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It's Not Murphy's Law, It's Newton's

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An intriguing essay in *Scientific American* about bread landing butter-side down when it falls off the table[1] referred to a journal[2] I don't have direct access to, so I thought I'd solve the riddle of "why the bread almost always lands butter-side down" myself. The results are described here.

A simple model of a slice of bread, toasted or not, buttered or jellied, is that of a uniform rectangular object. In our area, a slice of bread is essentially 11 cm square and about 1 cm thick. As Fig. 1 shows, the slice tips off the edge of the table due to the distance h that the center of mass is located beyond the table's edge. The slice rotates about the table edge and the forces acting on the bread are gravity, the normal force the edge of the table exerts on the slice, and the friction due to that normal force. The angle through which the slice has rotated is measured from the horizontal since that is the motion that concerns us. Gravity, acting at the center of mass, provides the torque that causes the rotation; static friction keeps the slice from sliding until the slice has rotated through a sufficient angle so that the sliding forces can overcome friction.

Using the parallel axis theorem, the moment of inertia, I, of the slice is,

$$I = (1/12)\,Ml^2 + Mh^2$$

where l is the length of the side of the slice (0.11 m) and M is its mass. The rotational equation of motion is

$$Mgh\cos(\theta) = I\frac{d^2\theta}{dt^2} \qquad (1)$$

A simplification used here is that h is the lever arm for gravity's torque (i.e., the bread has no thickness). The acceleration of the center of mass is determined by the net force acting on the system. The net force is due to gravity and the normal force (F_n) the table exerts on the slice because until the slice begins to slide the forces perpendicular to F_n add to zero. Applying Newton's Second Law to this situation yields

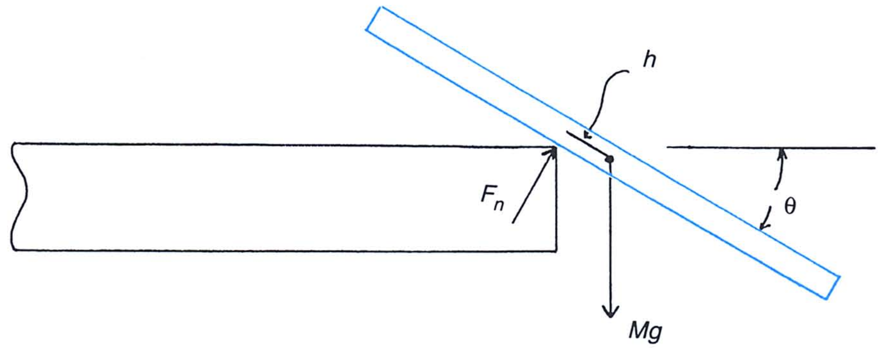

Fig. 1. Physical arrangement of a slice of toast falling off the table's edge.

$$Mg\cos(\theta) - F_n = Mh\frac{d^2\theta}{dt^2} \qquad (2)$$

The acceleration of the center of mass, a_{cm}, is related to the rotational acceleration, $\frac{d^2\theta}{dt^2}$, through the equation $a_{cm} = h\frac{d^2\theta}{dt^2}$. Solving Eq. (2) for F_n yields

$$F_n = Mg\cos(\theta) - Mh\frac{d^2\theta}{dt^2} \qquad (3)$$

The frictional force the table exerts on the slice is given by

$$F_h = Mg\sin(\theta) + Mh\left(\frac{d\theta}{dt}\right)^2 \qquad (4)$$

the first term being the component of gravity in that direction and the second the centripetal force needed for the rotation of the slice. As long as these two terms are less than the maximum force that static friction can provide, the slice will not slide. That condition fails when $F_h = \mu_s F_n$ or $\mu_s = F_h/F_n$.

Substituting Eqs. (3) and (4) into this provides

$$\mu_s = \frac{Mg\sin(\theta) + Mh\left(\frac{d\theta}{dt}\right)^2}{Mg\cos(\theta) - Mh\frac{d^2\theta}{dt^2}} \qquad (5)$$

When static friction can no longer hold the slice, it will begin to slide along the table top until the rotational motion lifts the slice off the edge of the table. During this time gravity continues to exert a torque, making the slice rotate even faster. The equations of motion during that short time are nonlinear and need numerical methods to solve, but we will see that in the majority of situations the rotation caused by the torque before slipping begins is sufficient to have the bread land butter-side down.

