1]
109
110
111
             put_a(task,
                   aval.
112
                   asub,
113
114
                   ptrb,
                   ptre,
115
116
                   numvar,
                   basis)
117
118
             # now solve rhs
119
             b = [1, -2]
120
             bsub = [ 0, 1 ]
121
             nz = task.solvewithbasis(0,2,bsub,b)
122
             print("\nSolution to Bx = b:\n")
124
125
             # Print solution and show correspondents
             # to original variables in the problem
126
             for i in range(nz):
127
128
               if basis[bsub[i]] < numcon:</pre>
                 print("This should never happen")
129
               else:
130
                 print("x%d = %d" % (basis[bsub[i]] - numcon, b[bsub[i]]))
131
132
             b[0]
133
                   = 7
             bsub[0] = 0
134
135
             nz = task.solvewithbasis(0,1,bsub,b);
136
137
             print("\nSolution to Bx = b:\n")
138
             # Print solution and show correspondents
139
140
             # to original variables in the problem
             for i in range(nz):
141
               if basis[bsub[i]] < numcon:</pre>
142
143
                 print ("This should never happen")
144
                 print ("x%d = %d" % (basis[bsub[i]] - numcon, b[bsub[i]] ))
145
146
```

The most important step in the above example is the definition of the basic solution using the Task.putsolutioni function, where we define the status key for each variable. The actual values of the variables are not important and can be selected arbitrarily, so we set them to zero. All variables corresponding to columns in the linear system we want to solve are set to basic and the slack variables for the constraints, which are all non-basic, are set to their bound.

The program produces the output:

```
Solution to Bx = b: x1 = 1 x0 = 3 Solution to Bx = b: x1 = 7 x0 = 7 and we can verify that x_0 = 2, x_1 = -4 is indeed a solution to (7.5).
```

# 7.3 Calling BLAS/LAPACK routines from MOSEK

Sometimes users need to perform linear algebra operations that involve dense matrices and vectors. Also MOSEK uses extensively high-performance linear algebra routines from the BLAS and LAPACK packages and some of this routine are included in the package shipped to the users.

MOSEK makes available to the user some BLAS and LAPACK routines by MOSEK functions that

- use MOSEK data types and response code:
- keep BLAS/LAPACK naming convention.

Therefore the user can leverage on efficient linear algebra routines, with a simplified interface, with no need for additional packages. In the following table we list BLAS functions:

Name	MOSEK name	Expression
AXPY	Env.axpy	$y = \alpha x + y$
DOT	Env.dot	$x^Ty$
GEMV	Env.gemv	$y = \alpha Ax + \beta y$
GEMM	Env.gemm	$C = \alpha AB + \beta C$
SYRK	Env.syrk	$C = \alpha A A^T + \beta C$

Function from LAPACK are listed below:

Name	MOSEK name	Description	
POTRF Env.potrf		Cholesky factorization	
SYEVD	Env.syevd	Eigen-values of a symmetric matrix	
SYEIG	Env.syeig	Eigen-values and eigen-vectors of a symmetric matrix	

A detailed list of the available routines follows. All code snippets are taken from the example blas-lapack distributed with MOSEK and listed below. All code snippets assume a valid MOSEK environment named env is available. For more details please refer to Section 7.3.1.

### Scaled Vectors Addiction (AXPY)

It computes the sum of a scaled vector x with a second vector y, i.e.

$$y = \alpha x + y,\tag{7.6}$$

where  $\alpha$  are two scalars and  $x, y \in \mathbb{R}^n$ . It is available through the Env.axpy. This routine may use optimized loop unrolling. Note that the results overwrites y. For example, we may use the following code:

#### Inner Product (DOT)

Given two vectors  $x, y \in \mathbb{R}^n$ , it computes the inner product (or dot product) defined as

$$x^{T} \cdot y = \sum_{i=0}^{n-1} x_{i} y_{i} = y^{T} \cdot x. \tag{7.7}$$

The inner product is a special case of the generalized matrix-vector multiplication. MOSEK provide access to BLAS implementation by the Env.dot function.

For example we may want to perform the dot product among two arrays x, y of the same dimension we can write

#### Generalized Matrix-Vector Multiplication (GEMV)

This function performs matrix-vector operations of the form

$$y = \alpha Ax + \beta y, \tag{7.8}$$

or

$$y = \alpha A^T x + \beta y. (7.9)$$

where  $\alpha, \beta$  are two scalars and  $A \in \mathbb{R}^{m \times n}$ , Dimension of x and y must be compatible with those of A depending whether it is transpose or not. MOSEK provides access to GEMV by the

Env.gemv function. Please note that the result overwrites the vector y. Expression (7.8) can be calculated as

```
| blas_lapack.py | env.gemv(mosek.transpose.no, m, n, alpha, A, x, beta,z)
```

### Generalized Matrix-Matrix Multiplication (GEMM)

This function perform a matrix-matrix multiplication followed by an addition. Given matrices A, B and C of compatible dimensions, and two scalars  $\alpha, \beta$  it performs the following

$$C = \alpha AB + \beta C. \tag{7.10}$$

Matrices A and B can be considered transposed or not, and their dimensions must be compatible accordingly.

MOSEK provides access to GEMM by the  ${\tt Env.gemm}$  function. Please note that the result overwrites the matrix C.

```
_____[blas_lapack.py]______env.gemm(mosek.transpose.no,mosek.transpose.no,m,n,k,alpha,A,B,beta,C)
```

#### Symmetric rank-k update (SYRK)

Given a symmetric matrix  $\in \mathbb{R}^{n \times n}$ , two scalars  $\alpha, \beta$  and a matrix A of rank k, this function computes either

$$C = \beta C + \alpha A^T A,\tag{7.11}$$

withfor  $A \in \mathbb{R}^{k \times n}$ , or

$$C = \beta C + \alpha A A^T, \tag{7.12}$$

for  $A \in \mathbb{R}^{k \times n}$ . The corresponding routine provided by MOSEK is **Env.syrk**. The matrix C only needs to be specified as triangular. Note also that the result ovewrites C in the relevant upper or lower triangular part, accordingly with the way it has been input.

#### Eigenvalue Computation (SYEIG)

This function returns the eigenvalues of a given square matrix A. MOSEK provides access to SYEIG by the Env. syeig function.

```
env.syeig(mosek.uplo.lo,m,Q,v) [blas_lapack.py]
```

Eigenvalue Decomposition (SYEVD)

Given a symmetric matrix A, this function returns its eigenvalue decomposition, i.e. a diagonal matrix V and a lower triangular matrix U, of the same dimension as A, such that

$$A = UVU^T$$
.

The diagonal of V contains the eigenvalues of A, while U is formed by the orthonormal eigenvectors of A stored column-wise. Note that U will ovewrites A. MOSEK provides access to SYEVD by the Env.syevd function.

```
env.syevd(mosek.uplo.lo,m,Q,v) [blas_lapack.py]
```

#### Cholesky Factorization (POTRF)

This function computes the Cholesky factorization of a symmetric positive-definite matrix  $A \in \mathbb{R}^{n \times n}$ , i.e. it return a lower triangular matrix U such that

$$A = U^T U$$
.

It is available through the function Env.potrf. Note that The result will overwrite the lower triangle of A.

# 7.3.1 A working example

The following code shows how to call the BLAS/LAPACK routines provided by MOSEK. The code has no practical purpose and it is only meant to show which kind of input the routines accept.

```
--[ blas_lapack.py ]---
    # Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
    # File:
                 blas_lapack.py
    # Purpose: To demonstrate how to call /LAPACK routines for whose MOSEK provides simplified interfaces.
7
    import mosek
10
    with mosek.Env() as env:
11
12
        n=3
13
        m=2
14
        k=3
15
16
        alpha=2.0
17
        beta=0.5
18
```

```
x=[1.0,1.0,1.0]
         y=[1.0,2.0,3.0]
21
         z=[1.0,1.0]
22
         v=[0.0,0.0]
23
         #A has m=2 rows and k=3 cols
24
         A=[1.0,1.0,2.0,2.0,3.,3.]
         #B has k=3 rows and n=3 cols
26
         B=[1.0,1.0,1.0,1.0,1.0,1.0,1.0,1.0,1.0]
27
         C=[ 0.0 for i in range(n*m)]
28
         D=[ 1.0,1.0,1.0,1.0]
29
30
         Q=[ 1.0,0.0,0.0,2.0]
31
32
         xy = []
33
       routines
34
35
         env.dot(n,x,y)
36
37
         env.axpy(n,alpha,x,y)
38
39
40
         env.gemv(mosek.transpose.no, m, n, alpha, A, x, beta,z)
41
42
         env.gemm(mosek.transpose.no,mosek.transpose.no,m,n,k,alpha,A,B,beta,C)
43
         env.syrk(mosek.uplo.lo, mosek.transpose.no, n,k,alpha, A, beta,D)
44
45
    # LAPACK routines
46
47
         env.potrf(mosek.uplo.lo,m,Q)
48
         env.syeig(mosek.uplo.lo,m,Q,v)
50
51
         env.syevd(mosek.uplo.lo,m,Q,v)
52
```

# 7.4 Automatic reformulation of QCQP problems in conic form

Despite that MOSEK can solve quadratic and quadratically constrained convex problems, as detailed in Section 5.5, it often performs better when the problems are reformulated in conic form. Moreover, the conic formulation can rely on a more sound duality theory. For this reason MOSEK provides a tool to reformulate automatically QCQP problem as Conic Quadratic problems.

We recall that QCQP problems that MOSEK can solve are of the form:

minimize 
$$\frac{1}{2}x^{T}Q^{o}x + c^{T}x + c^{f}$$
subject to  $l_{k}^{c} \leq \frac{1}{2}x^{T}Q^{k}x + \sum_{j=0}^{n-1}a_{k,j}x_{j} \leq u_{k}^{c}, \quad k = 0, \dots, m-1,$ 
subject to  $l_{k}^{cl} \leq \sum_{j=0}^{n-1}a_{k,j}^{l}x_{j} \leq u_{k}^{cl}, \quad k = 0, \dots, m_{l} - 1,$ 
 $l_{j}^{x} \leq x_{j} \leq u_{j}^{x}, \quad j = 0, \dots, n-1.$ 

$$(7.13)$$

Without loss of generality it is assumed that  $Q^o$  and  $Q^k$  are all symmetric because

$$x^T Q x = 0.5 x^T (Q + Q^T) x.$$

The reformulation is not in general unique. The approach followed in Task.toconic is to introduce additional variables, linear constraints and second order cones to obtain a larger but equivalent problem in which the original variables are preserved.

This allows the user to recover the original variable and constraint values, as well as their dual values, with no convertion or additional effort.

The reformulated model will contain:

- one second-order cone for each quadratic constraint,
- one secod-order cone if the objective function is quadratic,
- each quadratic constraint will contain no coefficients and upper/lower bounds will be set to ∞, −∞ respectively.

It is important to notice that **Task.toconic** modified the input task in-place: this means that if the reformulation is not possible, i.e. the problem is not conic representable, the state of the task is in general undefined. The user should consider cloning the task.

# 7.4.1 Quadratic constraint reformulation

Let assume that the k-th constraint has some quadratic terms, i.e. it can be written in the form

$$l_k^c \le \frac{1}{2} x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j \le u_k^c.$$

First we note that either  $l_k^c = -\infty$  or  $l_k^c = -\infty$  must hold, otherwise either the constraint can be dropped, or the constraint is not convex. Thus

$$\frac{1}{2}x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j \le u_k^c.$$

can be considered without loss of generality. Introducing an additional variable  $y_k$  we obtain the equivalent form

$$\begin{array}{lcl} x^T Q^k x & \leq & 2y_k, \\ \sum_{j=0}^{n-1} a_{k,j} x_j - u_k^c & = & y_k. \end{array}$$

If  $Q^k$  is positive semidefinite, we can compute its Cholesky factorization  $F^k$  and write

$$||F^k x||_2 \le 2y_k,$$
  
 $\sum_{j=0}^{n-1} a_{k,j} x_j - u_k^c = y_k.$ 

The first constraint defines a second-order cone of dimension, i.e.

$$||F^k x||_2 \le 2y_k \Leftrightarrow (y_k, Fx) \in \mathcal{Q}_r^{2+n}.$$

Thus, the constraint can be cast as

$$\sum_{j=0}^{n-1} a_{k,j} x_j - u_k^c = y_k,$$

$$z = Fx,$$

$$(1, y_k, z) \in \mathcal{Q}_r^{2+n}.$$

A similar approach is followed to deal with the case in which  $Q^k$  has exactly one negative eigenvalue. Moreover, some special cases, as such  $Q^k$  being diagonal, are taken into account.

# 7.4.2 Objective function reformulation

Let us assume that the objective function of problem (7.13) contains a quadratic terms, i.e. the matrix  $Q_o$  is not null.

From a logical point of view, we can introduce an additional free variable t and remove the quadratic term from the objective function, which reads

$$x_n + a^T x + c. (7.14)$$

The next step is to introduce a quadratic constraint of the form

$$\frac{1}{2}x^T Q_o x \le t. (7.15)$$

where  $Q_m = Q_o$ . The problem has now a linear objective function, as required for any COP. The quadratic constraint can be converted as in Section 7.4.1.

In practice the transformation will not introduce any additional quadratic constraint, but a second order cone will be included along with the additional linear constraints.

# Chapter 8

# A case study

# 8.1 Portfolio optimization

# 8.1.1 Introduction

In this section the Markowitz portfolio optimization problem and variants are implemented using the MOSEK optimizer API.

An alternative to using the optimizer API is the Fusion API which is much simpler to use because it makes it possible to implement the model almost as stated on paper. It is not uncommon that an optimization problem can be implemented using the Fusion API in 1/10th of the time implementing it using the optimizer API. On the other hand, a well implemented model in the optimizer API will usually run faster than the same Fusion model.

Since it so fast to implement a model in Fusion it can be attractive to implement a model in Fusion first because that way the results from the Fusion based code can be used to validate the results of the optimizer API implementation.

Subsequently the following MATLAB inspired notation will be employed. The : operator is used as follows

$$i: j = \{i, i+1, \dots, j\}$$

and hence

$$x_{2:4} = \left[ \begin{array}{c} x_2 \\ x_3 \\ x_4 \end{array} \right]$$

If x and y are two column vectors, then

$$[x;y] = \begin{bmatrix} x \\ y \end{bmatrix}$$

Furthermore, if  $f \in \mathbb{R}^{m \times n}$  then

$$f(:) = \begin{bmatrix} f_{1,1} \\ f_{2,1} \\ f_{m-1,n} \\ f_{m,n} \end{bmatrix}$$

i.e. f(:) stacks the columns of the matrix f.

# 8.1.2 A basic portfolio optimization model

The classical Markowitz portfolio optimization problem considers investing in n stocks or assets held over a period of time. Let  $x_j$  denote the amount invested in asset j, and assume a stochastic model where the return of the assets is a random variable r with known mean

$$\mu = \mathbf{E}r$$

and covariance

$$\Sigma = \mathbf{E}(r - \mu)(r - \mu)^T.$$

The return of the investment is also a random variable  $y = r^T x$  with mean (or expected return)

$$\mathbf{E}y = \mu^T x$$

and variance (or risk)

$$\mathbf{E}(y - \mathbf{E}y)^2 = x^T \Sigma x.$$

The problem facing the investor is to rebalance the portfolio to achieve a good compromise between risk and expected return, e.g., maximize the expected return subject to a budget constraint and an upper bound (denoted  $\gamma$ ) on the tolerable risk. This leads to the optimization problem

maximize 
$$\mu^T x$$
  
subject to  $e^T x = w + e^T x^0$ ,  
 $x^T \Sigma x \leq \gamma^2$ ,  
 $x \geq 0$ . (8.1)

The variables x denote the investment i.e.  $x_j$  is the amount invested in asset j and  $x_j^0$  is the initial holding of asset j. Finally, w is the initial amount of cash available.

A popular choice is  $x^0 = 0$  and w = 1 because then  $x_j$  may be interpretated as the relative amount of the total portfolio that is invested in asset j.

Since e is the vector of all ones then

$$e^T x = \sum_{j=1}^n x_j$$

is the total investment. Clearly, the total amount invested must be equal to the initial wealth, which is

$$w + e^T x^0$$
.

This leads to the first constraint

$$e^T x = w + e^T x^0.$$

The second constraint

$$x^T \Sigma x < \gamma^2$$

ensures that the variance, or the risk, is bounded by  $\gamma^2$ . Therefore,  $\gamma$  specifies an upper bound of the standard deviation the investor is willing to undertake. Finally, the constraint

$$x_j \geq 0$$

excludes the possibility of short-selling. This constraint can of course be excluded if short-selling is allowed.

The covariance matrix  $\Sigma$  is positive semidefinite by definition and therefore there exist a matrix G such that

$$\Sigma = GG^T. \tag{8.2}$$

In general the choice of G is **not** unique and one possible choice of G is the Cholesky factorization of  $\Sigma$ . However, in many cases another choice is better for efficiency reasons as discussed in Section 8.1.4. For a given G we have that

$$x^{T} \Sigma x = x^{T} G G^{T} x$$
$$= \|G^{T} x\|^{2}.$$

Hence, we may write the risk constraint as

$$\gamma \geq \left\|G^Tx\right\|$$

or equivalently

$$[\gamma; G^T x] \in Q^{n+1}.$$

where  $Q^{n+1}$  is the n+1 dimensional quadratic cone. Therefore, problem (8.1) can be written as

maximize 
$$\mu^T x$$
  
subject to  $e^T x = w + e^T x^0$ ,  
 $[\gamma; G^T x] \in Q^{n+1}$ ,  
 $x \geq 0$ ,
$$(8.3)$$

which is a conic quadratic optimization problem that can easily be solved using MOSEK. Subsequently we will use the example data

$$\mu = \left[ \begin{array}{c} 0.1073 \\ 0.0737 \\ 0.0627 \end{array} \right]$$

and

$$\Sigma = 0.1 \left[ \begin{array}{ccc} 0.2778 & 0.0387 & 0.0021 \\ 0.0387 & 0.1112 & -0.0020 \\ 0.0021 & -0.0020 & 0.0115 \end{array} \right]$$

This implies

$$G^T = \sqrt{0.1} \begin{bmatrix} 0.5271 & 0.0734 & 0.0040 \\ 0 & 0.3253 & -0.0070 \\ 0 & 0 & 0.1069 \end{bmatrix}$$

using 5 figures of accuracy. Moreover, let

$$x^0 = \left[ \begin{array}{c} 0.0 \\ 0.0 \\ 0.0 \end{array} \right]$$

and

$$w = 1.0.$$

The data has been taken from [5].

#### 8.1.2.1 Why a conic formulation?

The problem (8.1) is a convex quadratically constrained optimization problems that can be solved directly using MOSEK, then why reformulate it as a conic quadratic optimization problem? The main reason for choosing a conic model is that it is more robust and usually leads to a shorter solution times. For instance it is not always easy to determine whether the Q matrix in (8.1) is positive semidefinite due to the presence of rounding errors. It is also very easy to make a mistake so Q becomes indefinite. These causes of problems are completely eliminated in the conic formulation.

Moreover, observe the constraint

$$||G^Tx|| \leq \gamma$$

is nicer than

$$x^T \Sigma x < \gamma^2$$

for small and values of  $\gamma$ . For instance assume a  $\gamma$  of 10000 then  $\gamma^2$  would 1.0e8 which introduces a scaling issue in the model. Hence, using conic formulation it is possible to work with the standard deviation instead of the variance, which usually gives rise to a better scaled model.

#### 8.1.2.2 Implementing the portfolio model

The model (8.3) can not be implemented as stated using the MOSEK optimizer API because the API requires the problem to be on the form

maximize 
$$c^T \hat{x}$$
  
subject to  $l^c \leq A\hat{x} \leq u^c$ ,  
 $l^x \leq \hat{x} \leq u^x$ ,  
 $\hat{x} \in K$  (8.4)

where  $\hat{x}$  is referred to as the API variable.

The first step in bringing (8.3) to the form (8.4) is the reformulation

maximize 
$$\mu^T x$$
  
subject to  $e^T x = w + e^T x^0$ ,  
 $G^T x - t = 0$   
 $[s;t] \in Q^{n+1}$ ,  
 $x \geq 0$ ,  
 $s = 0$ . (8.5)

where s is an additional scalar variable and t is a n dimensional vector variable. The next step is to define a mapping of the variables

$$\hat{x} = [x; s; t] = \begin{bmatrix} x \\ s \\ t \end{bmatrix}. \tag{8.6}$$

Hence, the API variable  $\hat{x}$  is concatenation of model variables x, s and t. In Table (8.1) the details of the concatenation are specified. For instance it can be seen that

$$\hat{x}_{n+2} = t_1.$$

because the offset of the t variable is n+2.

Given the ordering of the variables specified by (8.6) the data should be defined as follows

Variable	Length	Offset
$\overline{x}$	n	1
s	1	n+1
t	$\mathbf{n}$	n+2

Figure 8.1: Storage layout of the  $\hat{x}$  variable.

$$\begin{array}{lll} c & = & \left[ \begin{array}{ccc} \mu^T & 0 & 0_{n,1} \end{array} \right]^T, \\ A & = & \left[ \begin{array}{ccc} e^T & 0 & 0_{n,1} \\ G^T & 0_{n,1} & -I_n \end{array} \right], \\ l^c & = & \left[ \begin{array}{ccc} w + e^T x^0 & 0_{1,n} \end{array} \right]^T, \\ u^c & = & \left[ \begin{array}{ccc} w + e^T x^0 & 0_{1,n} \end{array} \right]^T, \\ l^x & = & \left[ \begin{array}{ccc} 0_{1,n} & \gamma & -\infty_{n,1} \end{array} \right]^T, \\ u^x & = & \left[ \begin{array}{ccc} \infty_{n,1} & \gamma & \infty_{n,1} \end{array} \right]^T. \end{array}$$

The next step is to consider how the columns of A is defined. The following pseudo code

$$\begin{array}{ll} for & j=1:n \\ & \hat{x}_j=x_j \\ & A_{1,j}=1.0 \\ & A_{2:(n+1),j}=G_{j,1:n}^T \\ \\ \hat{x}_{n+1}=s & \\ \\ for & j=1:n \\ & \hat{x}_{n+1+j}=t_j \\ & A_{n+1+j,n+1+j}=-1.0 \end{array}$$

show how to construct each column of A.

In the above discussion index origin 1 is employed, i.e., the first position in a vector is 1. The Python programming language employs 0 as index origin and that should be kept in mind when reading the example code.

```
def streamprinter(text):
16
        print("%s" % text),
17
    if __name__ == '__main__':
19
            = 3
21
      gamma = 0.05
22
           = [0.1073, 0.0737, 0.0627]
23
      mu
           = [[0.1667, 0.0232, 0.0013],
24
                [0.0000, 0.1033, -0.0022],
25
               [0.0000, 0.0000, 0.0338]]
26
27
      x0
           = [0.0, 0.0, 0.0]
           = 1.0
28
29
      inf = 0.0 # This value has no significance
30
31
      with mosek.Env() as env:
          with env.Task(0.0) as task:
33
              task.set_Stream(mosek.streamtype.log,streamprinter)
34
35
              rtemp = w
36
              for j in range(0,n):
37
                  rtemp += x0[j]
38
               # Constraints.
40
               task.appendcons(1+n)
41
               task.putconbound(0,mosek.boundkey.fx,rtemp,rtemp)
42
              task.putconname(0,"budget")
43
              task.putconboundlist(range(1+0,1+n),n*[mosek.boundkey.fx],n*[0.0],n*[0.0])
45
               for j in range(1,1+n) :
46
                   task.putconname(j,"GT[%d]" % j)
47
48
               # Variables.
              task.appendvars(1+2*n)
50
51
              # Offset of variables into the API variable.
52
              offsetx = 0
53
54
              offsets = n
              offsett = n+1
55
56
              # x variables.
57
               task.putclist(range(offsetx+0,offsetx+n),mu)
58
59
               task.putaijlist(n*[0],range(offsetx+0,offsetx+n),n*[1.0])
               for j in range(0,n):
60
                   task.putaijlist(n*[1+j],range(offsetx+0,offsetx+n),GT[j])
62
               task.putvarboundlist(range(offsetx+0,offsetx+n),n*[mosek.boundkey.lo],n*[0.0],n*[inf])
63
64
               for j in range(0,n):
                   task.putvarname(offsetx+j,"x[%d]" % (1+j))
65
               # s variable.
67
               task.putvarbound(offsets+0,mosek.boundkey.fx,gamma,gamma)
69
               task.putvarname(offsets+0,"s")
70
71
               # t variables.
               task.putaijlist(range(1,n+1),range(offsett+0,offsett+n),n*[-1.0])
72
```

62

```
task.putvarboundlist(range(offsett+0,offsett+n),n*[mosek.boundkey.fr],n*[-inf],n*[inf])
73
               for j in range(0,n):
74
                   task.putvarname(offsett+j,"t[%d]" % (1+j))
75
               task.appendcone(mosek.conetype.quad,0.0,[offsets] + range(offsett,offsett+n))
77
               task.putconename(0,"stddev")
79
               task.putobjsense(mosek.objsense.maximize)
80
81
               # Turn all log output off.
82
83
               task.putintparam(mosek.iparam.log,1)
84
               # Dump the problem to a human readable OPF file.
85
               #task.writedata("dump.opf")
86
87
88
               task.optimize()
89
               # Display the solution summary for quick inspection of results.
90
               task.solutionsummary(mosek.streamtype.msg)
91
92
               expret = 0.0
93
                      = zeros(n,float)
94
               task.getxxslice(mosek.soltype.itr,offsetx+0,offsetx+n,x)
               for j in range(0,n):
96
                   expret += mu[j]*x[j]
97
98
               stddev = zeros(1,float)
99
100
               task.getxxslice(mosek.soltype.itr,offsets+0,offsets+1,stddev)
101
               print("\nExpected return %e for gamma %e\n" % (expret,stddev[0]))
   The above code produce the result
    Interior-point solution summary
      Problem status : PRIMAL_AND_DUAL_FEASIBLE
      Solution status : OPTIMAL
                                         Viol. con: 2e-008
Viol. con: 0e+000
      Primal. obj: 7.4766497707e-002
                                                               var: 0e+000
                                                                              cones: 3e-009
                obj: 7.4766522618e-002
                                                               var: 4e-008
                                                                              cones: 0e+000
    Expected return 7.476650e-02 for gamma 5.000000e-02
    The source code should be self-explanatory but a few comments are nevertheless in place. In the lines
                                             -[ case_portfolio_1.py ]-
    # Offset of variables into the API variable.
52
    offsetx = 0
    offsets = n
54
    offsett = n+1
    offsets into the MOSEK API variables are stored and those offsets are used later. The code
                                             -[ case_portfolio_1.py ]-
    task.putclist(range(offsetx+0,offsetx+n),mu)
59
    task.putaijlist(n*[0],range(offsetx+0,offsetx+n),n*[1.0])
    for j in range(0,n):
60
61
         task.putaijlist(n*[1+j],range(offsetx+0,offsetx+n),GT[j])
```

```
task.putvarboundlist(range(offsetx+0,offsetx+n),n*[mosek.boundkey.lo],n*[0.0],n*[inf])

for j in range(0,n):
    task.putvarname(offsetx+j,"x[%d]" % (1+j))

sets up the data for x variables. For instance

[ case_portfolio_1.py ]

task.putclist(range(offsetx+0,offsetx+n),mu)

inputs the objective coefficients for the x variables. Moreover, the code

[ case_portfolio_1.py ]

for j in range(0,n):
    task.putvarname(offsetx+j,"x[%d]" % (1+j))
```

assigns meaningful names to the API variables. This is not needed but it makes debugging easier.

#### 8.1.2.3 Debugging tips

Implementing an optimization model in optimizer can be cumbersome and error-prone and it is very easy to make mistakes. In order to check the implemented code for mistakes it is very useful to dump the problem to a file in a human readable form for visual inspection. The line

```
case_portfolio_1.py ]

#task.writedata("dump.opf")
```

does that and this will produce a file with the content

```
Written by MOSEK version 7.0.0.86
  Date 01-10-13
  Time 08:15:47
[/comment]
[hints]
  [hint NUMVAR] 7 [/hint]
  [hint NUMCON] 4 [/hint]
  [hint NUMANZ] 12 [/hint]
  [hint NUMQNZ] O [/hint]
  [hint NUMCONE] 1 [/hint]
[/hints]
[variables disallow_new_variables]
  'x[1]' 'x[2]' 'x[3]' s 't[1]'
  't[2]' 't[3]'
[/variables]
[objective maximize]
  1.073e-001 'x[1]' + 7.37e-002 'x[2]' + 6.27000000000001e-002 'x[3]'
[/objective]
[constraints]
  [con 'budget'] 'x[1]' + 'x[2]' + 'x[3]' = 1e+000 [/con]
```

Observe that since the API variables have been given meaningful names it is easy to see the model is correct.

#### 8.1.3 The efficient frontier

The portfolio computed by the Markowitz model is efficient in the sense that there is no other portfolio giving a strictly higher return for the same amount of risk. An efficient portfolio is also sometimes called a Pareto optimal portfolio. Clearly, an investor should only invest in efficient portfolios and therefore it may be relevant to present the investor with all efficient portfolios so the investor can choose the portfolio that has the desired tradeoff between return and risk.

Given a nonnegative  $\alpha$  then the problem

maximize 
$$\mu^T x - \alpha s$$
  
subject to  $e^T x = w + e^T x^0$ ,  
 $[s; G^T x] \in Q^{n+1}$ ,  
 $x \geq 0$ . (8.7)

computes efficient portfolios. Note that the objective maximizes the expected return while maximizing  $-\alpha$  times the standard deviation. Hence, the standard deviation is minimized while  $\alpha$  specifies the tradeoff between expected return and risk.

Ideally the problem 8.7 should be solved for all values  $\alpha \geq 0$  but in practice that is computationally too costly.

Using the example data from Section 8.1.2, the optimal values of return and risk for several  $\alpha$ s are listed below:

```
Expected return 1.073000e-01 for gamma 7.261311e-01 Expected return 1.032557e-01 for gamma 1.499440e-01 Expected return 6.975524e-02 for gamma 3.735435e-02 Expected return 6.766068e-02 for gamma 3.382809e-02 Expected return 6.679238e-02 for gamma 3.281319e-02 Expected return 6.598822e-02 for gamma 3.214199e-02 Expected return 6.560055e-02 for gamma 3.191601e-02 Expected return 6.537354e-02 for gamma 3.181398e-02 Expected return 6.52238e-02 for gamma 3.175861e-02 Expected return 6.503462e-02 for gamma 3.17556e-02 Expected return 6.503462e-02 for gamma 3.170391e-02 Expected return 6.497237e-02 for gamma 3.168923e-02 Expected return 6.497237e-02 for gamma 3.168923e-02
```

#### 8.1.3.1 Example code

The following example code demonstrates how to compute the efficient portfolios for several values of  $\alpha$ .

```
-[ case_portfolio_2.py ]---
    ....
1
      File : case_portfolio_2.py
      Copyright : Copyright (c) MOSEK ApS, Denmark. All rights reserved.
      Description: Implements a basic portfolio optimization model.
    import mosek
10
11
    try:
        from numpy import zeros
12
    except ImportError:
13
        from mosek.array import zeros
14
15
16
    def streamprinter(text):
        print("%s" % text),
17
18
    if __name__ == '__main__':
19
20
              = 3
21
       gamma = 0.05
22
              = [0.1073, 0.0737, 0.0627]
23
      mu
              = [[0.1667, 0.0232, 0.0013],
24
                 [0.0000, 0.1033, -0.0022],
25
                 [0.0000, 0.0000, 0.0338]]
26
      x0
              = [0.0, 0.0, 0.0]
27
              = 1.0
28
      alphas = [0.0, 0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5]
29
30
              = 0.0 # This value has no significance
31
32
       with mosek.Env() as env:
33
          with env.Task(0,0) as task:
34
              task.set_Stream(mosek.streamtype.log,streamprinter)
35
               rtemp = w
37
               for j in range(0,n):
38
                   rtemp += x0[j]
39
40
41
               # Constraints.
               task.appendcons(1+n)
42
               task.putconbound(0,mosek.boundkey.fx,rtemp,rtemp)
43
               task.putconname(0,"budget")
44
45
               task.putconboundlist(range(1+0,1+n),n*[mosek.boundkey.fx],n*[0.0],n*[0.0])
46
               for j in range(1,1+n) :
47
                   task.putconname(j,"GT[%d]" % j)
49
               # Variables.
               task.appendvars(1+2*n)
51
```

```
offsetx = 0  # Offset of variable x into the API variable.
53
               offsets = n  # Offset of variable x into the API variable.
54
               offsett = n+1 # Offset of variable t into the API variable.
55
               # x variables.
57
               task.putclist(range(offsetx+0,offsetx+n),mu)
               task.putaijlist(n*[0],range(offsetx+0,offsetx+n),n*[1.0])
59
               for j in range(0,n):
60
61
                   task.putaijlist(n*[1+j],range(offsetx+0,offsetx+n),GT[j])
62
               task.putvarboundlist(range(offsetx+0,offsetx+n),n*[mosek.boundkey.lo],n*[0.0],n*[inf])
63
               for j in range(0,n):
64
65
                    task.putvarname(offsetx+j,"x[%d]" % (1+j))
66
67
               task.putvarbound(offsets+0,mosek.boundkey.fr,gamma,gamma)
68
               task.putvarname(offsets+0,"s")
69
               # t variables.
71
               task.putaijlist(range(1,n+1),range(offsett+0,offsett+n),n*[-1.0])
72
               task.putvarboundlist(range(offsett+0,offsett+n),n*[mosek.boundkey.fr],n*[-inf],n*[inf])
73
               for j in range(0,n):
74
                    task.putvarname(offsett+j,"t[%d]" % (1+j))
75
76
               task.appendcone(mosek.conetype.quad,0.0,[offsets] + range(offsett,offsett+n))
77
               task.putconename(0, "stddev")
78
79
80
               task.putobjsense(mosek.objsense.maximize)
81
               # Turn all log output off.
               task.putintparam(mosek.iparam.log,0)
83
84
85
               for alpha in alphas:
                    # Dump the problem to a human readable OPF file.
86
                   #task.writedata("dump.opf")
88
                   task.putcj(offsets+0,-alpha);
89
90
                   task.optimize()
91
92
                   # Display the solution summary for quick inspection of results.
93
                   # task.solutionsummary(mosek.streamtype.msg)
94
95
                   solsta = task.getsolsta(mosek.soltype.itr)
96
97
                   if solsta in [mosek.solsta.optimal, mosek.solsta.near_optimal]:
98
                       expret = 0.0
                              = zeros(n.float)
100
                       task.getxxslice(mosek.soltype.itr,offsetx+0,offsetx+n,x)
101
102
                       for j in range(0,n):
                            expret += mu[j]*x[j]
103
104
                       stddev = zeros(1,float)
105
                       task.getxxslice(mosek.soltype.itr,offsets+0,offsets+1,stddev)
106
107
                       print("\nExpected return %e for gamma %e" % (expret,stddev[0])),
108
109
                   else:
```

### 8.1.4 Improving the computational efficiency

In practice it is often important to solve the portfolio problem in a short amount of time; this section it is discusses what can be done at the modelling stage to improve the computational efficiency.

The computational cost is of course to some extent dependent on the number of constraints and variables in the optimization problem. However, in practice a more important factor is the number nonzeros used to represent the problem. Indeed it is often better to focus at the number of nonzeros in G (see (8.2)) and try to reduce that number by for instance changing the choice of G.

In other words, if the computational efficiency should be improved then it is always good idea to start with focusing at the covariance matrix. As an example assume that

$$\Sigma = D + VV^T$$

where D is positive definite diagonal matrix. Moreover, V is a matrix with n rows and p columns. Such a model for the covariance matrix is called a factor model and usually p is much smaller than n. In practice p tends be a small number say less than 100 independent of n.

One possible choice for G is the Cholesky factorization of  $\Sigma$  which requires storage proportional to n(n+1)/2. However, another choice is

$$G^T = \left[ \begin{array}{c} D^{1/2} \\ V^T \end{array} \right]$$

because then

$$GG^T = D + VV^T$$
.

This choice requires storage proportional to n + pn which is much less than for the Cholesky choice of G. Indeed assuming p is a constant then the difference in storage requirements is a factor of n.

The example above exploits the so-called factor structure and demonstrates that an alternative choice of G may lead to a significant reduction in the amount of storage used to represent the problem. This will in most cases also lead to a significant reduction in the solution time.

The lesson to be learned is that it is important to investigate how the covariance is formed. Given this knowledge it might be possible to make a special choice for G that helps reducing the storage requirements and enhance the computational efficiency.

# 8.1.5 Slippage cost

The basic Markowitz portfolio model assumes that there are no costs associated with trading the assets and that the returns of the assets is independent of the amount traded. None of those assumptions are usually valid in practice. Therefore, a more realistic model is

maximize 
$$\mu^T x$$
  
subject to  $e^T x + \sum_{j=1}^n C_j (x_j - x_j^0) = w + e^T x^0,$   
 $x^T \Sigma x \leq \gamma^2,$   
 $x \leq 0,$ 

$$(8.8)$$

where the function

$$C_j(x_j - x_j^0)$$

specifies the transaction costs when the holding of asset j is changed from its initial value.

#### 8.1.5.1 Market impact costs

If the initial wealth is fairly small and short selling is not allowed, then the holdings will be small. Therefore, the amount traded of each asset must also be small. Hence, it is reasonable to assume that the prices of the assets is independent of the amount traded. However, if a large volume of an assert is sold or purchased it can be expected that the price change and hence the expected return also change. This effect is called market impact costs. It is common to assume that market impact costs for asset j can be modelled by

$$m_j \sqrt{|x_j - x_j^0|}$$

where  $m_j$  is a constant that is estimated in some way. See [6][p. 452] for details. To summarize then

$$C_j(x_j - x_j^0) = m_j |x_j - x_j^0| \sqrt{|x_j - x_j^0|} = m_j |x_j - x_j^0|^{3/2}.$$

From [7] it is known

$$\{(c,z): c > z^{3/2}, z > 0\} = \{(c,z): [v;c;z], [z;1/8;v] \in Q_x^3\}$$

where  $Q_r^3$  is the 3 dimensional rotated quadratic cone implying

$$z_{j} = |x_{j} - x_{j}^{0}|,$$

$$[v_{j}; c_{j}; z_{j}], [z_{j}; 1/8; v_{j}] \in Q_{r}^{3},$$

$$\sum_{j=1}^{n} C_{j}(x_{j} - x_{j}^{0}) = \sum_{j=1}^{n} c_{j}.$$

Unfortunately this set of constraints is nonconvex due to the constraint

$$z_j = |x_j - x_j^0| (8.9)$$

but in many cases that constraint can safely be replaced by the relaxed constraint

$$z_j \ge |x_j - x_j^0| \tag{8.10}$$

which is convex. If for instance the universe of assets contains a risk free asset with a positive return then

$$z_i > |x_i - x_i^0| \tag{8.11}$$

cannot hold for an optimal solution because that would imply the solution is not optimal.

Now assume that the optimal solution has the property that (8.11) holds then the market impact cost within the model is larger than the true market impact cost and hence money are essentially considered garbage and removed by generating transaction costs. This may happen if a portfolio with very small risk is requested because then the only way to obtain a small risk is to get rid of some of the assets by generating transaction costs. Here it is assumed this is not the case and hence the models (8.9) and (8.10) are equivalent.

Formula (8.10) is replaced by constraints

$$\begin{array}{rcl}
z_j & \geq & x_j - x_j^0, \\
z_j & \geq & -(x_j - x_j^0).
\end{array}$$
(8.12)

Now we have

maximize 
$$\mu^{T}x$$
  
subject to  $e^{T}x + m^{T}c = w + e^{T}x^{0}$ ,  
 $z_{j} \geq x_{j} - x_{j}^{0}$ ,  $j = 1, ..., n$ ,  
 $z_{j} \geq x_{j}^{0} - x_{j}$ ,  $j = 1, ..., n$ ,  
 $[\gamma; G^{T}x] \in Q^{n+1}$ ,  
 $[v_{j}; c_{j}; z_{j}] \in Q_{r}^{3}$ ,  $j = 1, ..., n$ ,  
 $[z_{j}; 1/8; v_{j}] \in Q_{r}^{3}$ ,  $j = 1, ..., n$ ,  
 $x \geq 0$ . (8.13)

The revised budget constraint

$$e^{T}x = w + e^{T}x^{0} - m^{T}c$$

specifies that the total investment must be equal to the initial wealth minus the transaction costs. Moreover, observe the variables v and z are some auxiliary variables that model the market impact cost. Indeed it holds

$$z_j \ge |x_j - x_j^0|$$

and

$$c_j \ge z_j^{3/2}$$
.

Before proceeding it should be mentioned that transaction costs of the form

$$c_j \ge z_j^{p/q}$$

where p and q are both integers and  $p \ge q$  can be modelled using quadratic cones. See [7] for details. One more reformulation of (8.13) is needed,

maximize 
$$\mu^{T}x$$
  
subject to  $e^{T}x + m^{T}c = w + e^{T}x^{0}$ ,  
 $G^{T}x - t = 0$ ,  
 $z_{j} - x_{j} \geq -x_{j}^{0}$ ,  $j = 1, ..., n$ ,  
 $[v_{j}; c_{j}; z_{j}] - f_{j,1:3} = 0$ ,  $j = 1, ..., n$ ,  
 $[z_{j}; 0; v_{j}] - g_{j,1:3} = [0; -1/8; 0]$ ,  $j = 1, ..., n$ ,  
 $[s; t] \in Q^{n+1}$ ,  
 $f_{j,1:3}^{T} \in Q_{r}^{3}$ ,  $j = 1, ..., n$ ,  
 $g_{j,1:3}^{T} \in Q_{r}^{3}$ ,  $j = 1, ..., n$ ,  
 $x \geq 0$ ,  
 $s = \gamma$ ,

where  $f, g \in \mathbb{R}^{n \times 3}$ . These additional variables f and g are only introduced to bring the problem on the API standard form.

The formulation (8.14) is not the most compact possible. However, the MOSEK presolve will automatically make it more compact and since it is easier to implement (8.14) than a more compact form then the form (8.14) is preferred.

The first step in developing the optimizer API implementation is to chose an ordering of the variables. In this case the ordering

$$\hat{x} = \begin{bmatrix} x \\ s \\ t \\ c \\ v \\ z \\ f^{T}(:) \\ g^{T}(:) \end{bmatrix}$$

will be used. Note  $f^T(:)$  means the rows of f are transposed and stacked on top of each other to form a long column vector. The Table 8.2 shows the mapping between the  $\hat{x}$  and the model variables.

The next step is to consider how the columns of A is defined. Reusing the idea in Section 8.1.2 then the following pseudo code describes the setup of A.

Variable	Length	Offset
$\overline{x}$	n	1
s	1	n+1
t	n	n+2
c	$\mathbf{n}$	2n+2
v	$\mathbf{n}$	3n+2
z	$\mathbf{n}$	4n+2
$f(:)^T$	3n	7n+2
$g(:)^T$	3n	10n+2

Figure 8.2: Storage layout for the  $\hat{x}$ 

$$\begin{array}{lll} for & j=1:n \\ & \hat{x}_j=x_j \\ & A_{1,j}=1.0 \\ & A_{2:n+1,j}=G_{j,1:n}^T \\ & A_{n+1+j,j}=-1.0 \\ & A_{2n+1+j,j}=1.0 \\ & \hat{x}_{n+1}=s \\ \\ for & j=1:n \\ & \hat{x}_{n+1+j}=t_j \\ & A_{1+j,n+1+j}=-1.0 \\ \\ for & j=1:n \\ & \hat{x}_{2n+1+j}=c_j \\ & A_{1,2n+1+j}=m_j \\ & A_{3n+1+3(j-1)+2,2n+1+j}=1.0 \\ \\ for & j=1:n \\ & \hat{x}_{3n+1+j}=v_j \\ & A_{3n+1+3(j-1)+3,3n+1+j}=1.0 \\ \\ for & j=1:n \\ & \hat{x}_{4n+1+j}=z_j \\ & A_{1+n+j,4n+1+j}=1.0 \\ & A_{6n+1+3(j-1)+3,3n+1+j}=1.0 \\ \\ for & j=1:n \\ & \hat{x}_{4n+1+j}=1.0 \\ & A_{3n+1+3(j-1)+1,4n+1+j}=1.0 \\ & A_{3n+1+3(j-1)+1,4n+1+j}=1.0 \\ \\ for & j=1:n \\ & \hat{x}_{7n+1+3(j-1)+1}=f_{j,1} \\ & A_{3n+1+3(j-1)+1,7n+(3(j-1)+1}=-1.0 \\ & \hat{x}_{7n+1+3(j-1)+2}=f_{j,2} \\ & A_{3n+1+3(j-1)+2,7n+(3(j-1)+2}=-1.0 \\ & \hat{x}_{7n+1+3(j-1)+3}=f_{j,3} \\ & A_{3n+1+3(j-1)+3,7n+(3(j-1)+3}=-1.0 \\ \\ for & j=1:n \\ \end{array}$$

 $\hat{x}_{10n+1+3(j-1)+1} = g_{j,1}$ 

 $\hat{x}_{10n+1+3(j-1)+2} = g_{j,2}$ 

 $\hat{x}_{10n+1+3(j-1)+3} = g_{j,3}$ 

 $A_{6n+1+3(j-1)+1,7n+(3(j-1)+1} = -1.0$ 

 $A_{6n+1+3(j-1)+2,7n+(3(j-1)+2)} = -1.0$ 

 $A_{6n+1+3(j-1)+3,7n+(3(j-1)+3)} = -1.0$ 

The following example code demonstrates how to implement the model (8.14).

```
—[ case_portfolio_3.py ]—
      File : case_portfolio_3.py
2
      Copyright : Copyright (c) MOSEK ApS, Denmark. All rights reserved.
      Description: Implements a basic portfolio optimization model.
9
    import mosek
10
11
        from numpy import zeros
12
13
    except ImportError:
        from mosek.array import zeros
14
15
    def streamprinter(text):
        print("%s" % text),
17
18
    if __name__ == '__main__':
19
20
            = 3
21
      gamma = 0.05
22
           = [0.1073, 0.0737, 0.0627]
           = [[0.1667, 0.0232, 0.0013],
24
               [0.0000, 0.1033, -0.0022],
25
               [0.0000, 0.0000, 0.0338]]
26
      x0
            = [0.0, 0.0, 0.0]
27
            = 1.0
28
      W
            = [0.01, 0.01, 0.01]
29
      # This value has no significance.
31
      inf = 0.0
32
33
      with mosek.Env() as env:
34
          with env.Task(0,0) as task:
35
              task.set_Stream(mosek.streamtype.log,streamprinter)
36
37
              rtemp = w
38
               for j in range(0,n):
39
                   rtemp += x0[j]
41
               # Constraints.
42
               {\tt task.appendcons(1+9*n)}
43
               task.putconbound(0,mosek.boundkey.fx,rtemp,rtemp)
44
45
               task.putconname(0,"budget")
46
               task.putconboundlist(range(1+0,1+n),n*[mosek.boundkey.fx],n*[0.0],n*[0.0])
47
               for j in range(1,1+n) :
48
                   task.putconname(j,"GT[%d]" % j)
49
50
               task.putconboundlist(range(1+n,1+2*n),n*[mosek.boundkey.lo],[-x0[j] for j in range(0,n)],n*[inf])
51
               for i in range(0,n):
                   task.putconname(1+n+i, "zabs1[%d]" % (1+i))
53
               task.putconboundlist(range(1+2*n,1+3*n),n*[mosek.boundkey.lo],x0,n*[inf])
55
```

```
56
               for i in range(0,n):
                    task.putconname(1+2*n+i,"zabs2[\%d]" \% (1+i))
57
58
               task.putconboundlist(range(1+3*n,1+3*n+3*n),3*n*[mosek.boundkey.fx],3*n*[0.],3*n*[0.0])
59
               for i in range(0,n):
60
                   for k in range(0,n):
                        task.putconname(1+3*n+3*i+k,"f[%d,%d]" % (1+i,1+k))
62
63
64
               task.putconboundlist(range(1+6*n,1+9*n),3*n*[mosek.boundkey.fx],
                                     3*[0.0, -1.0/8.0, 0.0], 3*[0.0, -1.0/8.0, 0.0])
65
               for i in range(0,n) :
                   for k in range(0,n):
67
68
                        task.putconname(1+6*n+3*i+k,"g[%d,%d]" % (1+i,1+k))
69
               # Offset of variables into the API variable.
70
71
               offsetx = 0
               offsets = n
72
               offsett = n+1
73
               offsetc = 2*n+1
74
               offsetv = 3*n+1
75
               offsetz = 4*n+1
76
               offsetf = 5*n+1
77
78
               offsetg = 8*n+1
79
               # Variables.
80
               \verb|task.appendvars(1+11*n)|
81
82
83
               # x variables.
               task.putclist(range(offsetx+0,offsetx+n),mu)
84
85
               task.putaijlist(n*[0],range(offsetx+0,offsetx+n),n*[1.0])
86
               for j in range(0,n):
                    task.putaijlist(n*[1+j],range(offsetx+0,offsetx+n),GT[j])
87
88
                    task.putaij(1+n+j,offsetx+j,-1.0)
                    task.putaij(1+2*n+j,offsetx+j,1.0)
89
               task.putvarboundlist(range(offsetx+0,offsetx+n),n*[mosek.boundkey.lo],n*[0.0],n*[inf])
91
               for j in range(0,n):
92
                   task.putvarname(offsetx+j,"x[%d]" % (1+j))
93
94
               # s variable.
95
               task.putvarbound(offsets+0,mosek.boundkey.fx,gamma,gamma)
96
               task.putvarname(offsets+0,"s")
97
98
               # t variables.
               task.putaijlist(range(1,n+1),range(offsett+0,offsett+n),n*[-1.0])\\
100
               task.putvarboundlist(range(offsett+0,offsett+n),n*[mosek.boundkey.fr],n*[-inf],n*[inf])
101
               for j in range(0,n):
102
                    task.putvarname(offsett+j,"t[%d]" % (1+j))
103
104
105
               # c variables.
               task.putaijlist(n*[0],range(offsetc,offsetc+n),m)
106
               task.putaijlist(range(1+3*n+1,1+6*n+1,3),range(offsetc,offsetc+n),n*[1.0])
107
               task.putvarboundlist(range(offsetc,offsetc+n),n*[mosek.boundkey.fr],n*[-inf],n*[inf])
108
               for j in range(0,n):
109
110
                    task.putvarname(offsetc+j,"c[%d]" % (1+j))
111
112
               # v variables.
               task.putaijlist(range(1+3*n+0,1+6*n+0,3),range(offsetv,offsetv+n),n*[1.0])
113
```

171

```
114
               task.putaijlist(range(1+6*n+2,1+9*n+2,3),range(offsetv,offsetv+n),n*[1.0])
               task.putvarboundlist(range(offsetv,offsetv+n),n*[mosek.boundkey.fr],n*[-inf],n*[inf])
115
                for j in range(0,n):
116
                    task.putvarname(offsetv+j,"v[%d]" % (1+j))
117
118
               # z variables.
119
               {\tt task.putaijlist(range(1+1*n,1+2*n),range(offsetz,offsetz+n),n*[1.0])}
120
                task.putaijlist(range(1+2*n,1+3*n),range(offsetz,offsetz+n),n*[1.0])
121
122
               task.putaijlist(range(1+3*n+2,1+6*n+2,3),range(offsetz,offsetz+n),n*[1.0])
               task.putaijlist(range(1+6*n+0,1+9*n+0,3),range(offsetz,offsetz+n),n*[1.0])
123
124
               task.putvarboundlist(range(offsetz,offsetz+n),n*[mosek.boundkey.fr],n*[-inf],n*[inf])
               for j in range(0,n):
125
                    task.putvarname(offsetz+j,"z[%d]" % (1+j))
126
127
               # f variables.
128
129
               for j in range(0,n):
                    for k in range(0,n):
130
                        task.putaij(1+3*n+3*j+k,offsetf+3*j+k,-1.0)
131
                        task.putvarbound(offsetf+3*j+k,mosek.boundkey.fr,-inf,inf)
132
                        task.putvarname(offsetf+3*j+k,"f[%d,%d]" % (1+j,1+k))
133
134
               # g variables.
135
               for j in range(0,n):
136
                    for k in range(0,n):
137
                        task.putaij(1+6*n+3*j+k,offsetg+3*j+k,-1.0)
138
                        task.putvarbound(offsetg+3*j+k,mosek.boundkey.fr,-inf,inf)
139
                        task.putvarname(offsetg+3*j+k, "g[%d,%d]" % (1+j,1+k))
140
141
               task.appendcone(mosek.conetype.quad,0.0,[offsets] + range(offsett,offsett+n))
142
143
               task.putconename(0, "stddev")
144
               for k in range(0,n):
145
                    task.appendconeseq(mosek.conetype.rquad,0.0,3,offsetf+3*k)
146
                    task.putconename(1+k, "f[%d]" % (1+k))
147
               for k in range(0,n):
149
                    task.appendconeseq(mosek.conetype.rquad,0.0,3,offsetg+3*k)
150
                    task.putconename(1+n+k, "g[%d]" % (1+k))
151
152
               task.putobjsense(mosek.objsense.maximize)
153
154
                # Turn all log output off.
155
               # task.putintparam(mosek.iparam.log,0)
156
157
158
               # Dump the problem to a human readable OPF file.
               #task.writedata("dump.opf")
159
160
               task.optimize()
161
162
               # Display the solution summary for quick inspection of results.
163
               task.solutionsummary(mosek.streamtype.msg)
164
165
               expret = 0.0
166
                       = zeros(n,float)
167
168
               task.getxxslice(mosek.soltype.itr,offsetx+0,offsetx+n,x)
               for j in range(0,n):
169
170
                    expret += mu[j]*x[j]
```

```
stddev = zeros(1,float)
               task.getxxslice(mosek.soltype.itr,offsets+0,offsets+1,stddev)
173
174
               print("\nExpected return %e for gamma %e\n" % (expret,stddev[0]))
    The example code above produces the result
     Interior-point solution summary
       Problem status : PRIMAL_AND_DUAL_FEASIBLE
       Solution status : OPTIMAL
       Primal. obj: 7.4390660228e-002
                                         Viol. con: 2e-007
                                                               var: 0e+000
                                                                               cones: 1e-009
                obj: 7.4390669047e-002
                                         Viol. con: 1e-008 var: 1e-008
       Dual.
                                                                               cones: 0e+000
     Expected return 7.439066e-02 for gamma 5.000000e-02
    If the problem is dumped to an OPF formatted file, then it has the following content.
     [comment]
        Written by MOSEK version 7.0.0.86
        Date 01-10-13
        Time 07:47:34
     [/comment]
     [hints]
       [hint NUMVAR] 34 [/hint]
       [hint NUMCON] 28 [/hint]
       [hint NUMANZ] 60 [/hint]
       [hint NUMQNZ] 0 [/hint]
       [hint NUMCONE] 7 [/hint]
     [/hints]
     [variables disallow_new_variables]
       'x[1]' 'x[2]' 'x[3]' s 't[1]'
       't[2]' 't[3]' 'c[1]' 'c[2]' 'c[3]'
       'v[1]' 'v[2]' 'v[3]' 'z[1]' 'z[2]'
       'z[3]' 'f[1,1]' 'f[1,2]' 'f[1,3]' 'f[2,1]'
       'f[2,2]' 'f[2,3]' 'f[3,1]' 'f[3,2]' 'f[3,3]'
       'g[1,1]' 'g[1,2]' 'g[1,3]' 'g[2,1]' 'g[2,2]'
       'g[2,3]' 'g[3,1]' 'g[3,2]' 'g[3,3]'
     [/variables]
     [objective maximize]
        1.073e-001 'x[1]' + 7.37e-002 'x[2]' + 6.2700000000000001e-002 'x[3]'
     [/objective]
     [constraints]
       [con 'budget'] 'x[1]' + 'x[2]' + 'x[3]' + 1e-002 'c[1]' + 1e-002 'c[2]'
          + 1e-002 'c[3]' = 1e+000 [/con]
        [con 'GT[1]'] \quad 1.667e-001 'x[1]' + 2.32e-002 'x[2]' + 1.3e-003 'x[3]' - 't[1]' = 0e+000 \ [/con] ] 
       [con 'GT[2]'] 1.033e-001 'x[2]' - 2.2e-003 'x[3]' - 't[2]' = 0e+000 [/con]
[con 'GT[3]'] 3.38e-002 'x[3]' - 't[3]' = 0e+000 [/con]
       [con 'zabs1[1]'] Oe+000 <= - 'x[1]' + 'z[1]' [/con]
       [con 'zabs1[2]'] Oe+000 \leftarrow - 'x[2]' + 'z[2]' [/con]
       [con 'zabs1[3]'] Oe+000 <= - 'x[3]' + 'z[3]' [/con]
       [con 'zabs2[1]'] 0e+000 <= 'x[1]' + 'z[1]' [/con]
       [con 'zabs2[2]'] 0e+000 <= 'x[2]' + 'z[2]' [/con]
```

[con 'zabs2[3]'] 0e+000 <= 'x[3]' + 'z[3]' [/con]
[con 'f[1,1]'] 'v[1]' - 'f[1,1]' = 0e+000 [/con]</pre>

```
[con 'f[1,2]'] 'c[1]' - 'f[1,2]' = 0e+000 [/con]
  [con 'f[1,3]'] 'z[1]' - 'f[1,3]' = 0e+000 [/con]
                      'v[2]' - 'f[2,1]' = 0e+000 [/con]
  [con 'f[2,1]']
  [con 'f[2,2]'] 'c[2]' - 'f[2,2]' = 0e+000 [/con]
  [con 'f[2,3]'] 'z[2]' - 'f[2,3]' = 0e+000 [/con]
  [con 'f[3,1]'] 'v[3]' - 'f[3,1]' = 0e+000 [/con]
  [con 'f[3,2]'] 'c[3]' - 'f[3,2]' = 0e+000 [/con]
  [con 'f[3,3]'] 'z[3]' - 'f[3,3]' = 0e+000 [/con]
[con 'g[1,1]'] 'z[1]' - 'g[1,1]' = 0e+000 [/con]
  [con 'g[1,2]'] - 'g[1,2]' = -1.25e-001 [/con]
  [con 'g[1,3]'] 'v[1]' - 'g[1,3]' = 0e+000 [/con]
[con 'g[2,1]'] 'z[2]' - 'g[2,1]' = 0e+000 [/con]
[con 'g[2,2]'] - 'g[2,2]' = -1.25e-001 [/con]
  [con 'g[2,3]'] 'v[2]' - 'g[2,3]' = 0e+000 [/con]
[con 'g[3,1]'] 'z[3]' - 'g[3,1]' = 0e+000 [/con]
  [con 'g[3,2]'] - 'g[3,2]' = -1.25e-001 [/con]
  [con 'g[3,3]'] 'v[3]' - 'g[3,3]' = 0e+000 [/con]
[/constraints]
[bounds]
                   0 <= * [/b]
  [b]
  [b]
                         s = 5e-002 [/b]
                         't[1]','t[2]','t[3]','c[1]','c[2]','c[3]' free [/b]
  [b]
                         'v[1]','v[2]','v[3]','z[1]','z[2]','z[3]' free [/b]
  [b]
  [b]
                         'f[1,1]','f[1,2]','f[1,3]','f[2,1]','f[2,2]','f[2,3]' free [/b]
                          \tt 'f[3,1]', 'f[3,2]', 'f[3,3]', 'g[1,1]', 'g[1,2]', 'g[1,3]' \ free \ [/b]
  [b]
                          'g[2,1]','g[2,2]','g[2,3]','g[3,1]','g[3,2]','g[3,3]' free [/b]
  [cone quad 'stddev'] s, 't[1]', 't[2]', 't[3]' [/cone]
  [cone rquad 'f[1]'] 'f[1,1]', 'f[1,2]', 'f[1,3]' [/cone]
  [cone rquad 'f[2]'] 'f[2,1]', 'f[2,2]', 'f[2,3]' [/cone]
  [cone rquad 'f[3]'] 'f[3,1]', 'f[3,2]', 'f[3,3]' [/cone] [cone rquad 'g[1]'] 'g[1,1]', 'g[1,2]', 'g[1,3]' [/cone] [cone rquad 'g[2]'] 'g[2,1]', 'g[2,2]', 'g[2,3]' [/cone] [cone rquad 'g[3]'] 'g[3,1]', 'g[3,2]', 'g[3,3]' [/cone]
[/bounds]
```

The file verifies that the correct problem has been setup.

# Chapter 9

# Usage guidelines

The purpose of this chapter is to present some general guidelines to follow when using MOSEK.

## 9.1 Verifying the results

The main purpose of MOSEK is to solve optimization problems and therefore the most fundamental question to be asked is whether the solution reported by MOSEK is a solution to the desired optimization problem.

There can be several reasons why it might be not case. The most prominent reasons are:

- A wrong problem. The problem inputted to MOSEK is simply not the right problem, i.e. some of the data may have been corrupted or the model has been incorrectly built.
- Numerical issues. The problem is badly scaled or otherwise badly posed.
- Other reasons. E.g. not enough memory or an explicit user request to stop.

The first step in verifying that MOSEK reports the expected solution is to inspect the solution summary generated by MOSEK. The solution summary provides information about

- the problem and solution statuses,
- objective value and infeasibility measures for the primal solution, and
- objective value and infeasibility measures for the dual solution, where applicable.

By inspecting the solution summary it can be verified that MOSEK produces a feasible solution, and, in the continuous case, the optimality can be checked using the dual solution. Furthermore, the problem itself ca be inspected using the problem analyzer discussed in section 13.1.

If the summary reports conflicting information (e.g. a solution status that does not match the actual solution), or the cause for terminating the solver before a solution was found cannot be traced back to

the reasons stated above, it may be caused by a bug in the solver; in this case, please contact MOSEK support.

#### 9.1.1 Verifying primal feasibility

If it has been verified that MOSEK solves the problem correctly but the solution is still not as expected, next step is to verify that the primal solution satisfies all the constraints. Hence, using the original problem it must be determined whether the solution satisfies all the required constraints in the model. For instance assume that the problem has the constraints

$$x_1 + 2x_2 + x_3 \le 1,$$
  
 $x_1, x_2, x_3 \ge 0$ 

and MOSEK reports the optimal solution

$$x_1 = x_2 = x_3 = 1.$$

Then clearly the solution violates the constraints. The most likely explanation is that the model does not match the problem entered into MOSEK, for instance

$$x_1 - 2x_2 + x_3 \le 1$$

may have been inputted instead of

$$x_1 + 2x_2 + x_3 \le 1$$
.

A good way to debug such an issue is to dump the problem to OPF file and check whether the violated constraint has been specified correctly.

#### 9.1.2 Verifying optimality

Verifying that a feasible solution is optimal can be harder. However, for continuous problems optimality can verified using a dual solution. Normally, MOSEK will report a dual solution; if that is feasible and has the same objective value as the primal solution, then the primal solution must be optimal.

An alternative method is to find another primal solution that has better objective value than the one reported to MOSEK. If that is possible then either the problem is badly posed or there is bug in MOSEK.

# 9.2 Turn on logging

While developing a new application it is recommended to turn on logging, so that error and diagnostics messages are displayed. See example in section 5.2 for instructions on turning log output on. You should also always cache and handle any exceptions thrown by MOSEK.

More log information can be obtained by modifying one or more of the parameters:

```
• iparam.log,
```

- iparam.log\_intpnt,
- iparam.log\_mio,
- iparam.log\_cut\_second\_opt,
- iparam.log\_sim, and
- iparam.log\_sim\_minor.

By default MOSEK will reduce the amount of log information after the first optimization on a given task. To get full log output on subsequent optimizations set:

```
iparam.log_cut_second_opt 0
```

# 9.3 Writing task data to a file

If something is wrong with a problem or a solution, one option is to output the problem to an OPF file and inspect it by hand. Use the Task.writedata function to write a task to a file immediately before optimizing, for example as follows:

```
task.writedata("taskdump.opf")
task.optimizetrm()
```

This will write the problem in task to the file taskdump.opf. Inspecting the text file taskdump.opf may reveal what is wrong in the problem setup.

# 9.4 Important API limitations

#### 9.4.1 Thread safety

The MOSEK API is thread safe in the sense that any number of threads may use it simultaneously. However, the individual tasks and environments may *only* be accessed from at most one thread at a time.

# Chapter 10

# Problem formulation and solutions

In this chapter we will discuss the following issues:

- The formal definitions of the problem types that MOSEK can solve.
- The solution information produced by MOSEK.
- The information produced by MOSEK if the problem is infeasible.

# 10.1 Linear optimization

A linear optimization problem can be written as

where

- $\bullet$  m is the number of constraints.
- $\bullet$  *n* is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part of the objective function.
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.

- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.

A primal solution (x) is (primal) feasible if it satisfies all constraints in (10.1). If (10.1) has at least one primal feasible solution, then (10.1) is said to be (primal) feasible.

In case (10.1) does not have a feasible solution, the problem is said to be (primal) infeasible.

#### 10.1.1 Duality for linear optimization

Corresponding to the primal problem (10.1), there is a dual problem

maximize 
$$(l^{c})^{T} s_{l}^{c} - (u^{c})^{T} s_{u}^{c} + (l^{x})^{T} s_{l}^{x} - (u^{x})^{T} s_{u}^{x} + c^{f}$$
subject to 
$$A^{T} y + s_{l}^{x} - s_{u}^{x} = c,$$

$$- y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \geq 0.$$

$$(10.2)$$

If a bound in the primal problem is plus or minus infinity, the corresponding dual variable is fixed at 0, and we use the convention that the product of the bound value and the corresponding dual variable is 0. E.g.

$$l_i^x = -\infty \implies (s_l^x)_j = 0$$
 and  $l_i^x \cdot (s_l^x)_j = 0$ .

This is equivalent to removing variable  $(s_l^x)_j$  from the dual problem.

A solution

$$(y, s_l^c, s_u^c, s_l^x, s_u^x)$$

to the dual problem is feasible if it satisfies all the constraints in (10.2). If (10.2) has at least one feasible solution, then (10.2) is (dual) feasible, otherwise the problem is (dual) infeasible.

#### 10.1.1.1 A primal-dual feasible solution

A solution

$$(x, y, s_l^c, s_u^c, s_l^x, s_u^x)$$

is denoted a *primal-dual feasible solution*, if (x) is a solution to the primal problem (10.1) and  $(y, s_l^c, s_u^c, s_l^x, s_u^x)$  is a solution to the corresponding dual problem (10.2).

#### 10.1.1.2 The duality gap

Let

$$(x^*, y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

be a primal-dual feasible solution, and let

$$(x^c)^* := Ax^*.$$

For a primal-dual feasible solution we define the *duality gap* as the difference between the primal and the dual objective value,

$$c^{T}x^{*} + c^{f} - \left((l^{c})^{T}(s_{l}^{c})^{*} - (u^{c})^{T}(s_{u}^{c})^{*} + (l^{x})^{T}(s_{l}^{x})^{*} - (u^{x})^{T}(s_{u}^{x})^{*} + c^{f}\right)$$

$$= \sum_{i=0}^{m-1} \left[ (s_{l}^{c})_{i}^{*}((x_{i}^{c})^{*} - l_{i}^{c}) + (s_{u}^{c})_{i}^{*}(u_{i}^{c} - (x_{i}^{c})^{*}) \right] + \sum_{j=0}^{m-1} \left[ (s_{l}^{x})_{j}^{*}(x_{j} - l_{j}^{x}) + (s_{u}^{x})_{j}^{*}(u_{j}^{x} - x_{j}^{*}) \right]$$

$$\geq 0$$

$$(10.3)$$

where the first relation can be obtained by transposing and multiplying the dual constraints (10.2) by  $x^*$  and  $(x^c)^*$  respectively, and the second relation comes from the fact that each term in each sum is nonnegative. It follows that the primal objective will always be greater than or equal to the dual objective.

#### 10.1.1.3 When the objective is to be maximized

When the objective sense of problem (10.1) is maximization, i.e.

the objective sense of the dual problem changes to minimization, and the domain of all dual variables changes sign in comparison to (10.2). The dual problem thus takes the form

$$\begin{array}{lll} \text{minimize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x &= c, \\ & - y + s_l^c - s_u^c &= 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x &\leq 0. \end{array}$$

This means that the duality gap, defined in (10.3) as the primal minus the dual objective value, becomes nonpositive. It follows that the dual objective will always be greater than or equal to the primal objective.

#### 10.1.1.4 An optimal solution

It is well-known that a linear optimization problem has an optimal solution if and only if there exist feasible primal and dual solutions so that the duality gap is zero, or, equivalently, that the *complementarity conditions* 

$$\begin{array}{rclcrcl} (s_l^c)_i^*((x_i^c)^*-l_i^c) & = & 0, & i=0,\dots,m-1, \\ (s_u^c)_i^*(u_i^c-(x_i^c)^*) & = & 0, & i=0,\dots,m-1, \\ (s_l^x)_j^*(x_j^*-l_j^x) & = & 0, & j=0,\dots,n-1, \\ (s_u^x)_j^*(u_j^x-x_j^*) & = & 0, & j=0,\dots,n-1, \end{array}$$

are satisfied.

If (10.1) has an optimal solution and MOSEK solves the problem successfully, both the primal and dual solution are reported, including a status indicating the exact state of the solution.

#### 10.1.2 Infeasibility for linear optimization

#### 10.1.2.1 Primal infeasible problems

If the problem (10.1) is infeasible (has no feasible solution), MOSEK will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the modified dual problem

maximize 
$$(l^{c})^{T} s_{l}^{c} - (u^{c})^{T} s_{u}^{c} + (l^{x})^{T} s_{l}^{x} - (u^{x})^{T} s_{u}^{x}$$
subject to 
$$A^{T} y + s_{l}^{x} - s_{u}^{x} = 0,$$

$$- y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \geq 0,$$

$$(10.4)$$

such that the objective value is strictly positive, i.e. a solution

$$(y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

to (10.4) so that

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* > 0.$$

Such a solution implies that (10.4) is unbounded, and that its dual is infeasible. As the constraints to the dual of (10.4) is identical to the constraints of problem (10.1), we thus have that problem (10.1) is also infeasible.

#### 10.1.2.2 Dual infeasible problems

If the problem (10.2) is infeasible (has no feasible solution), MOSEK will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the modified primal problem

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right. \text{ and } \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right.$$

and

$$\hat{l}_j^x = \left\{ \begin{array}{ll} 0 & \text{if } l_j^x > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right. \text{ and } \hat{u}_j^x := \left\{ \begin{array}{ll} 0 & \text{if } u_j^x < \infty, \\ \infty & \text{otherwise,} \end{array} \right.$$

such that the objective value  $c^T x$  is strictly negative.

Such a solution implies that (10.5) is unbounded, and that its dual is infeasible. As the constraints to the dual of (10.5) is identical to the constraints of problem (10.2), we thus have that problem (10.2) is also infeasible.

#### 10.1.2.3 Primal and dual infeasible case

In case that both the primal problem (10.1) and the dual problem (10.2) are infeasible, MOSEK will report only one of the two possible certificates — which one is not defined (MOSEK returns the first certificate found).

## 10.2 Conic quadratic optimization

Conic quadratic optimization is an extensions of linear optimization (see Section 10.1) allowing conic domains to be specified for subsets of the problem variables. A conic quadratic optimization problem can be written as

minimize 
$$c^T x + c^f$$
  
subject to  $l^c \le Ax \le u^c$ ,  
 $l^x \le x \le u^x$ ,  
 $x \in \mathcal{C}$ , (10.6)

where set  $\mathcal{C}$  is a Cartesian product of convex cones, namely  $\mathcal{C} = \mathcal{C}_1 \times \cdots \times \mathcal{C}_p$ . Having the domain restriction,  $x \in \mathcal{C}$ , is thus equivalent to

$$x^t \in \mathcal{C}_t \subset \mathbb{R}^{n_t}$$
,

where  $x = (x^1, ..., x^p)$  is a partition of the problem variables. Please note that the *n*-dimensional Euclidean space  $\mathbb{R}^n$  is a cone itself, so simple linear variables are still allowed.

MOSEK supports only a limited number of cones, specifically:

• The  $\mathbb{R}^n$  set.

• The quadratic cone:

$$Q_n = \left\{ x \in \mathbb{R}^n : x_1 \ge \sqrt{\sum_{j=2}^n x_j^2} \right\}.$$

• The rotated quadratic cone:

$$Q_n^r = \left\{ x \in \mathbb{R}^n : 2x_1 x_2 \ge \sum_{j=3}^n x_j^2, \ x_1 \ge 0, \ x_2 \ge 0 \right\}.$$

Although these cones may seem to provide only limited expressive power they can be used to model a wide range of problems as demonstrated in [7].

#### 10.2.1 Duality for conic quadratic optimization

The dual problem corresponding to the conic quadratic optimization problem (10.6) is given by

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + s_n^x & = c, \\ & - y + s_l^c - s_u^c & = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x & \geq 0, \\ & s_n^x \in \mathcal{C}^*, \end{array} \tag{10.7}$$

where the dual cone  $C^*$  is a Cartesian product of the cones

$$\mathcal{C}^* = \mathcal{C}_1^* \times \cdots \times \mathcal{C}_n^*,$$

where each  $C_t^*$  is the dual cone of  $C_t$ . For the cone types MOSEK can handle, the relation between the primal and dual cone is given as follows:

• The  $\mathbb{R}^n$  set:

$$\mathcal{C}_t = \mathbb{R}^{n_t} \iff \mathcal{C}_t^* = \{ s \in \mathbb{R}^{n_t} : s = 0 \}.$$

• The quadratic cone:

$$\mathcal{C}_t = \mathcal{Q}_{n_t} \iff \mathcal{C}_t^* = \mathcal{Q}_{n_t} = \left\{ s \in \mathbb{R}^{n_t} : s_1 \ge \sqrt{\sum_{j=2}^{n_t} s_j^2} \right\}.$$

• The rotated quadratic cone:

$$C_t = Q_{n_t}^r \iff C_t^* = Q_{n_t}^r = \left\{ s \in \mathbb{R}^{n_t} : 2s_1 s_2 \ge \sum_{j=3}^{n_t} s_j^2, \ s_1 \ge 0, \ s_2 \ge 0 \right\}.$$

Please note that the dual problem of the dual problem is identical to the original primal problem.

#### 10.2.2 Infeasibility for conic quadratic optimization

In case MOSEK finds a problem to be infeasible it reports a certificate of the infeasibility. This works exactly as for linear problems (see Section 10.1.2).

#### 10.2.2.1 Primal infeasible problems

If the problem (10.6) is infeasible, MOSEK will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the problem

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x$$
  
subject to  $A^T y + s_l^x - s_u^x + s_n^x = 0,$   
 $-y + s_l^c - s_u^c = 0,$   
 $s_l^c, s_u^c, s_l^x, s_u^x \geq 0,$   
 $s_n^c \in \mathcal{C}^*,$  (10.8)

such that the objective value is strictly positive.

#### 10.2.2.2 Dual infeasible problems

If the problem (10.7) is infeasible, MOSEK will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

minimize 
$$c^T x$$
  
subject to  $\hat{l}^c \leq Ax \leq \hat{u}^c$ ,  
 $\hat{l}^x \leq x \leq \hat{u}^x$ , (10.9)

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right. \text{ and } \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right.$$

and

$$\hat{l}_j^x = \left\{ \begin{array}{ll} 0 & \text{if } l_j^x > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right. \text{ and } \hat{u}_j^x := \left\{ \begin{array}{ll} 0 & \text{if } u_j^x < \infty, \\ \infty & \text{otherwise,} \end{array} \right.$$

such that the objective value is strictly negative.

## 10.3 Semidefinite optimization

Semidefinite optimization is an extension of conic quadratic optimization (see Section 10.2) allowing positive semidefinite matrix variables to be used in addition to the usual scalar variables. A semidefinite optimization problem can be written as

minimize 
$$\sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle + c^f$$
 subject to  $l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle \leq u_i^c, \quad i = 0, \dots, m-1$  
$$(10.10)$$
 
$$l_j^x \leq \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle \leq u_i^c, \quad i = 0, \dots, m-1$$
 
$$x \in \mathcal{C}, \overline{X}_j \in \mathcal{S}_{r_j}^+, \qquad \qquad j = 0, \dots, n-1$$
 the problem has  $p$  symmetric positive semidefinite variables  $\overline{X}_j \in \mathcal{S}_{r_j}^+$  of dimension  $r_j$  with

where the problem has p symmetric positive semidefinite variables  $\overline{X}_j \in \mathcal{S}_{r_j}^+$  of dimension  $r_j$  with symmetric coefficient matrices  $\overline{C}_j \in \mathcal{S}_{r_j}$  and  $\overline{A}_{i,j} \in \mathcal{S}_{r_j}$ . We use standard notation for the matrix inner product, i.e., for  $U, V \in \mathbb{R}^{m \times n}$  we have

$$\langle U, V \rangle := \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} U_{ij} V_{ij}.$$

With semidefinite optimization we can model a wide range of problems as demonstrated in [7].

#### 10.3.1 Duality for semidefinite optimization

The dual problem corresponding to the semidefinite optimization problem (10.10) is given by

maximize 
$$(l^{c})^{T} s_{l}^{c} - (u^{c})^{T} s_{u}^{c} + (l^{x})^{T} s_{l}^{x} - (u^{x})^{T} s_{u}^{x} + c^{f}$$
subject to 
$$C_{j} - \sum_{i=0}^{m} y_{i} \overline{A}_{ij} = \overline{S}_{j}, \quad j = 0, \dots, p - 1$$

$$s_{l}^{c} - s_{u}^{c} = y,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \geq 0,$$

$$s_{n}^{x} \in \mathcal{C}^{*}, \ \overline{S}_{j} \in \mathcal{S}_{r_{j}}^{+}, \qquad j = 0, \dots, p - 1$$

$$(10.11)$$

where  $A \in \mathbb{R}^{m \times n}$ ,  $A_{ij} = a_{ij}$ , which is similar to the dual problem for conic quadratic optimization (see Section 10.7), except for the addition of dual constraints

$$(\overline{C}_j - \sum_{i=0}^m y_i \overline{A}_{ij}) \in \mathcal{S}_{r_j}^+.$$

Note that the dual of the dual problem is identical to the original primal problem.

#### 10.3.2 Infeasibility for semidefinite optimization

In case MOSEK finds a problem to be infeasible it reports a certificate of the infeasibility. This works exactly as for linear problems (see Section 10.1.2).

#### 10.3.2.1 Primal infeasible problems

If the problem (10.10) is infeasible, MOSEK will report a certificate of primal infeasibility: The dual solution reported is a certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the problem

maximize 
$$(l^{c})^{T} s_{l}^{c} - (u^{c})^{T} s_{u}^{c} + (l^{x})^{T} s_{l}^{x} - (u^{x})^{T} s_{u}^{x}$$
subject to 
$$\sum_{i=0}^{m-1} y_{i} \overline{A}_{ij} + \overline{S}_{j} = 0, \qquad j = 0, \dots, p-1$$

$$-y + s_{l}^{c} - s_{u}^{c} = 0, \qquad j = 0, \dots, p-1$$

$$-y + s_{l}^{c} - s_{u}^{c} = 0, \qquad j = 0, \dots, p-1$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \geq 0,$$

$$s_{n}^{x} \in \mathcal{C}^{*}, \ \overline{S}_{j} \in \mathcal{S}_{r_{j}}^{+}, \qquad j = 0, \dots, p-1$$

such that the objective value is strictly positive.

#### 10.3.2.2 Dual infeasible problems

If the problem (10.11) is infeasible, MOSEK will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

minimize 
$$\sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \langle \overline{C}_j, \overline{X}_j \rangle$$
subject to  $\hat{l}_i^c \leq \sum_{j=1} a_{ij} x_j + \sum_{j=0}^{p-1} \langle \overline{A}_{ij}, \overline{X}_j \rangle \leq \hat{u}_i^c, \quad i = 0, \dots, m-1$ 

$$\hat{l}^x \leq \frac{x}{x \in \mathcal{C}, \ \overline{X}_j \in \mathcal{S}_{r_j}^+, \qquad j = 0, \dots, p-1$$

$$(10.13)$$

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right. \text{ and } \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right.$$

and

$$\hat{l}_j^x = \begin{cases} 0 & \text{if } l_j^x > -\infty, \\ -\infty & \text{otherwise,} \end{cases} \text{ and } \hat{u}_j^x := \begin{cases} 0 & \text{if } u_j^x < \infty, \\ \infty & \text{otherwise,} \end{cases}$$

such that the objective value is strictly negative.

# 10.4 Quadratic and quadratically constrained optimization

A convex quadratic and quadratically constrained optimization problem is an optimization problem of the form

minimize 
$$\frac{1}{2}x^{T}Q^{o}x + c^{T}x + c^{f}$$
subject to  $l_{k}^{c} \leq \frac{1}{2}x^{T}Q^{k}x + \sum_{j=0}^{n-1} a_{kj}x_{j} \leq u_{k}^{c}, \quad k = 0, \dots, m-1,$ 

$$l_{j}^{x} \leq x_{j} \leq u_{j}^{x}, \quad j = 0, \dots, n-1,$$
(10.14)

where  $Q^o$  and all  $Q^k$  are symmetric matrices. Moreover for convexity,  $Q^o$  must be a positive semidefinite matrix and  $Q^k$  must satisfy

$$\begin{array}{rcl} -\infty < l_k^c & \Rightarrow & Q^k \text{ is negative semidefinite,} \\ u_k^c < \infty & \Rightarrow & Q^k \text{ is positive semidefinite,} \\ -\infty < l_k^c \le u_k^c < \infty & \Rightarrow & Q^k = 0. \end{array}$$

The convexity requirement is very important and it is strongly recommended that MOSEK is applied to convex problems only.

Note that any convex quadratic and quadratically constrained optimization problem can be reformulated as a conic optimization problem. It is our experience that for the majority of practical applications it is better to cast them as conic problems because

- the resulting problem is convex by construction, and
- the conic optimizer is more efficient than the optimizer for general quadratic problems.

See [7] for further details.

#### 10.4.1 Duality for quadratic and quadratically constrained optimization

The dual problem corresponding to the quadratic and quadratically constrained optimization problem (10.14) is given by

maximize 
$$(l^{c})^{T} s_{l}^{c} - (u^{c})^{T} s_{u}^{c} + (l^{x})^{T} s_{l}^{x} - (u^{x})^{T} s_{u}^{x} + \frac{1}{2} x^{T} \left( \sum_{k=0}^{m-1} y_{k} Q^{k} - Q^{o} \right) x + c^{f}$$
subject to 
$$A^{T} y + s_{l}^{x} - s_{u}^{x} + \left( \sum_{k=0}^{m-1} y_{k} Q^{k} - Q^{o} \right) x = c,$$

$$- y + s_{l}^{c} - s_{u}^{c} \qquad = 0,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \qquad \geq 0.$$

$$(10.15)$$

The dual problem is related to the dual problem for linear optimization (see Section 10.2), but depend on variable x which in general can not be eliminated. In the solutions reported by MOSEK, the value of x is the same for the primal problem (10.14) and the dual problem (10.15).

# 10.4.2 Infeasibility for quadratic and quadratically constrained optimization

In case MOSEK finds a problem to be infeasible it reports a certificate of the infeasibility. This works exactly as for linear problems (see Section 10.1.2).

#### 10.4.2.1 Primal infeasible problems

If the problem (10.14) with all  $Q^k = 0$  is infeasible, MOSEK will report a certificate of primal infeasibility. As the constraints is the same as for a linear problem, the certificate of infeasibility is the same as for linear optimization (see Section 10.1.2.1).

#### 10.4.2.2 Dual infeasible problems

If the problem (10.15) with all  $Q^k = 0$  is infeasible, MOSEK will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

minimize 
$$c^{T}x$$
subject to 
$$\hat{l}^{c} \leq Ax \leq \hat{u}^{c},$$

$$0 \leq Q^{o}x \leq 0,$$

$$\hat{l}^{x} \leq x \leq \hat{u}^{x},$$

$$(10.16)$$

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right. \text{ and } \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right.$$

and

$$\hat{l}_j^x = \left\{ \begin{array}{ll} 0 & \text{if } l_j^x > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right. \text{ and } \hat{u}_j^x := \left\{ \begin{array}{ll} 0 & \text{if } u_j^x < \infty, \\ \infty & \text{otherwise,} \end{array} \right.$$

such that the objective value is strictly negative.

# Chapter 11

# The optimizers for continuous problems

The most essential part of MOSEK is the optimizers. Each optimizer is designed to solve a particular class of problems i.e. linear, conic, or general nonlinear problems. The purpose of the present chapter is to discuss which optimizers are available for the continuous problem classes and how the performance of an optimizer can be tuned, if needed.

This chapter deals with the optimizers for *continuous problems* with no integer variables.

# 11.1 How an optimizer works

When the optimizer is called, it roughly performs the following steps:

#### Presolve:

Preprocessing to reduce the size of the problem.

#### Dualizer:

Choosing whether to solve the primal or the dual form of the problem.

#### Scaling

Scaling the problem for better numerical stability.

#### Optimize:

Solve the problem using selected method.

The first three preprocessing steps are transparent to the user, but useful to know about for tuning purposes. In general, the purpose of the preprocessing steps is to make the actual optimization more efficient and robust.

#### 11.1.1 Presolve

Before an optimizer actually performs the optimization the problem is preprocessed using the so-called presolve. The purpose of the presolve is to

- remove redundant constraints,
- eliminate fixed variables,
- remove linear dependencies,
- substitute out (implied) free variables, and
- reduce the size of the optimization problem in general.

After the presolved problem has been optimized the solution is automatically postsolved so that the returned solution is valid for the original problem. Hence, the presolve is completely transparent. For further details about the presolve phase, please see [8], [9].

It is possible to fine-tune the behavior of the presolve or to turn it off entirely. If presolve consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This is done by setting the parameter <code>iparam.presolve\_use</code> to <code>presolvemode.off</code>.

The two most time-consuming steps of the presolve are

- the eliminator, and
- the linear dependency check.

Therefore, in some cases it is worthwhile to disable one or both of these.

#### 11.1.1.1 Numerical issues in the presolve

During the presolve the problem is reformulated so that it hopefully solves faster. However, in rare cases the presolved problem may be harder to solve then the original problem. The presolve may also be infeasible although the orinal problem is not.

If it is suspected that presolved problem is much harder to solve than the original then it is suggested to first turn the eliminator off by setting the parameter <code>iparam.presolve\_eliminator\_use</code>. If that does not help, then trying to turn presolve off may help.

Since all computations are done in finite prescision then the presolve employs some tolerances when concluding a variable is fixed or constraint is redundant. If it happens that MOSEK incorrectly concludes a problem is primal or dual infeasible, then it is worthwhile to try to reduce the parameters dparam.presolve\_tol\_x and dparam.presolve\_tol\_s. However, if actually help reducing the parameters then this should be taken as an indication of the problem is badly formulated.

#### 11.1.1.2 Eliminator

The purpose of the eliminator is to eliminate free and implied free variables from the problem using substitution. For instance, given the constraints

$$y = \sum x_j,$$
  
$$y, x \ge 0,$$

y is an implied free variable that can be substituted out of the problem, if deemed worthwhile.

If the eliminator consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This can be done with the parameter <code>iparam.presolve\_eliminator\_use</code> to <code>onoffkey.off</code>.

In rare cases the eliminator may cause that the problem becomes much hard to solve.

#### 11.1.1.3 Linear dependency checker

The purpose of the linear dependency check is to remove linear dependencies among the linear equalities. For instance, the three linear equalities

$$\begin{array}{rcl} x_1 + x_2 + x_3 & = & 1, \\ x_1 + 0.5x_2 & = & 0.5, \\ 0.5x_2 + x_3 & = & 0.5 \end{array}$$

contain exactly one linear dependency. This implies that one of the constraints can be dropped without changing the set of feasible solutions. Removing linear dependencies is in general a good idea since it reduces the size of the problem. Moreover, the linear dependencies are likely to introduce numerical problems in the optimization phase.

It is best practise to build models without linear dependencies. If the linear dependencies are removed at the modeling stage, the linear dependency check can safely be disabled by setting the parameter iparam.presolve\_lindep\_use to onoffkey.off.

#### 11.1.2 Dualizer

All linear, conic, and convex optimization problems have an equivalent dual problem associated with them. MOSEK has built-in heuristics to determine if it is most efficient to solve the primal or dual problem. The form (primal or dual) solved is displayed in the MOSEK log. Should the internal heuristics not choose the most efficient form of the problem it may be worthwhile to set the dualizer manually by setting the parameters:

- iparam.intpnt\_solve\_form: In case of the interior-point optimizer.
- iparam.sim\_solve\_form: In case of the simplex optimizer.

Note that currently only linear problems may be dualized.

#### 11.1.3 Scaling

Problems containing data with large and/or small coefficients, say 1.0e+9 or 1.0e-7, are often hard to solve. Significant digits may be truncated in calculations with finite precision, which can result in the optimizer relying on inaccurate calculations. Since computers work in finite precision, extreme coefficients should be avoided. In general, data around the same "order of magnitude" is preferred, and we will refer to a problem, satisfying this loose property, as being well-scaled. If the problem is not well scaled, MOSEK will try to scale (multiply) constraints and variables by suitable constants. MOSEK solves the scaled problem to improve the numerical properties.

The scaling process is transparent, i.e. the solution to the original problem is reported. It is important to be aware that the optimizer terminates when the termination criterion is met on the scaled problem, therefore significant primal or dual infeasibilities may occur after unscaling for badly scaled problems. The best solution to this problem is to reformulate it, making it better scaled.

By default MOSEK heuristically chooses a suitable scaling. The scaling for interior-point and simplex optimizers can be controlled with the parameters <code>iparam.intpnt\_scaling</code> and <code>iparam.sim\_scaling</code> respectively.

#### 11.1.4 Using multiple threads

The interior-point optimizers in MOSEK have been parallelized. This means that if you solve linear, quadratic, conic, or general convex optimization problem using the interior-point optimizer, you can take advantage of multiple CPU's.

By default MOSEK will automatically select the number of threads to be employed when solving the problem. However, the number of threads employed can be changed by setting the parameter <code>iparam.num\_threads</code>. This should never exceed the number of cores on the computer.

The speed-up obtained when using multiple threads is highly problem and hardware dependent, and consequently, it is advisable to compare single threaded and multi threaded performance for the given problem type to determine the optimal settings.

For small problems, using multiple threads is not be worthwhile and may even be counter productive.

# 11.2 Linear optimization

#### 11.2.1 Optimizer selection

Two different types of optimizers are available for linear problems: The default is an interior-point method, and the alternatives are simplex methods. The optimizer can be selected using the parameter iparam.optimizer.

#### 11.2.2 The interior-point optimizer

The purpose of this section is to provide information about the algorithm employed in MOSEK interiorpoint optimizer.

In order to keep the discussion simple it is assumed that MOSEK solves linear optimization problems on standard form

minimize 
$$c^T x$$
  
subject to  $Ax = b$ ,  $x \ge 0$ . (11.1)

This is in fact what happens inside MOSEK; for efficiency reasons MOSEK converts the problem to standard form before solving, then convert it back to the input form when reporting the solution.

Since it is not known beforehand whether problem (11.1) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason that MOSEK solves the so-called homogeneous model

$$Ax - b\tau = 0,$$

$$A^{T}y + s - c\tau = 0,$$

$$-c^{T}x + b^{T}y - \kappa = 0,$$

$$x, s, \tau, \kappa \geq 0,$$

$$(11.2)$$

where y and s correspond to the dual variables in (11.1), and  $\tau$  and  $\kappa$  are two additional scalar variables. Note that the homogeneous model (11.2) always has solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one.

Any solution

$$(x^*, y^*, s^*, \tau^*, \kappa^*)$$

to the homogeneous model (11.2) satisfies

$$x_{i}^{*}s_{i}^{*} = 0$$
 and  $\tau^{*}\kappa^{*} = 0$ .

Moreover, there is always a solution that has the property

$$\tau^* + \kappa^* > 0.$$

First, assume that  $\tau^* > 0$ . It follows that

$$A\frac{x^*}{\tau^*} = b,$$

$$A^T \frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} = c,$$

$$-c^T \frac{x^*}{\tau^*} + b^T \frac{y^*}{\tau^*} = 0,$$

$$x^*, s^*, \tau^*, \kappa^* \ge 0.$$

This shows that  $\frac{x^*}{\tau^*}$  is a primal optimal solution and  $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$  is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left(\frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau_*}\right)$$

is a primal-dual optimal solution.

On other hand, if  $\kappa^* > 0$  then

$$\begin{array}{rcl}
Ax^* & = & 0, \\
A^T y^* + s^* & = & 0, \\
-c^T x^* + b^T y^* & = & \kappa^*, \\
x^*, s^*, \tau^*, \kappa^* & \ge & 0.
\end{array}$$

This implies that at least one of

$$-c^T x^* > 0 \tag{11.3}$$

or

$$b^T y^* > 0 \tag{11.4}$$

is satisfied. If (11.3) is satisfied then  $x^*$  is a certificate of dual infeasibility, whereas if (11.4) is satisfied then  $y^*$  is a certificate of dual infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm [10].

#### 11.2.2.1 Interior-point termination criterion

For efficiency reasons it is not practical to solve the homogeneous model exactly. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In every iteration, k, of the interior-point algorithm a trial solution

$$(x^k,y^k,s^k,\tau^k,\kappa^k)$$

to homogeneous model is generated where

$$x^k, s^k, \tau^k, \kappa^k > 0.$$

Whenever the trial solution satisfies the criterion

$$\left\| A \frac{x^{k}}{\tau^{k}} - b \right\|_{\infty} \leq \epsilon_{p} (1 + \|b\|_{\infty}),$$

$$\left\| A^{T} \frac{y^{k}}{\tau^{k}} + \frac{s^{k}}{\tau^{k}} - c \right\|_{\infty} \leq \epsilon_{d} (1 + \|c\|_{\infty}), \text{ and}$$

$$\min \left( \frac{(x^{k})^{T} s^{k}}{(\tau^{k})^{2}}, \left| \frac{c^{T} x^{k}}{\tau^{k}} - \frac{b^{T} y^{k}}{\tau^{k}} \right| \right) \leq \epsilon_{g} \max \left( 1, \frac{\min(\left| c^{T} x^{k} \right|, \left| b^{T} y^{k} \right|)}{\tau^{k}} \right),$$

$$(11.5)$$

the interior-point optimizer is terminated and

$$\frac{(x^k,y^k,s^k)}{\tau^k}$$

is reported as the primal-dual optimal solution. The interpretation of (11.5) is that the optimizer is terminated if

- $\frac{x^k}{\tau^k}$  is approximately primal feasible,
- $\bullet \ \left(\frac{y^k}{\tau^k}, \frac{s^k}{\tau^k}\right)$  is approximately dual feasible, and
- the duality gap is almost zero.

On the other hand, if the trial solution satisfies

$$-\epsilon_i c^T x^k > \frac{\|c\|_{\infty}}{\max(1, \|b\|_{\infty})} \|Ax^k\|_{\infty}$$

then the problem is declared dual infeasible and  $x^k$  is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows: First assume that  $||Ax^k||_{\infty} = 0$ ; then  $x^k$  is an exact certificate of dual infeasibility. Next assume that this is not the case, i.e.

$$||Ax^k||_{\infty} > 0,$$

and define

$$\bar{x} := \epsilon_i \frac{\max(1, ||b||_{\infty})}{||Ax^k||_{\infty} ||c||_{\infty}} x^k.$$

It is easy to verify that

$$||A\bar{x}||_{\infty} = \epsilon_i \frac{\max(1, ||b||_{\infty})}{||c||_{\infty}} \text{ and } -c^T \bar{x} > 1,$$

which shows  $\bar{x}$  is an approximate certificate of dual infeasibility where  $\epsilon_i$  controls the quality of the approximation. A smaller value means a better approximation.

Tolerance	Parameter name
$\epsilon_p$	dparam.intpnt_tol_pfeas
$\epsilon_d$	dparam.intpnt_tol_dfeas
$\epsilon_g$	dparam.intpnt_tol_rel_gap
$\epsilon_i$	${\tt dparam.intpnt\_tol\_infeas}$

Table 11.1: Parameters employed in termination criterion.

Finally, if

$$\epsilon_i b^T y^k > \frac{\|b\|_{\infty}}{\max(1, \|c\|_{\infty})} \|A^T y^k + s^k\|_{\infty}$$

then  $y^k$  is reported as a certificate of primal infeasibility.

It is possible to adjust the tolerances  $\epsilon_p$ ,  $\epsilon_d$ ,  $\epsilon_g$  and  $\epsilon_i$  using parameters; see table 11.1 for details. The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (11.5) reveals that quality of the solution is dependent on  $||b||_{\infty}$  and  $||c||_{\infty}$ ; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by MOSEK will converge toward optimality and primal and dual feasibility at the same rate [10]. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances,  $\epsilon_p$ ,  $\epsilon_d$  and  $\epsilon_q$ , has to be relaxed together to achieve an effect.

In some cases the interior-point method terminates having found a solution not too far from meeting the optimality condition (11.5). A solution is defined as near optimal if scaling  $\epsilon_p$ ,  $\epsilon_d$  and  $\epsilon_g$  by any number  $\epsilon_n \in [1.0, +\infty]$  conditions (11.5) are satisfied.

A near optimal solution is therefore of lower quality but still potentially valuable. If for instance the solver stalls, i.e. it can make no more significant progress towards the optimal solution, a near optimal solution could be available and be good enough for the user.

The basis identification discussed in section 11.2.2.2 requires an optimal solution to work well; hence basis identification should turned off if the termination criterion is relaxed.

To conclude the discussion in this section, relaxing the termination criterion is usually is not worthwhile.

#### 11.2.2.2 Basis identification

An interior-point optimizer does not return an optimal basic solution unless the problem has a unique primal and dual optimal solution. Therefore, the interior-point optimizer has an optimal post-processing step that computes an optimal basic solution starting from the optimal interior-point solution. More information about the basis identification procedure may be found in [11].

Please note that a basic solution is often more accurate than an interior-point solution.

By default MOSEK performs a basis identification. However, if a basic solution is not needed, the

basis identification procedure can be turned off. The parameters

- iparam.intpnt\_basis,
- iparam.bi\_ignore\_max\_iter, and
- iparam.bi\_ignore\_num\_error

controls when basis identification is performed.

#### 11.2.2.3 The interior-point log

Below is a typical log output from the interior-point optimizer presented:

