Final Self-Reflection

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1. What did you learn?

When I first started the class, I was only really familiar with P and NP. I now feel like I have a deeper understanding of a variety complexity classes and a different ways to characterize them. The example that still sticks out to me the most are the $\exists$ and $\forall$ operators, which tie complexity into my area of research (logic). This also relates to probably my largest epiphany in this class: Non-deterministic versions of a complexity class can be thought of a as a certain kind of prefix to languages in the original class. This has been a much more helpful characterization for me than thinking about non-determinism in machines!

Proofs in complexity theory were a bit of a mystery to me when I started the class. I would often see results like "if $P \neq NP$ then blah blah blah" and wondered how we can get results that are founded on things we can't even prove! I now have a better idea of how it is possible to

1. pad languages to change their complexity classes while preserving results in the original class
2. build new languages by diagonlizing against families of machines
3. relativize languages using oracles

I learned that there a lot of open questions in this field. A's project on Ladner's theorem really stands out to me as an example. It is surprising that we can be fairly confident that a complexity class exests but have no examples of languages that belong to it.

2. How did your goals turn out?

My original goal for the class was to learn about the nature of proofs in complexity theory and I am satisfied with what I was able to take away from (most of) a semester! I think the question of how complexity theory relates to other areas of math–when viewed as a formal system–is still open. That is an exciting starting place for research. I feel like I am walking away with a better mental map of the field than when I started and there isn't much more I could ask for. I hope that I will have more opportunities in my research career to work with the building blocks of complexity theory and see how the field coalesces more into the broader world of math during my lifetime.

One particular thing I took away was the prevalence of diagonalization arguments, which depend on Gödel-like encodings. Being so central, I think studying some of the work related to recursive and effective toposes is an important next step in my complexity journey. I'm excited to see if there are methods of converting complexity arguments into expository and formally correct diagrams that make the complexity of the problem obvious from inspection.

3. What grade would you assign yourself?

I think my overall grade in terms my goals and what I was able to achieve is a **B+**. I learned most of what I wanted to learn, but didn't take some of those ideas as deep as I would have liked. For example, during my project I wanted to get more into the proofs themselves and not just cover the basic data definitions, to see if there were relationships to other formal proofs I had seen. My learning log is also somewhat sparse, but I'm not docking myself too much for this because intensive note-taking is not my learning style.

As always my full learning log is available at https://github.com/littlebenlittle/cu-boulder-fall-23/tree/main/complexity-learning-log.