Solving Eq. (1) for $\frac{d^2\theta}{dt^2}$ and integrating yields

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{2g}{h\left(1+\frac{l^2}{12h^2}\right)} \sin(\theta) \tag{6}$$

Substituting this and $\frac{d^2\theta}{dt^2}$ into Eq. (5) yields

$$\mu_s = [1 + 36\,(h/l)^2] \tan(\theta)$$

or

$$\tan(\theta) = \frac{\mu_s}{1 + 36\left(\frac{h}{0.11}\right)^2} \tag{7}$$

when taking into account the size of a slice of bread. Equation (7) allows us to determine the angle at which sliding begins for a given overhang (h) and coefficient of static friction, μ_s. Table I shows this angle for various values of both h and μ_s.

Substituting these various angles into Eq. (5) yields the angular velocity of the slice of bread when it begins to slide off the table. Table II shows these velocities.

If we assume that when sliding begins there is no more torque and the rotational velocity at that time is all the slice acquires, the angle through which the slice rotates is determined only by the time it takes the slice to fall to the floor. This is determined by the table height, 30 in or 0.76 m for a typical table. That time is 0.394 s [$t = \sqrt{2(0.76/9.8)} = 0.39$ s]. Using this time, the angular velocities of Table II, and the angles of Table I yield Table III, the angle through which the slice rotates as it falls to the floor.

The combinations of h and μ_s to the right and above the lines in Table III are such that the slice strikes the floor butter-side down. If the torque involved during the slipping stage is taken into account, we see that an even larger combination of h and μ_s will lead to the butter-side landing down.

We could ask, "How low or how tall must the tabletop be if the slice is to land butter-side up?" This question is answered by assuming that the slice must rotate at least half way around (less than 90° for the lowest tabletop and 270° for the taller tabletop) for it to land butter-side up. Here the question comes down to which conditions of overhang (h) and static friction lead to the highest low table and to the lowest high table. In these situations the highest low table is 0.7 m (27.5 in), which occurs when the overhang is 0.05 m and the static friction is 0.5; the lowest high table is 1.19 m (47 in), which occurs when the overhang is 0.01 m and the static friction is 0.6.

Tables of calculated data for various combinations of h (in meters) and μ_s.

I. Angle (θ) in degrees at which sliding begins.

		μ_s					
		0.1	0.2	0.3	0.4	0.5	0.6
	0.01	4.41	8.79	13.02	17.13	21.07	24.82
	0.02	2.61	5.22	7.8	10.35	12.86	15.32
h	0.03	1.56	3.11	4.66	6.21	7.74	9.27
	0.04	0.99	1.99	2.98	3.97	4.96	5.95
	0.05	0.68	0.36	2.04	2.71	3.39	4.07

II. Angular velocity (rad/s) when sliding begins.

	0.01	3.69	5.19	6.31	7.22	7.97	8.62
	0.02	3.56	5.03	6.15	7.07	7.87	8.58
h	0.03	2.89	4.09	5.01	5.77	6.44	7.04
	0.04	2.28	3.23	3.95	4.56	5.10	5.58
	0.05	1.82	2.57	3.15	3.64	4.06	4.45

III. Rotation angle (degrees) through which slice turns.

	0.01	87	126	155	180	201	215
	0.02	83	118	147	170	191	209
h	0.03	67	95	118	136	153	168
	0.04	53	75	92	107	120	132
	0.05	42	59	73	85	95	104

All of this is approachable experimentally by using a thin book of the approximate dimensions in lieu of buttered bread. If you can video it and then obtain data from one frame at a time, it should be possible to get some measure of how good this simple model is.

References

1. I. Stewart, "Mathematical recreations," *Sci. Am.* **273** (6), 104–106 (December 1995).
2. R. Matthews, "Tumbling toast, Murphy's Law and the fundamental constants," *Eur. J. Phys.* **16** (4), 172–176 (July 1995).

Physics Is Phun Pun Quiz

(Match item on left with description on right)

sonar	**condition requiring knock**
Nobel	**1/100th of a perfect test score**
proton	**textbook case of indigestion**
centigrade	**...and yet so far**
ideal gas	**in Rome directory**
Avogadro's number	**2000-pound professional**

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