```
Optimizer - threads
Optimizer - solved problem
                                   : the dual
Optimizer - Constraints
                                   : 2
Optimizer - Cones
                                   : 0
Optimizer - Scalar variables
                                 : 6
                                                       conic
Optimizer - Semi-definite variables: 0
                                                      scalarized
                                                                            : 0
                                  : 0.00
          - setup time
                                                      dense det. time
          - ML order time
Factor
                                   : 0.00
                                                      GP order time
                                                                             : 0.00
Factor
          - nonzeros before factor : 3
                                                      after factor
                                                                             : 3
Factor
          - dense dim.
                                   : 0
                                                      flops
                                                                             : 7.00e+001
                   GFEAS
                                                                            MU
ITE PFEAS
            DFEAS
                              PRSTATUS
                                        POBJ
                                                          DOBJ
   1.0e+000 8.6e+000 6.1e+000 1.00e+000 0.00000000e+000 -2.208000000e+003 1.0e+000 0.00
   1.1e+000 2.5e+000 1.6e-001 0.00e+000 -7.901380925e+003 -7.394611417e+003 2.5e+000 0.00
   1.4e-001 3.4e-001 2.1e-002 8.36e-001 -8.113031650e+003 -8.055866001e+003 3.3e-001 0.00
   2.4e-002 5.8e-002 3.6e-003 1.27e+000 -7.777530698e+003 -7.766471080e+003 5.7e-002 0.01
   1.3e-004 3.2e-004 2.0e-005 1.08e+000 -7.668323435e+003 -7.668207177e+003 3.2e-004 0.01
   1.3e-008 3.2e-008 2.0e-009 1.00e+000 -7.668000027e+003 -7.668000015e+003 3.2e-008 0.01
   1.3e-012 3.2e-012 2.0e-013 1.00e+000 -7.667999994e+003 -7.667999994e+003 3.2e-012 0.01
```

The first line displays the number of threads used by the optimizer and second line tells that the optimizer choose to solve the dual problem rather than the primal problem. The next line displays the problem dimensions as seen by the optimizer, and the "Factor..." lines show various statistics. This is followed by the iteration log.

Using the same notation as in section 11.2.2 the columns of the iteration log has the following meaning:

- ITE: Iteration index.
- PFEAS:  $||Ax^k b\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards to zero but may stall at low level due to rounding errors.
- DFEAS:  $||A^Ty^k + s^k c\tau^k||_{\infty}$ . The numbers in this column should converge monotonically toward to zero but may stall at low level due to rounding errors.
- GFEAS:  $\|-cx^k+b^Ty^k-\kappa^k\|_{\infty}$ . The numbers in this column should converge monotonically toward to zero but may stall at low level due to rounding errors.
- PRSTATUS: This number converge to 1 if the problem has an optimal solution whereas it converge to -1 if that is not the case.

- POBJ:  $c^T x^k / \tau^k$ . An estimate for the primal objective value.
- DOBJ:  $b^T y^k / \tau^k$ . An estimate for the dual objective value.
- MU:  $\frac{(x^k)^T s^k + \tau^k \kappa^k}{n+1}$  . The numbers in this column should always converge monotonically to zero.
- TIME: Time spend since the optimization started.

#### 11.2.3 The simplex based optimizer

An alternative to the interior-point optimizer is the simplex optimizer.

The simplex optimizer uses a different method that allows exploiting an initial guess for the optimal solution to reduce the solution time. Depending on the problem it may be faster or slower to use an initial guess; see section 11.2.4 for a discussion.

MOSEK provides both a primal and a dual variant of the simplex optimizer — we will return to this later.

#### 11.2.3.1 Simplex termination criterion

The simplex optimizer terminates when it finds an optimal basic solution or an infeasibility certificate. A basic solution is optimal when it is primal and dual feasible; see (10.1) and (10.2) for a definition of the primal and dual problem. Due the fact that to computations are performed in finite precision MOSEK allows violation of primal and dual feasibility within certain tolerances. The user can control the allowed primal and dual infeasibility with the parameters <code>dparam.basis\_tol\_x</code> and <code>dparam.basis\_tol\_s</code>.

#### 11.2.3.2 Starting from an existing solution

When using the simplex optimizer it may be possible to reuse an existing solution and thereby reduce the solution time significantly. When a simplex optimizer starts from an existing solution it is said to perform a *hot-start*. If the user is solving a sequence of optimization problems by solving the problem, making modifications, and solving again, MOSEK will hot-start automatically.

Setting the parameter iparam.optimizer to optimizertype.free\_simplex instructs MOSEK to select automatically between the primal and the dual simplex optimizers. Hence, MOSEK tries to choose the best optimizer for the given problem and the available solution.

By default MOSEK uses presolve when performing a hot-start. If the optimizer only needs very few iterations to find the optimal solution it may be better to turn off the presolve.

#### 11.2.3.3 Numerical difficulties in the simplex optimizers

Though MOSEK is designed to minimize numerical instability, completely avoiding it is impossible when working in finite precision. MOSEK counts a "numerical unexpected behavior" event inside the optimizer as a *set-back*. The user can define how many set-backs the optimizer accepts; if that number

is exceeded, the optimization will be aborted. Set-backs are implemented to avoid long sequences where the optimizer tries to recover from an unstable situation.

Set-backs are, for example, repeated singularities when factorizing the basis matrix, repeated loss of feasibility, degeneracy problems (no progress in objective) and other events indicating numerical difficulties. If the simplex optimizer encounters a lot of set-backs the problem is usually badly scaled; in such a situation try to reformulate into a better scaled problem. Then, if a lot of set-backs still occur, trying one or more of the following suggestions may be worthwhile:

- Raise tolerances for allowed primal or dual feasibility: Hence, increase the value of
  - dparam.basis\_tol\_x, and
  - dparam.basis\_tol\_s.
- Raise or lower pivot tolerance: Change the dparam.simplex\_abs\_tol\_piv parameter.
- Switch optimizer: Try another optimizer.
- Switch off crash: Set both iparam.sim\_primal\_crash and iparam.sim\_dual\_crash to 0.
- Experiment with other pricing strategies: Try different values for the parameters
  - iparam.sim\_primal\_selection and
  - iparam.sim\_dual\_selection.
- If you are using hot-starts, in rare cases switching off this feature may improve stability. This is controlled by the <code>iparam.sim.hotstart</code> parameter.
- Increase maximum set-backs allowed controlled by iparam.sim\_max\_num\_setbacks.
- If the problem repeatedly becomes infeasible try switching off the special degeneracy handling. See the parameter iparam.sim\_degen for details.

#### 11.2.4 The interior-point or the simplex optimizer?

Given a linear optimization problem, which optimizer is the best: The primal simplex, the dual simplex or the interior-point optimizer?

It is impossible to provide a general answer to this question, however, the interior-point optimizer behaves more predictably — it tends to use between 20 and 100 iterations, almost independently of problem size — but cannot perform hot-start, while simplex can take advantage of an initial solution, but is less predictable for cold-start. The interior-point optimizer is used by default.

#### 11.2.5 The primal or the dual simplex variant?

MOSEK provides both a primal and a dual simplex optimizer. Predicting which simplex optimizer is faster is impossible, however, in recent years the dual optimizer has seen several algorithmic and computational improvements, which, in our experience, makes it faster on average than the primal simplex optimizer. Still, it depends much on the problem structure and size.

Setting the iparam.optimizer parameter to optimizertype.free\_simplex instructs MOSEK to choose which simplex optimizer to use automatically.

To summarize, if you want to know which optimizer is faster for a given problem type, you should try all the optimizers.

Alternatively, use the concurrent optimizer presented in Section 11.6.3.

### 11.3 Linear network optimization

#### 11.3.1 Network flow problems

Linear optimization problems with network flow structure can often be solved significantly faster with a specialized version of the simplex method [12] than with the general solvers.

MOSEK includes a network simplex solver which frequently solves network problems significantly faster than the standard simplex optimizers.

To use the network simplex optimizer, do the following:

- Input the network flow problem as an ordinary linear optimization problem.
- Set the parameters
  - iparam.optimizer to optimizertype.network\_primal\_simplex.
- Optimize the problem using Task.optimize.

MOSEK will automatically detect the network structure and apply the specialized simplex optimizer.

# 11.4 Conic optimization

#### 11.4.1 The interior-point optimizer

For conic optimization problems only an interior-point type optimizer is available. The interior-point optimizer is an implementation of the so-called homogeneous and self-dual algorithm. For a detailed description of the algorithm, please see [13].

#### 11.4.1.1 Interior-point termination criteria

The parameters controlling when the conic interior-point optimizer terminates are shown in Table 11.2.

Parameter name	Purpose
dparam.intpnt_co_tol_pfeas	Controls primal feasibility
dparam.intpnt_co_tol_dfeas	Controls dual feasibility
dparam.intpnt_co_tol_rel_gap	Controls relative gap
dparam.intpnt_tol_infeas	Controls when the problem is declared infeasible
dparam.intpnt_co_tol_mu_red	Controls when the complementarity is reduced enough

Table 11.2: Parameters employed in termination criterion.

## 11.5 Nonlinear convex optimization

#### 11.5.1 The interior-point optimizer

For quadratic, quadratically constrained, and general convex optimization problems an interior-point type optimizer is available. The interior-point optimizer is an implementation of the homogeneous and self-dual algorithm. For a detailed description of the algorithm, please see [14], [15].

#### 11.5.1.1 The convexity requirement

Continuous nonlinear problems are required to be convex. For quadratic problems MOSEK test this requirement before optimizing. Specifying a non-convex problem results in an error message.

The following parameters are available to control the convexity check:

- iparam.check\_convexity: Turn convexity check on/off.
- dparam.check\_convexity\_rel\_tol: Tolerance for convexity check.
- iparam.log\_check\_convexity: Turn on more log information for debugging.

#### 11.5.1.2 The differentiability requirement

The nonlinear optimizer in MOSEK requires both first order and second order derivatives. This of course implies care should be taken when solving problems involving non-differentiable functions.

For instance, the function

$$f(x) = x^2$$

is differentiable everywhere whereas the function

$$f(x) = \sqrt{x}$$

is only differentiable for x>0. In order to make sure that MOSEK evaluates the functions at points where they are differentiable, the function domains must be defined by setting appropriate variable bounds.

Parameter name	Purpose
dparam.intpnt_nl_tol_pfeas	Controls primal feasibility
dparam.intpnt_nl_tol_dfeas	Controls dual feasibility
dparam.intpnt_nl_tol_rel_gap	Controls relative gap
dparam.intpnt_tol_infeas	Controls when the problem is declared infeasible
dparam.intpnt_nl_tol_mu_red	Controls when the complementarity is reduced enough

Table 11.3: Parameters employed in termination criteria.

In general, if a variable is not ranged MOSEK will only evaluate that variable at points strictly within the bounds. Hence, imposing the bound

$$x \ge 0$$

in the case of  $\sqrt{x}$  is sufficient to guarantee that the function will only be evaluated in points where it is differentiable.

However, if a function is differentiable on closed a range, specifying the variable bounds is not sufficient. Consider the function

$$f(x) = \frac{1}{x} + \frac{1}{1 - x}. (11.6)$$

In this case the bounds

$$0 \le x \le 1$$

will not guarantee that MOSEK only evaluates the function for x between 0 and 1. To force MOSEK to strictly satisfy both bounds on ranged variables set the parameter <code>iparam.intpnt\_starting\_point</code> to <code>startpointtype.satisfy\_bounds</code>.

For efficiency reasons it may be better to reformulate the problem than to force MOSEK to observe ranged bounds strictly. For instance, (11.6) can be reformulated as follows

$$f(x) = \frac{1}{x} + \frac{1}{y}$$

$$0 = 1 - x - y$$

$$0 \le x$$

$$0 \le y.$$

#### 11.5.1.3 Interior-point termination criteria

The parameters controlling when the general convex interior-point optimizer terminates are shown in Table 11.3.

## 11.6 Solving problems in parallel

If a computer has multiple CPUs, or has a CPU with multiple cores, it is possible for MOSEK to take advantage of this to speed up solution times.

#### 11.6.1 Thread safety

The MOSEK API is thread-safe provided that a task is only modified or accessed from one thread at any given time — accessing two separate tasks from two separate threads at the same time is safe. Sharing an environment between threads is safe.

#### 11.6.2 The parallelized interior-point optimizer

The interior-point optimizer is capable of using multiple CPUs or cores. This implies that whenever the MOSEK interior-point optimizer solves an optimization problem, it will try to divide the work so that each core gets a share of the work. The user decides how many coress MOSEK should exploit.

It is not always possible to divide the work equally, and often parts of the computations and the coordination of the work is processed sequentially, even if several cores are present. Therefore, the speed-up obtained when using multiple cores is highly problem dependent. However, as a rule of thumb, if the problem solves very quickly, i.e. in less than 60 seconds, it is not advantageous to use the parallel option.

The iparam.num\_threads parameter sets the number of threads (and therefore the number of cores) that the interior point optimizer will use.

#### 11.6.3 The concurrent optimizer

An alternative to the parallel interior-point optimizer is the *concurrent optimizer*. The idea of the concurrent optimizer is to run multiple optimizers on the same problem concurrently, for instance, it allows you to apply the interior-point and the dual simplex optimizers to a linear optimization problem concurrently. The concurrent optimizer terminates when the first of the applied optimizers has terminated successfully, and it reports the solution of the fastest optimizer. In that way a new optimizer has been created which essentially performs as the fastest of the interior-point and the dual simplex optimizers. Hence, the concurrent optimizer is the best one to use if there are multiple optimizers available in MOSEK for the problem and you cannot say beforehand which one will be faster.

Note in particular that any solution present in the task will also be used for hot-starting the simplex algorithms. One possible scenario would therefore be running a hot-start dual simplex in parallel with interior point, taking advantage of both the stability of the interior-point method and the ability of the simplex method to use an initial solution.

By setting the

iparam.optimizer

parameter to

Optimizer	Associated	Default
	parameter	priority
optimizertype.intpnt	iparam.concurrent_priority_intpnt	4
optimizertype.free_simplex	iparam.concurrent_priority_free_simplex	3
optimizertype.primal_simplex	<pre>iparam.concurrent_priority_primal_simplex</pre>	2
${\tt optimizertype.dual\_simplex}$	<pre>iparam.concurrent_priority_dual_simplex</pre>	1

Table 11.4: Default priorities for optimizer selection in concurrent optimization.

```
optimizertype.concurrent
```

the concurrent optimizer chosen.

The number of optimizers used in parallel is determined by the

```
iparam.concurrent_num_optimizers.
```

parameter. Moreover, the optimizers are selected according to a preassigned priority with optimizers having the highest priority being selected first. The default priority for each optimizer is shown in Table 11.6.3. For example, setting the <code>iparam.concurrent\_num\_optimizers</code> parameter to 2 tells the concurrent optimizer to the apply the two optimizers with highest priorities: In the default case that means the interior-point optimizer and one of the simplex optimizers.

#### 11.6.3.1 Concurrent optimization through the API

The following example shows how to call the concurrent optimizer through the API.

```
Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
#
#
   File:
             concurrent1.py
#
   Purpose: To demonstrate how to optimize in parallel using the
#
             concurrent optimizer.
##
import sys
import mosek
from mosek.array import array
# Since the actual value of Infinity is ignores, we define it solely
# for symbolic purposes:
inf = 0.0
# Define a stream printer to grab output from MOSEK
def streamprinter(text):
    sys.stdout.write(text)
    sys.stdout.flush()
```

```
# We might write everything directly as a script, but it looks nicer
# to create a function.
def main (args):
    # Open MOSEK and create an environment and task
    # Create a MOSEK environment
    env = mosek.Env ()
    # Attach a printer to the environment
    env.set_Stream (mosek.streamtype.log, streamprinter)
    # Create a task
    task = env.Task(0.0)
    # Attach a printer to the task
    task.set_Stream (mosek.streamtype.log, streamprinter)
    task.readdata(args[0])
    task.putintparam(mosek.iparam.optimizer,
                     mosek.optimizertype.concurrent)
    task.putintparam(mosek.iparam.concurrent_num_optimizers, 2)
    task.optimize()
    task.solutionsummary(mosek.streamtype.msg)
# call the main function
   main (sys.argv[1:])
except mosek. Exception as e:
   print ("ERROR: %s" % str(e.code))
    if msg is not None:
       print ("\t%s" % e.msg)
   sys.exit(1)
except:
    import traceback
    traceback.print_exc()
    sys.exit(1)
sys.exit(0)
```

#### 11.6.4 A more flexible concurrent optimizer

MOSEK also provides a more flexible method of concurrent optimization by using the function <code>Task.optimizeconcurrent</code>. The main advantages of this function are that it allows the calling application to assign arbitrary values to the parameters of each tasks, and that call-back functions can be attached to each task. This may be useful in the following situation: Assume that you know the primal simplex optimizer to be the best optimizer for your problem, but that you do not know which of the available selection strategies (as defined by the <code>iparam.sim\_primal\_selection</code> parameter) is the best. In this case you can solve the problem with the primal simplex optimizer using several different selection strategies concurrently.

An example demonstrating the usage of the **Task.optimizeconcurrent** function is included below. The example solves a single problem using the interior-point and primal simplex optimizers in parallel.

##

```
Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
#
  File:
              concurrent2.py
#
# Purpose: To demonstrate a more flexible interface for concurrent optimization.
##
import sys
import mosek
from mosek.array import array
# Since the actual value of Infinity is ignores, we define it solely
# for symbolic purposes:
inf = 0.0
# Define a stream printer to grab output from MOSEK
class streamprinter:
    def __init__(self,prefix):
        self.prefix = str(prefix)
    def __call__(self,text):
        #sys.stdout.write (self.prefix + text)
        sys.stdout.write (self.prefix + text)
        sys.stdout.flush()
        pass
# We might write everything directly as a script, but it looks nicer
# to create a function.
def main (args):
    # Open MOSEK and create an environment and task
    # Create a MOSEK environment
    env = mosek.Env ()
    # Attach a printer to the environment
    env.set_Stream (mosek.streamtype.log, streamprinter("[env]"))
    # Create a task
    task = env.Task(0,0)
    # Attach a printer to the task
   task.set_Stream (mosek.streamtype.log, streamprinter("simplex: "))
    # Create a task
    task_list = [env.Task(0,0)]
    # Attach a printer to the task
    task_list[0].set_Stream(mosek.streamtype.log, streamprinter("intpnt: "))
   task.readdata(args[0]);
    # Assign different parameter values to each task.
    # In this case different optimizers.
    task.putintparam(mosek.iparam.optimizer,
                           mosek.optimizertype.primal_simplex)
    task_list[0].putintparam(mosek.iparam.optimizer,
                             mosek.optimizertype.intpnt)
```

```
# Optimize task and task_list[0] in parallel.
# The problem data i.e. C, A, etc.
# is copied from task to task_list[0].
task.optimizeconcurrent(task_list)

task.solutionsummary(mosek.streamtype.log)

# call the main function
try:
    main (sys.argv[1:])
except mosek.Exception as e:
    print ("ERROR: %s" % str(e.errno))
    if e.msg is not None:
        print ("\t%s" % e.msg)
    sys.exit(1)
except:
    import traceback
    traceback.print_exc()
    sys.exit(1)
```

# Chapter 12

# The optimizers for mixed-integer problems

A problem is a mixed-integer optimization problem when one or more of the variables are constrained to be integer valued. MOSEK contains two optimizers for mixed integer problems that is capable for solving mixed-integer

- linear,
- quadratic and quadratically constrained, and
- conic

#### problems.

Readers unfamiliar with integer optimization are recommended to consult some relevant literature, e.g. the book [16] by Wolsey.

# 12.1 Some concepts and facts related to mixed-integer optimization

It is important to understand that in a worst-case scenario, the time required to solve integer optimization problems grows exponentially with the size of the problem. For instance, assume that a problem contains n binary variables, then the time required to solve the problem in the worst case may be proportional to  $2^n$ . The value of  $2^n$  is huge even for moderate values of n.

In practice this implies that the focus should be on computing a near optimal solution quickly rather than at locating an optimal solution. Even if the problem is only solved approximately, it is important to know how far the approximate solution is from an optimal one. In order to say something about the goodness of an approximate solution then the concept of a relaxation is important.

Name	Run-to-run deterministic	Parallelized	Strength	Cost
Mixed-integer conic	Yes	Yes	Conic	Free add-on
Mixed-integer	No	Partial	Linear	Payed add-on

Table 12.1: Mixed-integer optimizers.

The mixed-integer optimization problem

$$z^* = \underset{\text{subject to}}{\text{minimize}} c^T x$$

$$x \ge 0$$

$$x_j \in \mathbb{Z}, \qquad \forall j \in \mathcal{J},$$

$$(12.1)$$

has the continuous relaxation

$$\underline{z} = \text{minimize} \quad c^T x$$
subject to  $Ax = b$ ,  $x \ge 0$  (12.2)

The continuos relaxation is identical to the mixed-integer problem with the restriction that some variables must be integer removed.

There are two important observations about the continuous relaxation. Firstly, the continuous relaxation is usually much faster to optimize than the mixed-integer problem. Secondly if  $\hat{x}$  is any feasible solution to (12.1) and

$$\bar{z} := c^T \hat{x}$$

then

$$\underline{z} \le z^* \le \bar{z}$$
.

This is an important observation since if it is only possible to find a near optimal solution within a reasonable time frame then the quality of the solution can nevertheless be evaluated. The value  $\underline{z}$  is a lower bound on the optimal objective value. This implies that the obtained solution is no further away from the optimum than  $\overline{z} - \underline{z}$  in terms of the objective value.

Whenever a mixed-integer problem is solved MOSEK reports this lower bound so that the quality of the reported solution can be evaluated.

## 12.2 The mixed-integer optimizers

MOSEK includes two mixed-integer optimizers which are compared in Table 12.1. Both optimizers can handle problems with linear, quadratic objective and constraints and conic constraints. However, a problem must not contain both quadratic objective and constraints and conic constraints.

The mixed-integer conic optimizer is specialized for solving linear and conic optimization problems. It can also solve pure quadratic and quadratically constrained problems, these problems are automatically converted to conic problems before being solved. Whereas the mixed-integer optimizer deals with quadratic and quadratically constrained problems directly.

The mixed-integer conic optimizer is run-to-run deterministic. This means that if a problem is solved twice on the same computer with identical options then the obtained solution will be bit-for-bit identical for the two runs. However, if a time limit is set then this may not be case since the time taken to solve a problem is not deterministic. Moreover, the mixed-integer conic optimizer is parallelized i.e. it can exploit multiple cores during the optimization. Finally, the mixed-integer conic optimizer is a free addon to the continuous optimizers. However, for some linear problems the mixed-integer optimizer may outperform the mixed-integer conic optimizer. On the other hand the mixed-integer conic optimizer is included with continuous optimizers free of charge and usually the fastest for conic problems.

None of the mixed-integer optimizers handles symmetric matrix variables i.e semi-definite optimization problems.

#### 12.3 The mixed-integer conic optimizer

The mixed-integer conic optimizer is employed by setting the parameter iparam.optimizer to optimizertype.mixed\_int\_
The mixed-integer conic employs three phases:

#### Presolve:

In this phase the optimizer tries to reduce the size of the problem using preprocessing techniques. Moreover, it strengthens the continuous relaxation, if possible.

#### Heuristic:

Using heuristics the optimizer tries to guess a good feasible solution.

#### Optimization:

The optimal solution is located using a variant of the branch-and-cut method.

#### 12.3.1 Presolve

In the preprocessing stage redundant variables and constraints are removed. The presolve stage can be turned off using the <code>iparam.mio\_presolve\_use</code> parameter.

#### 12.3.2 Heuristic

Initially, the integer optimizer tries to guess a good feasible solution using a heuristic.

#### 12.3.3 The optimization phase

This phase solves the problem using the branch and cut algorithm.

#### 12.3.4 Caveats

The mixed-integer conic optimizer ignores the parameter

#### iparam.mio\_cont\_sol:

The user should fix all the integer variables at their optimal value and reoptimize instead of relying in this option.

#### 12.4 The mixed-integer optimizer

The mixed-integer optimizer is employed by setting the parameter iparam.optimizer to optimizertype.mixed\_int. In the following it is briefly described how the optimizer works.

The process of solving an integer optimization problem can be split in three phases:

#### Presolve:

In this phase the optimizer tries to reduce the size of the problem using preprocessing techniques. Moreover, it strengthens the continuous relaxation, if possible.

#### Heuristic:

Using heuristics the optimizer tries to guess a good feasible solution.

#### Optimization:

The optimal solution is located using a variant of the branch-and-cut method.

#### 12.4.1 Presolve

In the preprocessing stage redundant variables and constraints are removed. The presolve stage can be turned off using the <code>iparam.mio\_presolve\_use</code> parameter.

#### 12.4.2 Heuristic

Initially, the integer optimizer tries to guess a good feasible solution using different heuristics:

- First a very simple rounding heuristic is employed.
- Next, if deemed worthwhile, the feasibility pump heuristic is used.
- Finally, if the two previous stages did not produce a good initial solution, more sophisticated heuristics are used.

The following parameters can be used to control the effort made by the integer optimizer to find an initial feasible solution.

- iparam.mio\_heuristic\_level: Controls how sophisticated and computationally expensive a heuristic to employ.
- dparam.mio\_heuristic\_time: The minimum amount of time to spend in the heuristic search.
- iparam.mio\_feaspump\_level: Controls how aggressively the feasibility pump heuristic is used.

#### 12.4.3 The optimization phase

This phase solves the problem using the branch and cut algorithm.

#### 12.5 Termination criterion

In general, it is time consuming to find an exact feasible and optimal solution to an integer optimization problem, though in many practical cases it may be possible to find a sufficiently good solution. Therefore, the mixed-integer optimizer employs a relaxed feasibility and optimality criterion to determine when a satisfactory solution is located.

A candidate solution that is feasible to the continuous relaxation is said to be an integer feasible solution if the criterion

$$\min(|x_i| - |x_i|, \lceil x_i \rceil - |x_i|) \le \max(\delta_1, \delta_2 |x_i|) \ \forall j \in \mathcal{J}$$

is satisfied.

Whenever the integer optimizer locates an integer feasible solution it will check if the criterion

$$\bar{z} - \underline{z} \leq \max(\delta_3, \delta_4 \max(1, |\bar{z}|))$$

is satisfied. If this is the case, the integer optimizer terminates and reports the integer feasible solution as an optimal solution. Please note that  $\underline{z}$  is a valid lower bound determined by the integer optimizer during the solution process, i.e.

$$z < z^*$$
.

The lower bound z normally increases during the solution process.

#### 12.5.1 Relaxed termination

If an optimal solution cannot be located within a reasonable time, it may be advantageous to employ a relaxed termination criterion after some time. Whenever the integer optimizer locates an integer feasible solution and has spent at least the number of seconds defined by the <code>dparam.mio\_disable\_term\_time</code> parameter on solving the problem, it will check whether the criterion

$$\bar{z} - z \leq \max(\delta_5, \delta_6 \max(1, |\bar{z}|))$$

Tolerance	Parameter name
$\delta_1$	dparam.mio_tol_abs_relax_int
$\delta_2$	dparam.mio_tol_rel_relax_int
$\delta_3$	dparam.mio_tol_abs_gap
$\delta_4$	dparam.mio_tol_rel_gap
$\delta_5$	dparam.mio_near_tol_abs_gap
$\delta_6$	dparam.mio_near_tol_rel_gap

Table 12.2: Integer optimizer tolerances.

Parameter name	Delayed	Explanation
iparam.mio_max_num_branches	Yes	Maximum number of branches allowed.
iparam.mio_max_num_relaxs	Yes	Maximum number of relaxations allowed.
iparam.mio_max_num_solutions	Yes	Maximum number of feasible integer solutions allowed.

Table 12.3: Parameters affecting the termination of the integer optimizer.

is satisfied. If it is satisfied, the optimizer will report that the candidate solution is **near optimal** and then terminate. Please note that since this criteria depends on timing, the optimizer will not be run to run deterministic.

#### 12.5.2 Important parameters

All  $\delta$  tolerances can be adjusted using suitable parameters — see Table 12.2. In Table 12.3 some other parameters affecting the integer optimizer termination criterion are shown. Please note that if the effect of a parameter is delayed, the associated termination criterion is applied only after some time, specified by the dparam.mio\_disable\_term\_time parameter.

## 12.6 How to speed up the solution process

As mentioned previously, in many cases it is not possible to find an optimal solution to an integer optimization problem in a reasonable amount of time. Some suggestions to reduce the solution time are:

- Relax the termination criterion: In case the run time is not acceptable, the first thing to do is to relax the termination criterion see Section 12.5 for details.
- Specify a good initial solution: In many cases a good feasible solution is either known or easily computed using problem specific knowledge. If a good feasible solution is known, it is usually worthwhile to use this as a starting point for the integer optimizer.
- Improve the formulation: A mixed-integer optimization problem may be impossible to solve in one form and quite easy in another form. However, it is beyond the scope of this manual

to discuss good formulations for mixed-integer problems. For discussions on this topic see for example [16].

### 12.7 Understanding solution quality

To determine the quality of the solution one should check the following:

- The solution status key returned by MOSEK.
- The *optimality gap*: A measure for how much the located solution can deviate from the optimal solution to the problem.
- Feasibility. How much the solution violates the constraints of the problem.

The *optimality gap* is a measure for how close the solution is to the optimal solution. The optimality gap is given by

```
\epsilon = |(\text{objective value of feasible solution}) - (\text{objective bound})|.
```

The objective value of the solution is guarantied to be within  $\epsilon$  of the optimal solution.

The optimality gap can be retrieved through the solution item dinfitem.mio\_obj\_abs\_gap. Often it is more meaningful to look at the optimality gap normalized with the magnitude of the solution. The relative optimality gap is available in dinfitem.mio\_obj\_rel\_gap.

# Chapter 13

# The analyzers

#### 13.1 The problem analyzer

The problem analyzer prints a detailed survey of the

- linear constraints and objective
- quadratic constraints
- conic constraints
- variables

of the model.

In the initial stages of model formulation the problem analyzer may be used as a quick way of verifying that the model has been built or imported correctly. In later stages it can help revealing special structures within the model that may be used to tune the optimizer's performance or to identify the causes of numerical difficulties.

The problem analyzer is run from the command line using the -anapro argument and produces something similar to the following (this is the problemanalyzer's survey of the aflow30a problem from the MIPLIB 2003 collection, see Appendix G for more examples):

#### Analyzing the problem

```
Constraints
                                                    Variables
                                                                 421
                 421
 upper bd:
                          ranged : all
                                                     cont:
 fixed
                                                     bin :
Objective, min cx
   range: min |c|: 0.00000
                           min |c|>0: 11.0000
                                                    max |c|: 500.000
 distrib:
                |c|
                            vars
```

```
[11, 100)
                              150
          [100, 500]
                              271
Constraint matrix A has
       479 rows (constraints)
       842 columns (variables)
      2091 (0.518449%) nonzero entries (coefficients)
Row nonzeros, A_i
   range: min A_i: 2 (0.23753%)
                                   max A_i: 34 (4.038%)
 distrib:
                 A_{-}i
                           rows
                                        rows%
                                                     acc%
                  2
                                        87.89
                                                    87.89
                             421
             [8, 15]
                                         4.18
                                                    92.07
            [16, 31]
                              30
                                         6.26
                                                    98.33
            [32, 34]
                                         1.67
                                                   100.00
Column nonzeros. Ali
  range: min A|j: 2 (0.417537%)
                                     max Alj: 3 (0.626305%)
 distrib:
                 Аlj
                             cols
                                        cols%
                                                     acc%
                   2
                             435
                                        51.66
                                                    51.66
                   3
                             407
                                        48.34
                                                   100.00
A nonzeros, A(ij)
                                    max |A(ij)|: 100.000
  range: min |A(ij)|: 1.00000
 distrib:
              A(ij)
                          coeffs
             [1, 10)
                            1670
                             421
           [10, 100]
Constraint bounds, lb <= Ax <= ub
 distrib:
                 |b|
                                                  ubs
                   0
                                                  421
             [1, 10]
                                  58
                                                   58
Variable bounds, lb <= x <= ub
 distrib:
                 |b|
                                  lbs
                                                  ubs
                   0
                                  842
             [1, 10)
                                                  421
           [10, 100]
                                                  421
```

The survey is divided into six different sections, each described below. To keep the presentation short with focus on key elements the analyzer generally attempts to display information on issues relevant for the current model only: E.g., if the model does not have any conic constraints (this is the case in the example above) or any integer variables, those parts of the analysis will not appear.

#### 13.1.1 General characteristics

The first part of the survey consists of a brief summary of the model's linear and quadratic constraints (indexed by i) and variables (indexed by j). The summary is divided into three subsections:

#### Constraints

int:

```
upper bd:
           The number of upper bounded constraints, \sum_{j=0}^{n-1} a_{ij} x_j \leq u_i^c
           The number of lower bounded constraints, l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j
      ranged:
           The number of ranged constraints, l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j \leq u_i^c
      fixed:
           The number of fixed constraints, l_i^c = \sum_{j=0}^{n-1} a_{ij} x_j = u_i^c
      free:
           The number of free constraints
Bounds
      upper bd:
           The number of upper bounded variables, x_j \leq u_i^x
      lower bd:
           The number of lower bounded variables, l_k^x \leq x_j
           The number of ranged variables, l_k^x \leq x_j \leq u_j^x
      fixed:
           The number of fixed variables, l_k^x = x_j = u_j^x
      free:
           The number of free variables
Variables
      cont:
           The number of continuous variables, x_j \in \mathbb{R}
      bin:
           The number of binary variables, x_j \in \{0, 1\}
```

The number of general integer variables,  $x_j \in \mathbb{Z}$ 

Only constraints, bounds and domains actually in the model will be reported on, cf. appendix G; if all entities in a section turn out to be of the same kind, the number will be replaced by all for brevity.

#### 13.1.2 Objective

The second part of the survey focuses on (the linear part of) the objective, summarizing the optimization sense and the coefficients' absolute value range and distribution. The number of 0 (zero) coefficients is singled out (if any such variables are in the problem).

The range is displayed using three terms:

min |c|:

The minimum absolute value among all coeffecients

min |c|>0:

The minimum absolute value among the nonzero coefficients

max |c|:

The maximum absolute value among the coefficients

If some of these extrema turn out to be equal, the display is shortened accordingly:

- If min |c| is greater than zero, the min |c|?0 term is obsolete and will not be displayed
- If only one or two different coefficients occur this will be displayed using all and an explicit listing of the coefficients

The absolute value distribution is displayed as a table summarizing the numbers by orders of magnitude (with a ratio of 10). Again, the number of variables with a coefficient of 0 (if any) is singled out. Each line of the table is headed by an interval (half-open intervals including their lower bounds), and is followed by the number of variables with their objective coefficient in this interval. Intervals with no elements are skipped.

#### 13.1.3 Linear constraints

The third part of the survey displays information on the nonzero coefficients of the linear constraint matrix.

Following a brief summary of the matrix dimensions and the number of nonzero coefficients in total, three sections provide further details on how the nonzero coefficients are distributed by row-wise count (A\_i), by column-wise count (A|j), and by absolute value (|A(ij)|). Each section is headed by a brief display of the distribution's range (min and max), and for the row/column-wise counts the corresponding densities are displayed too (in parentheses).

The distribution tables single out three particularly interesting counts: zero, one, and two nonzeros per row/column; the remaining row/column nonzeros are displayed by orders of magnitude (ratio 2). For each interval the relative and accumulated relative counts are also displayed.

Note that constraints may have both linear and quadratic terms, but the empty rows and columns reported in this part of the survey relate to the linear terms only. If empty rows and/or columns are found in the linear constraint matrix, the problem is analyzed further in order to determine if the

corresponding constraints have any quadratic terms or the corresponding variables are used in conic or quadratic constraints; cf. the last two examples of appendix G.

The distribution of the absolute values, |A(ij)|, is displayed just as for the objective coefficients described above.

#### 13.1.4 Constraint and variable bounds

The fourth part of the survey displays distributions for the absolute values of the finite lower and upper bounds for both constraints and variables. The number of bounds at 0 is singled out and, otherwise, displayed by orders of magnitude (with a ratio of 10).

#### 13.1.5 Quadratic constraints

The fifth part of the survey displays distributions for the nonzero elements in the gradient of the quadratic constraints, i.e. the nonzero row counts for the column vectors Qx. The table is similar to the tables for the linear constraints' nonzero row and column counts described in the survey's third part.

Note: Quadratic constraints may also have a linear part, but that will be included in the linear constraints survey; this means that if a problem has one or more pure quadratic constraints, part three of the survey will report an equal number of linear constraint rows with 0 (zero) nonzeros, cf. the last example in appendix G. Likewise, variables that appear in quadratic terms only will be reported as empty columns (0 nonzeros) in the linear constraint report.

#### 13.1.6 Conic constraints

The last part of the survey summarizes the model's conic constraints. For each of the two types of cones, quadratic and rotated quadratic, the total number of cones are reported, and the distribution of the cones' dimensions are displayed using intervals. Cone dimensions of 2, 3, and 4 are singled out.

## 13.2 Analyzing infeasible problems

When developing and implementing a new optimization model, the first attempts will often be either infeasible, due to specification of inconsistent constraints, or unbounded, if important constraints have been left out.

In this chapter we will

- go over an example demonstrating how to locate infeasible constraints using the MOSEK infeasibility report tool,
- discuss in more general terms which properties that may cause infeasibilities, and
- present the more formal theory of infeasible and unbounded problems.



Figure 13.1: Supply, demand and cost of transportation.

Furthermore, chapter 14 contains a discussion on a specific method for repairing infeasibility problems where infeasibilities are caused by model parameters rather than errors in the model or the implementation.

#### 13.2.1 Example: Primal infeasibility

A problem is said to be *primal infeasible* if no solution exists that satisfy all the constraints of the problem.

As an example of a primal infeasible problem consider the problem of minimizing the cost of transportation between a number of production plants and stores: Each plant produces a fixed number of goods, and each store has a fixed demand that must be met. Supply, demand and cost of transportation per unit are given in figure 13.1. The problem represented in figure 13.1 is infeasible, since the total demand

$$2300 = 1100 + 200 + 500 + 500$$

exceeds the total supply

$$2200 = 200 + 1000 + 1000$$

If we denote the number of transported goods from plant i to store j by  $x_{ij}$ , the problem can be formulated as the LP:

Solving the problem (13.1) using MOSEK will result in a solution, a solution status and a problem status. Among the log output from the execution of MOSEK on the above problem are the lines:

Basic solution

Problem status : PRIMAL\_INFEASIBLE
Solution status : PRIMAL\_INFEASIBLE\_CER

The first line indicates that the problem status is primal infeasible. The second line says that a certificate of the infeasibility was found. The certificate is returned in place of the solution to the problem.

#### 13.2.2 Locating the cause of primal infeasibility

Usually a primal infeasible problem status is caused by a mistake in formulating the problem and therefore the question arises: "What is the cause of the infeasible status?" When trying to answer this question, it is often advantageous to follow these steps:

- Remove the objective function. This does not change the infeasible status but simplifies the problem, eliminating any possibility of problems related to the objective function.
- Consider whether your problem has some necessary conditions for feasibility and examine if these are satisfied, e.g. total supply should be greater than or equal to total demand.
- Verify that coefficients and bounds are reasonably sized in your problem.

If the problem is still primal infeasible, some of the constraints must be relaxed or removed completely. The MOSEK infeasibility report (Section 13.2.4) may assist you in finding the constraints causing the infeasibility.

Possible ways of relaxing your problem include:

• Increasing (decreasing) upper (lower) bounds on variables and constraints.

• Removing suspected constraints from the problem.

Returning to the transportation example, we discover that removing the fifth constraint

$$x_{12} = 200$$

makes the problem feasible.

#### 13.2.3 Locating the cause of dual infeasibility

A problem may also be *dual infeasible*. In this case the primal problem is often unbounded, mening that feasible solutions exists such that the objective tends towards infinity. An example of a dual infeasible and primal unbounded problem is:

minimize 
$$x_1$$
 subject to  $x_1 \le 5$ .

To resolve a dual infeasibility the primal problem must be made more restricted by

- Adding upper or lower bounds on variables or constraints.
- Removing variables.
- Changing the objective.

#### 13.2.3.1 A cautious note

The problem

$$\begin{array}{ll} \text{minimize} & 0\\ \text{subject to} & 0 \leq x_1, \\ & x_j \leq x_{j+1}, \quad j=1,\dots,n-1, \\ & x_n \leq -1 \end{array}$$

is clearly infeasible. Moreover, if any one of the constraints are dropped, then the problem becomes feasible.

This illustrates the worst case scenario that all, or at least a significant portion, of the constraints are involved in the infeasibility. Hence, it may not always be easy or possible to pinpoint a few constraints which are causing the infeasibility.

#### 13.2.4 The infeasibility report

MOSEK includes functionality for diagnosing the cause of a primal or a dual infeasibility. It can be turned on by setting the <code>iparam.infeas\_report\_auto</code> to <code>onoffkey.on</code>. This causes MOSEK to print a report on variables and constraints involved in the infeasibility.

The iparam.infeas\_report\_level parameter controls the amount of information presented in the infeasibility report. The default value is 1.

#### 13.2.4.1 Example: Primal infeasibility

We will reuse the example (13.1) located in infeas.lp:

```
\stackrel{\backslash}{} An example of an infeasible linear problem.
minimize
 obj: + 1 x11 + 2 x12 + 1 x13
      + 4 x21 + 2 x22 + 5 x23
      + 4 x31 + 1 x32 + 2 x33
st
  s0: + x11 + x12
                        <= 200
  s1: + x23 + x24
                        <= 1000
  s2: + x31 +x33 + x34 <= 1000
  d1: + x11 + x31
                         = 1100
  d2: + x12
                         = 200
  d3: + x23 + x33
                         = 500
  d4: + x24 + x34
                          = 500
bounds
end
```

Using the command line (please remeber it accepts options following the C API format)

```
mosek -d iparam.infeas_report_auto onoffkey.on infeas.lp
```

MOSEK produces the following infeasibility report

MOSEK PRIMAL INFEASIBILITY REPORT.

Problem status: The problem is primal infeasible

The following constraints are involved in the primal infeasibility.

Index	Name	Lower bound	Upper bound	Dual lower	Dual upper
0	s0	NONE	2.000000e+002	0.000000e+000	1.000000e+000
2	s2	NONE	1.000000e+003	0.000000e+000	1.000000e+000
3	d1	1.100000e+003	1.100000e+003	1.000000e+000	0.000000e+000
4	d2	2.000000e+002	2.000000e+002	1.000000e+000	0.000000e+000

The following bound constraints are involved in the infeasibility.

Index	Name	Lower bound	Upper bound	Dual lower	Dual upper
8	x33	0.000000e+000	NONE	1.000000e+000	0.000000e+000
10	x34	0.000000e+000	NONE	1.000000e+000	0.000000e+000

The infeasibility report is divided into two sections where the first section shows which constraints that are important for the infeasibility. In this case the important constraints are the ones named s0, s2, d1, and d2. The values in the columns "Dual lower" and "Dual upper" are also useful, since a non-zero dual lower value for a constraint implies that the lower bound on the constraint is important for the infeasibility. Similarly, a non-zero dual upper value implies that the upper bound on the constraint is important for the infeasibility.

It is also possible to obtain the infeasible subproblem. The command line

```
mosek -d iparam.infeas_report_auto onoffkey.on infeas.lp -info rinfeas.lp
```

produces the files rinfeas.bas.inf.lp. In this case the content of the file rinfeas.bas.inf.lp is

```
minimize
Obj: + CFIXVAR
s0: + x11 + x12 <= 200
s2: + x31 + x33 + x34 <= 1e+003
d1: + x11 + x31 = 1.1e+003
d2: + x12 = 200
bounds
x11 free
x12 free
x13 free
 x21 free
 x22 free
x23 free
x31 free
x32 free
x24 free
CFIXVAR = 0e+000
end
```

which is an optimization problem. This problem is identical to (13.1), except that the objective and some of the constraints and bounds have been removed. Executing the command

```
mosek -d MSK_IPAR_INFEAS_REPORT_AUTO MSK_ON infeas.bas.inf.lp
```

demonstrates that the reduced problem is **primal infeasible**. Since the reduced problem is usually smaller than original problem, it should be easier to locate the cause of the infeasibility in this rather than in the original (13.1).

#### 13.2.4.2 Example: Dual infeasibility

The example problem

```
maximize - 200 y1 - 1000 y2 - 1000 y3
        - 1100 y4 - 200 y5 - 500 y6
        - 500 y7
subject to
  x11: y1+y4 < 1
  x12: y1+y5 < 2
  x23: y2+y6 < 5
  x24: y2+y7 < 2
  x31: y3+y4 < 1
  x33: y3+y6 < 2
  x44: y3+y7 < 1
bounds
  y1 < 0
  y2 < 0
  y3 < 0
  y4 free
  y5 free
  y6 free
  y7 free
end
```

is dual infeasible. This can be verified by proving that

```
y1=-1, y2=-1, y3=0, y4=1, y5=1
```

is a certificate of dual infeasibility. In this example the following infeasibility report is produced (slightly edited):

The following constraints are involved in the infeasibility.

Index	Name	Activity	Objective	Lower bound	Upper bound
0	x11	-1.000000e+00		NONE	1.000000e+00
4	x31	-1.000000e+00		NONE	1.000000e+00

The following variables are involved in the infeasibility.

```
Upper bound
Index
         Name
                                           Objective
                                                            Lower bound
                          Activity
         y4
                          -1.000000e+00
                                           -1.100000e+03
                                                             NONE
                                                                              NONE
Interior-point solution
Problem status : DUAL_INFEASIBLE
Solution status : DUAL_INFEASIBLE_CER
                                       eq. infeas.: 0.00e+00 max bound infeas.: 0.00e+00 cone infeas.: 0.00e+00
Primal - objective: 1.1000000000e+03
      - objective: 0.0000000000e+00
                                       eq. infeas.: 0.00e+00 max bound infeas.: 0.00e+00 cone infeas.: 0.00e+00
```

Let  $x^*$  denote the reported primal solution. MOSEK states

- that the problem is dual infeasible,
- that the reported solution is a certificate of dual infeasibility, and
- that the infeasibility measure for  $x^*$  is approximately zero.

Since it was an maximization problem, this implies that

$$c^t x^* > 0. (13.2)$$

For a minimization problem this inequality would have been reversed — see (13.5).

From the infeasibility report we see that the variable y4, and the constraints x11 and x33 are involved in the infeasibility since these appear with non-zero values in the "Activity" column.

One possible strategy to "fix" the infeasibility is to modify the problem so that the certificate of infeasibility becomes invalid. In this case we may do one the following things:

- Put a lower bound in y3. This will directly invalidate the certificate of dual infeasibility.
- Increase the object coefficient of y3. Changing the coefficients sufficiently will invalidate the inequality (13.2) and thus the certificate.
- Put lower bounds on x11 or x31. This will directly invalidate the certificate of infeasibility.

Please note that modifying the problem to invalidate the reported certificate does *not* imply that the problem becomes dual feasible — the infeasibility may simply "move", resulting in a new infeasibility.

More often, the reported certificate can be used to give a hint about errors or inconsistencies in the model that produced the problem.

#### 13.2.5 Theory concerning infeasible problems

This section discusses the theory of infeasibility certificates and how MOSEK uses a certificate to produce an infeasibility report. In general, MOSEK solves the problem

minimize 
$$c^T x + c^f$$
  
subject to  $l^c \le Ax \le u^c$ ,  $l^x \le x \le u^x$  (13.3)

where the corresponding dual problem is

maximize 
$$(l^{c})^{T} s_{l}^{c} - (u^{c})^{T} s_{u}^{c}$$

$$+ (l^{x})^{T} s_{l}^{x} - (u^{x})^{T} s_{u}^{x} + c^{f}$$
subject to 
$$A^{T} y + s_{l}^{x} - s_{u}^{x} = c,$$

$$- y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \ge 0.$$

$$(13.4)$$

We use the convension that for any bound that is not finite, the corresponding dual variable is fixed at zero (and thus will have no influence on the dual problem). For example

$$l_i^x = -\infty \implies (s_l^x)_j = 0$$

#### 13.2.6 The certificate of primal infeasibility

A certificate of primal infeasibility is any solution to the homogenized dual problem

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c \\ & + (l^x)^T s_l^x - (u^x)^T s_u^x \\ \text{subject to} & A^T y + s_l^x - s_u^x & = & 0, \\ & - y + s_l^c - s_u^c & = & 0, \\ s_l^c, s_u^c, s_l^x, s_u^x \geq 0. & \end{array}$$

with a positive objective value. That is,  $(s_l^{c*}, s_u^{c*}, s_u^{r*}, s_u^{x*})$  is a certificate of primal infeasibility if

$$(l^c)^T s_l^{c*} - (u^c)^T s_u^{c*} + (l^x)^T s_l^{x*} - (u^x)^T s_u^{x*} > 0$$

and

$$\begin{array}{lll} A^T y + s_l^{x*} - s_u^{x*} & = & 0, \\ - y + s_l^{c*} - s_u^{c*} & = & 0, \\ s_l^{c*}, s_u^{x*}, s_l^{x*}, s_u^{x*} \geq 0. & & \end{array}$$

The well-known Farkas Lemma tells us that (13.3) is infeasible if and only if a certificate of primal infeasibility exists.

Let  $(s_l^{c*}, s_u^{c*}, s_l^{c*}, s_u^{x*}, s_u^{x*})$  be a certificate of primal infeasibility then

$$(s_l^{c*})_i > 0((s_u^{c*})_i > 0)$$

implies that the lower (upper) bound on the i th constraint is important for the infeasibility. Furthermore,

$$(s_l^{x*})_j > 0((s_u^{x*})_i > 0)$$

implies that the lower (upper) bound on the i th variable is important for the infeasibility.

#### 13.2.7 The certificate of dual infeasibility

A certificate of dual infeasibility is any solution to the problem

with negative objective value, where we use the definitions

$$\bar{l}_i^c := \left\{ \begin{array}{ll} 0, & l_i^c > -\infty, \\ -\infty, & \text{otherwise,} \end{array} \right., \ \bar{u}_i^c := \left\{ \begin{array}{ll} 0, & u_i^c < \infty, \\ \infty, & \text{otherwise,} \end{array} \right.$$

and

$$\bar{l}_i^x := \left\{ \begin{array}{ll} 0, & l_i^x > -\infty, \\ -\infty, & \text{otherwise,} \end{array} \right. \text{ and } \bar{u}_i^x := \left\{ \begin{array}{ll} 0, & u_i^x < \infty, \\ \infty, & \text{otherwise.} \end{array} \right.$$

Stated differently, a certificate of dual infeasibility is any  $x^*$  such that

$$c^{T}x^{*} < 0,$$

$$\bar{l}^{c} \leq Ax^{*} \leq \bar{u}^{c},$$

$$\bar{l}^{x} \leq x^{*} \leq \bar{u}^{x}$$

$$(13.5)$$

The well-known Farkas Lemma tells us that (13.4) is infeasible if and only if a certificate of dual infeasibility exists.

Note that if  $x^*$  is a certificate of dual infeasibility then for any j such that

$$x_{i}^{*} \neq 0,$$

variable j is involved in the dual infeasibility.

# Chapter 14

# Primal feasibility repair

Section 13.2.2 discusses how MOSEK treats infeasible problems. In particular, it is discussed which information MOSEK returns when a problem is infeasible and how this information can be used to pinpoint the cause of the infeasibility.

In this section we discuss how to repair a primal infeasible problem by relaxing the constraints in a controlled way. For the sake of simplicity we discuss the method in the context of linear optimization.

#### 14.1 Manual repair

Subsequently we discuss an automatic method for repairing an infeasible optimization problem. However, it should be observed that the best way to repair an infeasible problem usually depends on what the optimization problem models. For instance in many optimization problem it does not make sense to relax the constraints  $x \geq 0$  e.g. it is not possible to produce a negative quantity. Hence, whatever automatic method MOSEK provides it will never be as good as a method that exploits knowledge about what is being modelled. This implies that it is usually better to remove the underlying cause of infeasibility at the modelling stage.

Indeed consider the example

minimize subject to 
$$x_1 + x_2 = 1,$$
  $x_3 + x_4 = 1,$   $x_3 + x_4 = 1,$   $x_4 - x_2 - x_4 = -1,$   $x_5 - x_2 - x_4 = -1,$   $x_6 - x_8 -$ 

then if we add the equalities together we obtain the implied equality

$$0 = \epsilon$$

which is infeasible for any  $\epsilon \neq 0$ . Here the infeasibility is caused by a linear dependency in the constraint matrix and that the right-hand side does not match if  $\epsilon \neq 0$ . Observe even if the problem is feasible then just a tiny perturbation to the right-hand side will make the problem infeasible. Therefore, even though the problem can be repaired then a much more robust solution is to avoid problems with linear dependent constraints. Indeed if a problem contains linear dependencies then the problem is either infeasible or contains redundant constraints. In the above case any of the equality constraints can be removed while not changing the set of feasible solutions.

To summarize linear dependencies in the constraints can give rise to infeasible problems and therefore it is better to avoid them. Note that most network flow models usually is formulated with one linear dependent constraint.

Next consider the problem

minimize subject to 
$$x_1 - 0.01x_2 = 0$$
  $x_2 - 0.01x_3 = 0$   $x_3 - 0.01x_4 = 0$   $x_1 \ge -1.0e - 9$   $x_1 \le 1.0e - 9$   $x_4 \le -1.0e - 4$ 

Now the MOSEK presolve for the sake of efficiency fix variables (and constraints) that has tight bounds where tightness is controlled by the parameter dparam.presolve\_tol\_x. Since, the bounds

$$-1.0e - 9 < x_1 < 1.0e - 9$$

are tight then the MOSEK presolve will fix variable  $x_1$  at the mid point between the bounds i.e. at 0. It easy to see that this implies  $x_4 = 0$  too which leads to the incorrect conclusion that the problem is infeasible. Observe tiny change of the size 1.0e-9 make the problem switch from feasible to infeasible. Such a problem is inherently unstable and is hard to solve. We normally call such a problem ill-posed. In general it is recommended to avoid ill-posed problems, but if that is not possible then one solution to this issue is to reduce the parameter to say dparam.presolve\_tol\_x to say 1.0e-10. This will at least make sure that the presolve does not make the wrong conclusion.

## 14.2 Automatic repair

In this section we will describe the idea behind a method that automatically can repair an infeasible probem. The main idea can be described as follows.

Consider the linear optimization problem with m constraints and n variables

which is assumed to be infeasible.

One way of making the problem feasible is to reduce the lower bounds and increase the upper bounds. If the change is sufficiently large the problem becomes feasible. Now an obvious idea is to compute the optimal relaxation by solving an optimization problem. The problem

minimize 
$$p(v_{l}^{c}, v_{u}^{c}, v_{l}^{x}, v_{u}^{x})$$
subject to 
$$l^{c} \leq Ax + v_{l}^{c} - v_{u}^{c} \leq u^{c},$$

$$l^{x} \leq x + v_{l}^{x} - v_{u}^{x} \leq u^{x},$$

$$v_{l}^{c}, v_{u}^{c}, v_{l}^{x}, v_{u}^{x} \geq 0$$

$$(14.4)$$

does exactly that. The additional variables  $(v_l^c)_i$ ,  $(v_u^c)_i$ ,  $(v_u^c)_j$  and  $(v_u^c)_j$  are elasticity variables because they allow a constraint to be violated and hence add some elasticity to the problem. For instance, the elasticity variable  $(v_l^c)_i$  controls how much the lower bound  $(l^c)_i$  should be relaxed to make the problem feasible. Finally, the so-called penalty function

$$p(v_l^c, v_u^c, v_l^x, v_u^x)$$

is chosen so it penalize changes to bounds. Given the weights

- $w_l^c \in \mathbb{R}^m$  (associated with  $l^c$ ),
- $w_u^c \in \mathbb{R}^m$  (associated with  $u^c$ ),
- $w_l^x \in \mathbb{R}^n$  (associated with  $l^x$ ),
- $w_u^x \in \mathbb{R}^n$  (associated with  $u^x$ ),

then a natural choice is

$$p(v_l^c, v_u^c, v_u^x, v_u^x) = (w_l^c)^T v_l^c + (w_u^c)^T v_u^c + (w_l^x)^T v_l^x + (w_u^x)^T v_u^x.$$
(14.5)

Hence, the penalty function p() is a weighted sum of the relaxation and therefore the problem (14.4) keeps the amount of relaxation at a minimum. Please observe that

- the problem (14.6) is always feasible.
- a negative weight implies problem (14.6) is unbounded. For this reason if the value of a weight is negative MOSEK fixes the associated elasticity variable to zero. Clearly, if one or more of the weights are negative may imply that it is not possible repair the problem.

A simple choice of weights is to let them all to be 1, but of course that does not take into account that constraints may have different importance.

#### **14.2.1** Caveats

Observe if the infeasible problem

minimize 
$$x + z$$
  
subject to  $x = -1,$   
 $x > 0$  (14.6)

is repaired then it will be unbounded. Hence, a repaired problem may not have an optimal solution.

Another and more important caveat is that only a minimial repair is performed i.e. the repair that just make the problem feasible. Hence, the repaired problem is barely feasible and that sometimes make the repaired problem hard to solve.

#### 14.3 Feasibility repair in MOSEK

MOSEK includes a function that repair an infeasible problem using the idea described in the previous section simply by passing a set of weights to MOSEK. This can be used for linear and conic optimization problems, possibly having integer constrained variables.

#### 14.3.1 An example using the command line tool

Consider the example linear optimization

minimize 
$$-10x_1$$
  $-9x_2$ , subject to  $7/10x_1$  +  $1x_2$   $\leq 630$ ,  $1/2x_1$  +  $5/6x_2$   $\leq 600$ ,  $1x_1$  +  $2/3x_2$   $\leq 708$ ,  $1/10x_1$  +  $1/4x_2$   $\leq 135$ ,  $x_1$ ,  $x_2 \geq 650$  (14.7)

which is infeasible. Now suppose we wish to use MOSEK to suggest a modification to the bounds that makes the problem feasible.

Given the assumption that all weights are 1 then the command

```
mosek -primalrepair -d MSK_IPAR_LOG_FEAS_REPAIR 3 feasrepair.lp
```

will form the repaired problem and solve it. The parameter

```
MSK_IPAR_LOG_FEAS_REPAIR
```

controls the amount of log output from the repair. A value of 2 causes the optimal repair to printed out.

The output from running the above command is:

```
Copyright (c) 1998-2013 MOSEK ApS, Denmark. WWW: http://mosek.com
```

Open file 'feasrepair.lp'

Read summary

Type : LO (linear optimization problem)

Objective sense : min

```
Constraints : 4
  Scalar variables : 2
  Matrix variables : 0
  Time
             : 0.0
Computer
  Platform
                          : Windows/64-X86
  Cores
Problem
  Name
  Objective sense
                       : min
  Type
Constraints
  Туре
                           : LO (linear optimization problem)
                         : 4
  Cones
                          : 0
  Scalar variables : 2
  Matrix variables
                           : 0
  Integer variables
                           : 0
Primal feasibility repair started.
Optimizer started.
Interior-point optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator started.
Total number of eliminations : 2
Eliminator terminated.
Eliminator - tries
                                     : 1
                                                                                     : 0.00
                                                            time
Eliminator - elim's
                                     : 2
Lin. dep. - tries
Lin. dep. - number
                                    : 1
                                                                                     : 0.00
                                                            time
                                      : 0
Presolve terminated. Time: 0.00
Optimizer - threads : 1
Optimizer - solved problem : the primal
Optimizer - Constraints : 2
Optimizer - Cones : 0
Optimizer - Scalar variables : 6
                                                      conic : 0
scalarized : 0
dense det. time : 0.00
GP order time : 0.00
after factor : 3
flops : 5.40e
DOBJ MU
                                                                                    : 0
                                                           conic
Optimizer - Semi-definite variables: 0
Factor - setup time : 0.00
Factor - ML order time : 0.00
Factor
                                     : 0.00
                                                                                   : 0.00
: 3
: 5.40e+001
Factor
           - nonzeros before factor : 3
Factor - dense dim. : 0
ITE PFEAS DFEAS GFEAS PRSTATUS POBJ
0 2.7e+001 1.0e+000 4.8e+000 1.00e+000 4.195228609e+000 0.000000000e+000 1.0e+000 0.00
1 2.4e+001 8.6e-001 1.5e+000 0.00e+000 1.227497414e+001 1.504971820e+001 2.6e+000 0.00 2 2.6e+000 9.7e-002 1.7e-001 -6.19e-001 4.363064729e+001 4.648523094e+001 3.0e-001 0.00
3 4.7e-001 1.7e-002 3.1e-002 1.24e+000 4.256803136e+001 4.298540657e+001 5.2e-002 0.00
5 \quad 8.7 e-008 \ 3.2 e-009 \ 5.7 e-009 \ 1.00 e+000 \quad 4.249999999 e+001 \quad 4.250000008 e+001 \quad 9.7 e-009 \ 0.00 \\
    8.7e-012 3.2e-013 5.7e-013 1.00e+000 4.250000000e+001 4.250000000e+001 9.7e-013 0.00
Basis identification started.
Primal basis identification phase started.
ITER
           TIME.
0
           0.00
Primal basis identification phase terminated. Time: 0.00
Dual basis identification phase started.
```

```
0.00
Dual basis identification phase terminated. Time: 0.00
Basis identification terminated. Time: 0.00
Interior-point optimizer terminated. Time: 0.00.
Optimizer terminated. Time: 0.03
Basic solution summary
  Problem status : PRIMAL_AND_DUAL_FEASIBLE
  Solution status : OPTIMAL
  Primal. obj: 4.2500000000e+001 Viol. con: 1e-013 var: 0e+000
  Dual. obj: 4.2500000000e+001 Viol. con: 0e+000 var: 5e-013
Optimal objective value of the penalty problem: 4.250000000000e+001
Repairing bounds.
Increasing the upper bound -2.25e+001 on constraint 'c4' (3) with 1.35e+002.
Decreasing the lower bound 6.50e+002 on variable 'x2' (4) with 2.00e+001.
Primal feasibility repair terminated.
Optimizer started.
Interior-point optimizer started.
Presolve started.
Presolve terminated. Time: 0.00
Interior-point optimizer terminated. Time: 0.00.
Optimizer terminated. Time: 0.00
Interior-point solution summary
  Problem status : PRIMAL_AND_DUAL_FEASIBLE
  Solution status : OPTIMAL
  Primal. obj: -5.6700000000e+003 Viol. con: 0e+000 var: 0e+000
          obj: -5.6700000000e+003 Viol. con: 0e+000 var: 0e+000
Basic solution summary
  Problem status : PRIMAL_AND_DUAL_FEASIBLE
  Solution status : OPTIMAL
  Primal. obj: -5.6700000000e+003 Viol. con: 0e+000 var: 0e+000
  Dual. obj: -5.6700000000e+003 Viol. con: 0e+000 var: 0e+000
Optimizer summary
  Optimizer
                                                      time: 0.00
    Interior-point - iterations : 0
                                                    time: 0.00
       time: 0.00

- iterations: 0 time: 0.00

Dual - iterations: 0 time: 0.00

Clean primal - iterations: 0 time: 0.00

Clean dual - iterations: 0 time: 0.00

Clean primal-dual - iterations: 0 time: 0.00

plex - rimal simpley
                                                     time: 0.00
      Basis identification -
    Simplex
     Primal simplex
                           Dual simplex
      Primal-dual simplex - iterations : 0
                             - relaxations: 0
    Mixed integer
                                                      time: 0.00
```

reports the optimal repair. In this case it is to increase the upper bound on constraint c4 by 1.35e2 and decrease the lower bound on variable x2 by 20.

#### 14.3.2 Feasibility repair using the API

The function Task.primalrepair can be used to repair an infeasible problem. Details about the function Task.primalrepair can be seen in the reference.

#### 14.3.2.1 An example

Consider once again the example (14.7) then

```
——[feasrepairex1.py]—
    #
        Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
2
        File:
                   feasrepairex1.py
                     To demonstrate how to use the MSK_relaxprimal function to
    #
        Purpose:
                    locate the cause of an infeasibility.
        Syntax: On command line
    #
9
                python feasrepairex1.py feasrepair.lp
10
                 feasrepair.lp is located in mosek\<version>\tools\examples.
11
12
13
    import sys
14
    import mosek
15
    # Since the actual value of Infinity is ignores, we define it solely
17
18
    # for symbolic purposes:
    inf = 0.0
19
20
    # Define a stream printer to grab output from MOSEK
21
    def streamprinter(text):
22
23
        sys.stdout.write(text)
        sys.stdout.flush()
24
    def main (inputfile):
26
        # Make a MOSEK environment
27
        with mosek.Env () as env:
28
          with env. Task(0,0) as task:
29
            # Attach a printer to the task
            task.set_Stream (mosek.streamtype.log, streamprinter)
31
            # Read data
33
            task.readdata(inputfile)
34
            task.putintparam(mosek.iparam.log_feas_repair,3)
36
37
            task.primalrepair(None, None, None, None)
38
39
             sum_viol = task.getdouinf(mosek.dinfitem.primal_repair_penalty_obj)
40
             print ("Minimized sum of violations = %e" % sum_viol)
41
             task.optimize()
43
             task.solutionsummary(mosek.streamtype.msg)
45
```

```
# call the main function
try:
main (sys.argv[1])
except Exception as e:
print (e)
raise
```

will produce the same output as the command line tool discussed in Section 14.3.1.

# Chapter 15

# Sensitivity analysis

#### 15.1 Introduction

Given an optimization problem it is often useful to obtain information about how the optimal objective value changes when the problem parameters are perturbed. E.g, assume that a bound represents a capacity of a machine. Now, it may be possible to expand the capacity for a certain cost and hence it is worthwhile knowing what the value of additional capacity is. This is precisely the type of questions the sensitivity analysis deals with.

Analyzing how the optimal objective value changes when the problem data is changed is called sensitivity analysis.

#### 15.2 Restrictions

Currently, sensitivity analysis is only available for continuous linear optimization problems. Moreover, MOSEK can only deal with perturbations in bounds and objective coefficients.

#### 15.3 References

The book [1] discusses the classical sensitivity analysis in Chapter 10 whereas the book [17] presents a modern introduction to sensitivity analysis. Finally, it is recommended to read the short paper [18] to avoid some of the pitfalls associated with sensitivity analysis.

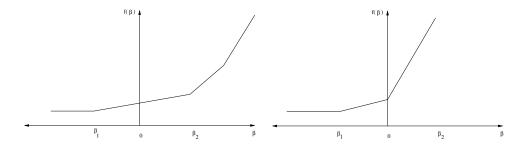


Figure 15.1: The optimal value function  $f_{l_i^c}(\beta)$ . Left:  $\beta = 0$  is in the interior of linearity interval. Right:  $\beta = 0$  is a breakpoint.

#### 15.4 Sensitivity analysis for linear problems

#### 15.4.1 The optimal objective value function

Assume that we are given the problem

$$z(l^{c}, u^{c}, l^{x}, u^{x}, c) = \underset{\text{subject to}}{\text{minimize}} c^{T}x$$

$$subject to \quad l^{c} \leq \underset{l^{x} < x < u^{x}}{Ax} \leq u^{c}, \quad (15.1)$$

and we want to know how the optimal objective value changes as  $l_i^c$  is perturbed. To answer this question we define the perturbed problem for  $l_i^c$  as follows

$$f_{l_i^c}(\beta) = \text{minimize} \qquad c^T x$$
  
 $\text{subject to} \quad l^c + \beta e_i \leq Ax \leq u^c,$   
 $l^x \leq x \leq u^x,$ 

where  $e_i$  is the *i* th column of the identity matrix. The function

$$f_{l^c}(\beta) \tag{15.2}$$

shows the optimal objective value as a function of  $\beta$ . Please note that a change in  $\beta$  corresponds to a perturbation in  $l_i^c$  and hence (15.2) shows the optimal objective value as a function of  $l_i^c$ .

It is possible to prove that the function (15.2) is a piecewise linear and convex function, i.e. the function may look like the illustration in Figure 15.1. Clearly, if the function  $f_{l_i^c}(\beta)$  does not change much when  $\beta$  is changed, then we can conclude that the optimal objective value is insensitive to changes in  $l_i^c$ . Therefore, we are interested in the rate of change in  $f_{l_i^c}(\beta)$  for small changes in  $\beta$ — specificly the gradient

$$f'_{l_i^c}(0),$$

which is called the *shadow price* related to  $l_i^c$ . The shadow price specifies how the objective value changes for small changes in  $\beta$  around zero. Moreover, we are interested in the *linearity interval* 

$$\beta \in [\beta_1, \beta_2]$$

for which

$$f'_{l_i^c}(\beta) = f'_{l_i^c}(0).$$

Since  $f_{l_i^c}$  is not a smooth function  $f'_{l_i^c}$  may not be defined at 0, as illustrated by the right example in figure 15.1. In this case we can define a left and a right shadow price and a left and a right linearity interval.

The function  $f_{l_i^c}$  considered only changes in  $l_i^c$ . We can define similar functions for the remaining parameters of the z defined in (15.1) as well:

$$\begin{array}{lcl} f_{u_i^c}(\beta) & = & z(l^c, u^c + \beta e_i, l^x, u^x, c), & i = 1, \dots, m, \\ f_{l_j^x}(\beta) & = & z(l^c, u^c, l^x + \beta e_j, u^x, c), & j = 1, \dots, n, \\ f_{u_j^x}(\beta) & = & z(l^c, u^c, l^x, u^x + \beta e_j, c), & j = 1, \dots, n, \\ f_{c_j}(\beta) & = & z(l^c, u^c, l^x, u^x, c + \beta e_j), & j = 1, \dots, n. \end{array}$$

Given these definitions it should be clear how linearity intervals and shadow prices are defined for the parameters  $u_i^c$  etc.

### 15.4.1.1 Equality constraints

In MOSEK a constraint can be specified as either an equality constraint or a ranged constraint. If constraint i is an equality constraint, we define the optimal value function for this as

$$f_{e^c}(\beta) = z(l^c + \beta e_i, u^c + \beta e_i, l^x, u^x, c)$$

Thus for an equality constraint the upper and the lower bounds (which are equal) are perturbed simultaneously. Therefore, MOSEK will handle sensitivity analysis differently for a ranged constraint with  $l_i^c = u_i^c$  and for an equality constraint.

# 15.4.2 The basis type sensitivity analysis

The classical sensitivity analysis discussed in most textbooks about linear optimization, e.g. [1], is based on an optimal basic solution or, equivalently, on an optimal basis. This method may produce misleading results [17] but is **computationally cheap**. Therefore, and for historical reasons this method is available in MOSEK We will now briefly discuss the basis type sensitivity analysis. Given an optimal basic solution which provides a partition of variables into basic and non-basic variables, the basis type sensitivity analysis computes the linearity interval  $[\beta_1, \beta_2]$  so that the basis remains optimal for the perturbed problem. A shadow price associated with the linearity interval is also computed. However, it is well-known that an optimal basic solution may not be unique and therefore the result depends on the optimal basic solution employed in the sensitivity analysis. This implies that the computed interval is only a subset of the largest interval for which the shadow price is constant. Furthermore, the optimal objective value function might have a breakpoint for  $\beta = 0$ . In this case the basis type sensitivity method will only provide a subset of either the left or the right linearity interval.

In summary, the basis type sensitivity analysis is computationally cheap but does not provide complete information. Hence, the results of the basis type sensitivity analysis should be used with care.

# 15.4.3 The optimal partition type sensitivity analysis

Another method for computing the complete linearity interval is called the *optimal partition type sensitivity analysis*. The main drawback of the optimal partition type sensitivity analysis is that it is computationally expensive compared to the basis type analysts. This type of sensitivity analysis is currently provided as an experimental feature in MOSEK.

Given the optimal primal and dual solutions to (15.1), i.e.  $x^*$  and  $((s_l^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*)$  the optimal objective value is given by

$$z^* := c^T x^*.$$

The left and right shadow prices  $\sigma_1$  and  $\sigma_2$  for  $l_i^c$  are given by this pair of optimization problems:

$$\begin{array}{lll} \sigma_1 & = & \text{minimize} & e_i^T s_l^c \\ & & \text{subject to} & A^T (s_l^c - s_u^c) + s_l^x - s_u^x & = c, \\ & & (l_c)^T (s_l^c) - (u_c)^T (s_u^c) + (l_x)^T (s_l^x) - (u_x)^T (s_u^x) & = z^*, \\ & & s_l^c, s_u^c, s_l^c, s_u^x \geq 0 \end{array}$$

and

$$\sigma_2 = \text{maximize} \qquad e_i^T s_l^c \\ \text{subject to} \qquad A^T (s_l^c - s_u^c) + s_l^x - s_u^x \qquad = c, \\ (l_c)^T (s_l^c) - (u_c)^T (s_u^c) + (l_x)^T (s_l^x) - (u_x)^T (s_u^x) \qquad = z^*, \\ s_l^c, s_u^c, s_l^c, s_u^x \geq 0.$$

These two optimization problems make it easy to interpret the shadow price. Indeed, if  $((s_l^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*)$  is an arbitrary optimal solution then

$$(s_{i}^{c})_{i}^{*} \in [\sigma_{1}, \sigma_{2}].$$

Next, the linearity interval  $[\beta_1, \beta_2]$  for  $l_i^c$  is computed by solving the two optimization problems

$$\beta_1 = \underset{\text{subject to}}{\text{minimize}} \qquad \beta \\ \text{subject to} \quad l^c + \beta e_i \leq \underset{c}{Ax} \leq u^c, \\ c^T x - \sigma_1 \beta = z^*, \\ l^x \leq x \leq u^x,$$

and

$$\beta_2 = \underset{\text{subject to}}{\text{maximize}} \qquad \beta \\ \text{subject to} \quad l^c + \beta e_i \leq \underset{c}{Ax} \leq u^c, \\ c^T x - \sigma_2 \beta = z^*, \\ l^x < x < u^x.$$

The linearity intervals and shadow prices for  $u_i^c$ ,  $l_i^x$ , and  $u_i^x$  are computed similarly to  $l_i^c$ .

The left and right shadow prices for  $c_j$  denoted  $\sigma_1$  and  $\sigma_2$  respectively are computed as follows:

$$\sigma_1 = \underset{\text{subject to}}{\text{minimize}} \qquad e_j^T x \\ \text{subject to} \quad l^c + \beta e_i \leq \underset{c}{Ax} \leq u^c, \\ c^T x = z^*, \\ l^x \leq x \leq u^x$$

and

$$\sigma_2 = \underset{\text{subject to}}{\text{maximize}} \qquad e_j^T x \\ \text{subject to} \quad l^c + \beta e_i \leq \underset{c}{Ax} \leq u^c, \\ c^T x = z^*, \\ l^x < x < u^x.$$

Once again the above two optimization problems make it easy to interpret the shadow prices. Indeed, if  $x^*$  is an arbitrary primal optimal solution, then

$$x_i^* \in [\sigma_1, \sigma_2].$$

The linearity interval  $[\beta_1, \beta_2]$  for a  $c_j$  is computed as follows:

$$\begin{array}{lll} \beta_1 & = & \text{minimize} & \beta \\ & & \text{subject to} & A^T(s_l^c - s_u^c) + s_l^x - s_u^x & = & c + \beta e_j, \\ & & & (l_c)^T(s_l^c) - (u_c)^T(s_u^c) + (l_x)^T(s_l^x) - (u_x)^T(s_u^x) - \sigma_1 \beta & \leq & z^*, \\ & & & & s_l^c, s_u^c, s_l^c, s_u^x \geq 0 \end{array}$$

and

$$\begin{array}{lll} \beta_2 & = & \text{maximize} & \beta \\ & & \text{subject to} & A^T(s_l^c - s_u^c) + s_l^x - s_u^x & = & c + \beta e_j, \\ & & & (l_c)^T(s_l^c) - (u_c)^T(s_u^c) + (l_x)^T(s_l^x) - (u_x)^T(s_u^x) - \sigma_2\beta & \leq & z^*, \\ & & & s_l^c, s_u^c, s_l^c, s_u^c \geq 0. \end{array}$$

# 15.4.4 Example: Sensitivity analysis

As an example we will use the following transportation problem. Consider the problem of minimizing the transportation cost between a number of production plants and stores. Each plant supplies a number of goods and each store has a given demand that must be met. Supply, demand and cost of transportation per unit are shown in Figure 15.2. If we denote the number of transported goods from location i to location j by  $x_{ij}$ , problem can be formulated as the linear optimization problem minimize

$$1x_{11} + 2x_{12} + 5x_{23} + 2x_{24} + 1x_{31} + 2x_{33} + 1x_{34}$$

subject to

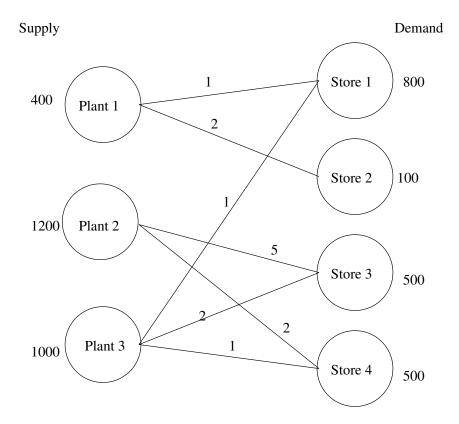


Figure 15.2: Supply, demand and cost of transportation.

Basis	type
Dasis	uype

Basis type						
Con.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$		
1	-300.00	0.00	3.00	3.00		
2	-700.00	$+\infty$	0.00	0.00		
3	-500.00	0.00	3.00	3.00		
4	-0.00	500.00	4.00	4.00		
5	-0.00	300.00	5.00	5.00		
6	-0.00	700.00	5.00	5.00		
7	-500.00	700.00	2.00	2.00		
Var.	$eta_1$	$eta_2$	$\sigma_1$	$\sigma_2$		
$x_{11}$	$-\infty$	300.00	0.00	0.00		
$x_{12}$	$-\infty$	100.00	0.00	0.00		
$x_{23}$	$-\infty$	0.00	0.00	0.00		
$x_{24}$	$-\infty$	500.00	0.00	0.00		
$x_{31}$	$-\infty$	500.00	0.00	0.00		
$x_{33}$	$-\infty$	500.00	0.00	0.00		
$x_{34}$	-0.000000	500.00	2.00	2.00		

Optimal partition type

-	-			
Con.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
1	-300.00	500.00	3.00	1.00
2	-700.00	$+\infty$	-0.00	-0.00
3	-500.00	500.00	3.00	1.00
4	-500.00	500.00	2.00	4.00
5	-100.00	300.00	3.00	5.00
6	-500.00	700.00	3.00	5.00
7	-500.00	700.00	2.00	2.00
Var.	$eta_1$	$eta_2$	$\sigma_1$	$\sigma_2$
$x_{11}$	$-\infty$	300.00	0.00	0.00
$x_{12}$	$-\infty$	100.00	0.00	0.00
$x_{23}$	$-\infty$	500.00	0.00	2.00
$x_{24}$	$-\infty$	500.00	0.00	0.00
$x_{31}$	$-\infty$	500.00	0.00	0.00
$x_{33}$	$-\infty$	500.00	0.00	0.00
$x_{34}$	$-\infty$	500.00	0.00	2.00

Table 15.1: Ranges and shadow prices related to bounds on constraints and variables. Left: Results for the basis type sensitivity analysis. Right: Results for the optimal partition type sensitivity analysis.

The basis type and the optimal partition type sensitivity results for the transportation problem are shown in Table 15.1 and 15.2 respectively. Examining the results from the optimal partition type sensitivity analysis we see that for constraint number 1 we have  $\sigma_1 \neq \sigma_2$  and  $\beta_1 \neq \beta_2$ . Therefore, we have a left linearity interval of [-300, 0] and a right interval of [0, 500]. The corresponding left and right shadow prices are 3 and 1 respectively. This implies that if the upper bound on constraint 1 increases by

$$\beta \in [0, \beta_1] = [0, 500]$$

then the optimal objective value will decrease by the value

$$\sigma_2\beta = 1\beta.$$

Correspondingly, if the upper bound on constraint 1 is decreased by

ъ.	
Basis	type
Dasis	UVDC

	J 1			
Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$c_1$	$-\infty$	3.00	300.00	300.00
$c_2$	$-\infty$	$\infty$	100.00	100.00
$c_3$	-2.00	$\infty$	0.00	0.00
$c_4$	$-\infty$	2.00	500.00	500.00
$c_5$	-3.00	$\infty$	500.00	500.00
$c_6$	$-\infty$	2.00	500.00	500.00
$c_7$	-2.00	$\infty$	0.00	0.00

Optimal partition type

Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$c_1$	$-\infty$	3.00	300.00	300.00
$c_2$	$-\infty$	$\infty$	100.00	100.00
$c_3$	-2.00	$\infty$	0.00	0.00
$c_4$	$-\infty$	2.00	500.00	500.00
$c_5$	-3.00	$\infty$	500.00	500.00
$c_6$	$-\infty$	2.00	500.00	500.00
$c_7$	-2.00	$\infty$	0.00	0.00

Table 15.2: Ranges and shadow prices related to the objective coefficients. Left: Results for the basis type sensitivity analysis. Right: Results for the optimal partition type sensitivity analysis.

$$\beta \in [0, 300]$$

then the optimal objective value will increase by the value

$$\sigma_1\beta=3\beta$$
.

# 15.5 Sensitivity analysis from the MOSEK API

MOSEK provides the functions Task.primalsensitivity and Task.dualsensitivity for performing sensitivity analysis. The code below gives an example of its use.

```
---- sensitivity.py]--
    ##
        Copyright: Copyright (c) MOSEK ApS, Denmark. All rights reserved.
    #
        File:
                    sensitivity.py
                   To demonstrate how to perform sensitivity
    #
        analysis from the API on a small problem:
        minimize
11
        obj: +1 x11 + 2 x12 + 5 x23 + 2 x24 + 1 x31 + 2 x33 + 1 x34
12
13
    #
        st
               + x11 + x12
        c1:
14
15
                                                  x31 +
    #
        c3:
                                                                          <= 1000
16
    #
        c4:
17
    #
                                                                          = 100
18
        c5:
    #
        c6:
                                                                          = 500
19
20
                                                                         = 500
    #
21
    #
        The example uses basis type sensitivity analysis.
22
    ##
23
24
```

```
import sys
26
    import mosek
    # If numpy is installed, use that, otherwise use the
29
    # Mosek's array module.
31
        from numpy import array,zeros,ones
32
    except ImportError:
33
        from mosek.array import array, zeros, ones
34
35
36
37
    # Since the actual value of Infinity is ignores, we define it solely
    # for symbolic purposes:
38
    inf = 0.0
    # Define a stream printer to grab output from MOSEK
41
    def streamprinter(text):
        sys.stdout.write(text)
43
        sys.stdout.flush()
44
45
46
    # We might write everything directly as a script, but it looks nicer
48
    # to create a function.
    def main ():
50
        # Create a MOSEK environment
51
52
        env = mosek.Env ()
        # Attach a printer to the environment
53
        env.set_Stream (mosek.streamtype.log, streamprinter)
55
        # Create a task
56
        task = env.Task(0,0)
        # Attach a printer to the task
58
        task.set_Stream (mosek.streamtype.log, streamprinter)
60
        # Set up data
61
62
        bkc = [ mosek.boundkey.up,mosek.boundkey.up,
63
                mosek.boundkey.up,mosek.boundkey.fx,
                mosek.boundkey.fx, mosek.boundkey.fx,
65
                mosek.boundkey.fx ]
66
        blc = [ -inf, -inf, -inf, 800., 100., 500., 500.]
67
        buc = [ 400., 1200., 1000., 800., 100., 500., 500.]
        bkx = [ mosek.boundkey.lo,mosek.boundkey.lo,
70
                mosek.boundkey.lo,mosek.boundkey.lo,
                mosek.boundkey.lo,mosek.boundkey.lo,
72
                mosek.boundkey.lo ]
73
        c = [1.0,2.0,5.0,2.0,1.0,2.0,1.0]
74
        blx = [0.0,0.0,0.0,0.0,0.0,0.0,0.0]
75
76
        bux = [ inf,inf,inf,inf,inf,inf]
77
        ptrb = [0,2,4,6,8,10,12]
79
        ptre = [ 2,4,6,8,10,12,14 ]
        sub = [0,3,0,4,1,5,1,6,2,3,2,5,2,6]
80
        val = [1.0,1.0,1.0,1.0,1.0,1.0,1.0]
```

```
1.0,1.0,1.0,1.0,1.0,1.0,1.0]
83
84
         numcon = len(bkc)
85
         numvar = len(bkx)
86
         numanz = len(val)
87
         # Input linear data
89
90
         task.inputdata(numcon,numvar,
91
                         c,0.0,
                         ptrb, ptre, sub, val,
92
93
                         bkc, blc, buc,
                         bkx, blx, bux)
94
95
         # Set objective sense
         task.putobjsense(mosek.objsense.minimize)
96
97
98
         # Optimize
         task.optimize();
99
100
         # Analyze upper bound on c1 and the equality constraint on c4
101
         subi = [0, 3]
102
         marki = [ mosek.mark.up, mosek.mark.up ]
103
104
         \# Analyze lower bound on the variables x12 and x31
105
         subj = [ 1, 4 ]
106
         markj = [ mosek.mark.lo, mosek.mark.lo ]
107
108
         leftpricei = zeros(2,float)
109
         rightpricei = zeros(2,float)
110
         leftrangei = zeros(2,float)
111
112
         rightrangei = zeros(2,float)
         leftpricej = zeros(2,float)
113
         rightpricej = zeros(2,float)
114
         leftrangej = zeros(2,float)
115
         rightrangej = zeros(2,float)
116
118
         task.primalsensitivity( subi,
119
                                  marki.
120
                                   subj,
121
122
                                   markj,
                                  leftpricei,
123
                                   rightpricei,
124
                                  leftrangei,
125
                                  rightrangei,
126
127
                                   leftpricej,
                                   rightpricej,
128
129
                                   leftrangej,
                                  rightrangej)
130
131
         print ('Results from sensitivity analysis on bounds:')
132
         print ('\tleftprice | rightprice | leftrange | rightrange ')
133
         print ('For constraints:')
134
135
         for i in range(2):
136
             print ('\t%10f %10f %10f %10f' % (leftpricei[i],
137
                                                       rightpricei[i],
138
139
                                                       leftrangei[i],
                                                       rightrangei[i]))
140
```

```
141
         print ('For variables:')
142
         for i in range(2):
143
                              %10f %10f %10f' % (leftpricej[i],
144
             print ('\t%10f
                                                        rightpricej[i],
145
                                                        leftrangej[i],
                                                        rightrangej[i]))
147
148
149
         leftprice = zeros(2,float)
150
151
         rightprice = zeros(2,float)
         leftrange = zeros(2,float)
152
153
         rightrange = zeros(2,float)
                    = array([ 2, 5 ])
154
         subc
155
         task.dualsensitivity( subc,
                                 leftprice.
157
                                 rightprice,
158
                                 leftrange.
159
                                 rightrange)
160
161
         print ('Results from sensitivity analysis on objective coefficients:')
162
163
         for i in range(2):
164
             print ('\t%10f
                                %10f
                                      %10f %10f' % (leftprice[i],
165
166
                                                        rightprice[i],
                                                        leftrange[i],
167
168
                                                        rightrange[i]))
169
170
         return None
171
     # call the main function
172
173
         main ()
174
     except mosek.Exception as e:
175
         print ("ERROR: %s" % str(e.errno))
176
177
         if e.msg is not None:
             print ("\t%s" % e.msg)
178
         sys.exit(1)
179
180
     except:
         import traceback
181
         traceback.print_exc()
182
         sys.exit(1)
183
```

# 15.6 Sensitivity analysis with the command line tool

A sensitivity analysis can be performed with the MOSEK command line tool using the command mosek myproblem.mps -sen sensitivity.ssp

where sensitivity.ssp is a file in the format described in the next section. The ssp file describes which parts of the problem the sensitivity analysis should be performed on.

By default results are written to a file named myproblem.sen. If necessary, this filename can be

```
* A comment
BOUNDS CONSTRAINTS
U|L|LU [cname1]
U|L|LU [cname2]-[cname3]
BOUNDS VARIABLES
U|L|LU [vname1]
U|L|LU [vname2]-[vname3]
OBJECTIVE VARIABLES
[vname1]
[vname2]-[vname3]
```

Figure 15.3: The sensitivity analysis file format.

changed by setting the

MSK\_SPAR\_SENSITIVITY\_RES\_FILE\_NAME

parameter By default a basis type sensitivity analysis is performed. However, the type of sensitivity analysis (basis or optimal partition) can be changed by setting the parameter

MSK\_IPAR\_SENSITIVITY\_TYPE

appropriately. Following values are accepted for this parameter:

- MSK\_SENSITIVITY\_TYPE\_BASIS
- MSK\_SENSITIVITY\_TYPE\_OPTIMAL\_PARTITION

It is also possible to use the command line
mosek myproblem.mps -d MSK\_IPAR\_SENSITIVITY\_ALL MSK\_ON

in which case a sensitivity analysis on all the parameters is performed.

# 15.6.1 Sensitivity analysis specification file

MOSEK employs an MPS like file format to specify on which model parameters the sensitivity analysis should be performed. As the optimal partition type sensitivity analysis can be computationally expensive it is important to limit the sensitivity analysis. The format of the sensitivity specification file is shown in figure 15.3, where capitalized names are keywords, and names in brackets are names of the constraints and variables to be included in the analysis.

The sensitivity specification file has three sections, i.e.

- BOUNDS CONSTRAINTS: Specifies on which bounds on constraints the sensitivity analysis should be performed.
- BOUNDS VARIABLES: Specifies on which bounds on variables the sensitivity analysis should be performed.
- OBJECTIVE VARIABLES: Specifies on which objective coefficients the sensitivity analysis should be performed.

Figure 15.4: Example of the sensitivity file format.

A line in the body of a section must begin with a whitespace. In the BOUNDS sections one of the keys L, U, and LU must appear next. These keys specify whether the sensitivity analysis is performed on the lower bound, on the upper bound, or on both the lower and the upper bound respectively. Next, a single constraint (variable) or range of constraints (variables) is specified.

Recall from Section 15.4.1.1 that equality constraints are handled in a special way. Sensitivity analysis of an equality constraint can be specified with either L, U, or LU, all indicating the same, namely that upper and lower bounds (which are equal) are perturbed simultaneously.

As an example consider

```
BOUNDS CONSTRAINTS
L "cons1"
U "cons2"
LU "cons3"-"cons6"
```

which requests that sensitivity analysis is performed on the lower bound of the constraint named cons1, on the upper bound of the constraint named cons2, and on both lower and upper bound on the constraints named cons3 to cons6.

It is allowed to use indexes instead of names, for instance

```
BOUNDS CONSTRAINTS
L "cons1"
U 2
LU 3 - 6
```

The character "\*" indicates that the line contains a comment and is ignored.

### 15.6.2 Example: Sensitivity analysis from command line

As an example consider the sensitivity.ssp file shown in Figure 15.4. The command mosek transport.lp -sen sensitivity.ssp -d iparam.sensitivity\_type sensitivitytype.basis produces the transport.sen file shown below.

```
        BOUNDS CONSTRAINTS

        INDEX
        NAME
        BOUND
        LEFTRANGE
        RIGHTRANGE
        LEFTPRICE
        RIGHTPRICE

        0
        c1
        UP
        -6.574875e-18
        5.000000e+02
        1.000000e+00
        1.000000e+00
```

2 3 4 5	c3 c4 c5 c6	UP FIX FIX FIX	-6.574875e-18 -5.000000e+02 -1.000000e+02 -5.000000e+02	5.000000e+02 6.574875e-18 6.574875e-18 6.574875e-18	1.000000e+00 2.000000e+00 3.000000e+00 3.000000e+00	1.000000e+00 2.000000e+00 3.000000e+00 3.000000e+00	
BOUNDS V	/ARIABLES						
INDEX	NAME	BOUND	LEFTRANGE	RIGHTRANGE	LEFTPRICE	RIGHTPRICE	
2	x23	LO	-6.574875e-18	5.000000e+02	2.000000e+00	2.000000e+00	
3	x24	LO	-inf	5.000000e+02	0.000000e+00	0.000000e+00	
4	x31	LO	-inf	5.000000e+02	0.000000e+00	0.000000e+00	
0	x11	LO	-inf	3.000000e+02	0.000000e+00	0.000000e+00	
OBJECTIVE VARIABLES							
INDEX	NAME		LEFTRANGE	RIGHTRANGE	LEFTPRICE	RIGHTPRICE	
0	x11		-inf	1.000000e+00	3.000000e+02	3.000000e+02	
2	x23		-2.000000e+00	+inf	0.000000e+00	0.000000e+00	

# 15.6.3 Controlling log output

Setting the parameter

MSK\_IPAR\_LOG\_SENSITIVITY

to 1 or 0 (default) controls whether or not the results from sensitivity calculations are printed to the message stream.

The parameter

MSK\_IPAR\_LOG\_SENSITIVITY\_OPT

controls the amount of debug information on internal calculations from the sensitivity analysis.

# Appendix A

# API reference

This chapter lists all functionality in the MOSEK Python API.

Environment constructor and destructor

- Env()
- Env.\_\_del\_\_()

Task constructors and destructor

- Task(Env env)
- Task(Task task)
- Task(Env env, int maxnumcon, int maxnumvar)
- Task.\_\_del\_\_()

## Exceptions

# Callback functions

# Bounds

- Task.putconboundlist
- Task.putvarboundlist

Operate on data associated with the conic constraints

- Task.appendcone
- Task.putcone
- Task.removecones

Operate on data associated with the constraints

- Task.appendcons
- Task.getnumcon
- Task.putconbound
- Task.removecons

# Task diagnostics

- Task.checkconvexity
- Task.getprobtype
- Task.optimizersummary
- Task.printdata
- Task.printparam
- Task.solutionsummary
- Task.updatesolutioninfo

### Reading and writing data files

- Task.readsolution
- Task.writedata
- Task.writesolution

### Obtaining solution values

- Task.getbarsj
- Task.getbarxj
- Task.getskcslice
- Task.getskxslice
- Task.getslcslice
- Task.getslxslice
- Task.getsnxslice
- Task.getsucslice
- Task.getsuxslice
- Task.getxcslice
- Task.getxxslice
- Task.getyslice

# Diagnosing infeasibility

- Task.getinfeasiblesubproblem
- Task.primalrepair

### • Task.relaxprimal

Obtain information about the solutions.

- Task.getdualobj
- Task.getdviolbarvar
- Task.getdviolcon
- Task.getdviolcones
- Task.getdviolvar
- Task.getprimalobj
- Task.getprosta
- Task.getpviolbarvar
- Task.getpviolcon
- Task.getpviolcones
- Task.getpviolvar
- Task.getsolsta
- Task.getsolutioninfo
- Task.solutiondef

Linear algebra utility functions for performing linear algebra operations

- Env.axpy
- Env.dot
- Env.gemm
- Env.gemv
- Env.potrf
- Env.syeig
- Env.syevd
- Env.syrk

Management of the environment

- Env.licensecleanup
- Env.putlicensedebug
- Env.putlicensepath
- Env.putlicensewait

### Naming

• Task.putbarvarname

- Task.putconename
- Task.putconname
- Task.putobjname
- Task.puttaskname
- Task.putvarname

Operate on data associated with objective.

- Task.putcfix
- Task.putobjsense

### Optimization

- Task.optimizeconcurrent
- Task.optimize

Setting task parameter values

- Task.putdouparam
- Task.putintparam
- Task.putstrparam

### Inputting solution values

- Task.putbarsj
- Task.putbarxj
- Task.putskcslice
- Task.putskxslice
- Task.putslcslice
- Task.putslxslice
- Task.putsnxslice
- Task.putsolution
- Task.putsolutioni
- Task.putsucslice
- Task.putsuxslice
- Task.putxcslice
- Task.putxxslice
- Task.putyslice

Operate on data associated with scalar variables

• Task.appendvars

- Task.getnumvar
- Task.putacol
- Task.putaij
- Task.putarow
- Task.putcj
- Task.putqcon
- Task.putqconk
- Task.putqobj
- Task.putqobjij
- Task.putvarbound
- Task.putvartype
- Task.removevars

# Sensitivity analysis

- Task.dualsensitivity
- Task.primalsensitivity
- Task.sensitivityreport

### Optimizer statistics

- Task.getdouinf
- Task.getintinf
- Task.getlintinf

### Output stream functions

• Task.linkfiletostream

Operate on data associated with symmetric matrix variables

- Task.appendbarvars
- Task.appendsparsesymmat
- Task.putbaraij
- Task.putbarcj

# Alphabetic list of functions

# A.1 Exceptions

```
mosek.Exception(Exception)
```

Base exception class for all MOSEK exceptions.

```
mosek.Error(mosek.Exception)
```

Exception class used for all error response codes from MOSEK.

```
mosek.Warning(mosek.Exception)
```

Exception class used for all warning response codes from MOSEK.

# A.2 Class Task

# A.2.1 Task.analyzenames()

```
Task.analyzenames(
    whichstream,
    nametype)
```

Analyze the names and issue an error for the first invalid name.

Arguments

```
nametype: nametype

The type of names e.g. valid in MPS or LP files.
```

whichstream : streamtype

Index of the stream.

Description:

The function analyzes the names and issue an error if a name is invalid.

# A.2.2 Task.analyzeproblem()

```
Task.analyzeproblem(whichstream)
```

Analyze the data of a task.

Arguments

```
whichstream: streamtype
Index of the stream.
```

### Description:

The function analyzes the data of task and writes out a report.

# A.2.3 Task.analyzesolution()

```
Task.analyzesolution(
    whichstream,
    whichsol)
```

Print information related to the quality of the solution.

### Arguments

```
whichsol: soltype
Selects a solution.
whichstream: streamtype
Index of the stream.
```

### Description:

Print information related to the quality of the solution and other solution statistics.

By default this function prints information about the largest infeasibilities in the solution, the primal (and possibly dual) objective value and the solution status.

Following parameters can be used to configure the printed statistics:

- iparam.ana\_sol\_basis. Enables or disables printing of statistics specific to the basis solution (condition number, number of basic variables etc.). Default is on.
- iparam.ana\_sol\_print\_violated. Enables or disables listing names of all constraints (both primal and dual) which are violated by the solution. Default is off.
- dparam.ana\_sol\_infeas\_tol. The tolerance defining when a constraint is considered violated. If a constraint is violated more than this, it will be listed in the summary.

### See also

- Task getpviolcon Computes the violation of a primal solution for a list of xc variables.
- Task getpviolvar Computes the violation of a primal solution for a list of x variables.
- Task.getpviolbarvar Computes the violation of a primal solution for a list of barx variables.
- Task.getpviolcones Computes the violation of a solution for set of conic constraints.
- Task.getdviolcon Computes the violation of a dual solution associated with a set of constraints.
- Task.getdviolvar Computes the violation of a dual solution associated with a set of x variables.

- Task.getdviolbarvar Computes the violation of dual solution for a set of barx variables.
- Task.getdviolcones Computes the violation of a solution for set of dual conic constraints.
- iparam.ana\_sol\_basis Controls whether the basis matrix is analyzed in solaution analyzer.

# A.2.4 Task.appendbarvars()

Task.appendbarvars(dim)

Appends a semidefinite variable of dimension dim to the problem.

Arguments

```
dim : int[]
```

Dimension of symmetric matrix variables to be added.

Description:

Appends a positive semidefinite matrix variable of dimension dim to the problem.

# A.2.5 Task.appendcone()

```
Task.appendcone(
    conetype,
    conepar,
    submem)
```

Appends a new cone constraint to the problem.

Arguments

```
conepar : double
```

This argument is currently not used. Can be set to 0.0.

conetype : conetype

Specifies the type of the cone.

submem : int[]

Variable subscripts of the members in the cone.

Description:

Appends a new conic constraint to the problem. Hence, add a constraint

to the problem where C is a convex cone.  $\hat{x}$  is a subset of the variables which will be specified by the argument submem.

Depending on the value of conetype this function appends a normal (conetype.quad) or rotated quadratic cone (conetype.rquad). Define

$$\hat{x} = x_{\mathtt{submem}[0]}, \dots, x_{\mathtt{submem}[\mathtt{nummem}-1]}$$

- . Depending on the value of conetype this function appends one of the constraints:
  - Quadratic cone (conetype.quad):

$$\hat{x}_0 \geq \sqrt{\sum_{i=1}^{i < \text{nummem}} \hat{x}_i^2}$$

• Rotated quadratic cone (conetype.rquad):

$$2 \hat{x}_0 \hat{x}_1 \geq \sum_{i=2}^{i < \text{nummem}} \hat{x}_i^2, \quad \hat{x}_0, \hat{x}_1 \geq 0$$

Please note that the sets of variables appearing in different conic constraints must be disjoint.

For an explained code example see Section 5.3.

See also

- Task.appendconeseq Appends a new conic constraint to the problem.
- Task.appendconesseq Appends multiple conic constraints to the problem.

# A.2.6 Task.appendconeseq()

```
Task.appendconeseq(
    conetype,
    conepar,
    nummem,
    j)
```

Appends a new conic constraint to the problem.

### Arguments

```
conepar : double
```

This argument is currently not used. Can be set to 0.0.

```
conetype : conetype
```

Specifies the type of the cone.

```
j: int
```

Index of the first variable in the conic constraint.

nummem : int

Dimension of the conic constraint to be appended.

### Description:

Appends a new conic constraint to the problem. The function assumes the members of cone are sequential where the first emeber has index j and the last j+nummem-1.

See also

- Task.appendcone Appends a new cone constraint to the problem.
- Task.appendconesseq Appends multiple conic constraints to the problem.

# A.2.7 Task.appendconesseq()

```
Task.appendconesseq(
    conetype,
    conepar,
    nummem,
    j)
```

Appends multiple conic constraints to the problem.

### Arguments

```
conepar : double[]
   This argument is currently not used. Can be set to 0.0.
conetype : conetype
   Specifies the type of the cone.
j : int
   Index of the first variable in the first cone to be appended.
nummem : int[]
   Number of member variables in the cone.
```

### Description:

Appends a number conic constraints to the problem. The kth cone is assumed to be of dimension nummem[k]. Moreover, is is assumed that the first variable of the first cone has index j and the index of the variable in each cone are sequential. Finally, it assumed in the second cone is the last index of first cone plus one and so forth.

See also

- Task.appendcone Appends a new cone constraint to the problem.
- Task.appendconeseq Appends a new conic constraint to the problem.

# A.2.8 Task.appendcons()

Task.appendcons(num)

Appends a number of constraints to the optimization task.

# Arguments

```
num : int
```

Number of constraints which should be appended.

### Description:

Appends a number of constraints to the model. Appended constraints will be declared free. Please note that MOSEK will automatically expand the problem dimension to accommodate the additional constraints.

See also

• Task.removecons The function removes a number of constraints.

# A.2.9 Task.appendsparsesymmat()

```
idx = Task.appendsparsesymmat(
    dim,
    subi,
    subj,
    valij)
```

Appends a general sparse symmetric matrix to the vector E of symmetric matrixes.

#### Arguments

```
dim : int
```

Dimension of the symmetric matrix that is appended.

```
idx : long
```

Each matrix that is appended to E is assigned a unique index i.e.  $\mathtt{idx}$  that can be used for later reference.

```
subi : int[]
```

Row subscript in the triplets.

subj : int[]

Column subscripts in the triplets.

valij : double[]

Values of each triplet.

### Description:

MOSEK maintains a storage of symmetric data matrixes that is used to build the  $\bar{c}$  and  $\bar{A}$ . The storage can be thought of as a vector of symmetric matrixes denoted E. Hence,  $E_i$  is a symmetric matrix of certain dimension.

This functions appends a general sparse symmetric matrix on triplet form to the vector E of symmetric matrixes. The vectors  $\mathtt{subi}$ ,  $\mathtt{subj}$ , and  $\mathtt{valij}$  contains the row subscripts, column subscripts and values of each element in the symmetric matrix to be appended. Since the matrix that is appended is symmetric then only the lower triangular part should be specified. Moreover, duplicates are not allowed.

Observe the function reports the index (position) of the appended matrix in E. This index should be used for later references to the appended matrix.

# A.2.10 Task.appendstat()

Task.appendstat()

Appends a record the statistics file.

### Description:

Appends a record to the statistics file.

# A.2.11 Task.appendvars()

Task.appendvars(num)

Appends a number of variables to the optimization task.

#### Arguments

num : int

Number of variables which should be appended.

### Description:

Appends a number of variables to the model. Appended variables will be fixed at zero. Please note that MOSEK will automatically expand the problem dimension to accommodate the additional variables.

See also

• Task.removevars The function removes a number of variables.

### A.2.12 Task.basiscond()

nrmbasis,nrminvbasis = Task.basiscond()

Computes conditioning information for the basis matrix.

### Arguments

nrmbasis : double

An estimate for the 1 norm of the basis.

nrminvbasis : double

An estimate for the 1 norm of the inverse of the basis.

#### Description:

If a basic solution is available and it defines a nonsingular basis, then this function computes the 1-norm estimate of the basis matrix and an 1-norm estimate for the inverse of the basis matrix. The 1-norm estimates are computed using the method outlined in [19].

By defintion the 1-norm condition number of a matrix B is defined as

$$\kappa_1(B) := \|B\|_1 \|B^{-1}\|.$$

Moreover, the larger the condition number is the harder it is to solve linear equation systems involving B. Given estimates for  $||B||_1$  and  $||B^{-1}||_1$  it is also possible to estimate  $\kappa_1(B)$ .

# A.2.13 Task.checkconvexity()

Task.checkconvexity()

Checks if a quadratic optimization problem is convex.

# Description:

This function checks if a quadratic optimization problem is convex. The amount of checking is controlled by iparam.check\_convexity.

The function throws an exception if the problem is not convex.

See also

• iparam.check\_convexity Specify the level of convexity check on quadratic problems

# A.2.14 Task.checkmem()

```
Task.checkmem(
    file,
    line)
```

Checks the memory allocated by the task.

### Arguments

file : str

File from which the function is called.

line : int

Line in the file from which the function is called.

### Description:

Checks the memory allocated by the task.

# A.2.15 Task.chgbound()

```
Task.chgbound(
    accmode,
    i,
    lower,
    finite,
    value)
```

Changes the bounds for one constraint or variable.

#### Arguments

#### accmode: accmode

Defines if operations are performed row-wise (constraint-oriented) or column-wise (variable-oriented).

finite : int

If non-zero, then value is assumed to be finite.

i: int

Index of the constraint or variable for which the bounds should be changed.

lower : int

If non-zero, then the lower bound is changed, otherwise the upper bound is changed.

value : double

New value for the bound.

### Description:

Changes a bound for one constraint or variable. If accmode equals accmode.con, a constraint bound is changed, otherwise a variable bound is changed.

If lower is non-zero, then the lower bound is changed as follows:

$$\text{new lower bound} = \left\{ \begin{array}{ll} -\infty, & \texttt{finite} = 0, \\ \texttt{value} & \text{otherwise}. \end{array} \right.$$

Otherwise if lower is zero, then

$$\text{new upper bound} = \left\{ \begin{array}{ll} \infty, & \text{finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Please note that this function automatically updates the bound key for bound, in particular, if the lower and upper bounds are identical, the bound key is changed to fixed.

See also

- Task.putbound Changes the bound for either one constraint or one variable.
- dparam.data\_tol\_bound\_inf Data tolerance threshold.
- dparam.data\_tol\_bound\_wrn Data tolerance threshold.

# A.2.16 Task.commitchanges()

Task.commitchanges()

Commits all cached problem changes.

### Description:

Commits all cached problem changes to the task. It is usually not necessary explicitly to call this function since changes will be committed automatically when required.

### A.2.17 Task.deletesolution()

Task.deletesolution(whichsol)

Undefines a solution and frees the memory it uses.

Arguments

```
whichsol: soltype Selects a solution.
```

### Description:

Undefines a solution and frees the memory it uses.

# A.2.18 Task.dualsensitivity()

```
Task.dualsensitivity(
    subj,
    leftpricej,
    rightpricej,
    leftrangej,
    rightrangej)
```

Performs sensitivity analysis on objective coefficients.

### Arguments

```
leftpricej : double[]
  leftpricej[j] is the left shadow price for the coefficients with index subj[j].

leftrangej : double[]
  leftrangej[j] is the left range \beta_1 for the coefficient with index subj[j].

rightpricej : double[]
  rightpricej[j] is the right shadow price for the coefficients with index subj[j].

rightrangej : double[]
  rightrangej[j] is the right range \beta_2 for the coefficient with index subj[j].

subj : int[]
  Index of objective coefficients to analyze.
```

#### Description:

Calculates sensitivity information for objective coefficients. The indexes of the coefficients to analyze are

```
\{ \mathtt{subj}[i] | i \in 0, \dots, \mathtt{numj} - 1 \}
```

The results are returned so that e.g leftprice[j] is the left shadow price of the objective coefficient with index subj[j].

The type of sensitivity analysis to perform (basis or optimal partition) is controlled by the parameter <code>iparam.sensitivity\_type</code>.

For an example, please see Section 15.5.

### See also

- Task.primalsensitivity Perform sensitivity analysis on bounds.
- Task.sensitivityreport Creates a sensitivity report.
- iparam.sensitivity\_type Controls which type of sensitivity analysis is to be performed.
- iparam.log\_sensitivity Control logging in sensitivity analyzer.
- iparam.log\_sensitivity\_opt Control logging in sensitivity analyzer.

# A.2.19 Task.getacol()

```
nzj = Task.getacol(
    j,
    subj,
    valj)
```

Obtains one column of the linear constraint matrix.

# Arguments

j: int

Index of the column.

nzj: int

Number of non-zeros in the column obtained.

subj : int[]

Index of the non-zeros in the column obtained.

valj : double[]

Numerical values of the column obtained.

# Description:

Obtains one column of A in a sparse format.

# A.2.20 Task.getacolnumnz()

```
nzj = Task.getacolnumnz(i)
```

Obtains the number of non-zero elements in one column of the linear constraint matrix

### Arguments

i: int

Index of the column.

nzj: int

Number of non-zeros in the jth row or column of A.

### Description:

Obtains the number of non-zero elements in one column of A.

# A.2.21 Task.getacolslicetrip()

```
Task.getacolslicetrip(
   first,
   last,
   subi,
   subj,
   val)
```

Obtains a sequence of columns from the coefficient matrix in triplet format.

# Arguments

```
first : int
    Index of the first column in the sequence.

last : int
    Index of the last column in the sequence plus one.

subi : int[]
    Constraint subscripts.

subj : int[]
    Column subscripts.

val : double[]
    Values.
```

### Description:

Obtains a sequence of columns from A in a sparse triplet format.

# A.2.22 Task.getaij()

```
aij = Task.getaij(
    i,
    j)
```

Obtains a single coefficient in linear constraint matrix.

### Arguments

```
\label{eq:aij:double} \text{The required coefficient $a_{i,j}$.} i : int
```

Row index of the coefficient to be returned.

#### j: int

Column index of the coefficient to be returned.

### Description:

Obtains a single coefficient in A.

# A.2.23 Task.getapiecenumnz()

```
numnz = Task.getapiecenumnz(
    firsti,
    lasti,
    firstj,
    lastj)
```

Obtains the number non-zeros in a rectangular piece of the linear constraint matrix.

#### Arguments

firsti : int

Index of the first row in the rectangular piece.

firstj: int

Index of the first column in the rectangular piece.

lasti : int

Index of the last row plus one in the rectangular piece.

lastj : int

Index of the last column plus one in the rectangular piece.

numnz : int

Number of non-zero A elements in the rectangular piece.

### Description:

Obtains the number non-zeros in a rectangular piece of A, i.e. the number

```
|\{(i,j):\ a_{i,j}\neq 0,\ \mathtt{firsti}\leq i\leq \mathtt{lasti}-1,\ \mathtt{firstj}\leq j\leq \mathtt{lastj}-1\}|
```

where  $|\mathcal{I}|$  means the number of elements in the set  $\mathcal{I}$ .

This function is not an efficient way to obtain the number of non-zeros in one row or column. In that case use the function Task.getarownumnz or Task.getacolnumnz.

# A.2.24 Task.getarow()

```
nzi = Task.getarow(
    i,
    subi,
    vali)
```

Obtains one row of the linear constraint matrix.

# Arguments

#### i: int

Index of the row or column.

nzi : int

Number of non-zeros in the row obtained.

subi : int[]

Index of the non-zeros in the row obtained.

vali : double[]

Numerical values of the row obtained.

# Description:

Obtains one row of A in a sparse format.

# A.2.25 Task.getarownumnz()

```
nzi = Task.getarownumnz(i)
```

Obtains the number of non-zero elements in one row of the linear constraint matrix

### Arguments

#### i: int

Index of the row or column.

nzi : int

Number of non-zeros in the ith row of A.

### Description:

Obtains the number of non-zero elements in one row of A.

# A.2.26 Task.getarowslicetrip()

```
Task.getarowslicetrip(
    first,
    last,
    subi,
    subj,
    val)
```

Obtains a sequence of rows from the coefficient matrix in triplet format.

# Arguments

```
first : int
    Index of the first row or column in the sequence.
last : int
    Index of the last row or column in the sequence plus one.
subi : int[]
    Constraint subscripts.
subj : int[]
    Column subscripts.
val : double[]
    Values.
```

## Description:

Obtains a sequence of rows from A in a sparse triplet format.

# A.2.27 Task.getaslice()

```
Task.getaslice(
    accmode,
    first,
    last,
    ptrb,
    ptre,
    sub,
    val)
```

Obtains a sequence of rows or columns from the coefficient matrix.

# Arguments

```
accmode : accmode
```

Defines whether a column slice or a row slice is requested.

first : int

Index of the first row or column in the sequence.

last: int

Index of the last row or column in the sequence **plus one**.

ptrb : long[]

ptrb[t] is an index pointing to the first element in the tth row or column obtained.

ptre : long[]

ptre[t] is an index pointing to the last element plus one in the tth row or column obtained.

sub : int[]

Contains the row or column subscripts.

val : double∏

Contains the coefficient values.

### Description:

Obtains a sequence of rows or columns from A in sparse format.

See also

• Task.getaslicenumnz Obtains the number of non-zeros in a slice of rows or columns of the coefficient matrix.

# A.2.28 Task.getaslicenumnz()

```
numnz = Task.getaslicenumnz(
    accmode,
    first,
    last)
```

Obtains the number of non-zeros in a slice of rows or columns of the coefficient matrix.

# Arguments

```
accmode : accmode
```

Defines whether non-zeros are counted in a column slice or a row slice.

first : int

Index of the first row or column in the sequence.

last: int

Index of the last row or column **plus one** in the sequence.

numnz : long

Number of non-zeros in the slice.

#### Description:

Obtains the number of non-zeros in a slice of rows or columns of A.

# A.2.29 Task.getbarablocktriplet()

```
num = Task.getbarablocktriplet(
    subi,
    subj,
    subk,
    subl,
    valijkl)
```

Obtains barA in block triplet form.

# Arguments

num : long

Number of elements in the block triplet form.

subi : int[]

Constraint index.

subj : int[]

Symmetric matrix variable index.

subk : int[]

Block row index.

subl : int[]

Block column index.

valijkl : double[]

A list indexes of the elements from symmetric matrix storage that appers in the weighted sum.

### Description:

Obtains  $\bar{A}$  in block triplet form.

# A.2.30 Task.getbaraidx()

```
i,j,num = Task.getbaraidx(
   idx,
   sub,
   weights)
```

Obtains information about an element barA.

# Arguments

### i : int

Row index of the element at position idx.

idx : long

Position of the element in the vectorized form.

j: int

Column index of the element at position idx.

num : long

Number of terms in weighted sum that forms the element.

sub : long[]

A list indexes of the elements from symmetric matrix storage that appears in the weighted sum.

weights : double[]

The weights associated with each term in the weighted sum.

### Description:

Obtains information about an element in  $\bar{A}$ . Since  $\bar{A}$  is a sparse matrix of symmetric matrixes then only the nonzero elements in  $\bar{A}$  are stored in order to save space. Now  $\bar{A}$  is stored vectorized form i.e. as one long vector. This function makes it possible to obtain information such as the row index and the column index of a particular element of the vectorized form of  $\bar{A}$ .

Please observe if one element of  $\bar{A}$  is inputted multiple times then it may be stored several times in vectorized form. In that case the element with the highest index is the one that is used.

# A.2.31 Task.getbaraidxij()

```
i,j = Task.getbaraidxij(idx)
```

Obtains information about an element barA.

### Arguments

i : int

Row index of the element at position idx.

idx : long

Position of the element in the vectorized form.

j: int

Column index of the element at position idx.

### Description:

Obtains information about an element in  $\bar{A}$ . Since  $\bar{A}$  is a sparse matrix of symmetric matrixes only the nonzero elements in  $\bar{A}$  are stored in order to save space. Now  $\bar{A}$  is stored vectorized form i.e. as one long vector. This function makes it possible to obtain information such as the row index and the column index of a particular element of the vectorized form of  $\bar{A}$ .

Please note that if one element of  $\bar{A}$  is inputted multiple times then it may be stored several times in vectorized form. In that case the element with the highest index is the one that is used.

### A.2.32 Task.getbaraidxinfo()

```
num = Task.getbaraidxinfo(idx)
```

Obtains the number terms in the weighted sum that forms a particular element in barA.

#### Arguments

idx: long

The internal position of the element that should be obtained information for.

num : long

Number of terms in the weighted sum that forms the specified element in  $\bar{A}$ .

#### Description:

Each nonzero element in  $\bar{A}_{ij}$  is formed as a weighted sum of symmtric matrixes. Using this function the number terms in the weighted sum can be obtained. See description of Task.appendsparsesymmat for details about the weighted sum.

## A.2.33 Task.getbarasparsity()

```
numnz = Task.getbarasparsity(idxij)
```

Obtains the sparsity pattern of the barA matrix.

#### Arguments

#### idxij : long[]

Position of each nonzero element in the vectorized form of  $A_{ij}$ . Hence, idxij[k] is the vector position of the element in row subi[k] and column subj[k] of  $\bar{A}_{ij}$ .

numnz : long

Number of nonzero elements in  $\bar{A}$ .

#### Description:

The matrix  $\bar{A}$  is assumed to be a sparse matrix of symmetric matrixes. This implies that many of elements in  $\bar{A}$  is likely to be zero matrixes. Therefore, in order to save space only nonzero elements in  $\bar{A}$  are stored on vectorized form. This function is used to obtain the sparsity pattern of  $\bar{A}$  and the position of each nonzero element in the vectorized form of  $\bar{A}$ .

## A.2.34 Task.getbarcblocktriplet()

```
num = Task.getbarcblocktriplet(
    subj,
    subk,
    subl,
    valijkl)
```

Obtains barc in block triplet form.

## Arguments

```
num : long
```

Number of elements in the block triplet form.

subj : int[]

Symmetric matrix variable index.

subk : int[]

Block row index.

subl : int[]

Block column index.

valijkl : double[]

A list indexes of the elements from symmetric matrix storage that appers in the weighted sum.

### Description:

Obtains  $\bar{C}$  in block triplet form.

## A.2.35 Task.getbarcidx()

```
j,num = Task.getbarcidx(
   idx,
   sub,
   weights)
```

Obtains information about an element in barc.

#### Arguments

```
idx: long
```

Index of the element that should be obtained information about.

j : int

Row index in  $\bar{c}$ .

num : long

Number of terms in the weighted sum.

sub : long[]

Elements appearing the weighted sum.

weights : double[]

Weights of terms in the weighted sum.

### Description:

Obtains information about an element in  $\bar{c}$ .

## A.2.36 Task.getbarcidxinfo()

```
num = Task.getbarcidxinfo(idx)
```

Obtains information about an element in barc.

### Arguments

idx: long

Index of element that should be obtained information about. The value is an index of a symmetric sparse variable.

num : long

Number of terms that appears in weighted that forms the requested element.

### Description:

Obtains information about about the  $\bar{c}_{ij}$ .

### A.2.37 Task.getbarcidxj()

```
j = Task.getbarcidxj(idx)
```

Obtains the row index of an element in barc.

#### Arguments

```
idx: long
```

Index of the element that should be obtained information about.

j: int

Row index in  $\bar{c}$ .

#### Description:

Obtains the row index of an element in  $\bar{c}$ .

## A.2.38 Task.getbarcsparsity()

```
numnz = Task.getbarcsparsity(idxj)
```

Get the positions of the nonzero elements in barc.

#### Arguments

```
idxj : long[]
```

Internal positions of the nonzeros elements in  $\bar{c}$ .

numnz : long

Number of nonzero elements in  $\bar{C}$ .

#### Description:

Internally only the nonzero elements of  $\bar{c}$  is stored

in a vector. This function returns which elements  $\bar{c}$  that are nonzero (in subj) and their internal position (in idx). Using the position detailed information about each nonzero  $\bar{C}_j$  can be obtained using Task.getbarcidxinfo and Task.getbarcidx.

## A.2.39 Task.getbarsj()

```
Task.getbarsj(
    whichsol,
    j,
    barsj)
```

Obtains the dual solution for a semidefinite variable.

#### Arguments

```
\begin{array}{l} {\rm barsj: \ double[]} \\ {\rm \ Value \ of \ } \bar{s}_j. \\ {\rm j: \ int} \\ {\rm \ Index \ of \ the \ semidefinite \ variable.} \\ {\rm whichsol: \ soltype} \\ {\rm \ Selects \ a \ solution.} \end{array}
```

#### Description:

Obtains the dual solution for a semidefinite variable.

## A.2.40 Task.getbarvarname()

```
name = Task.getbarvarname(i)
```

Obtains a name of a semidefinite variable.

#### Arguments

i : int

Index.

name : str

The requested name is copied to this buffer.

Description:

Obtains a name of a semidefinite variable.

See also

• Task.getbarvarnamelen Obtains the length of a name of a semidefinite variable.

## A.2.41 Task.getbarvarnameindex()

```
asgn,index = Task.getbarvarnameindex(somename)
```

Obtains the index of name of semidefinite variable.

Arguments

asgn: int

Is non-zero if the name somename is assigned to a semidefinite variable.

index : int

If the name somename is assigned to a semidefinite variable, then index is the name of the constraint.

somename : str

The requested name is copied to this buffer.

Description:

Obtains the index of name of semidefinite variable.

See also

• Task.getbarvarname Obtains a name of a semidefinite variable.

## A.2.42 Task.getbarvarnamelen()

```
len = Task.getbarvarnamelen(i)
```

Obtains the length of a name of a semidefinite variable.

### Arguments

```
i : int
    Index.
len : int
```

Returns the length of the indicated name.

### Description:

Obtains the length of a name of a semidefinite variable.

See also

• Task.getbarvarname Obtains a name of a semidefinite variable.

## A.2.43 Task.getbarxj()

```
Task.getbarxj(
    whichsol,
    j,
    barxj)
```

Obtains the primal solution for a semidefinite variable.

#### Arguments

```
{f barxj: double[]}
{f Value \ of \ ar{X}_j.}
{f j: int}
{f Index \ of \ the \ semidefinite \ variable.}}
{f which sol: \ soltype}
{f Selects \ a \ solution.}
```

### Description:

Obtains the primal solution for a semidefinite variable.

## A.2.44 Task.getbound()

```
bk,bl,bu = Task.getbound(
    accmode,
    i)
```

Obtains bound information for one constraint or variable.

#### Arguments

#### accmode : accmode

Defines if operations are performed row-wise (constraint-oriented) or column-wise (variable-oriented).

bk : boundkey

Bound keys.

bl : double

Values for lower bounds.

bu : double

Values for upper bounds.

i : int

Index of the constraint or variable for which the bound information should be obtained.

### Description:

Obtains bound information for one constraint or variable.

## A.2.45 Task.getboundslice()

```
Task.getboundslice(
    accmode,
    first,
    last,
    bk,
    bl,
    bu)
```

Obtains bounds information for a sequence of variables or constraints.

### Arguments

#### accmode : accmode

Defines if operations are performed row-wise (constraint-oriented) or column-wise (variable-oriented).

bk : mosek.boundkey[]

Bound keys.

bl : double[]

Values for lower bounds.

bu : double[]

Values for upper bounds.

first : int

First index in the sequence.

last: int

Last index plus 1 in the sequence.

## Description:

Obtains bounds information for a sequence of variables or constraints.

## A.2.46 Task.getc()

Task.getc(c)

Obtains all objective coefficients.

Arguments

#### c : double[]

Linear terms of the objective as a dense vector. The lengths is the number of variables.

Description:

Obtains all objective coefficients c.

## A.2.47 Task.getcfix()

```
cfix = Task.getcfix()
```

Obtains the fixed term in the objective.

Arguments

cfix : double

Fixed term in the objective.

Description:

Obtains the fixed term in the objective.

## A.2.48 Task.getcj()

```
cj = Task.getcj(j)
```

Obtains one coefficient of c.

### Arguments

cj : double

The value of  $c_j$ .

j: int

Index of the variable for which c coefficient should be obtained.

### Description:

Obtains one coefficient of c.

See also

• Task.getcslice Obtains a sequence of coefficients from the objective.

## A.2.49 Task.getconbound()

```
bk,bl,bu = Task.getconbound(i)
```

Obtains bound information for one constraint.

### Arguments

bk: boundkey

Bound keys.

bl : double

Values for lower bounds.

bu : double

Values for upper bounds.

i: int

Index of the constraint for which the bound information should be obtained.

## Description:

Obtains bound information for one constraint.

## A.2.50 Task.getconboundslice()

```
Task.getconboundslice(
    first,
    last,
    bk,
    bl,
    bu)
```

Obtains bounds information for a slice of the constraints.

### Arguments

```
bk : mosek.boundkey[]
    Bound keys.
bl : double[]
    Values for lower bounds.
bu : double[]
    Values for upper bounds.
first : int
    First index in the sequence.
last : int
    Last index plus 1 in the sequence.
```

## Description:

Obtains bounds information for a slice of the constraints.

## A.2.51 Task.getcone()

```
conetype,conepar,nummem = Task.getcone(
    k,
    submem)
```

Obtains a conic constraint.

## Arguments

```
conepar : double
   This argument is currently not used. Can be set to 0.0.
conetype : conetype
   Specifies the type of the cone.
```

#### k: int

Index of the cone constraint.

nummem : int

Number of member variables in the cone.

submem : int[]

Variable subscripts of the members in the cone.

### Description:

Obtains a conic constraint.

## A.2.52 Task.getconeinfo()

```
conetype,conepar,nummem = Task.getconeinfo(k)
```

Obtains information about a conic constraint.

### Arguments

conepar : double

This argument is currently not used. Can be set to 0.0.

conetype : conetype

Specifies the type of the cone.

k : int

Index of the conic constraint.

nummem : int

Number of member variables in the cone.

### Description:

Obtains information about a conic constraint.

### A.2.53 Task.getconename()

```
name = Task.getconename(i)
```

Obtains a name of a cone.

## Arguments

i : int

Index.

#### name: str

Is assigned the required name.

### Description:

Obtains a name of a cone.

See also

• Task.getconnamelen Obtains the length of a name of a constraint variable.

### A.2.54 Task.getconenameindex()

```
asgn,index = Task.getconenameindex(somename)
```

Checks whether the name somename has been assigned to any cone.

### Arguments

#### asgn: int

Is non-zero if the name somename is assigned to a cone.

#### index : int

If the name somename is assigned to a cone, then index is the name of the cone.

#### somename : str

The name which should be checked.

### Description:

Checks whether the name somename has been assigned to any cone. If it has been assigned to cone, then index of the cone is reported.

## A.2.55 Task.getconenamelen()

```
len = Task.getconenamelen(i)
```

Obtains the length of a name of a cone.

#### Arguments

#### i: int

Index.

#### len: int

Returns the length of the indicated name.

#### Description:

Obtains the length of a name of a cone.

See also

• Task.getbarvarname Obtains a name of a semidefinite variable.

## A.2.56 Task.getconname()

```
name = Task.getconname(i)
```

Obtains a name of a constraint.

Arguments

```
i: int Index.
```

name : str

Is assigned the required name.

Description:

Obtains a name of a constraint.

See also

• Task.getconnamelen Obtains the length of a name of a constraint variable.

## A.2.57 Task.getconnameindex()

```
asgn,index = Task.getconnameindex(somename)
```

Checks whether the name somename has been assigned to any constraint.

## Arguments

```
asgn: int
```

Is non-zero if the name somename is assigned to a constraint.

index : int

If the name somename is assigned to a constraint, then index is the name of the constraint.

somename : str

The name which should be checked.

### Description:

Checks whether the name somename has been assigned to any constraint. If it has been assigned to constraint, then index of the constraint is reported.

## A.2.58 Task.getconnamelen()

```
len = Task.getconnamelen(i)
```

Obtains the length of a name of a constraint variable.

### Arguments

i : int
 Index.
len : int

Returns the length of the indicated name.

### Description:

Obtains the length of a name of a constraint variable.

See also

• Task.getbarvarname Obtains a name of a semidefinite variable.

## A.2.59 Task.getcslice()

```
Task.getcslice(
    first,
    last,
    c)
```

Obtains a sequence of coefficients from the objective.

#### Arguments

```
c : double[]
```

Linear terms of the objective as a dense vector. The lengths is the number of variables.

first : int

First index in the sequence.

last : int

Last index plus 1 in the sequence.

### Description:

Obtains a sequence of elements in c.

### A.2.60 Task.getdbi()

```
Task.getdbi(
    whichsol,
    accmode,
    sub,
    dbi)
```

Deprecated.

### Arguments

accmode : accmode

If set to accmode.con then sub contains constraint indexes, otherwise variable indexes.

dbi : double[]

Dual bound infeasibility. If acmode is accmode.con then

$$\mathtt{dbi}[i] = \max(-(s_l^c)_{\mathtt{sub}[i]}, -(s_u^c)_{\mathtt{sub}[i]}, 0)$$
 for  $i = 0, \dots, \mathtt{len} - \mathtt{1}$ 

else

$$\mathtt{dbi}[i] = \max(-(s^x_l)_{\mathtt{sub}[i]}, -(s^x_u)_{\mathtt{sub}[i]}, 0) \ \text{for} \ i = 0, \dots, \mathtt{len} - \mathbf{1}.$$

sub : int[]

Indexes of constraints or variables.

whichsol: soltype Selects a solution.

Description:

Deprecated.

Obtains the dual bound infeasibility.

### A.2.61 Task.getdcni()

```
Task.getdcni(
    whichsol,
    sub,
    dcni)
```

Deprecated.

Arguments

```
dcni : double[]
   dcni[i] contains dual cone infeasibility for the cone with index sub[i].
sub : int[]
   Constraint indexes to calculate equation infeasibility for.
whichsol : soltype
   Selects a solution.
```

### Description:

Deprecated.

Obtains the dual cone infeasibility.

## A.2.62 Task.getdeqi()

```
Task.getdeqi(
    whichsol,
    accmode,
    sub,
    deqi,
    normalize)
```

Deprecated.

### Arguments

```
accmode : accmode
```

If set to accmode.con the dual equation infeasibilities corresponding to constraints are retrieved. Otherwise for a variables.

```
deqi : double[]
```

Dual equation infeasibilities corresponding to constraints or variables.

```
normalize : int
```

If non-zero, normalize with largest absolute value of the input data used to compute the individual infeasibility.

```
sub : int[]
```

Indexes of constraints or variables.

```
whichsol: soltype Selects a solution.
```

### Description:

Deprecated.

Optains the dual equation infeasibility. If acmode is accmode.con then

$$\mathtt{pbi}[i] = \left| (-y + s_l^c - s_u^c)_{\mathtt{sub}[i]} \right| \text{ for } i = 0, \dots, \mathtt{len} - \mathtt{1}$$

If acmode is accmode.var then

$$\mathtt{pbi}[i] = \left| (A^Ty + s^x_l - s^x_u - c)_{\mathtt{sub}[i]} \right| \ \ \mathtt{for} \ \ i = 0, \dots, \mathtt{len-1}$$

## A.2.63 Task.getdimbarvarj()

dimbarvarj = Task.getdimbarvarj(j)

Obtains the dimension of a symmetric matrix variable.

#### Arguments

dimbarvarj : int

The dimension of the j'th semidefinite variable.

j: int

Index of the semidefinite variable whose dimension is requested.

### Description:

Obtains the dimension of a symmetric matrix variable.

## A.2.64 Task.getdouinf()

dvalue = Task.getdouinf(whichdinf)

Obtains a double information item.

#### Arguments

dvalue : double

The value of the required double information item.

whichdinf : dinfitem

A double float information item.

### Description:

Obtains a double information item from the task information database.

## A.2.65 Task.getdouparam()

```
parvalue = Task.getdouparam(param)
```

Obtains a double parameter.

#### Arguments

```
param : dparam
    Which parameter.
parvalue : double
    Parameter value.
```

#### Description:

Obtains the value of a double parameter.

## A.2.66 Task.getdualobj()

```
dualobj = Task.getdualobj(whichsol)
```

Computes the dual objective value associated with the solution.

### Arguments

```
dualobj : double
   Objective value corresponding to the dual solution.
whichsol : soltype
   Selects a solution.
```

### Description:

Computes the dual objective value associated with the solution. Note if the solution is a primal infeasibility certificate, then the fixed term in the objective value is not included.

### A.2.67 Task.getdviolbarvar()

```
Task.getdviolbarvar(
    whichsol,
    sub,
    viol)
```

Computes the violation of dual solution for a set of barx variables.

#### Arguments

sub : int[]

An array of indexes of  $\bar{X}$  variables.

viol : double[]

viol[k] is violation of the solution for the constraint  $\bar{S}_{sub[k]} \in \mathcal{S}$ .

whichsol: soltype Selects a solution.

### Description:

Let  $(\bar{S}_j)^*$  be the value of variable  $\bar{S}_j$  for the specified solution. Then the dual violation of the solution associated with variable  $\bar{S}_j$  is given by

$$\max(-\lambda_{\min}(\bar{S}_j), 0.0).$$

Both when the solution is a certificate of primal infeasibility or when it is dual feasibile solution the violation should be small.

## A.2.68 Task.getdviolcon()

```
Task.getdviolcon(
    whichsol,
    sub,
    viol)
```

Computes the violation of a dual solution associated with a set of constraints.

## Arguments

sub : int[]

An array of indexes of constraints.

viol : double[]

viol[k] is the violation of dual solution associated with the constraint sub[k].

whichsol: soltype
Selects a solution.

#### Description:

The violation of the dual solution associated with the i'th constraint is computed as follows

$$\max(\rho((s_l^c)_i^*, (b_l^c)_i), \rho((s_u^c)_i^*, -(b_u^c)_i), |-y_i + (s_l^c)_i^* - (s_u^c)_i^*|)$$

where

$$\rho(x,l) = \begin{cases} -x, & l > -\infty, \\ |x|, & \text{otherwise} \end{cases}$$

Both when the solution is a certificate of primal infeasibility or it is a dual feasibible solution the violation should be small.

## A.2.69 Task.getdviolcones()

```
Task.getdviolcones(
    whichsol,
    sub,
    viol)
```

Computes the violation of a solution for set of dual conic constraints.

#### Arguments

sub : int[]

An array of indexes of  $\bar{X}$  variables.

viol : double[]

viol[k] violation of the solution associated with sub[k]'th dual conic constraint.

whichsol: soltype
Selects a solution.

### Description:

Let  $(s_n^x)^*$  be the value of variable  $(s_n^x)$  for the specified solution. For simplicity let us assume that  $s_n^x$  is a member of quadratic cone, then the violation is computed as follows

$$\left\{ \begin{array}{ll} \max(0, \|(s_n^x)_{2;n}\|^* - (s_n^x)_1^*)/\sqrt{2}, & (s_n^x)^* \geq - \|(s_n^x)_{2:n}^*\|\,, \\ \|(s_n^x)^*\|\,, & \text{otherwise.} \end{array} \right.$$

Both when the solution is a certificate of primal infeasibility or when it is a dual feasibile solution the violation should be small.

### A.2.70 Task.getdviolvar()

```
Task.getdviolvar(
    whichsol,
    sub,
    viol)
```

Computes the violation of a dual solution associated with a set of x variables.

#### Arguments

sub : int[]

An array of indexes of x variables.

viol : double[]

viol[k] is the maximal violation of the solution for the constraints  $(s_l^x)_{\text{sub}[k]} \geq 0$  and  $(s_u^x)_{\text{sub}[k]} \geq 0$ .

whichsol: soltype
Selects a solution.

#### Description:

The violation fo dual solution associated with the j'th variable is computed as follows

$$\max(\rho((s_l^x)_i^*, (b_l^x)_i), \rho((s_u^x)_i^*, -(b_u^x)_i), |\sum j = 0^{numcon-1} a_{ij} y_i + (s_l^x)_i^* - (s_u^x)_i^* - \tau c_j|)$$

where

$$\rho(x,l) = \begin{cases} -x, & l > -\infty, \\ |x|, & \text{otherwise} \end{cases}$$

 $\tau=0$  if the the solution is certificate of dual infeasibility and  $\tau=1$  otherwise. The formula for computing the violation is only shown for linear case but is generalized approxiately for the more general problems.

### A.2.71 Task.getinfeasiblesubproblem()

inftask = Task.getinfeasiblesubproblem(whichsol)

Obtains an infeasible sub problem.

### Arguments

inftask : Task

A new task containing the infeasible subproblem.

whichsol : soltype

Which solution to use when determining the infeasible subproblem.

### Description:

Given the solution is a certificate of primal or dual infeasibility then a primal or dual infeasible subproblem is obtained respectively. The subproblem tend to be much smaller than the original problem and hence it easier to locate the infeasibility inspecting the subproblem than the original problem.

For the procedure to be useful then it is important to assigning meaningful names to constraints, variables etc. in the original task because those names will be duplicated in the subproblem.

The function is only applicable to linear and conic quadrtic optimization problems.

For more information see Section 13.2.

See also

- iparam.infeas\_prefer\_primal Controls which certificate is used if both primal- and dual-certificate of infeasibility is available.
- Task.relaxprimal Deprecated.

## A.2.72 Task.getinti()

```
Task.getinti(
    whichsol,
    sub,
    inti)
```

Deprecated.

Arguments

```
inti : double[]
```

inti[i] contains integer infeasibility of variable sub[i].

sub : int[]

Variable indexes for which to calculate the integer infeasibility.

whichsol: soltype Selects a solution.

Description:

Deprecated.

Obtains the primal equation infeasibility.

$$peqi[i] = |(Ax - x^c)_{sub[i]}|$$
 for  $i = 0, ..., len - 1$ .

### A.2.73 Task.getintinf()

```
ivalue = Task.getintinf(whichiinf)
```

Obtains an integer information item.

#### Arguments

```
ivalue : int
```

The value of the required integer information item.

whichiinf : iinfitem

Specifies an information item.

#### Description:

Obtains an integer information item from the task information database.

## A.2.74 Task.getintparam()

```
parvalue = Task.getintparam(param)
```

Obtains an integer parameter.

#### Arguments

```
param: iparam
Which parameter.

parvalue: int
Parameter value.
```

### Description:

Obtains the value of an integer parameter.

## A.2.75 Task.getlenbarvarj()

```
lenbarvarj = Task.getlenbarvarj(j)
```

Obtains the length if the j'th semidefinite variables.

## Arguments

## j : int

Index of the semidefinite variable whose length if requested.

```
lenbarvarj : long
```

Number of scalar elements in the lower triangular part of the semidefinite variable.

## Description:

Obtains the length of the jth semidefinite variable i.e. the number of elements in the triangular part.

## A.2.76 Task.getlintinf()

```
ivalue = Task.getlintinf(whichliinf)
```

Obtains an integer information item.

#### Arguments

ivalue : long

The value of the required integer information item.

whichliinf : liinfitem

Specifies an information item.

#### Description:

Obtains an integer information item from the task information database.

### A.2.77 Task.getmaxnumanz()

```
maxnumanz = Task.getmaxnumanz()
```

Obtains number of preallocated non-zeros in the linear constraint matrix.

#### Arguments

```
maxnumanz : long
```

Number of preallocated non-zero linear matrix elements.

### Description:

Obtains number of preallocated non-zeros in A. When this number of non-zeros is reached MOSEK will automatically allocate more space for A.

## A.2.78 Task.getmaxnumbarvar()

```
maxnumbarvar = Task.getmaxnumbarvar()
```

Obtains the number of semidefinite variables.

### Arguments

```
maxnumbarvar : int
```

Obtains maximum number of semidefinite variable currently allowed.

#### Description:

Obtains the number of semidefinite variables.

### A.2.79 Task.getmaxnumcon()

```
maxnumcon = Task.getmaxnumcon()
```

Obtains the number of preallocated constraints in the optimization task.

#### Arguments

```
maxnumcon : int
```

Number of preallocated constraints in the optimization task.

#### Description:

Obtains the number of preallocated constraints in the optimization task. When this number of constraints is reached MOSEK will automatically allocate more space for constraints.

### A.2.80 Task.getmaxnumcone()

```
maxnumcone = Task.getmaxnumcone()
```

Obtains the number of preallocated cones in the optimization task.

#### Arguments

```
maxnumcone : int
```

Number of preallocated conic constraints in the optimization task.

### Description:

Obtains the number of preallocated cones in the optimization task. When this number of cones is reached MOSEK will automatically allocate space for more cones.

### A.2.81 Task.getmaxnumqnz()

```
maxnumqnz = Task.getmaxnumqnz()
```

Obtains the number of preallocated non-zeros for all quadratic terms in objective and constraints.

### Arguments

```
maxnumqnz : long
```

Number of non-zero elements preallocated in quadratic coefficient matrixes.

### Description:

Obtains the number of preallocated non-zeros for Q (both objective and constraints). When this number of non-zeros is reached MOSEK will automatically allocate more space for Q.

## A.2.82 Task.getmaxnumvar()

```
maxnumvar = Task.getmaxnumvar()
```

Obtains the maximum number variables allowed.

#### Arguments

```
maxnumvar : int
```

Number of preallocated variables in the optimization task.

#### Description:

Obtains the number of preallocated variables in the optimization task. When this number of variables is reached MOSEK will automatically allocate more space for constraints.

## A.2.83 Task.getmemusage()

```
meminuse,maxmemuse = Task.getmemusage()
```

Obtains information about the amount of memory used by a task.

#### Arguments

maxmemuse : long

Maximum amount of memory used by the task until now.

meminuse : long

Amount of memory currently used by the task.

#### Description:

Obtains information about the amount of memory used by a task.

### A.2.84 Task.getnumanz()

```
numanz = Task.getnumanz()
```

Obtains the number of non-zeros in the coefficient matrix.

### Arguments

```
numanz : int
```

Number of non-zero elements in the linear constraint matrix.

#### Description:

Obtains the number of non-zeros in A.

### A.2.85 Task.getnumanz64()

```
numanz = Task.getnumanz64()
```

Obtains the number of non-zeros in the coefficient matrix.

Arguments

numanz : long

Number of non-zero elements in the linear constraint matrix.

Description:

Obtains the number of non-zeros in A.

### A.2.86 Task.getnumbarablocktriplets()

```
num = Task.getnumbarablocktriplets()
```

Obtains an upper bound on the number of scalar elements in the block triplet form of bara.

Arguments

num : long

Number elements in the block triplet form of  $\bar{A}$ .

Description:

Obtains an upper bound on the number of elements in the block triplet form of  $\bar{A}$ .

### A.2.87 Task.getnumbaranz()

```
nz = Task.getnumbaranz()
```

Get the number of nonzero elements in barA.

Arguments

nz: long

The number of nonzero elements in  $\bar{A}$  i.e. the number of  $\bar{a}_{ij}$  elements that is nonzero.

Description:

Get the number of nonzero elements in  $\bar{A}$ .

## A.2.88 Task.getnumbarcblocktriplets()

```
num = Task.getnumbarcblocktriplets()
```

Obtains an upper bound on the number of elements in the block triplet form of barc.

#### Arguments

```
num : long
```

An upper bound on the number elements in the block trip let form of  $\bar{c}$ .

#### Description:

Obtains an upper bound on the number of elements in the block triplet form of  $\bar{C}$ .

## A.2.89 Task.getnumbarcnz()

```
nz = Task.getnumbarcnz()
```

Obtains the number of nonzero elements in barc.

### Arguments

```
nz : long
```

The number of nonzeros in  $\bar{c}$  i.e. the number of elements  $\bar{c}_i$  that is diffrent from 0.

### Description:

Obtains the number of nonzero elements in  $\bar{c}$ .

### A.2.90 Task.getnumbarvar()

```
numbarvar = Task.getnumbarvar()
```

Obtains the number of semidefinite variables.

### Arguments

```
numbarvar : int
```

Number of semidefinite variable in the problem.

#### Description:

Obtains the number of semidefinite variables.

## A.2.91 Task.getnumcon()

```
numcon = Task.getnumcon()
```

Obtains the number of constraints.

Arguments

numcon: int

Number of constraints.

Description:

Obtains the number of constraints.

## A.2.92 Task.getnumcone()

```
numcone = Task.getnumcone()
```

Obtains the number of cones.

Arguments

numcone : int

Number conic constraints.

Description:

Obtains the number of cones.

### A.2.93 Task.getnumconemem()

```
nummem = Task.getnumconemem(k)
```

Obtains the number of members in a cone.

Arguments

k: int

Index of the cone.

nummem : int

Number of member variables in the cone.

Description:

Obtains the number of members in a cone.

## A.2.94 Task.getnumintvar()

```
numintvar = Task.getnumintvar()
```

Obtains the number of integer-constrained variables.

Arguments

```
numintvar : int
```

Number of integer variables.

Description:

Obtains the number of integer-constrained variables.

## A.2.95 Task.getnumparam()

```
numparam = Task.getnumparam(partype)
```

Obtains the number of parameters of a given type.

Arguments

numparam : int

Identical to the number of parameters of the type partype.

partype : parametertype

Parameter type.

Description:

Obtains the number of parameters of a given type.

## A.2.96 Task.getnumqconknz()

```
numqcnz = Task.getnumqconknz(k)
```

Obtains the number of non-zero quadratic terms in a constraint.

Arguments

#### k : int

Index of the constraint for which the number of non-zero quadratic terms should be obtained.

```
numqcnz : int
```

Number of quadratic terms.

### Description:

Obtains the number of non-zero quadratic terms in a constraint.

## A.2.97 Task.getnumqconknz64()

```
numqcnz = Task.getnumqconknz64(k)
```

Obtains the number of non-zero quadratic terms in a constraint.

### Arguments

#### k: int

Index of the constraint for which the number quadratic terms should be obtained.

### numqcnz : long

Number of quadratic terms.

### Description:

Obtains the number of non-zero quadratic terms in a constraint.

## A.2.98 Task.getnumqobjnz()

```
numqonz = Task.getnumqobjnz()
```

Obtains the number of non-zero quadratic terms in the objective.

#### Arguments

```
numqonz : long
```

Number of non-zero elements in the quadratic objective terms.

### Description:

Obtains the number of non-zero quadratic terms in the objective.

### A.2.99 Task.getnumsymmat()

```
num = Task.getnumsymmat()
```

Get the number of symmetric matrixes stored.

Arguments

num : long

Returns the number of symmetric sparse matrixes.

Description:

Get the number of symmetric matrixes stored in the vector E.

## A.2.100 Task.getnumvar()

```
numvar = Task.getnumvar()
```

Obtains the number of variables.

Arguments

numvar : int

Number of variables.

Description:

Obtains the number of variables.

## A.2.101 Task.getobjname()

```
objname = Task.getobjname()
```

Obtains the name assigned to the objective function.

Arguments

objname : str

Assigned the objective name.

Description:

Obtains the name assigned to the objective function.

## A.2.102 Task.getobjnamelen()

```
len = Task.getobjnamelen()
```

Obtains the length of the name assigned to the objective function.

Arguments

```
len: int
```

Assigned the length of the objective name.

Description:

Obtains the length of the name assigned to the objective function.

## A.2.103 Task.getobjsense()

```
sense = Task.getobjsense()
```

Gets the objective sense.

Arguments

```
sense : objsense
```

The returned objective sense.

Description:

Gets the objective sense of the task.

See also

• Task.putobjsense Sets the objective sense.

## A.2.104 Task.getpbi()

```
Task.getpbi(
    whichsol,
    accmode,
    sub,
    pbi,
    normalize)
```

Deprecated.

#### Arguments

accmode : accmode

If set to accmode.var return bound infeasibility for x otherwise for  $x^c$ .

normalize : int

If non-zero, normalize with largest absolute value of the input data used to compute the individual infeasibility.

pbi : double[]

Bound infeasibility for x or  $x^c$ .

sub : int[]

An array of constraint or variable indexes.

whichsol: soltype Selects a solution.

#### Description:

Deprecated.

Obtains the primal bound infeasibility. If acmode is accmode.con then

$$\mathtt{pbi}[i] = \max(x^c_{\mathtt{sub[i]}} - u^c_{\mathtt{sub[i]}}, l^c_{\mathtt{sub[i]}} - x^c_{\mathtt{sub[i]}}, 0) \text{ for } i = 0, \dots, \mathtt{len-1}$$

If acmode is accmode.var then

$$\mathtt{pbi}[i] = \max(x_{\mathtt{sub[i]}} - u^x_{\mathtt{sub[i]}}, l^x_{\mathtt{sub[i]}} - x_{\mathtt{sub[i]}}, 0) \text{ for } i = 0, \dots, \mathtt{len-1}$$

## A.2.105 Task.getpcni()

```
Task.getpcni(
    whichsol,
    sub,
    pcni)
```

Deprecated.

### Arguments

pcni : double[]

pcni[i] contains primal cone infeasibility for the cone with index sub[i].

sub : int[

Constraint indexes for which to calculate the equation infeasibility.

whichsol: soltype
Selects a solution.

### Description:

Deprectaed.

## A.2.106 Task.getpeqi()

```
Task.getpeqi(
    whichsol,
    sub,
    peqi,
    normalize)
```

Deprecated.

#### Arguments

#### normalize : int

If non-zero, normalize with largest absolute value of the input data used to compute the individual infeasibility.

peqi : double[]

peqi[i] contains equation infeasibility of constraint sub[i].

sub : int[]

Constraint indexes for which to calculate the equation infeasibility.

whichsol: soltype Selects a solution.

#### Description:

Deprecated.

Obtains the primal equation infeasibility.

$$peqi[i] = |(Ax - x^c)_{sub[i]}|$$
 for  $i = 0, ..., len - 1$ .

## A.2.107 Task.getprimalobj()

```
primalobj = Task.getprimalobj(whichsol)
```

Computes the primal objective value for the desired solution.

### Arguments

primalobj : double

Objective value corresponding to the primal solution.

whichsol: soltype Selects a solution.

### Description:

Computes the primal objective value for the desired solution. Note if the solution is an infeasibility certificate, then the fixed term in the objective is not included.

## A.2.108 Task.getprobtype()

# A.2.109 Task.getprosta()

Obtains the problem type.

```
prosta = Task.getprosta(whichsol)
```

Obtains the problem status.

Arguments

Description:

```
prosta : prosta
    Problem status.
whichsol : soltype
    Selects a solution.
```

Description:

Obtains the problem status.

## A.2.110 Task.getpviolbarvar()

```
Task.getpviolbarvar(
    whichsol,
    sub,
    viol)
```

Computes the violation of a primal solution for a list of barx variables.

Arguments

sub : int[]

An array of indexes of  $\bar{X}$  variables.

viol : double[]

viol[k] is how much the solution violate the constraint  $\bar{X}_{sub[k]} \in \mathcal{S}^+$ .

whichsol: soltype
Selects a solution.

#### Description:

Let  $(\bar{X}_j)^*$  be the value of variable  $\bar{X}_j$  for the specified solution. Then the primal violation of the solution associated with variable  $\bar{X}_j$  is given by

$$\max(-\lambda_{\min}(\bar{X}_j), 0.0).$$

# A.2.111 Task.getpviolcon()

```
Task.getpviolcon(
    whichsol,
    sub,
    viol)
```

Computes the violation of a primal solution for a list of xc variables.

#### Arguments

sub : int[]

An array of indexes of constraints.

viol : double[]

viol[k] associated with the solution for the sub[k]'th constraint.

whichsol: soltype Selects a solution.

## Description:

The primal violation of the solution associated of constraint is computed by

$$\max(l_i^c \tau - (x_i^c)^*), (x_i^c)^* \tau - u_i^c \tau, |\sum_{j=0}^{numvar-1} a_{ij} x_j^* - x_i^c|)$$

where  $\tau$  is defined as follows. If the solution is a certificate of dual infeasibility, then  $\tau=0$  and otherwise  $\tau=1$ . Both when the solution is a valid certificate of dual infeasibility or when it is primal feasibile solution the violation should be small. The above is only shown for linear case but is appropriately generalized for the other cases.

# A.2.112 Task.getpviolcones()

```
Task.getpviolcones(
    whichsol,
    sub,
    viol)
```

Computes the violation of a solution for set of conic constraints.

#### Arguments

```
sub : int[]
```

An array of indexes of  $\bar{X}$  variables.

viol : double[]

viol[k] violation of the solution associated with sub[k]'th conic constraint.

whichsol: soltype Selects a solution.

## Description:

Let  $x^*$  be the value of variable x for the specified solution. For simplicity let us assume that x is a member of quadratic cone, then the violation is computed as follows

$$\begin{cases} \max(0, \|x_{2;n}\| - x_1) / \sqrt{2}, & x_1 \ge - \|x_{2:n}\|, \\ \|x\|, & \text{otherwise.} \end{cases}$$

Both when the solution is a certificate of dual infeasibility or when it is a primal feasibile solution the violation should be small.

# A.2.113 Task.getpviolvar()

```
Task.getpviolvar(
    whichsol,
    sub,
    viol)
```

Computes the violation of a primal solution for a list of x variables.

#### Arguments

sub : int[]

An array of indexes of x variables.

viol : double[]

viol[k] is the violation associated the solution for variable  $x_i$ .

whichsol: soltype Selects a solution.

#### Description:

Let  $x_j^*$  be the value of variable  $x_j$  for the specified solution. Then the primal violation of the solution associated with variable  $x_j$  is given by

$$\max(l_{j}^{x}\tau - x_{j}^{*}, x_{j}^{*} - u_{j}^{x}\tau).$$

where  $\tau$  is defined as follows. If the solution is a certificate of dual infeasibility, then  $\tau = 0$  and otherwise  $\tau = 1$ . Both when the solution is a valid certificate of dual infeasibility or when it is primal feasibile solution the violation should be small.

# A.2.114 Task.getqconk()

```
numqcnz = Task.getqconk(
    k,
    qcsubi,
    qcsubj,
    qcval)
```

Obtains all the quadratic terms in a constraint.

#### Arguments

### k : int

Which constraint.

numqcnz: long

Number of quadratic terms.

qcsubi : int[]

Row subscripts for quadratic constraint matrix.

qcsubj : int[]

Column subscripts for quadratic constraint matrix.

qcval : double[]

Quadratic constraint coefficient values.

#### Description:

Obtains all the quadratic terms in a constraint. The quadratic terms are stored sequentially qcsubi, qcsubj, and qcval.

# A.2.115 Task.getqobj()

```
numqonz = Task.getqobj(
    qosubi,
    qosubj,
    qoval)
```

Obtains all the quadratic terms in the objective.

## Arguments

```
numqonz : int
```

Number of non-zero elements in the quadratic objective terms.

qosubi : int[]

Row subscripts for quadratic objective coefficients.

qosubj : int[]

Column subscripts for quadratic objective coefficients.

qoval : double[]

Quadratic objective coefficient values.

#### Description:

Obtains the quadratic terms in the objective. The required quadratic terms are stored sequentially in qosubi, qosubj, and qoval.

# A.2.116 Task.getqobj64()

```
numqonz = Task.getqobj64(
    qosubi,
    qosubj,
    qoval)
```

Obtains all the quadratic terms in the objective.

## Arguments

numqonz : long

Number of non-zero elements in the quadratic objective terms.

qosubi : int[]

Row subscripts for quadratic objective coefficients.

qosubj : int[]

Column subscripts for quadratic objective coefficients.

```
qoval : double[]
```

Quadratic objective coefficient values.

### Description:

Obtains the quadratic terms in the objective. The required quadratic terms are stored sequentially in qosubi, qosubj, and qoval.

# A.2.117 Task.getqobjij()

```
qoij = Task.getqobjij(
    i,
    j)
```

Obtains one coefficient from the quadratic term of the objective

## Arguments

i : int

Row index of the coefficient.

j: int

Column index of coefficient.

qoij : double

The required coefficient.

## Description:

Obtains one coefficient  $q_{ij}^o$  in the quadratic term of the objective.

# A.2.118 Task.getreducedcosts()

```
Task.getreducedcosts(
    whichsol,
    first,
    last,
    redcosts)
```

Obtains the difference of (slx-sux) for a sequence of variables.

```
first: int See formula (A.1) for the definition.
```

last: int

See formula (A.1) for the definition.

redcosts : double[]

The reduced costs in the required sequence of variables are stored sequentially in redcosts starting at redcosts[0].

whichsol: soltype
Selects a solution.

## Description:

Computes the reduced costs for a sequence of variables and return them in the variable redcosts i.e.

$$redcosts[j-first] = (s_l^x)_j - (s_u^x)_j, \ j = first, \dots, last - 1.$$
(A.1)

# A.2.119 Task.getskc()

```
Task.getskc(
    whichsol,
    skc)
```

Obtains the status keys for the constraints.

#### Arguments

skc : mosek.stakey[]

Status keys for the constraints.

whichsol: soltype
Selects a solution.

# Description:

Obtains the status keys for the constraints.

See also

• Task.getskcslice Obtains the status keys for the constraints.

# A.2.120 Task.getskcslice()

```
Task.getskcslice(
   whichsol,
   first,
   last,
   skc)
```

Obtains the status keys for the constraints.

### Arguments

```
first : int
```

First index in the sequence.

last : int

Last index plus 1 in the sequence.

skc : mosek.stakey[]

Status keys for the constraints.

whichsol: soltype
Selects a solution.

## Description:

Obtains the status keys for the constraints.

See also

• Task.getskc Obtains the status keys for the constraints.

# A.2.121 Task.getskx()

```
Task.getskx(
    whichsol,
    skx)
```

Obtains the status keys for the scalar variables.

## Arguments

```
skx : mosek.stakey[]
```

Status keys for the variables.

whichsol: soltype
Selects a solution.

## Description:

Obtains the status keys for the scalar variables.

See also

• Task.getskxslice Obtains the status keys for the variables.

# A.2.122 Task.getskxslice()

```
Task.getskxslice(
    whichsol,
    first,
    last,
    skx)
```

Obtains the status keys for the variables.

## Arguments

```
first : int
   First index in the sequence.
last : int
   Last index plus 1 in the sequence.
skx : mosek.stakey[]
   Status keys for the variables.
whichsol : soltype
   Selects a solution.
```

# Description:

Obtains the status keys for the variables.

# A.2.123 Task.getslc()

```
Task.getslc(
    whichsol,
    slc)
```

Obtains the slc vector for a solution.

## Arguments

```
\begin{array}{ll} {\rm slc} \ : \ {\rm double} \ [] \\ {\rm The} \ s_l^c \ {\rm vector}. \\ \\ {\rm whichsol} \ : \ {\rm soltype} \\ {\rm Selects} \ {\rm a \ solution}. \end{array}
```

#### Description:

Obtains the  $s_l^c$  vector for a solution.

See also

• Task.getslcslice Obtains a slice of the slc vector for a solution.

# A.2.124 Task.getslcslice()

```
Task.getslcslice(
   whichsol,
   first,
   last,
   slc)
```

Obtains a slice of the slc vector for a solution.

## Arguments

```
first : int
    First index in the sequence.
last : int
    Last index plus 1 in the sequence.
slc : double[]
    Dual variables corresponding to the lower bounds on the constraints.
whichsol : soltype
```

Selects a solution.

# Description:

Obtains a slice of the  $s_l^c$  vector for a solution.

See also

• Task.getslc Obtains the slc vector for a solution.

# A.2.125 Task.getslx()

```
Task.getslx(
    whichsol,
    slx)
```

Obtains the slx vector for a solution.

```
slx : double[]
    The s_l^x vector.

whichsol : soltype
    Selects a solution.
```

## Description:

Obtains the  $s_l^x$  vector for a solution.

See also

• Task.getslx Obtains the slx vector for a solution.

# A.2.126 Task.getslxslice()

```
Task.getslxslice(
    whichsol,
    first,
    last,
    slx)
```

Obtains a slice of the slx vector for a solution.

## Arguments

```
first : int
```

First index in the sequence.

last : int

Last index plus 1 in the sequence.

slx : double[]

Dual variables corresponding to the lower bounds on the variables.

whichsol: soltype
Selects a solution.

# Description:

Obtains a slice of the  $s_l^x$  vector for a solution.

See also

 $\bullet$  Task.getslx Obtains the slx vector for a solution.

# A.2.127 Task.getsnx()

```
Task.getsnx(
    whichsol,
    snx)
```

Obtains the snx vector for a solution.

## Arguments

```
\operatorname{snx}: \operatorname{double}[]
\operatorname{The} s_n^x \operatorname{vector}.
\operatorname{whichsol}: \operatorname{soltype}
\operatorname{Selects} \operatorname{a} \operatorname{solution}.
```

## Description:

Obtains the  $s_n^x$  vector for a solution.

See also

• Task.getsnxslice Obtains a slice of the snx vector for a solution.

# A.2.128 Task.getsnxslice()

```
Task.getsnxslice(
    whichsol,
    first,
    last,
    snx)
```

Obtains a slice of the snx vector for a solution.

## Arguments

```
first : int
   First index in the sequence.
```

last: int

Last index plus 1 in the sequence.

snx : double[]

Dual variables corresponding to the conic constraints on the variables.

whichsol: soltype
Selects a solution.

## Description:

Obtains a slice of the  $s_n^x$  vector for a solution.

See also

• Task.getsnx Obtains the snx vector for a solution.

# A.2.129 Task.getsolsta()

```
solsta = Task.getsolsta(whichsol)
```

Obtains the solution status.

## Arguments

solsta : solsta
 Solution status.
whichsol : soltype
 Selects a solution.

# Description:

Obtains the solution status.

# A.2.130 Task.getsolution()

```
prosta,solsta = Task.getsolution(
    whichsol,
    skc,
    skx,
    skn,
    xc,
    xx,
    y,
    slc,
    suc,
    slx,
    sux,
    snx)
```

Obtains the complete solution.

```
prosta : prosta
    Problem status.

skc : mosek.stakey[]
    Status keys for the constraints.

skn : mosek.stakey[]
    Status keys for the conic constraints.

skx : mosek.stakey[]
    Status keys for the variables.
```

slc : double[]

Dual variables corresponding to the lower bounds on the constraints.

slx : double[]

Dual variables corresponding to the lower bounds on the variables.

snx : double[]

Dual variables corresponding to the conic constraints on the variables.

solsta : solsta

Solution status.

suc : double[]

Dual variables corresponding to the upper bounds on the constraints.

sux : double[]

Dual variables corresponding to the upper bounds on the variables.

whichsol : soltype

Selects a solution.

xc : double[]

Primal constraint solution.

xx : double[]

Primal variable solution.

y : double[]

Vector of dual variables corresponding to the constraints.

#### Description:

Obtains the complete solution.

Consider the case of linear programming. The primal problem is given by

and the corresponding dual problem is

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c \\ & + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x & = c, \\ & - y + s_l^c - s_u^c & = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0. \end{array}$$

In this case the mapping between variables and arguments to the function is as follows:

xx:

Corresponds to variable x.

```
y:
    Corresponds to variable y.

slc:
    Corresponds to variable s_l^c.

suc:
    Corresponds to variable s_u^c.

slx:
    Corresponds to variable s_u^x.

sux:
    Corresponds to variable s_u^x.

xc:
    Corresponds to Ax.
```

The meaning of the values returned by this function depend on the *solution status* returned in the argument solsta. The most important possible values of solsta are:

#### solsta.optimal

An optimal solution satisfying the optimality criteria for continuous problems is returned.

```
solsta.integer_optimal
```

An optimal solution satisfying the optimality criteria for integer problems is returned.

#### solsta.prim\_feas

A solution satisfying the feasibility criteria.

### solsta.prim\_infeas\_cer

A primal certificate of infeasibility is returned.

#### solsta.dual\_infeas\_cer

A dual certificate of infeasibility is returned.

See also

- Task.getsolutioni Obtains the solution for a single constraint or variable.
- Task.getsolutionslice Obtains a slice of the solution.

# A.2.131 Task.getsolutioni()

```
sk,x,sl,su,sn = Task.getsolutioni(
   accmode,
   i,
   whichsol)
```

Obtains the solution for a single constraint or variable.

#### Arguments

accmode : accmode

If set to accmode.con the solution information for a constraint is retrieved. Otherwise for a variable.

i: int

Index of the constraint or variable.

sk: stakey

Status key of the constraint of variable.

sl : double

Solution value of the dual variable associated with the lower bound.

sn : double

Solution value of the dual variable associated with the cone constraint.

su : double

Solution value of the dual variable associated with the upper bound.

whichsol : soltype

Selects a solution.

x : double

Solution value of the primal variable.

Description:

Obtains the primal and dual solution information for a single constraint or variable.

See also

- Task.getsolution Obtains the complete solution.
- Task.getsolutionslice Obtains a slice of the solution.

# A.2.132 Task.getsolutioninf()

prosta, solsta, primalobj, maxpbi, maxpcni, maxpeqi, maxinti, dualobj, maxdcni, maxdcni, maxdeqi = Task.getsolutioninf(whichsol)

Deprecated

Arguments

dualobj : double

Value of the dual objective.

$$(l^c)^T s_l^c - (u^c)^T s_u^c + c^f$$

maxdbi : double

Maximum infeasibility in bounds on dual variables.

$$\max\{0, \max_{i \in \{0, \dots, n-1\}} - (s_l^x)_i, \max_{i \in \{0, \dots, n-1\}} - (s_u^x)_i, \max_{i \in \{0, \dots, m-1\}} - (s_l^c)_i, \max_{i \in \{0, \dots, m-1\}} - (s_u^c)_i\}$$

maxdcni: double

Maximum infeasibility in the dual conic constraints.

maxdeqi : double

Maximum infeasibility in the dual equality constraints.

$$\max \left\{ \|A^T y + s_l^x - s_u^x - c\|_{\infty}, \|-y + s_l^c - s_u^c\|_{\infty} \right\}$$

maxinti : double

Maximum infeasibility in integer constraints.

$$\max_{i \in \{0,\dots,n-1\}} (\min(x_i - \lfloor x_i \rfloor, \lceil x_i \rceil - x_i)).$$

maxpbi : double

Maximum infeasibility in primal bounds on variables.

$$\max\{0, \max_{i \in 1, \dots, n-1}(x_i - u_i^x), \max_{i \in 1, \dots, n-1}(l_i^x - x_i), \max_{i \in 1, \dots, n-1}(x_i^c - u_i^c), \max_{i \in 1, \dots, n-1}(l_i^c - x_i^c)\}$$

maxpcni : double

Maximum infeasibility in the primal conic constraints.

maxpeqi : double

Maximum infeasibility in primal equality constraints.

$$||Ax - x^c||_{\infty}$$

primalobj : double

Value of the primal objective.

$$c^T x + c^f$$

prosta: prosta

Problem status.

solsta : solsta

Solution status.

whichsol : soltype

Selects a solution.

Description:

Deprecated. Use Task.getsolutioninfo instead.

## A.2.133 Task.getsolutioninfo()

pobj,pviolcon,pviolvar,pviolbarvar,pviolcone,pviolitg,dobj,dviolcon,dviolvar,dviolbarvar,dviolcone = Task.getsolutioninfo(wl

Obtains information about of a solution.

Arguments

dobj : double

Dual objective value as computed as computed by Task.getdualobj.

dviolbarvar : double

Maximal violation of the dual solution associated with the  $\bar{s}$  variable as computed by as computed by Task.getdviolbarvar.

dviolcon : double

Maximal violation of the dual solution associated with the  $x^c$  variable as computed by as computed by Task.getdviolcon.

dviolcone : double

Maximal violation of the dual solution associated with the dual conic constraints as computed by Task.getdviolcones.

dviolvar : double

Maximal violation of the dual solution associated with the x variable as computed by as computed by Task.getdviolvar.

pobj : double

The primal objective value as computed by Task.getprimalobj.

pviolbarvar : double

Maximal primal violation of solution for the  $\bar{X}$  variables where the violations are computed by Task.getpviolbarvar.

pviolcon : double

Maximal primal violation of the solution associated with the  $x^c$  variables where the violations are computed by Task.getpviolcon.

pviolcone : double

Maximal primal violation of solution for the conic constraints where the violations are computed by Task.getpviolcones.

pviolitg: double

Maximal violation in the integer constraints. The violation for an integer constrained variable  $x_i$  is given by

$$\min(x_i - |x_i|, \lceil x_i \rceil - x_i).$$

This number is always zero for the interior-point and the basic solutions.

pviolvar : double

Maximal primal violation of the solution for the  $x^x$  variables where the violations are computed by Task.getpviolvar.

whichsol: soltype Selects a solution.

#### Description:

Obtains information about a solution.

#### See also

- Task.getsolsta Obtains the solution status.
- Task.getprimalobj Computes the primal objective value for the desired solution.
- Task.getpviolcon Computes the violation of a primal solution for a list of xc variables.
- Task.getpviolvar Computes the violation of a primal solution for a list of x variables.
- Task.getpviolbarvar Computes the violation of a primal solution for a list of barx variables.
- Task.getpviolcones Computes the violation of a solution for set of conic constraints.
- Task.getdualobj Computes the dual objective value associated with the solution.
- Task.getdviolcon Computes the violation of a dual solution associated with a set of constraints.
- Task.getdviolvar Computes the violation of a dual solution associated with a set of x variables.
- Task.getdviolbarvar Computes the violation of dual solution for a set of barx variables.
- Task.getdviolcones Computes the violation of a solution for set of dual conic constraints.

## A.2.134 Task.getsolutionslice()

```
Task.getsolutionslice(
   whichsol,
   solitem,
   first,
   last,
   values)
```

Obtains a slice of the solution.

#### Arguments

#### first : int

Index of the first value in the slice.

last: int

Value of the last index+1 in the slice, e.g. if xx[5,...,9] is required last should be 10.

solitem : solitem

Which part of the solution is required.

values : double[]

The values in the required sequence are stored sequentially in values starting at values [0].

whichsol: soltype Selects a solution.

## Description:

Obtains a slice of the solution.

Consider the case of linear programming. The primal problem is given by

$$\begin{array}{lllll} \text{minimize} & & c^Tx+c^f \\ \text{subject to} & l^c & \leq & Ax & \leq & u^c, \\ & l^x & \leq & x & \leq & u^x. \end{array}$$

and the corresponding dual problem is

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c \\ & + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x & = c, \\ & - y + s_l^c - s_u^c & = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0. \end{array}$$

The solitem argument determines which part of the solution is returned:

#### solitem.xx:

The variable values return x.

#### solitem.y:

The variable values return y.

#### solitem.slc:

The variable values return  $s_I^c$ .

#### solitem.suc:

The variable values return  $s_u^c$ .

#### solitem.slx:

The variable values return  $s_l^x$ .

### solitem.sux:

The variable values return  $s_u^x$ .

A conic optimization problem has the same primal variables as in the linear case. Recall that the dual of a conic optimization problem is given by:

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c \\ & + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + s_n^x & = & c, \\ & - y + s_l^c - s_u^c & = & 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x & \geq & 0, \\ & s_n^x \in \mathcal{C}^* & \end{array}$$

This introduces one additional dual variable  $s_n^x$ . This variable can be acceded by selecting solitem as solitem.snx.

The meaning of the values returned by this function also depends on the *solution status* which can be obtained with Task.getsolsta. Depending on the solution status value will be:

#### solsta.optimal

A part of the optimal solution satisfying the optimality criteria for continuous problems.

#### solsta.integer\_optimal

A part of the optimal solution satisfying the optimality criteria for integer problems.

#### solsta.prim\_feas

A part of the solution satisfying the feasibility criteria.

#### solsta.prim\_infeas\_cer

A part of the primal certificate of infeasibility.

## solsta.dual\_infeas\_cer

A part of the dual certificate of infeasibility.

See also

- Task.getsolution Obtains the complete solution.
- Task.getsolutioni Obtains the solution for a single constraint or variable.

# A.2.135 Task.getsparsesymmat()

```
Task.getsparsesymmat(
    idx,
    subi,
    subj,
    valij)
```

Gets a single symmetric matrix from the matrix store.

#### Arguments

```
idx: long
```

Index of the matrix to get.

```
subi : int[]
```

Row subscripts of the matrix non-zero elements.

subj : int[]

Column subscripts of the matrix non-zero elements.

valij : double[]

Coefficients of the matrix non-zero elements.

#### Description:

Get a single symmetric matrix from the matrix store.

# A.2.136 Task.getstrparam()

```
len,parvalue = Task.getstrparam(param)
```

Obtains the value of a string parameter.

## Arguments

len : int

The length of the parameter value.

param : sparam
Which parameter.

parvalue : str

If this is not NULL, the parameter value is stored here.

## Description:

Obtains the value of a string parameter.

# A.2.137 Task.getstrparamlen()

```
len = Task.getstrparamlen(param)
```

Obtains the length of a string parameter.

#### Arguments

len : int

The length of the parameter value.

param : sparam
Which parameter.

# Description:

Obtains the length of a string parameter.

# A.2.138 Task.getsuc()

```
Task.getsuc(
    whichsol,
    suc)
```

Obtains the suc vector for a solution.

## Arguments

```
\operatorname{suc}:\operatorname{double}[]
\operatorname{The} s_u^c \operatorname{vector}.
\operatorname{whichsol}:\operatorname{soltype}
\operatorname{Selects} \operatorname{a} \operatorname{solution}.
```

# Description:

Obtains the  $s_u^c$  vector for a solution.

See also

• Task.getsucslice Obtains a slice of the suc vector for a solution.

# A.2.139 Task.getsucslice()

```
Task.getsucslice(
   whichsol,
   first,
   last,
   suc)
```

Obtains a slice of the suc vector for a solution.

```
first : int
    First index in the sequence.
last : int
    Last index plus 1 in the sequence.
suc : double[]
    Dual variables corresponding to the upper bounds on the constraints.
whichsol : soltype
    Selects a solution.
```

## Description:

Obtains a slice of the  $\boldsymbol{s}_{u}^{c}$  vector for a solution.

See also

• Task.getsuc Obtains the suc vector for a solution.

# A.2.140 Task.getsux()

```
Task.getsux(
    whichsol,
    sux)
```

Obtains the sux vector for a solution.

# Arguments

```
\begin{array}{lll} & \text{sux} : & \text{double[]} \\ & \text{The } s^x_u \text{ vector.} \\ \\ & \text{whichsol} : & \text{soltype} \\ & \text{Selects a solution.} \end{array}
```

Description:

Obtains the  $s_u^x$  vector for a solution.

See also

• Task.getsuxslice Obtains a slice of the sux vector for a solution.

# A.2.141 Task.getsuxslice()

```
Task.getsuxslice(
    whichsol,
    first,
    last,
    sux)
```

Obtains a slice of the sux vector for a solution.

# Arguments

```
first : int
```

First index in the sequence.

last: int

Last index plus 1 in the sequence.

sux : double[]

Dual variables corresponding to the upper bounds on the variables.

whichsol: soltype
Selects a solution.

## Description:

Obtains a slice of the  $s_u^x$  vector for a solution.

See also

• Task.getsux Obtains the sux vector for a solution.

# A.2.142 Task.getsymmatinfo()

```
dim,nz,type = Task.getsymmatinfo(idx)
```

Obtains information of a matrix from the symmetric matrix storage E.

## Arguments

dim : int

Returns the dimension of the requested matrix.

idx : long

Index of the matrix that is requested information about.

nz : long

Returns the number of non-zeros in the requested matrix.

type : symmattype

Returns the type of the requested matrix.

#### Description:

MOSEK maintains a vector denoted E of symmetric data matrixes. This function makes it possible to obtain important information about an data matrix in E.

# A.2.143 Task.gettaskname()

```
taskname = Task.gettaskname()
```

Obtains the task name.

## Arguments

```
taskname : str
```

Is assigned the task name.

Description:

Obtains the name assigned to the task.

# A.2.144 Task.gettasknamelen()

```
len = Task.gettasknamelen()
```

Obtains the length the task name.

Arguments

len: int

Returns the length of the task name.

Description:

Obtains the length the task name.

See also

• Task.getbarvarname Obtains a name of a semidefinite variable.

# A.2.145 Task.getvarbound()

```
bk,bl,bu = Task.getvarbound(i)
```

Obtains bound information for one variable.

Arguments

bk: boundkey

Bound keys.

bl : double

Values for lower bounds.

bu : double

Values for upper bounds.

i : int

Index of the variable for which the bound information should be obtained.

Description:

Obtains bound information for one variable.

# A.2.146 Task.getvarboundslice()

```
Task.getvarboundslice(
    first,
    last,
    bk,
    bl,
    bu)
```

Obtains bounds information for a slice of the variables.

# Arguments

```
bk : mosek.boundkey[]
    Bound keys.
bl : double[]
    Values for lower bounds.
bu : double[]
    Values for upper bounds.
first : int
    First index in the sequence.
last : int
    Last index plus 1 in the sequence.
```

## Description:

Obtains bounds information for a slice of the variables.

# A.2.147 Task.getvarbranchdir()

```
direction = Task.getvarbranchdir(j)
```

Obtains the branching direction for a variable.

## Arguments

```
direction: branchdir
The branching direction assigned to variable j.
j: int
Index of the variable.
```

## Description:

Obtains the branching direction for a given variable j.

# A.2.148 Task.getvarbranchpri()

```
priority = Task.getvarbranchpri(j)
```

Obtains the branching priority for a variable.

Arguments

```
j: int
```

Index of the variable.

priority: int

The branching priority assigned to variable j.

Description:

Obtains the branching priority for a given variable j.

# A.2.149 Task.getvarname()

```
name = Task.getvarname(j)
```

Obtains a name of a variable.

Arguments

j: int

Index.

name : str

Is assigned the required name.

Description:

Obtains a name of a variable.

# A.2.150 Task.getvarnameindex()

```
asgn,index = Task.getvarnameindex(somename)
```

Checks whether the name somename has been assigned to any variable.

asgn: int

Is non-zero if the name somename is assigned to a variable.

index : int

If the name somename is assigned to a variable, then index is the name of the variable.

somename : str

The name which should be checked.

## Description:

Checks whether the name somename has been assigned to any variable. If it has been assigned to variable, then index of the variable is reported.

# A.2.151 Task.getvarnamelen()

```
len = Task.getvarnamelen(i)
```

Obtains the length of a name of a variable variable.

## Arguments

i: int

Index.

len : int

Returns the length of the indicated name.

#### Description:

Obtains the length of a name of a variable variable.

See also

• Task.getbarvarname Obtains a name of a semidefinite variable.

# A.2.152 Task.getvartype()

```
vartype = Task.getvartype(j)
```

Gets the variable type of one variable.

# Arguments

j: int

Index of the variable.

```
vartype: variable type Variable type of variable j.
```

## Description:

Gets the variable type of one variable.

# A.2.153 Task.getvartypelist()

```
Task.getvartypelist(
    subj,
    vartype)
```

Obtains the variable type for one or more variables.

## Arguments

```
subj : int[]
```

A list of variable indexes.

vartype : mosek.variabletype[]

The variables types corresponding to the variables specified by subj.

## Description:

Obtains the variable type of one or more variables.

Upon return vartype[k] is the variable type of variable subj[k].

# A.2.154 Task.getxc()

```
Task.getxc(
    whichsol,
    xc)
```

Obtains the xc vector for a solution.

#### Arguments

```
whichsol : soltype Selects a solution. xc : double[] The x^c vector.
```

#### Description:

Obtains the  $x^c$  vector for a solution.

See also

• Task.getxcslice Obtains a slice of the xc vector for a solution.

# A.2.155 Task.getxcslice()

```
Task.getxcslice(
    whichsol,
    first,
    last,
    xc)
```

Obtains a slice of the xc vector for a solution.

## Arguments

```
first : int
    First index in the sequence.

last : int
    Last index plus 1 in the sequence.

whichsol : soltype
    Selects a solution.

xc : double[]
    Primal constraint solution.
```

# Description:

Obtains a slice of the  $x^c$  vector for a solution.

See also

• Task.getxc Obtains the xc vector for a solution.

# A.2.156 Task.getxx()

```
Task.getxx(
    whichsol,
    xx)
```

Obtains the xx vector for a solution.

```
whichsol : soltype
Selects a solution.

xx : double[]
The x^x vector.
```

## Description:

Obtains the  $x^x$  vector for a solution.

See also

• Task.getxxslice Obtains a slice of the xx vector for a solution.

# A.2.157 Task.getxxslice()

```
Task.getxxslice(
    whichsol,
    first,
    last,
    xx)
```

Obtains a slice of the xx vector for a solution.

## Arguments

first : int

First index in the sequence.

last : int

Last index plus 1 in the sequence.

whichsol: soltype
Selects a solution.

xx : double[]

Primal variable solution.

# Description:

Obtains a slice of the  $x^x$  vector for a solution.

See also

• Task.getxx Obtains the xx vector for a solution.

# A.2.158 Task.gety()

```
Task.gety(
    whichsol,
    y)
```

Obtains the y vector for a solution.

## Arguments

```
whichsol : soltype
    Selects a solution.
y : double[]
    The y vector.
```

Description:

Obtains the y vector for a solution.

See also

• Task.getyslice Obtains a slice of the y vector for a solution.

# A.2.159 Task.getyslice()

```
Task.getyslice(
    whichsol,
    first,
    last,
    y)
```

Obtains a slice of the y vector for a solution.

#### Arguments

```
first : int
    First index in the sequence.
last : int
    Last index plus 1 in the sequence.
whichsol : soltype
    Selects a solution.
y : double[]
```

Vector of dual variables corresponding to the constraints.

#### Description:

Obtains a slice of the y vector for a solution.

See also

• Task.gety Obtains the y vector for a solution.

## A.2.160 Task.initbasissolve()

```
Task.initbasissolve(basis)
```

Prepare a task for basis solver.

## Arguments

```
basis : int[]
```

The array of basis indexes to use.

The array is interpreted as follows: If  $\mathtt{basis}[i] \leq numcon - 1$ , then  $x^c_{\mathtt{basis}[i]}$  is in the basis at position i, otherwise  $x_{\mathtt{basis}[i]-\mathtt{numcon}}$  is in the basis at position i.

## Description:

Prepare a task for use with the Task.solvewithbasis function.

This function should be called

- immediately before the first call to Task.solvewithbasis, and
- immediately before any subsequent call to Task.solvewithbasis if the task has been modified.

If the basis is singular i.e. not invertible, then

the exception rescode.err\_basis\_singular is generated.

# A.2.161 Task.inputdata()

```
Task.inputdata(
    maxnumcon,
    maxnumvar,
    c,
    cfix,
    aptrb,
    aptre,
    asub,
    aval,
    bkc,
    blc,
    buc,
    bkx,
    blx,
    bux)
```

Input the linear part of an optimization task in one function call.

### Arguments

aptrb : long[]

Row or column end pointers.

aptre : long[]

Row or column start pointers.

asub : int[]

Coefficient subscripts.

aval : double[]

Coefficient coefficient values.

bkc : boundkey

Bound keys for the constraints.

bkx: boundkey

Bound keys for the variables.

blc : double[]

Lower bounds for the constraints.

blx : double[]

Lower bounds for the variables.

buc : double[]

Upper bounds for the constraints.

bux : double[]

Upper bounds for the variables.

c : double[]

Linear terms of the objective as a dense vector. The lengths is the number of variables.

cfix : double

Fixed term in the objective.

maxnumcon: int

Number of preallocated constraints in the optimization task.

maxnumvar : int

Number of preallocated variables in the optimization task.

# Description:

Input the linear part of an optimization problem.

The non-zeros of A are inputted column-wise in the format described in Section 5.13.3.2.

For an explained code example see Section 5.2 and Section 5.13.3.

# A.2.162 Task.isdouparname()

```
param = Task.isdouparname(parname)
```

Checks a double parameter name.

Arguments

param : dparam
 Which parameter.
parname : str
 Parameter name.

 ${\bf Description:}$ 

Checks whether parname is a valid double parameter name.

# A.2.163 Task.isintparname()

```
param = Task.isintparname(parname)
```

Checks an integer parameter name.

Arguments

param : iparam
 Which parameter.
parname : str
 Parameter name.

Description:

Checks whether parname is a valid integer parameter name.

# A.2.164 Task.isstrparname()

```
param = Task.isstrparname(parname)
```

Checks a string parameter name.

```
param : sparam
   Which parameter.
parname : str
   Parameter name.
```

#### Description:

Checks whether parname is a valid string parameter name.

# A.2.165 Task.linkfiletostream()

```
Task.linkfiletostream(
    whichstream,
    filename,
    append)
```

Directs all output from a task stream to a file.

## Arguments

```
append : int
```

If this argument is 0 the output file will be overwritten, otherwise text is append to the output file.

```
filename : str
```

The name of the file where text from the stream defined by whichstream is written.

```
whichstream: streamtype
Index of the stream.
```

## Description:

Directs all output from a task stream to a file.

# A.2.166 Task.onesolutionsummary()

```
Task.onesolutionsummary(
    whichstream,
    whichsol)
```

Prints a short summary for the specified solution.

```
whichsol: soltype
Selects a solution.
```

```
whichstream: streamtype
Index of the stream.
```

### Description:

Prints a short summary for a specified solution.

### A.2.167 Task.optimize()

```
trmcode = Task.optimize()
```

Optimizes the problem.

Arguments

```
trmcode : rescode
```

Is either rescode.ok or a termination response code.

#### Description:

Calls the optimizer. Depending on the problem type and the selected optimizer this will call one of the optimizers in MOSEK. By default the interior point optimizer will be selected for continuous problems. The optimizer may be selected manually by setting the parameter <code>iparam.optimizer</code>.

See also

- Task.optimizeconcurrent Optimize a given task with several optimizers concurrently.
- Task.getsolution Obtains the complete solution.
- Task.getsolutioni Obtains the solution for a single constraint or variable.
- Task.getsolutioninfo Obtains information about of a solution.
- iparam.optimizer Controls which optimizer is used to optimize the task.

### A.2.168 Task.optimizeconcurrent()

Task.optimizeconcurrent(taskarray)

Optimize a given task with several optimizers concurrently.

### Arguments

```
taskarray : mosek.Task[]
An array of num tasks.
```

#### Description:

Solves several instances of the same problem in parallel, with unique parameter settings for each task. The argument task contains the problem to be solved. taskarray is a pointer to an array of num empty tasks. The task task and the num tasks pointed to by taskarray are solved in parallel. That is num + 1 threads are started with one optimizer in each. Each of the tasks can be initialized with different parameters, e.g different selection of solver.

All the concurrently running tasks are stopped when the optimizer successfully terminates for one of the tasks. After the function returns task contains the solution found by the task that finished first.

After Task.optimizeconcurrent returns task holds the optimal solution of the task which finished first. If all the concurrent optimizations finished without providing an optimal solution the error code from the solution of the task task is returned.

In summary a call to Task.optimizeconcurrent does the following:

- All data except task parameters (iparam, dparam and sparam) in task is copied to each of the tasks in taskarray. In particular this means that any solution in task is copied to the other tasks. Call-back functions are not copied.
- The tasks task and the num tasks in taskarray are started in parallel.
- When a task finishes providing an optimal solution (or a certificate of infeasibility) its solution is copied to task and all other tasks are stopped.

Observe the concurrent optimizer is not deterministic.

For an explained code example see Section 11.6.4.

#### A.2.169 Task.optimizersummary()

Task.optimizersummary(whichstream)

Prints a short summary with optimizer statistics for last optimization.

#### Arguments

whichstream: streamtype
Index of the stream.

#### Description:

Prints a short summary with optimizer statistics for last optimization.

### A.2.170 Task.primalrepair()

```
Task.primalrepair(
    wlc,
    wuc,
    wlx,
    wux)
```

The function repairs a primal infeasible optimization problem by adjusting the bounds on the constraints and variables.

#### Arguments

#### wlc : double[]

 $(w_l^c)_i$  is the weight associated with relaxing the lower bound on constraint i. If the weight is negative, then the lower bound is not relaxed. Moreover, if the argument is None, then all the weights are assumed to be 1.

#### wlx : double[]

 $(w_l^x)_j$  is the weight associated with relaxing the upper bound on constraint j. If the weight is negative, then the lower bound is not relaxed. Moreover, if the argument is None, then all the weights are assumed to be 1.

#### wuc : double[]

 $(w_u^c)_i$  is the weight associated with relaxing the upper bound on constraint i. If the weight is negative, then the upper bound is not relaxed. Moreover, if the argument is None, then all the weights are assumed to be 1.

#### wux : double[]

 $(w_l^x)_i$  is the weight associated with relaxing the upper bound on variable j. If the weight is negative, then the upper bound is not relaxed. Moreover, if the argument is None, then all the weights are assumed to be 1.

#### Description:

The function repairs a primal infeasible optimization problem by adjusting the bounds on the constraints and variables where the adjustment is computed as the minimal weighted sum relaxation to the bounds on the constraints and variables.

The function is applicable to linear and conic problems possibly having integer constrained variables.

Observe that when computing the minimal weighted relaxation then the termination tolerance specified by the parameters of the task is employed. For instance the parameter <code>iparam.mio\_mode</code> can be used make MOSEK ignore the integer constraints during the repair leading to a possibly a much faster repair. However, the drawback is of course that the repaired problem may not have integer feasible solution.

Note the function modifies the bounds on the constraints and variables. If this is not a desired feature, then apply the function to a cloned task.

See also

- iparam.primal\_repair\_optimizer Controls which optimizer that is used to find the optimal repair.
- iparam.log\_feas\_repair Controls the amount of output printed when performing feasibility repair. A value higher than one means extensive logging.
- dinfitem.primal\_repair\_penalty\_obj The optimal objective value of the penalty function.

### A.2.171 Task.primalsensitivity()

```
Task.primalsensitivity(
    subi,
    marki,
    subj,
    markj,
    leftpricei,
    rightpricei,
    leftrangei,
    rightrangei,
    leftpricej,
    rightpricej,
    rightpricej,
    rightpricej,
    rightrangej,
    rightrangej)
```

Perform sensitivity analysis on bounds.

#### Arguments

```
leftpricei : double[]
```

leftpricei[i] is the left shadow price for the upper/lower bound (indicated by marki[i])
of the constraint with index subi[i].

```
leftpricej : double[]
```

leftpricej[j] is the left shadow price for the upper/lower bound (indicated by marki[j])
on variable subj[j].

#### leftrangei : double[]

leftrangei[i] is the left range for the upper/lower bound (indicated by marki[i]) of the
constraint with index subi[i].

#### leftrangej : double[]

leftrangej[j] is the left range for the upper/lower bound (indicated by marki[j]) on
variable subj[j].

```
marki : mark
```

The value of marki[i] specifies for which bound (upper or lower) on constraint subi[i] sensitivity analysis should be performed.

#### markj : mark

The value of markj[j] specifies for which bound (upper or lower) on variable subj[j] sensitivity analysis should be performed.

#### rightpricei : double[]

rightpricei[i] is the right shadow price for the upper/lower bound (indicated by marki[i]) of the constraint with index subi[i].

#### rightpricej : double[]

rightpricej[j] is the right shadow price for the upper/lower bound (indicated by marki[j])
on variable subj[j] .

#### rightrangei : double[]

rightrangei[i] is the right range for the upper/lower bound (indicated by marki[i]) of the constraint with index subi[i].

#### rightrangej : double[]

rightrangej[j] is the right range for the upper/lower bound (indicated by marki[j]) on variable subj[j].

#### subi : int[]

Indexes of bounds on constraints to analyze.

#### subj : int[]

Indexes of bounds on variables to analyze.

#### Description:

Calculates sensitivity information for bounds on variables and constraints.

For details on sensitivity analysis and the definitions of *shadow price* and *linearity interval* see chapter 15.

The constraints for which sensitivity analysis is performed are given by the data structures:

- subi Index of constraint to analyze.
- marki Indicate for which bound of constraint subi[i] sensitivity analysis is performed. If marki[i] = mark.up the upper bound of constraint subi[i] is analyzed, and if marki[i] = mark.lo the lower bound is analyzed. If subi[i] is an equality constraint, either mark.lo or mark.up can be used to select the constraint for sensitivity analysis.

Consider the problem:

$$\begin{array}{lll} \text{minimize} & x_1 + x_2 \\ \text{subject to} - 1 \leq & x_1 - x_2 & \leq & 1, \\ & x_1 & = & 0, \\ & x_1 \geq 0, x_2 \geq 0 & \end{array}$$

Suppose that

- numi = 1;
- subi = [0];

• marki = [mark.up]

then

leftpricei[0], rightpricei[0], leftrangei[0] and rightrangei[0] will contain the sensitivity information for the upper bound on constraint 0 given by the expression:

$$x_1 - x_2 \le 1$$

Similarly, the variables for which to perform sensitivity analysis are given by the structures:

- subj Index of variables to analyze.
- markj Indicate for which bound of variable subi[j] sensitivity analysis is performed. If markj[j] = mark.up the upper bound of constraint subi[j] is analyzed, and if markj[j] = mark.lo the lower bound is analyzed. If subi[j] is an equality constraint, either mark.lo or mark.up can be used to select the constraint for sensitivity analysis.

For an example, please see Section 15.5.

The type of sensitivity analysis to be performed (basis or optimal partition) is controlled by the parameter <code>iparam.sensitivity\_type</code>.

See also

- Task.dualsensitivity Performs sensitivity analysis on objective coefficients.
- Task.sensitivityreport Creates a sensitivity report.
- iparam.sensitivity\_type Controls which type of sensitivity analysis is to be performed.
- iparam.log\_sensitivity Control logging in sensitivity analyzer.
- iparam.log\_sensitivity\_opt Control logging in sensitivity analyzer.

#### A.2.172 Task.printdata()

```
Task.printdata(
    whichstream,
    firsti,
    lasti,
    firstj,
    lastj,
    firstk,
    lastk,
    с,
    qo,
    a,
    qc,
    bc.
    bх,
    vartype,
    cones)
```

Prints a part of the problem data to a stream.

### Arguments

a: int

If non-zero A is printed.

bc : int

If non-zero the constraints bounds are printed.

bx : int

If non-zero the variable bounds are printed.

c: int

If non-zero c is printed.

cones : int

If non-zero the conic data is printed.

firsti : int

Index of first constraint for which data should be printed.

firstj: int

Index of first variable for which data should be printed.

firstk: int

Index of first cone for which data should be printed.

lasti : int

Index of last constraint plus 1 for which data should be printed.

lastj: int

Index of last variable plus 1 for which data should be printed.

lastk: int

Index of last cone plus 1 for which data should be printed.

qc : int

If non-zero  $Q^k$  is printed for the relevant constraints.

qo: int

If non-zero  $Q^o$  is printed.

vartype : int

If non-zero the variable types are printed.

whichstream : streamtype

Index of the stream.

#### Description:

Prints a part of the problem data to a stream. This function is normally used for debugging purposes only, e.g. to verify that the correct data has been inputted.

### A.2.173 Task.printparam()

Task.printparam()

Prints the current parameter settings.

#### Description:

Prints the current parameter settings to the message stream.

### A.2.174 Task.putacol()

```
Task.putacol(
    j,
    subj,
    valj)
```

Replaces all elements in one column of A.

Arguments

j: int

Index of column in A.

subj : int[]

Row indexes of non-zero values in column j of A.

valj : double[]

New non-zero values of column j in A.

#### Description:

Replaces all entries in column j of A. Assuming that there are no duplicate subscripts in subj, assignment is performed as follows:

$$A_{\mathrm{subj}[k],j} = \mathrm{valj}[k], \quad k = 0, \dots, \mathrm{nzj} - 1$$

All other entries in column j are set to zero.

See also

- Task.putarow Replaces all elements in one row of A.
- Task.putaij Changes a single value in the linear coefficient matrix.
- Task.putmaxnumanz The function changes the size of the preallocated storage for linear coefficients.

### A.2.175 Task.putacollist()

```
Task.putacollist(
    sub,
    ptrb,
    ptre,
    asub,
    aval)
```

Replaces all elements in several columns the linear constraint matrix by new values.

#### Arguments

```
asub : int[]
  asub contains the new variable indexes.
aval : double[]
```

Coefficient coefficient values.

ptrb : long[]

Array of pointers to the first element in the columns stored in asub and aval.

For an explanation of the meaning of ptrb see Section 5.13.3.2.

```
ptre : long[]
```

Array of pointers to the last element plus one in the columns stored in asub and aval. For an explanation of the meaning of ptre see Section 5.13.3.2.

```
sub · int[]
```

Indexes of columns that should be replaced. sub should not contain duplicate values.

#### Description:

Replaces all elements in a set of columns of A. The elements are replaced as follows

```
\label{eq:constraints} \begin{array}{ll} \texttt{for} & i = 0, \dots, num - 1 \\ & a_{\texttt{asub}[k], \texttt{sub}[i]} = \texttt{aval}[k], & k = \texttt{aptrb}[i], \dots, \texttt{aptre}[i] - 1. \end{array}
```

See also

• Task.putmaxnumanz The function changes the size of the preallocated storage for linear coefficients.

#### A.2.176 Task.putacolslice()

```
Task.putacolslice(
    first,
    last,
    ptrb,
    ptre,
    asub,
    aval)
```

Replaces all elements in several columns the linear constraint matrix by new values.

#### Arguments

asub : int[]

asub contains the new variable indexes.

aval : double[]

Coefficient coefficient values.

first : int

First column in the slice.

last: int

Last column plus one in the slice.

ptrb : long[]

Array of pointers to the first element in the columns stored in asub and aval.

For an explanation of the meaning of ptrb see Section 5.13.3.2.

ptre : long[]

Array of pointers to the last element plus one in the columns stored in asub and aval.

For an explanation of the meaning of ptre see Section 5.13.3.2.

#### Description:

Replaces all elements in a set of columns of A.

See also

• Task.putmaxnumanz The function changes the size of the preallocated storage for linear coefficients.

#### **A.2.177** Task.putaij()

```
Task.putaij(
    i,
    j,
    aij)
```

Changes a single value in the linear coefficient matrix.

#### Arguments

```
aij : double

New coefficient for a_{i,j}.
```

i : int

Index of the constraint in which the change should occur.

#### j: int

Index of the variable in which the change should occur.

#### Description:

Changes a coefficient in A using the method

$$a_{ij} = aij.$$

See also

- Task.putarow Replaces all elements in one row of A.
- Task.putacol Replaces all elements in one column of A.
- Task.putaij Changes a single value in the linear coefficient matrix.
- Task.putmaxnumanz The function changes the size of the preallocated storage for linear coefficients.

### A.2.178 Task.putaijlist()

```
Task.putaijlist(
    subi,
    subj,
    valij)
```

Changes one or more coefficients in the linear constraint matrix.

#### Arguments

subi : int[]

Constraint indexes in which the change should occur.

subj : int[]

Variable indexes in which the change should occur.

valij : double[]

New coefficient values for  $a_{i,j}$ .

#### Description:

Changes one or more coefficients in A using the method

$$a_{\texttt{subi}[\texttt{k}],\texttt{subj}[\texttt{k}]} = \texttt{valij}[\texttt{k}], \quad k = 0, \dots, \texttt{num} - 1.$$

See also

- Task.putarow Replaces all elements in one row of A.
- Task.putacol Replaces all elements in one column of A.
- Task.putaij Changes a single value in the linear coefficient matrix.
- Task.putmaxnumanz The function changes the size of the preallocated storage for linear coefficients.

### A.2.179 Task.putarow()

```
Task.putarow(
    i,
    subi,
    vali)
```

Replaces all elements in one row of A.

#### Arguments

```
i : int
    Index of row in A.
subi : int[]
    Row indexes of non-zero values in row i of A.
vali : double[]
    New non-zero values of row i in A.
```

#### Description:

Replaces all entries in row i of A. Assuming that there are no duplicate subscripts in subi, assignment is performed as follows:

$$A_{\mathtt{i},\mathtt{subi}[k]} = \mathtt{vali}[k], \quad k = 0, \dots, \mathtt{nzi} - 1$$

All other entries in row i are set to zero.

See also

- Task.putacol Replaces all elements in one column of A.
- Task.putaij Changes a single value in the linear coefficient matrix.
- Task.putmaxnumanz The function changes the size of the preallocated storage for linear coefficients.

### A.2.180 Task.putarowlist()

```
Task.putarowlist(
    sub,
    ptrb,
    ptre,
    asub,
    aval)
```

Replaces all elements in several rows the linear constraint matrix by new values.

#### Arguments

```
asub : int[]
```

asub contains the new variable indexes.

aval : double[]

Coefficient coefficient values.

ptrb : long[]

Array of pointers to the first element in the rows stored in asub and aval.

For an explanation of the meaning of ptrb see Section 5.13.3.2.

ptre : long[]

Array of pointers to the last element plus one in the rows stored in asub and aval.

For an explanation of the meaning of ptre see Section 5.13.3.2.

sub : int[]

Indexes of rows or columns that should be replaced. sub should not contain duplicate values.

#### Description:

Replaces all elements in a set of rows of A. The elements are replaced as follows

$$\begin{aligned} & \text{for} \quad i = \texttt{first}, \dots, \texttt{last} - 1 \\ & \quad a_{\texttt{sub[i]}, \texttt{asub[}k]} = \texttt{aval}[k], \quad k = \texttt{aptrb}[i], \dots, \texttt{aptre}[i] - 1. \end{aligned}$$

See also

• Task.putmaxnumanz The function changes the size of the preallocated storage for linear coefficients.

### A.2.181 Task.putbarablocktriplet()

```
Task.putbarablocktriplet(
   num,
   subi,
   subj,
   subk,
   subl,
   valijkl)
```

Inputs barA in block triplet form.

#### Arguments

num : long

Number of elements in the block triplet form.

subi : int[]

Constraint index.

```
subj : int[]
```

Symmetric matrix variable index.

subk : int[]

Block row index.

subl : int[]

Block column index.

valijkl : double[]

The numerical value associated with the block triplet.

#### Description:

Inputs the  $\bar{A}$  in block triplet form.

### A.2.182 Task.putbaraij()

```
Task.putbaraij(
    i,
    j,
    sub,
    weights)
```

Inputs an element of barA.

#### Arguments

i : int

Row index of  $\bar{A}$ .

j: int

Column index of  $\bar{A}$ .

sub : long[]

See argument weights for an explenation.

weights : double[]

weights[k] times sub[k]'th term of E is added to  $\bar{A}_{ij}$ .

### Description:

This function puts one element associated with  $\bar{X}_j$  in the  $\bar{A}$  matrix.

Each element in the  $\bar{A}$  matrix is a weighted sum of symmetric matrixes, i.e.  $\bar{A}_{ij}$  is a symmetric matrix with dimensions as  $\bar{X}_j$ . By default all elements in  $\bar{A}$  are 0, so only non-zero elements need be added.

Setting the same elements again will overwrite the earlier entry.

The symmetric matrixes themselves are defined separately using the funtion Task.appendsparsesymmat.

#### A.2.183Task.putbarcblocktriplet()

```
Task.putbarcblocktriplet(
    num,
    subj,
    subk,
    subl,
    valjkl)
     Inputs barC in block triplet form.
 Arguments
      num : long
          Number of elements in the block triplet form.
      subj : int[]
          Symmetric matrix variable index.
      subk : int[]
          Block row index.
      subl : int[]
          Block column index.
      valjkl : double[]
          The numerical value associated with the block triplet.
 Description:
     Inputs the \bar{C} in block triplet form.
A.2.184
           Task.putbarcj()
```

```
Task.putbarcj(
    sub,
    weights)
```

Changes one element in barc.

Arguments

```
j: int
```

Index of the element in  $\bar{c}$  that should be changed.

```
sub : long[]
```

sub is list of indexes of those symmetric matrixes appearing in sum.

```
weights : double[]
```

The weights of the terms in the weighted sum that forms  $c_j$ .

#### Description:

This function puts one element associated with  $\bar{X}_j$  in the  $\bar{c}$  vector.

Each element in the  $\bar{c}$  vector is a weighted sum of symmetric matrixes, i.e.  $\bar{c}_j$  is a symmetric matrix with dimensions as  $\bar{X}_j$ . By default all elements in  $\bar{c}$  are 0, so only non-zero elements need be added.

Setting the same elements again will overwrite the earlier entry.

The symmetric matrixes themselves are defined separately using the funtion Task.appendsparsesymmat.

### A.2.185 Task.putbarsj()

```
Task.putbarsj(
    whichsol,
    j,
    barsj)
```

Sets the dual solution for a semidefinite variable.

#### Arguments

```
\begin{array}{l} \texttt{barsj: double[]} \\ & \texttt{Value of } \bar{s}_j. \\ \texttt{j: int} \\ & \texttt{Index of the semidefinite variable.} \\ \texttt{whichsol: soltype} \\ & \texttt{Selects a solution.} \end{array}
```

#### Description:

Sets the dual solution for a semidefinite variable.

### A.2.186 Task.putbarvarname()

```
Task.putbarvarname(
    j,
    name)
```

Puts the name of a semidefinite variable.

#### Arguments

```
j : int
    Index of the variable.
name : str
    The variable name.
```

Description:

Puts the name of a semidefinite variable.

See also

• Task.getbarvarnamelen Obtains the length of a name of a semidefinite variable.

# A.2.187 Task.putbarxj()

```
Task.putbarxj(
    whichsol,
    j,
    barxj)
```

Sets the primal solution for a semidefinite variable.

Arguments

```
\begin{array}{ll} \mathtt{barxj} \; : \; \; \mathtt{double[]} \\ & \; \; \mathtt{Value} \; \mathrm{of} \; \bar{X}_j. \\ \mathtt{j} \; : \; \; \mathtt{int} \\ & \; \; \; \mathtt{Index} \; \mathrm{of} \; \mathtt{the} \; \mathtt{semidefinite} \; \mathtt{variable}. \\ \mathtt{whichsol} \; : \; \; \; \; \mathtt{soltype} \\ & \; \; \; \mathtt{Selects} \; \mathtt{a} \; \mathtt{solution}. \end{array}
```

Description:

Sets the primal solution for a semidefinite variable.

### A.2.188 Task.putbound()

```
Task.putbound(
    accmode,
    i,
    bk,
    bl,
    bu)
```

Changes the bound for either one constraint or one variable.

#### Arguments

#### accmode: accmode

Defines whether the bound for a constraint or a variable is changed.

bk: boundkey

New bound key.

bl : double

New lower bound.

bu : double

New upper bound.

i : int

Index of the constraint or variable.

#### Description:

Changes the bounds for either one constraint or one variable.

If the a bound value specified is numerically larger than <code>dparam.data\_tol\_bound\_inf</code> it is considered infinite and the bound key is changed accordingly. If a bound value is numerically larger than <code>dparam.data\_tol\_bound\_wrn</code>, a warning will be displayed, but the bound is inputted as specified.

See also

• Task.putboundlist Changes the bounds of constraints or variables.

## A.2.189 Task.putboundlist()

```
Task.putboundlist(
    accmode,
    sub,
    bk,
    bl,
    bu)
```

Changes the bounds of constraints or variables.

#### Arguments

#### accmode : accmode

Defines whether bounds for constraints (accmode.con) or variables (accmode.var) are changed.

bk: boundkey

Constraint or variable index sub[t] is assigned the bound key bk[t].

```
bl : double[]
```

Constraint or variable index sub[t] is assigned the lower bound bl[t].

bu : double[]

Constraint or variable index sub[t] is assigned the upper bound bu[t].

sub : int[]

Subscripts of the bounds that should be changed.

#### Description:

Changes the bounds for either some constraints or variables. If multiple bound changes are specified for a constraint or a variable, only the last change takes effect.

See also

- Task.putbound Changes the bound for either one constraint or one variable.
- dparam.data\_tol\_bound\_inf Data tolerance threshold.
- dparam.data\_tol\_bound\_wrn Data tolerance threshold.

### A.2.190 Task.putboundslice()

```
Task.putboundslice(
    con,
    first,
    last,
    bk,
    bl,
    bu)
```

Modifies bounds.

### Arguments

```
bk: boundkey
Bound keys.
bl: double[]
```

Values for lower bounds.

bu : double[]

Values for upper bounds.

con: accmode

Defines whether bounds for constraints (accmode.con) or variables (accmode.var) are changed.

first : int

First index in the sequence.

```
last : int
```

Last index plus 1 in the sequence.

### Description:

Changes the bounds for a sequence of variables or constraints.

See also

- Task.putbound Changes the bound for either one constraint or one variable.
- dparam.data\_tol\_bound\_inf Data tolerance threshold.
- dparam.data\_tol\_bound\_wrn Data tolerance threshold.

### A.2.191 Task.putcfix()

```
Task.putcfix(cfix)
```

Replaces the fixed term in the objective.

#### Arguments

```
cfix : double
```

Fixed term in the objective.

### Description:

Replaces the fixed term in the objective by a new one.

### A.2.192 Task.putcj()

```
Task.putcj(
    j,
    cj)
```

Modifies one linear coefficient in the objective.

#### Arguments

```
cj : double
```

New value of  $c_i$ .

j: int

Index of the variable for which c should be changed.

#### Description:

Modifies one coefficient in the linear objective vector c, i.e.

$$c_{\mathbf{j}} = \mathbf{c}\mathbf{j}$$
.

See also

- Task.putclist Modifies a part of the linear objective coefficients.
- Task.putcslice Modifies a slice of the linear objective coefficients.

### A.2.193 Task.putclist()

```
Task.putclist(
    subj,
    val)
```

Modifies a part of the linear objective coefficients.

#### Arguments

subj : int[]

Index of variables for which c should be changed.

val : double[]

New numerical values for coefficients in c that should be modified.

#### Description:

Modifies elements in the linear term c in the objective using the principle

$$c_{\mathtt{subj[t]}} = \mathtt{val[t]}, \quad t = 0, \dots, \mathtt{num} - 1.$$

If a variable index is specified multiple times in subj only the last entry is used.

### A.2.194 Task.putconbound()

Task.putconbound(
 i,
 bk,
 bl,
 bu)

Changes the bound for one constraint.

#### Arguments

bk: boundkey

New bound key.

bl : double

New lower bound.

bu : double

New upper bound.

i: int

Index of the constraint.

#### Description:

Changes the bounds for one constraint.

If the a bound value specified is numerically larger than <code>dparam.data\_tol\_bound\_inf</code> it is considered infinite and the bound key is changed accordingly. If a bound value is numerically larger than <code>dparam.data\_tol\_bound\_wrn</code>, a warning will be displayed, but the bound is inputted as specified.

See also

• Task.putconboundslice Changes the bounds for a slice of the constraints.

### A.2.195 Task.putconboundlist()

```
Task.putconboundlist(
    sub,
    bkc,
    blc,
    buc)
```

Changes the bounds of a list of constraints.

#### Arguments

bkc: boundkey

New bound keys.

blc : double[]

New lower bound values.

buc : double[]

New upper bound values.

sub : int[]

List constraints indexes.

#### Description:

Changes the bounds for a list of constraints. If multiple bound changes are specified for a constraint, then only the last change takes effect.

#### See also

- Task.putconbound Changes the bound for one constraint.
- Task.putconboundslice Changes the bounds for a slice of the constraints.
- dparam.data\_tol\_bound\_inf Data tolerance threshold.
- dparam.data\_tol\_bound\_wrn Data tolerance threshold.

### A.2.196 Task.putconboundslice()

```
Task.putconboundslice(
    first,
    last,
    bk,
    bl,
    bu)
```

Changes the bounds for a slice of the constraints.

#### Arguments

```
bk : boundkey
```

New bound keys.

bl : double[]

New lower bounds.

bu : double[]

New upper bounds.

first : int

Index of the first constraint in the slice.

last : int

Index of the last constraint in the slice plus 1.

#### Description:

Changes the bounds for a slice of the constraints.

### See also

- Task.putconbound Changes the bound for one constraint.
- Task.putconboundlist Changes the bounds of a list of constraints.

### A.2.197 Task.putcone()

```
Task.putcone(
    k,
    conetype,
    conepar,
    submem)

Replaces a conic constraint.

Arguments

conepar: double
    This argument is currently not used. Can be set to 0.0.

conetype: conetype
    Specifies the type of the cone.

k: int
    Index of the cone.

submem: int[]

Variable subscripts of the members in the cone.
```

#### Description:

Replaces a conic constraint.

### A.2.198 Task.putconename()

```
Task.putconename(
    j,
    name)
```

Puts the name of a cone.

## Arguments

```
j : int
    Index of the variable.
name : str
    The variable name.
```

### Description:

Puts the name of a cone.

### A.2.199 Task.putconname()

```
Task.putconname(
    i,
    name)
```

Puts the name of a constraint.

### Arguments

```
i : int
```

Index of the variable.

name : str

The variable name.

Description:

Puts the name of a constraint.

### A.2.200 Task.putcslice()

```
Task.putcslice(
    first,
    last,
    slice)
```

Modifies a slice of the linear objective coefficients.

#### Arguments

first : int

First element in the slice of c.

last : int

Last element plus 1 of the slice in c to be changed.

slice : double[]

New numerical values for coefficients in c that should be modified.

#### Description:

Modifies a slice in the linear term c in the objective using the principle

$$c_{j} = slice[j - first], j = first, .., last - 1$$

### A.2.201 Task.putdouparam()

```
Task.putdouparam(
param,
parvalue)

Sets a double parameter.

Arguments

param: dparam
Which parameter.

parvalue: double
Parameter value.
```

### Description:

Sets the value of a double parameter.

### A.2.202 Task.putintparam()

```
Task.putintparam(
    param,
    parvalue)

Sets an integer parameter.

Arguments

param : iparam
    Which parameter.

parvalue : int
    Parameter value.
```

### Description:

Sets the value of an integer parameter.

### A.2.203 Task.putmaxnumanz()

Task.putmaxnumanz(maxnumanz)

The function changes the size of the preallocated storage for linear coefficients.

#### Arguments

#### maxnumanz: long

New size of the storage reserved for storing A.

#### Description:

MOSEK stores only the non-zero elements in A. Therefore, MOSEK cannot predict how much storage is required to store A. Using this function it is possible to specify the number of non-zeros to preallocate for storing A.

If the number of non-zeros in the problem is known, it is a good idea to set maxnumanz slightly larger than this number, otherwise a rough estimate can be used. In general, if A is inputted in many small chunks, setting this value may speed up the the data input phase.

It is not mandatory to call this function, since MOSEK will reallocate internal structures whenever it is necessary.

See also

• iinfitem.sto\_num\_a\_realloc Number of times the storage for storing the linear coefficient matrix has been changed.

### A.2.204 Task.putmaxnumbarvar()

 ${\tt Task.putmaxnumbarvar(maxnumbarvar)}$ 

Sets the number of preallocated symmetric matrix variables in the optimization task.

#### Arguments

#### maxnumbarvar : int

The maximum number of semidefinite variables.

#### Description:

Sets the number of preallocated symmetric matrix variables in the optimization task. When this number of variables is reached MOSEK will automatically allocate more space for variables.

It is not mandatory to call this function, since its only function is to give a hint of the amount of data to preallocate for efficiency reasons.

Please note that maxnumbarvar must be larger than the current number of variables in the task.

### A.2.205 Task.putmaxnumcon()

Task.putmaxnumcon(maxnumcon)

Sets the number of preallocated constraints in the optimization task.

#### Arguments

maxnumcon: int

Number of preallocated constraints in the optimization task.

### Description:

Sets the number of preallocated constraints in the optimization task. When this number of constraints is reached MOSEK will automatically allocate more space for constraints.

It is never mandatory to call this function, since MOSEK will reallocate any internal structures whenever it is required.

Please note that maxnumcon must be larger than the current number of constraints in the task.

### A.2.206 Task.putmaxnumcone()

Task.putmaxnumcone(maxnumcone)

Sets the number of preallocated conic constraints in the optimization task.

#### Arguments

maxnumcone : int

Number of preallocated conic constraints in the optimization task.

#### Description:

Sets the number of preallocated conic constraints in the optimization task. When this number of conic constraints is reached MOSEK will automatically allocate more space for conic constraints.

It is never mandatory to call this function, since MOSEK will reallocate any internal structures whenever it is required.

Please note that maxnumcon must be larger than the current number of constraints in the task.

### A.2.207 Task.putmaxnumqnz()

Task.putmaxnumqnz(maxnumqnz)

Changes the size of the preallocated storage for quadratic terms.

#### Arguments

#### maxnumqnz : long

Number of non-zero elements preallocated in quadratic coefficient matrixes.

#### Description:

MOSEK stores only the non-zero elements in Q. Therefore, MOSEK cannot predict how much storage is required to store Q. Using this function it is possible to specify the number non-zeros to preallocate for storing Q (both objective and constraints).

It may be advantageous to reserve more non-zeros for Q than actually needed since it may improve the internal efficiency of MOSEK, however, it is never worthwhile to specify more than the double of the anticipated number of non-zeros in Q.

It is never mandatory to call this function, since its only function is to give a hint of the amount of data to preallocate for efficiency reasons.

### A.2.208 Task.putmaxnumvar()

Task.putmaxnumvar(maxnumvar)

Sets the number of preallocated variables in the optimization task.

#### Arguments

#### maxnumvar : int

Number of preallocated variables in the optimization task.

#### Description:

Sets the number of preallocated variables in the optimization task. When this number of variables is reached MOSEK will automatically allocate more space for variables.

It is never mandatory to call this function, since its only function is to give a hint of the amount of data to preallocate for efficiency reasons.

Please note that maxnumvar must be larger than the current number of variables in the task.

### A.2.209 Task.putnadouparam()

```
Task.putnadouparam(
    paramname,
    parvalue)
```

Sets a double parameter.

### Arguments

```
paramname : str
   Name of a parameter.
parvalue : double
   Parameter value.
```

#### Description:

Sets the value of a named double parameter.

### A.2.210 Task.putnaintparam()

```
Task.putnaintparam(
    paramname,
    parvalue)
```

Sets an integer parameter.

### Arguments

```
paramname : str
   Name of a parameter.
parvalue : int
   Parameter value.
```

#### Description:

Sets the value of a named integer parameter.

# A.2.211 Task.putnastrparam()

```
Task.putnastrparam(
    paramname,
    parvalue)
```

Sets a string parameter.

#### Arguments

paramname : str
 Name of a parameter.
parvalue : str
 Parameter value.

Description:

Sets the value of a named string parameter.

### A.2.212 Task.putobjname()

Task.putobjname(objname)

Assigns a new name to the objective.

Arguments

objname: str
Name of the objective.

Description:

Assigns the name given by objname to the objective function.

### A.2.213 Task.putobjsense()

Task.putobjsense(sense)

Sets the objective sense.

Arguments

sense : objsense

The objective sense of the task. The values objsense.maximize and objsense.minimize means that the problem is maximized or minimized respectively.

Description:

Sets the objective sense of the task.

See also

• Task.getobjsense Gets the objective sense.

## A.2.214 Task.putparam()

```
Task.putparam(
    parname,
    parvalue)
```

Modifies the value of parameter.

#### Arguments

```
parname : str
   Parameter name.
parvalue : str
   Parameter value.
```

#### Description:

Checks if a parname is valid parameter name. If it is, the parameter is assigned the value specified by parvalue.

### A.2.215 Task.putqcon()

```
Task.putqcon(
    qcsubk,
    qcsubi,
    qcsubj,
    qcval)
```

Replaces all quadratic terms in constraints.

### Arguments

```
qcsubi : int[]
   Row subscripts for quadratic constraint matrix.
qcsubj : int[]
   Column subscripts for quadratic constraint matrix.
qcsubk : int[]
   Constraint subscripts for quadratic coefficients.
qcval : double[]
```

Quadratic constraint coefficient values.

Description:

Replaces all quadratic entries in the constraints. Consider constraints on the form:

$$l_k^c \le \frac{1}{2} \sum_{i=0}^{numvar-1} \sum_{j=0}^{numvar-1} q_{ij}^k x_i x_j + \sum_{j=0}^{numvar-1} a_{kj} x_j \le u_k^c, \ k = 0, \dots, m-1.$$

The function assigns values to q such that:

$$q_{\texttt{qcsubi}[\texttt{t}],\texttt{qcsubj}[\texttt{t}]}^{\texttt{qcsubk}[\texttt{t}]} = \texttt{qcval}[\texttt{t}], \ t = 0, \dots, \texttt{numqcnz} - 1.$$

and

$$q_{\texttt{qcsubj[t]},\texttt{qcsubi[t]}}^{\texttt{qcsubk[t]}} = \texttt{qcval[t]}, \ t = 0, \dots, \texttt{numqcnz} - 1.$$

Values not assigned are set to zero.

Please note that duplicate entries are added together.

See also

- Task.putqconk Replaces all quadratic terms in a single constraint.
- Task.putmaxnumqnz Changes the size of the preallocated storage for quadratic terms.

### A.2.216 Task.putqconk()

```
Task.putqconk(
    k,
    qcsubi,
    qcsubj,
    qcval)
```

Replaces all quadratic terms in a single constraint.

Arguments

#### k: int

The constraint in which the new Q elements are inserted.

qcsubi : int[]

Row subscripts for quadratic constraint matrix.

qcsubj : int[]

Column subscripts for quadratic constraint matrix.

qcval : double[]

Quadratic constraint coefficient values.

#### Description:

Replaces all the quadratic entries in one constraint k of the form:

$$l_k^c \leq \frac{1}{2} \sum_{i=0}^{numvar-1} \sum_{j=0}^{numvar-1} q_{ij}^k x_i x_j + \sum_{j=0}^{numvar-1} a_{kj} x_j \leq u_k^c.$$

It is assumed that  $Q^k$  is symmetric, i.e.  $q^k_{ij}=q^k_{ji}$ , and therefore, only the values of  $q^k_{ij}$  for which  $i\geq j$  should be inputted to MOSEK. To be precise, MOSEK uses the following procedure

1.  $Q^k = 0$ 2. for t = 0 to numqonz - 13.  $q_{\texttt{qcsubi[t],qcsubj[t]}}^k = q_{\texttt{qcsubi[t],qcsubj[t]}}^k + \texttt{qcval[t]}$ 3.  $q_{\texttt{qcsubj[t],qcsubi[t]}}^k = q_{\texttt{qcsubj[t],qcsubi[t]}}^k + \texttt{qcval[t]}$ 

#### Please note that:

- For large problems it is essential for the efficiency that the function Task.putmaxnumqnz is employed to specify an appropriate maxnumqnz.
- Only the lower triangular part should be specified because  $Q^k$  is symmetric. Specifying values for  $q_{ij}^k$  where i < j will result in an error.
- Only non-zero elements should be specified.
- The order in which the non-zero elements are specified is insignificant.
- Duplicate elements are added together. Hence, it is recommended not to specify the same element multiple times in qosubi, qosubj, and qoval.

For a code example see Section 5.5.2.

See also

- Task.putqcon Replaces all quadratic terms in constraints.
- Task.putmaxnumqnz Changes the size of the preallocated storage for quadratic terms.

### A.2.217 Task.putqobj()

```
Task.putqobj(
qosubi,
qosubj,
qoval)
```

Replaces all quadratic terms in the objective.

#### Arguments

```
qosubi : int[]
```

Row subscripts for quadratic objective coefficients.

qosubj : int[]

Column subscripts for quadratic objective coefficients.

qoval : double[]

Quadratic objective coefficient values.

#### Description:

Replaces all the quadratic terms in the objective

$$\frac{1}{2} \sum_{i=0}^{numvar-1} \sum_{j=0}^{numvar-1} q_{ij}^{o} x_i x_j + \sum_{j=0}^{numvar-1} c_j x_j + c^f.$$

It is assumed that  $Q^o$  is symmetric, i.e.  $q^o_{ij}=q^o_{ji}$ , and therefore, only the values of  $q^o_{ij}$  for which  $i\geq j$  should be specified. To be precise, MOSEK uses the following procedure

- 1.  $Q^o = 0$

- 2. for t=0 to numqonz-13.  $q_{\text{qosubi[t],qosubj[t]}}^o = q_{\text{qosubi[t],qosubj[t]}}^o + \text{qoval[t]}$ 3.  $q_{\text{qosubj[t],qosubi[t]}}^o = q_{\text{qosubj[t],qosubi[t]}}^o + \text{qoval[t]}$

Please note that:

- $\bullet$  Only the lower triangular part should be specified because  $Q^o$  is symmetric. Specifying values for  $q_{ij}^o$  where i < j will result in an error.
- Only non-zero elements should be specified.
- The order in which the non-zero elements are specified is insignificant.
- Duplicate entries are added to together.

For a code example see Section 5.5.1.

#### A.2.218 Task.putqobjij()

```
Task.putqobjij(
    qoij)
```

Replaces one coefficient in the quadratic term in the objective.

#### Arguments

i: int

Row index for the coefficient to be replaced.

int

Column index for the coefficient to be replaced.

```
qoij : double

The new value for q_{ij}^o.
```

#### Description:

Replaces one coefficient in the quadratic term in the objective. The function performs the assignment

$$q_{\mathtt{i}\mathtt{j}}^o = \mathtt{qoij}.$$

Only the elements in the lower triangular part are accepted. Setting  $q_{ij}$  with j > i will cause an error.

Please note that replacing all quadratic element, one at a time, is more computationally expensive than replacing all elements at once. Use Task.putqobj instead whenever possible.

### A.2.219 Task.putskc()

```
Task.putskc(
    whichsol,
    skc)
```

Sets the status keys for the constraints.

#### Arguments

```
skc : stakey
    Status keys for the constraints.
whichsol : soltype
    Selects a solution.
```

### Description:

Sets the status keys for the constraints.

See also

• Task.putskcslice Sets the status keys for the constraints.

### A.2.220 Task.putskcslice()

```
Task.putskcslice(
    whichsol,
    first,
    last,
    skc)
```

Sets the status keys for the constraints.

#### Arguments

```
first : int
```

First index in the sequence.

last : int

Last index plus 1 in the sequence.

skc : stakey

Status keys for the constraints.

whichsol: soltype
Selects a solution.

### Description:

Sets the status keys for the constraints.

See also

• Task.putskc Sets the status keys for the constraints.

### A.2.221 Task.putskx()

```
Task.putskx(
    whichsol,
    skx)
```

Sets the status keys for the scalar variables.

### Arguments

```
skx: stakey
```

Status keys for the variables.

whichsol: soltype Selects a solution.

### Description:

Sets the status keys for the scalar variables.

See also

• Task.putskxslice Sets the status keys for the variables.

Task.putskxslice(

# A.2.222 Task.putskxslice()

```
whichsol,
first,
last,
skx)

Sets the status keys for the variables.

Arguments

first: int
First index in the sequence.
last: int
Last index plus 1 in the sequence.
skx: stakey
Status keys for the variables.
```

### Description:

Sets the status keys for the variables.

# A.2.223 Task.putslc()

whichsol: soltype Selects a solution.

```
Task.putslc(
    whichsol,
    slc)
```

Sets the slc vector for a solution.

### Arguments

```
\begin{array}{ll} {\rm slc} \ : \ {\rm double} \ [] \\ {\rm The} \ s_l^c \ {\rm vector}. \\ \\ {\rm whichsol} \ : \ {\rm soltype} \\ {\rm Selects} \ {\rm a} \ {\rm solution}. \end{array}
```

#### Description:

Sets the  $s_I^c$  vector for a solution.

See also

• Task.putslcslice Sets a slice of the slc vector for a solution.

### A.2.224 Task.putslcslice()

```
Task.putslcslice(
    whichsol,
    first,
    last,
    slc)
```

Sets a slice of the slc vector for a solution.

### Arguments

```
first : int
   First index in the sequence.
last : int
   Last index plus 1 in the sequence.
slc : double[]
```

Dual variables corresponding to the lower bounds on the constraints.

```
whichsol: soltype
Selects a solution.
```

## Description:

Sets a slice of the  $s_l^c$  vector for a solution.

See also

• Task.putslc Sets the slc vector for a solution.

# A.2.225 Task.putslx()

```
Task.putslx(
    whichsol,
    slx)
```

Sets the slx vector for a solution.

# Arguments

```
slx : double[]
    The s_l^x vector.

whichsol : soltype
    Selects a solution.
```

### Description:

Sets the  $s_l^x$  vector for a solution.

See also

• Task.putslx Sets the slx vector for a solution.

# A.2.226 Task.putslxslice()

```
Task.putslxslice(
    whichsol,
    first,
    last,
    slx)
```

Sets a slice of the slx vector for a solution.

### Arguments

```
first : int
```

First index in the sequence.

last : int

Last index plus 1 in the sequence.

slx : double[]

Dual variables corresponding to the lower bounds on the variables.

whichsol: soltype Selects a solution.

### Description:

Sets a slice of the  $s_l^x$  vector for a solution.

See also

• Task.putslx Sets the slx vector for a solution.

# A.2.227 Task.putsnx()

```
Task.putsnx(
    whichsol,
    sux)
```

Sets the snx vector for a solution.

### Arguments

```
\operatorname{sux}:\operatorname{double}[]
The s_n^x vector.

whichsol: soltype
Selects a solution.
```

### Description:

Sets the  $s_n^x$  vector for a solution.

See also

• Task.putsnxslice Sets a slice of the snx vector for a solution.

# A.2.228 Task.putsnxslice()

```
Task.putsnxslice(
    whichsol,
    first,
    last,
    snx)
```

Sets a slice of the snx vector for a solution.

#### Arguments

```
first : int
```

First index in the sequence.

last: int

Last index plus 1 in the sequence.

snx : double[]

Dual variables corresponding to the conic constraints on the variables.

whichsol: soltype
Selects a solution.

### Description:

Sets a slice of the  $s_n^x$  vector for a solution.

See also

• Task.putsnx Sets the snx vector for a solution.

### A.2.229 Task.putsolution()

```
Task.putsolution(
   whichsol,
    skc,
   skx,
   skn,
   хc,
   хх,
   у,
   slc,
   suc,
   slx,
    sux,
    snx)
    Inserts a solution.
Arguments
     skc : stakey
         Status keys for the constraints.
     skn: stakey
         Status keys for the conic constraints.
     skx: stakey
         Status keys for the variables.
     slc : double[]
         Dual variables corresponding to the lower bounds on the constraints.
     slx : double[]
         Dual variables corresponding to the lower bounds on the variables.
     snx : double[]
         Dual variables corresponding to the conic constraints on the variables.
     suc : double[]
         Dual variables corresponding to the upper bounds on the constraints.
     sux : double[]
         Dual variables corresponding to the upper bounds on the variables.
     whichsol : soltype
         Selects a solution.
     xc : double[]
         Primal constraint solution.
     xx : double[]
         Primal variable solution.
```

#### y : double[]

Vector of dual variables corresponding to the constraints.

### Description:

Inserts a solution into the task.

# A.2.230 Task.putsolutioni()

```
Task.putsolutioni(
    accmode,
    i,
    whichsol,
    sk,
    x,
    sl,
    su,
    sn)
```

Sets the primal and dual solution information for a single constraint or variable.

#### Arguments

```
accmode: accmode
```

If set to accmode.con the solution information for a constraint is modified. Otherwise for a variable.

i : int

Index of the constraint or variable.

sk: stakey

Status key of the constraint or variable.

sl : double

Solution value of the dual variable associated with the lower bound.

sn : double

Solution value of the dual variable associated with the cone constraint.

su : double

Solution value of the dual variable associated with the upper bound.

```
whichsol : soltype
```

Selects a solution.

x : double

Solution value of the primal constraint or variable.

#### Description:

Sets the primal and dual solution information for a single constraint or variable.

# A.2.231 Task.putsolutionyi()

```
Task.putsolutionyi(
    i,
    whichsol,
    y)
```

Inputs the dual variable of a solution.

### Arguments

#### i: int

Index of the dual variable.

whichsol: soltype Selects a solution.

y : double

Solution value of the dual variable.

### Description:

Inputs the dual variable of a solution.

See also

• Task.putsolutioni Sets the primal and dual solution information for a single constraint or variable.

# A.2.232 Task.putstrparam()

```
Task.putstrparam(
param,
parvalue)
```

Sets a string parameter.

### Arguments

```
param: sparam
Which parameter.

parvalue: str
Parameter value.
```

### Description:

Sets the value of a string parameter.

# A.2.233 Task.putsuc()

```
Task.putsuc(
    whichsol,
    suc)
```

Sets the suc vector for a solution.

### Arguments

```
\operatorname{suc}:\operatorname{double}[]
\operatorname{The} s_u^c \operatorname{vector}.
\operatorname{whichsol}:\operatorname{soltype}
\operatorname{Selects} \operatorname{a} \operatorname{solution}.
```

### Description:

Sets the  $s_u^c$  vector for a solution.

See also

• Task.putsucslice Sets a slice of the suc vector for a solution.

# A.2.234 Task.putsucslice()

```
Task.putsucslice(
    whichsol,
    first,
    last,
    suc)
```

Sets a slice of the suc vector for a solution.

#### Arguments

```
first : int
    First index in the sequence.
last : int
    Last index plus 1 in the sequence.
suc : double[]
    Dual variables corresponding to the upper bounds on the constraints.
whichsol : soltype
    Selects a solution.
```

### Description:

Sets a slice of the  $\boldsymbol{s}_{u}^{c}$  vector for a solution.

See also

• Task.putsuc Sets the suc vector for a solution.

# A.2.235 Task.putsux()

```
Task.putsux(
    whichsol,
    sux)
```

Sets the sux vector for a solution.

### Arguments

```
\begin{array}{lll} & \text{sux} \ : \ \text{double[]} \\ & \text{The } s^x_u \text{ vector.} \\ \\ & \text{whichsol} \ : \ \text{soltype} \\ & \text{Selects a solution.} \end{array}
```

### Description:

Sets the  $s_u^x$  vector for a solution.

See also

• Task.putsuxslice Sets a slice of the sux vector for a solution.

# A.2.236 Task.putsuxslice()

```
Task.putsuxslice(
   whichsol,
   first,
   last,
   sux)
```

Sets a slice of the sux vector for a solution.

# Arguments

```
first : int
```

First index in the sequence.

```
last : int
```

Last index plus 1 in the sequence.

sux : double[]

Dual variables corresponding to the upper bounds on the variables.

whichsol: soltype
Selects a solution.

### Description:

Sets a slice of the  $s_u^x$  vector for a solution.

See also

• Task.putsux Sets the sux vector for a solution.

# A.2.237 Task.puttaskname()

Task.puttaskname(taskname)

Assigns a new name to the task.

Arguments

taskname : str

Name assigned to the task.

Description:

Assigns the name taskname to the task.

### A.2.238 Task.putvarbound()

```
Task.putvarbound(
```

j, bk,

bl,

bu)

Changes the bound for one variable.

### Arguments

bk: boundkey

New bound key.

bl : double

New lower bound.

bu : double

New upper bound.

j: int

Index of the variable.

### Description:

Changes the bounds for one variable.

If the a bound value specified is numerically larger than <code>dparam.data\_tol\_bound\_inf</code> it is considered infinite and the bound key is changed accordingly. If a bound value is numerically larger than <code>dparam.data\_tol\_bound\_wrn</code>, a warning will be displayed, but the bound is inputted as specified.

See also

• Task.putvarboundslice Changes the bounds for a slice of the variables.

# A.2.239 Task.putvarboundlist()

```
Task.putvarboundlist(
    sub,
    bkx,
    blx,
    bux)
```

Changes the bounds of a list of variables.

#### Arguments

```
bkx : boundkey
   New bound keys.
blx : double[]
   New lower bound values.
```

bux : double[]

New upper bound values.

sub : int[]

List of variable indexes.

#### Description:

Changes the bounds for one or more variables. If multiple bound changes are specified for a variable, then only the last change takes effect.

#### See also

- Task.putvarbound Changes the bound for one variable.
- Task.putvarboundslice Changes the bounds for a slice of the variables.
- dparam.data\_tol\_bound\_inf Data tolerance threshold.
- dparam.data\_tol\_bound\_wrn Data tolerance threshold.

# A.2.240 Task.putvarboundslice()

```
Task.putvarboundslice(
    first,
    last,
    bk,
    bl,
    bu)
```

Changes the bounds for a slice of the variables.

#### Arguments

bk : boundkeyNew bound keys.

bl : double[]

New lower bounds.

bu : double[]

New upper bounds.

first : int

Index of the first variable in the slice.

last: int

Index of the last variable in the slice plus 1.

### Description:

Changes the bounds for a slice of the variables.

### See also

• Task.putconbound Changes the bound for one constraint.

# A.2.241 Task.putvarbranchorder()

```
Task.putvarbranchorder(
    j,
    priority,
    direction)
```

Assigns a branching priority and direction to a variable.

#### Arguments

```
direction : branchdir
    Specifies the preferred branching direction for variable j.
j : int
    Index of the variable.
priority : int
    The branching priority that should be assigned to variable j.
```

#### Description:

The purpose of the function is to assign a branching priority and direction. The higher priority that is assigned to an integer variable the earlier the mixed integer optimizer will branch on the variable. The branching direction controls if the optimizer branches up or down on the variable.

# A.2.242 Task.putvarname()

```
Task.putvarname(
    j,
    name)
```

Puts the name of a variable.

#### Arguments

```
j : int
    Index of the variable.
name : str
    The variable name.
```

# Description:

Puts the name of a variable.

# A.2.243 Task.putvartype()

```
Task.putvartype(
    j,
    vartype)
```

Sets the variable type of one variable.

### Arguments

```
j : int
    Index of the variable.
vartype : variabletype
    The new variable type.
```

### Description:

Sets the variable type of one variable.

See also

• Task.putvartypelist Sets the variable type for one or more variables.

# A.2.244 Task.putvartypelist()

```
Task.putvartypelist(
    subj,
    vartype)
```

Sets the variable type for one or more variables.

#### Arguments

subj : int[]

A list of variable indexes for which the variable type should be changed.

vartype : variabletype

A list of variable types that should be assigned to the variables specified by subj. See section variabletype for the possible values of vartype.

#### Description:

Sets the variable type for one or more variables, i.e. variable number subj[k] is assigned the variable type vartype[k].

If the same index is specified multiple times in subj only the last entry takes effect.

See also

• Task.putvartype Sets the variable type of one variable.

# A.2.245 Task.putxc()

```
Task.putxc(
    whichsol,
    xc)
```

Sets the xc vector for a solution.

### Arguments

```
whichsol : soltype Selects a solution.

xc : double[]

The x^c vector.
```

### Description:

Sets the  $x^c$  vector for a solution.

See also

• Task.putxcslice Sets a slice of the xc vector for a solution.

# A.2.246 Task.putxcslice()

```
Task.putxcslice(
    whichsol,
    first,
    last,
    xc)
```

Sets a slice of the xc vector for a solution.

#### Arguments

```
first : int
   First index in the sequence.
last : int
   Last index plus 1 in the sequence.
whichsol : soltype
   Selects a solution.
xc : double[]
   Primal constraint solution.
```

### Description:

Sets a slice of the  $x^c$  vector for a solution.

See also

• Task.putxc Sets the xc vector for a solution.

# A.2.247 Task.putxx()

```
Task.putxx(
    whichsol,
    xx)
```

Sets the xx vector for a solution.

### Arguments

```
whichsol: soltype
Selects a solution.

xx: double[]
The x^x vector.
```

Description:

Sets the  $x^x$  vector for a solution.

See also

• Task.putxxslice Obtains a slice of the xx vector for a solution.

# A.2.248 Task.putxxslice()

```
Task.putxxslice(
   whichsol,
   first,
   last,
   xx)
```

Obtains a slice of the xx vector for a solution.

# Arguments

```
first : int
```

First index in the sequence.

y)

last : int

```
Last index plus 1 in the sequence.
      whichsol : soltype
          Selects a solution.
      xx : double[]
          Primal variable solution.
 Description:
     Obtains a slice of the x^x vector for a solution.
 See also
        • Task.putxx Sets the xx vector for a solution.
A.2.249
            Task.puty()
Task.puty(
    whichsol,
     Sets the y vector for a solution.
 Arguments
      whichsol : soltype
          Selects a solution.
      y : double[]
          The y vector.
 Description:
     Sets the y vector for a solution.
 See also
       • Task.putyslice Sets a slice of the y vector for a solution.
A.2.250
            Task.putyslice()
Task.putyslice(
    whichsol,
    first,
    last,
```

Sets a slice of the y vector for a solution.

### Arguments

first : int

First index in the sequence.

last : int

Last index plus 1 in the sequence.

whichsol: soltype Selects a solution.

y : double[]

Vector of dual variables corresponding to the constraints.

### Description:

Sets a slice of the y vector for a solution.

See also

• Task.puty Sets the y vector for a solution.

# A.2.251 Task.readbranchpriorities()

Task.readbranchpriorities(filename)

Reads branching priority data from a file.

### Arguments

filename: str

Data is read from the file filename.

# ${\bf Description:}$

Reads branching priority data from a file.

See also

• Task.writebranchpriorities Writes branching priority data to a file.

### A.2.252 Task.readdata()

```
Task.readdata(filename)
```

Reads problem data from a file.

Arguments

```
filename : str
```

Data is read from the file filename.

Description:

Reads an optimization problem and associated data from a file.

See also

• iparam.read\_data\_format Format of the data file to be read.

### A.2.253 Task.readdataformat()

```
Task.readdataformat(
    filename,
    format,
    compress)
```

Reads problem data from a file.

Arguments

compress : compresstype
File compression type.

filename : str

Data is read from the file filename.

format : dataformat
File data format.

Description:

Reads an optimization problem and associated data from a file.

See also

• iparam.read\_data\_format Format of the data file to be read.

### A.2.254 Task.readparamfile()

```
Task.readparamfile()
```

Reads a parameter file.

Description:

Reads a parameter file.

### A.2.255 Task.readsolution()

```
Task.readsolution(
    whichsol,
    filename)
```

Reads a solution from a file.

### Arguments

```
filename: str
A valid file name.
whichsol: soltype
Selects a solution.
```

Description:

Reads a solution file and inserts the solution into the solution whichsol.

# A.2.256 Task.readsummary()

```
Task.readsummary(whichstream)
```

Prints information about last file read.

Arguments

```
whichstream: streamtype
Index of the stream.
```

Description:

Prints a short summary of last file that was read.

### A.2.257 Task.readtask()

```
Task.readtask(filename)
```

Load task data from a file.

#### Arguments

```
filename: str
Input file name.
```

#### Description:

Load task data from a file, replacing any data that already is in the task object. All problem data are resorted, but if the file contains solutions, the solution status after loading a file is still unknown, even if it was optimal or otherwise well-defined when the file was dumped.

See section F.4 for a description of the Task format.

### A.2.258 Task.relaxprimal()

```
relaxedtask = Task.relaxprimal(
   wlc,
   wuc,
   wlx,
   wux)
```

Deprecated.

### Arguments

# relaxedtask : Task The returned task.

#### wlc : double[]

Weights associated with lower bounds on the activity of constraints. If negative, the bound is strictly enforced, i.e. if  $(w_l^c)_i < 0$ , then  $(v_l^c)_i$  is fixed to zero. On return wlc[i] contains the relaxed bound.

### wlx : double[]

Weights associated with lower bounds on the activity of variables. If negative, the bound is strictly enforced, i.e. if  $(w_l^x)_j < 0$  then  $(v_l^x)_j$  is fixed to zero. On return wlx[i] contains the relaxed bound.

```
wuc : double[]
```

Weights associated with upper bounds on the activity of constraints. If negative, the bound is strictly enforced, i.e. if  $(w_u^c)_i < 0$ , then  $(v_u^c)_i$  is fixed to zero. On return wuc[i] contains the relaxed bound.

#### wux : double[]

Weights associated with upper bounds on the activity of variables. If negative, the bound is strictly enforced, i.e. if  $(w_u^x)_j < 0$  then  $(v_u^x)_j$  is fixed to zero. On return wux[i] contains the relaxed bound.

#### Description:

Deprecated. Please use Task.primalrepair instead.

See also

- dparam.feasrepair\_tol Tolerance for constraint enforcing upper bound on sum of weighted violations in feasibility repair.
- iparam.feasrepair\_optimize Controls which type of feasibility analysis is to be performed.
- sparam.feasrepair\_name\_separator Feasibility repair name separator.
- sparam.feasrepair\_name\_prefix Feasibility repair name prefix.

### A.2.259 Task.removebarvars()

Task.removebarvars(subset)

The function removes a number of symmetric matrix.

#### Arguments

```
subset : int[]
```

Indexes of symmetric matrix which should be removed.

#### Description:

The function removes a subset of the symmetric matrix from the optimization task. This implies that the existing symmetric matrix are renumbered, for instance if constraint 5 is removed then constraint 6 becomes constraint 5 and so forth.

See also

• Task.appendbarvars Appends a semidefinite variable of dimension dim to the problem.

### A.2.260 Task.removecones()

Task.removecones(subset)

Removes a conic constraint from the problem.

#### Arguments

subset : int[]

Indexes of cones which should be removed.

#### Description:

Removes a number conic constraint from the problem. In general, it is much more efficient to remove a cone with a high index than a low index.

### A.2.261 Task.removecons()

Task.removecons(subset)

The function removes a number of constraints.

#### Arguments

subset : int[]

Indexes of constraints which should be removed.

#### Description:

The function removes a subset of the constraints from the optimization task. This implies that the existing constraints are renumbered, for instance if constraint 5 is removed then constraint 6 becomes constraint 5 and so forth.

See also

• Task.appendcons Appends a number of constraints to the optimization task.

#### A.2.262 Task.removevars()

Task.removevars(subset)

The function removes a number of variables.

### Arguments

subset : int[]

Indexes of variables which should be removed.

#### Description:

The function removes a subset of the variables from the optimization task. This implies that the existing variables are renumbered, for instance if constraint 5 is removed then constraint 6 becomes constraint 5 and so forth.

#### See also

• Task. appendvars Appends a number of variables to the optimization task.

### A.2.263 Task.resizetask()

```
Task.resizetask(
    maxnumcon,
    maxnumvar,
    maxnumcone,
    maxnumanz,
    maxnumqnz)
```

Resizes an optimization task.

#### Arguments

maxnumanz : long

New maximum number of non-zeros in A.

maxnumcon: int

New maximum number of constraints.

maxnumcone : int

New maximum number of cones.

maxnumqnz : long

New maximum number of non-zeros in all Q matrixes.

maxnumvar : int

New maximum number of variables.

#### Description:

Sets the amount of preallocated space assigned for each type of data in an optimization task.

It is never mandatory to call this function, since its only function is to give a hint of the amount of data to preallocate for efficiency reasons.

Please note that the procedure is **destructive** in the sense that all existing data stored in the task is destroyed.

#### See also

- $\bullet \ \, \textbf{Task.putmaxnumvar} \ \, \textbf{Sets the number of preallocated variables in the optimization task}.$
- Task.putmaxnumcon Sets the number of preallocated constraints in the optimization task.
- Task.putmaxnumcone Sets the number of preallocated conic constraints in the optimization task.
- Task.putmaxnumanz The function changes the size of the preallocated storage for linear coefficients.
- Task.putmaxnumqnz Changes the size of the preallocated storage for quadratic terms.

### A.2.264 Task.sensitivityreport()

Task.sensitivityreport(whichstream)

Creates a sensitivity report.

#### Arguments

```
whichstream: streamtype
Index of the stream.
```

#### Description:

Reads a sensitivity format file from a location given by sparam.sensitivity\_file\_name and writes the result to the stream whichstream. If sparam.sensitivity\_res\_file\_name is set to a non-empty string, then the sensitivity report is also written to a file of this name.

#### See also

- Task.dualsensitivity Performs sensitivity analysis on objective coefficients.
- Task.primalsensitivity Perform sensitivity analysis on bounds.
- iparam.log\_sensitivity Control logging in sensitivity analyzer.
- iparam.log\_sensitivity\_opt Control logging in sensitivity analyzer.
- iparam.sensitivity\_type Controls which type of sensitivity analysis is to be performed.

#### A.2.265 Task.setdefaults()

Task.setdefaults()

Resets all parameters values.

### Description:

Resets all the parameters to their default values.

#### A.2.266 Task.solutiondef()

```
isdef = Task.solutiondef(whichsol)
```

Checks whether a solution is defined.

Arguments

```
isdef : int
```

Is non-zero if the requested solution is defined.

whichsol: soltype Selects a solution.

### Description:

Checks whether a solution is defined.

### A.2.267 Task.solutionsummary()

Task.solutionsummary(whichstream)

Prints a short summary of the current solutions.

Arguments

```
whichstream: streamtype
Index of the stream.
```

Description:

Prints a short summary of the current solutions.

#### A.2.268 Task.solvewithbasis()

```
numnz = Task.solvewithbasis(
    transp,
    numnz,
    sub,
    val)
```

Solve a linear equation system involving a basis matrix.

### Arguments

numnz : int

As input it is the number of non-zeros in b. As output it is the number of non-zeros in  $\bar{X}$ .

 $\mathtt{sub}: \mathtt{int}[]$ 

As input it contains the positions of the non-zeros in b, i.e.

$$b[\operatorname{sub}[k]] \neq 0, \ k = 0, \dots, numnz[0] - 1.$$

As output it contains the positions of the non-zeros in  $\bar{X}$ . It is important that sub has room for numcon elements.

transp: int

If this argument is non-zero, then (A.3) is solved. Otherwise the system (A.2) is solved.

#### val : double[]

As input it is the vector b. Although the positions of the non-zero elements are specified in sub it is required that val[i] = 0 if b[i] = 0. As output val is the vector  $\bar{X}$ .

Please note that val is a dense vector — not a packed sparse vector. This implies that val has room for numcon elements.

#### Description:

If a basic solution is available, then exactly numcon basis variables are defined. These numcon basis variables are denoted the basis. Associated with the basis is a basis matrix denoted B. This function solves either the linear equation system

$$B\bar{X} = b \tag{A.2}$$

or the system

$$B^T \bar{X} = b \tag{A.3}$$

for the unknowns  $\bar{X}$ , with b being a user-defined vector.

In order to make sense of the solution  $\bar{X}$  it is important to know the ordering of the variables in the basis because the ordering specifies how B is constructed. When calling Task.initbasissolve an ordering of the basis variables is obtained, which can be used to deduce how MOSEK has constructed B. Indeed if the kth basis variable is variable  $x_i$  it implies that

$$B_{i,k} = A_{i,j}, i = 0, \dots, numcon - 1.$$

Otherwise if the kth basis variable is variable  $x_i^c$  it implies that

$$B_{i,k} = \left\{ \begin{array}{ll} -1, & i = j, \\ 0, & i \neq j. \end{array} \right.$$

Given the knowledge of how B is constructed it is possible to interpret the solution  $\bar{X}$  correctly. Please note that this function exploits the sparsity in the vector b to speed up the computations.

#### See also

- Task.initbasissolve Prepare a task for basis solver.
- iparam.basis\_solve\_use\_plus\_one Controls the sign of the columns in the basis matrix corresponding to slack variables.

# A.2.269 Task.startstat()

```
Task.startstat()
```

Starts the statistics file.

Description:

Starts the statistics file.

# A.2.270 Task.stopstat()

```
Task.stopstat()
```

Stops the statistics file.

Description:

Stops the statistics file.

# A.2.271 Task.strtoconetype()

```
conetype = Task.strtoconetype(str)
```

Obtains a cone type code.

Arguments

conetype : conetype

The cone type corresponding to the string str.

str : str

String corresponding to the cone type code codetype.

Description:

Obtains cone type code corresponding to a cone type string.

### A.2.272 Task.strtosk()

```
sk = Task.strtosk(str)
```

Obtains a status key.

#### Arguments

sk: int

Status key corresponding to the string.

str: str

Status key string.

#### Description:

Obtains the status key corresponding to an explanatory string.

### A.2.273 Task.toconic()

Task.toconic()

Inplace reformulation of a QCQP to a COP

#### Description:

This function tries to reformulate a given Quadratically Constrained Quadratic Optimization problem (QCQP) as a Conic Quadratic Optimization problem (CQO). The first step of the reformulation is to convert the quadratic term of the objective function as a constraint, if any. Then the following steps are repeated for each quadratic constraint:

- a conic constraint is added along with a suitable number of auxiliary variables and constraints;
- the original quadratic constraint is not removed, but all its coefficients are zeroed out.

Note that the reformulation preserves all the original variables.

The conversion is performed in-place, i.e. the task passed as argument is modified on exit. That also means that if the reformulation fails, i.e. the given QCQP is not representable as a CQO, then the task has an undefined state. In some cases, users may want to clone the task to ensure a clean copy is preserved.

# A.2.274 Task.updatesolutioninfo()

```
Task.updatesolutioninfo(whichsol)
```

Update the information items related to the solution.

Arguments

```
whichsol: soltype Selects a solution.
```

Description:

Update the information items related to the solution.

# A.2.275 Task.writebranchpriorities()

```
Task.writebranchpriorities(filename)
```

Writes branching priority data to a file.

Arguments

```
filename : str
```

Data is written to the file filename.

Description:

Writes branching priority data to a file.

See also

• Task.readbranchpriorities Reads branching priority data from a file.

### A.2.276 Task.writedata()

Task.writedata(filename)

Writes problem data to a file.

Arguments

#### filename: str

Data is written to the file filename if it is a nonempty string. Otherwise data is written to the file specified by sparam.data\_file\_name.

#### Description:

Writes problem data associated with the optimization task to a file in one of four formats:

LP:

A text based row oriented format. File extension .1p. See Appendix F.2.

MPS:

A text based column oriented format. File extension .mps. See Appendix F.1.

OPF:

A text based row oriented format. File extension .opf. Supports more problem types than MPS and LP. See Appendix F.3.

TASK:

A MOSEK specific binary format for fast reading and writing. File extension .task.

By default the data file format is determined by the file name extension. This behaviour can be overridden by setting the <code>iparam.write\_data\_format</code> parameter.

MOSEK is able to read and write files in a compressed format (gzip). To write in the compressed format append the extension ".gz". E.g to write a gzip compressed MPS file use the extension mps.gz.

Please note that MPS, LP and OPF files require all variables to have unique names. If a task contains no names, it is possible to write the file with automaticly generated anonymous names by setting the <code>iparam.write\_generic\_names</code> parameter to <code>onoffkey.on</code>.

Please note that if a general nonlinear function appears in the problem then such function *cannot* be written to file and MOSEK will issue a warning.

See also

• iparam.write\_data\_format Controls the output file format.

### A.2.277 Task.writeparamfile()

Task.writeparamfile(filename)

Writes all the parameters to a parameter file.

Arguments

filename : str

The name of parameter file.

Description:

Writes all the parameters to a parameter file.

### A.2.278 Task.writesolution()

```
Task.writesolution(
    whichsol,
    filename)
```

Write a solution to a file.

#### Arguments

```
filename: str
A valid file name.
whichsol: soltype
Selects a solution.
```

#### Description:

Saves the current basic, interior-point, or integer solution to a file.

See also

- iparam.write\_sol\_ignore\_invalid\_names Controls whither the user specified names are employed even if they are invalid names.
- iparam.write\_sol\_head Controls solution file format.
- iparam.write\_sol\_constraints Controls the solution file format.
- iparam.write\_sol\_variables Controls the solution file format.
- iparam.write\_sol\_barvariables Controls the solution file format.
- iparam.write\_bas\_head Controls the basic solution file format.
- iparam.write\_bas\_constraints Controls the basic solution file format.
- iparam.write\_bas\_variables Controls the basic solution file format.

### A.2.279 Task.writetask()

Task.writetask(filename)

Write a complete binary dump of the task data.

### Arguments

```
filename: str
Output file name.
```

### Description:

Write a binary dump of the task data. This format saves all problem data, but not callbackfunktions and general non-linear terms.

See section F.4 for a description of the Task format.

# A.3 Class Env

# A.3.1 Env.axpy()

```
Env.axpy(
    n,
    alpha,
    x,
    y)

Adds alpha times x to y.

Arguments

alpha: double
    The scalar that multiplies x.

n: int
    Length of the vectors.

x: double[]
    The vector.

y: double[]
    The vector.
```

### A.3.2 Env.checkinlicense()

Env.checkinlicense(feature)

Adds  $\alpha x$  to y.

Check in a license feature from the license server ahead of time.

Arguments

Description:

```
feature : feature
```

Feature to check in to the license system.

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#### Description:

Check in a license feature to the license server. By default all licenses consumed by functions using a single environment is kept checked out for the lifetime of the MOSEK environment. This function checks in a given license feature to the license server immidiatly.

If the given license feature is not checked out or is in use by a call to Task.optimize calling this function has no effect.

Please note that returning a license to the license server incurs a small overhead, so frequent calls to this function should be avoided.

# A.3.3 Env.checkoutlicense()

Env.checkoutlicense(feature)

Check out a license feature from the license server ahead of time.

#### Arguments

#### feature : feature

Feature to check out from the license system.

#### Description:

Check out a license feature from the license server. Normally the required license features will be automatically checked out the first time it is needed by the function **Task.optimize**. This function can be used to check out one or more features ahead of time.

The license will remain checked out for the lifetime of the MOSEK environment or until the function Env.checkinlicense is called.

If a given feature is already checked out when this function is called, only one feature will be checked out from the license server.

#### **A.3.4** Env.dot()

Computes the inner product of two vectors.

#### Arguments

#### n : int

Length of the vectors.

x : double[]

The x vector.

xty: double

The result of the inner product between x and y.

y : double[]

The y vector.

## Description:

Computes the inner product of two vectors x, y of length  $n \geq 0$ , i.e

$$x \cdot y = \sum_{i=1} x_i y_i.$$

Note that if n = 0, then the results of the operation is 0.

### A.3.5 Env.echointro()

Env.echointro(longver)

Prints an intro to message stream.

Arguments

longver: int

If non-zero, then the intro is slightly longer.

Description:

Prints an intro to message stream.

# A.3.6 Env.gemm()

Env.gemm(
 transa,
 transb,
 m,
 n,
 k,
 alpha,
 a,
 b,
 beta,
 c)

Performs a dense matrix multiplication.

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### Arguments

a : double[]

The pointer to the array storing matrix A in a column-major format.

alpha: double

A scalar value multipling the result of the matrix multiplication.

b : double[]

Indicates the number of rows of matrix B and columns of matrix A.

beta : double

A scalar value that multiplies C.

c : double[]

The pointer to the array storing matrix C in a column-major format.

k: int

Specifies the number of columns of the matrix A and the number of rows of the matrix B.

m: int

Indicates the number of rows of matrices A and C.

n: int

Indicates the number of columns of matrices B and C.

transa : transpose

Indicates whether the matrix A must be transposed.

transb : transpose

Indicates whether the matrix B must be transposed.

### Description:

Performs a matrix multiplication plus addition of dense matrices. Given A, B and C of compatible dimensions, this function computes

$$C := \alpha op(A)op(B) + \beta C$$

where  $\alpha, \beta$  are two scalar values. The function op(X) return X if transX is YES, or  $X^T$  if set to NO. Dimensions of A, b must therefore match those of C.

The result of this operation is stored in C.

### A.3.7 Env.gemv()

```
Env.gemv(
    transa,
    m,
    n,
    alpha,
    a,
    x,
    beta,
```

Computes dense matrix times a dense vector product.

## Arguments

a : double[]

A pointer to the array storing matrix A in a column-major format.

alpha: double

A scalar value multipling the matrix A.

beta : double

A scalar value multipling the vector y.

m : int

Specifies the number of rows of the matrix A.

n : int

Specifies the number of columns of the matrix A.

transa : transpose

Indicates whether the matrix A must be transposed.

x : double[]

A pointer to the array storing the vector x.

y : double[]

A pointer to the array storing the vector y.

### Description:

Computes the multiplication of a scaled dense matrix times a dense vector product, plus a scaled dense vector. In formula

$$y = \alpha Ax + \beta y$$
,

or if trans is set to transpose.yes

$$y = \alpha A^T x + \beta y,$$

where  $\alpha, \beta$  are scalar values. A is an  $n \times m$  matrix,  $x \in \mathbb{R}^m$  and  $y \in \mathbb{R}^n$ .

Note that the result is stored overwriting y.

# A.3.8 Env.getcodedesc()

symname,str = Env.getcodedesc(code)

Obtains a short description of a response code.

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### Arguments

code : rescode

A valid MOSEK response code.

str: str

Obtains a short description of a response code.

symname: str

Symbolic name corresponding to code.

### Description:

Obtains a short description of the meaning of the response code given by code.

# A.3.9 Env.getversion()

```
major,minor,build,revision = Env.getversion()
```

Obtains MOSEK version information.

### Arguments

build: int

Build number.

major : int

Major version number.

minor : int

Minor version number.

revision : int

Revision number.

### Description:

Obtains MOSEK version information.

# A.3.10 Env.licensecleanup()

Env.licensecleanup()

Stops all threads and delete all handles used by the license system.

### Description:

Stops all threads and delete all handles used by the license system. If this function is called, it must be called as the last MOSEK API call. No other MOSEK API calls are valid after this.

### A.3.11 Env.linkfiletostream()

```
Env.linkfiletostream(
    whichstream,
    filename,
    append)
```

Directs all output from a stream to a file.

### Arguments

```
append: int
```

If this argument is non-zero, the output is appended to the file.

filename: str

Sends all output from the stream defined by whichstream to the file given by filename.

```
whichstream: streamtype
Index of the stream.
```

### Description:

Directs all output from a stream to a file.

# **A.3.12** Env.potrf()

```
Env.potrf(
uplo,
n,
```

Computes a Cholesky factorization a dense matrix.

### Arguments

### a : double[]

A symmetric matrix stored in column-major order. Only the lower or the upper triangular part is used, accordingly with the uplo parameter. It will contain the result on exit.

n : int

Dimension of the symmetric matrix.

```
uplo: uplo
```

Indicates whether the upper or lower triangular part of the matrix is stored.

### Description:

Computes a Cholesky factorization of a real symmetric positive definite dense matrix.

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## A.3.13 Env.putdllpath()

Env.putdllpath(dllpath)

Sets the path to the DLL/shared libraries that MOSEK is loading.

Arguments

dllpath : str

A path to where the MOSEK dynamic link/shared libraries are located. If dllpath is NULL, then MOSEK assumes that the operating system can locate the libraries.

Description:

Sets the path to the DLL/shared libraries that MOSEK are loading.

# A.3.14 Env.putkeepdlls()

Env.putkeepdlls(keepdlls)

Controls whether explicitly loaded DLLs should be kept.

Arguments

keepdlls : int

Controls whether explicitly loaded DLLs should be kept.

Description:

Controls whether explicitly loaded DLLs should be kept when they no longer are in use.

### A.3.15 Env.putlicensecode()

Env.putlicensecode(code)

The purpose of this function is to input a runtime license code.

Arguments

code : int[]

A runtime license code.

Description:

The purpose of this function is to input a runtime license code.

# A.3.16 Env.putlicensedebug()

Env.putlicensedebug(licdebug)

Enables debug information for the license system.

Arguments

```
licdebug: int
```

If this argument is non-zero, then MOSEK will print debug info regarding the license checkout.

Description:

If licdebug is non-zero, then MOSEK will print debug info regarding the license checkout.

# A.3.17 Env.putlicensepath()

Env.putlicensepath(licensepath)

Set the path to the license file.

Arguments

licensepath: str

A path specifycing where to search for the license.

Description:

Set the path to the license file.

### A.3.18 Env.putlicensewait()

Env.putlicensewait(licwait)

Control whether mosek should wait for an available license if no license is available.

Arguments

licwait : int

If this argument is non-zero, then MOSEK will wait for a license if no license is available. Moreover, licwait-1 is the number of milliseconds to wait between each check for an available license.

A.3. CLASS ENV 379

### Description:

If licwait is non-zero, then MOSEK will wait for a license if no license is available. Moreover, licwait-1 is the number of milliseconds to wait between each check for an available license.

# A.3.19 Env.syeig()

```
Env.syeig(
    uplo,
    n,
    a,
    w)
```

Computes all eigenvalues of a symmetric dense matrix.

### Arguments

```
a : double[]
```

A symmetric matrix stored in column-major order. Only the lower-triangular part is used.

 $\mathtt{n}$ : int

Dimension of the symmetric input matrix.

```
uplo : uplo
```

Indicates whether the upper or lower triangular part is used.

w : double[]

Array of minimum dimension n where eigenvalues will be stored.

### Description:

Computes all eigenvalues of a real symmetric matrix A. Eigenvalues are stored in the w array.

### A.3.20 Env.syevd()

```
Env.syevd(
uplo,
n,
a,
w)
```

Computes all the eigenvalue and eigenvectors of a symmetric dense matrix, and thus its eigenvalue decomposition.

### Arguments

#### a : double[]

A symmetric matrix stored in column-major order. Only the lower-triangular part is used. It will be overwritten on exit.

#### n: int

Dimension of symmetric input matrix.

### uplo: uplo

Indicates whether the upper or lower triangular part is used.

### w : double[]

An array where eigenvalues will be stored. Its lenght must be at least the dimension of the input matrix.

### Description:

Computes all the eigenvalues and eigenvectors a real symmetric matrix.

Given the input matrix  $A \in \mathbb{R}^{n \times n}$ , this function returns a vector  $w \in \mathbb{R}^n$  containing the eigenvalues of A and the corresponding eigenvectors, stored in A as well.

Therefore, this function compute the eigenvalue decomposition of A as

$$A = UVU^T$$
.

where V = diag(w) and U contains the eigen-vectors of A.

### A.3.21 Env.syrk()

```
Env.syrk(
uplo,
trans,
n,
k,
alpha,
a,
beta,
c)
```

Performs a rank-k update of a symmetric matrix.

### Arguments

### a : double[]

The pointer to the array storing matrix A in a column-major format.

### alpha: double

A scalar value multipling the result of the matrix multiplication.

### beta : double

A scalar value that multiplies C.

c : double[]

The pointer to the array storing matrix C in a column-major format.

k : int

Indicates the number of rows or columns of A, and its rank.

n : int

Specifies the order of C.

trans: transpose

Indicates whether the matrix A must be transposed.

uplo: uplo

Indicates whether the upper or lower triangular part of C is stored.

### Description:

Performs a symmetric rank-k update for a symmetric matrix.

Given a symmetric matrix  $C \in \mathbb{R}^{n \times n}$ , two scalars  $\alpha, \beta$  and a matrix A of rank  $k \leq n$ , it computes either

$$C = \alpha A A^T + \beta C,$$

or

$$C = \alpha A^T A + \beta C.$$

In the first case  $A \in \mathbb{R}^{k \times n}$ , in the second  $A \in \mathbb{R}^{n \times k}$ .

Note that the results overwrite the matrix C.

# A.4 Callback functions and related methods

Callbacks from task methods are performed by attaching a function to the task object.

### A.4.1 Progress callback

When MOSEK is optimizing or doing other tasks that may potentially take a long time, it is possible to recieve periodical calls from MOSEK indicating the current status. This achieved by attaching a callback handler to the task. A callback handler is simply a function that accepts one integer argument and returns an integer.

The argument is an integer indicating the current location in MOSEK. If the function returns a non-zero value or raises an exception, MOSEK will attempt to stop the current task and return as fast as possible.

```
def myCallback(caller, dinf, iinf, liinf):
    print "Caller: %d, intpnt iteration = %d" % (caller, iinf[iinfitem.intpnt_iter])
```

```
return 0
task.set_Progress(myCallback)
```

The arguments dinf, iinf and liinf are arrays of values containing information items for the current state of the solver.

The Progress callback can be detached by calling

```
task.set_Progress(None)
```

NOTE: Due to the way the Python garbage collector works, it is necessary to hold a reference to the callback function for the duration of the lifetime of the Task object. This means that a construction as

```
#NOTE: WRONG way to attach callback!
task.set_Progress(lambda caller, dinf, liinf; print "Caller : %d" % caller)
```

will not work.

### A.4.2 Stream callback

Text written to specific MOSEK streams can be intercepted by attaching a stream handler to the specific stream. A stream handler is simply a function that accepts one string argument.

To attach a stream callback to a Task, use:

```
def myStream(msg):
    sys.stdout.write(msg)
    sys.stdout.flush()
task.set_Stream(streamtype.log,myStream)
```

The Stream object can be detached by calling

```
task.set_Stream(None)
```

In this example we attached to the log stream; see streamtype for other options.

NOTE: As for Progress callbacks, it is necessary to hold a reference to the callback function for the duration of the lifetime of the Task or ttEnv object.

# A.5 Dispose and garbage collection

The garbage collecter in Python will automatically dispose and reuse both Task and Env objects, but since these objects may allocate substantial amounts of memory or reserve MOSEK licenses outside

Python, in some cases it is desireable to explicitly free up this memory when the task or environment will not be used anymore.

The native memory associated with Task and Env can be free'd by calling the \_\_del\_\_() method:

```
task.__del__()
env.__del__()
```

Notice that even if an Env is disposed before all Tasks that belongs to it were disposed, its memory will not be free'd before that last task is disposed.

# A.6 All functions by name

### Task.analyzenames

Analyze the names and issue an error for the first invalid name.

#### Task.analyzeproblem

Analyze the data of a task.

#### Task.analyzesolution

Print information related to the quality of the solution.

#### Task.appendbarvars

Appends a semidefinite variable of dimension dim to the problem.

### Task.appendcone

Appends a new cone constraint to the problem.

### Task.appendconeseq

Appends a new conic constraint to the problem.

### Task.appendconesseq

Appends multiple conic constraints to the problem.

### Task.appendcons

Appends a number of constraints to the optimization task.

#### Task.appendsparsesymmat

Appends a general sparse symmetric matrix to the vector E of symmetric matrixes.

### Task.appendstat

Appends a record the statistics file.

### Task.appendvars

Appends a number of variables to the optimization task.

#### Env.axpy

Adds alpha times x to y.

### Task.basiscond

Computes conditioning information for the basis matrix.

### Task.checkconvexity

Checks if a quadratic optimization problem is convex.

#### Env.checkinlicense

Check in a license feature from the license server ahead of time.

### Task.checkmem

Checks the memory allocated by the task.

#### Env.checkoutlicense

Check out a license feature from the license server ahead of time.

### Task.chgbound

Changes the bounds for one constraint or variable.

#### Task.commitchanges

Commits all cached problem changes.

#### Task.deletesolution

Undefines a solution and frees the memory it uses.

### ${\tt Env.dot}$

Computes the inner product of two vectors.

### Task.dualsensitivity

Performs sensitivity analysis on objective coefficients.

### Env.echointro

Prints an intro to message stream.

#### Env.gemm

Performs a dense matrix multiplication.

### Env.gemv

Computes dense matrix times a dense vector product.

# Task.getacol

Obtains one column of the linear constraint matrix.

### Task.getacolnumnz

Obtains the number of non-zero elements in one column of the linear constraint matrix

### Task.getacolslicetrip

Obtains a sequence of columns from the coefficient matrix in triplet format.

### Task.getaij

Obtains a single coefficient in linear constraint matrix.

### Task.getarow

Obtains one row of the linear constraint matrix.

### Task.getarownumnz

Obtains the number of non-zero elements in one row of the linear constraint matrix

### Task.getarowslicetrip

Obtains a sequence of rows from the coefficient matrix in triplet format.

### Task.getaslice

Obtains a sequence of rows or columns from the coefficient matrix.

### Task.getbarablocktriplet

Obtains barA in block triplet form.

#### Task.getbaraidx

Obtains information about an element barA.

### Task.getbaraidxij

Obtains information about an element barA.

### ${\tt Task.getbaraidxinfo}$

Obtains the number terms in the weighted sum that forms a particular element in barA.

### Task.getbarasparsity

Obtains the sparsity pattern of the barA matrix.

### Task.getbarcblocktriplet

Obtains barc in block triplet form.

### Task.getbarcidx

Obtains information about an element in barc.

### Task.getbarcidxinfo

Obtains information about an element in barc.

#### Task.getbarcidxj

Obtains the row index of an element in barc.

### Task.getbarcsparsity

Get the positions of the nonzero elements in barc.

#### Task.getbarsj

Obtains the dual solution for a semidefinite variable.

#### Task.getbarvarname

Obtains a name of a semidefinite variable.

### Task.getbarvarnameindex

Obtains the index of name of semidefinite variable.

### Task.getbarvarnamelen

Obtains the length of a name of a semidefinite variable.

### Task.getbarxj

Obtains the primal solution for a semidefinite variable.

### Task.getbound

Obtains bound information for one constraint or variable.

### Task.getboundslice

Obtains bounds information for a sequence of variables or constraints.

### Task.getc

Obtains all objective coefficients.

### Task.getcfix

Obtains the fixed term in the objective.

### Task.getcj

Obtains one coefficient of c.

### Env.getcodedesc

Obtains a short description of a response code.

### Task.getconbound

Obtains bound information for one constraint.

### Task.getconboundslice

Obtains bounds information for a slice of the constraints.

### Task.getcone

Obtains a conic constraint.

#### Task.getconeinfo

Obtains information about a conic constraint.

### Task.getconename

Obtains a name of a cone.

### Task.getconenameindex

Checks whether the name somename has been assigned to any cone.

### Task.getconenamelen

Obtains the length of a name of a cone.

### Task.getconname

Obtains a name of a constraint.

### Task.getconnameindex

Checks whether the name somename has been assigned to any constraint.

#### Task.getconnamelen

Obtains the length of a name of a constraint variable.

### Task.getcslice

Obtains a sequence of coefficients from the objective.

### Task.getdbi

Deprecated.

### Task.getdcni

Deprecated.

### Task.getdeqi

Deprecated.

### Task.getdimbarvarj

Obtains the dimension of a symmetric matrix variable.

### Task.getdouinf

Obtains a double information item.

### Task.getdouparam

Obtains a double parameter.

### Task.getdualobj

Computes the dual objective value associated with the solution.

### Task.getdviolbarvar

Computes the violation of dual solution for a set of barx variables.

# Task.getdviolcon

Computes the violation of a dual solution associated with a set of constraints.

### Task.getdviolcones

Computes the violation of a solution for set of dual conic constraints.

### Task.getdviolvar

Computes the violation of a dual solution associated with a set of x variables.

### Task.getinfeasiblesubproblem

Obtains an infeasible sub problem.

### Task.getinti

Deprecated.

### Task.getintinf

Obtains an integer information item.

### Task.getintparam

Obtains an integer parameter.

### Task.getlenbarvarj

Obtains the length if the j'th semidefinite variables.

### Task.getlintinf

Obtains an integer information item.

### Task.getmaxnumanz

Obtains number of preallocated non-zeros in the linear constraint matrix.

### Task.getmaxnumbarvar

Obtains the number of semidefinite variables.

### ${\tt Task.getmaxnumcon}$

Obtains the number of preallocated constraints in the optimization task.

### Task.getmaxnumcone

Obtains the number of preallocated cones in the optimization task.

# ${\tt Task.getmaxnumqnz}$

Obtains the number of preallocated non-zeros for all quadratic terms in objective and constraints.

### Task.getmaxnumvar

Obtains the maximum number variables allowed.

### Task.getmemusage

Obtains information about the amount of memory used by a task.

#### Task.getnumanz

Obtains the number of non-zeros in the coefficient matrix.

### Task.getnumanz64

Obtains the number of non-zeros in the coefficient matrix.

### Task.getnumbarablocktriplets

Obtains an upper bound on the number of scalar elements in the block triplet form of bara.

#### Task.getnumbaranz

Get the number of nonzero elements in barA.

### Task.getnumbarcblocktriplets

Obtains an upper bound on the number of elements in the block triplet form of barc.

### Task.getnumbarcnz

Obtains the number of nonzero elements in barc.

#### Task.getnumbarvar

Obtains the number of semidefinite variables.

### Task.getnumcon

Obtains the number of constraints.

### Task.getnumcone

Obtains the number of cones.

#### Task.getnumconemem

Obtains the number of members in a cone.

### Task.getnumintvar

Obtains the number of integer-constrained variables.

#### Task.getnumparam

Obtains the number of parameters of a given type.

### Task.getnumqconknz

Obtains the number of non-zero quadratic terms in a constraint.

### Task.getnumqconknz64

Obtains the number of non-zero quadratic terms in a constraint.

### Task.getnumqobjnz

Obtains the number of non-zero quadratic terms in the objective.

### Task.getnumsymmat

Get the number of symmetric matrixes stored.

### Task.getnumvar

Obtains the number of variables.

#### Task.getobjname

Obtains the name assigned to the objective function.

### Task.getobjnamelen

Obtains the length of the name assigned to the objective function.

### Task.getobjsense

Gets the objective sense.

### Task.getpbi

Deprecated.

### Task.getpcni

Deprecated.

### Task.getpeqi

Deprecated.

### Task.getprimalobj

Computes the primal objective value for the desired solution.

### Task.getprobtype

Obtains the problem type.

### Task.getprosta

Obtains the problem status.

### Task.getpviolbarvar

Computes the violation of a primal solution for a list of barx variables.

### Task.getpviolcon

Computes the violation of a primal solution for a list of xc variables.

### Task.getpviolcones

Computes the violation of a solution for set of conic constraints.

### Task.getpviolvar

Computes the violation of a primal solution for a list of x variables.

### Task.getqconk

Obtains all the quadratic terms in a constraint.

### Task.getqobj

Obtains all the quadratic terms in the objective.

# Task.getqobj64

Obtains all the quadratic terms in the objective.

### Task.getqobjij

Obtains one coefficient from the quadratic term of the objective

### Task.getreducedcosts

Obtains the difference of (slx-sux) for a sequence of variables.

### Task.getskc

Obtains the status keys for the constraints.

### Task.getskcslice

Obtains the status keys for the constraints.

### Task.getskx

Obtains the status keys for the scalar variables.

#### Task.getskxslice

Obtains the status keys for the variables.

### Task.getslc

Obtains the slc vector for a solution.

### Task.getslcslice

Obtains a slice of the slc vector for a solution.

#### Task.getslx

Obtains the slx vector for a solution.

### Task.getslxslice

Obtains a slice of the slx vector for a solution.

### ${\tt Task.getsnx}$

Obtains the snx vector for a solution.

### Task.getsnxslice

Obtains a slice of the snx vector for a solution.

### Task.getsolsta

Obtains the solution status.

### Task.getsolution

Obtains the complete solution.

### Task.getsolutioni

Obtains the solution for a single constraint or variable.

# Task.getsolutioninf

Deprecated

### Task.getsolutioninfo

Obtains information about of a solution.

### Task.getsolutionslice

Obtains a slice of the solution.

### Task.getsparsesymmat

Gets a single symmetric matrix from the matrix store.

### Task.getstrparam

Obtains the value of a string parameter.

### Task.getstrparamlen

Obtains the length of a string parameter.

#### Task.getsuc

Obtains the suc vector for a solution.

### Task.getsucslice

Obtains a slice of the suc vector for a solution.

### Task.getsux

Obtains the sux vector for a solution.

#### Task.getsuxslice

Obtains a slice of the sux vector for a solution.

### Task.getsymmatinfo

Obtains information of a matrix from the symmetric matrix storage E.

### Task.gettaskname

Obtains the task name.

### Task.gettasknamelen

Obtains the length the task name.

### Task.getvarbound

Obtains bound information for one variable.

### Task.getvarboundslice

Obtains bounds information for a slice of the variables.

### Task.getvarbranchdir

Obtains the branching direction for a variable.

# Task.getvarbranchpri

Obtains the branching priority for a variable.

### Task.getvarname

Obtains a name of a variable.

### Task.getvarnameindex

Checks whether the name somename has been assigned to any variable.

### Task.getvarnamelen

Obtains the length of a name of a variable variable.

### Task.getvartype

Gets the variable type of one variable.

### Task.getvartypelist

Obtains the variable type for one or more variables.

### Task.getxc

Obtains the xc vector for a solution.

### Task.getxcslice

Obtains a slice of the xc vector for a solution.

### Task.getxx

Obtains the xx vector for a solution.

#### Task.getxxslice

Obtains a slice of the xx vector for a solution.

### Task.gety

Obtains the y vector for a solution.

### Task.getyslice

Obtains a slice of the y vector for a solution.

### Task.initbasissolve

Prepare a task for basis solver.

### Task.inputdata

Input the linear part of an optimization task in one function call.

### Task.isdouparname

Checks a double parameter name.

### Task.isintparname

Checks an integer parameter name.

# Task.isstrparname

Checks a string parameter name.

### Env.licensecleanup

Stops all threads and delete all handles used by the license system.

#### Env.linkfiletostream

Directs all output from a stream to a file.

### Task.linkfiletostream

Directs all output from a task stream to a file.

### Task.onesolutionsummary

Prints a short summary for the specified solution.

### Task.optimizeconcurrent

Optimize a given task with several optimizers concurrently.

#### Task.optimizersummary

Prints a short summary with optimizer statistics for last optimization.

### Task.optimize

Optimizes the problem.

### Env.potrf

Computes a Cholesky factorization a dense matrix.

### Task.primalrepair

The function repairs a primal infeasible optimization problem by adjusting the bounds on the constraints and variables.

### Task.primalsensitivity

Perform sensitivity analysis on bounds.

### Task.printdata

Prints a part of the problem data to a stream.

#### Task.printparam

Prints the current parameter settings.

#### Task.putacol

Replaces all elements in one column of A.

### Task.putacollist

Replaces all elements in several columns the linear constraint matrix by new values.

### Task.putacolslice

Replaces all elements in several columns the linear constraint matrix by new values.

### Task.putaij

Changes a single value in the linear coefficient matrix.

### Task.putaijlist

Changes one or more coefficients in the linear constraint matrix.

### Task.putarow

Replaces all elements in one row of A.

### Task.putarowlist

Replaces all elements in several rows the linear constraint matrix by new values.

### ${\tt Task.putbarablocktriplet}$

Inputs barA in block triplet form.

### Task.putbaraij

Inputs an element of barA.

### Task.putbarcblocktriplet

Inputs barC in block triplet form.

### Task.putbarcj

Changes one element in barc.

### Task.putbarsj

Sets the dual solution for a semidefinite variable.

### Task.putbarvarname

Puts the name of a semidefinite variable.

### Task.putbarxj

Sets the primal solution for a semidefinite variable.

### Task.putbound

Changes the bound for either one constraint or one variable.

### Task.putboundlist

Changes the bounds of constraints or variables.

### Task.putboundslice

Modifies bounds.

### Task.putcfix

Replaces the fixed term in the objective.

# Task.putcj

Modifies one linear coefficient in the objective.

### Task.putclist

Modifies a part of the linear objective coefficients.

### Task.putconbound

Changes the bound for one constraint.

### Task.putconboundlist

Changes the bounds of a list of constraints.

### Task.putconboundslice

Changes the bounds for a slice of the constraints.

### Task.putcone

Replaces a conic constraint.

### Task.putconename

Puts the name of a cone.

### Task.putconname

Puts the name of a constraint.

### Task.putcslice

Modifies a slice of the linear objective coefficients.

### Task.putdouparam

Sets a double parameter.

### Task.putintparam

Sets an integer parameter.

#### Env.putlicensecode

The purpose of this function is to input a runtime license code.

### Env.putlicensedebug

Enables debug information for the license system.

### Env.putlicensepath

Set the path to the license file.

### Env.putlicensewait

Control whether mosek should wait for an available license if no license is available.

### Task.putmaxnumanz

The function changes the size of the preallocated storage for linear coefficients.

#### Task.putmaxnumbarvar

Sets the number of preallocated symmetric matrix variables in the optimization task.

### Task.putmaxnumcon

Sets the number of preallocated constraints in the optimization task.

### Task.putmaxnumcone

Sets the number of preallocated conic constraints in the optimization task.

### Task.putmaxnumqnz

Changes the size of the preallocated storage for quadratic terms.

### Task.putmaxnumvar

Sets the number of preallocated variables in the optimization task.

### Task.putnadouparam

Sets a double parameter.

### Task.putnaintparam

Sets an integer parameter.

### Task.putnastrparam

Sets a string parameter.

### Task.putobjname

Assigns a new name to the objective.

### Task.putobjsense

Sets the objective sense.

### Task.putparam

Modifies the value of parameter.

### Task.putqcon

Replaces all quadratic terms in constraints.

### Task.putqconk

Replaces all quadratic terms in a single constraint.

### Task.putqobj

Replaces all quadratic terms in the objective.

### Task.putqobjij

Replaces one coefficient in the quadratic term in the objective.

### Task.putskc

Sets the status keys for the constraints.

# Task.putskcslice

Sets the status keys for the constraints.

### Task.putskx

Sets the status keys for the scalar variables.

### Task.putskxslice

Sets the status keys for the variables.

### Task.putslc

Sets the slc vector for a solution.

### Task.putslcslice

Sets a slice of the slc vector for a solution.

### Task.putslx

Sets the slx vector for a solution.

### Task.putslxslice

Sets a slice of the slx vector for a solution.

### Task.putsnx

Sets the snx vector for a solution.

### Task.putsnxslice

Sets a slice of the snx vector for a solution.

#### Task.putsolution

Inserts a solution.

### Task.putsolutioni

Sets the primal and dual solution information for a single constraint or variable.

### Task.putstrparam

Sets a string parameter.

### Task.putsuc

Sets the suc vector for a solution.

### Task.putsucslice

Sets a slice of the suc vector for a solution.

### Task.putsux

Sets the sux vector for a solution.

### Task.putsuxslice

Sets a slice of the sux vector for a solution.

### Task.puttaskname

Assigns a new name to the task.

### Task.putvarbound

Changes the bound for one variable.

### Task.putvarboundlist

Changes the bounds of a list of variables.

### Task.putvarboundslice

Changes the bounds for a slice of the variables.

### Task.putvarbranchorder

Assigns a branching priority and direction to a variable.

### Task.putvarname

Puts the name of a variable.

### Task.putvartype

Sets the variable type of one variable.

### Task.putvartypelist

Sets the variable type for one or more variables.

### Task.putxc

Sets the xc vector for a solution.

#### Task.putxcslice

Sets a slice of the xc vector for a solution.

### Task.putxx

Sets the xx vector for a solution.

### Task.putxxslice

Obtains a slice of the xx vector for a solution.

### Task.puty

Sets the y vector for a solution.

### Task.putyslice

Sets a slice of the y vector for a solution.

### Task.readbranchpriorities

Reads branching priority data from a file.

### Task.readdata

Reads problem data from a file.

### Task.readdataformat

Reads problem data from a file.

### Task.readparamfile

Reads a parameter file.

### Task.readsolution

Reads a solution from a file.

### Task.readsummary

Prints information about last file read.

### Task.relaxprimal

Deprecated.

### Task.removebarvars

The function removes a number of symmetric matrix.

#### Task.removecones

Removes a conic constraint from the problem.

#### Task.removecons

The function removes a number of constraints.

#### Task.removevars

The function removes a number of variables.

### Task.sensitivityreport

Creates a sensitivity report.

### Task.setdefaults

Resets all parameters values.

### Task.solutiondef

Checks whether a solution is defined.

### Task.solutionsummary

Prints a short summary of the current solutions.

### Task.solvewithbasis

Solve a linear equation system involving a basis matrix.

#### Task.startstat

Starts the statistics file.

### Task.stopstat

Stops the statistics file.

### Env.syeig

Computes all eigenvalues of a symmetric dense matrix.

### Env.syevd

Computes all the eigenvalue and eigenvectors of a symmetric dense matrix, and thus its eigenvalue decomposition.

### Env.syrk

Performs a rank-k update of a symmetric matrix.

### Task.updatesolutioninfo

Update the information items related to the solution.

#### Task.writebranchpriorities

Writes branching priority data to a file.

#### Task.writedata

Writes problem data to a file.

### Task.writeparamfile

Writes all the parameters to a parameter file.

### Task.writesolution

Write a solution to a file.

# A.6.1 Env()

Env()

The environment construction takes no arguments. Please note that each process should only construct one environment, even when multiple task object are constructed.

# A.6.2 Task()

The task object is created from an environment object and, optionally, the problem maximum dimensions. The the dimensions are not given they default to 0, put they can be changed afterwards.

If a Task object is given instead of an Env object, the new task is created using the data from the old task. Callback objects are not copied.

# Appendix B

# **Parameters**

Parameters grouped by functionality.

Analysis parameters.

Parameters controlling the behaviour of the problem and solution analyzers.

• dparam.ana\_sol\_infeas\_tol. If a constraint violates its bound with an amount larger than this value, the constraint name, index and violation will be printed by the solution analyzer.

Basis identification parameters.

- iparam.bi\_clean\_optimizer. Controls which simplex optimizer is used in the clean-up phase.
- iparam.bi\_ignore\_max\_iter. Turns on basis identification in case the interior-point optimizer is terminated due to maximum number of iterations.
- iparam.bi\_ignore\_num\_error. Turns on basis identification in case the interior-point optimizer is terminated due to a numerical problem.
- iparam.bi\_max\_iterations. Maximum number of iterations after basis identification.
- iparam.intpnt\_basis. Controls whether basis identification is performed.
- iparam.log\_bi. Controls the amount of output printed by the basis identification procedure. A higher level implies that more information is logged.
- iparam.log\_bi\_freq. Controls the logging frequency.
- dparam.sim\_lu\_tol\_rel\_piv. Relative pivot tolerance employed when computing the LU factorization of the basis matrix.

Behavior of the optimization task.

Parameters defining the behavior of an optimization task when loading data.

- sparam.feasrepair\_name\_prefix. Feasibility repair name prefix.
- sparam.feasrepair\_name\_separator. Feasibility repair name separator.

• sparam.feasrepair\_name\_wsumviol. Feasibility repair name violation name.

Conic interior-point method parameters.

Parameters defining the behavior of the interior-point method for conic problems.

- dparam.intpnt\_co\_tol\_dfeas. Dual feasibility tolerance used by the conic interior-point optimizer.
- dparam.intpnt\_co\_tol\_infeas. Infeasibility tolerance for the conic solver.
- dparam.intpnt\_co\_tol\_mu\_red. Optimality tolerance for the conic solver.
- dparam.intpnt\_co\_tol\_near\_rel. Optimality tolerance for the conic solver.
- dparam.intpnt\_co\_tol\_pfeas. Primal feasibility tolerance used by the conic interior-point optimizer.
- dparam.intpnt\_co\_tol\_rel\_gap. Relative gap termination tolerance used by the conic interior-point optimizer.

### Data check parameters.

These parameters defines data checking settings and problem data tolerances, i.e. which values are rounded to 0 or infinity, and which values are large or small enough to produce a warning.

- dparam.data\_tol\_aij. Data tolerance threshold.
- dparam.data\_tol\_aij\_huge. Data tolerance threshold.
- dparam.data\_tol\_aij\_large. Data tolerance threshold.
- dparam.data\_tol\_bound\_inf. Data tolerance threshold.
- dparam.data\_tol\_bound\_wrn. Data tolerance threshold.
- dparam.data\_tol\_c\_huge. Data tolerance threshold.
- dparam.data\_tol\_cj\_large. Data tolerance threshold.
- dparam.data\_tol\_qij. Data tolerance threshold.
- dparam.data\_tol\_x. Data tolerance threshold.
- iparam.log\_check\_convexity. Controls logging in convexity check on quadratic problems. Set to a positive value to turn logging on.

If a quadratic coefficient matrix is found to violate the requirement of PSD (NSD) then a list of negative (positive) pivot elements is printed. The absolute value of the pivot elements is also shown.

#### Data input/output parameters.

Parameters defining the behavior of data readers and writers.

- sparam.bas\_sol\_file\_name. Name of the bas solution file.
- sparam.data\_file\_name. Data are read and written to this file.
- sparam.debug\_file\_name. MOSEK debug file.
- sparam.int\_sol\_file\_name. Name of the int solution file.
- sparam.itr\_sol\_file\_name. Name of the itr solution file.

- iparam.log\_file. If turned on, then some log info is printed when a file is written or read.
- sparam.mio\_debug\_string. For internal use only.
- sparam.param\_comment\_sign. Solution file comment character.
- sparam.param\_read\_file\_name. Modifications to the parameter database is read from this file
- sparam.param\_write\_file\_name. The parameter database is written to this file.
- sparam.read\_mps\_bou\_name. Name of the BOUNDS vector used. An empty name means that the first BOUNDS vector is used.
- sparam.read\_mps\_obj\_name. Objective name in the MPS file.
- sparam.read.mps\_ran\_name. Name of the RANGE vector used. An empty name means that the first RANGE vector is used.
- sparam.read\_mps\_rhs\_name. Name of the RHS used. An empty name means that the first RHS vector is used.
- sparam.sol\_filter\_xc\_low. Solution file filter.
- sparam.sol\_filter\_xc\_upr. Solution file filter.
- sparam.sol\_filter\_xx\_low. Solution file filter.
- sparam.sol\_filter\_xx\_upr. Solution file filter.
- sparam.stat\_file\_name. Statistics file name.
- sparam.stat\_key. Key used when writing the summary file.
- sparam.stat\_name. Name used when writing the statistics file.
- sparam.write\_lp\_gen\_var\_name. Added variable names in the LP files.

### Debugging parameters.

These parameters defines that can be used when debugging a problem.

• iparam.auto\_sort\_a\_before\_opt. Controls whether the elements in each column of A are sorted before an optimization is performed.

### Dual simplex optimizer parameters.

Parameters defining the behavior of the dual simplex optimizer for linear problems.

- iparam.sim\_dual\_crash. Controls whether crashing is performed in the dual simplex optimizer.
- iparam.sim\_dual\_restrict\_selection. Controls how aggressively restricted selection is used.
- iparam.sim\_dual\_selection. Controls the dual simplex strategy.

### Feasibility repair parameters.

• dparam.feasrepair\_tol. Tolerance for constraint enforcing upper bound on sum of weighted violations in feasibility repair.

Infeasibility report parameters.

• iparam.log\_infeas\_ana. Controls log level for the infeasibility analyzer.

Interior-point method parameters.

Parameters defining the behavior of the interior-point method for linear, conic and convex problems.

- iparam.bi\_ignore\_max\_iter. Turns on basis identification in case the interior-point optimizer is terminated due to maximum number of iterations.
- iparam.bi\_ignore\_num\_error. Turns on basis identification in case the interior-point optimizer is terminated due to a numerical problem.
- dparam.check\_convexity\_rel\_tol. Convexity check tolerance.
- iparam.intpnt\_basis. Controls whether basis identification is performed.
- dparam.intpnt\_co\_tol\_dfeas. Dual feasibility tolerance used by the conic interior-point optimizer.
- dparam.intpnt\_co\_tol\_infeas. Infeasibility tolerance for the conic solver.
- dparam.intpnt\_co\_tol\_mu\_red. Optimality tolerance for the conic solver.
- dparam.intpnt\_co\_tol\_near\_rel. Optimality tolerance for the conic solver.
- dparam.intpnt\_co\_tol\_pfeas. Primal feasibility tolerance used by the conic interior-point optimizer.
- dparam.intpnt\_co\_tol\_rel\_gap. Relative gap termination tolerance used by the conic interior-point optimizer.
- iparam.intpnt\_diff\_step. Controls whether different step sizes are allowed in the primal and dual space.
- iparam.intpnt\_max\_iterations. Controls the maximum number of iterations allowed in the interior-point optimizer.
- iparam.intpnt\_max\_num\_cor. Maximum number of correction steps.
- iparam.intpnt\_max\_num\_refinement\_steps. Maximum number of steps to be used by the iterative search direction refinement.
- dparam.intpnt\_nl\_merit\_bal. Controls if the complementarity and infeasibility is converging to zero at about equal rates.
- dparam.intpnt\_nl\_tol\_dfeas. Dual feasibility tolerance used when a nonlinear model is solved.
- dparam.intpnt\_nl\_tol\_mu\_red. Relative complementarity gap tolerance.
- dparam.intpnt\_nl\_tol\_near\_rel. Nonlinear solver optimality tolerance parameter.
- dparam.intpnt\_nl\_tol\_pfeas. Primal feasibility tolerance used when a nonlinear model is solved.
- dparam.intpnt\_nl\_tol\_rel\_gap. Relative gap termination tolerance for nonlinear problems.

- dparam.intpnt\_nl\_tol\_rel\_step. Relative step size to the boundary for general nonlinear optimization problems.
- iparam.intpnt\_off\_col\_trh. Controls the aggressiveness of the offending column detection.
- iparam.intpnt\_order\_method. Controls the ordering strategy.
- iparam.intpnt\_regularization\_use. Controls whether regularization is allowed.
- iparam.intpnt\_scaling. Controls how the problem is scaled before the interior-point optimizer is used.
- iparam.intpnt\_solve\_form. Controls whether the primal or the dual problem is solved.
- iparam.intpnt\_starting\_point. Starting point used by the interior-point optimizer.
- dparam.intpnt\_tol\_dfeas. Dual feasibility tolerance used for linear and quadratic optimization problems.
- dparam.intpnt\_tol\_dsafe. Controls the interior-point dual starting point.
- dparam.intpnt\_tol\_infeas. Nonlinear solver infeasibility tolerance parameter.
- dparam.intpnt\_tol\_mu\_red. Relative complementarity gap tolerance.
- dparam.intpnt\_tol\_path. interior-point centering aggressiveness.
- dparam.intpnt\_tol\_pfeas. Primal feasibility tolerance used for linear and quadratic optimization problems.
- dparam.intpnt\_tol\_psafe. Controls the interior-point primal starting point.
- dparam.intpnt\_tol\_rel\_gap. Relative gap termination tolerance.
- dparam.intpnt\_tol\_rel\_step. Relative step size to the boundary for linear and quadratic optimization problems.
- dparam.intpnt\_tol\_step\_size. If the step size falls below the value of this parameter, then the interior-point optimizer assumes that it is stalled. In other words the interior-point optimizer does not make any progress and therefore it is better stop.
- iparam.log\_intpnt. Controls the amount of log information from the interior-point optimizers.
- iparam.log\_presolve. Controls amount of output printed by the presolve procedure. A higher level implies that more information is logged.
- dparam.qcqo\_reformulate\_rel\_drop\_tol. This parameter determines when columns are dropped in incomplete cholesky factorization doing reformulation of quadratic problems.

#### License manager parameters.

- iparam.cache\_license. Control license caching.
- iparam.license\_debug. Controls the license manager client debugging behavior.
- iparam.license\_pause\_time. Controls license manager client behavior.
- iparam.license\_suppress\_expire\_wrns. Controls license manager client behavior.
- iparam.license\_wait. Controls if MOSEK should queue for a license if none is available.

### Logging parameters.

- iparam.log. Controls the amount of log information.
- iparam.log\_bi. Controls the amount of output printed by the basis identification procedure. A higher level implies that more information is logged.
- iparam.log\_bi\_freq. Controls the logging frequency.
- iparam.log\_concurrent. Controls amount of output printed by the concurrent optimizer.
- iparam.log\_expand. Controls the amount of logging when a data item such as the maximum number constrains is expanded.
- iparam.log\_factor. If turned on, then the factor log lines are added to the log.
- iparam.log\_feas\_repair. Controls the amount of output printed when performing feasibility repair. A value higher than one means extensive logging.
- iparam.log\_file. If turned on, then some log info is printed when a file is written or read.
- iparam.log head. If turned on, then a header line is added to the log.
- iparam.log\_infeas\_ana. Controls log level for the infeasibility analyzer.
- iparam.log\_intpnt. Controls the amount of log information from the interior-point optimizers.
- iparam.log\_mio. Controls the amount of log information from the mixed-integer optimizers.
- iparam.log\_mio\_freq. The mixed-integer solver logging frequency.
- iparam.log nonconvex. Controls amount of output printed by the nonconvex optimizer.
- iparam.log\_optimizer. Controls the amount of general optimizer information that is logged.
- iparam.log\_order. If turned on, then factor lines are added to the log.
- iparam.log\_param. Controls the amount of information printed out about parameter changes.
- iparam.log\_presolve. Controls amount of output printed by the presolve procedure. A higher level implies that more information is logged.
- iparam.log\_response. Controls amount of output printed when response codes are reported. A higher level implies that more information is logged.
- iparam.log\_sim. Controls the amount of log information from the simplex optimizers.
- iparam.log\_sim\_freq. Controls simplex logging frequency.
- iparam.log\_sim\_network\_freq. Controls the network simplex logging frequency.
- iparam.log\_storage. Controls the memory related log information.

### Mixed-integer optimization parameters.

- iparam.log\_mio. Controls the amount of log information from the mixed-integer optimizers.
- iparam.log\_mio\_freq. The mixed-integer solver logging frequency.
- iparam.mio\_branch\_dir. Controls whether the mixed-integer optimizer is branching up or down by default.

- iparam.mio\_construct\_sol. Controls if an initial mixed integer solution should be constructed from the values of the integer variables.
- iparam.mio\_cont\_sol. Controls the meaning of interior-point and basic solutions in mixed integer problems.
- iparam.mio\_cut\_cg. Controls whether CG cuts should be generated.
- iparam.mio\_cut\_cmir. Controls whether mixed integer rounding cuts should be generated.
- iparam.mio\_cut\_level\_root. Controls the cut level employed by the mixed-integer optimizer at the root node.
- iparam.mio\_cut\_level\_tree. Controls the cut level employed by the mixed-integer optimizer in the tree.
- dparam.mio\_disable\_term\_time. Certain termination criteria is disabled within the mixed-integer optimizer for period time specified by the parameter.
- iparam.mio\_feaspump\_level. Controls the feasibility pump heuristic which is used to construct a good initial feasible solution.
- iparam.mio\_heuristic\_level. Controls the heuristic employed by the mixed-integer optimizer to locate an initial integer feasible solution.
- dparam.mio\_heuristic\_time. Time limit for the mixed-integer heuristics.
- iparam.mio\_hotstart. Controls whether the integer optimizer is hot-started.
- iparam.mio\_keep\_basis. Controls whether the integer presolve keeps bases in memory.
- iparam.mio\_max\_num\_branches. Maximum number of branches allowed during the branch and bound search.
- iparam.mio\_max\_num\_relaxs. Maximum number of relaxations in branch and bound search.
- iparam.mio\_max\_num\_solutions. Controls how many feasible solutions the mixed-integer optimizer investigates.
- dparam.mio\_max\_time. Time limit for the mixed-integer optimizer.
- dparam.mio\_max\_time\_aprx\_opt. Time limit for the mixed-integer optimizer.
- dparam.mio\_near\_tol\_abs\_gap. Relaxed absolute optimality tolerance employed by the mixed-integer optimizer.
- dparam.mio\_near\_tol\_rel\_gap. The mixed-integer optimizer is terminated when this tolerance is satisfied.
- iparam.mio\_node\_optimizer. Controls which optimizer is employed at the non-root nodes in the mixed-integer optimizer.
- iparam.mio\_node\_selection. Controls the node selection strategy employed by the mixed-integer optimizer.
- iparam.mio\_optimizer\_mode. An exprimental feature.
- iparam.mio\_presolve\_aggregate. Controls whether problem aggregation is performed in the mixed-integer presolve.
- iparam.mio\_presolve\_probing. Controls whether probing is employed by the mixed-integer presolve.

- iparam.mio\_presolve\_use. Controls whether presolve is performed by the mixed-integer optimizer.
- iparam.mio\_probing\_level. Controls the amount of probing employed by the mixed-integer optimizer in presolve.
- dparam.mio\_rel\_add\_cut\_limited. Controls cut generation for mixed-integer optimizer.
- dparam.mio\_rel\_gap\_const. This value is used to compute the relative gap for the solution to an integer optimization problem.
- iparam.mio\_rins\_max\_nodes. Maximum number of nodes in each call to the RINS heuristic.
- iparam.mio\_root\_optimizer. Controls which optimizer is employed at the root node in the mixed-integer optimizer.
- iparam.mio\_strong\_branch. The depth from the root in which strong branching is employed.
- dparam.mio\_tol\_abs\_gap. Absolute optimality tolerance employed by the mixed-integer optimizer.
- dparam.mio\_tol\_abs\_relax\_int. Integer constraint tolerance.
- dparam.mio\_tol\_feas. Feasibility tolerance for mixed integer solver. Any solution with maximum infeasibility below this value will be considered feasible.
- dparam.mio\_tol\_max\_cut\_frac\_rhs. Controls cut generation for mixed-integer optimizer.
- dparam.mio\_tol\_min\_cut\_frac\_rhs. Controls cut generation for mixed-integer optimizer.
- dparam.mio\_tol\_rel\_dual\_bound\_improvement. Controls cut generation for mixed-integer optimizer.
- dparam.mio\_tol\_rel\_gap. Relative optimality tolerance employed by the mixed-integer optimizer.
- dparam.mio\_tol\_rel\_relax\_int. Integer constraint tolerance.
- dparam.mio\_tol\_x. Absolute solution tolerance used in mixed-integer optimizer.
- iparam.mio\_use\_multithreaded\_optimizer. Controls wheter the new multithreaded optimizer should be used for Mixed integer problems.

Network simplex optimizer parameters.

Parameters defining the behavior of the network simplex optimizer for linear problems.

- iparam.log\_sim\_network\_freq. Controls the network simplex logging frequency.
- iparam.sim\_refactor\_freq. Controls the basis refactoring frequency.

Non-convex solver parameters.

- iparam.log\_nonconvex. Controls amount of output printed by the nonconvex optimizer.
- iparam.nonconvex\_max\_iterations. Maximum number of iterations that can be used by the nonconvex optimizer.
- dparam.nonconvex\_tol\_feas. Feasibility tolerance used by the nonconvex optimizer.
- dparam.nonconvex\_tol\_opt. Optimality tolerance used by the nonconvex optimizer.

Nonlinear convex method parameters.

Parameters defining the behavior of the interior-point method for nonlinear convex problems.

- dparam.intpnt\_nl\_merit\_bal. Controls if the complementarity and infeasibility is converging to zero at about equal rates.
- dparam.intpnt\_nl\_tol\_dfeas. Dual feasibility tolerance used when a nonlinear model is solved.
- dparam.intpnt\_nl\_tol\_mu\_red. Relative complementarity gap tolerance.
- dparam.intpnt\_nl\_tol\_near\_rel. Nonlinear solver optimality tolerance parameter.
- dparam.intpnt\_nl\_tol\_pfeas. Primal feasibility tolerance used when a nonlinear model is solved.
- dparam.intpnt\_nl\_tol\_rel\_gap. Relative gap termination tolerance for nonlinear problems.
- dparam.intpnt\_nl\_tol\_rel\_step. Relative step size to the boundary for general nonlinear optimization problems.
- dparam.intpnt\_tol\_infeas. Nonlinear solver infeasibility tolerance parameter.
- iparam.log\_check\_convexity. Controls logging in convexity check on quadratic problems. Set to a positive value to turn logging on.

If a quadratic coefficient matrix is found to violate the requirement of PSD (NSD) then a list of negative (positive) pivot elements is printed. The absolute value of the pivot elements is also shown.

### Optimization system parameters.

Parameters defining the overall solver system environment. This includes system and platform related information and behavior.

- iparam.license\_wait. Controls if MOSEK should queue for a license if none is available.
- iparam.log\_storage. Controls the memory related log information.
- iparam.num\_threads. Controls the number of threads employed by the optimizer. If set to 0 the number of threads used will be equal to the number of cores detected on the machine.

#### Output information parameters.

- iparam.license\_suppress\_expire\_wrns. Controls license manager client behavior.
- iparam.log. Controls the amount of log information.
- iparam.log\_bi. Controls the amount of output printed by the basis identification procedure. A higher level implies that more information is logged.
- iparam.log\_bi\_freq. Controls the logging frequency.
- iparam.log\_expand. Controls the amount of logging when a data item such as the maximum number constrains is expanded.
- iparam.log\_factor. If turned on, then the factor log lines are added to the log.

- iparam.log\_feas\_repair. Controls the amount of output printed when performing feasibility repair. A value higher than one means extensive logging.
- iparam.log\_file. If turned on, then some log info is printed when a file is written or read.
- iparam.log\_head. If turned on, then a header line is added to the log.
- iparam.log\_infeas\_ana. Controls log level for the infeasibility analyzer.
- iparam.log\_intpnt. Controls the amount of log information from the interior-point optimizers.
- iparam.log\_mio. Controls the amount of log information from the mixed-integer optimizers.
- iparam.log\_mio\_freq. The mixed-integer solver logging frequency.
- iparam.log\_nonconvex. Controls amount of output printed by the nonconvex optimizer.
- iparam.log\_optimizer. Controls the amount of general optimizer information that is logged.
- iparam.log\_order. If turned on, then factor lines are added to the log.
- iparam.log\_param. Controls the amount of information printed out about parameter changes.
- iparam.log\_response. Controls amount of output printed when response codes are reported. A higher level implies that more information is logged.
- iparam.log\_sim. Controls the amount of log information from the simplex optimizers.
- iparam.log\_sim\_freq. Controls simplex logging frequency.
- iparam.log\_sim\_minor. Currently not in use.
- iparam.log\_sim\_network\_freq. Controls the network simplex logging frequency.
- iparam.log\_storage. Controls the memory related log information.
- iparam.max\_num\_warnings. A negtive number means all warnings are logged. Otherwise the parameter specifies the maximum number times each warning is logged.
- iparam.warning\_level. Deprecated and not in use

### Overall solver parameters.

- iparam.bi\_clean\_optimizer. Controls which simplex optimizer is used in the clean-up phase.
- iparam.concurrent\_num\_optimizers. The maximum number of simultaneous optimizations that will be started by the concurrent optimizer.
- iparam.concurrent\_priority\_dual\_simplex. Priority of the dual simplex algorithm when selecting solvers for concurrent optimization.
- iparam.concurrent\_priority\_free\_simplex. Priority of the free simplex optimizer when selecting solvers for concurrent optimization.
- iparam.concurrent\_priority\_intpnt. Priority of the interior-point algorithm when selecting solvers for concurrent optimization.
- iparam.concurrent\_priority\_primal\_simplex. Priority of the primal simplex algorithm when selecting solvers for concurrent optimization.

- iparam.infeas\_prefer\_primal. Controls which certificate is used if both primal- and dual-certificate of infeasibility is available.
- iparam.license\_wait. Controls if MOSEK should queue for a license if none is available.
- iparam.mio\_cont\_sol. Controls the meaning of interior-point and basic solutions in mixed integer problems.
- iparam.mio\_local\_branch\_number. Controls the size of the local search space when doing local branching.
- iparam.mio\_mode. Turns on/off the mixed-integer mode.
- iparam.optimizer. Controls which optimizer is used to optimize the task.
- iparam.presolve\_level. Currently not used.
- iparam.presolve\_use. Controls whether the presolve is applied to a problem before it is optimized.
- iparam.solution\_callback. Indicates whether solution call-backs will be performed during the optimization.

#### Presolve parameters.

- iparam.presolve\_elim\_fill. Maximum amount of fill-in in the elimination phase.
- iparam.presolve\_eliminator\_max\_num\_tries. Control the maximum number of times the eliminator is tried.
- iparam.presolve\_eliminator\_use. Controls whether free or implied free variables are eliminated from the problem.
- iparam.presolve\_level. Currently not used.
- iparam.presolve\_lindep\_abs\_work\_trh. Controls linear dependency check in presolve.
- iparam.presolve\_lindep\_rel\_work\_trh. Controls linear dependency check in presolve.
- iparam.presolve\_lindep\_use. Controls whether the linear constraints are checked for linear dependencies.
- dparam.presolve\_tol\_abs\_lindep. Absolute tolerance employed by the linear dependency checker.
- dparam.presolve\_tol\_aij. Absolute zero tolerance employed for constraint coefficients in the presolve.
- dparam.presolve\_tol\_rel\_lindep. Relative tolerance employed by the linear dependency checker.
- dparam.presolve\_tol\_s. Absolute zero tolerance employed for slack variables in the presolve.
- dparam.presolve\_tol\_x. Absolute zero tolerance employed for variables in the presolve.
- iparam.presolve\_use. Controls whether the presolve is applied to a problem before it is optimized.

#### Primal simplex optimizer parameters.

Parameters defining the behavior of the primal simplex optimizer for linear problems.

- iparam.sim\_primal\_crash. Controls the simplex crash.
- iparam.sim\_primal\_restrict\_selection. Controls how aggressively restricted selection is used
- iparam.sim\_primal\_selection. Controls the primal simplex strategy.

Progress call-back parameters.

• iparam.solution\_callback. Indicates whether solution call-backs will be performed during the optimization.

Simplex optimizer parameters.

Parameters defining the behavior of the simplex optimizer for linear problems.

- dparam.basis\_rel\_tol\_s. Maximum relative dual bound violation allowed in an optimal basic solution.
- dparam.basis\_tol\_s. Maximum absolute dual bound violation in an optimal basic solution.
- dparam.basis\_tol\_x. Maximum absolute primal bound violation allowed in an optimal basic solution.
- iparam.log\_sim. Controls the amount of log information from the simplex optimizers.
- iparam.log\_sim\_freq. Controls simplex logging frequency.
- iparam.log\_sim\_minor. Currently not in use.
- iparam.sim\_basis\_factor\_use. Controls whether a (LU) factorization of the basis is used in a hot-start. Forcing a refactorization sometimes improves the stability of the simplex optimizers, but in most cases there is a performance penanlty.
- iparam.sim\_degen. Controls how aggressively degeneration is handled.
- iparam.sim\_dual\_phaseone\_method. An exprimental feature.
- iparam.sim\_exploit\_dupvec. Controls if the simplex optimizers are allowed to exploit duplicated columns.
- iparam.sim.hotstart. Controls the type of hot-start that the simplex optimizer perform.
- iparam.sim\_integer. An exprimental feature.
- dparam.sim\_lu\_tol\_rel\_piv. Relative pivot tolerance employed when computing the LU factorization of the basis matrix.
- iparam.sim\_max\_iterations. Maximum number of iterations that can be used by a simplex optimizer.
- iparam.sim\_max\_num\_setbacks. Controls how many set-backs that are allowed within a simplex optimizer.
- iparam.sim\_non\_singular. Controls if the simplex optimizer ensures a non-singular basis, if possible.
- iparam.sim\_primal\_phaseone\_method. An exprimental feature.
- iparam.sim\_reformulation. Controls if the simplex optimizers are allowed to reformulate the problem.

- iparam.sim\_save\_lu. Controls if the LU factorization stored should be replaced with the LU factorization corresponding to the initial basis.
- iparam.sim\_scaling. Controls how much effort is used in scaling the problem before a simplex optimizer is used.
- iparam.sim\_scaling\_method. Controls how the problem is scaled before a simplex optimizer is used.
- iparam.sim\_solve\_form. Controls whether the primal or the dual problem is solved by the primal-/dual- simplex optimizer.
- iparam.sim\_stability\_priority. Controls how high priority the numerical stability should be given.
- iparam.sim\_switch\_optimizer. Controls the simplex behavior.
- dparam.simplex\_abs\_tol\_piv. Absolute pivot tolerance employed by the simplex optimizers.

### Solution input/output parameters.

Parameters defining the behavior of solution reader and writer.

- sparam.bas\_sol\_file\_name. Name of the bas solution file.
- sparam.int\_sol\_file\_name. Name of the int solution file.
- sparam.itr\_sol\_file\_name. Name of the itr solution file.
- iparam.sol\_filter\_keep\_basic. Controls the license manager client behavior.
- sparam.sol\_filter\_xc\_low. Solution file filter.
- sparam.sol\_filter\_xc\_upr. Solution file filter.
- sparam.sol\_filter\_xx\_low. Solution file filter.
- sparam.sol\_filter\_xx\_upr. Solution file filter.

#### Termination criterion parameters.

Parameters which define termination and optimality criteria and related information.

- dparam.basis\_rel\_tol\_s. Maximum relative dual bound violation allowed in an optimal basic solution.
- dparam.basis\_tol\_s. Maximum absolute dual bound violation in an optimal basic solution.
- dparam.basis\_tol\_x. Maximum absolute primal bound violation allowed in an optimal basic solution.
- iparam.bi\_max\_iterations. Maximum number of iterations after basis identification.
- dparam.intpnt\_co\_tol\_dfeas. Dual feasibility tolerance used by the conic interior-point optimizer.
- dparam.intpnt\_co\_tol\_infeas. Infeasibility tolerance for the conic solver.
- dparam.intpnt\_co\_tol\_mu\_red. Optimality tolerance for the conic solver.
- dparam.intpnt\_co\_tol\_near\_rel. Optimality tolerance for the conic solver.

- dparam.intpnt\_co\_tol\_pfeas. Primal feasibility tolerance used by the conic interior-point optimizer.
- dparam.intpnt\_co\_tol\_rel\_gap. Relative gap termination tolerance used by the conic interior-point optimizer.
- iparam.intpnt\_max\_iterations. Controls the maximum number of iterations allowed in the interior-point optimizer.
- dparam.intpnt\_nl\_tol\_dfeas. Dual feasibility tolerance used when a nonlinear model is solved.
- dparam.intpnt\_nl\_tol\_mu\_red. Relative complementarity gap tolerance.
- dparam.intpnt\_nl\_tol\_near\_rel. Nonlinear solver optimality tolerance parameter.
- dparam.intpnt\_nl\_tol\_pfeas. Primal feasibility tolerance used when a nonlinear model is solved.
- dparam.intpnt\_nl\_tol\_rel\_gap. Relative gap termination tolerance for nonlinear prob-
- dparam.intpnt\_tol\_dfeas. Dual feasibility tolerance used for linear and quadratic optimization problems.
- dparam.intpnt\_tol\_infeas. Nonlinear solver infeasibility tolerance parameter.
- dparam.intpnt\_tol\_mu\_red. Relative complementarity gap tolerance.
- dparam.intpnt\_tol\_pfeas. Primal feasibility tolerance used for linear and quadratic optimization problems.
- dparam.intpnt\_tol\_rel\_gap. Relative gap termination tolerance.
- dparam.lower\_obj\_cut. Objective bound.
- dparam.lower\_obj\_cut\_finite\_trh. Objective bound.
- dparam.mio\_disable\_term\_time. Certain termination criteria is disabled within the mixed-integer optimizer for period time specified by the parameter.
- iparam.mio\_max\_num\_branches. Maximum number of branches allowed during the branch and bound search.
- iparam.mio\_max\_num\_solutions. Controls how many feasible solutions the mixed-integer optimizer investigates.
- dparam.mio\_max\_time. Time limit for the mixed-integer optimizer.
- dparam.mio\_near\_tol\_rel\_gap. The mixed-integer optimizer is terminated when this tolerance is satisfied.
- dparam.mio\_rel\_gap\_const. This value is used to compute the relative gap for the solution to an integer optimization problem.
- dparam.mio\_tol\_rel\_gap. Relative optimality tolerance employed by the mixed-integer optimizer.
- dparam.optimizer\_max\_time. Solver time limit.
- iparam.sim\_max\_iterations. Maximum number of iterations that can be used by a simplex optimizer.

- dparam.upper\_obj\_cut. Objective bound.
- dparam.upper\_obj\_cut\_finite\_trh. Objective bound.
- Integer parameters
- Double parameters
- String parameters

# B.1 dparam: Double parameters

# B.1.1 dparam.ana\_sol\_infeas\_tol

## Corresponding constant:

dparam.ana\_sol\_infeas\_tol

### Description:

If a constraint violates its bound with an amount larger than this value, the constraint name, index and violation will be printed by the solution analyzer.

### Possible Values:

Any number between 0.0 and  $+\inf$ .

### Default value:

1e-6

# B.1.2 dparam.basis\_rel\_tol\_s

### Corresponding constant:

dparam.basis\_rel\_tol\_s

### Description:

Maximum relative dual bound violation allowed in an optimal basic solution.

## Possible Values:

Any number between 0.0 and  $+\inf$ .

#### Default value:

## B.1.3 dparam.basis\_tol\_s

### Corresponding constant:

dparam.basis\_tol\_s

### **Description:**

Maximum absolute dual bound violation in an optimal basic solution.

### Possible Values:

Any number between 1.0e-9 and  $+\inf$ .

### Default value:

1.0e-6

# B.1.4 dparam.basis\_tol\_x

### Corresponding constant:

dparam.basis\_tol\_x

### **Description:**

Maximum absolute primal bound violation allowed in an optimal basic solution.

#### Possible Values:

Any number between 1.0e-9 and +inf.

### Default value:

1.0e-6

# B.1.5 dparam.check\_convexity\_rel\_tol

### Corresponding constant:

dparam.check\_convexity\_rel\_tol

### **Description:**

This parameter controls when the full convexity check declares a problem to be non-convex. Increasing this tolerance relaxes the criteria for declaring the problem non-convex.

A problem is declared non-convex if negative (positive) pivot elements are detected in the cholesky factor of a matrix which is required to be PSD (NSD). This parameter controles how much this non-negativity requirement may be violated.

If  $d_i$  is the pivot element for column i, then the matrix Q is considered to not be PSD if:

$$d_i \leq -|Q_{ii}| * \texttt{check\_convexity\_rel\_tol}$$

Any number between 0 and +inf.

### Default value:

1e-10

## B.1.6 dparam.data\_tol\_aij

## Corresponding constant:

dparam.data\_tol\_aij

### Description:

Absolute zero tolerance for elements in A. If any value  $A_{ij}$  is smaller than this parameter in absolute terms MOSEK will treat the values as zero and generate a warning.

#### Possible Values:

Any number between 1.0e-16 and 1.0e-6.

### Default value:

1.0e-12

# B.1.7 dparam.data\_tol\_aij\_huge

### Corresponding constant:

dparam.data\_tol\_aij\_huge

### Description:

An element in A which is larger than this value in absolute size causes an error.

### Possible Values:

Any number between 0.0 and  $+\inf$ .

### Default value:

1.0e20

# B.1.8 dparam.data\_tol\_aij\_large

### Corresponding constant:

dparam.data\_tol\_aij\_large

### **Description:**

An element in A which is larger than this value in absolute size causes a warning message to be printed.

Any number between 0.0 and +inf.

#### Default value:

1.0e10

# B.1.9 dparam.data\_tol\_bound\_inf

### Corresponding constant:

dparam.data\_tol\_bound\_inf

### **Description:**

Any bound which in absolute value is greater than this parameter is considered infinite.

## Possible Values:

Any number between 0.0 and  $+\inf$ .

### Default value:

1.0e16

## B.1.10 dparam.data\_tol\_bound\_wrn

### Corresponding constant:

dparam.data\_tol\_bound\_wrn

## Description:

If a bound value is larger than this value in absolute size, then a warning message is issued.

#### Possible Values:

Any number between 0.0 and  $+\inf$ .

### Default value:

1.0e8

# B.1.11 dparam.data\_tol\_c\_huge

### Corresponding constant:

dparam.data\_tol\_c\_huge

### Description:

An element in c which is larger than the value of this parameter in absolute terms is considered to be huge and generates an error.

Any number between 0.0 and +inf.

### Default value:

1.0e16

# B.1.12 dparam.data\_tol\_cj\_large

## Corresponding constant:

dparam.data\_tol\_cj\_large

### Description:

An element in c which is larger than this value in absolute terms causes a warning message to be printed.

### Possible Values:

Any number between 0.0 and +inf.

### Default value:

1.0e8

# B.1.13 dparam.data\_tol\_qij

### Corresponding constant:

dparam.data\_tol\_qij

## Description:

Absolute zero tolerance for elements in Q matrixes.

### Possible Values:

Any number between 0.0 and  $+\inf$ .

### Default value:

1.0e-16

### B.1.14 dparam.data\_tol\_x

### Corresponding constant:

 $dparam.data\_tol\_x$ 

### Description:

Zero tolerance for constraints and variables i.e. if the distance between the lower and upper bound is less than this value, then the lower and lower bound is considered identical.

Any number between 0.0 and  $+\inf$ .

#### Default value:

1.0e-8

# B.1.15 dparam.feasrepair\_tol

## Corresponding constant:

 $dparam.feasrepair\_tol$ 

### Description:

Tolerance for constraint enforcing upper bound on sum of weighted violations in feasibility repair.

### Possible Values:

Any number between 1.0e-16 and 1.0e+16.

### Default value:

1.0e-10

# B.1.16 dparam.intpnt\_co\_tol\_dfeas

## Corresponding constant:

 $dparam.intpnt\_co\_tol\_dfeas$ 

## Description:

Dual feasibility tolerance used by the conic interior-point optimizer.

### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

1.0e-8

### See also:

• dparam.intpnt\_co\_tol\_near\_rel Optimality tolerance for the conic solver.

# B.1.17 dparam.intpnt\_co\_tol\_infeas

### Corresponding constant:

dparam.intpnt\_co\_tol\_infeas

### Description:

Controls when the conic interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

#### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

1.0e-10

### B.1.18 dparam.intpnt\_co\_tol\_mu\_red

### Corresponding constant:

dparam.intpnt\_co\_tol\_mu\_red

### Description:

Relative complementarity gap tolerance feasibility tolerance used by the conic interior-point optimizer.

### Possible Values:

Any number between 0.0 and 1.0.

#### Default value:

1.0e-8

# B.1.19 dparam.intpnt\_co\_tol\_near\_rel

### Corresponding constant:

dparam.intpnt\_co\_tol\_near\_rel

### Description:

If MOSEK cannot compute a solution that has the prescribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.

### Possible Values:

Any number between 1.0 and  $+\inf$ .

### Default value:

1000

## B.1.20 dparam.intpnt\_co\_tol\_pfeas

### Corresponding constant:

 $dparam.intpnt\_co\_tol\_pfeas$ 

### Description:

Primal feasibility tolerance used by the conic interior-point optimizer.

#### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

1.0e-8

### See also:

• dparam.intpnt\_co\_tol\_near\_rel Optimality tolerance for the conic solver.

## B.1.21 dparam.intpnt\_co\_tol\_rel\_gap

### Corresponding constant:

dparam.intpnt\_co\_tol\_rel\_gap

### Description:

Relative gap termination tolerance used by the conic interior-point optimizer.

### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

1.0e-7

#### See also:

• dparam.intpnt\_co\_tol\_near\_rel Optimality tolerance for the conic solver.

## B.1.22 dparam.intpnt\_nl\_merit\_bal

## Corresponding constant:

dparam.intpnt\_nl\_merit\_bal

# Description:

Controls if the complementarity and infeasibility is converging to zero at about equal rates.

### Possible Values:

Any number between 0.0 and 0.99.

### Default value:

1.0e-4

## B.1.23 dparam.intpnt\_nl\_tol\_dfeas

### Corresponding constant:

dparam.intpnt\_nl\_tol\_dfeas

## Description:

Dual feasibility tolerance used when a nonlinear model is solved.

### Possible Values:

Any number between 0.0 and 1.0.

#### Default value:

1.0e-8

## B.1.24 dparam.intpnt\_nl\_tol\_mu\_red

### Corresponding constant:

 $dparam.intpnt_nl_tol_mu_red$ 

### Description:

Relative complementarity gap tolerance.

### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

1.0e-12

# B.1.25 dparam.intpnt\_nl\_tol\_near\_rel

### Corresponding constant:

dparam.intpnt\_nl\_tol\_near\_rel

### **Description:**

If the MOSEK nonlinear interior-point optimizer cannot compute a solution that has the prescribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.

### Possible Values:

Any number between 1.0 and  $+\inf$ .

### Default value:

1000.0

# B.1.26 dparam.intpnt\_nl\_tol\_pfeas

### Corresponding constant:

 $dparam.intpnt_nl_tol_pfeas$ 

### Description:

Primal feasibility tolerance used when a nonlinear model is solved.

### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

1.0e-8

# B.1.27 dparam.intpnt\_nl\_tol\_rel\_gap

## Corresponding constant:

 $dparam.intpnt\_nl\_tol\_rel\_gap$ 

## Description:

Relative gap termination tolerance for nonlinear problems.

#### Possible Values:

Any number between 1.0e-14 and +inf.

### Default value:

1.0e-6

# B.1.28 dparam.intpnt\_nl\_tol\_rel\_step

### Corresponding constant:

 $dparam.intpnt_nl\_tol\_rel\_step$ 

### Description:

Relative step size to the boundary for general nonlinear optimization problems.

### Possible Values:

Any number between 1.0e-4 and 0.9999999.

### Default value:

0.995

# B.1.29 dparam.intpnt\_tol\_dfeas

### Corresponding constant:

dparam.intpnt\_tol\_dfeas

### Description:

Dual feasibility tolerance used for linear and quadratic optimization problems.

### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

1.0e-8

# B.1.30 dparam.intpnt\_tol\_dsafe

### Corresponding constant:

dparam.intpnt\_tol\_dsafe

### Description:

Controls the initial dual starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly.

### Possible Values:

Any number between 1.0e-4 and +inf.

#### Default value:

1.0

## B.1.31 dparam.intpnt\_tol\_infeas

### Corresponding constant:

dparam.intpnt\_tol\_infeas

### **Description:**

Controls when the optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

#### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

# B.1.32 dparam.intpnt\_tol\_mu\_red

### Corresponding constant:

dparam.intpnt\_tol\_mu\_red

### Description:

Relative complementarity gap tolerance.

### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

1.0e-16

# B.1.33 dparam.intpnt\_tol\_path

### Corresponding constant:

dparam.intpnt\_tol\_path

### Description:

Controls how close the interior-point optimizer follows the central path. A large value of this parameter means the central is followed very closely. On numerical unstable problems it may be worthwhile to increase this parameter.

### Possible Values:

Any number between 0.0 and 0.9999.

### Default value:

1.0e-8

# B.1.34 dparam.intpnt\_tol\_pfeas

### Corresponding constant:

dparam.intpnt\_tol\_pfeas

## Description:

Primal feasibility tolerance used for linear and quadratic optimization problems.

### Possible Values:

Any number between 0.0 and 1.0.

#### Default value:

## B.1.35 dparam.intpnt\_tol\_psafe

### Corresponding constant:

dparam.intpnt\_tol\_psafe

### Description:

Controls the initial primal starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it may be worthwhile to increase this value.

### Possible Values:

Any number between 1.0e-4 and  $+\inf$ .

### Default value:

1.0

## B.1.36 dparam.intpnt\_tol\_rel\_gap

## Corresponding constant:

dparam.intpnt\_tol\_rel\_gap

### Description:

Relative gap termination tolerance.

### Possible Values:

Any number between 1.0e-14 and  $+\inf$ .

### Default value:

1.0e-8

# B.1.37 dparam.intpnt\_tol\_rel\_step

### Corresponding constant:

dparam.intpnt\_tol\_rel\_step

## Description:

Relative step size to the boundary for linear and quadratic optimization problems.

### Possible Values:

Any number between 1.0e-4 and 0.999999.

### Default value:

0.9999

## B.1.38 dparam.intpnt\_tol\_step\_size

### Corresponding constant:

dparam.intpnt\_tol\_step\_size

### Description:

If the step size falls below the value of this parameter, then the interior-point optimizer assumes that it is stalled. In other words the interior-point optimizer does not make any progress and therefore it is better stop.

### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

1.0e-6

# B.1.39 dparam.lower\_obj\_cut

### Corresponding constant:

dparam.lower\_obj\_cut

### Description:

If either a primal or dual feasible solution is found proving that the optimal objective value is outside, the interval [dparam.lower\_obj\_cut, dparam.upper\_obj\_cut], then MOSEK is terminated.

### Possible Values:

Any number between -inf and +inf.

#### Default value:

-1.0e30

#### See also:

• dparam.lower\_obj\_cut\_finite\_trh Objective bound.

# B.1.40 dparam.lower\_obj\_cut\_finite\_trh

### Corresponding constant:

dparam.lower\_obj\_cut\_finite\_trh

### Description:

If the lower objective cut is less than the value of this parameter value, then the lower objective cut i.e. dparam.lower\_obj\_cut is treated as  $-\infty$ .

Any number between -inf and +inf.

#### Default value:

-0.5e30

## B.1.41 dparam.mio\_disable\_term\_time

### Corresponding constant:

dparam.mio\_disable\_term\_time

### Description:

The termination criteria governed by

- iparam.mio\_max\_num\_relaxs
- iparam.mio\_max\_num\_branches
- dparam.mio\_near\_tol\_abs\_gap
- dparam.mio\_near\_tol\_rel\_gap

is disabled the first n seconds. This parameter specifies the number n. A negative value is identical to infinity i.e. the termination criteria are never checked.

### Possible Values:

Any number between  $-\inf$  and  $+\inf$ .

#### Default value:

-1.0

### See also:

- iparam.mio\_max\_num\_relaxs Maximum number of relaxations in branch and bound search.
- iparam.mio\_max\_num\_branches Maximum number of branches allowed during the branch and bound search.
- dparam.mio\_near\_tol\_abs\_gap Relaxed absolute optimality tolerance employed by the mixed-integer optimizer.
- dparam.mio\_near\_tol\_rel\_gap The mixed-integer optimizer is terminated when this tolerance is satisfied.

# B.1.42 dparam.mio\_heuristic\_time

### Corresponding constant:

dparam.mio\_heuristic\_time

### Description:

Minimum amount of time to be used in the heuristic search for a good feasible integer solution. A negative values implies that the optimizer decides the amount of time to be spent in the heuristic.

#### Possible Values:

Any number between -inf and +inf.

### Default value:

-1.0

# B.1.43 dparam.mio\_max\_time

### Corresponding constant:

dparam.mio\_max\_time

### Description:

This parameter limits the maximum time spent by the mixed-integer optimizer. A negative number means infinity.

### Possible Values:

Any number between -inf and +inf.

### Default value:

-1.0

# B.1.44 dparam.mio\_max\_time\_aprx\_opt

### Corresponding constant:

dparam.mio\_max\_time\_aprx\_opt

## Description:

Number of seconds spent by the mixed-integer optimizer before the dparam.mio\_tol\_rel\_relax\_int is applied.

### Possible Values:

Any number between 0.0 and  $+\inf$ .

### Default value:

60

## B.1.45 dparam.mio\_near\_tol\_abs\_gap

### Corresponding constant:

dparam.mio\_near\_tol\_abs\_gap

### **Description:**

Relaxed absolute optimality tolerance employed by the mixed-integer optimizer. This termination criteria is delayed. See dparam.mio\_disable\_term\_time for details.

### Possible Values:

Any number between 0.0 and +inf.

#### Default value:

0.0

### See also:

• dparam.mio\_disable\_term\_time Certain termination criteria is disabled within the mixed-integer optimizer for period time specified by the parameter.

# B.1.46 dparam.mio\_near\_tol\_rel\_gap

### Corresponding constant:

dparam.mio\_near\_tol\_rel\_gap

### Description:

The mixed-integer optimizer is terminated when this tolerance is satisfied. This termination criteria is delayed. See dparam.mio\_disable\_term\_time for details.

### Possible Values:

Any number between 0.0 and  $+\inf$ .

### Default value:

1.0e-3

#### See also:

• dparam.mio\_disable\_term\_time Certain termination criteria is disabled within the mixed-integer optimizer for period time specified by the parameter.

## B.1.47 dparam.mio\_rel\_add\_cut\_limited

### Corresponding constant:

dparam.mio\_rel\_add\_cut\_limited

### Description:

Controls how many cuts the mixed-integer optimizer is allowed to add to the problem. Let  $\alpha$  be the value of this parameter and m the number constraints, then mixed-integer optimizer is allowed to  $\alpha m$  cuts.

### Possible Values:

Any number between 0.0 and 2.0.

### Default value:

0.75

## B.1.48 dparam.mio\_rel\_gap\_const

## Corresponding constant:

dparam.mio\_rel\_gap\_const

### Description:

This value is used to compute the relative gap for the solution to an integer optimization problem.

### Possible Values:

Any number between 1.0e-15 and  $+\inf$ .

### Default value:

1.0e-10

# B.1.49 dparam.mio\_tol\_abs\_gap

### Corresponding constant:

dparam.mio\_tol\_abs\_gap

## Description:

Absolute optimality tolerance employed by the mixed-integer optimizer.

### Possible Values:

Any number between 0.0 and  $+\inf$ .

### Default value:

0.0

## B.1.50 dparam.mio\_tol\_abs\_relax\_int

### Corresponding constant:

dparam.mio\_tol\_abs\_relax\_int

## Description:

Absolute relaxation tolerance of the integer constraints. I.e.  $\min(|x| - \lfloor x \rfloor, \lceil x \rceil - |x|)$  is less than the tolerance then the integer restrictions assumed to be satisfied.

### Possible Values:

Any number between 1e-9 and  $+\inf$ .

### Default value:

1.0e-5

# B.1.51 dparam.mio\_tol\_feas

### Corresponding constant:

dparam.mio\_tol\_feas

### Description:

Feasibility tolerance for mixed integer solver. Any solution with maximum infeasibility below this value will be considered feasible.

### Possible Values:

Any number between 0.0 and  $+\inf$ .

### Default value:

1.0e-7

# B.1.52 dparam.mio\_tol\_max\_cut\_frac\_rhs

### Corresponding constant:

dparam.mio\_tol\_max\_cut\_frac\_rhs

### Description:

Maximum value of fractional part of right hand side to generate CMIR and CG cuts for. A value of 0.0 means that the value is selected automatically.

#### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

0.0

## B.1.53 dparam.mio\_tol\_min\_cut\_frac\_rhs

### Corresponding constant:

dparam.mio\_tol\_min\_cut\_frac\_rhs

### Description:

Minimum value of fractional part of right hand side to generate CMIR and CG cuts for. A value of 0.0 means that the value is selected automatically.

#### Possible Values:

Any number between 0.0 and 1.0.

#### Default value:

0.0

## B.1.54 dparam.mio\_tol\_rel\_dual\_bound\_improvement

### Corresponding constant:

 $dparam.mio\_tol\_rel\_dual\_bound\_improvement$ 

### **Description:**

If the relative improvement of the dual bound is smaller than this value, the solver will terminate the root cut generation. A value of 0.0 means that the value is selected automatically.

### Possible Values:

Any number between 0.0 and 1.0.

### Default value:

0.0

# B.1.55 dparam.mio\_tol\_rel\_gap

## Corresponding constant:

dparam.mio\_tol\_rel\_gap

## Description:

Relative optimality tolerance employed by the mixed-integer optimizer.

#### Possible Values:

Any number between 0.0 and  $+\inf$ .

#### Default value:

## B.1.56 dparam.mio\_tol\_rel\_relax\_int

### Corresponding constant:

dparam.mio\_tol\_rel\_relax\_int

### Description:

Relative relaxation tolerance of the integer constraints. I.e  $(\min(|x| - \lfloor x \rfloor, \lceil x \rceil - |x|))$  is less than the tolerance times |x| then the integer restrictions assumed to be satisfied.

### Possible Values:

Any number between 0.0 and  $+\inf$ .

### Default value:

1.0e-6

## B.1.57 dparam.mio\_tol\_x

### Corresponding constant:

dparam.mio\_tol\_x

### Description:

Absolute solution tolerance used in mixed-integer optimizer.

### Possible Values:

Any number between 0.0 and  $+\inf$ .

### Default value:

1.0e-6

## B.1.58 dparam.nonconvex\_tol\_feas

### Corresponding constant:

 $dparam.nonconvex\_tol\_feas$ 

### **Description:**

Feasibility tolerance used by the nonconvex optimizer.

#### Possible Values:

Any number between 0.0 and +inf.

### Default value:

## B.1.59 dparam.nonconvex\_tol\_opt

### Corresponding constant:

 $dparam.nonconvex\_tol\_opt$ 

## Description:

Optimality tolerance used by the nonconvex optimizer.

### Possible Values:

Any number between 0.0 and  $+\inf$ .

#### Default value:

1.0e-7

# B.1.60 dparam.optimizer\_max\_time

### Corresponding constant:

dparam.optimizer\_max\_time

### Description:

Maximum amount of time the optimizer is allowed to spent on the optimization. A negative number means infinity.

### Possible Values:

Any number between -inf and +inf.

### Default value:

-1.0

## B.1.61 dparam.presolve\_tol\_abs\_lindep

## Corresponding constant:

 $dparam.presolve\_tol\_abs\_lindep$ 

### **Description:**

Absolute tolerance employed by the linear dependency checker.

#### Possible Values:

Any number between 0.0 and +inf.

### Default value:

# B.1.62 dparam.presolve\_tol\_aij

### Corresponding constant:

dparam.presolve\_tol\_aij

### Description:

Absolute zero tolerance employed for  $a_{ij}$  in the presolve.

### Possible Values:

Any number between 1.0e-15 and  $+\inf$ .

### Default value:

1.0e-12

# B.1.63 dparam.presolve\_tol\_rel\_lindep

### Corresponding constant:

dparam.presolve\_tol\_rel\_lindep

### Description:

Relative tolerance employed by the linear dependency checker.

### Possible Values:

Any number between 0.0 and  $+\inf$ .

## Default value:

1.0e-10

## B.1.64 dparam.presolve\_tol\_s

### Corresponding constant:

 $dparam.presolve\_tol\_s$ 

### Description:

Absolute zero tolerance employed for  $s_i$  in the presolve.

### Possible Values:

Any number between 0.0 and +inf.

### Default value:

## B.1.65 dparam.presolve\_tol\_x

### Corresponding constant:

 $dparam.presolve\_tol\_x$ 

## Description:

Absolute zero tolerance employed for  $x_j$  in the presolve.

#### Possible Values:

Any number between 0.0 and +inf.

## Default value:

1.0e-8

# B.1.66 dparam.qcqo\_reformulate\_rel\_drop\_tol

### Corresponding constant:

dparam.qcqo\_reformulate\_rel\_drop\_tol

### **Description:**

This parameter determines when columns are dropped in incomplete cholesky factorization doing reformulation of quadratic problems.

#### Possible Values:

Any number between 0 and +inf.

### Default value:

1e-15

## B.1.67 dparam.sim\_lu\_tol\_rel\_piv

## Corresponding constant:

dparam.sim\_lu\_tol\_rel\_piv

### **Description:**

Relative pivot tolerance employed when computing the LU factorization of the basis in the simplex optimizers and in the basis identification procedure.

A value closer to 1.0 generally improves numerical stability but typically also implies an increase in the computational work.

#### Possible Values:

Any number between 1.0e-6 and 0.999999.

### Default value:

0.01

## B.1.68 dparam.simplex\_abs\_tol\_piv

### Corresponding constant:

dparam.simplex\_abs\_tol\_piv

### Description:

Absolute pivot tolerance employed by the simplex optimizers.

### Possible Values:

Any number between 1.0e-12 and  $+\inf$ .

### Default value:

1.0e-7

# B.1.69 dparam.upper\_obj\_cut

### Corresponding constant:

dparam.upper\_obj\_cut

### Description:

If either a primal or dual feasible solution is found proving that the optimal objective value is outside, [dparam.lower\_obj\_cut, dparam.upper\_obj\_cut], then MOSEK is terminated.

### Possible Values:

Any number between -inf and +inf.

#### Default value:

1.0e30

### See also:

• dparam.upper\_obj\_cut\_finite\_trh Objective bound.

# B.1.70 dparam.upper\_obj\_cut\_finite\_trh

### Corresponding constant:

 $dparam.upper\_obj\_cut\_finite\_trh$ 

### **Description:**

If the upper objective cut is greater than the value of this value parameter, then the the upper objective cut  $dparam.upper_obj_cut$  is treated as  $\infty$ .

### Possible Values:

Any number between  $-\inf$  and  $+\inf$ .

### Default value:

0.5e30

# B.2 iparam: Integer parameters

# B.2.1 iparam.alloc\_add\_qnz

## Corresponding constant:

 $iparam.alloc\_add\_qnz$ 

### Description:

Additional number of Q non-zeros that are allocated space for when numanz exceeds maxnumqnz during addition of new Q entries.

#### Possible Values:

Any number between 0 and +inf.

### Default value:

5000

## B.2.2 iparam.ana\_sol\_basis

## Corresponding constant:

iparam.ana\_sol\_basis

### **Description:**

Controls whether the basis matrix is analyzed in solaution analyzer.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

### Default value:

onoffkey.on

## B.2.3 iparam.ana\_sol\_print\_violated

### Corresponding constant:

iparam.ana\_sol\_print\_violated

### **Description:**

Controls whether a list of violated constraints is printed when calling Task.analyzesolution. All constraints violated by more than the value set by the parameter dparam.ana\_sol\_infeas\_tol will be printed.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.off
```

## B.2.4 iparam.auto\_sort\_a\_before\_opt

### Corresponding constant:

 $iparam.auto\_sort\_a\_before\_opt$ 

### Description:

Controls whether the elements in each column of A are sorted before an optimization is performed. This is not required but makes the optimization more deterministic.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

### Default value:

```
onoffkey.off
```

## B.2.5 iparam.auto\_update\_sol\_info

### Corresponding constant:

 $iparam.auto\_update\_sol\_info$ 

### Description:

Controls whether the solution information items are automatically updated after an optimization is performed.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

### Default value:

```
onoffkey.off
```

## B.2.6 iparam.basis\_solve\_use\_plus\_one

### Corresponding constant:

iparam.basis\_solve\_use\_plus\_one

### Description:

If a slack variable is in the basis, then the corresponding column in the basis is a unit vector with -1 in the right position. However, if this parameter is set to <code>onoffkey.on</code>, -1 is replaced by 1.

This has significance for the results returned by the Task.solvewithbasis function.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.off

## B.2.7 iparam.bi\_clean\_optimizer

#### Corresponding constant:

iparam.bi\_clean\_optimizer

### **Description:**

Controls which simplex optimizer is used in the clean-up phase.

#### Possible values:

- optimizertype.concurrent The optimizer for nonconvex nonlinear problems.
- optimizertype.conic The optimizer for problems having conic constraints.
- optimizertype.dual\_simplex The dual simplex optimizer is used.
- optimizertype.free The optimizer is chosen automatically.
- optimizertype.free\_simplex One of the simplex optimizers is used.
- optimizertype.intpnt The interior-point optimizer is used.
- $\bullet$  optimizer type.mixed\_int The mixed-integer optimizer.
- optimizertype.mixed\_int\_conic The mixed-integer optimizer for conic and linear problems.
- optimizertype.network\_primal\_simplex The network primal simplex optimizer is used. It is only applicable to pure network problems.
- optimizertype.nonconvex The optimizer for nonconvex nonlinear problems.
- optimizertype.primal\_dual\_simplex The primal dual simplex optimizer is used.
- optimizertype.primal\_simplex The primal simplex optimizer is used.

#### Default value:

```
optimizertype.free
```

# B.2.8 iparam.bi\_ignore\_max\_iter

## Corresponding constant:

iparam.bi\_ignore\_max\_iter

## **Description:**

If the parameter <code>iparam.intpnt\_basis</code> has the value <code>basindtype.no\_error</code> and the interior-point optimizer has terminated due to maximum number of iterations, then basis identification is performed if this parameter has the value <code>onoffkey.on</code>.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.off

# B.2.9 iparam.bi\_ignore\_num\_error

## Corresponding constant:

iparam.bi\_ignore\_num\_error

## **Description:**

If the parameter <code>iparam.intpnt\_basis</code> has the value <code>basindtype.no\_error</code> and the interior-point optimizer has terminated due to a numerical problem, then basis identification is performed if this parameter has the value <code>onoffkey.on</code>.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.off

# B.2.10 iparam.bi\_max\_iterations

## Corresponding constant:

iparam.bi\_max\_iterations

## **Description:**

Controls the maximum number of simplex iterations allowed to optimize a basis after the basis identification.

## Possible Values:

Any number between 0 and +inf.

#### Default value:

1000000

# B.2.11 iparam.cache\_license

# Corresponding constant:

iparam.cache\_license

## Description:

Specifies if the license is kept checked out for the lifetime of the mosek environment (on) or returned to the server immediately after the optimization (off).

Check-in and check-out of licenses have an overhead. Frequent communication with the license server should be avoided.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.on

# B.2.12 iparam.check\_convexity

## Corresponding constant:

 $iparam.check\_convexity$ 

## **Description:**

Specify the level of convexity check on quadratic problems

### Possible values:

- checkconvexitytype.full Perform a full convexity check.
- checkconvexitytype.none No convexity check.
- checkconvexitytype.simple Perform simple and fast convexity check.

## Default value:

checkconvexitytype.full

## B.2.13 iparam.compress\_statfile

## Corresponding constant:

iparam.compress\_statfile

## Description:

Control compression of stat files.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.on

# B.2.14 iparam.concurrent\_num\_optimizers

## Corresponding constant:

iparam.concurrent\_num\_optimizers

## Description:

The maximum number of simultaneous optimizations that will be started by the concurrent optimizer.

## Possible Values:

Any number between 0 and +inf.

## Default value:

2

# B.2.15 iparam.concurrent\_priority\_dual\_simplex

## Corresponding constant:

iparam.concurrent\_priority\_dual\_simplex

## Description:

Priority of the dual simplex algorithm when selecting solvers for concurrent optimization.

## Possible Values:

Any number between 0 and +inf.

## Default value:

# B.2.16 iparam.concurrent\_priority\_free\_simplex

## Corresponding constant:

iparam.concurrent\_priority\_free\_simplex

## **Description:**

Priority of the free simplex optimizer when selecting solvers for concurrent optimization.

## Possible Values:

Any number between 0 and +inf.

## Default value:

3

# B.2.17 iparam.concurrent\_priority\_intpnt

## Corresponding constant:

iparam.concurrent\_priority\_intpnt

## Description:

Priority of the interior-point algorithm when selecting solvers for concurrent optimization.

## Possible Values:

Any number between 0 and +inf.

## Default value:

4

# B.2.18 iparam.concurrent\_priority\_primal\_simplex

## Corresponding constant:

iparam.concurrent\_priority\_primal\_simplex

## **Description:**

Priority of the primal simplex algorithm when selecting solvers for concurrent optimization.

## Possible Values:

Any number between 0 and  $+\inf$ .

#### Default value:

## B.2.19 iparam.feasrepair\_optimize

## Corresponding constant:

iparam.feasrepair\_optimize

## **Description:**

Controls which type of feasibility analysis is to be performed.

# Possible values:

- feasrepairtype.optimize\_combined Minimize with original objective subject to minimal weighted violation of bounds.
- feasrepairtype.optimize\_none Do not optimize the feasibility repair problem.
- feasrepairtype.optimize\_penalty Minimize weighted sum of violations.

## Default value:

feasrepairtype.optimize\_none

## B.2.20 iparam.infeas\_generic\_names

## Corresponding constant:

iparam.infeas\_generic\_names

#### Description:

Controls whether generic names are used when an infeasible subproblem is created.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.off

# B.2.21 iparam.infeas\_prefer\_primal

# Corresponding constant:

iparam.infeas\_prefer\_primal

## **Description:**

If both certificates of primal and dual infeasibility are supplied then only the primal is used when this option is turned on.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.on
```

# B.2.22 iparam.infeas\_report\_auto

# Corresponding constant:

 $iparam.infeas\_report\_auto$ 

## **Description:**

Controls whether an infeasibility report is automatically produced after the optimization if the problem is primal or dual infeasible.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.off
```

# B.2.23 iparam.infeas\_report\_level

## Corresponding constant:

iparam.infeas\_report\_level

## Description:

Controls the amount of information presented in an infeasibility report. Higher values imply more information.

## Possible Values:

Any number between 0 and +inf.

## Default value:

## B.2.24 iparam.intpnt\_basis

## Corresponding constant:

iparam.intpnt\_basis

## Description:

Controls whether the interior-point optimizer also computes an optimal basis.

## Possible values:

- basindtype.always Basis identification is always performed even if the interior-point optimizer terminates abnormally.
- basindtype.if\_feasible Basis identification is not performed if the interior-point optimizer terminates with a problem status saying that the problem is primal or dual infeasible.
- basindtype.never Never do basis identification.
- basindtype.no\_error Basis identification is performed if the interior-point optimizer terminates without an error.
- basindtype.reservered Not currently in use.

#### Default value:

basindtype.always

## See also:

- iparam.bi\_ignore\_max\_iter Turns on basis identification in case the interior-point optimizer is terminated due to maximum number of iterations.
- iparam.bi\_ignore\_num\_error Turns on basis identification in case the interior-point optimizer is terminated due to a numerical problem.

# B.2.25 iparam.intpnt\_diff\_step

## Corresponding constant:

 $iparam.intpnt\_diff\_step$ 

#### **Description:**

Controls whether different step sizes are allowed in the primal and dual space.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.on

# B.2.26 iparam.intpnt\_factor\_debug\_lvl

## Corresponding constant:

 $iparam.intpnt\_factor\_debug\_lvl$ 

## Description:

Controls factorization debug level.

## Possible Values:

Any number between 0 and +inf.

## Default value:

0

# B.2.27 iparam.intpnt\_factor\_method

# Corresponding constant:

 $iparam.intpnt\_factor\_method$ 

## Description:

Controls the method used to factor the Newton equation system.

## Possible Values:

Any number between 0 and +inf.

## Default value:

0

# B.2.28 iparam.intpnt\_hotstart

## Corresponding constant:

 $iparam.intpnt\_hotstart$ 

## Description:

Currently not in use.

## Possible values:

- intpnthotstart.dual The interior-point optimizer exploits the dual solution only.
- intpnthotstart.none The interior-point optimizer performs a coldstart.
- intpnthotstart.primal The interior-point optimizer exploits the primal solution only.
- intpnthotstart.primal\_dual The interior-point optimizer exploits both the primal and dual solution.

## Default value:

intpnthotstart.none

# B.2.29 iparam.intpnt\_max\_iterations

## Corresponding constant:

 $iparam.intpnt\_max\_iterations$ 

## **Description:**

Controls the maximum number of iterations allowed in the interior-point optimizer.

## Possible Values:

Any number between 0 and +inf.

#### Default value:

400

# B.2.30 iparam.intpnt\_max\_num\_cor

## Corresponding constant:

iparam.intpnt\_max\_num\_cor

## **Description:**

Controls the maximum number of correctors allowed by the multiple corrector procedure. A negative value means that MOSEK is making the choice.

## Possible Values:

Any number between -1 and +inf.

# Default value:

-1

# B.2.31 iparam.intpnt\_max\_num\_refinement\_steps

## Corresponding constant:

iparam.intpnt\_max\_num\_refinement\_steps

# Description:

Maximum number of steps to be used by the iterative refinement of the search direction. A negative value implies that the optimizer Chooses the maximum number of iterative refinement steps.

## Possible Values:

Any number between -inf and +inf.

## Default value:

-1

# B.2.32 iparam.intpnt\_off\_col\_trh

## Corresponding constant:

iparam.intpnt\_off\_col\_trh

## **Description:**

Controls how many offending columns are detected in the Jacobian of the constraint matrix.

1 means aggressive detection, higher values mean less aggressive detection.

0 means no detection.

## Possible Values:

Any number between 0 and  $+\inf$ .

#### Default value:

40

# B.2.33 iparam.intpnt\_order\_method

## Corresponding constant:

iparam.intpnt\_order\_method

## Description:

Controls the ordering strategy used by the interior-point optimizer when factorizing the Newton equation system.

### Possible values:

- orderingtype.appminloc Approximate minimum local fill-in ordering is employed.
- orderingtype.experimental This option should not be used.
- orderingtype.force\_graphpar Always use the graph partitioning based ordering even if it is worse that the approximate minimum local fill ordering.
- orderingtype.free The ordering method is chosen automatically.
- orderingtype.none No ordering is used.
- orderingtype.try\_graphpar Always try the the graph partitioning based ordering.

## Default value:

```
orderingtype.free
```

# B.2.34 iparam.intpnt\_regularization\_use

## Corresponding constant:

iparam.intpnt\_regularization\_use

## Description:

Controls whether regularization is allowed.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.on
```

# B.2.35 iparam.intpnt\_scaling

## Corresponding constant:

iparam.intpnt\_scaling

## Description:

Controls how the problem is scaled before the interior-point optimizer is used.

## Possible values:

- scalingtype.aggressive A very aggressive scaling is performed.
- scalingtype.free The optimizer chooses the scaling heuristic.
- scalingtype.moderate A conservative scaling is performed.
- scalingtype.none No scaling is performed.

## Default value:

```
scalingtype.free
```

# B.2.36 iparam.intpnt\_solve\_form

## Corresponding constant:

 $iparam.intpnt\_solve\_form$ 

## **Description:**

Controls whether the primal or the dual problem is solved.

## Possible values:

- solveform.dual The optimizer should solve the dual problem.
- solveform.free The optimizer is free to solve either the primal or the dual problem.
- solveform.primal The optimizer should solve the primal problem.

#### Default value:

solveform.free

# B.2.37 iparam.intpnt\_starting\_point

# Corresponding constant:

iparam.intpnt\_starting\_point

## **Description:**

Starting point used by the interior-point optimizer.

#### Possible values:

- startpointtype.constant The optimizer constructs a starting point by assigning a constant value to all primal and dual variables. This starting point is normally robust.
- startpointtype.free The starting point is chosen automatically.
- startpointtype.guess The optimizer guesses a starting point.
- startpointtype.satisfy\_bounds The starting point is choosen to satisfy all the simple bounds on nonlinear variables. If this starting point is employed, then more care than usual should employed when choosing the bounds on the nonlinear variables. In particular very tight bounds should be avoided.

## Default value:

startpointtype.free

# B.2.38 iparam.lic\_trh\_expiry\_wrn

## Corresponding constant:

iparam.lic\_trh\_expiry\_wrn

## Description:

If a license feature expires in a numbers days less than the value of this parameter then a warning will be issued.

### Possible Values:

Any number between 0 and  $+\inf$ .

## Default value:

# B.2.39 iparam.license\_debug

## Corresponding constant:

iparam.license\_debug

# Description:

This option is used to turn on debugging of the incense manager.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.off

# B.2.40 iparam.license\_pause\_time

## Corresponding constant:

iparam.license\_pause\_time

## Description:

If iparam.license\_wait=onoffkey.on and no license is available, then MOSEK sleeps a number of milliseconds between each check of whether a license has become free.

#### Possible Values:

Any number between 0 and 1000000.

## Default value:

100

# B.2.41 iparam.license\_suppress\_expire\_wrns

## Corresponding constant:

iparam.license\_suppress\_expire\_wrns

# Description:

Controls whether license features expire warnings are suppressed.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.off
```

# B.2.42 iparam.license\_wait

## Corresponding constant:

iparam.license\_wait

## Description:

If all licenses are in use MOSEK returns with an error code. However, by turning on this parameter MOSEK will wait for an available license.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.off

# B.2.43 iparam.log

## Corresponding constant:

iparam.log

## **Description:**

Controls the amount of log information. The value 0 implies that all log information is suppressed. A higher level implies that more information is logged.

Please note that if a task is employed to solve a sequence of optimization problems the value of this parameter is reduced by the value of <code>iparam.log\_cut\_second\_opt</code> for the second and any subsequent optimizations.

## Possible Values:

Any number between 0 and +inf.

## Default value:

10

## See also:

• iparam.log\_cut\_second\_opt Controls the reduction in the log levels for the second and any subsequent optimizations.

# B.2.44 iparam.log\_bi

## Corresponding constant:

iparam.log\_bi

## Description:

Controls the amount of output printed by the basis identification procedure. A higher level implies that more information is logged.

## Possible Values:

Any number between 0 and +inf.

## Default value:

4

# B.2.45 iparam.log\_bi\_freq

## Corresponding constant:

iparam.log\_bi\_freq

## Description:

Controls how frequent the optimizer outputs information about the basis identification and how frequent the user-defined call-back function is called.

### Possible Values:

Any number between 0 and  $+\inf$ .

## Default value:

2500

# B.2.46 iparam.log\_check\_convexity

## Corresponding constant:

iparam.log\_check\_convexity

## **Description:**

Controls logging in convexity check on quadratic problems. Set to a positive value to turn logging on.

If a quadratic coefficient matrix is found to violate the requirement of PSD (NSD) then a list of negative (positive) pivot elements is printed. The absolute value of the pivot elements is also shown.

## Possible Values:

Any number between 0 and +inf.

### Default value:

0

# B.2.47 iparam.log\_concurrent

## Corresponding constant:

iparam.log\_concurrent

## Description:

Controls amount of output printed by the concurrent optimizer.

#### Possible Values:

Any number between 0 and +inf.

#### Default value:

1

# B.2.48 iparam.log\_cut\_second\_opt

# Corresponding constant:

iparam.log\_cut\_second\_opt

## Description:

If a task is employed to solve a sequence of optimization problems, then the value of the log levels is reduced by the value of this parameter. E.g <code>iparam.log</code> and <code>iparam.log\_sim</code> are reduced by the value of this parameter for the second and any subsequent optimizations.

## Possible Values:

Any number between 0 and +inf.

## Default value:

1

## See also:

- iparam.log Controls the amount of log information.
- iparam.log\_intpnt Controls the amount of log information from the interior-point optimizers.
- iparam.log\_mio Controls the amount of log information from the mixed-integer optimizers.
- iparam.log\_sim Controls the amount of log information from the simplex optimizers.

# B.2.49 iparam.log\_expand

## Corresponding constant:

iparam.log\_expand

## Description:

Controls the amount of logging when a data item such as the maximum number constrains is expanded.

## Possible Values:

Any number between 0 and +inf.

## Default value:

0

# B.2.50 iparam.log\_factor

## Corresponding constant:

iparam.log\_factor

## Description:

If turned on, then the factor log lines are added to the log.

## Possible Values:

Any number between 0 and +inf.

#### Default value:

1

# B.2.51 iparam.log\_feas\_repair

## Corresponding constant:

iparam.log\_feas\_repair

## Description:

Controls the amount of output printed when performing feasibility repair. A value higher than one means extensive logging.

## Possible Values:

Any number between 0 and +inf.

## Default value:

# B.2.52 iparam.log\_file

## Corresponding constant:

 $iparam.log\_file$ 

## Description:

If turned on, then some log info is printed when a file is written or read.

## Possible Values:

Any number between 0 and +inf.

## Default value:

1

# B.2.53 iparam.log\_head

# Corresponding constant:

iparam.log\_head

## **Description:**

If turned on, then a header line is added to the log.

## Possible Values:

Any number between 0 and  $+\inf$ .

# Default value:

1

# B.2.54 iparam.log\_infeas\_ana

## Corresponding constant:

iparam.log\_infeas\_ana

## Description:

Controls amount of output printed by the infeasibility analyzer procedures. A higher level implies that more information is logged.

# Possible Values:

Any number between 0 and +inf.

## Default value:

# B.2.55 iparam.log\_intpnt

## Corresponding constant:

 $iparam.log\_intpnt$ 

## Description:

Controls amount of output printed printed by the interior-point optimizer. A higher level implies that more information is logged.

## Possible Values:

Any number between 0 and  $+\inf$ .

## Default value:

4

# B.2.56 iparam.log\_mio

## Corresponding constant:

iparam.log\_mio

## Description:

Controls the log level for the mixed-integer optimizer. A higher level implies that more information is logged.

## Possible Values:

Any number between 0 and +inf.

## Default value:

4

# B.2.57 iparam.log\_mio\_freq

## Corresponding constant:

iparam.log\_mio\_freq

## Description:

Controls how frequent the mixed-integer optimizer prints the log line. It will print line every time iparam.log\_mio\_freq relaxations have been solved.

# Possible Values:

A integer value.

## Default value:

# B.2.58 iparam.log\_nonconvex

# Corresponding constant:

 $iparam.log\_nonconvex$ 

## Description:

Controls amount of output printed by the nonconvex optimizer.

## Possible Values:

Any number between 0 and +inf.

## Default value:

1

# B.2.59 iparam.log\_optimizer

# Corresponding constant:

iparam.log\_optimizer

## Description:

Controls the amount of general optimizer information that is logged.

## Possible Values:

Any number between 0 and  $+\inf$ .

## Default value:

1

# B.2.60 iparam.log\_order

# Corresponding constant:

iparam.log\_order

## Description:

If turned on, then factor lines are added to the log.

## Possible Values:

Any number between 0 and +inf.

#### Default value:

# B.2.61 iparam.log\_param

## Corresponding constant:

 $iparam.log\_param$ 

## Description:

Controls the amount of information printed out about parameter changes.

## Possible Values:

Any number between 0 and +inf.

## Default value:

0

# B.2.62 iparam.log\_presolve

## Corresponding constant:

iparam.log\_presolve

## Description:

Controls amount of output printed by the presolve procedure. A higher level implies that more information is logged.

## Possible Values:

Any number between 0 and +inf.

#### Default value:

1

# B.2.63 iparam.log\_response

## Corresponding constant:

iparam.log\_response

## Description:

Controls amount of output printed when response codes are reported. A higher level implies that more information is logged.

## Possible Values:

Any number between 0 and +inf.

## Default value:

# B.2.64 iparam.log\_sensitivity

## Corresponding constant:

iparam.log\_sensitivity

## **Description:**

Controls the amount of logging during the sensitivity analysis. 0: Means no logging information is produced. 1: Timing information is printed. 2: Sensitivity results are printed.

## Possible Values:

Any number between 0 and  $+\inf$ .

#### Default value:

1

# B.2.65 iparam.log\_sensitivity\_opt

## Corresponding constant:

 $iparam.log\_sensitivity\_opt$ 

## Description:

Controls the amount of logging from the optimizers employed during the sensitivity analysis. 0 means no logging information is produced.

## Possible Values:

Any number between 0 and +inf.

## Default value:

0

## B.2.66 iparam.log\_sim

## Corresponding constant:

iparam.log\_sim

## Description:

Controls amount of output printed by the simplex optimizer. A higher level implies that more information is logged.

### Possible Values:

Any number between 0 and +inf.

## Default value:

# B.2.67 iparam.log\_sim\_freq

## Corresponding constant:

iparam.log\_sim\_freq

## Description:

Controls how frequent the simplex optimizer outputs information about the optimization and how frequent the user-defined call-back function is called.

## Possible Values:

Any number between 0 and  $+\inf$ .

#### Default value:

1000

# B.2.68 iparam.log\_sim\_minor

## Corresponding constant:

iparam.log\_sim\_minor

## Description:

Currently not in use.

## Possible Values:

Any number between 0 and +inf.

# Default value:

1

# B.2.69 iparam.log\_sim\_network\_freq

## Corresponding constant:

iparam.log\_sim\_network\_freq

# Description:

Controls how frequent the network simplex optimizer outputs information about the optimization and how frequent the user-defined call-back function is called. The network optimizer will use a logging frequency equal to <code>iparam.log\_sim\_freq</code> times <code>iparam.log\_sim\_network\_freq</code>.

# Possible Values:

Any number between 0 and +inf.

## Default value:

# B.2.70 iparam.log\_storage

## Corresponding constant:

iparam.log\_storage

# Description:

When turned on, MOSEK prints messages regarding the storage usage and allocation.

#### Possible Values:

Any number between 0 and +inf.

#### Default value:

0

## B.2.71 iparam.max\_num\_warnings

## Corresponding constant:

iparam.max\_num\_warnings

## Description:

A negtive number means all warnings are logged. Otherwise the parameter specifies the maximum number times each warning is logged.

#### Possible Values:

Any number between  $-\inf$  and  $+\inf$ .

## Default value:

6

# B.2.72 iparam.mio\_branch\_dir

## Corresponding constant:

iparam.mio\_branch\_dir

## **Description:**

Controls whether the mixed-integer optimizer is branching up or down by default.

#### Possible values:

- branchdir.down The mixed-integer optimizer always chooses the down branch first.
- branchdir.free The mixed-integer optimizer decides which branch to choose.
- branchdir.up The mixed-integer optimizer always chooses the up branch first.

## Default value:

branchdir.free

# B.2.73 iparam.mio\_branch\_priorities\_use

## Corresponding constant:

iparam.mio\_branch\_priorities\_use

## Description:

Controls whether branching priorities are used by the mixed-integer optimizer.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.on
```

# B.2.74 iparam.mio\_construct\_sol

## Corresponding constant:

iparam.mio\_construct\_sol

## Description:

If set to onoffkey.on and all integer variables have been given a value for which a feasible mixed integer solution exists, then MOSEK generates an initial solution to the mixed integer problem by fixing all integer values and solving the remaining problem.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.off
```

# B.2.75 iparam.mio\_cont\_sol

# Corresponding constant:

 $iparam.mio\_cont\_sol$ 

## **Description:**

Controls the meaning of the interior-point and basic solutions in mixed integer problems.

### Possible values:

- miocontsoltype.itg The reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. A solution is only reported in case the problem has a primal feasible solution.
- miocontsoltype.itg\_rel In case the problem is primal feasible then the reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. If the problem is primal infeasible, then the solution to the root node problem is reported.
- miocontsoltype.none No interior-point or basic solution are reported when the mixed-integer optimizer is used.
- miocontsoltype.root The reported interior-point and basic solutions are a solution to the root node problem when mixed-integer optimizer is used.

## Default value:

miocontsoltype.none

# B.2.76 iparam.mio\_cut\_cg

# Corresponding constant:

iparam.mio\_cut\_cg

## Description:

Controls whether CG cuts should be generated.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.on

## B.2.77 iparam.mio\_cut\_cmir

## Corresponding constant:

iparam.mio\_cut\_cmir

### Description:

Controls whether mixed integer rounding cuts should be generated.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.on

# B.2.78 iparam.mio\_cut\_level\_root

## Corresponding constant:

iparam.mio\_cut\_level\_root

## **Description:**

Controls the cut level employed by the mixed-integer optimizer at the root node. A negative value means a default value determined by the mixed-integer optimizer is used. By adding the appropriate values from the following table the employed cut types can be controlled.

GUB cover	+2
Flow cover	+4
Lifting	+8
Plant location	+16
Disaggregation	+32
Knapsack cover	+64
Lattice	+128
Gomory	+256
Coefficient reduction	+512
GCD	+1024
Obj. integrality	+2048

## Possible Values:

Any value.

## Default value:

-1

# B.2.79 iparam.mio\_cut\_level\_tree

## Corresponding constant:

 $iparam.mio\_cut\_level\_tree$ 

## Description:

Controls the cut level employed by the mixed-integer optimizer at the tree. See <code>iparam.mio\_cut\_level\_root</code> for an explanation of the parameter values.

## Possible Values:

Any value.

## Default value:

-1

# B.2.80 iparam.mio\_feaspump\_level

## Corresponding constant:

iparam.mio\_feaspump\_level

## Description:

Feasibility pump is a heuristic designed to compute an initial feasible solution. A value of 0 implies that the feasibility pump heuristic is not used. A value of -1 implies that the mixed-integer optimizer decides how the feasibility pump heuristic is used. A larger value than 1 implies that the feasibility pump is employed more aggressively. Normally a value beyond 3 is not worthwhile.

#### Possible Values:

Any number between -inf and 3.

#### Default value:

-1

## B.2.81 iparam.mio\_heuristic\_level

## Corresponding constant:

iparam.mio\_heuristic\_level

## **Description:**

Controls the heuristic employed by the mixed-integer optimizer to locate an initial good integer feasible solution. A value of zero means the heuristic is not used at all. A larger value than 0 means that a gradually more sophisticated heuristic is used which is computationally more expensive. A negative value implies that the optimizer chooses the heuristic. Normally a value around 3 to 5 should be optimal.

## Possible Values:

Any value.

## Default value:

-1

# B.2.82 iparam.mio\_hotstart

## Corresponding constant:

iparam.mio\_hotstart

## **Description:**

Controls whether the integer optimizer is hot-started.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.on
```

# $B.2.83 \quad iparam.mio\_keep\_basis$

## Corresponding constant:

 $iparam.mio_keep_basis$ 

## Description:

Controls whether the integer presolve keeps bases in memory. This speeds on the solution process at cost of bigger memory consumption.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.on
```

# B.2.84 iparam.mio\_local\_branch\_number

# Corresponding constant:

 $iparam.mio\_local\_branch\_number$ 

## Description:

Controls the size of the local search space when doing local branching.

## Possible Values:

Any number between  $-\inf$  and  $+\inf$ .

## Default value:

-1

# B.2.85 iparam.mio\_max\_num\_branches

# Corresponding constant:

 $iparam.mio\_max\_num\_branches$ 

## **Description:**

Maximum number of branches allowed during the branch and bound search. A negative value means infinite.

#### Possible Values:

Any number between -inf and +inf.

#### Default value:

-1

#### See also:

• dparam.mio\_disable\_term\_time Certain termination criteria is disabled within the mixed-integer optimizer for period time specified by the parameter.

# B.2.86 iparam.mio\_max\_num\_relaxs

## Corresponding constant:

iparam.mio\_max\_num\_relaxs

## **Description:**

Maximum number of relaxations allowed during the branch and bound search. A negative value means infinite.

## Possible Values:

Any number between -inf and +inf.

## Default value:

-1

#### See also:

• dparam.mio\_disable\_term\_time Certain termination criteria is disabled within the mixed-integer optimizer for period time specified by the parameter.

# B.2.87 iparam.mio\_max\_num\_solutions

## Corresponding constant:

iparam.mio\_max\_num\_solutions

## Description:

The mixed-integer optimizer can be terminated after a certain number of different feasible solutions has been located. If this parameter has the value n and n is strictly positive, then the mixed-integer optimizer will be terminated when n feasible solutions have been located.

#### Possible Values:

Any number between -inf and +inf.

#### Default value:

-1

## See also:

• dparam.mio\_disable\_term\_time Certain termination criteria is disabled within the mixed-integer optimizer for period time specified by the parameter.

# B.2.88 iparam.mio\_mode

## Corresponding constant:

iparam.mio\_mode

## Description:

Controls whether the optimizer includes the integer restrictions when solving a (mixed) integer optimization problem.

# Possible values:

- miomode.ignored The integer constraints are ignored and the problem is solved as a continuous problem.
- miomode.lazy Integer restrictions should be satisfied if an optimizer is available for the problem.
- miomode.satisfied Integer restrictions should be satisfied.

### Default value:

miomode.satisfied

# B.2.89 iparam.mio\_mt\_user\_cb

## Corresponding constant:

 $iparam.mio_mt_user_cb$ 

#### **Description:**

It true user callbacks are called from each thread used by this optimizer. If false the user callback is only called from a single thread.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.on

# B.2.90 iparam.mio\_node\_optimizer

## Corresponding constant:

iparam.mio\_node\_optimizer

## Description:

Controls which optimizer is employed at the non-root nodes in the mixed-integer optimizer.

#### Possible values:

- optimizertype.concurrent The optimizer for nonconvex nonlinear problems.
- optimizertype.conic The optimizer for problems having conic constraints.
- optimizertype.dual\_simplex The dual simplex optimizer is used.
- optimizertype.free The optimizer is chosen automatically.
- optimizertype.free\_simplex One of the simplex optimizers is used.
- optimizertype.intpnt The interior-point optimizer is used.
- optimizertype.mixed\_int The mixed-integer optimizer.
- optimizertype.mixed\_int\_conic The mixed-integer optimizer for conic and linear problems.
- optimizertype.network\_primal\_simplex The network primal simplex optimizer is used. It is only applicable to pure network problems.
- optimizertype.nonconvex The optimizer for nonconvex nonlinear problems.
- optimizertype.primal\_dual\_simplex The primal dual simplex optimizer is used.
- optimizertype.primal\_simplex The primal simplex optimizer is used.

## Default value:

optimizertype.free

## B.2.91 iparam.mio\_node\_selection

## Corresponding constant:

iparam.mio\_node\_selection

#### **Description:**

Controls the node selection strategy employed by the mixed-integer optimizer.

## Possible values:

- mionodeseltype.best The optimizer employs a best bound node selection strategy.
- mionodeseltype.first The optimizer employs a depth first node selection strategy.
- mionodeseltype.free The optimizer decides the node selection strategy.
- mionodeseltype.hybrid The optimizer employs a hybrid strategy.

- mionodeseltype.pseudo The optimizer employs selects the node based on a pseudo cost estimate.
- mionodeseltype.worst The optimizer employs a worst bound node selection strategy.

# Default value:

```
mionodeseltype.free
```

# B.2.92 iparam.mio\_optimizer\_mode

# Corresponding constant:

 $iparam.mio\_optimizer\_mode$ 

## Description:

An exprimental feature.

## Possible Values:

Any number between 0 and 1.

## Default value:

0

# B.2.93 iparam.mio\_presolve\_aggregate

# Corresponding constant:

 $iparam.mio\_presolve\_aggregate$ 

## Description:

Controls whether the presolve used by the mixed-integer optimizer tries to aggregate the constraints.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.on
```

# B.2.94 iparam.mio\_presolve\_probing

## Corresponding constant:

iparam.mio\_presolve\_probing

## **Description:**

Controls whether the mixed-integer presolve performs probing. Probing can be very time consuming.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.on

# B.2.95 iparam.mio\_presolve\_use

## Corresponding constant:

 $iparam.mio\_presolve\_use$ 

## Description:

Controls whether presolve is performed by the mixed-integer optimizer.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

### Default value:

onoffkey.on

# B.2.96 iparam.mio\_probing\_level

# Corresponding constant:

 $iparam.mio\_probing\_level$ 

## Description:

Controls the amount of probing employed by the mixed-integer optimizer in presolve.

- -1 The optimizer chooses the level of probing employed.
- 0 Probing is disabled.
- 1 A low amount of probing is employed.

- 2 A medium amount of probing is employed.
- 3 A high amount of probing is employed.

## Possible Values:

An integer value in the range of -1 to 3.

### Default value:

-1

# B.2.97 iparam.mio\_rins\_max\_nodes

## Corresponding constant:

iparam.mio\_rins\_max\_nodes

## Description:

Controls the maximum number of nodes allowed in each call to the RINS heuristic. The default value of -1 means that the value is determined automatically. A value of zero turns off the heuristic.

## Possible Values:

Any number between -1 and  $+\inf$ .

## Default value:

-1

# B.2.98 iparam.mio\_root\_optimizer

## Corresponding constant:

iparam.mio\_root\_optimizer

# Description:

Controls which optimizer is employed at the root node in the mixed-integer optimizer.

# Possible values:

- optimizertype.concurrent The optimizer for nonconvex nonlinear problems.
- optimizertype.conic The optimizer for problems having conic constraints.
- optimizertype.dual\_simplex The dual simplex optimizer is used.
- $\bullet$  optimizer type.free The optimizer is chosen automatically.
- optimizertype.free\_simplex One of the simplex optimizers is used.
- optimizertype.intpnt The interior-point optimizer is used.
- optimizertype.mixed\_int The mixed-integer optimizer.

- optimizertype.mixed\_int\_conic The mixed-integer optimizer for conic and linear problems.
- optimizertype.network\_primal\_simplex The network primal simplex optimizer is used. It is only applicable to pure network problems.
- optimizertype.nonconvex The optimizer for nonconvex nonlinear problems.
- optimizertype.primal\_dual\_simplex The primal dual simplex optimizer is used.
- optimizertype.primal\_simplex The primal simplex optimizer is used.

## Default value:

```
optimizertype.free
```

# B.2.99 iparam.mio\_strong\_branch

#### Corresponding constant:

iparam.mio\_strong\_branch

# Description:

The value specifies the depth from the root in which strong branching is used. A negative value means that the optimizer chooses a default value automatically.

#### Possible Values:

Any number between -inf and +inf.

#### Default value:

-1

## B.2.100 iparam.mio\_use\_multithreaded\_optimizer

## Corresponding constant:

 $iparam.mio\_use\_multithreaded\_optimizer$ 

## Description:

Controls wheter the new multithreaded optimizer should be used for Mixed integer problems.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.off
```

## B.2.101 iparam.mt\_spincount

## Corresponding constant:

iparam.mt\_spincount

## Description:

Set the number of iterations to spin before sleeping.

### Possible Values:

Any integer greater or equal to 0.

#### Default value:

0

## B.2.102 iparam.nonconvex\_max\_iterations

## Corresponding constant:

 $iparam.nonconvex\_max\_iterations$ 

### **Description:**

Maximum number of iterations that can be used by the nonconvex optimizer.

#### Possible Values:

Any number between 0 and  $+\inf$ .

## Default value:

100000

## B.2.103 iparam.num\_threads

## Corresponding constant:

iparam.num\_threads

#### **Description:**

Controls the number of threads employed by the optimizer. If set to 0 the number of threads used will be equal to the number of cores detected on the machine.

#### Possible Values:

Any integer greater or equal to 0.

#### Default value:

# B.2.104 iparam.opf\_max\_terms\_per\_line

## Corresponding constant:

iparam.opf\_max\_terms\_per\_line

## Description:

The maximum number of terms (linear and quadratic) per line when an OPF file is written.

#### Possible Values:

Any number between 0 and +inf.

#### Default value:

5

# B.2.105 iparam.opf\_write\_header

## Corresponding constant:

iparam.opf\_write\_header

### Description:

Write a text header with date and MOSEK version in an OPF file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.on

## B.2.106 iparam.opf\_write\_hints

## Corresponding constant:

iparam.opf\_write\_hints

#### Description:

Write a hint section with problem dimensions in the beginning of an OPF file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.on
```

## B.2.107 iparam.opf\_write\_parameters

## Corresponding constant:

 $iparam.opf\_write\_parameters$ 

## Description:

Write a parameter section in an OPF file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.off

# B.2.108 iparam.opf\_write\_problem

# Corresponding constant:

 $iparam.opf\_write\_problem$ 

## Description:

Write objective, constraints, bounds etc. to an OPF file.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.on

# B.2.109 iparam.opf\_write\_sol\_bas

## Corresponding constant:

 $iparam.opf\_write\_sol\_bas$ 

#### **Description:**

If iparam.opf\_write\_solutions is onoffkey.on and a basic solution is defined, include the basic solution in OPF files.

## Possible values:

• onoffkey.off Switch the option off.

• onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.on
```

# B.2.110 iparam.opf\_write\_sol\_itg

## Corresponding constant:

iparam.opf\_write\_sol\_itg

### Description:

If <code>iparam.opf\_write\_solutions</code> is <code>onoffkey.on</code> and an integer solution is defined, write the integer solution in OPF files.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.on
```

# B.2.111 iparam.opf\_write\_sol\_itr

## Corresponding constant:

 $iparam.opf\_write\_sol\_itr$ 

## Description:

If iparam.opf\_write\_solutions is onoffkey.on and an interior solution is defined, write the interior solution in OPF files.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.on
```

# B.2.112 iparam.opf\_write\_solutions

### Corresponding constant:

iparam.opf\_write\_solutions

## **Description:**

Enable inclusion of solutions in the OPF files.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.off

## B.2.113 iparam.optimizer

## Corresponding constant:

iparam.optimizer

#### **Description:**

The paramter controls which optimizer is used to optimize the task.

#### Possible values:

- optimizertype.concurrent The optimizer for nonconvex nonlinear problems.
- optimizertype.conic The optimizer for problems having conic constraints.
- optimizertype.dual\_simplex The dual simplex optimizer is used.
- optimizertype.free The optimizer is chosen automatically.
- optimizertype.free\_simplex One of the simplex optimizers is used.
- optimizertype.intpnt The interior-point optimizer is used.
- optimizertype.mixed\_int The mixed-integer optimizer.
- optimizertype.mixed\_int\_conic The mixed-integer optimizer for conic and linear problems.
- optimizertype.network\_primal\_simplex The network primal simplex optimizer is used. It is only applicable to pure network problems.
- optimizertype.nonconvex The optimizer for nonconvex nonlinear problems.
- optimizertype.primal\_dual\_simplex The primal dual simplex optimizer is used.
- optimizertype.primal\_simplex The primal simplex optimizer is used.

#### Default value:

optimizertype.free

## B.2.114 iparam\_read\_case\_name

### Corresponding constant:

iparam\_read\_case\_name

## Description:

If turned on, then names in the parameter file are case sensitive.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

### Default value:

```
onoffkey.on
```

# B.2.115 iparam\_read\_ign\_error

# Corresponding constant:

iparam.param\_read\_ign\_error

### Description:

If turned on, then errors in paramter settings is ignored.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.off
```

## B.2.116 iparam.presolve\_elim\_fill

## Corresponding constant:

iparam.presolve\_elim\_fill

## **Description:**

Controls the maximum amount of fill-in that can be created during the elimination phase of the presolve. This parameter times (numcon+numvar) denotes the amount of fill-in.

### Possible Values:

Any number between 0 and +inf.

## Default value:

## B.2.117 iparam.presolve\_eliminator\_max\_num\_tries

## Corresponding constant:

iparam.presolve\_eliminator\_max\_num\_tries

#### **Description:**

Control the maximum number of times the eliminator is tried.

#### Possible Values:

A negative value implies MOSEK decides maximum number of times.

#### Default value:

-1

## B.2.118 iparam.presolve\_eliminator\_use

## Corresponding constant:

 $iparam.presolve\_eliminator\_use$ 

## Description:

Controls whether free or implied free variables are eliminated from the problem.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.on

## B.2.119 iparam.presolve\_level

### Corresponding constant:

 $iparam.presolve\_level$ 

## Description:

Currently not used.

### Possible Values:

Any number between -inf and +inf.

### Default value:

-1

## B.2.120 iparam.presolve\_lindep\_abs\_work\_trh

## Corresponding constant:

iparam.presolve\_lindep\_abs\_work\_trh

### Description:

The linear dependency check is potentially computationally expensive.

#### Possible Values:

Any number between 0 and  $+\inf$ .

### Default value:

100

## B.2.121 iparam.presolve\_lindep\_rel\_work\_trh

## Corresponding constant:

 $iparam.presolve\_lindep\_rel\_work\_trh$ 

## Description:

The linear dependency check is potentially computationally expensive.

#### Possible Values:

Any number between 0 and +inf.

#### Default value:

100

## B.2.122 iparam.presolve\_lindep\_use

## Corresponding constant:

iparam.presolve\_lindep\_use

## **Description:**

Controls whether the linear constraints are checked for linear dependencies.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.on
```

## B.2.123 iparam.presolve\_max\_num\_reductions

### Corresponding constant:

iparam.presolve\_max\_num\_reductions

## Description:

Controls the maximum number reductions performed by the presolve. The value of the parameter is normally only changed in connection with debugging. A negative value implies that an infinite number of reductions are allowed.

#### Possible Values:

Any number between -inf and +inf.

#### Default value:

-1

## B.2.124 iparam.presolve\_use

## Corresponding constant:

iparam.presolve\_use

## Description:

Controls whether the presolve is applied to a problem before it is optimized.

#### Possible values:

- presolvemode.free It is decided automatically whether to presolve before the problem is optimized.
- presolvemode.off The problem is not presolved before it is optimized.
- presolvemode.on The problem is presolved before it is optimized.

#### Default value:

presolvemode.free

## B.2.125 iparam.primal\_repair\_optimizer

## Corresponding constant:

iparam.primal\_repair\_optimizer

## **Description:**

Controls which optimizer that is used to find the optimal repair.

#### Possible values:

• optimizertype.concurrent The optimizer for nonconvex nonlinear problems.

- optimizertype.conic The optimizer for problems having conic constraints.
- optimizertype.dual\_simplex The dual simplex optimizer is used.
- optimizertype.free The optimizer is chosen automatically.
- optimizertype.free\_simplex One of the simplex optimizers is used.
- optimizertype.intpnt The interior-point optimizer is used.
- optimizertype.mixed\_int The mixed-integer optimizer.
- optimizertype.mixed\_int\_conic The mixed-integer optimizer for conic and linear problems.
- optimizertype.network\_primal\_simplex The network primal simplex optimizer is used. It is only applicable to pure network problems.
- optimizertype.nonconvex The optimizer for nonconvex nonlinear problems.
- optimizertype.primal\_dual\_simplex The primal dual simplex optimizer is used.
- optimizertype.primal\_simplex The primal simplex optimizer is used.

### Default value:

```
optimizertype.free
```

# B.2.126 iparam.qo\_separable\_reformulation

## Corresponding constant:

iparam.qo\_separable\_reformulation

## Description:

Determine if Quadratic programing problems should be reformulated to separable form.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.off
```

## B.2.127 iparam.read\_anz

## Corresponding constant:

 $iparam.read\_anz$ 

### **Description:**

Expected maximum number of A non-zeros to be read. The option is used only by fast MPS and LP file readers.

#### Possible Values:

Any number between 0 and +inf.

#### Default value:

100000

## B.2.128 iparam.read\_con

### Corresponding constant:

 $iparam.read\_con$ 

# Description:

Expected maximum number of constraints to be read. The option is only used by fast MPS and LP file readers.

#### Possible Values:

Any number between 0 and +inf.

#### Default value:

10000

# B.2.129 iparam.read\_cone

## Corresponding constant:

iparam.read\_cone

## Description:

Expected maximum number of conic constraints to be read. The option is used only by fast MPS and LP file readers.

### Possible Values:

Any number between 0 and +inf.

## Default value:

2500

## B.2.130 iparam.read\_data\_compressed

## Corresponding constant:

 $iparam.read\_data\_compressed$ 

## Description:

If this option is turned on, it is assumed that the data file is compressed.

#### Possible values:

- compresstype.free The type of compression used is chosen automatically.
- compresstype.gzip The type of compression used is gzip compatible.
- compresstype.none No compression is used.

#### Default value:

```
compresstype.free
```

# B.2.131 iparam.read\_data\_format

## Corresponding constant:

 $iparam.read\_data\_format$ 

## Description:

Format of the data file to be read.

#### Possible values:

- dataformat.cb Conic benchmark format.
- dataformat.extension The file extension is used to determine the data file format.
- dataformat.free\_mps The data data a free MPS formatted file.
- dataformat.lp The data file is LP formatted.
- dataformat.mps The data file is MPS formatted.
- dataformat.op The data file is an optimization problem formatted file.
- dataformat.task Generic task dump file.
- dataformat.xml The data file is an XML formatted file.

#### Default value:

```
dataformat.extension
```

## B.2.132 iparam.read\_debug

## Corresponding constant:

iparam.read\_debug

### **Description:**

Turns on additional debugging information when reading files.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.off
```

# B.2.133 iparam.read\_keep\_free\_con

## Corresponding constant:

iparam.read\_keep\_free\_con

## Description:

Controls whether the free constraints are included in the problem.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.off

## B.2.134 iparam.read\_lp\_drop\_new\_vars\_in\_bou

### Corresponding constant:

iparam.read\_lp\_drop\_new\_vars\_in\_bou

## Description:

If this option is turned on, MOSEK will drop variables that are defined for the first time in the bounds section.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

### Default value:

onoffkey.off

## B.2.135 iparam.read\_lp\_quoted\_names

# Corresponding constant:

iparam.read\_lp\_quoted\_names

## Description:

If a name is in quotes when reading an LP file, the quotes will be removed.

## Possible values:

• onoffkey.off Switch the option off.

• onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.on
```

## B.2.136 iparam.read\_mps\_format

## Corresponding constant:

iparam.read\_mps\_format

## Description:

Controls how strictly the MPS file reader interprets the MPS format.

#### Possible values:

- mpsformat.free It is assumed that the input file satisfies the free MPS format. This implies that spaces are not allowed in names. Otherwise the format is free.
- mpsformat.relaxed It is assumed that the input file satisfies a slightly relaxed version of the MPS format.
- mpsformat.strict It is assumed that the input file satisfies the MPS format strictly.

#### Default value:

```
mpsformat.relaxed
```

## B.2.137 iparam.read\_mps\_keep\_int

## Corresponding constant:

iparam.read\_mps\_keep\_int

#### **Description:**

Controls whether MOSEK should keep the integer restrictions on the variables while reading the MPS file.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.on
```

## B.2.138 iparam.read\_mps\_obj\_sense

### Corresponding constant:

 $iparam.read\_mps\_obj\_sense$ 

# Description:

If turned on, the MPS reader uses the objective sense section. Otherwise the MPS reader ignores it.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.on

## B.2.139 iparam.read\_mps\_relax

### Corresponding constant:

iparam.read\_mps\_relax

### **Description:**

If this option is turned on, then mixed integer constraints are ignored when a problem is read.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.on

## B.2.140 iparam.read\_mps\_width

## Corresponding constant:

 $iparam.read\_mps\_width$ 

## Description:

Controls the maximal number of characters allowed in one line of the MPS file.

## Possible Values:

Any positive number greater than 80.

## Default value:

## B.2.141 iparam.read\_qnz

## Corresponding constant:

iparam.read\_qnz

## Description:

Expected maximum number of Q non-zeros to be read. The option is used only by MPS and LP file readers.

#### Possible Values:

Any number between 0 and +inf.

#### Default value:

20000

## B.2.142 iparam.read\_task\_ignore\_param

## Corresponding constant:

iparam.read\_task\_ignore\_param

### Description:

Controls whether MOSEK should ignore the parameter setting defined in the task file and use the default parameter setting instead.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.off

# B.2.143 iparam.read\_var

## Corresponding constant:

 $iparam.read\_var$ 

#### **Description:**

Expected maximum number of variable to be read. The option is used only by MPS and LP file readers.

### Possible Values:

Any number between 0 and +inf.

## Default value:

## B.2.144 iparam.sensitivity\_all

### Corresponding constant:

iparam.sensitivity\_all

### **Description:**

If set to <code>onoffkey.on</code>, then <code>Task.sensitivityreport</code> analyzes all bounds and variables instead of reading a specification from the file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.off

## B.2.145 iparam.sensitivity\_optimizer

#### Corresponding constant:

iparam.sensitivity\_optimizer

### **Description:**

Controls which optimizer is used for optimal partition sensitivity analysis.

#### Possible values:

- optimizertype.concurrent The optimizer for nonconvex nonlinear problems.
- optimizertype.conic The optimizer for problems having conic constraints.
- optimizertype.dual\_simplex The dual simplex optimizer is used.
- optimizertype.free The optimizer is chosen automatically.
- optimizertype.free\_simplex One of the simplex optimizers is used.
- optimizertype.intpnt The interior-point optimizer is used.
- optimizertype.mixed\_int The mixed-integer optimizer.
- optimizertype.mixed\_int\_conic The mixed-integer optimizer for conic and linear problems.
- optimizertype.network\_primal\_simplex The network primal simplex optimizer is used. It is only applicable to pure network problems.
- optimizertype.nonconvex The optimizer for nonconvex nonlinear problems.
- optimizertype.primal\_dual\_simplex The primal dual simplex optimizer is used.
- optimizertype.primal\_simplex The primal simplex optimizer is used.

#### Default value:

optimizertype.free\_simplex

## B.2.146 iparam.sensitivity\_type

## Corresponding constant:

iparam.sensitivity\_type

### Description:

Controls which type of sensitivity analysis is to be performed.

#### Possible values:

- sensitivitytype.basis Basis sensitivity analysis is performed.
- sensitivitytype.optimal\_partition Optimal partition sensitivity analysis is performed.

#### Default value:

sensitivitytype.basis

## B.2.147 iparam.sim\_basis\_factor\_use

## Corresponding constant:

iparam.sim\_basis\_factor\_use

### Description:

Controls whether a (LU) factorization of the basis is used in a hot-start. Forcing a refactorization sometimes improves the stability of the simplex optimizers, but in most cases there is a performance penantty.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.on

# B.2.148 iparam.sim\_degen

# Corresponding constant:

 $iparam.sim\_degen$ 

### **Description:**

Controls how aggressively degeneration is handled.

#### Possible values:

- simdegen.aggressive The simplex optimizer should use an aggressive degeneration strategy.
- simdegen.free The simplex optimizer chooses the degeneration strategy.
- simdegen.minimum The simplex optimizer should use a minimum degeneration strategy.
- simdegen.moderate The simplex optimizer should use a moderate degeneration strategy.
- simdegen.none The simplex optimizer should use no degeneration strategy.

#### Default value:

```
simdegen.free
```

## B.2.149 iparam.sim\_dual\_crash

### Corresponding constant:

 $iparam.sim\_dual\_crash$ 

## Description:

Controls whether crashing is performed in the dual simplex optimizer.

In general if a basis consists of more than (100-this parameter value)% fixed variables, then a crash will be performed.

#### Possible Values:

Any number between 0 and +inf.

### Default value:

90

# B.2.150 iparam.sim\_dual\_phaseone\_method

## Corresponding constant:

iparam.sim\_dual\_phaseone\_method

## Description:

An exprimental feature.

#### Possible Values:

Any number between 0 and 10.

#### Default value:

# B.2.151 iparam.sim\_dual\_restrict\_selection

### Corresponding constant:

iparam.sim\_dual\_restrict\_selection

### Description:

The dual simplex optimizer can use a so-called restricted selection/pricing strategy to chooses the outgoing variable. Hence, if restricted selection is applied, then the dual simplex optimizer first choose a subset of all the potential outgoing variables. Next, for some time it will choose the outgoing variable only among the subset. From time to time the subset is redefined.

A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

#### Possible Values:

Any number between 0 and 100.

#### Default value:

50

## B.2.152 iparam.sim\_dual\_selection

## Corresponding constant:

iparam.sim\_dual\_selection

## Description:

Controls the choice of the incoming variable, known as the selection strategy, in the dual simplex optimizer.

#### Possible values:

- simseltype.ase The optimizer uses approximate steepest-edge pricing.
- simseltype.devex The optimizer uses devex steepest-edge pricing (or if it is not available an approximate steep-edge selection).
- simseltype.free The optimizer chooses the pricing strategy.
- simseltype.full The optimizer uses full pricing.
- simseltype.partial The optimizer uses a partial selection approach. The approach is usually beneficial if the number of variables is much larger than the number of constraints.
- simseltype.se The optimizer uses steepest-edge selection (or if it is not available an approximate steep-edge selection).

#### Default value:

simseltype.free

## B.2.153 iparam.sim\_exploit\_dupvec

### Corresponding constant:

 $iparam.sim\_exploit\_dupvec$ 

### Description:

Controls if the simplex optimizers are allowed to exploit duplicated columns.

#### Possible values:

- simdupvec.free The simplex optimizer can choose freely.
- simdupvec.off Disallow the simplex optimizer to exploit duplicated columns.
- simdupvec.on Allow the simplex optimizer to exploit duplicated columns.

#### Default value:

```
simdupvec.off
```

## B.2.154 iparam.sim\_hotstart

## Corresponding constant:

iparam.sim\_hotstart

### **Description:**

Controls the type of hot-start that the simplex optimizer perform.

## Possible values:

- simhotstart.free The simplex optimize chooses the hot-start type.
- simhotstart.none The simplex optimizer performs a coldstart.
- simhotstart.status\_keys Only the status keys of the constraints and variables are used to choose the type of hot-start.

#### Default value:

```
simhotstart.free
```

## B.2.155 iparam.sim\_hotstart\_lu

## Corresponding constant:

iparam.sim\_hotstart\_lu

## Description:

Determines if the simplex optimizer should exploit the initial factorization.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.on

# B.2.156 iparam.sim\_integer

## Corresponding constant:

 $iparam.sim\_integer$ 

## Description:

An exprimental feature.

#### Possible Values:

Any number between 0 and 10.

## Default value:

0

# B.2.157 iparam.sim\_max\_iterations

## Corresponding constant:

 $iparam.sim\_max\_iterations$ 

#### **Description:**

Maximum number of iterations that can be used by a simplex optimizer.

#### Possible Values:

Any number between 0 and  $+\inf$ .

#### Default value:

10000000

## B.2.158 iparam.sim\_max\_num\_setbacks

### Corresponding constant:

 $iparam.sim_max_num_setbacks$ 

### Description:

Controls how many set-backs are allowed within a simplex optimizer. A set-back is an event where the optimizer moves in the wrong direction. This is impossible in theory but may happen due to numerical problems.

#### Possible Values:

Any number between 0 and  $+\inf$ .

#### Default value:

250

# B.2.159 iparam.sim\_non\_singular

## Corresponding constant:

 $iparam.sim\_non\_singular$ 

## Description:

Controls if the simplex optimizer ensures a non-singular basis, if possible.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

# Default value:

```
onoffkey.on
```

## B.2.160 iparam.sim\_primal\_crash

## Corresponding constant:

iparam.sim\_primal\_crash

## Description:

Controls whether crashing is performed in the primal simplex optimizer.

In general, if a basis consists of more than (100-this parameter value)% fixed variables, then a crash will be performed.

## Possible Values:

Any nonnegative integer value.

#### Default value:

## B.2.161 iparam.sim\_primal\_phaseone\_method

### Corresponding constant:

iparam.sim\_primal\_phaseone\_method

### Description:

An exprimental feature.

#### Possible Values:

Any number between 0 and 10.

### Default value:

0

# B.2.162 iparam.sim\_primal\_restrict\_selection

## Corresponding constant:

iparam.sim\_primal\_restrict\_selection

### **Description:**

The primal simplex optimizer can use a so-called restricted selection/pricing strategy to chooses the outgoing variable. Hence, if restricted selection is applied, then the primal simplex optimizer first choose a subset of all the potential incoming variables. Next, for some time it will choose the incoming variable only among the subset. From time to time the subset is redefined.

A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

### Possible Values:

Any number between 0 and 100.

#### Default value:

50

## B.2.163 iparam.sim\_primal\_selection

### Corresponding constant:

iparam.sim\_primal\_selection

## Description:

Controls the choice of the incoming variable, known as the selection strategy, in the primal simplex optimizer.

#### Possible values:

• simseltype.ase The optimizer uses approximate steepest-edge pricing.

- simseltype.devex The optimizer uses devex steepest-edge pricing (or if it is not available an approximate steep-edge selection).
- simseltype.free The optimizer chooses the pricing strategy.
- simseltype.full The optimizer uses full pricing.
- simseltype.partial The optimizer uses a partial selection approach. The approach is usually beneficial if the number of variables is much larger than the number of constraints.
- simseltype.se The optimizer uses steepest-edge selection (or if it is not available an approximate steep-edge selection).

#### Default value:

```
simseltype.free
```

## B.2.164 iparam.sim\_refactor\_freq

## Corresponding constant:

iparam.sim\_refactor\_freq

## Description:

Controls how frequent the basis is refactorized. The value 0 means that the optimizer determines the best point of refactorization.

It is strongly recommended NOT to change this parameter.

#### Possible Values:

Any number between 0 and  $+\inf$ .

### Default value:

0

## B.2.165 iparam.sim\_reformulation

### Corresponding constant:

 $iparam.sim\_reformulation$ 

### Description:

Controls if the simplex optimizers are allowed to reformulate the problem.

#### Possible values:

- simreform.aggressive The simplex optimizer should use an aggressive reformulation strategy.
- simreform.free The simplex optimizer can choose freely.
- simreform.off Disallow the simplex optimizer to reformulate the problem.

• simreform.on Allow the simplex optimizer to reformulate the problem.

#### Default value:

```
simreform.off
```

## B.2.166 iparam.sim\_save\_lu

## Corresponding constant:

iparam.sim\_save\_lu

### Description:

Controls if the LU factorization stored should be replaced with the LU factorization corresponding to the initial basis.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.off
```

# B.2.167 iparam.sim\_scaling

#### Corresponding constant:

iparam.sim\_scaling

## Description:

Controls how much effort is used in scaling the problem before a simplex optimizer is used.

#### Possible values:

- scalingtype.aggressive A very aggressive scaling is performed.
- scalingtype.free The optimizer chooses the scaling heuristic.
- scalingtype.moderate A conservative scaling is performed.
- scalingtype.none No scaling is performed.

## Default value:

```
scalingtype.free
```

# B.2.168 iparam.sim\_scaling\_method

### Corresponding constant:

iparam.sim\_scaling\_method

## **Description:**

Controls how the problem is scaled before a simplex optimizer is used.

#### Possible values:

- scalingmethod.free The optimizer chooses the scaling heuristic.
- scalingmethod.pow2 Scales only with power of 2 leaving the mantissa untouched.

#### Default value:

scalingmethod.pow2

## B.2.169 iparam.sim\_solve\_form

### Corresponding constant:

iparam.sim\_solve\_form

#### **Description:**

Controls whether the primal or the dual problem is solved by the primal-/dual- simplex optimizer.

#### Possible values:

- solveform.dual The optimizer should solve the dual problem.
- solveform.free The optimizer is free to solve either the primal or the dual problem.
- solveform.primal The optimizer should solve the primal problem.

### Default value:

solveform.free

## B.2.170 iparam.sim\_stability\_priority

## Corresponding constant:

iparam.sim\_stability\_priority

### Description:

Controls how high priority the numerical stability should be given.

### Possible Values:

Any number between 0 and 100.

## Default value:

## B.2.171 iparam.sim\_switch\_optimizer

### Corresponding constant:

iparam.sim\_switch\_optimizer

## Description:

The simplex optimizer sometimes chooses to solve the dual problem instead of the primal problem. This implies that if you have chosen to use the dual simplex optimizer and the problem is dualized, then it actually makes sense to use the primal simplex optimizer instead. If this parameter is on and the problem is dualized and furthermore the simplex optimizer is chosen to be the primal (dual) one, then it is switched to the dual (primal).

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

### Default value:

onoffkey.off

## B.2.172 iparam.sol\_filter\_keep\_basic

### Corresponding constant:

iparam.sol\_filter\_keep\_basic

## Description:

If turned on, then basic and super basic constraints and variables are written to the solution file independent of the filter setting.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.off

# B.2.173 iparam.sol\_filter\_keep\_ranged

## Corresponding constant:

iparam.sol\_filter\_keep\_ranged

### **Description:**

If turned on, then ranged constraints and variables are written to the solution file independent of the filter setting.

### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.off
```

## B.2.174 iparam.sol\_read\_name\_width

## Corresponding constant:

iparam.sol\_read\_name\_width

## **Description:**

When a solution is read by MOSEK and some constraint, variable or cone names contain blanks, then a maximum name width much be specified. A negative value implies that no name contain blanks.

#### Possible Values:

Any number between -inf and +inf.

### Default value:

-1

## B.2.175 iparam.sol\_read\_width

## Corresponding constant:

iparam.sol\_read\_width

## Description:

Controls the maximal acceptable width of line in the solutions when read by MOSEK.

## Possible Values:

Any positive number greater than 80.

## Default value:

## B.2.176 iparam.solution\_callback

## Corresponding constant:

iparam.solution\_callback

### Description:

Indicates whether solution call-backs will be performed during the optimization.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.off
```

## B.2.177 iparam.timing\_level

## Corresponding constant:

 $iparam.timing\_level$ 

## Description:

Controls the a amount of timing performed inside MOSEK.

## Possible Values:

Any integer greater or equal to 0.

## Default value:

1

## B.2.178 iparam.warning\_level

## Corresponding constant:

iparam.warning\_level

## Description:

Deprecated and not in use

### Possible Values:

Any number between 0 and  $+\inf$ .

### Default value:

## B.2.179 iparam.write\_bas\_constraints

### Corresponding constant:

 $iparam.write\_bas\_constraints$ 

## Description:

Controls whether the constraint section is written to the basic solution file.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

### Default value:

onoffkey.on

# B.2.180 iparam.write\_bas\_head

## Corresponding constant:

iparam.write\_bas\_head

### Description:

Controls whether the header section is written to the basic solution file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.on

## B.2.181 iparam.write\_bas\_variables

## Corresponding constant:

 $iparam.write\_bas\_variables$ 

#### Description:

Controls whether the variables section is written to the basic solution file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.on

# B.2.182 iparam.write\_data\_compressed

## Corresponding constant:

 $iparam.write\_data\_compressed$ 

## Description:

Controls whether the data file is compressed while it is written. 0 means no compression while higher values mean more compression.

## Possible Values:

Any number between 0 and +inf.

#### Default value:

0

## B.2.183 iparam.write\_data\_format

## Corresponding constant:

 $iparam.write\_data\_format$ 

## Description:

Controls the data format when a task is written using Task.writedata.

### Possible values:

- dataformat.cb Conic benchmark format.
- dataformat.extension The file extension is used to determine the data file format.
- dataformat.free\_mps The data data a free MPS formatted file.
- dataformat.lp The data file is LP formatted.
- dataformat.mps The data file is MPS formatted.
- dataformat.op The data file is an optimization problem formatted file.
- dataformat.task Generic task dump file.
- dataformat.xml The data file is an XML formatted file.

## Default value:

dataformat.extension

# B.2.184 iparam.write\_data\_param

### Corresponding constant:

 $iparam.write\_data\_param$ 

## **Description:**

If this option is turned on the parameter settings are written to the data file as parameters.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

### Default value:

onoffkey.off

# B.2.185 iparam.write\_free\_con

## Corresponding constant:

iparam.write\_free\_con

### Description:

Controls whether the free constraints are written to the data file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.off

## B.2.186 iparam.write\_generic\_names

## Corresponding constant:

 $iparam.write\_generic\_names$ 

#### Description:

Controls whether the generic names or user-defined names are used in the data file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.off

# B.2.187 iparam.write\_generic\_names\_io

### Corresponding constant:

 $iparam.write\_generic\_names\_io$ 

## Description:

Index origin used in generic names.

#### Possible Values:

Any number between 0 and +inf.

#### Default value:

1

# B.2.188 iparam.write\_ignore\_incompatible\_conic\_items

## Corresponding constant:

iparam.write\_ignore\_incompatible\_conic\_items

### **Description:**

If the output format is not compatible with conic quadratic problems this parameter controls if the writer ignores the conic parts or produces an error.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.off
```

# B.2.189 iparam.write\_ignore\_incompatible\_items

## Corresponding constant:

iparam.write\_ignore\_incompatible\_items

## Description:

Controls if the writer ignores incompatible problem items when writing files.

# Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.off
```

## B.2.190 iparam.write\_ignore\_incompatible\_nl\_items

## Corresponding constant:

iparam.write\_ignore\_incompatible\_nl\_items

### **Description:**

Controls if the writer ignores general non-linear terms or produces an error.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.off
```

# B.2.191 iparam.write\_ignore\_incompatible\_psd\_items

### Corresponding constant:

iparam.write\_ignore\_incompatible\_psd\_items

## Description:

If the output format is not compatible with semidefinite problems this parameter controls if the writer ignores the conic parts or produces an error.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

### Default value:

```
onoffkey.off
```

## B.2.192 iparam.write\_int\_constraints

#### Corresponding constant:

iparam.write\_int\_constraints

### **Description:**

Controls whether the constraint section is written to the integer solution file.

## Possible values:

• onoffkey.off Switch the option off.

• onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.on
```

## B.2.193 iparam.write\_int\_head

## Corresponding constant:

 $iparam.write\_int\_head$ 

## Description:

Controls whether the header section is written to the integer solution file.

## Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

# Default value:

```
onoffkey.on
```

# B.2.194 iparam.write\_int\_variables

## Corresponding constant:

iparam.write\_int\_variables

#### Description:

Controls whether the variables section is written to the integer solution file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.on
```

# B.2.195 iparam.write\_lp\_line\_width

## Corresponding constant:

 $iparam.write\_lp\_line\_width$ 

## Description:

Maximum width of line in an LP file written by MOSEK.

# Possible Values:

Any positive number.

#### Default value:

80

# B.2.196 iparam.write\_lp\_quoted\_names

# Corresponding constant:

 $iparam.write\_lp\_quoted\_names$ 

# Description:

If this option is turned on, then MOSEK will quote invalid LP names when writing an LP file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

```
onoffkey.on
```

# B.2.197 iparam.write\_lp\_strict\_format

# Corresponding constant:

 $iparam.write\_lp\_strict\_format$ 

# Description:

Controls whether LP output files satisfy the LP format strictly.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.off
```

# B.2.198 iparam.write\_lp\_terms\_per\_line

# Corresponding constant:

iparam.write\_lp\_terms\_per\_line

# Description:

Maximum number of terms on a single line in an LP file written by MOSEK. 0 means unlimited.

#### Possible Values:

Any number between 0 and +inf.

## Default value:

10

# B.2.199 iparam.write\_mps\_int

# Corresponding constant:

 $iparam.write\_mps\_int$ 

## Description:

Controls if marker records are written to the MPS file to indicate whether variables are integer restricted.

# Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

# Default value:

onoffkey.on

# B.2.200 iparam.write\_precision

# Corresponding constant:

iparam.write\_precision

# Description:

Controls the precision with which double numbers are printed in the MPS data file. In general it is not worthwhile to use a value higher than 15.

#### Possible Values:

Any number between 0 and +inf.

## Default value:

# B.2.201 iparam.write\_sol\_barvariables

## Corresponding constant:

iparam.write\_sol\_barvariables

# Description:

Controls whether the symmetric matrix variables section is written to the solution file.

# Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

## Default value:

onoffkey.on

# $B.2.202 \quad iparam.write\_sol\_constraints$

# Corresponding constant:

iparam.write\_sol\_constraints

## Description:

Controls whether the constraint section is written to the solution file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.on

# B.2.203 iparam.write\_sol\_head

# Corresponding constant:

 $iparam.write\_sol\_head$ 

#### Description:

Controls whether the header section is written to the solution file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

onoffkey.on

# B.2.204 iparam.write\_sol\_ignore\_invalid\_names

## Corresponding constant:

iparam.write\_sol\_ignore\_invalid\_names

# Description:

Even if the names are invalid MPS names, then they are employed when writing the solution file.

# Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.off
```

# B.2.205 iparam.write\_sol\_variables

## Corresponding constant:

iparam.write\_sol\_variables

## Description:

Controls whether the variables section is written to the solution file.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.on
```

# B.2.206 iparam.write\_task\_inc\_sol

# Corresponding constant:

iparam.write\_task\_inc\_sol

#### Description:

Controls whether the solutions are stored in the task file too.

#### Possible values:

- onoffkey.off Switch the option off.
- onoffkey.on Switch the option on.

#### Default value:

```
onoffkey.on
```

# B.2.207 iparam.write\_xml\_mode

## Corresponding constant:

 $iparam.write\_xml\_mode$ 

## Description:

Controls if linear coefficients should be written by row or column when writing in the XML file format.

# Possible values:

- xmlwriteroutputtype.col Write in column order.
- xmlwriteroutputtype.row Write in row order.

#### Default value:

xmlwriteroutputtype.row

# B.3 sparam: String parameter types

# B.3.1 sparam.bas\_sol\_file\_name

# Corresponding constant:

 $sparam.bas\_sol\_file\_name$ 

# Description:

Name of the bas solution file.

#### Possible Values:

Any valid file name.

## Default value:

11 11

# B.3.2 sparam.data\_file\_name

# Corresponding constant:

 $sparam.data\_file\_name$ 

# Description:

Data are read and written to this file.

# Possible Values:

Any valid file name.

# Default value:

# B.3.3 sparam.debug\_file\_name

# Corresponding constant:

sparam.debug\_file\_name

# Description:

MOSEK debug file.

#### Possible Values:

Any valid file name.

#### Default value:

11 11

# B.3.4 sparam.feasrepair\_name\_prefix

# Corresponding constant:

 $sparam.feasrepair\_name\_prefix$ 

# Description:

If the function Task.relaxprimal adds new constraints to the problem, then they are prefixed by the value of this parameter.

# Possible Values:

Any valid string.

# Default value:

"MSK-"

# B.3.5 sparam.feasrepair\_name\_separator

# Corresponding constant:

sparam.feasrepair\_name\_separator

# Description:

Separator string for names of constraints and variables generated by Task.relaxprimal.

#### Possible Values:

Any valid string.

#### Default value:

"-"

# B.3.6 sparam.feasrepair\_name\_wsumviol

# Corresponding constant:

sparam.feasrepair\_name\_wsumviol

## **Description:**

The constraint and variable associated with the total weighted sum of violations are each given the name of this parameter postfixed with CON and VAR respectively.

#### Possible Values:

Any valid string.

#### Default value:

"WSUMVIOL"

# B.3.7 sparam.int\_sol\_file\_name

# Corresponding constant:

sparam.int\_sol\_file\_name

# Description:

Name of the int solution file.

# Possible Values:

Any valid file name.

# Default value:

11 11

# B.3.8 sparam.itr\_sol\_file\_name

# Corresponding constant:

 $sparam.itr\_sol\_file\_name$ 

# **Description:**

Name of the itr solution file.

#### Possible Values:

Any valid file name.

#### Default value:

,, ,,

# B.3.9 sparam.mio\_debug\_string

# Corresponding constant:

sparam.mio\_debug\_string

# Description:

For internal use only.

#### Possible Values:

Any valid string.

#### Default value:

11 11

# B.3.10 sparam\_comment\_sign

# Corresponding constant:

 $sparam.param\_comment\_sign$ 

## **Description:**

Only the first character in this string is used. It is considered as a start of comment sign in the MOSEK parameter file. Spaces are ignored in the string.

# Possible Values:

Any valid string.

# Default value:

"%%"

# B.3.11 sparam.param\_read\_file\_name

# Corresponding constant:

sparam\_read\_file\_name

# **Description:**

Modifications to the parameter database is read from this file.

#### Possible Values:

Any valid file name.

#### Default value:

# B.3.12 sparam\_write\_file\_name

# Corresponding constant:

sparam\_write\_file\_name

# Description:

The parameter database is written to this file.

#### Possible Values:

Any valid file name.

# Default value:

11 11

# B.3.13 sparam.read\_mps\_bou\_name

# Corresponding constant:

 $sparam.read\_mps\_bou\_name$ 

# Description:

Name of the BOUNDS vector used. An empty name means that the first BOUNDS vector is used.

# Possible Values:

Any valid MPS name.

## Default value:

11 11

# B.3.14 sparam.read\_mps\_obj\_name

# Corresponding constant:

sparam.read\_mps\_obj\_name

# Description:

Name of the free constraint used as objective function. An empty name means that the first constraint is used as objective function.

# Possible Values:

Any valid MPS name.

## Default value:

# B.3.15 sparam.read\_mps\_ran\_name

# Corresponding constant:

sparam.read\_mps\_ran\_name

#### **Description:**

Name of the RANGE vector used. An empty name means that the first RANGE vector is used.

#### Possible Values:

Any valid MPS name.

#### Default value:

11 11

# B.3.16 sparam.read\_mps\_rhs\_name

# Corresponding constant:

 $sparam.read\_mps\_rhs\_name$ 

## **Description:**

Name of the RHS used. An empty name means that the first RHS vector is used.

# Possible Values:

Any valid MPS name.

# Default value:

11 11

# B.3.17 sparam.sensitivity\_file\_name

# Corresponding constant:

 $sparam.sensitivity\_file\_name$ 

#### **Description:**

If defined **Task.sensitivityreport** reads this file as a sensitivity analysis data file specifying the type of analysis to be done.

#### Possible Values:

Any valid string.

#### Default value:

# B.3.18 sparam.sensitivity\_res\_file\_name

## Corresponding constant:

sparam.sensitivity\_res\_file\_name

# Description:

If this is a nonempty string, then Task.sensitivityreport writes results to this file.

# Possible Values:

Any valid string.

#### Default value:

11 11

# B.3.19 sparam.sol\_filter\_xc\_low

# Corresponding constant:

sparam.sol\_filter\_xc\_low

# Description:

A filter used to determine which constraints should be listed in the solution file. A value of "0.5" means that all constraints having xc[i]>0.5 should be listed, whereas "+0.5" means that all constraints having xc[i]>=blc[i]+0.5 should be listed. An empty filter means that no filter is applied.

# Possible Values:

Any valid filter.

#### Default value:

11 11

# B.3.20 sparam.sol\_filter\_xc\_upr

# Corresponding constant:

sparam.sol\_filter\_xc\_upr

## Description:

A filter used to determine which constraints should be listed in the solution file. A value of "0.5" means that all constraints having xc[i]<0.5 should be listed, whereas "-0.5" means all constraints having xc[i]<=buc[i]-0.5 should be listed. An empty filter means that no filter is applied.

## Possible Values:

Any valid filter.

#### Default value:

" "

# B.3.21 sparam.sol\_filter\_xx\_low

## Corresponding constant:

sparam.sol\_filter\_xx\_low

# Description:

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j] >= 0.5 should be listed, whereas "+0.5" means that all constraints having xx[j] >= blx[j] + 0.5 should be listed. An empty filter means no filter is applied.

#### Possible Values:

Any valid filter.

#### Default value:

11 11

# B.3.22 sparam.sol\_filter\_xx\_upr

# Corresponding constant:

 $sparam.sol\_filter\_xx\_upr$ 

# Description:

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j]<0.5 should be printed, whereas "-0.5" means all constraints having xx[j]<-bux[j]-0.5 should be listed. An empty filter means no filter is applied.

# Possible Values:

Any valid file name.

#### Default value:

11 11

# B.3.23 sparam.stat\_file\_name

## Corresponding constant:

sparam.stat\_file\_name

## **Description:**

Statistics file name.

# Possible Values:

Any valid file name.

# Default value:

11 11

# B.3.24 sparam.stat\_key

# Corresponding constant:

sparam.stat\_key

# Description:

Key used when writing the summary file.

# Possible Values:

Any valid XML string.

#### Default value:

11 11

# B.3.25 sparam.stat\_name

# Corresponding constant:

sparam.stat\_name

# Description:

Name used when writing the statistics file.

## Possible Values:

Any valid XML string.

## Default value:

11 11

# B.3.26 sparam.write\_lp\_gen\_var\_name

# Corresponding constant:

sparam.write\_lp\_gen\_var\_name

# Description:

Sometimes when an LP file is written additional variables must be inserted. They will have the prefix denoted by this parameter.

Possible Values:

Any valid string.

Default value:

"xmskgen"

# Appendix C

# Response codes

Response codes ordered by name.

```
rescode.err_ad_invalid_codelist
```

The code list data was invalid.

# rescode.err\_ad\_invalid\_operand

The code list data was invalid. An unknown operand was used.

## rescode.err\_ad\_invalid\_operator

The code list data was invalid. An unknown operator was used.

#### rescode.err\_ad\_missing\_operand

The code list data was invalid. Missing operand for operator.

# ${\tt rescode.err\_ad\_missing\_return}$

The code list data was invalid. Missing return operation in function.

## rescode.err\_api\_array\_too\_small

An input array was too short.

#### rescode.err\_api\_cb\_connect

Failed to connect a callback object.

# rescode.err\_api\_fatal\_error

An internal error occurred in the API. Please report this problem.

# rescode.err\_api\_internal

An internal fatal error occurred in an interface function.

## rescode.err\_arg\_is\_too\_large

The value of a argument is too small.

## rescode.err\_arg\_is\_too\_small

The value of a argument is too small.

# rescode.err\_argument\_dimension

A function argument is of incorrect dimension.

# rescode.err\_argument\_is\_too\_large

The value of a function argument is too large.

# rescode.err\_argument\_lenneq

Incorrect length of arguments.

#### rescode.err\_argument\_perm\_array

An invalid permutation array is specified.

#### rescode.err\_argument\_type

Incorrect argument type.

#### rescode.err\_bar\_var\_dim

The dimension of a symmetric matrix variable has to greater than 0.

#### rescode.err\_basis

An invalid basis is specified. Either too many or too few basis variables are specified.

#### rescode.err\_basis\_factor

The factorization of the basis is invalid.

# ${\tt rescode.err\_basis\_singular}$

The basis is singular and hence cannot be factored.

#### rescode.err\_blank\_name

An all blank name has been specified.

#### rescode.err\_cannot\_clone\_nl

A task with a nonlinear function call-back cannot be cloned.

#### rescode.err\_cannot\_handle\_nl

A function cannot handle a task with nonlinear function call-backs.

# rescode.err\_cbf\_duplicate\_acoord

Duplicate index in ACOORD.

# rescode.err\_cbf\_duplicate\_bcoord

Duplicate index in BCOORD.

## rescode.err\_cbf\_duplicate\_con

Duplicate CON keyword.

rescode.err\_cbf\_duplicate\_int
 Duplicate INT keyword.

rescode.err\_cbf\_duplicate\_obj

Duplicate OBJ keyword.

rescode.err\_cbf\_duplicate\_objacoord

Duplicate index in OBJCOORD.

rescode.err\_cbf\_duplicate\_var
Duplicate VAR keyword.

rescode.err\_cbf\_invalid\_con\_type Invalid constraint type.

rescode.err\_cbf\_invalid\_domain\_dimension
Invalid domain dimension.

rescode.err\_cbf\_invalid\_int\_index Invalid INT index.

rescode.err\_cbf\_invalid\_var\_type
Invalid variable type.

rescode.err\_cbf\_no\_variables

No variables are specified.

rescode.err\_cbf\_no\_version\_specified

No version specified.

rescode.err\_cbf\_obj\_sense
An invalid objective sense is specified.

rescode.err\_cbf\_parse

An error occurred while parsing an CBF file.

rescode.err\_cbf\_syntax
Invalid syntax.

rescode.err\_cbf\_too\_few\_constraints
Too few constraints defined.

rescode.err\_cbf\_too\_few\_ints
Too few ints are specified.

rescode.err\_cbf\_too\_few\_variables
Too few variables defined.

#### rescode.err\_cbf\_too\_many\_constraints

Too many constraints specified.

#### rescode.err\_cbf\_too\_many\_ints

Too many ints are specified.

# rescode.err\_cbf\_too\_many\_variables

Too many variables specified.

## rescode.err\_cbf\_unsupported

Unsupported feature is present.

# rescode.err\_con\_q\_not\_nsd

The quadratic constraint matrix is not negative semidefinite as expected for a constraint with finite lower bound. This results in a nonconvex problem. The parameter <code>dparam.check\_convexity\_rel\_tol</code> can be used to relax the convexity check.

#### rescode.err\_con\_q\_not\_psd

The quadratic constraint matrix is not positive semidefinite as expected for a constraint with finite upper bound. This results in a nonconvex problem. The parameter <code>dparam.check\_convexity\_rel\_tol</code> can be used to relax the convexity check.

#### rescode.err\_concurrent\_optimizer

An unsupported optimizer was chosen for use with the concurrent optimizer.

#### rescode.err\_cone\_index

An index of a non-existing cone has been specified.

#### rescode.err\_cone\_overlap

A new cone which variables overlap with an existing cone has been specified.

#### rescode.err\_cone\_overlap\_append

The cone to be appended has one variable which is already member of another cone.

#### rescode.err\_cone\_rep\_var

A variable is included multiple times in the cone.

# rescode.err\_cone\_size

A cone with too few members is specified.

## rescode.err\_cone\_type

Invalid cone type specified.

## rescode.err\_cone\_type\_str

Invalid cone type specified.

# rescode.err\_data\_file\_ext

The data file format cannot be determined from the file name.

# rescode.err\_dup\_name

The same name was used multiple times for the same problem item type.

# rescode.err\_duplicate\_barvariable\_names

Two barvariable names are identical.

# rescode.err\_duplicate\_cone\_names

Two cone names are identical.

#### rescode.err\_duplicate\_constraint\_names

Two constraint names are identical.

# rescode.err\_duplicate\_variable\_names

Two variable names are identical.

# rescode.err\_end\_of\_file

End of file reached.

#### rescode.err\_factor

An error occurred while factorizing a matrix.

#### rescode.err\_feasrepair\_cannot\_relax

An optimization problem cannot be relaxed. This is the case e.g. for general nonlinear optimization problems.

# rescode.err\_feasrepair\_inconsistent\_bound

The upper bound is less than the lower bound for a variable or a constraint. Please correct this before running the feasibility repair.

# rescode.err\_feasrepair\_solving\_relaxed

The relaxed problem could not be solved to optimality. Please consult the log file for further details.

# rescode.err\_file\_license

Invalid license file.

#### rescode.err\_file\_open

Error while opening a file.

#### rescode.err\_file\_read

File read error.

## rescode.err\_file\_write

File write error.

#### rescode.err\_first

Invalid first.

#### rescode.err\_firsti

Invalid firsti.

#### rescode.err\_firstj

Invalid firstj.

#### rescode.err\_fixed\_bound\_values

A fixed constraint/variable has been specified using the bound keys but the numerical value of the lower and upper bound is different.

#### rescode.err\_flexlm

The FLEXIm license manager reported an error.

#### rescode.err\_global\_inv\_conic\_problem

The global optimizer can only be applied to problems without semidefinite variables.

# rescode.err\_huge\_aij

A numerically huge value is specified for an  $a_{i,j}$  element in A. The parameter dparam.data\_tol\_aij\_huge controls when an  $a_{i,j}$  is considered huge.

# rescode.err\_huge\_c

A huge value in absolute size is specified for one  $c_i$ .

#### rescode.err\_identical\_tasks

Some tasks related to this function call were identical. Unique tasks were expected.

# rescode.err\_in\_argument

A function argument is incorrect.

#### rescode.err\_index

An index is out of range.

#### rescode.err\_index\_arr\_is\_too\_large

An index in an array argument is too large.

## rescode.err\_index\_arr\_is\_too\_small

An index in an array argument is too small.

# rescode.err\_index\_is\_too\_large

An index in an argument is too large.

## rescode.err\_index\_is\_too\_small

An index in an argument is too small.

#### rescode.err\_inf\_dou\_index

A double information index is out of range for the specified type.

#### rescode.err\_inf\_dou\_name

A double information name is invalid.

#### rescode.err\_inf\_int\_index

An integer information index is out of range for the specified type.

#### rescode.err\_inf\_int\_name

An integer information name is invalid.

#### rescode.err\_inf\_lint\_index

A long integer information index is out of range for the specified type.

#### rescode.err\_inf\_lint\_name

A long integer information name is invalid.

# rescode.err\_inf\_type

The information type is invalid.

#### rescode.err\_infeas\_undefined

The requested value is not defined for this solution type.

#### rescode.err\_infinite\_bound

A numerically huge bound value is specified.

#### rescode.err\_int64\_to\_int32\_cast

An 32 bit integer could not cast to a 64 bit integer.

# rescode.err\_internal

An internal error occurred. Please report this problem.

# rescode.err\_internal\_test\_failed

An internal unit test function failed.

# rescode.err\_inv\_aptre

aptre[j] is strictly smaller than aptrb[j] for some j.

# rescode.err\_inv\_bk

Invalid bound key.

# rescode.err\_inv\_bkc

Invalid bound key is specified for a constraint.

## rescode.err\_inv\_bkx

An invalid bound key is specified for a variable.

#### rescode.err\_inv\_cone\_type

Invalid cone type code is encountered.

#### rescode.err\_inv\_cone\_type\_str

Invalid cone type string encountered.

#### rescode.err\_inv\_conic\_problem

The conic optimizer can only be applied to problems with linear objective and constraints. Many problems such convex quadratically constrained problems can easily be reformulated to conic problems. See the appropriate MOSEK manual for details.

#### rescode.err\_inv\_marki

Invalid value in marki.

#### rescode.err\_inv\_markj

Invalid value in markj.

#### rescode.err\_inv\_name\_item

An invalid name item code is used.

#### rescode.err\_inv\_numi

Invalid numi.

#### rescode.err\_inv\_numj

Invalid numj.

# rescode.err\_inv\_optimizer

An invalid optimizer has been chosen for the problem. This means that the simplex or the conic optimizer is chosen to optimize a nonlinear problem.

#### rescode.err\_inv\_problem

Invalid problem type. Probably a nonconvex problem has been specified.

# rescode.err\_inv\_qcon\_subi

Invalid value in qcsubi.

# rescode.err\_inv\_qcon\_subj

Invalid value in qcsubj.

# rescode.err\_inv\_qcon\_subk

Invalid value in qcsubk.

# ${\tt rescode.err\_inv\_qcon\_val}$

Invalid value in qcval.

#### rescode.err\_inv\_qobj\_subi

Invalid value in qosubi.

rescode.err\_inv\_qobj\_subj

Invalid value in qosubj.

rescode.err\_inv\_qobj\_val

Invalid value in qoval.

rescode.err\_inv\_sk

Invalid status key code.

rescode.err\_inv\_sk\_str

Invalid status key string encountered.

rescode.err\_inv\_skc

Invalid value in skc.

rescode.err\_inv\_skn

Invalid value in skn.

rescode.err\_inv\_skx

Invalid value in skx.

rescode.err\_inv\_var\_type

An invalid variable type is specified for a variable.

rescode.err\_invalid\_accmode

An invalid access mode is specified.

 $\tt rescode.err\_invalid\_aij$ 

 $a_{i,j}$  contains an invalid floating point value, i.e. a NaN or an infinite value.

rescode.err\_invalid\_ampl\_stub

Invalid AMPL stub.

rescode.err\_invalid\_barvar\_name

An invalid symmetric matrix variable name is used.

 ${\tt rescode.err\_invalid\_branch\_direction}$ 

An invalid branching direction is specified.

rescode.err\_invalid\_branch\_priority

An invalid branching priority is specified. It should be nonnegative.

 ${\tt rescode.err\_invalid\_compression}$ 

Invalid compression type.

rescode.err\_invalid\_con\_name

An invalid constraint name is used.

#### rescode.err\_invalid\_cone\_name

An invalid cone name is used.

#### rescode.err\_invalid\_file\_format\_for\_cones

The file format does not support a problem with conic constraints.

# rescode.err\_invalid\_file\_format\_for\_general\_nl

The file format does not support a problem with general nonlinear terms.

#### rescode.err\_invalid\_file\_format\_for\_sym\_mat

The file format does not support a problem with symmetric matrix variables.

#### rescode.err\_invalid\_file\_name

An invalid file name has been specified.

# rescode.err\_invalid\_format\_type

Invalid format type.

#### rescode.err\_invalid\_idx

A specified index is invalid.

## rescode.err\_invalid\_iomode

Invalid io mode.

# ${\tt rescode.err\_invalid\_max\_num}$

A specified index is invalid.

#### rescode.err\_invalid\_name\_in\_sol\_file

An invalid name occurred in a solution file.

# rescode.err\_invalid\_network\_problem

The problem is not a network problem as expected. The error occurs if a network optimizer is applied to a problem that cannot (easily) be converted to a network problem.

#### rescode.err\_invalid\_obj\_name

An invalid objective name is specified.

## rescode.err\_invalid\_objective\_sense

An invalid objective sense is specified.

## rescode.err\_invalid\_problem\_type

An invalid problem type.

#### rescode.err\_invalid\_sol\_file\_name

An invalid file name has been specified.

rescode.err\_invalid\_stream

An invalid stream is referenced.

rescode.err\_invalid\_surplus

Invalid surplus.

rescode.err\_invalid\_sym\_mat\_dim

A sparse symmetric matrix of invalid dimension is specified.

rescode.err\_invalid\_task

The task is invalid.

rescode.err\_invalid\_utf8

An invalid UTF8 string is encountered.

rescode.err invalid var name

An invalid variable name is used.

rescode.err\_invalid\_wchar

An invalid wchar string is encountered.

rescode.err\_invalid\_whichsol

whichsol is invalid.

rescode.err\_last

Invalid index last. A given index was out of expected range.

 $\verb"rescode.err_lasti"$ 

Invalid lasti.

rescode.err\_lastj

Invalid lastj.

rescode.err\_lau\_arg\_k

Invalid argument k.

rescode.err\_lau\_arg\_m

Invalid argument m.

rescode.err\_lau\_arg\_n

Invalid argument n.

rescode.err\_lau\_arg\_trans

Invalid argument trans.

rescode.err\_lau\_arg\_transa

Invalid argument transa.

# rescode.err\_lau\_arg\_transb

Invalid argument transb.

# rescode.err\_lau\_arg\_uplo

Invalid argument uplo.

#### rescode.err\_lau\_singular\_matrix

A matrix is singular.

#### rescode.err\_lau\_unknown

An unknown error.

#### rescode.err\_license

Invalid license.

#### rescode.err\_license\_cannot\_allocate

The license system cannot allocate the memory required.

#### rescode.err\_license\_cannot\_connect

MOSEK cannot connect to the license server. Most likely the license server is not up and running.

#### rescode.err\_license\_expired

The license has expired.

#### rescode.err\_license\_feature

A requested feature is not available in the license file(s). Most likely due to an incorrect license system setup.

#### rescode.err\_license\_invalid\_hostid

The host ID specified in the license file does not match the host ID of the computer.

#### rescode.err\_license\_max

Maximum number of licenses is reached.

## rescode.err\_license\_moseklm\_daemon

The MOSEKLM license manager daemon is not up and running.

#### rescode.err\_license\_no\_server\_line

There is no SERVER line in the license file. All non-zero license count features need at least one SERVER line.

# rescode.err\_license\_no\_server\_support

The license server does not support the requested feature. Possible reasons for this error include:

- The feature has expired.
- The feature's start date is later than today's date.

- The version requested is higher than feature's the highest supported version.
- A corrupted license file.

Try restarting the license and inspect the license server debug file, usually called lmgrd.log.

#### rescode.err\_license\_server

The license server is not responding.

#### rescode.err\_license\_server\_version

The version specified in the checkout request is greater than the highest version number the daemon supports.

# rescode.err\_license\_version

The license is valid for another version of MOSEK.

# rescode.err\_link\_file\_dll

A file cannot be linked to a stream in the DLL version.

## rescode.err\_living\_tasks

All tasks associated with an environment must be deleted before the environment is deleted. There are still some undeleted tasks.

#### rescode.err\_lower\_bound\_is\_a\_nan

The lower bound specificied is not a number (nan).

# rescode.err\_lp\_dup\_slack\_name

The name of the slack variable added to a ranged constraint already exists.

# rescode.err\_lp\_empty

The problem cannot be written to an LP formatted file.

# rescode.err\_lp\_file\_format

Syntax error in an LP file.

# rescode.err\_lp\_format

Syntax error in an LP file.

# rescode.err\_lp\_free\_constraint

Free constraints cannot be written in LP file format.

# rescode.err\_lp\_incompatible

The problem cannot be written to an LP formatted file.

## rescode.err\_lp\_invalid\_con\_name

A constraint name is invalid when used in an LP formatted file.

#### rescode.err\_lp\_invalid\_var\_name

A variable name is invalid when used in an LP formatted file.

#### rescode.err\_lp\_write\_conic\_problem

The problem contains cones that cannot be written to an LP formatted file.

#### rescode.err\_lp\_write\_geco\_problem

The problem contains general convex terms that cannot be written to an LP formatted file.

#### rescode.err\_lu\_max\_num\_tries

Could not compute the LU factors of the matrix within the maximum number of allowed tries.

#### rescode.err\_max\_len\_is\_too\_small

An maximum length that is too small has been specified.

#### rescode.err\_maxnumbarvar

The maximum number of semidefinite variables specified is smaller than the number of semidefinite variables in the task.

#### rescode.err\_maxnumcon

The maximum number of constraints specified is smaller than the number of constraints in the task.

#### rescode.err\_maxnumcone

The value specified for maxnumcone is too small.

#### rescode.err\_maxnumqnz

The maximum number of non-zeros specified for the Q matrixes is smaller than the number of non-zeros in the current Q matrixes.

#### rescode.err\_maxnumvar

The maximum number of variables specified is smaller than the number of variables in the task.

#### rescode.err\_mbt\_incompatible

The MBT file is incompatible with this platform. This results from reading a file on a 32 bit platform generated on a 64 bit platform.

# rescode.err\_mbt\_invalid

The MBT file is invalid.

## rescode.err\_mio\_internal

A fatal error occurred in the mixed integer optimizer. Please contact MOSEK support.

## rescode.err\_mio\_invalid\_node\_optimizer

An invalid node optimizer was selected for the problem type.

#### rescode.err\_mio\_invalid\_root\_optimizer

An invalid root optimizer was selected for the problem type.

#### rescode.err\_mio\_no\_optimizer

No optimizer is available for the current class of integer optimization problems.

#### rescode.err\_mio\_not\_loaded

The mixed-integer optimizer is not loaded.

# rescode.err\_missing\_license\_file

MOSEK cannot license file or a token server. See the MOSEK installation manual for details.

#### rescode.err\_mixed\_problem

The problem contains both conic and nonlinear constraints.

#### rescode.err\_mps\_cone\_overlap

A variable is specified to be a member of several cones.

## rescode.err\_mps\_cone\_repeat

A variable is repeated within the CSECTION.

#### rescode.err\_mps\_cone\_type

Invalid cone type specified in a CSECTION.

#### rescode.err\_mps\_duplicate\_q\_element

Duplicate elements is specified in a Q matrix.

# rescode.err\_mps\_file

An error occurred while reading an MPS file.

#### rescode.err\_mps\_inv\_bound\_key

An invalid bound key occurred in an MPS file.

# rescode.err\_mps\_inv\_con\_key

An invalid constraint key occurred in an MPS file.

# rescode.err\_mps\_inv\_field

A field in the MPS file is invalid. Probably it is too wide.

# rescode.err\_mps\_inv\_marker

An invalid marker has been specified in the MPS file.

# rescode.err\_mps\_inv\_sec\_name

An invalid section name occurred in an MPS file.

## rescode.err\_mps\_inv\_sec\_order

The sections in the MPS data file are not in the correct order.

## rescode.err\_mps\_invalid\_obj\_name

An invalid objective name is specified.

# rescode.err\_mps\_invalid\_objsense

An invalid objective sense is specified.

#### rescode.err\_mps\_mul\_con\_name

A constraint name was specified multiple times in the ROWS section.

#### rescode.err\_mps\_mul\_csec

Multiple CSECTIONs are given the same name.

# rescode.err\_mps\_mul\_qobj

The Q term in the objective is specified multiple times in the MPS data file.

#### rescode.err\_mps\_mul\_qsec

Multiple QSECTIONs are specified for a constraint in the MPS data file.

#### rescode.err\_mps\_no\_objective

No objective is defined in an MPS file.

## rescode.err\_mps\_non\_symmetric\_q

A non symmetric matrice has been speciefied.

# ${\tt rescode.err\_mps\_null\_con\_name}$

An empty constraint name is used in an MPS file.

# rescode.err\_mps\_null\_var\_name

An empty variable name is used in an MPS file.

# rescode.err\_mps\_splitted\_var

All elements in a column of the A matrix must be specified consecutively. Hence, it is illegal to specify non-zero elements in A for variable 1, then for variable 2 and then variable 1 again.

#### rescode.err\_mps\_tab\_in\_field2

A tab char occurred in field 2.

## rescode.err\_mps\_tab\_in\_field3

A tab char occurred in field 3.

# rescode.err\_mps\_tab\_in\_field5

A tab char occurred in field 5.

# rescode.err\_mps\_undef\_con\_name

An undefined constraint name occurred in an MPS file.

## rescode.err\_mps\_undef\_var\_name

An undefined variable name occurred in an MPS file.

#### rescode.err\_mul\_a\_element

An element in A is defined multiple times.

#### rescode.err\_name\_is\_null

The name buffer is a NULL pointer.

#### rescode.err\_name\_max\_len

A name is longer than the buffer that is supposed to hold it.

#### rescode.err\_nan\_in\_blc

 $l^c$  contains an invalid floating point value, i.e. a NaN.

#### rescode.err\_nan\_in\_blx

 $l^x$  contains an invalid floating point value, i.e. a NaN.

#### rescode.err\_nan\_in\_buc

 $u^c$  contains an invalid floating point value, i.e. a NaN.

#### rescode.err\_nan\_in\_bux

 $u^x$  contains an invalid floating point value, i.e. a NaN.

#### rescode.err\_nan\_in\_c

c contains an invalid floating point value, i.e. a NaN.

# rescode.err\_nan\_in\_double\_data

An invalid floating point value was used in some double data.

# ${\tt rescode.err\_negative\_append}$

Cannot append a negative number.

#### rescode.err\_negative\_surplus

Negative surplus.

# rescode.err\_newer\_dll

The dynamic link library is newer than the specified version.

#### rescode.err\_no\_bars\_for\_solution

There is no  $\bar{s}$  available for the solution specified. In particular note there are no  $\bar{s}$  defined for the basic and integer solutions.

## rescode.err\_no\_barx\_for\_solution

There is no  $\bar{X}$  available for the solution specified. In particular note there are no  $\bar{X}$  defined for the basic and integer solutions.

#### rescode.err\_no\_basis\_sol

No basic solution is defined.

#### rescode.err\_no\_dual\_for\_itg\_sol

No dual information is available for the integer solution.

#### rescode.err\_no\_dual\_infeas\_cer

A certificate of infeasibility is not available.

## rescode.err\_no\_dual\_info\_for\_itg\_sol

Dual information is not available for the integer solution.

#### rescode.err\_no\_init\_env

env is not initialized.

#### rescode.err\_no\_optimizer\_var\_type

No optimizer is available for this class of optimization problems.

## rescode.err\_no\_primal\_infeas\_cer

A certificate of primal infeasibility is not available.

#### rescode.err\_no\_snx\_for\_bas\_sol

 $s_n^x$  is not available for the basis solution.

#### rescode.err\_no\_solution\_in\_callback

The required solution is not available.

# rescode.err\_non\_unique\_array

An array does not contain unique elements.

# rescode.err\_nonconvex

The optimization problem is nonconvex.

# rescode.err\_nonlinear\_equality

The model contains a nonlinear equality which defines a nonconvex set.

#### rescode.err\_nonlinear\_functions\_not\_allowed

An operation that is invalid for problems with nonlinear functions defined has been attempted.

# rescode.err\_nonlinear\_ranged

The model contains a nonlinear ranged constraint which by definition defines a nonconvex set.

# ${\tt rescode.err\_nr\_arguments}$

Incorrect number of function arguments.

#### rescode.err\_null\_env

env is a NULL pointer.

#### rescode.err\_null\_pointer

An argument to a function is unexpectedly a NULL pointer.

#### rescode.err\_null\_task

task is a NULL pointer.

#### rescode.err\_numconlim

Maximum number of constraints limit is exceeded.

#### rescode.err\_numvarlim

Maximum number of variables limit is exceeded.

# rescode.err\_obj\_q\_not\_nsd

The quadratic coefficient matrix in the objective is not negative semidefinite as expected for a maximization problem. The parameter <code>dparam.check\_convexity\_rel\_tol</code> can be used to relax the convexity check.

#### rescode.err\_obj\_q\_not\_psd

The quadratic coefficient matrix in the objective is not positive semidefinite as expected for a minimization problem. The parameter <code>dparam.check\_convexity\_rel\_tol</code> can be used to relax the convexity check.

## rescode.err\_objective\_range

Empty objective range.

# rescode.err\_older\_dll

The dynamic link library is older than the specified version.

## rescode.err\_open\_dl

A dynamic link library could not be opened.

#### rescode.err\_opf\_format

Syntax error in an OPF file

# rescode.err\_opf\_new\_variable

Introducing new variables is now allowed. When a [variables] section is present, it is not allowed to introduce new variables later in the problem.

## rescode.err\_opf\_premature\_eof

Premature end of file in an OPF file.

# rescode.err\_optimizer\_license

The optimizer required is not licensed.

## rescode.err\_ord\_invalid

Invalid content in branch ordering file.

#### rescode.err\_ord\_invalid\_branch\_dir

An invalid branch direction key is specified.

#### rescode.err\_overflow

A computation produced an overflow i.e. a very large number.

#### rescode.err\_param\_index

Parameter index is out of range.

# rescode.err\_param\_is\_too\_large

The parameter value is too large.

#### rescode.err\_param\_is\_too\_small

The parameter value is too small.

#### rescode.err\_param\_name

The parameter name is not correct.

#### rescode.err\_param\_name\_dou

The parameter name is not correct for a double parameter.

## rescode.err\_param\_name\_int

The parameter name is not correct for an integer parameter.

## rescode.err\_param\_name\_str

The parameter name is not correct for a string parameter.

# rescode.err\_param\_type

The parameter type is invalid.

#### rescode.err\_param\_value\_str

The parameter value string is incorrect.

# rescode.err\_platform\_not\_licensed

A requested license feature is not available for the required platform.

#### rescode.err\_postsolve

An error occurred during the postsolve. Please contact MOSEK support.

# rescode.err\_pro\_item

An invalid problem is used.

# rescode.err\_prob\_license

The software is not licensed to solve the problem.

## rescode.err\_qcon\_subi\_too\_large

Invalid value in qcsubi.

#### rescode.err\_qcon\_subi\_too\_small

Invalid value in qcsubi.

# rescode.err\_qcon\_upper\_triangle

An element in the upper triangle of a  $Q^k$  is specified. Only elements in the lower triangle should be specified.

## rescode.err\_qobj\_upper\_triangle

An element in the upper triangle of  $Q^o$  is specified. Only elements in the lower triangle should be specified.

# rescode.err\_read\_format

The specified format cannot be read.

# rescode.err\_read\_lp\_missing\_end\_tag

Syntax error in LP file. Possibly missing End tag.

#### rescode.err\_read\_lp\_nonexisting\_name

A variable never occurred in objective or constraints.

#### rescode.err\_remove\_cone\_variable

A variable cannot be removed because it will make a cone invalid.

#### rescode.err\_repair\_invalid\_problem

The feasibility repair does not support the specified problem type.

## rescode.err\_repair\_optimization\_failed

Computation the optimal relaxation failed. The cause may have been numerical problems.

## rescode.err\_sen\_bound\_invalid\_lo

Analysis of lower bound requested for an index, where no lower bound exists.

#### rescode.err\_sen\_bound\_invalid\_up

Analysis of upper bound requested for an index, where no upper bound exists.

#### rescode.err\_sen\_format

Syntax error in sensitivity analysis file.

## rescode.err\_sen\_index\_invalid

Invalid range given in the sensitivity file.

# rescode.err\_sen\_index\_range

Index out of range in the sensitivity analysis file.

## rescode.err\_sen\_invalid\_regexp

Syntax error in regexp or regexp longer than 1024.

#### rescode.err\_sen\_numerical

Numerical difficulties encountered performing the sensitivity analysis.

#### rescode.err\_sen\_solution\_status

No optimal solution found to the original problem given for sensitivity analysis.

#### rescode.err\_sen\_undef\_name

An undefined name was encountered in the sensitivity analysis file.

## rescode.err\_sen\_unhandled\_problem\_type

Sensitivity analysis cannot be performed for the spcified problem. Sensitivity analysis is only possible for linear problems.

#### rescode.err\_size\_license

The problem is bigger than the license.

#### rescode.err\_size\_license\_con

The problem has too many constraints to be solved with the available license.

#### rescode.err\_size\_license\_intvar

The problem contains too many integer variables to be solved with the available license.

#### rescode.err\_size\_license\_numcores

The computer contains more cpu cores than the license allows for.

#### rescode.err\_size\_license\_var

The problem has too many variables to be solved with the available license.

#### rescode.err\_sol\_file\_invalid\_number

An invalid number is specified in a solution file.

#### rescode.err\_solitem

The solution item number solitem is invalid. Please note that solitem.snx is invalid for the basic solution.

# rescode.err\_solver\_probtype

Problem type does not match the chosen optimizer.

# rescode.err\_space

Out of space.

#### rescode.err\_space\_leaking

MOSEK is leaking memory. This can be due to either an incorrect use of MOSEK or a bug.

#### rescode.err\_space\_no\_info

No available information about the space usage.

#### rescode.err\_sym\_mat\_duplicate

A value in a symmetric matric as been specified more than once.

## rescode.err\_sym\_mat\_invalid\_col\_index

A column index specified for sparse symmetric maxtrix is invalid.

## rescode.err\_sym\_mat\_invalid\_row\_index

A row index specified for sparse symmetric maxtrix is invalid.

## rescode.err\_sym\_mat\_invalid\_value

The numerical value specified in a sparse symmetric matrix is not a value floating value.

#### rescode.err\_sym\_mat\_not\_lower\_tringular

Only the lower triangular part of sparse symmetric matrix should be specified.

## rescode.err\_task\_incompatible

The Task file is incompatible with this platform. This results from reading a file on a 32 bit platform generated on a 64 bit platform.

#### rescode.err\_task\_invalid

The Task file is invalid.

#### rescode.err\_thread\_cond\_init

Could not initialize a condition.

#### rescode.err\_thread\_create

Could not create a thread. This error may occur if a large number of environments are created and not deleted again. In any case it is a good practice to minimize the number of environments created.

## rescode.err\_thread\_mutex\_init

Could not initialize a mutex.

## rescode.err\_thread\_mutex\_lock

Could not lock a mutex.

#### rescode.err\_thread\_mutex\_unlock

Could not unlock a mutex.

## rescode.err\_toconic\_conversion\_fail

A constraint could not be converted in conic form.

## rescode.err\_too\_many\_concurrent\_tasks

Too many concurrent tasks specified.

#### rescode.err\_too\_small\_max\_num\_nz

The maximum number of non-zeros specified is too small.

#### rescode.err\_too\_small\_maxnumanz

The maximum number of non-zeros specified for A is smaller than the number of non-zeros in the current A.

## rescode.err\_unb\_step\_size

A step size in an optimizer was unexpectedly unbounded. For instance, if the step-size becomes unbounded in phase 1 of the simplex algorithm then an error occurs. Normally this will happen only if the problem is badly formulated. Please contact MOSEK support if this error occurs.

## rescode.err\_undef\_solution

MOSEK has the following solution types:

- an interior-point solution,
- an basic solution,
- and an integer solution.

Each optimizer may set one or more of these solutions; e.g by default a successful optimization with the interior-point optimizer defines the interior-point solution, and, for linear problems, also the basic solution. This error occurs when asking for a solution or for information about a solution that is not defined.

## rescode.err\_undefined\_objective\_sense

The objective sense has not been specified before the optimization.

#### rescode.err\_unhandled\_solution\_status

Unhandled solution status.

## rescode.err\_unknown

Unknown error.

## rescode.err\_upper\_bound\_is\_a\_nan

The upper bound specificied is not a number (nan).

## ${\tt rescode.err\_upper\_triangle}$

An element in the upper triangle of a lower triangular matrix is specified.

#### rescode.err\_user\_func\_ret

An user function reported an error.

## rescode.err\_user\_func\_ret\_data

An user function returned invalid data.

## rescode.err\_user\_nlo\_eval

The user-defined nonlinear function reported an error.

## rescode.err\_user\_nlo\_eval\_hessubi

The user-defined nonlinear function reported an invalid subscript in the Hessian.

## rescode.err\_user\_nlo\_eval\_hessubj

The user-defined nonlinear function reported an invalid subscript in the Hessian.

## rescode.err\_user\_nlo\_func

The user-defined nonlinear function reported an error.

## rescode.err\_whichitem\_not\_allowed

whichitem is unacceptable.

#### rescode.err\_whichsol

The solution defined by compwhich oldoes not exists.

## rescode.err\_write\_lp\_format

Problem cannot be written as an LP file.

## rescode.err\_write\_lp\_non\_unique\_name

An auto-generated name is not unique.

## rescode.err\_write\_mps\_invalid\_name

An invalid name is created while writing an MPS file. Usually this will make the MPS file unreadable.

## rescode.err\_write\_opf\_invalid\_var\_name

Empty variable names cannot be written to OPF files.

## rescode.err\_writing\_file

An error occurred while writing file

## rescode.err\_xml\_invalid\_problem\_type

The problem type is not supported by the XML format.

#### rescode.err\_y\_is\_undefined

The solution item y is undefined.

### rescode.ok

No error occurred.

## rescode.trm\_internal

The optimizer terminated due to some internal reason. Please contact MOSEK support.

## rescode.trm\_internal\_stop

The optimizer terminated for internal reasons. Please contact MOSEK support.

## ${\tt rescode.trm\_max\_iterations}$

The optimizer terminated at the maximum number of iterations.

#### rescode.trm\_max\_num\_setbacks

The optimizer terminated as the maximum number of set-backs was reached. This indicates numerical problems and a possibly badly formulated problem.

#### rescode.trm\_max\_time

The optimizer terminated at the maximum amount of time.

#### rescode.trm\_mio\_near\_abs\_gap

The mixed-integer optimizer terminated because the near optimal absolute gap tolerance was satisfied.

## rescode.trm\_mio\_near\_rel\_gap

The mixed-integer optimizer terminated because the near optimal relative gap tolerance was satisfied.

#### rescode.trm\_mio\_num\_branches

The mixed-integer optimizer terminated as to the maximum number of branches was reached.

#### rescode.trm\_mio\_num\_relaxs

The mixed-integer optimizer terminated as the maximum number of relaxations was reached.

#### rescode.trm\_num\_max\_num\_int\_solutions

The mixed-integer optimizer terminated as the maximum number of feasible solutions was reached.

## rescode.trm\_numerical\_problem

The optimizer terminated due to numerical problems.

## rescode.trm\_objective\_range

The optimizer terminated on the bound of the objective range.

## rescode.trm\_stall

The optimizer is terminated due to slow progress.

Stalling means that numerical problems prevent the optimizer from making reasonable progress and that it make no sense to continue. In many cases this happens if the problem is badly scaled or otherwise ill-conditioned. There is no guarantee that the solution will be (near) feasible or near optimal. However, often stalling happens near the optimum, and the returned solution may be of good quality. Therefore, it is recommended to check the status of then solution. If the solution near optimal the solution is most likely good enough for most practical purposes.

Please note that if a linear optimization problem is solved using the interior-point optimizer with basis identification turned on, the returned basic solution likely to have high accuracy, even though the optimizer stalled.

Some common causes of stalling are a) badly scaled models, b) near feasible or near infeasible problems and c) a non-convex problems. Case c) is only relevant for general non-linear problems. It is not possible in general for MOSEK to check if a specific problems is convex since such a check would be NP hard in itself. This implies that care should be taken when solving problems involving general user defined functions.

#### rescode.trm\_user\_callback

The optimizer terminated due to the return of the user-defined call-back function.

#### rescode.wrn\_ana\_almost\_int\_bounds

This warning is issued by the problem analyzer if a constraint is bound nearly integral.

#### rescode.wrn\_ana\_c\_zero

This warning is issued by the problem analyzer, if the coefficients in the linear part of the objective are all zero.

#### rescode.wrn\_ana\_close\_bounds

This warning is issued by problem analyzer, if ranged constraints or variables with very close upper and lower bounds are detected. One should consider treating such constraints as equalities and such variables as constants.

#### rescode.wrn\_ana\_empty\_cols

This warning is issued by the problem analyzer, if columns, in which all coefficients are zero, are found.

## rescode.wrn\_ana\_large\_bounds

This warning is issued by the problem analyzer, if one or more constraint or variable bounds are very large. One should consider omitting these bounds entirely by setting them to +inf or -inf.

## rescode.wrn\_construct\_invalid\_sol\_itg

The intial value for one or more of the integer variables is not feasible.

#### rescode.wrn\_construct\_no\_sol\_itg

The construct solution requires an integer solution.

## rescode.wrn\_construct\_solution\_infeas

After fixing the integer variables at the suggested values then the problem is infeasible.

## rescode.wrn\_dropped\_nz\_qobj

One or more non-zero elements were dropped in the Q matrix in the objective.

## rescode.wrn\_duplicate\_barvariable\_names

Two barvariable names are identical.

## rescode.wrn\_duplicate\_cone\_names

Two cone names are identical.

## rescode.wrn\_duplicate\_constraint\_names

Two constraint names are identical.

## rescode.wrn\_duplicate\_variable\_names

Two variable names are identical.

## rescode.wrn\_eliminator\_space

The eliminator is skipped at least once due to lack of space.

## rescode.wrn\_empty\_name

A variable or constraint name is empty. The output file may be invalid.

## rescode.wrn\_ignore\_integer

Ignored integer constraints.

## rescode.wrn\_incomplete\_linear\_dependency\_check

The linear dependency check(s) is not completed. Normally this is not an important warning unless the optimization problem has been formulated with linear dependencies which is bad practice.

## rescode.wrn\_large\_aij

A numerically large value is specified for an  $a_{i,j}$  element in A. The parameter dparam.data\_tol\_aij\_large controls when an  $a_{i,j}$  is considered large.

## rescode.wrn\_large\_bound

A numerically large bound value is specified.

## rescode.wrn\_large\_cj

A numerically large value is specified for one  $c_i$ .

## rescode.wrn\_large\_con\_fx

An equality constraint is fixed to a numerically large value. This can cause numerical problems.

## rescode.wrn\_large\_lo\_bound

A numerically large lower bound value is specified.

## rescode.wrn\_large\_up\_bound

A numerically large upper bound value is specified.

#### rescode.wrn\_license\_expire

The license expires.

## rescode.wrn\_license\_feature\_expire

The license expires.

## rescode.wrn\_license\_server

The license server is not responding.

## rescode.wrn\_lp\_drop\_variable

Ignored a variable because the variable was not previously defined. Usually this implies that a variable appears in the bound section but not in the objective or the constraints.

## rescode.wrn\_lp\_old\_quad\_format

Missing '/2' after quadratic expressions in bound or objective.

#### rescode.wrn\_mio\_infeasible\_final

The final mixed-integer problem with all the integer variables fixed at their optimal values is infeasible.

#### rescode.wrn\_mps\_split\_bou\_vector

A BOUNDS vector is split into several nonadjacent parts in an MPS file.

## rescode.wrn\_mps\_split\_ran\_vector

A RANGE vector is split into several nonadjacent parts in an MPS file.

## rescode.wrn\_mps\_split\_rhs\_vector

An RHS vector is split into several nonadjacent parts in an MPS file.

#### rescode.wrn\_name\_max\_len

A name is longer than the buffer that is supposed to hold it.

#### rescode.wrn\_no\_dualizer

No automatic dualizer is available for the specified problem. The primal problem is solved.

## rescode.wrn\_no\_global\_optimizer

No global optimizer is available.

### rescode.wrn\_no\_nonlinear\_function\_write

The problem contains a general nonlinear function in either the objective or the constraints. Such a nonlinear function cannot be written to a disk file. Note that quadratic terms when inputted explicitly can be written to disk.

## rescode.wrn\_nz\_in\_upr\_tri

Non-zero elements specified in the upper triangle of a matrix were ignored.

## rescode.wrn\_open\_param\_file

The parameter file could not be opened.

## rescode.wrn\_param\_ignored\_cmio

A parameter was ignored by the conic mixed integer optimizer.

## rescode.wrn\_param\_name\_dou

The parameter name is not recognized as a double parameter.

## rescode.wrn\_param\_name\_int

The parameter name is not recognized as a integer parameter.

#### rescode.wrn\_param\_name\_str

The parameter name is not recognized as a string parameter.

## rescode.wrn\_param\_str\_value

The string is not recognized as a symbolic value for the parameter.

## rescode.wrn\_presolve\_outofspace

The presolve is incomplete due to lack of space.

### rescode.wrn\_quad\_cones\_with\_root\_fixed\_at\_zero

For at least one quadratic cone the root is fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problems, or to fix all the variables in the cone to 0.

## rescode.wrn\_rquad\_cones\_with\_root\_fixed\_at\_zero

For at least one rotated quadratic cone at least one of the root variables are fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problems, or to fix all the variables in the cone to 0.

## rescode.wrn\_sol\_file\_ignored\_con

One or more lines in the constraint section were ignored when reading a solution file.

## rescode.wrn\_sol\_file\_ignored\_var

One or more lines in the variable section were ignored when reading a solution file.

#### rescode.wrn\_sol\_filter

Invalid solution filter is specified.

## rescode.wrn\_spar\_max\_len

A value for a string parameter is longer than the buffer that is supposed to hold it.

#### rescode.wrn\_too\_few\_basis\_vars

An incomplete basis has been specified. Too few basis variables are specified.

## rescode.wrn\_too\_many\_basis\_vars

A basis with too many variables has been specified.

#### rescode.wrn\_too\_many\_threads\_concurrent

The concurrent optimizer employs more threads than available. This will lead to poor performance.

#### rescode.wrn\_undef\_sol\_file\_name

Undefined name occurred in a solution.

## rescode.wrn\_using\_generic\_names

Generic names are used because a name is not valid. For instance when writing an LP file the names must not contain blanks or start with a digit.

#### rescode.wrn\_write\_changed\_names

Some names were changed because they were invalid for the output file format.

## rescode.wrn\_write\_discarded\_cfix

The fixed objective term could not be converted to a variable and was discarded in the output file.

## rescode.wrn\_zero\_aij

One or more zero elements are specified in A.

## ${\tt rescode.wrn\_zeros\_in\_sparse\_col}$

One or more (near) zero elements are specified in a sparse column of a matrix. It is redundant to specify zero elements. Hence, it may indicate an error.

## rescode.wrn\_zeros\_in\_sparse\_row

One or more (near) zero elements are specified in a sparse row of a matrix. It is redundant to specify zero elements. Hence it may indicate an error.

## Appendix D

## API constants

## D.1 Constraint or variable access modes

accmode.var

Access data by columns (variable oriented)

accmode.con

Access data by rows (constraint oriented)

## D.2 Basis identification

## basindtype.never

Never do basis identification.

## basindtype.always

Basis identification is always performed even if the interior-point optimizer terminates abnormally.

## basindtype.no\_error

Basis identification is performed if the interior-point optimizer terminates without an error.

## ${\tt basindtype.if\_feasible}$

Basis identification is not performed if the interior-point optimizer terminates with a problem status saying that the problem is primal or dual infeasible.

## basindtype.reservered

Not currently in use.

## D.3 Bound keys

## boundkey.lo

The constraint or variable has a finite lower bound and an infinite upper bound.

## boundkey.up

The constraint or variable has an infinite lower bound and an finite upper bound.

#### boundkey.fx

The constraint or variable is fixed.

## boundkey.fr

The constraint or variable is free.

## boundkey.ra

The constraint or variable is ranged.

## D.4 Specifies the branching direction.

#### branchdir.free

The mixed-integer optimizer decides which branch to choose.

## branchdir.up

The mixed-integer optimizer always chooses the up branch first.

#### branchdir.down

The mixed-integer optimizer always chooses the down branch first.

## D.5 Progress call-back codes

## callbackcode.begin\_bi

The basis identification procedure has been started.

## callbackcode.begin\_concurrent

Concurrent optimizer is started.

## callbackcode.begin\_conic

The call-back function is called when the conic optimizer is started.

## callbackcode.begin\_dual\_bi

The call-back function is called from within the basis identification procedure when the dual phase is started.

## callbackcode.begin\_dual\_sensitivity

Dual sensitivity analysis is started.

## callbackcode.begin\_dual\_setup\_bi

The call-back function is called when the dual BI phase is started.

## callbackcode.begin\_dual\_simplex

The call-back function is called when the dual simplex optimizer started.

## callbackcode.begin\_dual\_simplex\_bi

The call-back function is called from within the basis identification procedure when the dual simplex clean-up phase is started.

## callbackcode.begin\_full\_convexity\_check

Begin full convexity check.

## callbackcode.begin\_infeas\_ana

The call-back function is called when the infeasibility analyzer is started.

## callbackcode.begin\_intpnt

The call-back function is called when the interior-point optimizer is started.

#### callbackcode.begin\_license\_wait

Begin waiting for license.

#### callbackcode.begin\_mio

The call-back function is called when the mixed-integer optimizer is started.

## callbackcode.begin\_network\_dual\_simplex

The call-back function is called when the dual network simplex optimizer is started.

#### callbackcode.begin\_network\_primal\_simplex

The call-back function is called when the primal network simplex optimizer is started.

### callbackcode.begin\_network\_simplex

The call-back function is called when the simplex network optimizer is started.

## callbackcode.begin\_nonconvex

The call-back function is called when the nonconvex optimizer is started.

## callbackcode.begin\_optimizer

The call-back function is called when the optimizer is started.

## callbackcode.begin\_presolve

The call-back function is called when the presolve is started.

## callbackcode.begin\_primal\_bi

The call-back function is called from within the basis identification procedure when the primal phase is started.

## callbackcode.begin\_primal\_dual\_simplex

The call-back function is called when the primal-dual simplex optimizer is started.

## callbackcode.begin\_primal\_dual\_simplex\_bi

The call-back function is called from within the basis identification procedure when the primal-dual simplex clean-up phase is started.

## callbackcode.begin\_primal\_repair

Begin primal feasibility repair.

## callbackcode.begin\_primal\_sensitivity

Primal sensitivity analysis is started.

#### callbackcode.begin\_primal\_setup\_bi

The call-back function is called when the primal BI setup is started.

## callbackcode.begin\_primal\_simplex

The call-back function is called when the primal simplex optimizer is started.

## callbackcode.begin\_primal\_simplex\_bi

The call-back function is called from within the basis identification procedure when the primal simplex clean-up phase is started.

## callbackcode.begin\_qcqo\_reformulate

Begin QCQO reformulation.

## callbackcode.begin\_read

MOSEK has started reading a problem file.

## callbackcode.begin\_simplex

The call-back function is called when the simplex optimizer is started.

## callbackcode.begin\_simplex\_bi

The call-back function is called from within the basis identification procedure when the simplex clean-up phase is started.

## callbackcode.begin\_simplex\_network\_detect

The call-back function is called when the network detection procedure is started.

## callbackcode.begin\_write

MOSEK has started writing a problem file.

#### callbackcode.conic

The call-back function is called from within the conic optimizer after the information database has been updated.

#### callbackcode.dual\_simplex

The call-back function is called from within the dual simplex optimizer.

#### callbackcode.end\_bi

The call-back function is called when the basis identification procedure is terminated.

#### callbackcode.end\_concurrent

Concurrent optimizer is terminated.

#### callbackcode.end\_conic

The call-back function is called when the conic optimizer is terminated.

#### callbackcode.end\_dual\_bi

The call-back function is called from within the basis identification procedure when the dual phase is terminated.

#### callbackcode.end\_dual\_sensitivity

Dual sensitivity analysis is terminated.

## callbackcode.end\_dual\_setup\_bi

The call-back function is called when the dual BI phase is terminated.

## callbackcode.end\_dual\_simplex

The call-back function is called when the dual simplex optimizer is terminated.

#### callbackcode.end\_dual\_simplex\_bi

The call-back function is called from within the basis identification procedure when the dual clean-up phase is terminated.

## callbackcode.end\_full\_convexity\_check

End full convexity check.

## callbackcode.end\_infeas\_ana

The call-back function is called when the infeasibility analyzer is terminated.

## callbackcode.end\_intpnt

The call-back function is called when the interior-point optimizer is terminated.

## callbackcode.end\_license\_wait

End waiting for license.

#### callbackcode.end\_mio

The call-back function is called when the mixed-integer optimizer is terminated.

## callbackcode.end\_network\_dual\_simplex

The call-back function is called when the dual network simplex optimizer is terminated.

## callbackcode.end\_network\_primal\_simplex

The call-back function is called when the primal network simplex optimizer is terminated.

## callbackcode.end\_network\_simplex

The call-back function is called when the simplex network optimizer is terminated.

#### callbackcode.end\_nonconvex

The call-back function is called when the nonconvex optimizer is terminated.

## callbackcode.end\_optimizer

The call-back function is called when the optimizer is terminated.

## callbackcode.end\_presolve

The call-back function is called when the presolve is completed.

## callbackcode.end\_primal\_bi

The call-back function is called from within the basis identification procedure when the primal phase is terminated.

#### callbackcode.end\_primal\_dual\_simplex

The call-back function is called when the primal-dual simplex optimizer is terminated.

## $\verb|callbackcode.end_primal_dual_simplex_bi|\\$

The call-back function is called from within the basis identification procedure when the primal-dual clean-up phase is terminated.

#### callbackcode.end\_primal\_repair

End primal feasibility repair.

## ${\tt callbackcode.end\_primal\_sensitivity}$

Primal sensitivity analysis is terminated.

## callbackcode.end\_primal\_setup\_bi

The call-back function is called when the primal BI setup is terminated.

#### callbackcode.end\_primal\_simplex

The call-back function is called when the primal simplex optimizer is terminated.

## callbackcode.end\_primal\_simplex\_bi

The call-back function is called from within the basis identification procedure when the primal clean-up phase is terminated.

#### callbackcode.end\_qcqo\_reformulate

End QCQO reformulation.

#### callbackcode.end\_read

MOSEK has finished reading a problem file.

## callbackcode.end\_simplex

The call-back function is called when the simplex optimizer is terminated.

### callbackcode.end\_simplex\_bi

The call-back function is called from within the basis identification procedure when the simplex clean-up phase is terminated.

## callbackcode.end\_simplex\_network\_detect

The call-back function is called when the network detection procedure is terminated.

#### callbackcode.end\_write

MOSEK has finished writing a problem file.

#### callbackcode.im\_bi

The call-back function is called from within the basis identification procedure at an intermediate point.

#### callbackcode.im\_conic

The call-back function is called at an intermediate stage within the conic optimizer where the information database has not been updated.

#### callbackcode.im\_dual\_bi

The call-back function is called from within the basis identification procedure at an intermediate point in the dual phase.

#### callbackcode.im\_dual\_sensivity

The call-back function is called at an intermediate stage of the dual sensitivity analysis.

## callbackcode.im\_dual\_simplex

The call-back function is called at an intermediate point in the dual simplex optimizer.

#### callbackcode.im\_full\_convexity\_check

The call-back function is called at an intermediate stage of the full convexity check.

## callbackcode.im\_intpnt

The call-back function is called at an intermediate stage within the interior-point optimizer where the information database has not been updated.

## callbackcode.im\_license\_wait

MOSEK is waiting for a license.

### callbackcode.im\_lu

The call-back function is called from within the LU factorization procedure at an intermediate point.

#### callbackcode.im\_mio

The call-back function is called at an intermediate point in the mixed-integer optimizer.

#### callbackcode.im\_mio\_dual\_simplex

The call-back function is called at an intermediate point in the mixed-integer optimizer while running the dual simplex optimizer.

## callbackcode.im\_mio\_intpnt

The call-back function is called at an intermediate point in the mixed-integer optimizer while running the interior-point optimizer.

#### callbackcode.im\_mio\_presolve

The call-back function is called at an intermediate point in the mixed-integer optimizer while running the presolve.

## callbackcode.im\_mio\_primal\_simplex

The call-back function is called at an intermediate point in the mixed-integer optimizer while running the primal simplex optimizer.

## callbackcode.im\_network\_dual\_simplex

The call-back function is called at an intermediate point in the dual network simplex optimizer.

### callbackcode.im\_network\_primal\_simplex

The call-back function is called at an intermediate point in the primal network simplex optimizer.

#### callbackcode.im\_nonconvex

The call-back function is called at an intermediate stage within the nonconvex optimizer where the information database has not been updated.

## callbackcode.im\_order

The call-back function is called from within the matrix ordering procedure at an intermediate point.

## callbackcode.im\_presolve

The call-back function is called from within the presolve procedure at an intermediate stage.

### callbackcode.im\_primal\_bi

The call-back function is called from within the basis identification procedure at an intermediate point in the primal phase.

## callbackcode.im\_primal\_dual\_simplex

The call-back function is called at an intermediate point in the primal-dual simplex optimizer.

## callbackcode.im\_primal\_sensivity

The call-back function is called at an intermediate stage of the primal sensitivity analysis.

#### callbackcode.im\_primal\_simplex

The call-back function is called at an intermediate point in the primal simplex optimizer.

## callbackcode.im\_qo\_reformulate

The call-back function is called at an intermediate stage of the conic quadratic reformulation.

#### callbackcode.im\_read

Intermediate stage in reading.

#### callbackcode.im\_simplex

The call-back function is called from within the simplex optimizer at an intermediate point.

#### callbackcode.im\_simplex\_bi

The call-back function is called from within the basis identification procedure at an intermediate point in the simplex clean-up phase. The frequency of the call-backs is controlled by the <code>iparam.log\_sim\_freq</code> parameter.

#### callbackcode.intpnt

The call-back function is called from within the interior-point optimizer after the information database has been updated.

#### callbackcode.new\_int\_mio

The call-back function is called after a new integer solution has been located by the mixed-integer optimizer.

#### callbackcode.noncovex

The call-back function is called from within the nonconvex optimizer after the information database has been updated.

## callbackcode.primal\_simplex

The call-back function is called from within the primal simplex optimizer.

#### callbackcode.read\_opf

The call-back function is called from the OPF reader.

## callbackcode.read\_opf\_section

A chunk of Q non-zeos has been read from a problem file.

#### callbackcode.update\_dual\_bi

The call-back function is called from within the basis identification procedure at an intermediate point in the dual phase.

## callbackcode.update\_dual\_simplex

The call-back function is called in the dual simplex optimizer.

## callbackcode.update\_dual\_simplex\_bi

The call-back function is called from within the basis identification procedure at an intermediate point in the dual simplex clean-up phase. The frequency of the call-backs is controlled by the <code>iparam.log\_sim\_freq</code> parameter.

#### callbackcode.update\_network\_dual\_simplex

The call-back function is called in the dual network simplex optimizer.

## callbackcode.update\_network\_primal\_simplex

The call-back function is called in the primal network simplex optimizer.

## callbackcode.update\_nonconvex

The call-back function is called at an intermediate stage within the nonconvex optimizer where the information database has been updated.

## callbackcode.update\_presolve

The call-back function is called from within the presolve procedure.

#### callbackcode.update\_primal\_bi

The call-back function is called from within the basis identification procedure at an intermediate point in the primal phase.

## callbackcode.update\_primal\_dual\_simplex

The call-back function is called in the primal-dual simplex optimizer.

## callbackcode.update\_primal\_dual\_simplex\_bi

The call-back function is called from within the basis identification procedure at an intermediate point in the primal-dual simplex clean-up phase. The frequency of the call-backs is controlled by the <code>iparam.log\_sim\_freq</code> parameter.

## ${\tt callbackcode.update\_primal\_simplex}$

The call-back function is called in the primal simplex optimizer.

## ${\tt callbackcode.update\_primal\_simplex\_bi}$

The call-back function is called from within the basis identification procedure at an intermediate point in the primal simplex clean-up phase. The frequency of the call-backs is controlled by the <code>iparam.log\_sim\_freq</code> parameter.

#### callbackcode.write\_opf

The call-back function is called from the OPF writer.

## D.6 Types of convexity checks.

checkconvexitytype.none

No convexity check.

checkconvexitytype.simple

Perform simple and fast convexity check.

checkconvexitytype.full

Perform a full convexity check.

## D.7 Compression types

compresstype.none

No compression is used.

compresstype.free

The type of compression used is chosen automatically.

compresstype.gzip

The type of compression used is gzip compatible.

## D.8 Cone types

conetype.quad

The cone is a quadratic cone.

conetype.rquad

The cone is a rotated quadratic cone.

## D.9 Data format types

dataformat.extension

The file extension is used to determine the data file format.

dataformat.mps

The data file is MPS formatted.

dataformat.lp

The data file is LP formatted.

#### dataformat.op

The data file is an optimization problem formatted file.

#### dataformat.xml

The data file is an XML formatted file.

## dataformat.free\_mps

The data data a free MPS formatted file.

#### dataformat.task

Generic task dump file.

#### dataformat.cb

Conic benchmark format.

## D.10 Double information items

#### dinfitem.bi\_clean\_dual\_time

Time spent within the dual clean-up optimizer of the basis identification procedure since its invocation.

## dinfitem.bi\_clean\_primal\_dual\_time

Time spent within the primal-dual clean-up optimizer of the basis identification procedure since its invocation.

## dinfitem.bi\_clean\_primal\_time

Time spent within the primal clean-up optimizer of the basis identification procedure since its invocation.

## dinfitem.bi\_clean\_time

Time spent within the clean-up phase of the basis identification procedure since its invocation.

## dinfitem.bi\_dual\_time

Time spent within the dual phase basis identification procedure since its invocation.

## dinfitem.bi\_primal\_time

Time spent within the primal phase of the basis identification procedure since its invocation.

## dinfitem.bi\_time

Time spent within the basis identification procedure since its invocation.

#### dinfitem.concurrent\_time

Time spent within the concurrent optimizer since its invocation.

#### dinfitem.intpnt\_dual\_feas

Dual feasibility measure reported by the interior-point optimizer. (For the interior-point optimizer this measure does not directly related to the original problem because a homogeneous model is employed.)

#### dinfitem.intpnt\_dual\_obj

Dual objective value reported by the interior-point optimizer.

#### dinfitem.intpnt\_factor\_num\_flops

An estimate of the number of flops used in the factorization.

## dinfitem.intpnt\_opt\_status

This measure should converge to +1 if the problem has a primal-dual optimal solution, and converge to -1 if problem is (strictly) primal or dual infeasible. Furthermore, if the measure converges to 0 the problem is usually ill-posed.

## dinfitem.intpnt\_order\_time

Order time (in seconds).

## dinfitem.intpnt\_primal\_feas

Primal feasibility measure reported by the interior-point optimizers. (For the interior-point optimizer this measure does not directly related to the original problem because a homogeneous model is employed).

## dinfitem.intpnt\_primal\_obj

Primal objective value reported by the interior-point optimizer.

#### dinfitem.intpnt\_time

Time spent within the interior-point optimizer since its invocation.

### dinfitem.mio\_cg\_seperation\_time

Separation time for CG cuts.

## dinfitem.mio\_cmir\_seperation\_time

Separation time for CMIR cuts.

## dinfitem.mio\_construct\_solution\_obj

If MOSEK has successfully constructed an integer feasible solution, then this item contains the optimal objective value corresponding to the feasible solution.

## dinfitem.mio\_dual\_bound\_after\_presolve

Value of the dual bound after presolve but before cut generation.

#### dinfitem.mio\_heuristic\_time

Time spent in the optimizer while solving the relaxtions.

## dinfitem.mio\_obj\_abs\_gap

Given the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the absolute gap defined by

```
|(objective value of feasible solution) – (objective bound)|.
```

Otherwise it has the value -1.0.

## dinfitem.mio\_obj\_bound

The best known bound on the objective function. This value is undefined until at least one relaxation has been solved: To see if this is the case check that <code>iinfitem.mio\_num\_relax</code> is stricly positive.

## dinfitem.mio\_obj\_int

The primal objective value corresponding to the best integer feasible solution. Please note that at least one integer feasible solution must have located i.e. check <u>iinfitem.mio\_num\_int\_solutions</u>.

#### dinfitem.mio\_obj\_rel\_gap

Given that the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the relative gap defined by

```
\frac{|(\text{objective value of feasible solution}) - (\text{objective bound})|}{\max(\delta, |(\text{objective value of feasible solution})|)}
```

where  $\delta$  is given by the paramater dparam.mio\_rel\_gap\_const. Otherwise it has the value -1.0.

## dinfitem.mio\_optimizer\_time

Time spent in the optimizer while solving the relaxtions.

## dinfitem.mio\_probing\_time

Total time for probing.

## ${\tt dinfitem.mio\_root\_cutgen\_time}$

Total time for cut generation.

#### dinfitem.mio\_root\_optimizer\_time

Time spent in the optimizer while solving the root relaxation.

#### dinfitem.mio\_root\_presolve\_time

Time spent in while presolveing the root relaxation.

## dinfitem.mio\_time

Time spent in the mixed-integer optimizer.

#### dinfitem.mio\_user\_obj\_cut

If the objective cut is used, then this information item has the value of the cut.

## dinfitem.optimizer\_time

Total time spent in the optimizer since it was invoked.

## dinfitem.presolve\_eli\_time

Total time spent in the eliminator since the presolve was invoked.

## dinfitem.presolve\_lindep\_time

Total time spent in the linear dependency checker since the presolve was invoked.

## ${\tt dinfitem.presolve\_time}$

Total time (in seconds) spent in the presolve since it was invoked.

## dinfitem.primal\_repair\_penalty\_obj

The optimal objective value of the penalty function.

## dinfitem.qcqo\_reformulate\_time

Time spent with conic quadratic reformulation.

#### dinfitem.rd\_time

Time spent reading the data file.

#### dinfitem.sim\_dual\_time

Time spent in the dual simplex optimizer since invoking it.

### dinfitem.sim\_feas

Feasibility measure reported by the simplex optimizer.

## ${\tt dinfitem.sim\_network\_dual\_time}$

Time spent in the dual network simplex optimizer since invoking it.

## dinfitem.sim\_network\_primal\_time

Time spent in the primal network simplex optimizer since invoking it.

## dinfitem.sim\_network\_time

Time spent in the network simplex optimizer since invoking it.

## dinfitem.sim\_obj

Objective value reported by the simplex optimizer.

## dinfitem.sim\_primal\_dual\_time

Time spent in the primal-dual simplex optimizer optimizer since invoking it.

## ${\tt dinfitem.sim\_primal\_time}$

Time spent in the primal simplex optimizer since invoking it.

#### dinfitem.sim\_time

Time spent in the simplex optimizer since invoking it.

## dinfitem.sol\_bas\_dual\_obj

Dual objective value of the basic solution.

## dinfitem.sol\_bas\_dviolcon

Maximal dual bound violation for  $x^c$  in the basic solution.

#### dinfitem.sol\_bas\_dviolvar

Maximal dual bound violation for  $x^x$  in the basic solution.

## dinfitem.sol\_bas\_primal\_obj

Primal objective value of the basic solution.

## dinfitem.sol\_bas\_pviolcon

Maximal primal bound violation for  $x^c$  in the basic solution.

## dinfitem.sol\_bas\_pviolvar

Maximal primal bound violation for  $x^x$  in the basic solution.

## dinfitem.sol\_itg\_primal\_obj

Primal objective value of the integer solution.

## dinfitem.sol\_itg\_pviolbarvar

Maximal primal bound violation for  $\bar{X}$  in the integer solution.

## dinfitem.sol\_itg\_pviolcon

Maximal primal bound violation for  $x^c$  in the integer solution.

## ${\tt dinfitem.sol\_itg\_pviolcones}$

Maximal primal violation for primal conic constraints in the integer solution.

## dinfitem.sol\_itg\_pviolitg

Maximal violation for the integer constraints in the integer solution.

## dinfitem.sol\_itg\_pviolvar

Maximal primal bound violation for  $x^x$  in the integer solution.

## dinfitem.sol\_itr\_dual\_obj

Dual objective value of the interior-point solution.

## dinfitem.sol\_itr\_dviolbarvar

Maximal dual bound violation for  $\bar{X}$  in the interior-point solution.

## dinfitem.sol\_itr\_dviolcon

Maximal dual bound violation for  $x^c$  in the interior-point solution.

## $dinfitem.sol_itr_dviolcones$

Maximal dual violation for dual conic constraints in the interior-point solution.

#### dinfitem.sol\_itr\_dviolvar

Maximal dual bound violation for  $x^x$  in the interior-point solution.

## dinfitem.sol\_itr\_primal\_obj

Primal objective value of the interior-point solution.

## dinfitem.sol\_itr\_pviolbarvar

Maximal primal bound violation for  $\bar{X}$  in the interior-point solution.

## dinfitem.sol\_itr\_pviolcon

Maximal primal bound violation for  $x^c$  in the interior-point solution.

## dinfitem.sol\_itr\_pviolcones

Maximal primal violation for primal conic constraints in the interior-point solution.

#### dinfitem.sol\_itr\_pviolvar

Maximal primal bound violation for  $x^x$  in the interior-point solution.

## D.11 Feasibility repair types

## feasrepairtype.optimize\_none

Do not optimize the feasibility repair problem.

## feasrepairtype.optimize\_penalty

Minimize weighted sum of violations.

## feasrepairtype.optimize\_combined

Minimize with original objective subject to minimal weighted violation of bounds.

## D.12 License feature

## feature.pts

Base system.

#### feature.pton

Nonlinear extension.

## feature.ptom

Mixed-integer extension.

## feature.ptox

Non-convex extension.

## D.13 Integer information items.

## iinfitem.ana\_pro\_num\_con

Number of constraints in the problem.

This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_con\_eq

Number of equality constraints.

This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_con\_fr

Number of unbounded constraints.

This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_con\_lo

Number of constraints with a lower bound and an infinite upper bound.

This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_con\_ra

Number of constraints with finite lower and upper bounds.

This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_con\_up

Number of constraints with an upper bound and an infinite lower bound.

This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_var

Number of variables in the problem.

This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_var\_bin

Number of binary (0-1) variables.

This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_var\_cont

Number of continuous variables.

This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_var\_eq

Number of fixed variables.

This value is set by Task.analyzeproblem.

#### iinfitem.ana\_pro\_num\_var\_fr

Number of free variables.

This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_var\_int

Number of general integer variables.

This value is set by Task.analyzeproblem.

#### iinfitem.ana\_pro\_num\_var\_lo

Number of variables with a lower bound and an infinite upper bound.

This value is set by Task.analyzeproblem.

#### iinfitem.ana\_pro\_num\_var\_ra

Number of variables with finite lower and upper bounds.

This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_var\_up

Number of variables with an upper bound and an infinite lower bound. This value is set by This value is set by Task.analyzeproblem.

## $\verb|iinfitem.concurrent_fastest_optimizer|\\$

The type of the optimizer that finished first in a concurrent optimization.

#### iinfitem.intpnt\_factor\_dim\_dense

Dimension of the dense sub system in factorization.

## iinfitem.intpnt\_iter

Number of interior-point iterations since invoking the interior-point optimizer.

## iinfitem.intpnt\_num\_threads

Number of threads that the interior-point optimizer is using.

## iinfitem.intpnt\_solve\_dual

Non-zero if the interior-point optimizer is solving the dual problem.

#### iinfitem.mio\_construct\_num\_roundings

Number of values in the integer solution that is rounded to an integer value.

## ${\tt iinfitem.mio\_construct\_solution}$

If this item has the value 0, then MOSEK did not try to construct an initial integer feasible solution. If the item has a positive value, then MOSEK successfully constructed an initial integer feasible solution.

## iinfitem.mio\_initial\_solution

Is non-zero if an initial integer solution is specified.

#### iinfitem.mio\_num\_active\_nodes

Number of active brabch bound nodes.

## iinfitem.mio\_num\_basis\_cuts

Number of basis cuts.

#### iinfitem.mio\_num\_branch

Number of branches performed during the optimization.

## iinfitem.mio\_num\_cardgub\_cuts

Number of cardgub cuts.

## iinfitem.mio\_num\_clique\_cuts

Number of clique cuts.

#### iinfitem.mio num coef redc cuts

Number of coef. redc. cuts.

## iinfitem.mio\_num\_contra\_cuts

Number of contra cuts.

## iinfitem.mio\_num\_disagg\_cuts

Number of diasagg cuts.

### iinfitem.mio\_num\_flow\_cover\_cuts

Number of flow cover cuts.

## ${\tt iinfitem.mio\_num\_gcd\_cuts}$

Number of gcd cuts.

## iinfitem.mio\_num\_gomory\_cuts

Number of Gomory cuts.

## $\verb|iinfitem.mio_num_gub_cover_cuts|$

Number of GUB cover cuts.

#### iinfitem.mio\_num\_int\_solutions

Number of integer feasible solutions that has been found.

## iinfitem.mio\_num\_knapsur\_cover\_cuts

Number of knapsack cover cuts.

## iinfitem.mio\_num\_lattice\_cuts

Number of lattice cuts.

## iinfitem.mio\_num\_lift\_cuts

Number of lift cuts.

## iinfitem.mio\_num\_obj\_cuts

Number of obj cuts.

## iinfitem.mio\_num\_plan\_loc\_cuts

Number of loc cuts.

#### iinfitem.mio\_num\_relax

Number of relaxations solved during the optimization.

#### iinfitem.mio\_numcon

Number of constraints in the problem solved be the mixed-integer optimizer.

#### iinfitem.mio\_numint

Number of integer variables in the problem solved be the mixed-integer optimizer.

#### iinfitem.mio\_numvar

Number of variables in the problem solved be the mixed-integer optimizer.

## iinfitem.mio\_obj\_bound\_defined

Non-zero if a valid objective bound has been found, otherwise zero.

#### iinfitem.mio\_total\_num\_cuts

Total number of cuts generated by the mixed-integer optimizer.

## iinfitem.mio\_user\_obj\_cut

If it is non-zero, then the objective cut is used.

## $\verb|iinfitem.opt_numcon||$

Number of constraints in the problem solved when the optimizer is called.

## iinfitem.opt\_numvar

Number of variables in the problem solved when the optimizer is called

## iinfitem.optimize\_response

The reponse code returned by optimize.

#### iinfitem.rd\_numbarvar

Number of variables read.

## iinfitem.rd\_numcon

Number of constraints read.

#### iinfitem.rd\_numcone

Number of conic constraints read.

## $iinfitem.rd\_numintvar$

Number of integer-constrained variables read.

#### iinfitem.rd\_numq

Number of nonempty Q matrixes read.

#### iinfitem.rd\_numvar

Number of variables read.

## iinfitem.rd\_protype

Problem type.

#### iinfitem.sim\_dual\_deg\_iter

The number of dual degenerate iterations.

#### iinfitem.sim\_dual\_hotstart

If 1 then the dual simplex algorithm is solving from an advanced basis.

#### iinfitem.sim\_dual\_hotstart\_lu

If 1 then a valid basis factorization of full rank was located and used by the dual simplex algorithm.

#### iinfitem.sim\_dual\_inf\_iter

The number of iterations taken with dual infeasibility.

#### iinfitem.sim\_dual\_iter

Number of dual simplex iterations during the last optimization.

## iinfitem.sim\_network\_dual\_deg\_iter

The number of dual network degenerate iterations.

## iinfitem.sim\_network\_dual\_hotstart

If 1 then the dual network simplex algorithm is solving from an advanced basis.

#### iinfitem.sim\_network\_dual\_hotstart\_lu

If 1 then a valid basis factorization of full rank was located and used by the dual network simplex algorithm.

## iinfitem.sim\_network\_dual\_inf\_iter

The number of iterations taken with dual infeasibility in the network optimizer.

#### iinfitem.sim\_network\_dual\_iter

Number of dual network simplex iterations during the last optimization.

## iinfitem.sim\_network\_primal\_deg\_iter

The number of primal network degenerate iterations.

## iinfitem.sim\_network\_primal\_hotstart

If 1 then the primal network simplex algorithm is solving from an advanced basis.

## iinfitem.sim\_network\_primal\_hotstart\_lu

If 1 then a valid basis factorization of full rank was located and used by the primal network simplex algorithm.

## iinfitem.sim\_network\_primal\_inf\_iter

The number of iterations taken with primal infeasibility in the network optimizer.

## iinfitem.sim\_network\_primal\_iter

Number of primal network simplex iterations during the last optimization.

#### iinfitem.sim\_numcon

Number of constraints in the problem solved by the simplex optimizer.

#### iinfitem.sim\_numvar

Number of variables in the problem solved by the simplex optimizer.

## iinfitem.sim\_primal\_deg\_iter

The number of primal degenerate iterations.

## iinfitem.sim\_primal\_dual\_deg\_iter

The number of degenerate major iterations taken by the primal dual simplex algorithm.

## iinfitem.sim\_primal\_dual\_hotstart

If 1 then the primal dual simplex algorithm is solving from an advanced basis.

#### iinfitem.sim\_primal\_dual\_hotstart\_lu

If 1 then a valid basis factorization of full rank was located and used by the primal dual simplex algorithm.

#### iinfitem.sim\_primal\_dual\_inf\_iter

The number of master iterations with dual infeasibility taken by the primal dual simplex algorithm.

#### iinfitem.sim\_primal\_dual\_iter

Number of primal dual simplex iterations during the last optimization.

## iinfitem.sim\_primal\_hotstart

If 1 then the primal simplex algorithm is solving from an advanced basis.

## iinfitem.sim\_primal\_hotstart\_lu

If 1 then a valid basis factorization of full rank was located and used by the primal simplex algorithm.

## iinfitem.sim\_primal\_inf\_iter

The number of iterations taken with primal infeasibility.

## iinfitem.sim\_primal\_iter

Number of primal simplex iterations during the last optimization.

#### iinfitem.sim\_solve\_dual

Is non-zero if dual problem is solved.

## iinfitem.sol\_bas\_prosta

Problem status of the basic solution. Updated after each optimization.

#### iinfitem.sol\_bas\_solsta

Solution status of the basic solution. Updated after each optimization.

#### iinfitem.sol\_int\_prosta

Deprecated.

#### iinfitem.sol\_int\_solsta

Degrecated.

## iinfitem.sol\_itg\_prosta

Problem status of the integer solution. Updated after each optimization.

#### iinfitem.sol\_itg\_solsta

Solution status of the integer solution. Updated after each optimization.

## iinfitem.sol\_itr\_prosta

Problem status of the interior-point solution. Updated after each optimization.

## iinfitem.sol\_itr\_solsta

Solution status of the interior-point solution. Updated after each optimization.

## iinfitem.sto\_num\_a\_cache\_flushes

Number of times the cache of A elements is flushed. A large number implies that maxnumanz is too small as well as an inefficient usage of MOSEK.

#### iinfitem.sto\_num\_a\_realloc

Number of times the storage for storing A has been changed. A large value may indicates that memory fragmentation may occur.

## iinfitem.sto\_num\_a\_transposes

Number of times the A matrix is transposed. A large number implies that maxnumanz is too small or an inefficient usage of MOSEK. This will occur in particular if the code alternate between accessing rows and columns of A.

## D.14 Information item types

```
inftype.dou_type
```

Is a double information type.

inftype.int\_type

Is an integer.

inftype.lint\_type

Is a long integer.

# D.15 Hot-start type employed by the interior-point optimizers.

## intpnthotstart.none

The interior-point optimizer performs a coldstart.

## intpnthotstart.primal

The interior-point optimizer exploits the primal solution only.

## intpnthotstart.dual

The interior-point optimizer exploits the dual solution only.

## intpnthotstart.primal\_dual

The interior-point optimizer exploits both the primal and dual solution.

## D.16 Input/output modes

## iomode.read

The file is read-only.

## iomode.write

The file is write-only. If the file exists then it is truncated when it is opened. Otherwise it is created when it is opened.

#### iomode.readwrite

The file is to read and written.

## D.17 Language selection constants

#### language.eng

English language selection

## language.dan

Danish language selection

## D.18 Long integer information items.

## liinfitem.bi\_clean\_dual\_deg\_iter

Number of dual degenerate clean iterations performed in the basis identification.

#### liinfitem.bi\_clean\_dual\_iter

Number of dual clean iterations performed in the basis identification.

## liinfitem.bi\_clean\_primal\_deg\_iter

Number of primal degenerate clean iterations performed in the basis identification.

## liinfitem.bi\_clean\_primal\_dual\_deg\_iter

Number of primal-dual degenerate clean iterations performed in the basis identification.

## liinfitem.bi\_clean\_primal\_dual\_iter

Number of primal-dual clean iterations performed in the basis identification.

## liinfitem.bi\_clean\_primal\_dual\_sub\_iter

Number of primal-dual subproblem clean iterations performed in the basis identification.

#### liinfitem.bi\_clean\_primal\_iter

Number of primal clean iterations performed in the basis identification.

## liinfitem.bi\_dual\_iter

Number of dual pivots performed in the basis identification.

## liinfitem.bi\_primal\_iter

Number of primal pivots performed in the basis identification.

## liinfitem.intpnt\_factor\_num\_nz

Number of non-zeros in factorization.

## liinfitem.mio\_intpnt\_iter

Number of interior-point iterations performed by the mixed-integer optimizer.

## liinfitem.mio\_simplex\_iter

Number of simplex iterations performed by the mixed-integer optimizer.

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#### liinfitem.rd\_numanz

Number of non-zeros in A that is read.

## liinfitem.rd\_numqnz

Number of Q non-zeros.

## D.19 Mark

#### mark.lo

The lower bound is selected for sensitivity analysis.

#### mark.up

The upper bound is selected for sensitivity analysis.

## D.20 Continuous mixed-integer solution type

## miocontsoltype.none

No interior-point or basic solution are reported when the mixed-integer optimizer is used.

## miocontsoltype.root

The reported interior-point and basic solutions are a solution to the root node problem when mixed-integer optimizer is used.

### miocontsoltype.itg

The reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. A solution is only reported in case the problem has a primal feasible solution.

## miocontsoltype.itg\_rel

In case the problem is primal feasible then the reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. If the problem is primal infeasible, then the solution to the root node problem is reported.

## D.21 Integer restrictions

### miomode.ignored

The integer constraints are ignored and the problem is solved as a continuous problem.

## miomode.satisfied

Integer restrictions should be satisfied.

### miomode.lazy

Integer restrictions should be satisfied if an optimizer is available for the problem.

## D.22 Mixed-integer node selection types

### mionodeseltype.free

The optimizer decides the node selection strategy.

## mionodeseltype.first

The optimizer employs a depth first node selection strategy.

## mionodeseltype.best

The optimizer employs a best bound node selection strategy.

#### mionodeseltype.worst

The optimizer employs a worst bound node selection strategy.

## mionodeseltype.hybrid

The optimizer employs a hybrid strategy.

## mionodeseltype.pseudo

The optimizer employs selects the node based on a pseudo cost estimate.

## D.23 MPS file format type

## mpsformat.strict

It is assumed that the input file satisfies the MPS format strictly.

#### mpsformat.relaxed

It is assumed that the input file satisfies a slightly relaxed version of the MPS format.

## mpsformat.free

It is assumed that the input file satisfies the free MPS format. This implies that spaces are not allowed in names. Otherwise the format is free.

## D.24 Message keys

```
msgkey.reading_file
```

msgkey.writing\_file

msgkey.mps\_selected

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## D.25 Name types

```
nametype.gen

General names. However, no duplicate and blank names are allowed.

nametype.mps

MPS type names.

nametype.lp

LP type names.
```

## D.26 Objective sense types

```
objsense.minimize

The problem should be minimized.

objsense.maximize

The problem should be maximized.
```

## D.27 On/off

```
onoffkey.off
Switch the option off.
onoffkey.on
Switch the option on.
```

## D.28 Optimizer types

```
optimizertype.free
The optimizer is chosen automatically.

optimizertype.intpnt
The interior-point optimizer is used.

optimizertype.conic
The optimizer for problems having conic constraints.

optimizertype.primal_simplex
The primal simplex optimizer is used.
```

## optimizertype.dual\_simplex

The dual simplex optimizer is used.

## optimizertype.primal\_dual\_simplex

The primal dual simplex optimizer is used.

## optimizertype.free\_simplex

One of the simplex optimizers is used.

#### optimizertype.network\_primal\_simplex

The network primal simplex optimizer is used. It is only applicable to pure network problems.

## optimizertype.mixed\_int\_conic

The mixed-integer optimizer for conic and linear problems.

## optimizertype.mixed\_int

The mixed-integer optimizer.

### optimizertype.concurrent

The optimizer for nonconvex nonlinear problems.

## optimizertype.nonconvex

The optimizer for nonconvex nonlinear problems.

## D.29 Ordering strategies

## orderingtype.free

The ordering method is chosen automatically.

## orderingtype.appminloc

Approximate minimum local fill-in ordering is employed.

## orderingtype.experimental

This option should not be used.

## orderingtype.try\_graphpar

Always try the the graph partitioning based ordering.

## orderingtype.force\_graphpar

Always use the graph partitioning based ordering even if it is worse that the approximate minimum local fill ordering.

### orderingtype.none

No ordering is used.

## D.30 Parameter type

```
parametertype.invalid_type
    Not a valid parameter.

parametertype.dou_type
    Is a double parameter.

parametertype.int_type
    Is an integer parameter.

parametertype.str_type
    Is a string parameter.
```

## D.31 Presolve method.

```
presolvemode.off
    The problem is not presolved before it is optimized.
presolvemode.on
    The problem is presolved before it is optimized.
presolvemode.free
    It is decided automatically whether to presolve before the problem is optimized.
```

## D.32 Problem data items

```
problemitem.var
Item is a variable.

problemitem.con
Item is a constraint.

problemitem.cone
Item is a cone.
```

## D.33 Problem types

```
problemtype.lo
```

The problem is a linear optimization problem.

#### problemtype.qo

The problem is a quadratic optimization problem.

## problemtype.qcqo

The problem is a quadratically constrained optimization problem.

### problemtype.geco

General convex optimization.

## problemtype.conic

A conic optimization.

## problemtype.mixed

General nonlinear constraints and conic constraints. This combination can not be solved by MOSEK.

## D.34 Problem status keys

## prosta.unknown

Unknown problem status.

### prosta.prim\_and\_dual\_feas

The problem is primal and dual feasible.

## prosta.prim\_feas

The problem is primal feasible.

## prosta.dual\_feas

The problem is dual feasible.

## prosta.prim\_infeas

The problem is primal infeasible.

## $prosta.dual\_infeas$

The problem is dual infeasible.

### prosta.prim\_and\_dual\_infeas

The problem is primal and dual infeasible.

## prosta.ill\_posed

The problem is ill-posed. For example, it may be primal and dual feasible but have a positive duality gap.

#### prosta.near\_prim\_and\_dual\_feas

The problem is at least nearly primal and dual feasible.

### prosta.near\_prim\_feas

The problem is at least nearly primal feasible.

## prosta.near\_dual\_feas

The problem is at least nearly dual feasible.

## prosta.prim\_infeas\_or\_unbounded

The problem is either primal infeasible or unbounded. This may occur for mixed-integer problems.

## D.35 Response code type

## rescodetype.ok

The response code is OK.

### rescodetype.wrn

The response code is a warning.

## rescodetype.trm

The response code is an optimizer termination status.

### rescodetype.err

The response code is an error.

## rescodetype.unk

The response code does not belong to any class.

## D.36 Scaling type

## scalingmethod.pow2

Scales only with power of 2 leaving the mantissa untouched.

## scalingmethod.free

The optimizer chooses the scaling heuristic.

## D.37 Scaling type

## scalingtype.free

The optimizer chooses the scaling heuristic.

## scalingtype.none

No scaling is performed.

## scalingtype.moderate

A conservative scaling is performed.

### scalingtype.aggressive

A very aggressive scaling is performed.

## D.38 Sensitivity types

### sensitivitytype.basis

Basis sensitivity analysis is performed.

## sensitivitytype.optimal\_partition

Optimal partition sensitivity analysis is performed.

## D.39 Degeneracy strategies

### simdegen.none

The simplex optimizer should use no degeneration strategy.

## simdegen.free

The simplex optimizer chooses the degeneration strategy.

### simdegen.aggressive

The simplex optimizer should use an aggressive degeneration strategy.

## simdegen.moderate

The simplex optimizer should use a moderate degeneration strategy.

#### simdegen.minimum

The simplex optimizer should use a minimum degeneration strategy.

## D.40 Exploit duplicate columns.

### simdupvec.off

Disallow the simplex optimizer to exploit duplicated columns.

## simdupvec.on

Allow the simplex optimizer to exploit duplicated columns.

### simdupvec.free

The simplex optimizer can choose freely.

## D.41 Hot-start type employed by the simplex optimizer

#### simhotstart.none

The simplex optimizer performs a coldstart.

#### simhotstart.free

The simplex optimize chooses the hot-start type.

### simhotstart.status\_keys

Only the status keys of the constraints and variables are used to choose the type of hot-start.

## D.42 Problem reformulation.

#### simreform.off

Disallow the simplex optimizer to reformulate the problem.

#### simreform.on

Allow the simplex optimizer to reformulate the problem.

#### simreform.free

The simplex optimizer can choose freely.

## simreform.aggressive

The simplex optimizer should use an aggressive reformulation strategy.

## D.43 Simplex selection strategy

## simseltype.free

The optimizer chooses the pricing strategy.

## simseltype.full

The optimizer uses full pricing.

## simseltype.ase

The optimizer uses approximate steepest-edge pricing.

#### simseltype.devex

The optimizer uses devex steepest-edge pricing (or if it is not available an approximate steep-edge selection).

### simseltype.se

The optimizer uses steepest-edge selection (or if it is not available an approximate steep-edge selection).

## simseltype.partial

The optimizer uses a partial selection approach. The approach is usually beneficial if the number of variables is much larger than the number of constraints.

## D.44 Solution items

#### solitem.xc

Solution for the constraints.

#### solitem.xx

Variable solution.

#### solitem.y

Lagrange multipliers for equations.

#### solitem.slc

Lagrange multipliers for lower bounds on the constraints.

#### solitem.suc

Lagrange multipliers for upper bounds on the constraints.

## solitem.slx

Lagrange multipliers for lower bounds on the variables.

## solitem.sux

Lagrange multipliers for upper bounds on the variables.

## solitem.snx

Lagrange multipliers corresponding to the conic constraints on the variables.

## D.45 Solution status keys

#### solsta.unknown

Status of the solution is unknown.

## solsta.optimal

The solution is optimal.

## solsta.prim\_feas

The solution is primal feasible.

#### solsta.dual\_feas

The solution is dual feasible.

#### solsta.prim\_and\_dual\_feas

The solution is both primal and dual feasible.

## solsta.prim\_infeas\_cer

The solution is a certificate of primal infeasibility.

#### solsta.dual\_infeas\_cer

The solution is a certificate of dual infeasibility.

## solsta.near\_optimal

The solution is nearly optimal.

### solsta.near\_prim\_feas

The solution is nearly primal feasible.

## solsta.near\_dual\_feas

The solution is nearly dual feasible.

## solsta.near\_prim\_and\_dual\_feas

The solution is nearly both primal and dual feasible.

## solsta.near\_prim\_infeas\_cer

The solution is almost a certificate of primal infeasibility.

## solsta.near\_dual\_infeas\_cer

The solution is almost a certificate of dual infeasibility.

## solsta.integer\_optimal

The primal solution is integer optimal.

## solsta.near\_integer\_optimal

The primal solution is near integer optimal.

## D.46 Solution types

```
soltype.itr
```

The interior solution.

soltype.bas

The basic solution.

soltype.itg

The integer solution.

## D.47 Solve primal or dual form

```
solveform.free
```

The optimizer is free to solve either the primal or the dual problem.

### solveform.primal

The optimizer should solve the primal problem.

solveform.dual

The optimizer should solve the dual problem.

## D.48 Status keys

```
stakey.unk
```

The status for the constraint or variable is unknown.

stakey.bas

The constraint or variable is in the basis.

stakey.supbas

The constraint or variable is super basic.

stakey.low

The constraint or variable is at its lower bound.

stakey.upr

The constraint or variable is at its upper bound.

stakey.fix

The constraint or variable is fixed.

stakey.inf

The constraint or variable is infeasible in the bounds.

## D.49 Starting point types

### startpointtype.free

The starting point is chosen automatically.

### startpointtype.guess

The optimizer guesses a starting point.

#### startpointtype.constant

The optimizer constructs a starting point by assigning a constant value to all primal and dual variables. This starting point is normally robust.

### startpointtype.satisfy\_bounds

The starting point is choosen to satisfy all the simple bounds on nonlinear variables. If this starting point is employed, then more care than usual should employed when choosing the bounds on the nonlinear variables. In particular very tight bounds should be avoided.

## D.50 Stream types

## streamtype.log

Log stream. Contains the aggregated contents of all other streams. This means that a message written to any other stream will also be written to this stream.

### streamtype.msg

Message stream. Log information relating to performance and progress of the optimization is written to this stream.

## streamtype.err

Error stream. Error messages are written to this stream.

## streamtype.wrn

Warning stream. Warning messages are written to this stream.

## D.51 Symmetric matrix types

## symmattype.sparse

Sparse symmetric matrix.

## D.52 Transposed matrix.

```
transpose.no
```

No transpose is applied.

transpose.yes

A transpose is applied.

## D.53 Triangular part of a symmetric matrix.

```
uplo.lo
```

Lower part.

uplo.up

Upper part

## D.54 Integer values

```
value.license_buffer_length
```

The length of a license key buffer.

value.max\_str\_len

Maximum string length allowed in MOSEK.

## D.55 Variable types

```
{\tt variable type\_cont}
```

Is a continuous variable.

variabletype.type\_int

Is an integer variable.

## D.56 XML writer output mode

xmlwriteroutputtype.row

Write in row order.

xmlwriteroutputtype.col

Write in column order.

# Appendix E

# Troubleshooting

When creating multiple tasks and running for a long time memory usage grows, and the Task and Env objects are never garbage collected.

The Task and Environment objects cannot always be automatically garbage collected by Python, when when they are no longer in use. There are two ways to ensure that they are destroyed. Use the with-statement:

```
with Task(env,0,0) as t:
    t.readdata("somefile.task")
    # use the task
```

This will ensure that the create Task is automatically destroyed when the with-statement exits. Alternatively, call the \_\_del\_\_() method directly when the object should not be used anymore.

# Appendix F

# Mosek file formats

MOSEK supports a range of problem and solution formats. The Task formats is MOSEK's native binary format and it supports all features that MOSEK supports. OPF is the corresponding ASCII format and this supports nearly all features (everything except semidefinite problems). In general, the text formats are significantly slower to read, but they can be examined and edited directly in any text editor.

MOSEK supports GZIP compression of files. Problem files with an additional ".gz" extension are assumed to be compressed when read, and is automatically compressed when written. For example, a file called

problem.mps.gz

will be read as a GZIP compressed MPS file.

## F.1 The MPS file format

MOSEK supports the standard MPS format with some extensions. For a detailed description of the MPS format see the book by Nazareth [2].

## F.1.1 MPS file structure

The version of the MPS format supported by MOSEK allows specification of an optimization problem on the form

$$l^{c} \leq Ax + q(x) \leq u^{c},$$

$$l^{x} \leq x \leq u^{x},$$

$$x \in \mathcal{C},$$

$$x_{\mathcal{I}} \text{ integer},$$
(F.1)

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = 1/2x^T Q^i x$$

where it is assumed that

$$Q^i = (Q^i)^T.$$

Please note the explicit 1/2 in the quadratic term and that  $Q^i$  is required to be symmetric.

- C is a convex cone.
- $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer-constrained variables.

An MPS file with one row and one column can be illustrated like this:

```
*2345678901234567890123456789012345678901234567890
OBJSENSE
    [objsense]
OBJNAME
    [objname]
ROWS
   [cname1]
COLUMNS
    [vname1]
               [cname1]
                           [value1]
                                         [vname3]
                                                    [value2]
               [cname1]
                           [value1]
                                         [cname2]
                                                    [value2]
    [name]
RANGES
               [cname1]
                           [value1]
                                         [cname2]
                                                    [value2]
    [name]
QSECTION
               [cname1]
               [vname2]
                           [value1]
                                         [vname3]
                                                    [value2]
    [vname1]
BOUNDS
 ?? [name]
                           [value1]
               [vname1]
CSECTION
               [kname1]
                           [value1]
                                         [ktype]
    [vname1]
ENDATA
```

Here the names in capitals are keywords of the MPS format and names in brackets are custom defined names or values. A couple of notes on the structure:

#### Fields:

All items surrounded by brackets appear in *fields*. The fields named "valueN" are numerical values. Hence, they must have the format

```
[+|-]XXXXXXX.XXXXXX[[e|E][+|-]XXX] where X = [0|1|2|3|4|5|6|7|8|9].
```

#### Sections:

The MPS file consists of several sections where the names in capitals indicate the beginning of a new section. For example, COLUMNS denotes the beginning of the columns section.

#### Comments:

Lines starting with an "\*" are comment lines and are ignored by MOSEK.

### Keys:

The question marks represent keys to be specified later.

#### Extensions:

The sections QSECTION and CSECTION are MOSEK specific extensions of the MPS format.

The standard MPS format is a fixed format, i.e. everything in the MPS file must be within certain fixed positions. MOSEK also supports a *free format*. See Section F.1.5 for details.

## F.1.1.1 Linear example lo1.mps

A concrete example of a MPS file is presented below:

```
* File: lo1.mps
NAME
              lo1
OBJSENSE
    MAX
ROWS
N obj
E c1
G c2
L c3
COLUMNS
    x1
              obj
                         3
    x1
              c1
              c2
                         2
    x1
    x2
              obj
    x2
              c1
                         1
    x2
              c2
                         1
    x2
              сЗ
                         2
              obj
    xЗ
    хЗ
              c1
                         2
    x3
                         3
              c2
    x4
              obj
    x4
              c2
                         1
```

RHS		
rhs	c1	30
rhs	c2	15
rhs	c3	25
RANGES		
BOUNDS		
UP bound	x2	10
ENDATA		

Subsequently each individual section in the MPS format is discussed.

#### F.1.1.2 NAME

In this section a name ([name]) is assigned to the problem.

## F.1.1.3 OBJSENSE (optional)

This is an optional section that can be used to specify the sense of the objective function. The OBJSENSE section contains one line at most which can be one of the following

MIN MINIMIZE MAX MAXIMIZE

It should be obvious what the implication is of each of these four lines.

## F.1.1.4 OBJNAME (optional)

This is an optional section that can be used to specify the name of the row that is used as objective function. The OBJNAME section contains one line at most which has the form

objname

objname should be a valid row name.

## F.1.1.5 ROWS

A record in the ROWS section has the form

? [cname1]

where the requirements for the fields are as follows:

Field	Starting	Maximum	Re-	Description
	position	width	quired	
?	2	1	Yes	Constraint key
[cname1]	5	8	Yes	Constraint name

Hence, in this section each constraint is assigned an unique name denoted by [cname1]. Please note that [cname1] starts in position 5 and the field can be at most 8 characters wide. An initial key (?)

must be present to specify the type of the constraint. The key can have the values E, G, L, or N with the following interpretation:

Constraint	$l_i^c$	$u_i^c$
type		
E	finite	$l_i^c$
G	finite	$\infty$
L	$-\infty$	finite
N	$-\infty$	$\infty$

In the MPS format an objective vector is not specified explicitly, but one of the constraints having the key N will be used as the objective vector c. In general, if multiple N type constraints are specified, then the first will be used as the objective vector c.

#### F.1.1.6 COLUMNS

In this section the elements of A are specified using one or more records having the form [vname1] [cname1] [value1] [value2]

where the requirements for each field are as follows:

Field	Starting	Maximum	Re-	Description
	position	width	quired	
[vname1]	5	8	Yes	Variable name
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

Hence, a record specifies one or two elements  $a_{ij}$  of A using the principle that [vname1] and [cname1] determines j and i respectively. Please note that [cname1] must be a constraint name specified in the ROWS section. Finally, [value1] denotes the numerical value of  $a_{ij}$ . Another optional element is specified by [cname2], and [value2] for the variable specified by [vname1]. Some important comments are:

- All elements belonging to one variable must be grouped together.
- Zero elements of A should not be specified.
- At least one element for each variable should be specified.

#### F.1.1.7 RHS (optional)

A record in this section has the format

[name] [cname1] [value1] [cname2] [value2]

where the requirements for each field are as follows:

Field	Starting	Maximum	Re-	Description
	position	width	quired	
[name]	5	8	Yes	Name of the RHS vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The interpretation of a record is that [name] is the name of the RHS vector to be specified. In general, several vectors can be specified. [cname1] denotes a constraint name previously specified in the ROWS section. Now, assume that this name has been assigned to the i th constraint and  $v_1$  denotes the value specified by [value1], then the interpretation of  $v_1$  is:

Constraint	$l_i^c$	$u_i^c$
type		
E	$v_1$	$v_1$
G	$v_1$	
L		$v_1$
N		

An optional second element is specified by [cname2] and [value2] and is interpreted in the same way. Please note that it is not necessary to specify zero elements, because elements are assumed to be zero.

### F.1.1.8 RANGES (optional)

A record in this section has the form

[name] [cname1] [value1] [cname2] [value2]

where the requirements for each fields are as follows:

Field	Starting	Maximum	Re-	Description
	position	width	quired	
[name]	5	8	Yes	Name of the RANGE vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The records in this section are used to modify the bound vectors for the constraints, i.e. the values in  $l^c$  and  $u^c$ . A record has the following interpretation: [name] is the name of the RANGE vector and [cname1] is a valid constraint name. Assume that [cname1] is assigned to the i th constraint and let  $v_1$  be the value specified by [value1], then a record has the interpretation:

Constraint	Sign of $v_1$	$l_i^c$	$u_i^c$
type			
E	-	$u_i^c + v_1$	
E	+		$l_i^c + v_1$
G	- or +		$l_i^c +  v_1 $
L	- or +	$u_i^c -  v_1 $	
N			

## F.1.1.9 QSECTION (optional)

Within the QSECTION the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic term belongs. A record in the QSECTION has the form

[vname1] [vname2] [value1] [vname3] [value2]

where the requirements for each field are:

Field	Starting	Maximum	Re-	Description
	position	width	quired	
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value
[vname3]	40	8	No	Variable name
[value2]	50	12	No	Numerical value

A record specifies one or two elements in the lower triangular part of the  $Q^i$  matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k th and j th variable, then  $Q^i_{kj}$  is assigned the value given by [value1] An optional second element is specified in the same way by the fields [vname1], [vname3], and [value2].

The example

has the following MPS file representation

```
* File: qo1.mps
NAME
              qo1
ROWS
N obj
G c1
COLUMNS
    x1
                         1.0
              c1
    x2
              obj
                         -1.0
    x2
                         1.0
              c1
    хЗ
                         1.0
RHS
              c1
                         1.0
```

QSECTION	obj	
x1	x1	2.0
x1	xЗ	-1.0
x2	x2	0.2
x3	x3	2.0
ENDATA		

Regarding the QSECTIONs please note that:

- Only one QSECTION is allowed for each constraint.
- The QSECTIONs can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QSECTION must already be specified in the COLUMNS section.
- ullet All entries specified in a QSECTION are assumed to belong to the lower triangular part of the quadratic term of Q.

## F.1.1.10 BOUNDS (optional)

In the BOUNDS section changes to the default bounds vectors  $l^x$  and  $u^x$  are specified. The default bounds vectors are  $l^x=0$  and  $u^x=\infty$ . Moreover, it is possible to specify several sets of bound vectors. A record in this section has the form

?? [name] [vname1] [value1]

where the requirements for each field are:

Field	Starting	Maximum	Re-	Description
	position	width	quired	
??	2	2	Yes	Bound key
[name]	5	8	Yes	Name of the BOUNDS vector
[vname1]	15	8	Yes	Variable name
[value1]	25	12	No	Numerical value

Hence, a record in the BOUNDS section has the following interpretation: [name] is the name of the bound vector and [vname1] is the name of the variable which bounds are modified by the record. ?? and [value1] are used to modify the bound vectors according to the following table:

??	$l_i^x$	$u_i^x$	Made integer
	3	3	(added to $\mathcal{J}$ )
FR	$-\infty$	$\infty$	No
FX	$v_1$	$v_1$	No
LO	$v_1$	unchanged	No
MI	$-\infty$	unchanged	No
PL	unchanged	$\infty$	No
UP	unchanged	$v_1$	No
${\tt BV}$	0	1	Yes
LI	$\lceil v_1 \rceil$	unchanged	Yes
UI	unchanged	$\lfloor v_1  floor$	Yes

 $v_1$  is the value specified by [value1].

## F.1.1.11 CSECTION (optional)

The purpose of the CSECTION is to specify the constraint

$$x \in \mathcal{C}$$
.

in (F.1).

It is assumed that  $\mathcal{C}$  satisfies the following requirements. Let

$$x^t \in \mathbb{R}^{n^t}, \ t = 1, \dots, k$$

be vectors comprised of parts of the decision variables x so that each decision variable is a member of exactly **one** vector  $x^t$ , for example

$$x^1 = \begin{bmatrix} x_1 \\ x_4 \\ x_7 \end{bmatrix} \text{ and } x^2 = \begin{bmatrix} x_6 \\ x_5 \\ x_3 \\ x_2 \end{bmatrix}.$$

Next define

$$\mathcal{C} := \left\{ x \in \mathbb{R}^n : \ x^t \in \mathcal{C}_t, \ t = 1, \dots, k \right\}$$

where  $C_t$  must have one of the following forms

•  $\mathbb{R}$  set:

$$\mathcal{C}_t = \{ x \in \mathbb{R}^{n^t} \}.$$

• Quadratic cone:

$$C_t = \left\{ x \in \mathbb{R}^{n^t} : x_1 \ge \sqrt{\sum_{j=2}^{n^t} x_j^2} \right\}.$$
 (F.2)

• Rotated quadratic cone:

$$C_t = \left\{ x \in \mathbb{R}^{n^t} : 2x_1 x_2 \ge \sum_{j=3}^{n^t} x_j^2, \ x_1, x_2 \ge 0 \right\}.$$
 (F.3)

In general, only quadratic and rotated quadratic cones are specified in the MPS file whereas membership of the  $\mathbb R$  set is not. If a variable is not a member of any other cone then it is assumed to be a member of an  $\mathbb R$  cone.

Next, let us study an example. Assume that the quadratic cone

$$x_4 \ge \sqrt{x_5^2 + x_8^2} \tag{F.4}$$

and the rotated quadratic cone

$$2x_3x_7 \ge x_1^2 + x_0^2, \ x_3, x_7 \ge 0, \tag{F.5}$$

should be specified in the MPS file. One CSECTION is required for each cone and they are specified as follows:

*	1 2	3	4	5	6
*23456789	012345678901	2345678901234	567890123456	7890123456	<del>3</del> 7890
CSECTION	konea	0.0	QUAD		
x4					
x5					
x8					
CSECTION	koneb	0.0	RQUAD		
x7					
x3					
x1					
x0					

This first CSECTION specifies the cone (F.4) which is given the name konea. This is a quadratic cone which is specified by the keyword QUAD in the CSECTION header. The 0.0 value in the CSECTION header is not used by the QUAD cone.

The second CSECTION specifies the rotated quadratic cone (F.5). Please note the keyword RQUAD in the CSECTION which is used to specify that the cone is a rotated quadratic cone instead of a quadratic cone. The 0.0 value in the CSECTION header is not used by the RQUAD cone.

In general, a CSECTION header has the format

CSECTION [kname1] [value1] [ktype]

where the requirement for each field are as follows:

Field	Starting	Maximum	Re-	Description
	position	width	quired	
[kname1]	5	8	Yes	Name of the cone
[value1]	15	12	No	Cone parameter
[ktype]	25		Yes	Type of the cone.

The possible cone type keys are:

Cone type key	Members	Interpretation.
QUAD	$\geq 1$	Quadratic cone i.e. $(F.2)$ .
RQUAD	> 2	Rotated quadratic cone i.e. (F.3).

Please note that a quadratic cone must have at least one member whereas a rotated quadratic cone must have at least two members. A record in the CSECTION has the format

#### [vname1]

where the requirements for each field are

Field	Starting	Maximum	Re-	Description
	position	width	quired	
[vname1]	2	8	Yes	A valid variable name

The most important restriction with respect to the CSECTION is that a variable must occur in only one CSECTION.

#### F.1.1.12 ENDATA

This keyword denotes the end of the MPS file.

## F.1.2 Integer variables

Using special bound keys in the BOUNDS section it is possible to specify that some or all of the variables should be integer-constrained i.e. be members of  $\mathcal{J}$ . However, an alternative method is available.

This method is available only for backward compatibility and we recommend that it is not used. This method requires that markers are placed in the COLUMNS section as in the example:

COLUMNS					
x1	obj	-10.0	c1	0.7	
x1	c2	0.5	c3	1.0	
x1	c4	0.1			
* Start of integer-constrained variables.					
MARKOOO 'MARKER			'INTORG'		
x2	obj	-9.0	c1	1.0	
x2	c2	0.833333333	c3	0.6666667	
x2	c4	0.25			
x3	obj	1.0	c6	2.0	
MARKO01	'MARKER'		'INTEND'		

st End of integer-constrained variables.

Please note that special marker lines are used to indicate the start and the end of the integer variables. Furthermore be aware of the following

- IMPORTANT: All variables between the markers are assigned a default lower bound of 0 and a default upper bound of 1. **This may not be what is intended.** If it is not intended, the correct bounds should be defined in the BOUNDS section of the MPS formatted file.
- MOSEK ignores field 1, i.e. MARKO001 and MARKO01, however, other optimization systems require them.
- Field 2, i.e. 'MARKER', must be specified including the single quotes. This implies that no row can be assigned the name 'MARKER'.

- Field 3 is ignored and should be left blank.
- Field 4, i.e. 'INTORG' and 'INTEND', must be specified.
- It is possible to specify several such integer marker sections within the COLUMNS section.

### F.1.3 General limitations

• An MPS file should be an ASCII file.

## F.1.4 Interpretation of the MPS format

Several issues related to the MPS format are not well-defined by the industry standard. However, MOSEK uses the following interpretation:

- If a matrix element in the COLUMNS section is specified multiple times, then the multiple entries are added together.
- If a matrix element in a QSECTION section is specified multiple times, then the multiple entries are added together.

### F.1.5 The free MPS format

MOSEK supports a free format variation of the MPS format. The free format is similar to the MPS file format but less restrictive, e.g. it allows longer names. However, it also presents two main limitations:

- By default a line in the MPS file must not contain more than 1024 characters. However, by modifying the parameter iparam.read\_mps\_width an arbitrary large line width will be accepted.
- A name must not contain any blanks.

To use the free MPS format instead of the default MPS format the MOSEK parameter <code>iparam.read\_mps\_format</code> should be changed.

## F.2 The LP file format

MOSEK supports the LP file format with some extensions i.e. MOSEK can read and write LP formatted files.

Please note that the LP format is not a completely well-defined standard and hence different optimization packages may interpret the same LP file in slightly different ways. MOSEK tries to emulate as closely as possible CPLEX's behavior, but tries to stay backward compatible.

The LP file format can specify problems on the form

$$\begin{array}{lll} \text{minimize/maximize} & & c^Tx + \frac{1}{2}q^o(x) \\ \text{subject to} & & l^c & \leq & Ax + \frac{1}{2}q(x) & \leq & u^c, \\ & l^x & \leq & x & \leq & u^x, \\ & & & x_{\mathcal{J}} \text{integer}, \end{array}$$

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear term in the objective.
- $q^o :\in \mathbb{R}^n \to \mathbb{R}$  is the quadratic term in the objective where

$$q^o(x) = x^T Q^o x$$

and it is assumed that

$$Q^o = (Q^o)^T.$$

- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = x^T Q^i x$$

where it is assumed that

$$Q^i = (Q^i)^T.$$

•  $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer constrained variables.

## F.2.1 The sections

An LP formatted file contains a number of sections specifying the objective, constraints, variable bounds, and variable types. The section keywords may be any mix of upper and lower case letters.

## F.2.1.1 The objective

The first section beginning with one of the keywords

max maximum maximize min minimum minimize

defines the objective sense and the objective function, i.e.

$$c^T x + \frac{1}{2} x^T Q^o x.$$

The objective may be given a name by writing

myname:

before the expressions. If no name is given, then the objective is named obj.

The objective function contains linear and quadratic terms. The linear terms are written as

```
4 x1 + x2 - 0.1 x3
```

and so forth. The quadratic terms are written in square brackets ([]) and are either squared or multiplied as in the examples

x1^2

and

x1 \* x2

There may be zero or more pairs of brackets containing quadratic expressions.

An example of an objective section is:

```
minimize myobj: 4 \times 1 + \times 2 - 0.1 \times 3 + [\times 1^2 + 2.1 \times 1 * \times 2]/2
```

Please note that the quadratic expressions are multiplied with  $\frac{1}{2}$ , so that the above expression means

minimize 
$$4x_1 + x_2 - 0.1 \cdot x_3 + \frac{1}{2}(x_1^2 + 2.1 \cdot x_1 \cdot x_2)$$

If the same variable occurs more than once in the linear part, the coefficients are added, so that  $4 \times 1 + 2 \times 1$  is equivalent to  $6 \times 1$ . In the quadratic expressions  $\times 1 \times 2$  is equivalent to  $\times 2 \times 1$  and as in the linear part, if the same variables multiplied or squared occur several times their coefficients are added.

#### F.2.1.2 The constraints

The second section beginning with one of the keywords

```
subj to
subject to
s.t.
```

st

defines the linear constraint matrix (A) and the quadratic matrices  $(Q^i)$ .

A constraint contains a name (optional), expressions adhering to the same rules as in the objective and a bound:

```
subject to con1: x1 + x2 + [x3^2]/2 \le 5.1
```

The bound type (here  $\leq$ ) may be any of  $\leq$ ,  $\leq$ ,  $\Rightarrow$ ,  $\Rightarrow$  ( $\leq$  and  $\leq$  mean the same), and the bound may be any number.

In the standard LP format it is not possible to define more than one bound, but MOSEK supports defining ranged constraints by using double-colon (''::'') instead of a single-colon (":") after the constraint name, i.e.

$$-5 \le x_1 + x_2 \le 5 \tag{F.6}$$

may be written as

```
con:: -5 < x_1 + x_2 < 5
```

By default MOSEK writes ranged constraints this way.

If the files must adhere to the LP standard, ranged constraints must either be split into upper bounded and lower bounded constraints or be written as en equality with a slack variable. For example the expression (F.6) may be written as

$$x_1 + x_2 - sl_1 = 0, -5 \le sl_1 \le 5.$$

#### **F.2.1.3** Bounds

Bounds on the variables can be specified in the bound section beginning with one of the keywords

bound bounds

The bounds section is optional but should, if present, follow the **subject to** section. All variables listed in the bounds section must occur in either the objective or a constraint.

The default lower and upper bounds are 0 and  $+\infty$ . A variable may be declared free with the keyword free, which means that the lower bound is  $-\infty$  and the upper bound is  $+\infty$ . Furthermore it may be assigned a finite lower and upper bound. The bound definitions for a given variable may be written in one or two lines, and bounds can be any number or  $\pm\infty$  (written as  $+\inf/-\inf/+\inf\inf$ ) as in the example

```
bounds
x1 free
x2 <= 5
0.1 <= x2
x3 = 42
2 <= x4 < +inf
```

## F.2.1.4 Variable types

The final two sections are optional and must begin with one of the keywords

```
binaries
binary
and
gen
general
```

Under general all integer variables are listed, and under binary all binary (integer variables with bounds 0 and 1) are listed:

```
general
x1 x2
binary
x3 x4
```

Again, all variables listed in the binary or general sections must occur in either the objective or a constraint.

## F.2.1.5 Terminating section

Finally, an LP formatted file must be terminated with the keyword

## F.2.1.6 Linear example lo1.lp

A simple example of an LP file is:

```
\ File: lo1.lp
maximize
obj: 3 x1 + x2 + 5 x3 + x4
subject to
c1: 3 x1 + x2 + 2 x3 = 30
c2: 2 x1 + x2 + 3 x3 + x4 >= 15
c3: 2 x2 + 3 x4 <= 25
bounds
0 <= x1 <= +infinity
0 <= x2 <= 10
0 <= x3 <= +infinity
0 <= x4 <= +infinity
end
```

## F.2.1.7 Mixed integer example milo1.lp

```
maximize
obj: x1 + 6.4e-01 x2
subject to
c1: 5e+01 x1 + 3.1e+01 x2 <= 2.5e+02
c2: 3e+00 x1 - 2e+00 x2 >= -4e+00
```

```
bounds
  0 <= x1 <= +infinity
  0 <= x2 <= +infinity
general
  x1 x2
end</pre>
```

## F.2.2 LP format peculiarities

### F.2.2.1 Comments

Anything on a line after a "\" is ignored and is treated as a comment.

#### **F.2.2.2** Names

A name for an objective, a constraint or a variable may contain the letters a-z, A-Z, the digits 0-9 and the characters

```
!"#$%&()/,.;?@_','|~
```

The first character in a name must not be a number, a period or the letter 'e' or 'E'. Keywords must not be used as names.

MOSEK accepts any character as valid for names, except '\0'. When writing a name that is not allowed in LP files, it is changed and a warning is issued.

The algorithm for making names LP valid works as follows: The name is interpreted as an utf-8 string. For a unicode character c:

- If c=='\_' (underscore), the output is '\_\_' (two underscores).
- If c is a valid LP name character, the output is just c.
- If c is another character in the ASCII range, the output is \_XX, where XX is the hexadecimal code for the character.
- If c is a character in the range 127—65535, the output is \_uxxxx, where xxxx is the hexadecimal code for the character.
- If c is a character above 65535, the output is \_UXXXXXXXX, where XXXXXXXX is the hexadecimal code for the character.

Invalid utf-8 substrings are escaped as '\_XX', and if a name starts with a period, 'e' or 'E', that character is escaped as '\_XX'.

## F.2.2.3 Variable bounds

Specifying several upper or lower bounds on one variable is possible but MOSEK uses only the tightest bounds. If a variable is fixed (with =), then it is considered the tightest bound.

## F.2.2.4 MOSEK specific extensions to the LP format

Some optimization software packages employ a more strict definition of the LP format that the one used by MOSEK. The limitations imposed by the strict LP format are the following:

- Quadratic terms in the constraints are not allowed.
- Names can be only 16 characters long.
- Lines must not exceed 255 characters in length.

If an LP formatted file created by MOSEK should satisfies the strict definition, then the parameter

```
iparam.write_lp_strict_format
```

should be set; note, however, that some problems cannot be written correctly as a strict LP formatted file. For instance, all names are truncated to 16 characters and hence they may loose their uniqueness and change the problem.

To get around some of the inconveniences converting from other problem formats, MOSEK allows lines to contain 1024 characters and names may have any length (shorter than the 1024 characters).

Internally in MOSEK names may contain any (printable) character, many of which cannot be used in LP names. Setting the parameters

```
iparam.read_lp_quoted_names
```

and

```
iparam.write_lp_quoted_names
```

allows MOSEK to use quoted names. The first parameter tells MOSEK to remove quotes from quoted names e.g, "x1", when reading LP formatted files. The second parameter tells MOSEK to put quotes around any semi-illegal name (names beginning with a number or a period) and fully illegal name (containing illegal characters). As double quote is a legal character in the LP format, quoting semi-illegal names makes them legal in the pure LP format as long as they are still shorter than 16 characters. Fully illegal names are still illegal in a pure LP file.

## F.2.3 The strict LP format

The LP format is not a formal standard and different vendors have slightly different interpretations of the LP format. To make MOSEK's definition of the LP format more compatible with the definitions of other vendors, use the parameter setting

```
iparam.write\_lp\_strict\_format = onoffkey.on
```

This setting may lead to truncation of some names and hence to an invalid LP file. The simple solution to this problem is to use the parameter setting

```
iparam.write_generic_names = onoffkey.on
```

which will cause all names to be renamed systematically in the output file.

## F.2.4 Formatting of an LP file

A few parameters control the visual formatting of LP files written by MOSEK in order to make it easier to read the files. These parameters are

```
iparam.write_lp_line_width
iparam.write_lp_terms_per_line
```

The first parameter sets the maximum number of characters on a single line. The default value is 80 corresponding roughly to the width of a standard text document.

The second parameter sets the maximum number of terms per line; a term means a sign, a coefficient, and a name (for example "+ 42 elephants"). The default value is 0, meaning that there is no maximum.

## F.2.4.1 Speeding up file reading

If the input file should be read as fast as possible using the least amount of memory, then it is important to tell MOSEK how many non-zeros, variables and constraints the problem contains. These values can be set using the parameters

```
iparam.read_con
iparam.read_var
iparam.read_anz
iparam.read_qnz
```

#### F.2.4.2 Unnamed constraints

Reading and writing an LP file with MOSEK may change it superficially. If an LP file contains unnamed constraints or objective these are given their generic names when the file is read (however unnamed constraints in MOSEK are written without names).

## F.3 The OPF format

The Optimization Problem Format (OPF) is an alternative to LP and MPS files for specifying optimization problems. It is row-oriented, inspired by the CPLEX LP format.

Apart from containing objective, constraints, bounds etc. it may contain complete or partial solutions, comments and extra information relevant for solving the problem. It is designed to be easily read and modified by hand and to be forward compatible with possible future extensions.

### F.3.1 Intended use

The OPF file format is meant to replace several other files:

- The LP file format. Any problem that can be written as an LP file can be written as an OPF file to; furthermore it naturally accommodates ranged constraints and variables as well as arbitrary characters in names, fixed expressions in the objective, empty constraints, and conic constraints.
- Parameter files. It is possible to specify integer, double and string parameters along with the problem (or in a separate OPF file).
- Solution files. It is possible to store a full or a partial solution in an OPF file and later reload it.

### F.3.2 The file format

The format uses tags to structure data. A simple example with the basic sections may look like this:

```
[comment]
  This is a comment. You may write almost anything here...
[/comment]

# This is a single-line comment.

[objective min 'myobj']
    x + 3 y + x^2 + 3 y^2 + z + 1
[/objective]

[constraints]
    [con 'con01'] 4 <= x + y [/con]
[/constraints]

[bounds]
    [b] -10 <= x,y <= 10 [/b]

[cone quad] x,y,z [/cone]
[/bounds]</pre>
```

A scope is opened by a tag of the form [tag] and closed by a tag of the form [/tag]. An opening tag may accept a list of unnamed and named arguments, for examples

```
[tag value] tag with one unnamed argument [/tag] [tag arg=value] tag with one named argument in quotes [/tag]
```

Unnamed arguments are identified by their order, while named arguments may appear in any order, but never before an unnamed argument. The value can be a quoted, single-quoted or double-quoted text string, i.e.

```
[tag 'value'] single-quoted value [/tag]
[tag arg='value'] single-quoted value [/tag]
```

```
[tag "value"] double-quoted value [/tag]
[tag arg="value"] double-quoted value [/tag]
```

#### F.3.2.1 Sections

The recognized tags are

- [comment] A comment section. This can contain *almost* any text: Between single quotes (') or double quotes (") any text may appear. Outside quotes the markup characters ([ and ]) must be prefixed by backslashes. Both single and double quotes may appear alone or inside a pair of quotes if it is prefixed by a backslash.
- [objective] The objective function: This accepts one or two parameters, where the first one (in the above example 'min') is either min or max (regardless of case) and defines the objective sense, and the second one (above 'myobj'), if present, is the objective name. The section may contain linear and quadratic expressions.

If several objectives are specified, all but the last are ignored.

• [constraints] This does not directly contain any data, but may contain the subsection 'con' defining a linear constraint.

[con] defines a single constraint; if an argument is present ([con NAME]) this is used as the name of the constraint, otherwise it is given a null-name. The section contains a constraint definition written as linear and quadratic expressions with a lower bound, an upper bound, with both or with an equality. Examples:

Constraint names are unique. If a constraint is specified which has the same name as a previously defined constraint, the new constraint replaces the existing one.

- [bounds] This does not directly contain any data, but may contain the subsections 'b' (linear bounds on variables) and cone' (quadratic cone).
  - [b]. Bound definition on one or several variables separated by comma (','). An upper or lower bound on a variable replaces any earlier defined bound on that variable. If only one bound (upper or lower) is given only this bound is replaced. This means that upper and lower bounds can be specified separately. So the OPF bound definition:

```
[b] x,y >= -10 [/b]
[b] x,y <= 10 [/b]
results in the bound</pre>
```

[cone]. Currently, the supported cones are the quadratic cone and the rotated quadratic cone (see section 5.3). A conic constraint is defined as a set of variables which belongs to a single unique cone.

A quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1^2 > \sum_{i=2}^n x_i^2.$$

A rotated quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1 x_2 > \sum_{i=3}^n x_i^2$$
.

A [bounds]-section example:

```
[bounds]

[b] 0 <= x,y <= 10 [/b] # ranged bound

[b] 10 >= x,y >= 0 [/b] # ranged bound

[b] 0 <= x,y <= inf [/b] # using inf

[b] x,y free [/b] # free variables

# Let (x,y,z,w) belong to the cone K

[cone quad] x,y,z,w [/cone] # quadratic cone

[cone rquad] x,y,z,w [/cone] # rotated quadratic cone

[/bounds]
```

By default all variables are free.

- [variables] This defines an ordering of variables as they should appear in the problem. This is simply a space-separated list of variable names.
- [integer] This contains a space-separated list of variables and defines the constraint that the listed variables must be integer values.
- [hints] This may contain only non-essential data; for example estimates of the number of variables, constraints and non-zeros. Placed before all other sections containing data this may reduce the time spent reading the file.

In the hints section, any subsection which is not recognized by MOSEK is simply ignored. In this section a hint in a subsection is defined as follows:

```
[hint ITEM] value [/hint]
```

where ITEM may be replaced by number of variables), numcon (number of linear/quadratic constraints), numanz (number of linear non-zeros in constraints) and numqnz (number of quadratic non-zeros in constraints).

• [solutions] This section can contain a set of full or partial solutions to a problem. Each solution must be specified using a [solution]-section, i.e.

Note that a [solution]-section must be always specified inside a [solutions]-section. The syntax of a [solution]-section is the following:

```
[solution SOLTYPE status=STATUS]...[/solution]
```

where SOLTYPE is one of the strings

- 'interior', a non-basic solution,
- 'basic', a basic solution,
- 'integer', an integer solution,

and STATUS is one of the strings

- 'UNKNOWN',
- 'OPTIMAL',
- 'INTEGER\_OPTIMAL',
- 'PRIM\_FEAS',
- 'DUAL\_FEAS',
- 'PRIM\_AND\_DUAL\_FEAS',
- 'NEAR\_OPTIMAL',
- 'NEAR\_PRIM\_FEAS',
- 'NEAR\_DUAL\_FEAS',
- 'NEAR\_PRIM\_AND\_DUAL\_FEAS',
- 'PRIM\_INFEAS\_CER',
- 'DUAL\_INFEAS\_CER',
- 'NEAR\_PRIM\_INFEAS\_CER',
- 'NEAR\_DUAL\_INFEAS\_CER',
- 'NEAR\_INTEGER\_OPTIMAL'.

Most of these values are irrelevant for input solutions; when constructing a solution for simplex hot-start or an initial solution for a mixed integer problem the safe setting is UNKNOWN.

A [solution]-section contains [con] and [var] sections. Each [con] and [var] section defines solution information for a single variable or constraint, specified as list of KEYWORD/value pairs, in any order, written as

#### KEYWORD=value

Allowed keywords are as follows:

- sk. The status of the item, where the value is one of the following strings:
  - \* LOW, the item is on its lower bound.
  - \* UPR, the item is on its upper bound.
  - \* FIX, it is a fixed item.
  - \* BAS, the item is in the basis.
  - \* SUPBAS, the item is super basic.

- \* UNK, the status is unknown.
- \* INF, the item is outside its bounds (infeasible).
- lvl Defines the level of the item.
- sl Defines the level of the dual variable associated with its lower bound.
- su Defines the level of the dual variable associated with its upper bound.
- sn Defines the level of the variable associated with its cone.
- y Defines the level of the corresponding dual variable (for constraints only).

A [var] section should always contain the items sk, lvl, sl and su. Items sl and su are not required for integer solutions.

A [con] section should always contain sk, lvl, sl, su and y.

An example of a solution section

```
[solution basic status=UNKNOWN]

[var x0] sk=LOW lvl=5.0 [/var]

[var x1] sk=UPR lvl=10.0 [/var]

[var x2] sk=SUPBAS lvl=2.0 sl=1.5 su=0.0 [/var]

[con c0] sk=LOW lvl=3.0 y=0.0 [/con]

[con c0] sk=UPR lvl=0.0 y=5.0 [/con]
```

• [vendor] This contains solver/vendor specific data. It accepts one argument, which is a vendor ID – for MOSEK the ID is simply mosek – and the section contains the subsection parameters defining solver parameters. When reading a vendor section, any unknown vendor can be safely ignored. This is described later.

Comments using the '#' may appear anywhere in the file. Between the '#' and the following line-break any text may be written, including markup characters.

#### F.3.2.2 Numbers

Numbers, when used for parameter values or coefficients, are written in the usual way by the printf function. That is, they may be prefixed by a sign (+ or -) and may contain an integer part, decimal part and an exponent. The decimal point is always '.' (a dot). Some examples are

```
1
1.0
.0
1.
1e10
1e+10
```

Some *invalid* examples are

```
e10  # invalid, must contain either integer or decimal part
.  # invalid
.e10  # invalid
```

More formally, the following standard regular expression describes numbers as used:

```
[+|-]?([0-9]+[.][0-9]*|[.][0-9]+)([eE][+|-]?[0-9]+)?
```

#### F.3.2.3 Names

Variable names, constraint names and objective name may contain arbitrary characters, which in some cases must be enclosed by quotes (single or double) that in turn must be preceded by a backslash. Unquoted names must begin with a letter (a-z or A-Z) and contain only the following characters: the letters a-z and A-Z, the digits 0-9, braces ({ and }) and underscore (\_).

Some examples of legal names:

```
an_unquoted_name
another_name{123}
'single quoted name'
"double quoted name"
"name with \\"quote\\" in it"
"name with []s in it"
```

#### F.3.3 Parameters section

In the vendor section solver parameters are defined inside the parameters subsection. Each parameter is written as

```
[p PARAMETER_NAME] value [/p]
```

where PARAMETER\_NAME is replaced by a MOSEK parameter name, usually of the form MSK\_IPAR\_..., MSK\_DPAR\_... or MSK\_SPAR\_..., and the value is replaced by the value of that parameter; both integer values and named values may be used. Some simple examples are:

```
[vendor mosek]
  [parameters]
  [p MSK_IPAR_OPF_MAX_TERMS_PER_LINE] 10  [/p]
  [p MSK_IPAR_OPF_WRITE_PARAMETERS] MSK_ON [/p]
  [p MSK_DPAR_DATA_TOL_BOUND_INF] 1.0e18 [/p]
  [/parameters]
[/vendor]
```

#### F.3.4 Writing OPF files from MOSEK

The function Task.writedata can be used to produce an OPF file from a task.

To write an OPF file set the parameter <code>iparam.write\_data\_format</code> to <code>dataformat.op</code> as this ensures that <code>OPF</code> format is used. Then modify the following parameters to define what the file should contain:

- iparam.opf\_write\_header, include a small header with comments.
- iparam.opf\_write\_hints, include hints about the size of the problem.
- iparam.opf\_write\_problem, include the problem itself objective, constraints and bounds.
- iparam.opf\_write\_solutions, include solutions if they are defined. If this is off, no solutions are included.
- iparam.opf\_write\_sol\_bas, include basic solution, if defined.

- iparam.opf\_write\_sol\_itg, include integer solution, if defined.
- iparam.opf\_write\_sol\_itr, include interior solution, if defined.
- iparam.opf\_write\_parameters, include all parameter settings.

### F.3.5 Examples

This section contains a set of small examples written in OPF and describing how to formulate linear, quadratic and conic problems.

#### F.3.5.1 Linear example lo1.opf

Consider the example:

having the bounds

In the OPF format the example is displayed as shown below:

```
[comment]
 The lo1 example in OPF format
[/comment]
[hints]
 [hint NUMVAR] 4 [/hint]
 [hint NUMCON] 3 [/hint]
 [hint NUMANZ] 9 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3 x4
[/variables]
[objective maximize 'obj']
  3 x1 + x2 + 5 x3 + x4
[/objective]
[constraints]
 [con 'c1'] 3 x1 + x2 + 2 x3
                                      = 30 [/con]
 [con 'c2'] 2 x1 + x2 + 3 x3 + x4 >= 15 [/con]
                 2 x2
 [con 'c3']
                             + 3 x4 <= 25 [/con]
[/constraints]
```

```
[bounds]

[b] 0 <= * [/b]

[b] 0 <= x2 <= 10 [/b]

[/bounds]
```

#### F.3.5.2 Quadratic example qol.opf

An example of a quadratic optimization problem is

$$\begin{array}{ll} \text{minimize} & x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2 \\ \text{subject to} & 1 & \leq & x_1 + x_2 + x_3, \\ & & x > 0. \end{array}$$

This can be formulated in opf as shown below.

```
The qo1 example in OPF format
[/comment]
[hints]
  [hint NUMVAR] 3 [/hint]
  [hint NUMCON] 1 [/hint]
  [hint NUMANZ] 3 [/hint]
  [hint NUMQNZ] 4 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3
[/variables]
[objective minimize 'obj']
 # The quadratic terms are often written with a factor of 1/2 as here,
 # but this is not required.
  - x2 + 0.5 ( 2.0 x1 ^ 2 - 2.0 x3 * x1 + 0.2 x2 ^ 2 + 2.0 x3 ^ 2 )
[/objective]
[constraints]
 [con 'c1'] 1.0 \le x1 + x2 + x3 [/con]
[/constraints]
[bounds]
  [b] 0 \le * [/b]
[/bounds]
```

#### F.3.5.3 Conic quadratic example cqo1.opf

Consider the example:

$$\begin{array}{lll} \text{minimize} & x_3 + x_4 + x_5 \\ \text{subject to} & x_0 + x_1 + 2x_2 & = & 1, \\ & x_0, x_1, x_2 & \geq & 0, \\ & x_3 \geq \sqrt{x_0^2 + x_1^2}, \\ & 2x_4x_5 \geq x_2^2. \end{array}$$

Please note that the type of the cones is defined by the parameter to [cone ...]; the content of the cone-section is the names of variables that belong to the cone.

```
[comment]
 The cqo1 example in OPF format.
[/comment]
[hints]
  [hint NUMVAR] 6 [/hint]
  [hint NUMCON] 1 [/hint]
  [hint NUMANZ] 3 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3 x4 x5 x6
[/variables]
[objective minimize 'obj']
  x4 + x5 + x6
[/objective]
[constraints]
  [con 'c1'] x1 + x2 + 2e+00 x3 = 1e+00 [/con]
[/constraints]
[bounds]
 # We let all variables default to the positive orthant
  [b] 0 \le * [/b]
 \mbox{\#}\xspace\ldots and change those that differ from the default
  [b] x4,x5,x6 free [/b]
 # Define quadratic cone: x4 \ge sqrt(x1^2 + x2^2)
  [cone quad 'k1'] x4, x1, x2 [/cone]
 # Define rotated quadratic cone: 2 x5 x6 >= x3^2
  [cone rquad 'k2'] x5, x6, x3 [/cone]
[/bounds]
```

#### F.3.5.4 Mixed integer example milo1.opf

Consider the mixed integer problem:

This can be implemented in OPF with:

```
[comment]
 The milo1 example in OPF format
[/comment]
[hints]
  [hint NUMVAR] 2 [/hint]
  [hint NUMCON] 2 [/hint]
  [hint NUMANZ] 4 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2
[/variables]
[objective maximize 'obj']
  x1 + 6.4e-1 x2
[/objective]
[constraints]
  [con 'c1'] 5e+1 x1 + 3.1e+1 x2 <= 2.5e+2 [/con]
  [con 'c2'] -4 \le 3 x1 - 2 x2 [/con]
[/constraints]
[bounds]
  [b] 0 \le * [/b]
[/bounds]
[integer]
 x1 x2
[/integer]
```

## F.4 The Task format

The Task format is MOSEK's native binary format. It contains a complete image of a MOSEK task, i.e.

- Problem data: Linear, conic quadratic, semidefinite and quadratic data
- Problem item names: Variable names, constraints names, cone names etc.
- Parameter settings
- Solutions

There are a few things to be aware of:

- The task format *does not* support General Convex problems since these are defined by arbitrary user-defined functions.
- Status of a solution read from a file will always be unknown.

```
* 1 2 3 4 5 6
*2345678901234567890123456789012345678901234567890
NAME [name]
?? [vname1] [value1]
ENDATA
```

Figure F.1: The standard ORD format.

The format is based on the TAR (USTar) file format. This means that the individual pieces of data in a .task file can be examined by unpacking it as a TAR file. Please note that the inverse may not work: Creating a file using TAR will most probably not create a valid MOSEK Task file since the order of the entries is important.

## F.5 The XML (OSiL) format

MOSEK can write data in the standard OSiL xml format. For a definition of the OSiL format please see <a href="http://www.optimizationservices.org/">http://www.optimizationservices.org/</a>. Only linear constraints (possibly with integer variables) are supported. By default output files with the extension .xml are written in the OSiL format.

The parameter  $iparam.write_xml_mode$  controls if the linear coefficients in the A matrix are written in row or column order.

## F.6 The ORD file format

An ORD formatted file specifies in which order the mixed integer optimizer branches on variables. The format of an ORD file is shown in Figure F.1. In the figure names in capitals are keywords of the ORD format, whereas names in brackets are custom names or values. The ?? is an optional key specifying the preferred branching direction. The possible keys are DN and UP which indicate that down or up is the preferred branching direction respectively. The branching direction key is optional and is left blank the mixed integer optimizer will decide whether to branch up or down.

## F.6.1 An example

A concrete example of a ORD file is presented below:

NAME	EXAMPLE	
DN x1		2
UP x2		1
x3		10
ENDATA		

This implies that the priorities 2, 1, and 10 are assigned to variable x1, x2, and x3 respectively. The higher the priority value assigned to a variable the earlier the mixed integer optimizer will branch on that variable. The key DN implies that the mixed integer optimizer first will branch down on variable whereas the key UP implies that the mixed integer optimizer will first branch up on a variable.

If no branch direction is specified for a variable then the mixed integer optimizer will automatically

choose the branching direction for that variable. Similarly, if no priority is assigned to a variable then it is automatically assigned the priority of 0.

### F.7 The solution file format

MOSEK provides one or two solution files depending on the problem type and the optimizer used. If a problem is optimized using the interior-point optimizer and no basis identification is required, then a file named probname.sol is provided. probname is the name of the problem and .sol is the file extension. If the problem is optimized using the simplex optimizer or basis identification is performed, then a file named probname.bas is created presenting the optimal basis solution. Finally, if the problem contains integer constrained variables then a file named probname.int is created. It contains the integer solution.

#### F.7.1 The basic and interior solution files

In general both the interior-point and the basis solution files have the format:

```
NAME
                    : cproblem name>
PROBLEM STATUS
                    : <status of the problem>
SOLUTION STATUS
                    : <status of the solution>
OBJECTIVE NAME
                    : <name of the objective function>
PRIMAL OBJECTIVE
                    : <pri>: <pri> corresponding to the solution>
DUAL OBJECTIVE
                    : <dual objective value corresponding to the solution>
CONSTRAINTS
INDEX
      NAME
                AT ACTIVITY
                               LOWER LIMIT
                                              UPPER LIMIT
                                                            DUAL LOWER.
                                                                         DUAL UPPER
       <name>
                ?? <a value>
                               <a value>
                                              <a value>
                                                            <a value>
                                                                         <a value>
VARIABLES
                AT ACTIVITY
                               I.OWER I.TMTT
                                              UPPER LIMIT
                                                                                       CONTC DUAL
INDEX NAME
                                                            DUAL LOWER
                                                                         DUAL UPPER
                ?? <a value>
                               <a value>
                                              <a value>
                                                            <a value>
                                                                                       <a value>
       <name>
                                                                         <a value>
```

In the example the fields? and <> will be filled with problem and solution specific information. As can be observed a solution report consists of three sections, i.e.

#### **HEADER**

In this section, first the name of the problem is listed and afterwards the problem and solution statuses are shown. In this case the information shows that the problem is primal and dual feasible and the solution is optimal. Next the primal and dual objective values are displayed.

#### CONSTRAINTS

Subsequently in the constraint section the following information is listed for each constraint:

#### INDEX

A sequential index assigned to the constraint by MOSEK

#### NAME

The name of the constraint assigned by the user.

Status key	Interpretation
UN	Unknown status
BS	Is basic
SB	Is superbasic
LL	Is at the lower limit (bound)
UL	Is at the upper limit (bound)
EQ	Lower limit is identical to upper limit
**	Is infeasible i.e. the lower limit is
	greater than the upper limit.

Table F.1: Status keys.

ΑT

The status of the constraint. In Table F.1 the possible values of the status keys and their interpretation are shown.

#### ACTIVITY

Given the i th constraint on the form

$$l_i^c \le \sum_{j=1}^n a_{ij} x_j \le u_i^c, \tag{F.7}$$

then activity denote the quantity  $\sum_{j=1}^{n} a_{ij}x_{j}^{*}$ , where  $x^{*}$  is the value for the x solution.

#### LOWER LIMIT

Is the quantity  $l_i^c$  (see (F.7)).

#### UPPER LIMIT

Is the quantity  $u_i^c$  (see (F.7)).

#### DUAL LOWER

Is the dual multiplier corresponding to the lower limit on the constraint.

#### DUAL UPPER

Is the dual multiplier corresponding to the upper limit on the constraint.

#### VARIABLES

The last section of the solution report lists information for the variables. This information has a similar interpretation as for the constraints. However, the column with the header [CONIC DUAL] is only included for problems having one or more conic constraints. This column shows the dual variables corresponding to the conic constraints.

## F.7.2 The integer solution file

The integer solution is equivalent to the basic and interior solution files except that no dual information is included.

## Appendix G

## Problem analyzer examples

This appendix presents a few examples of the output produced by the problem analyzer described in Section 13.1. The first two problems are taken from the MIPLIB 2003 collection, <a href="http://miplib.zib.de/">http://miplib.zib.de/</a>.

### $G.1 \quad air04$

```
Analyzing the problem
Constraints
                          Bounds
                                                    Variables
fixed : all
                          ranged : all
                                                     bin : all
Objective, min cx
  range: min |c|: 31.0000 max |c|: 2258.00
distrib: |c| vars [31, 100) 176 [100, 1e+03) 8084
   [1e+03, 2.26e+03]
                            644
Constraint matrix A has
       823 rows (constraints)
      8904 columns (variables)
     72965 (0.995703%) nonzero entries (coefficients)
Row nonzeros, A_i
  range: min A_i: 2 (0.0224618%)
                                   max A_i: 368 (4.13297%)
 distrib: A_i rows
                                              acc%
                                    rows%
           2 2

[3, 7] 4

[8, 15] 19

[16, 31] 80

[32, 63] 236

[64, 127] 289
                                      0.24
                                                   0.24
                                                  0.73
                                      0.49
                                      2.31
9.72
                                                   3.04
                                                  12.76
                                     28.68
                                                   41.43
                                     35.12
                                                  76.55
```

[128, 255] 186 22.60 99.15 7 0.85 100.00 [256, 368] Column nonzeros, A|j range: min A|j: 2 (0.243013%) max A|j: 15 (1.8226%) distrib: A|j cols
2 118 cols% 118 1.33 1.33 [3, 7] 2853 32.04 33.37 66.63 100.00 5933 [8, 15] A nonzeros, A(ij) range: all |A(ij)| = 1.00000 Constraint bounds, 1b <= Ax <= ub distrib: |b| lbs ubs [1, 10] 823 823 Variable bounds, lb <= x <= ub distrib: |b| lbs ubs 0 8904 [1, 10] 8904

## G.2 arki001

Analyzing the problem  $% \frac{1}{2}\left( \frac{1}{2}\right) =\frac{1}{2}\left( \frac{1}{2}\right)$ 

Constra	ints		Bounds			Varia	bles	
lower	bd:	82	lower	bd:	38	cont	: 85	0
upper	bd:	946	fixed	:	353	bin	: 41	5
fixed	:	20	free	:	1	int	: 12	3
			ranged	l :	996			

-----

Objective,  $\min \ cx$ 

-----

Constraint matrix  ${\tt A}$  has

1048 rows (constraints) 1388 columns (variables)

20439 (1.40511%) nonzero entries (coefficients)

Row nonzeros,  $A_{-i}$ 

range: min A\_i: 1 (0.0720461%) max A\_i: 1046 (75.3602%) distrib: A\_i rows rows% acc% 1 29 2.77 2.77

G.2. ARKI001 639

```
476
49
56
64
                               45.42
               2
                                          48.19
           [3, 7]
                                         52.86
                                4.68
5.34
           [8, 15]
                                           58.21
          [16, 31]
                                 6.11
                                           64.31
          [32, 63]
                                35.59
                                           99.90
                        373
       [1024, 1046]
                                 0.10
                                          100.00
                        1
Column nonzeros, A|j
  range: min A|j: 1 (0.0954198%)
                              max A|j: 29 (2.76718%)
 distrib: A|j cols
                               cols%
                                        acc%
          1 381
2 19
[3, 7] 38
[8, 15] 233
                                           27.45
                                 27.45
                                1.37
2.74
                                           28.82
                                           31.56
                                          48.34
                                16.79
          [16, 29]
                        717
                                51.66 100.00
A nonzeros, A(ij)
  range: min |A(ij)|: 0.000200000
                               max |A(ij)|: 2.33067e+07
 distrib: A(ij) coeffs
    [0.0002, 0.001)
      [0.001, 0.01)
                      1049
                      4553
8840
       [0.01, 0.1)
          [0.1, 1)
          [1, 10)
                      3822
         [10, 100)
      [100, 1e+03)
                        267
     [1e+03, 1e+04)
                        699
     [1e+04, 1e+05)
                        291
     [1e+05, 1e+06)
                         83
     [1e+06, 1e+07)
                         19
  [1e+07, 2.33e+07]
                         19
Constraint bounds, lb <= Ax <= ub
distrib: |b| lbs
                                          ubs
          [0.1, 1)
                                          386
          [1, 10)
                                          74
         [10, 100)
                          101
                                          456
                                          34
       [100, 1000)
      [1000, 10000)
                                          15
    [100000, 1e+06]
Variable bounds, lb <= x <= ub
distrib: |b|
                            lbs
                                          ubs
               0
                            974
                                          323
      [0.001, 0.01)
                                          19
         [0.1, 1)
                            370
                                          57
          [1, 10)
                           41
                                          704
         [10, 100]
                                          246
```

## G.3 Problem with both linear and quadratic constraints

```
Analyzing the problem
                                                  Variables
                         Bounds
Constraints
Constraints
lower bd: 40
upper bd: 121
                       upper bd:
                                          1
                                                  cont: all
                                      204
                        fixed : free :
fixed :
              5480
                                        5600
              161
 ranged :
                         ranged :
Objective, maximize cx
  range: all |c| in {0.00000, 15.4737}
 distrib:
                |c| vars
                 0
                          5844
            15.4737
                        1
Constraint matrix A has
     5802 rows (constraints)
     5845 columns (variables)
     6480 (0.0191079%) nonzero entries (coefficients)
Row nonzeros, A_i
  range: min A_i: 0 (0%) max A_i: 3 (0.0513259%)
 distrib:
               A_i rows rows%
                 0
                         80
                                     1.38
                                                1.38
                 0 80 1.38
1 5003 86.23
2 680 11.72
3 39 0.67
                                                87.61
                                                99.33
                                              100.00
0/80 empty rows have quadratic terms
Column nonzeros, Alj
  range: min A|j: 0 (0%) max A|j: 15 (0.258532%)
               A|j cols cols% acc% 0 204 3.49 3.49
 distrib:
                         5521 94.46
40 0.68
40 0.68
40 0.68
            1
2
[3, 7]
[8, 15]
                                                97.95
                                                98.63
                                               99.32
0/204 empty columns correspond to variables used in conic
and/or quadratic expressions only
A nonzeros, A(ij)
  range: min |A(ij)|: 2.02410e-05
                                  max |A(ij)|: 35.8400
 distrib:
            A(ij) coeffs
  [2.02e-05, 0.0001)
     [0.0001, 0.001)
       [0.001, 0.01)
                           305
         [0.01, 0.1)
                          176
            [0.1, 1)
                            40
            [1, 10)
                          5721
         [10, 35.8]
```

Constraint bo	unds, lb <=	Ax <= ub		
distrib:	b	lbs	ubs	
	0	5481	5600	
[1000,	10000)		1	
[10000,	100000)	2	1	
[1e+06,	1e+07)	78	40	
[1e+08,	1e+09]	120	120	
Variable boun	ds, lb <= x	<= ub		
distrib:	b	lbs	ubs	
	0	243	203	
[	0.1, 1)	1	1	
[1e+06,	1e+07)		40	
[1e+11,	1e+12]		1	
Quadratic constraints: 121				
Gradient nonz	eros, Qx			
range: min	Qx: 1 (0.01	171086%) ma	ax Qx: 2720 (46.5355%)	)
distrib:	Qx	cons	cons% acc%	
	1	40	33.06 33.06	
[6	4, 127]	80	66.12 99.17	
[2048	, 2720]	1	0.83 100.00	

## G.4 Problem with both linear and conic constraints

Rotated quadratic cones: 3600

dim

3600

```
distrib:
           A_i
                    rows
                              rows%
                                         acc%
                             14.20
                     3600
             1
                                        14.20
              2
                     10803
                               42.60
                                        56.79
                   3995
           [3, 7]
                              15.75
                                        72.55
                     6962
                              27.45
                                       100.00
Column nonzeros, A|j
 range: min A|j: 0 (0%) max A|j: 61 (0.240536%)
distrib: A|j
                      cols cols% acc%
                     3602
                              11.12
                                        11.12
                             33.33
                                      44.45
                    10800
              1
                             22.22
26
                    7200
                                       66.67
              2
           [3, 7]
                      7279
                               22.46
                                        89.13
          [8, 15]
                     3521
                             10.87
                                       100.00
         [32, 61]
                     1
                              0.00
                                      100.00
3600/3602 empty columns correspond to variables used in conic
and/or quadratic constraints only
A nonzeros, A(ij)
 distrib: A(ij) coeffs
[0.00833, 0.01) 57280
       [0.01, 0.1)
                     59
         [0.1, 1]
                   36000
Constraint bounds, 1b <= Ax <= ub
distrib: |b| lbs 0 21760
                                      ubs
                                     21760
         [0.1, 1]
                                      3600
Variable bounds, lb <= x <= ub
         |b|
distrib:
                          lbs
                                      ubs
                         3601
                                      3601
          [1, 10]
```

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